AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 13

Dr. Nicholas Smith

Last Updated: March 3, 2016 at 1:43pm

Wichita State University, Department of Aerospace Engineering

SCHEDULE

- 3 Mar - Section 1 Review, Homework 5 return
- 8 Mar - Exam 1
- 10 Mar - Exam return, Final Project discussion
- 22 Mar - Stress based fatigue, Homework 6 assigned
- 24 Mar - Stress based fatigue

OUTLINE

EXAM NOTES

EXAM

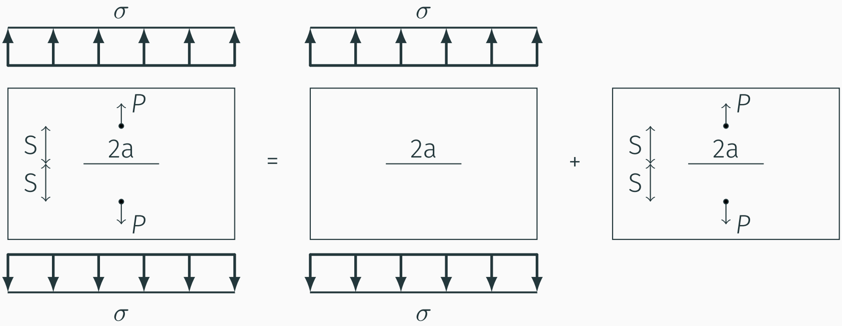
- 4 questions
- Bring a scientific calculator (or graphing)
- Specific equations or correction factors (i.e. modified MSD or β for stiffeners) will be given where needed
- Equation sheet is posted on Blackboard

STRESS INTENSITY

METHODS FOR FINDING STRESS INTENSITY

- Handbook lookup
- Superposition
- Compounding
- Stress concentration ratio
- Stress function method (Westergaard)
- Boundary collocation (approximation to Westergaard)
- Schwartz-Neumann alternating method (approximation to Westergaard)
- Finite elements (Direct, Modified Crack Closure)
- Boundary elements
- Experimental

- Since the stress intensity factor is derived using Linear Elasticity, the principle of superposition applies
- Multiple applied loads can be superposed to find the effective stress intensity factor of the combined loading

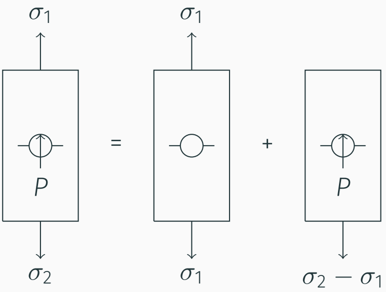

σ
P
S
2a
S
P
σ
=
σ
2a
σ
+
P
S
2a
S
P

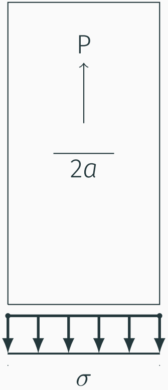
$$K_I = K_{I(\sigma)} + K_{I(P)}$$

$$K_I = \sigma\sqrt{\pi a} + \frac{P}{t\sqrt{\pi a}} \frac{1 - 0.5\left(\frac{a}{W}\right) + 0.975\left(\frac{a}{W}\right)^2 - 0.16\left(\frac{a}{W}\right)^3}{\sqrt{1 - \left(\frac{a}{W}\right)}}$$

- Sometimes, the superposition needed to solve a problem is not obvious
- It can be helpful to subtract a known solution from the problem

SUPERPOSITION




P
2a
σ

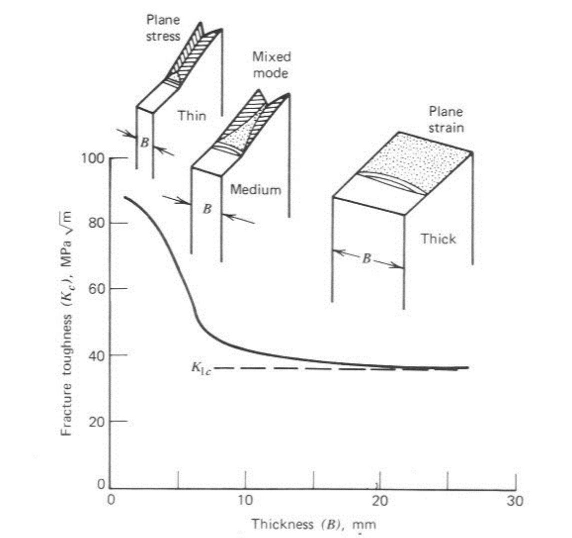
FRACTURE TOUGHNESS

- The critical load at which a cracked specimen fails produces a critical stress intensity factor
- The ”critical stress intensity factor” is known as K_c
- For Mode I, this is called K_{Ic}
- The critical stress intensity factor is also known as fracture toughness

$$K_{IC} = \sigma_c \sqrt{\pi a} \beta \tag{13.1}$$

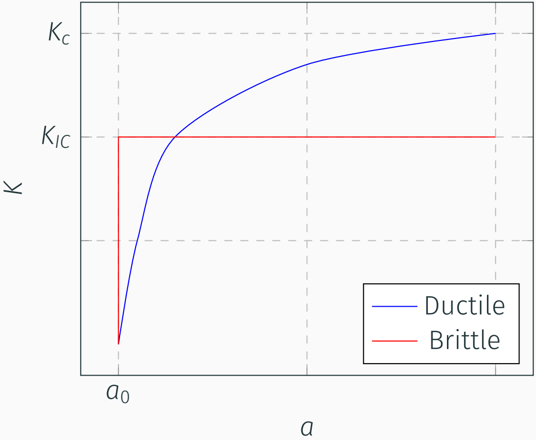
- NOTE: ”Fracture Toughness” can also refer to G_{Ic}, which is analogous to K_{Ic}, but not the same

- Fracture toughness is a material property, but it is only well-defined in certain conditions
- Brittle materials
- Plane strain (smaller plastic zone)
- In these cases ASTM E399-12 is used.

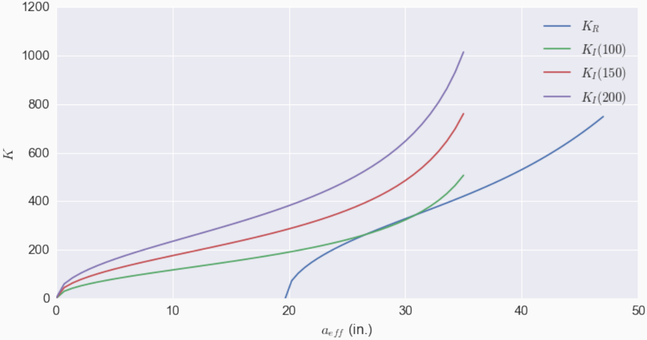

Plane
stress
Mixed
mode
Thin
Plane
strain
B
100
Medium
B
80
Thick
B
60
40
K_{Ic}
20
0
0
10
20
30
Fracture toughness (K_c), MPa $\sqrt{m}$
Thickness (B), mm

- ”Stable” vs. ”unstable” crack growth
- Stable crack growth means the crack extends only with increased load
- Unstable crack growth means the crack will continue to extend indefinitely under the same load
- For a perfectly brittle material, there is no stable crack growth, as soon as a critical load is reached, the crack will extend indefinitely
- For an elastic-plastic material, once the load is large enough to extend the crack, it will extend slightly
- The load must be continually increased until a critical value causes unstable crack growth

- During an experiment, we will record the crack length and applied load (P_i, a_i) each time we increase the load
- We can calculate a unique stress intensity factor K_{Ii} at each of these points
- These are then used to create a ”K-curve”, plotting K_I vs. a


K_c
K_{IC}
K
a_0
a
Ductile
Brittle

k_r CURVE

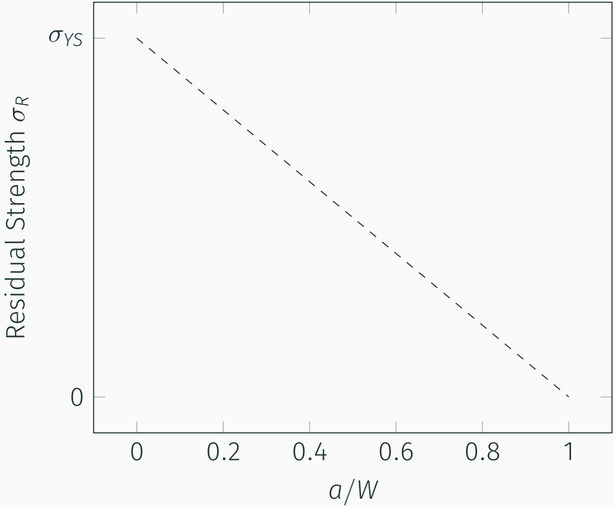


1200
1000
800
600
400
200
0
K
0
10
20
30
40
50
a_{eff} (in.)
K_R
$K_I(100)$
$K_I(150)$
$K_I(200)$

RESIDUAL STRENGTH

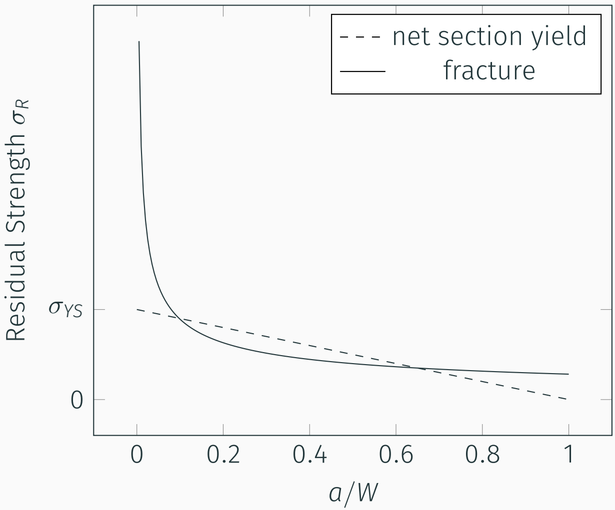
- In the last chapter we performed some basic residual strength analysis by checking for net section yield
- As the crack grows, the area of the sample decreases, increasing the net section stress
- The residual strength, σ_R is given in terms of the gross area, so as the crack grows the residual strength due to yield decreases
- We can relate the net-section stress to σ_R by

$$\sigma_R = \sigma_{YS} \frac{A_{net}}{A_{gross}} \tag{13.2}$$



σ_{YS}
0
Residual Strength σ_R
0
0.2
0.4
0.6
0.8
1
a/W

- For brittle fracture to occur, we need to satisfy the condition
- $$\sigma_R = \sigma_C = \frac{K_C}{\sqrt{\pi a}\beta} \tag{13.3}$$


net section yield
fracture
Residual Strength σ_R
σ_{YS}
0
0
0.2
0.4
0.6
0.8
1
a/W

- Within the same family of materials (i.e. Aluminum), there is generally a trade-off between yield stress and fracture toughness
- As we increase the yield strength, we decrease the fracture toughness (and vice versa)
- Consider a comparison of the following aluminum alloys
    1. 7178-T6, $K_C = 43$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 74$ksi
    2. 7075-T6, $K_C = 68$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 63$ksi
    3. 2024-T3, $K_C = 144$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 42$ksi

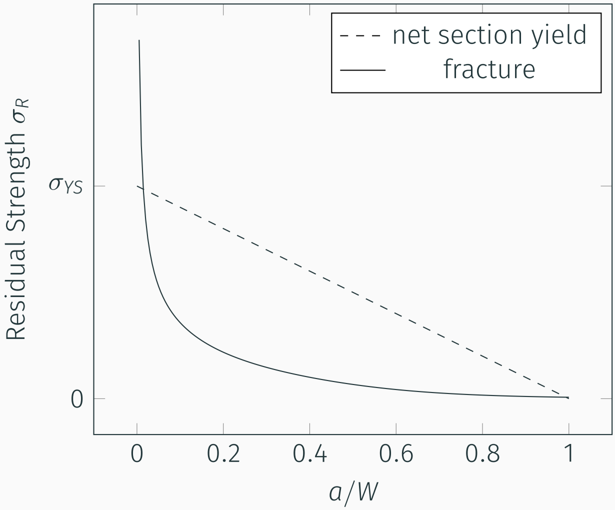
- As an example let us consider an edge-cracked panel with $W = 6''$ and $t = 0.1''$
- The net section yield condition will be given by
- $$\sigma_C = \sigma_{YS}\frac{W-a}{W} = \sigma_{YS}\frac{6-a}{6}$$
- And the fracture condition by

$$\sigma_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

With

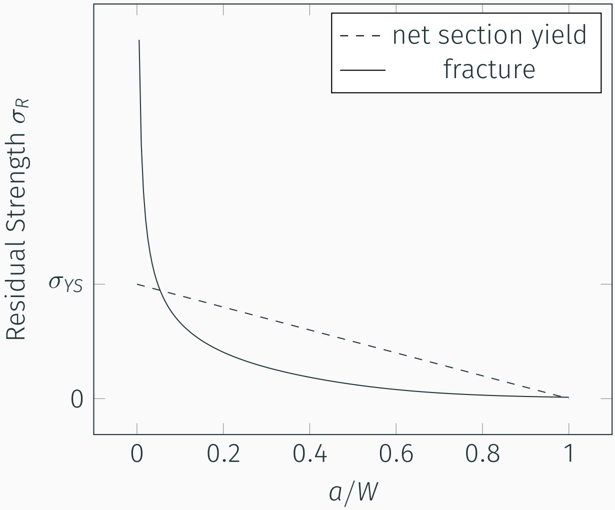
$$\beta = 1.12 - 0.231\left(\frac{a}{W}\right) + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4$$

7178-T6

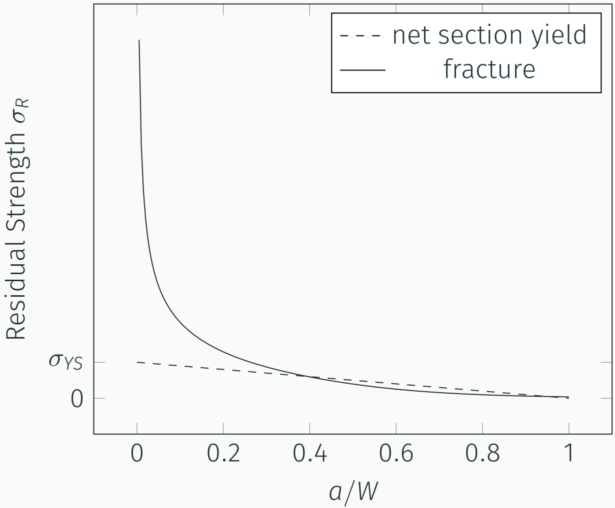


net section yield
fracture
Residual Strength σ_R
σ_{YS}
0
0
0.2
0.4
0.6
0.8
1
a/W

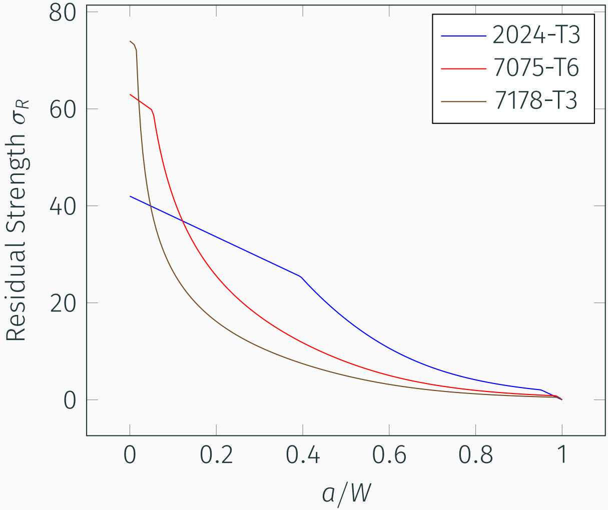
7075-T6



net section yield
fracture
Residual Strength σ_R
σ_{YS}
0
0
0.2
0.4
0.6
0.8
1
a/W



net section yield
fracture
Residual Strength σ_R
σ_{YS}
0
0
0.2
0.4
0.6
0.8
1
a/W



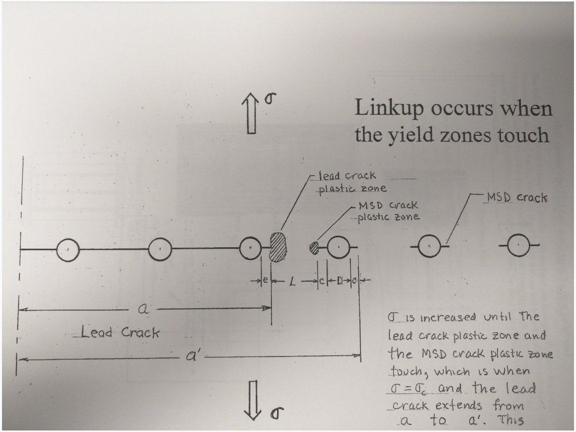
80
60
40
20
0
Residual Strength σ_R
0
0.2
0.4
0.6
0.8
1
a/W
2024-T3
7075-T6
7178-T3

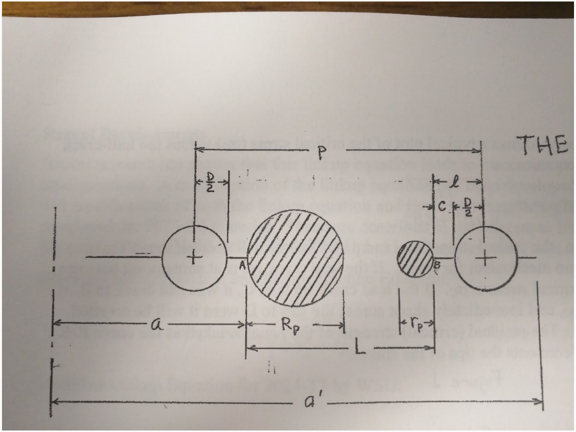
STIFFENERS

- In general, residual strength curves do NOT give any information about crack growth
- When σ_R is exceeded, the panel fails due to unstable crack growth
- Stiffeners reverse this trend to some extent, but causing some sections of residual strength curve to have positive slope
- When the slope of the residual strength curve is positive, crack growth is stable
- Thus in some cases, we can predict some amount of crack growth

MULTIPLE SITE DAMAGE

- Often damage can accumulate among multiple sources
- This is very common when there are a series of holes, each can develop cracks with a potential to link up
- ”link up” occurs when the plastic zones between two adjacent cracks touch


σ
Linkup occurs when the yield zones touch
lead crack plastic zone
MSD crack plastic zone
MSD crack
e
L
c
D
c
a
Lead Crack
a′
σ is increased until the lead crack plastic zone and the MSD crack plastic zone touch, which is when σ = σc and the lead crack extends from a to a′. This
σ


THE
P
D/2
ℓ
C
D/2
A
B
a
R_p
r_p
L
a'

- We know that

$$R_p = \frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2 \tag{13.4}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2 \tag{13.5}$$

- Where we define the stress intensity factors at a and L as

$$K_{Ia} = \sigma\sqrt{\pi a}\beta_a \tag{13.6}$$

$$K_{Il} = \sigma\sqrt{\pi l}\beta_l \tag{13.7}$$

- Since fast cracking occurs when $R_p + r_p = L$, we solve for the condition where $R_p + r_p < L$

$$\frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2 + \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2 < L \tag{13.8a}$$

$$\frac{1}{2\pi\sigma_{YS}^2}\left[K_{Ia}^2 + K_{Il}^2\right] < L \tag{13.8b}$$

$$\frac{1}{2\pi\sigma_{YS}^2}\left[\sigma^2\pi a\beta_a^2 + \sigma^2\pi l\beta_l^2\right] < L \tag{13.8c}$$

$$\frac{\sigma^2}{2\sigma_{YS}^2}\left[a\beta_a^2 + l\beta_l^2\right] < L \tag{13.8d}$$

$$\frac{\sigma_c^2}{2\sigma_{YS}^2}\left[a\beta_a^2 + l\beta_l^2\right] = L \tag{13.8e}$$

$$\sigma_c = \sigma_{YS}\sqrt{\frac{2L}{a\beta_a^2 + l\beta_l^2}} \tag{13.8f}$$

CAVEATS

- The link-up equation is not a good predictor for materials with a small plastic zone size
- Even for ductile materials, some fine tuning of the equation is needed
- In practice, MSD predictions are based on experiments

MIXED MODE FRACTURE

MAXIMUM CIRCUMFERENTIAL STRESS VS. MAXIMUM PRINCIPAL STRESS

- Maximum circumferential stress finds the principal stress direction near the crack tip
- Assumes crack will propagate due to maximum opening stress
- Maximum principal stress theory finds the maximum principal stress, neglecting crack tip stress field
- Also assumes crack propagates in Mode I direction

EXTRA CREDIT

DIGITIZING FIGURES!

- Our text has a lot of useful information that can be somewhat difficult to access
- There are programs online that can be used to semi-automatically trace figure lines
- I will give 25 points in Homework extra credit to students who trace a figure and send me the data
- Start out with a limit of one figure per student
- Use Google Doc to write the page of a figure you are working on (so we don't repeat)

- Google Doc: `https://docs.google.com/spreadsheets/d/1ay4HfJQG2mF-nyr3fgtDr0lHDQoWP30TIMynKGxiwp0/edit?usp=sharing`
- Chart tracer: `http://arohatgi.info/WebPlotDigitizer/app/?`

REVIEW PROBLEMS

REVIEW PROBLEMS

- p. 415 problem 6
- p. 418 problem 9
- p. 419 problem 10-11
- p. 421 problem 13
- p. 423 problem 17
- p. 424 problem 3
- p. 425 problem 5
- p. 426 problem 1
- p. 427 problem 3
- p. 429 problem 6
- p. 432 problem 9
- p. 433 problem 14
- p. 434 problem 3
- p. 437 problem 8