AE 737: Mechanics of Damage Tolerance

Lecture 16 - Strain based fatigue

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schedule

- 17 Mar - Strain-based fatigue
- 19 Mar - Crack growth, HW6 Due
- 23-27 Mar - Spring Break
- 31 Mar - Crack growth
- 2 Apr - Crack growth, HW7 Due

outline

- strain based fatigue
- variable amplitude strains
- mean stress effects
- general trends
- notches
- multiaxial loading
- other factors affecting fatigue
- crack growth rate
- crack growth rate equations

strain based fatigue

strain based fatigue

- The strain based fatigue method uses local stresses and strains (instead of global, nominal values)
- The strain-based method gives greater detail, and validity at lower cycles
- It is still valid for high cycle fatigue (but gives same result as stress-based fatigue)
- Does not include crack growth analysis or fracture mechanics

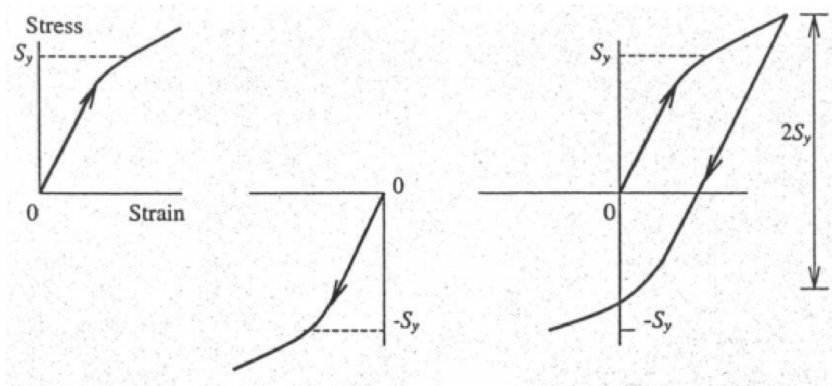
strain life curve

- Similar to the S-N curves in stress-based fatigue analysis, we can plot the cyclic strain amplitude vs. number of cycles to failure
- This is most commonly done using axial test machines (instead of rotating bending tests)
- The test is run in strain control (not load control)
- Generally plotted on log-log scale

plastic and elastic strain

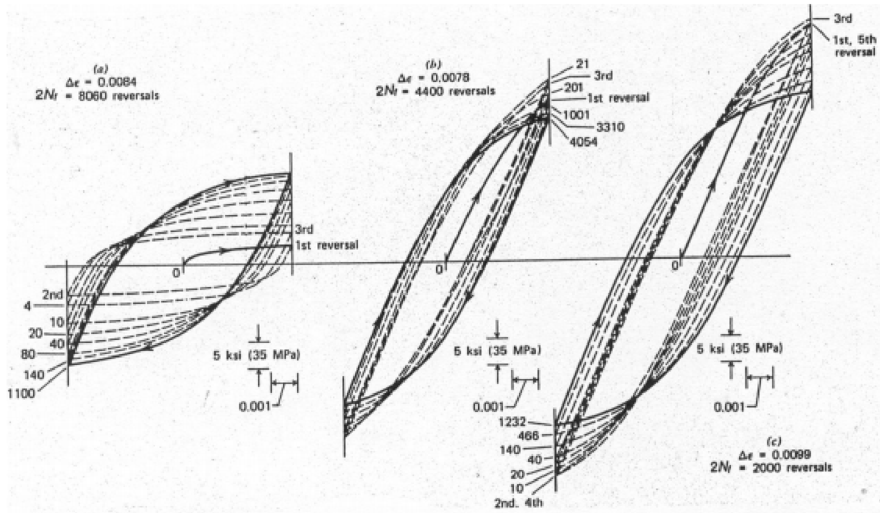
- We can separate the total strain into elastic and plastic components $\epsilon_a = \epsilon_{ea} + \epsilon_{pa}$

plastic strain



(a) (b) (c)

hysteresis loops



cyclic stress strain curve

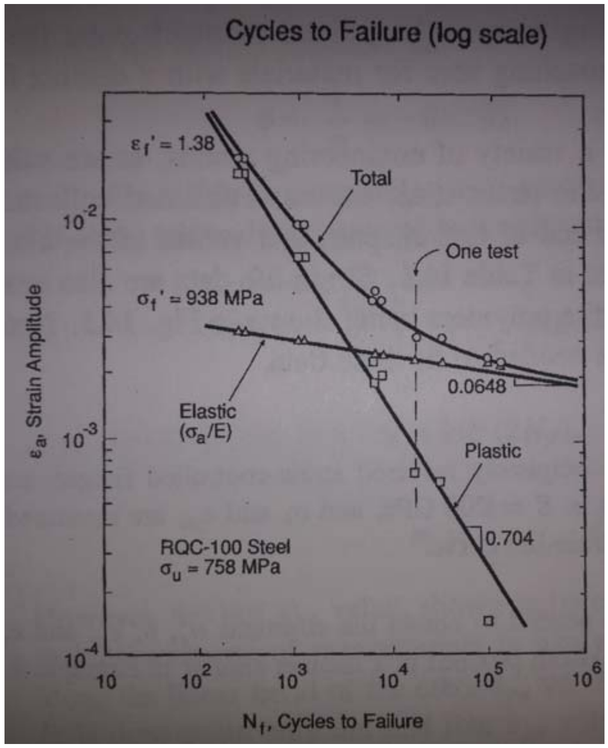
- While strain-life data will generally just report ϵ_a and ϵ_{pa}, some will also tabulate a form for the cyclic stress-strain curve

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

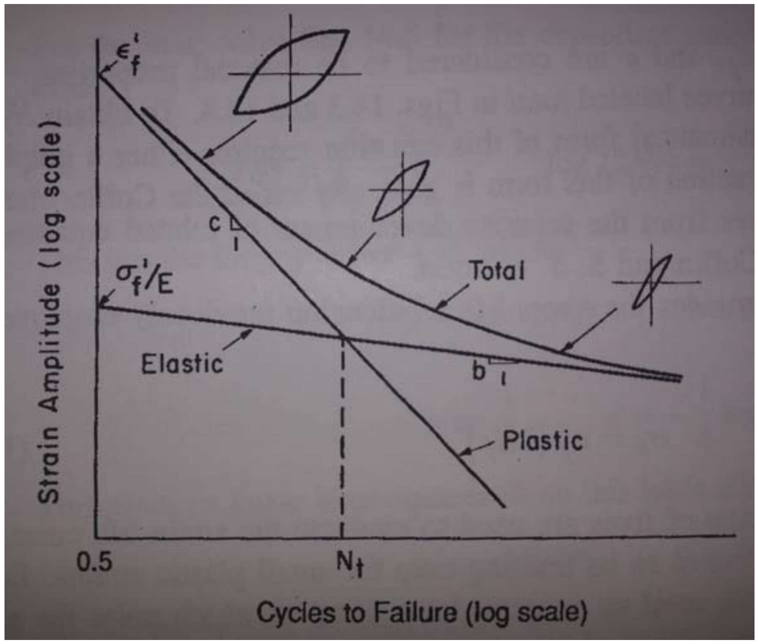
plastic and elastic strain

- On strain life curves, the strain is often plotted three times per each experiment
- Once for total strain, once for plastic strain, and once for elastic strain
- Since plastic strain and elastic strain vary by the number of cycles, a hysteresis loop from half the fatigue life is generally used
- This is considered representative of stable behavior

experimental data



trends



ϵ_f'
σ_f'/E
c
1
Total
Elastic
b
1
Plastic
Strain Amplitude (log scale)
0.5
N_t
Cycles to Failure (log scale)

lines

- We notice that the data for elastic and plastic strains are represented by straight lines, in the log-log scale
- If we recall the form used for a straight line in log-log plots for S-N curves: $\sigma_a = \sigma_f^{'}(2N_f)^b$
- We can convert this to find the elastic component of strain

$$\epsilon_{ea} = \frac{\sigma_f^{'}}{E}(2N_f)^b$$

lines

- We can use the same form with new constants for the plastic component of strain $\epsilon_{pa} = \epsilon_f^{'}(2N_f)^c$
- We can combine the elastic and plastic portions to find the total strain-life curve

$$\epsilon_a = \frac{\sigma_f^{'}}{E}(2N_f)^b + \epsilon_f^{'}(2N_f)^c$$

example

ϵ_a	σ_a (MPa)	ϵ_{pa}	N_f
0.0202	631	0.01695	227
0.0100	574	0.00705	1030
0.0045	505	0.00193	6450
0.0030	472	0.00064	22250
0.0023	455	(0.00010)	110000

transition life

- With the strain-based fatigue method we are better equipped to discuss the difference between high and low-cycle fatigue
- Low-cycle fatigue is dominated by plastic effects, while high-cycle fatigue has little plasticity
- We can find the intersection of the plastic strain and elastic strain lines
- This point is N_t, the transition fatigue life

$$N_t = \frac{1}{2}\left(\frac{\sigma_f'}{\epsilon_f'}\right)^{\frac{1}{c-b}}$$

inconsistencies in constants

- If we consider the equation for the cyclic stress train curve

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

- We can consider the plastic portion and solve for σ_a $\sigma_a = H' {\epsilon_{pa}}^{n'}$

inconsistencies in constants

- We can eliminate $2N_f$ from the plastic strain equation $\epsilon_{pa} = \epsilon_f^{'}(2N_f)^c$
- By solving the stress-life relationship for $2N_f$ $\sigma_a = \sigma_f^{'}(2N_f)^b$ and substituting that into the plastic strain

inconsistencies in constants

- We then compare with stress-life equations and find

$$H' = \frac{\sigma_f'}{(\epsilon_f')^{b/c}}$$

$$n' = \frac{b}{c}$$

inconsistencies in constants

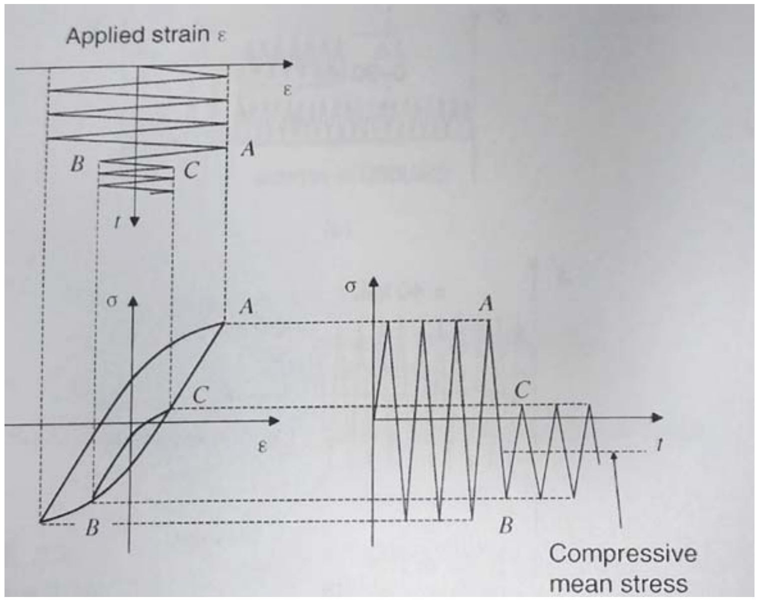
- However, in practice these constants are fit from different curves
- In some cases there can be large inconsistencies in these values
- One cause for this is data that do not lie on a straight line in the log-log domain
- For ductile materials at short lives, the true stresses and strains may differ significantly from engineering stress and strain

variable amplitude strains

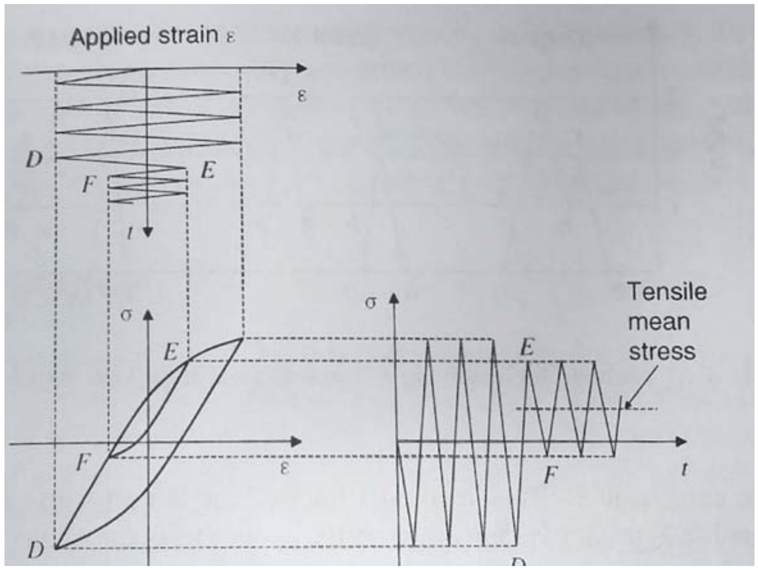
variable amplitude strains

- As with stresses, we can apply variable amplitude strains
- However, when the change is made will affect whether there is a tensile or compressive mean stress

compressive mean



tensile mean



mean stress effects

mean stress in strain-based fatigue

- In regions where plastic strain is significant, some applied mean stress is likely to be relaxed through cyclic plastic strain
- When the plastic strain is not significant, mean stress will exist
- Mean strain does not generally affect fatigue life

morrow approach

- Recall the Morrow approach for mean stress effects from the stress-based method

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f^{'}} = 1$$

- We can rearrange the equation such that

$$\sigma_a = \sigma_f^{'}\left[\left(1 - \frac{\sigma_m}{\sigma_f^{'}}\right)^{\frac{1}{b}}(2N_f)\right]^b$$

morrow approach

- If we compare to the stress-life equation ($\sigma_a = \sigma_f'(2N_f)^b$), we see that we can replace N_f with

$$N^* = N_f\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{1}{b}}$$

- We can now substitute N^* for N_f in the strain-life equation to find

$$\epsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)(2N_f)^b + \epsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{c}{b}}(2N_f)^c$$

morrow approach

- Graphically, we can use the Morrow approach very easily using only the zero-mean stress graph
- From the zero-mean stress graph, find the point corresponding to your applied strain
- For a non zero mean stress, this point represents (ϵ_a, N^*), we can now solve for N_f using the equation for N^*

modified morrow

- While the Morrow equation agrees very well with many data, some are better fit with a modification
- In the modified version, it is assumed that the mean stress has no effect on the plastic term

$$\epsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_f}{\sigma_f'}\right)(2N_f)^b + \epsilon_f'(2N_f)^c$$

- There is no convenient solution method for this form, and it generally must be solved numerically, or plotted with many families of σ_m

smith watson topper

- The Smith, Watson, and Topper approach assumes that the life for any given state is dependent on the product $\sigma_{max}\epsilon_a$
- After some manipulation, this gives

$$\sigma_{max}\epsilon_a = \frac{\left(\sigma_f'\right)^2}{E}(2N_f)^{2b} + \sigma_f'\epsilon_f'(2N_f)^{b+c}$$

- This method can also be solved graphically if a plot of $\sigma_{max}\epsilon_a$ is made using zero-mean data. All we need to do is find the new $\sigma_{max}\epsilon_a$ point to find a new N_f

comparison

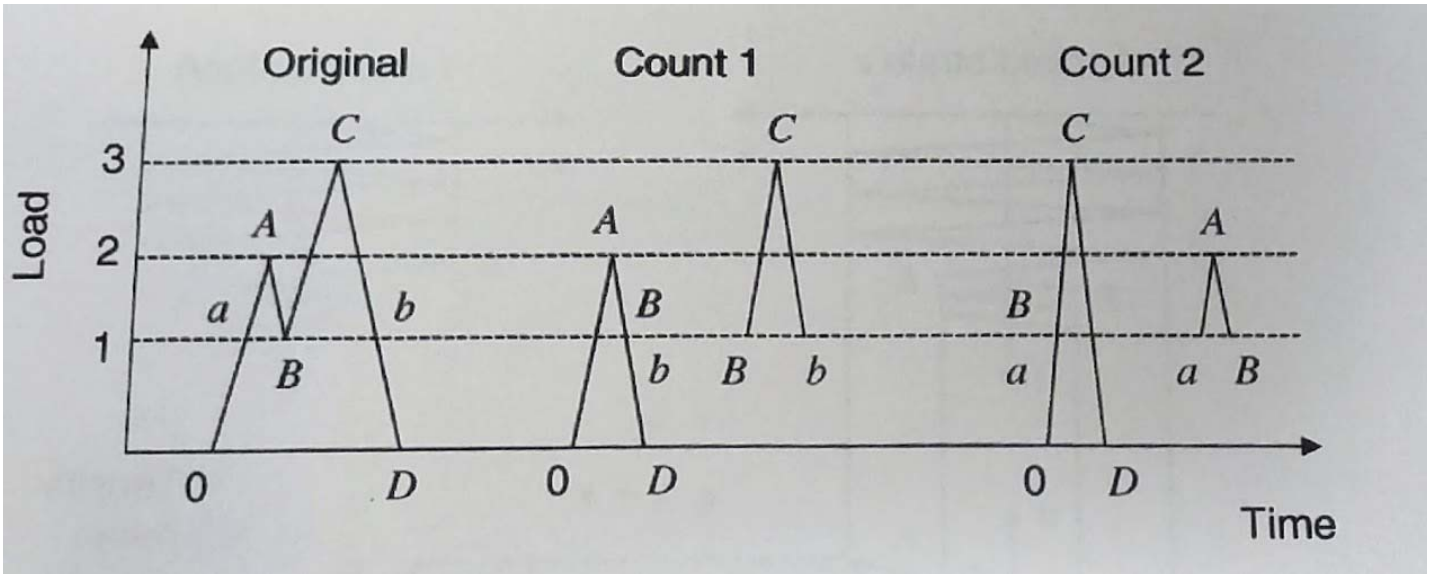
- All three methods discussed are in general use
- The Morrow method is very good for steel
- The modified Morrow method gives improved results in many materials
- The SWT approach is very good for general use, but is non-conservative with a compressive mean stress

example p. 285

cycle counting

- In all fatigue methods (stress, strain, and crack propagation) the way we count load cycles can have an effect on our results
- To avoid being non-conservative, we need to always count the largest amplitudes first
- We will discuss some specific cycle-counting algorithms during crack propagation

cycle counting



general trends

true fracture strength

- We can consider a tensile test as a fatigue test with $N_f = 0.5$
- We would then expect the true fracture strength $\tilde{\sigma}_f \approx \sigma_f^{'}$
- And similarly for strain $\tilde{\epsilon}_f \approx \epsilon_f^{'}$

ductile materials

- Since ductile materials experience large strains before failure, we expect relatively large $\epsilon_f^{'}$ and relatively small $\sigma_f^{'}$
- This will cause a less steep slope in the plastic strain line
- In turn this intersects with the elastic strain line much later, resulting a longer transition life for ductile materials

brittle materials

- Brittle materials exhibit the opposite effect, with relatively low $\epsilon_f^{'}$ and relatively high $\sigma_f^{'}$
- This results in a steeper plastic strain line
- And shorter transition life

tough materials

- Tough materials have intermediate values for both ϵ_f' and σ_f'
- This gives a transition life somewhere between brittle and ductile materials
- It is also noteworthy that strain-life for many metals pass through the point ϵ_a = 0.01 and N_f = 1000 cycles
- Steels also follow a trend with Brinell Hardness, the higher they are on the HB scale, the lower their transition life

typical property ranges

- Most common engineering materials have $-0.8 < c < -0.5$, with most values being very close to $c = -0.6$
- The elastic strain slope generally has $b = -0.085$
- A “steep” elastic slope is around $b = -0.12$, common in soft metals
- While “shallow” slopes are around $b = -0.05$, common for hardened metals

notches

fatigue notch factor

- We previously found expressions for stress-based fatigue analysis when notches are present
- Due to yielding, the notch sensitivity is not the same for stress and strain controlled fatigue analysis
- One simple approach to find the strain fatigue notch factor is to use

$$K_t = \sqrt{K_f^{\sigma} K_f^{\epsilon}}$$

multiaxial loading

multiaxial loading

- Multi-axial loading in strain-based fatigue analysis is still an active field of research
- We are currently only capable of handling proportional loads that are in-phase (i.e. have the same frequency)

multiaxial loading

- If we consider the principal directions where $\sigma_{2a} = \lambda\sigma_{1a}$, we find an expression for the strain-life as

$$\epsilon_{1a} = \frac{\frac{\sigma_f^{'}}{E}(1 - v\lambda)(2N_f)^b + \epsilon_f^{'}(1 - 0.5\lambda)(2N_f)^c}{\sqrt{1 - \lambda + \lambda^2}}$$

stress triaxiality factor

- Another approach is to consider the stress triaxiality factor

$$T = \frac{1 + \lambda}{\sqrt{1 - \lambda + \lambda^2}}$$

- Three notable cases of this are
    1. Pure planar shear: $\lambda = -1, T = 0$
    2. Uniaxial stress: $\lambda = 0, T = 1$
    3. Equal biaxial stress: $\lambda = 1, T = 2$

stress triaxiality factor

- Marloff suggests the following inclusion of stress triaxiality

$$\bar{\epsilon}_a = \frac{\sigma'_f}{E}(2N_f)^b + 2^{1-T}\epsilon'_f(2N_f)^c$$

other factors affecting fatigue

factors affecting fatigue life

- At temperatures above one-half the melting temperature (absolute scale), creep-relaxation is significant
- This will cause the strain/stress-life curves to become rate dependent
- Occurs at room temperature for many materials (lead, tin, many polymers)
- At a sufficiently elevated temperature for any material

surface finish

- High cycle fatigue is sensitive to surface finish, samples are generally polished
- Low cycle fatigue is not sensitive to surface finish or residual stress
- The plastic deformation tends to remove residual stresses
- In high-cycle fatigue, crack initiation is important (poor surface finish allows cracks to form earlier)
- When plastic deformation is present (low-cycle fatigue), cracks form relatively quickly regardless of surface finish

surface finish

- Since low-cycle fatigue has little effect from surface finish, we could modify the strain life curve by altering only the elastic portion
- If we define the surface effect factor, m_s, we can find a new b_s to replace b in the strain-life equation

$$b_s = \frac{\log\left(m_s (2N_e)^b\right)}{\log(2N_e)}$$

surface treatments

- Treatments which decrease fatigue life:
    - Electro-plating (chrome, +corrosion resistance, -fatigue life)
    - Grinding improves surface finish, but introduces surface tension, and heat generated can temper quench
    - Stamping introduces discontinuities and irregularities
    - Forging can refine grain structure and improve physical properties, but can cause decarburization in steels, which hurts fatigue life
    - Hot rolling can also cause decarburization

surface treatments

- Some treatments improve fatigue life:
  - Cold rolling improves surface finish, introduces residual compressive stress on surface (slows crack initiation on surface)
  - Shot peeing introduces many small divots on surface, which can be detrimental in corrosion, but it does cause a residual compressive stress on the surface

size

- Size can also have effects on fatigue life
- Larger parts are more susceptible to damage/imperfections at the same stress level
- This is why composites are often made from very small fibers (glass fiber, carbon fiber, ceramic-matrix composites)

size

- The exact effect of size will depend on material, one study for low carbon steels found

$$m_d = \left(\frac{d}{25.4\text{mm}}\right)^{-0.093}$$

- Which is then used to re-calculate material constants $\sigma_{fd}^{'} = m_d\sigma_f^{'}$, $\qquad \epsilon_{fd}^{'} = m_d\epsilon_f^{'}$

thermal fatigue

- Thermal loading can be introduced when two dissimilar parts are attached together, the coefficient of thermal expansion causes them to expand differently, introducing extra stresses due to the temperature change
- If the temperature is significantly different between two sides of a part thermal stresses can also be introduced

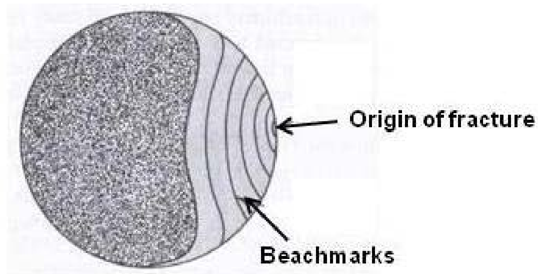
thermal fatigue

- Low temperatures generally cause a material to behave in a more brittle fashion, which alters the fatigue life
- High temperatures cause problems with creep-relaxation and can also affect the crystalline structure

crack growth rate

fracture surface

fracture surface



Fatigue Fracture with Beachmarks

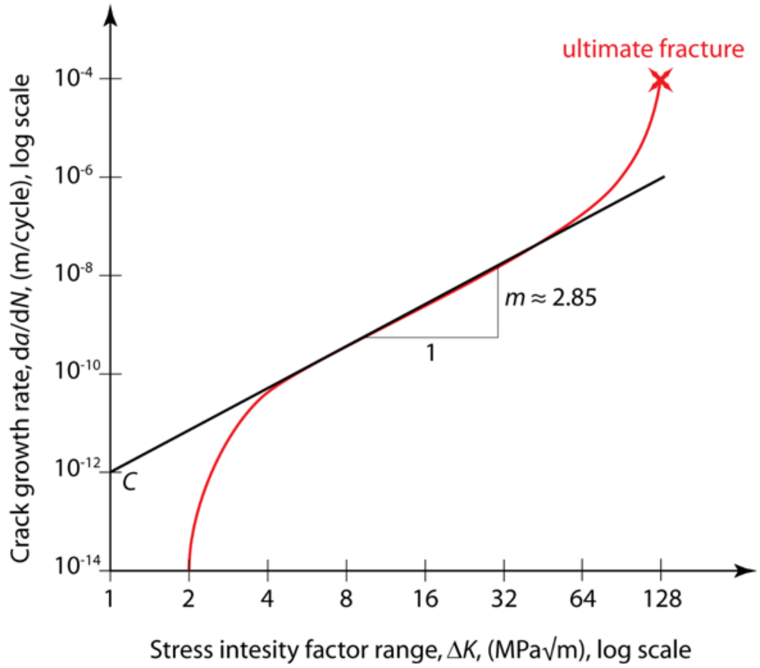
crack growth rate

- We can observe that fatigue damage occurs through crack propagation
- “cracks” and fracture mechanics have been omitted from all our fatigue discussion thus far
- It would be beneficial to predict at what rate a crack will extend

crack growth rate

- Crack growth rate can be measured experimentally
- Using a center-crack specimen, a fatigue load is applied
- The crack length is measured and plotted vs. the number of cycles
- The slope of this curve ($\frac{da}{dN}$) is then plotted vs. either K_{Imax} or ΔK_I on a log-log scale
- This chart is then commonly divided into three regions

da-dN vs K


ultimate fracture
Crack growth rate, da/dN, (m/cycle), log scale
10^-4
10^-6
10^-8
10^-10
10^-12
10^-14
C
m ≈ 2.85
1
1
2
4
8
16
32
64
128
Stress intesity factor range, ΔK, (MPa√m), log scale

region I

- In Region I crack growth is very slow and/or difficult to measure
- In many cases, da/dN corresponds to the spacing between atoms!
- The point at which the da/dN curve intersects the x-axis (usually with a relatively vertical slope) is called the fatigue threshold
- Typically 3-15 $\text{ksi}\sqrt{\text{in}}$ for steel
- 3-6 $\text{ksi}\sqrt{\text{in}}$ for aluminum

region II

- Most important region for general engineering analysis
- Once a crack is present, most of the growth and life occurs in Region II
- Generally linear in the log-log scale

region III

- Unstable crack growth
- Usually neglected (we expect failure before Region III fully develops in actual parts)
- Can be significant for parts where we expect high stress and relatively short life

crack growth rate curve

- The crack growth rate curve is considered a material property
- The same considerations for thickness apply as with fracture toughness (K_c vs. K_{Ic})
- Is also a function of the load ratio, $R = \sigma_{min}/\sigma_{max}$

R effects

- While the x-axis can be either ΔK or K_{max}, the shape of the data is the same
- When we look at the effects of load ratio, R, the axis causes some differences on the plot
- With ΔK on the x-axis, increasing R will shift the curve up and to the left, shifting the fatigue threshold and fracture toughness on the graph as well

R effects

- With K_{max} on the x-axis, increasing R shifts the curve down and to the right, but fatigue threshold and fracture toughness keep same values
- In general, R dependence vanishes for $R > 0.8$ or $R < -0.3$. This effect is known as the band width

crack growth rate equations

crack growth rate equations

- There are many crack growth rate equations of varying complexity
- The “best” form to use will depend on design needs

growth equations

- The important features in curve-fit equations are
    1. Region II curve fit (linear on log-log scale)
    2. Region I curve fit (fatigue threshold)
    3. Region III curve fit (critical stress intensity)
    4. Stress ratio effects
    5. Band width of R-curves

paris law

- The original
- Fits the linear portion (Region II)
- Does not fit Region I, Region III, or have R-dependence

$$\frac{da}{dN} = C(\Delta K)^n$$

- Note: this assumes the x-axis is ΔK, but $\Delta K = (1 - R)K_{max}$, so we can easily convert

walker

- Region II is usually all that is needed for engineering, but R-dependence is often an important effect to capture
- Walker modified the Paris law to account for R-dependence

$$\frac{da}{dN} = C\left[(1 - R)^m K_{max}\right]^n$$

- Gives a good fit for Region II with R-dependence and band width

forman

- The Forman equation was developed to capture the effects of Region II and Region III
- Also includes the effects of R, but does not control the band width of R effects

$$\frac{da}{dN} = \frac{C\left[(1-R)K_{max}\right]^n}{(1-R)K_c - (1-R)K_{max}}$$

modified forman

- The Forman equation can be modified to include the effect of band width

$$\frac{da}{dN} = \frac{C\left[(1-R)^m K_{max}\right]^n}{\left[(1-R)^m K_c - (1-R)^m K_{max}\right]^L}$$

collipriest

- The Collipriest equation fits Regions I, II and III, but has no R-dependence

$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1}\left[\frac{\log\left(\frac{K_{max}^2}{K_o K_c}\right)}{\log(K_c / K_o)}\right]$$

modified collipriest

- Following the same methods as before, we can modify the Collipriest equation for R-dependence and band width control

$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1}\left[\frac{\log\left(\frac{(1-R)^m K_{max}^2}{K_o K_c}\right)}{\log(K_c/K_o)}\right]$$

- For a cleaner graph, experimental data at different R-values is sometimes plotted vs. K_{eff}
- $K^*_{eff} = (1 - R)^m K_{max}$

nasgrow growth rate equation

- A very complicated curve fit is provided in the NASGROW growth rate equation

$$\frac{da}{dN} = C\left[\frac{1-f}{1-R}\Delta K\right]^n \frac{\left[1 - \frac{\Delta K_{th}}{\Delta K}\right]}{\left[1 - \frac{K_{max}}{K_{crit}}\right]}$$

- The curve fit parameters can be found in p. 307 of your text (or the NASGROLW/AFGROW documentation)

boeing-walker growth rate equation

- The Boeing-Walker growth equation is given as (for $R \geq 0$)

$$\frac{da}{dN} = 10^{-4}\left(\frac{1}{mT}\right)^{p}\left[K_{max}(1 - R)^{q}\right]^{p}$$

conversion of constants

- Much of the data available to us is from Boeing, and given in terms of the Boeing-Walker equation
- We can re-write some other equations to more easily convert parameters between the various equations
- Walker-Boeing:

$$\frac{da}{dN} = 10^{-4}\left(\frac{1}{mT}\right)^{p}\left[\Delta K(1-R)^{q-1}\right]^{p}$$

- Walker-AFGROW:

$$\frac{da}{dN} = C_w\left[\Delta K(1-R)^{m-1}\right]^{n_w}$$

- Forman:

$$\frac{da}{dN} = \frac{C_F}{(1-R)K_c-\Delta K}(\Delta K)^{n_f}$$

conversion of constants

Walker-Boeing	Walker-AFGROW	Forman
$10^{-4}\left(\frac{1}{mT}\right)^{p}$	$C_w = 10^{-4}\left(\frac{1}{mT}\right)^{p}$	$C_F = (K_c - 1)10^{-4}\left(\frac{1}{mT}\right)^{p}$
q	$m = q$	
p	$n_w = p$	$n_f = p$