ae 737 - mechanics of damage tolerance

Lecture 17

Dr. Nicholas Smith

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Wichita State University, Department of Aerospace Engineering

SCHEDULE

- 29 Mar - Influence of notches on fatigue, Homework 7 assigned, Homework 6 due
- 31 Mar - Strain based fatigue, project abstract due
- 5 Apr - Crack Growth, Homework 7 due, Homework 8 assigned
- 7 Apr - Crack Growth

1. fatigue review
2. influence of notches
3. strain based fatigue

FATIGUE REVIEW

- A part from AISI 4340 in a typical ”block” undergoes 100,000 cycles with $\sigma_{min} = 0$ ksi and $\sigma_{max} = 100$ ksi and an additional 10 cycles with $\sigma_{min} = 50$ ksi and $\sigma_{max} = 200$ ksi
- How many ”blocks” can this part support before failure?

GROUP 2

- Use the S-N-P chart on p. 245 for 7075-T6 Aluminum
- What is the probability of failure for 30 ksi at 10^6 cycles?
- To ensure that 99% of parts do not fail, after how many cycles should a fully reversed load of 35 ksi be inspected?
- How many cycles could the same part sustain if only 50% of parts are needed?

GROUP 3

- The fatigue limit for AISI 4142 steel is 58 ksi for completely reversed fatigue loads.
- What is the fatigue limit for fatigue loads with $\sigma_m = 10, 20, 30$ ksi?

- A material made of 2024-T4 Aluminum undergoes the following load cycle
    - $\sigma_{x,min} = 10$, $\sigma_{x,max} = 50$
    - $\sigma_{y,min} = -20$, $\sigma_{y,max} = 20$
    - $\tau_{xy,min} = 0$, $\tau_{xy,max} = 30$
- How many cycles can it support before failure?

INFLUENCE OF NOTCHES

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- This relates local stress to the average, nominal stress
- The stress intensity factor can be used to characterize the ”strength” of a notch

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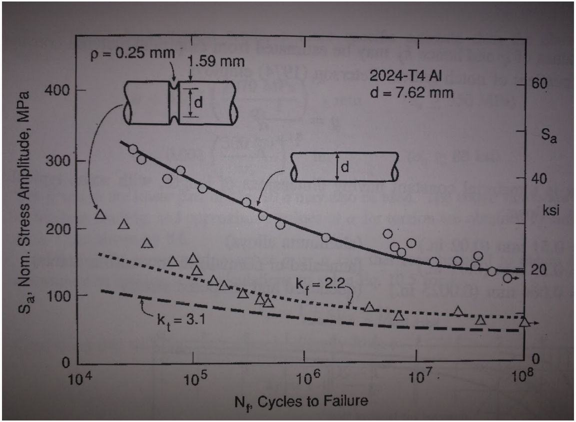
- The maximum possible fatigue notch factor is $k_f = k_t$


ρ = 0.25 mm
1.59 mm
2024-T4 Al
d = 7.62 mm
d
d
S_a, Nom. Stress Amplitude, MPa
400
300
200
100
0
60
S_a
40
ksi
20
0
k_f = 2.2
k_t = 3.1
10^4
10^5
10^6
10^7
10^8
N_f, Cycles to Failure

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- When $k_f = k_t$, $q = 1$, in which case the notch has its maximum effect

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Table 1: Table of α values for Peterson notch sensitivity equation

Material	α (mm)	α (in)
Aluminum alloys	0.51	0.02
Annealed or low-carbon steels	0.25	0.01
Quenched and tempered steels	0.064	0.0025

- For high-strength steels, a more specific α estimate can be found

$$\alpha = 0.025\left(\frac{2070}{\sigma_u}\right)^{1.8} \qquad \text{mm} \qquad \sigma_u \geq 550\text{ MPa} \tag{17.4}$$

$$\alpha = 0.001\left(\frac{300}{\sigma_u}\right)^{1.8} \qquad \text{in} \qquad \sigma_u \geq 80\text{ ksi} \tag{17.5}$$

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$$\alpha_{torsion} = 0.6\alpha \tag{17.6}$$

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$$\log \beta = -\frac{\sigma_u - 134}{586} \qquad \text{mm} \qquad \sigma_u \leq 1520 \text{ MPa} \tag{17.8}$$

$$\log \beta = -\frac{\sigma_u + 100}{85} \qquad \text{in} \qquad \sigma_u \leq 220 \text{ ksi} \tag{17.9}$$

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- For aluminum use the chart MPa (ksi) and mm (in.)

S_u	150 (22)	300 (43)	600 (87)
β	2 (0.08)	0.6 (0.025)	0.5 (0.015)

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NOTCH SENSITIVITY FACTORS

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- Physical complexities are not fully modeled by these methods
- All of these have been developed for relatively ”mild” notches
- For sharp notches, best results are found by treating the notch as a crack

- Find the notch sensitivity factor for the following scenario

$$\begin{aligned} \rho &= 0.25 \text{ in.} \\ \sigma_m &= 0 \text{ ksi} \\ K_t &= 3.0 \\ \sigma_u &= 84 \text{ ksi} \end{aligned}$$

STRAIN BASED FATIGUE

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- Does not include crack growth analysis or fracture mechanics

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- The test is run in strain control (not load control)
- Generally plotted on log-log scale

- We can separate the total strain into elastic and plastic components

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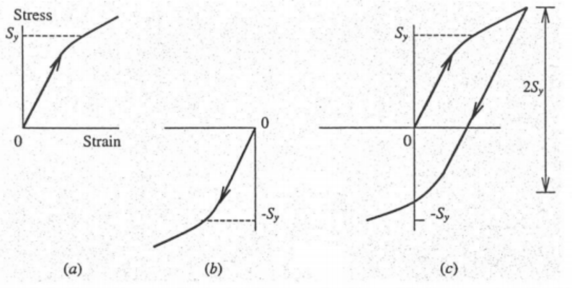
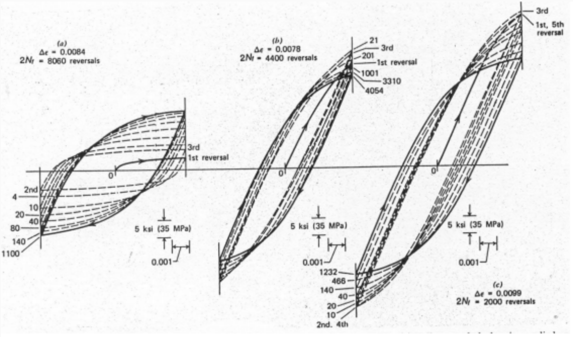
$$\epsilon_a = \epsilon_{ea} + \epsilon_{pa} \tag{17.10}$$

PLASTIC STRAIN



HYSTERESIS LOOPS



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$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} \tag{17.11}$$

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PLASTIC AND ELASTIC STRAIN

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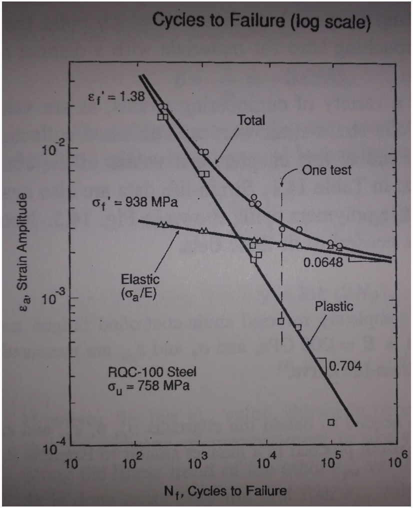
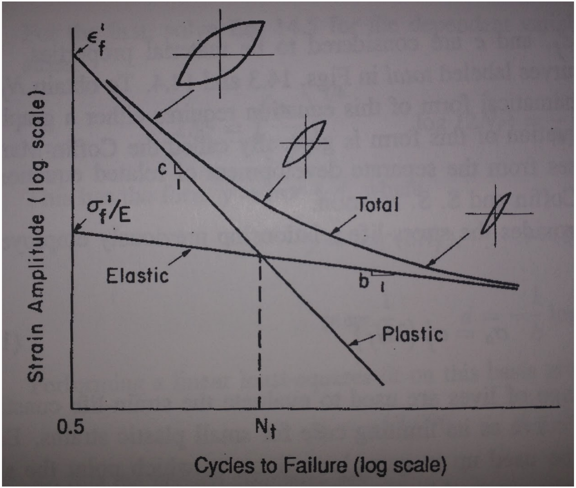
- Once for total strain, once for plastic strain, and once for elastic strain
- Since plastic strain and elastic strain vary by the number of cycles, a hysteresis loop from half the fatigue life is generally used
- This is considered representative of stable behavior



Cycles to Failure (log scale)
εf' = 1.38
Total
One test
σf' = 938 MPa
0.0648
Elastic
(σa/E)
Plastic
0.704
RQC-100 Steel
σu = 758 MPa
εa, Strain Amplitude
10^-2
10^-3
10^-4
10
10^2
10^3
10^4
10^5
10^6
Nf, Cycles to Failure



ϵ_f'
σ_f'/E
Strain Amplitude (log scale)
c
1
Total
Elastic
b
1
Plastic
0.5
N_t
Cycles to Failure (log scale)

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$$\epsilon_{ea} = \frac{\sigma_f'}{E}(2N_f)^b \tag{17.13}$$

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$$\epsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \epsilon_f'(2N_f)^c \tag{17.15}$$