

AE 737 - MECHANICS OF DAMAGE TOLERANCE

LECTURE 19

Dr. Nicholas Smith

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SCHEDULE

- 5 Apr - Crack Growth, Homework 7 due, Homework 8 assigned
- 7 Apr - Crack Growth, Stress Spectrum
- 12 Apr - Retardation, Boeing Commercial Method
- 14 Apr - Exam Review, Homework 8 Due
- 19 Apr - Exam 2
- 21 Apr - Exam Solutions, Damage Tolerance

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FINAL PROJECT CLARIFICATION

- Some of the wording I used in the final project description is ambiguous, and based on what I learned in my fatigue class (not your text)
- By "crack growth" I intended "crack nucleation," which is a phrase to describe stress and strain based fatigue analysis
- "Crack propagation" is what I intended by what your text calls "crack growth", and refers to fracture mechanics-based fatigue analysis

1. mean stress effects
2. multiaxial loading
3. crack growth rate
4. crack growth rate equations

MEAN STRESS EFFECTS

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- Mean strain does not generally affect fatigue life

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$$\sigma_a = \sigma'_f \left[\left(1 - \frac{\sigma_m}{\sigma'_f} \right)^{\frac{1}{b}} (2N_f) \right]^b \quad (19.2)$$

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$$\epsilon_a = \frac{\sigma'_f}{E} \left(1 - \frac{\sigma_m}{\sigma'_f} \right) (2N_f)^b + \epsilon'_f \left(1 - \frac{\sigma_m}{\sigma'_f} \right)^{\frac{c}{b}} (2N_f)^c \quad (19.4)$$

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- From the zero-mean stress graph, find the point corresponding to your applied strain
- For a non zero mean stress, this point represents (ϵ_a, N^*) , we can now solve for N_f using 19.3

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- There is no convenient solution method for this form, and it generally must be solved numerically, or plotted with many families of σ_m

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- This method can also be solved graphically if a plot of $\sigma_{max}\epsilon_a$ is made using zero-mean data. All we need to do is find the new $\sigma_{max}\epsilon_a$ point to find a new N_f

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- The SWT approach is very good for general use, but is non-conservative with a compressive mean stress

MULTIAXIAL LOADING

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$$\epsilon_{1a} = \frac{\frac{\sigma'_f}{E}(1 - \nu\lambda)(2N_f)^b + \epsilon'_f(1 - 0.5\lambda)(2N_f)^c}{\sqrt{1 - \lambda + \lambda^2}} \quad (19.7)$$

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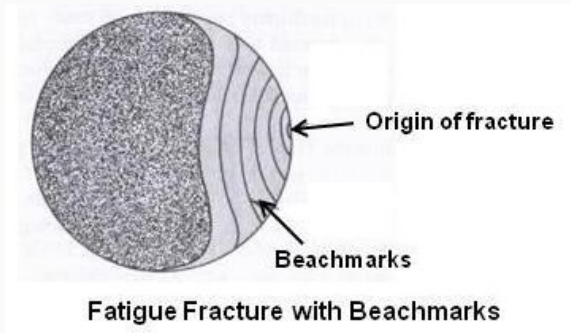
$$\bar{\epsilon}_a = \frac{\sigma'_f}{E} (2N_f)^b + 2^{1-T} \epsilon'_f (2N_f)^c \quad (19.9)$$

CRACK GROWTH RATE

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- It would be beneficial to predict at what rate a crack will extend

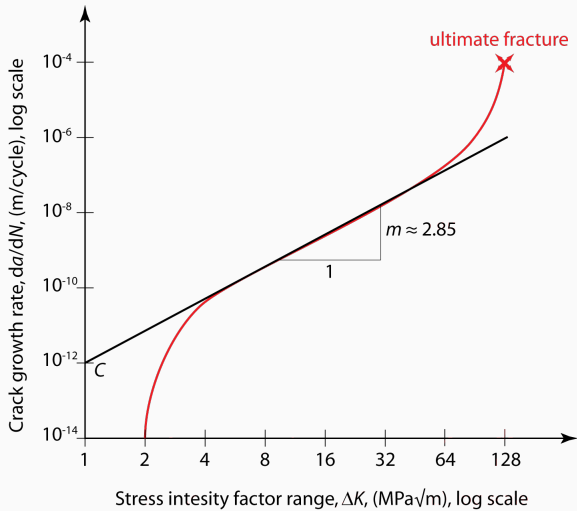
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- This chart is then commonly divided into three regions



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- In general, R dependence vanishes for $R > 0.8$ or $R < -0.3$. This effect is known as the band width

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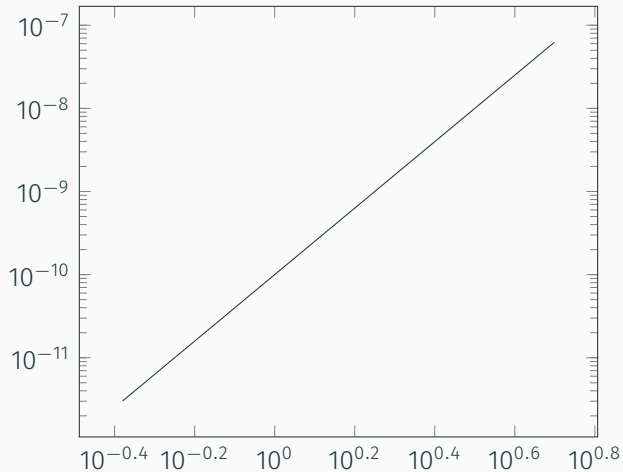
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- Note: this assumes the x-axis is ΔK , but $\Delta K = (1 - R)K_{max}$, so we can easily convert

PARIS LAW



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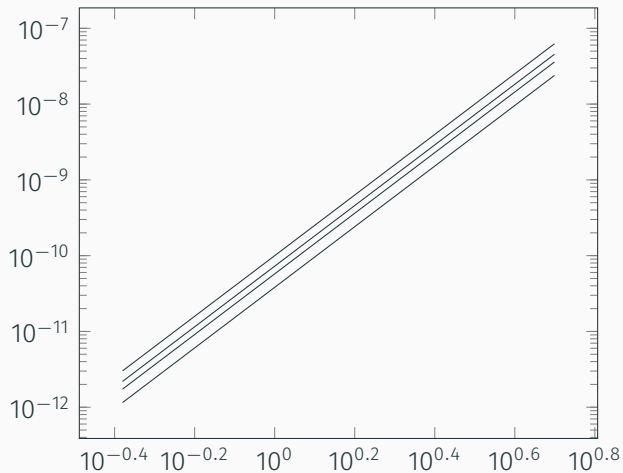
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- Gives a good fit for Region II with R-dependence and band width



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$$\frac{da}{dN} = \frac{C [(1 - R)K_{max}]^n}{(1 - R)K_c - (1 - R)K_{max}} \quad (19.12)$$

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$$\frac{da}{dN} = \frac{C [(1 - R)^m K_{max}]^n}{[(1 - R)^m K_c - (1 - R)^m K_{max}]^L} \quad (19.13)$$

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$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1} \left[\frac{\log \left(\frac{K_{max}^2}{K_o K_c} \right)}{\log(K_c/K_o)} \right] \quad (19.14)$$

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$$K_{eff} = (1 - R)^m K_{max} \quad (19.16)$$

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$$\frac{da}{dN} = C \left[\frac{1-f}{1-R} \Delta K \right]^n \frac{\left[1 - \frac{\Delta K_{th}}{\Delta K} \right]}{\left[1 - \frac{K_{max}}{K_{crit}} \right]} \quad (19.17)$$

NASGROW GROWTH RATE EQUATION

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- The curve fit parameters can be found in p. 307 of your text (or the NASGROW/AFGROW documentation)

- The Boeing-Walker growth equation is given as (for $R \geq 0$)

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$$\frac{da}{dN} = 10^{-4} \left(\frac{1}{mT} \right)^p [K_m a x (1 - R)^q]^p \quad (19.18)$$

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- Forman:

$$\frac{da}{dN} = \frac{C_F}{(1 - R)K_c - \Delta K} (\Delta K)^{n_f} \quad (19.21)$$

CONVERSION OF CONSTANTS

Walker-Boeing	Walker-AFGROW	Forman
$10^{-4} \left(\frac{1}{mT}\right)^p$	$C_w = 10^{-4} \left(\frac{1}{mT}\right)^p$	$C_F = (K_c - 1)10^{-4} \left(\frac{1}{mT}\right)^p$
q	$m = q$	
p	$n_w = p$	$n_f = p$