AE 737: Mechanics of Damage Tolerance

Lecture 3 - Superposition, Compounding

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schedule

- 29 Jan - Superposition, Compounding
- 31 Jan - Curved Boundaries, Homework 1 Due
- 5 Feb - Plastic Zone
- 7 Feb - Plastic Zone

office hours

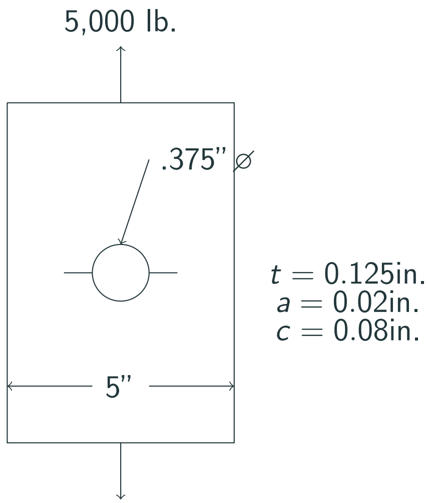
- With 12/21 students reporting, TODO
- As a back-up, AE 333 Office Hours are TODO
- Take advantage of office hours, this is time that I have already set aside for you
- If the regular office hours do not work for your schedule, send me an e-mail and we can work out a time to meet

outline

- Review
- Superposition
- Compounding

review

example



example

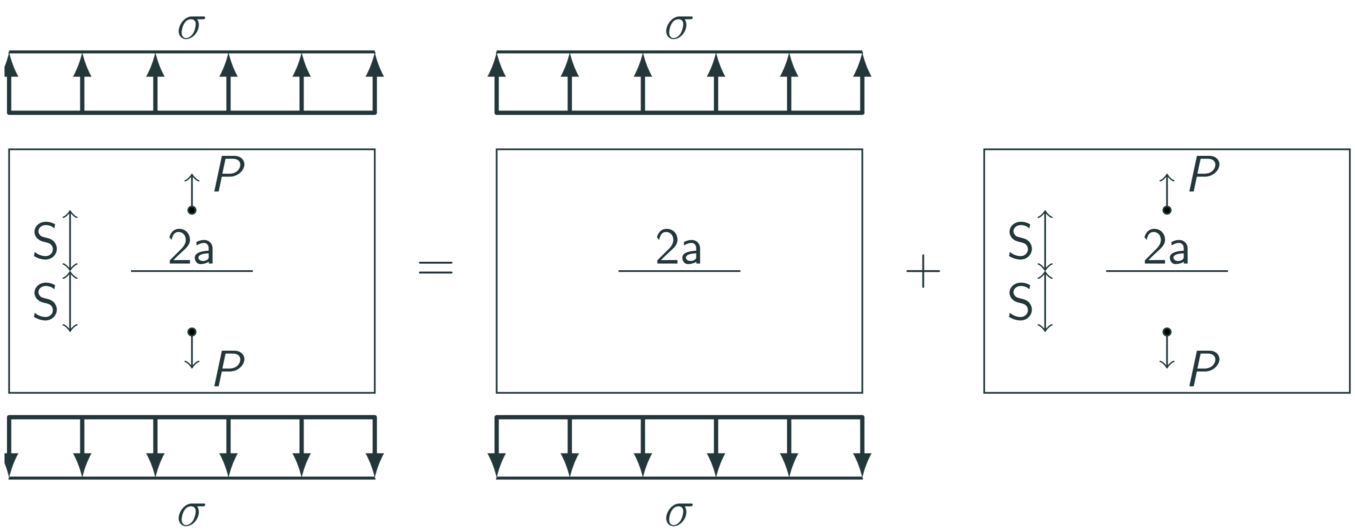
- Case 1 - symmetric through cracks
- Case 2 - single through crack
- Case 3 - symmetric corner cracks
- Case 4 - single corner crack
- Case 5 - symmetric surface cracks
- Case 6 - single surface crack
- Viewable **here**

superposition

superposition

- Since the stress intensity factor is derived using Linear Elasticity, the principle of superposition applies
- Multiple applied loads can be superposed to find the effective stress intensity factor of the combined loading

superposition



superposition

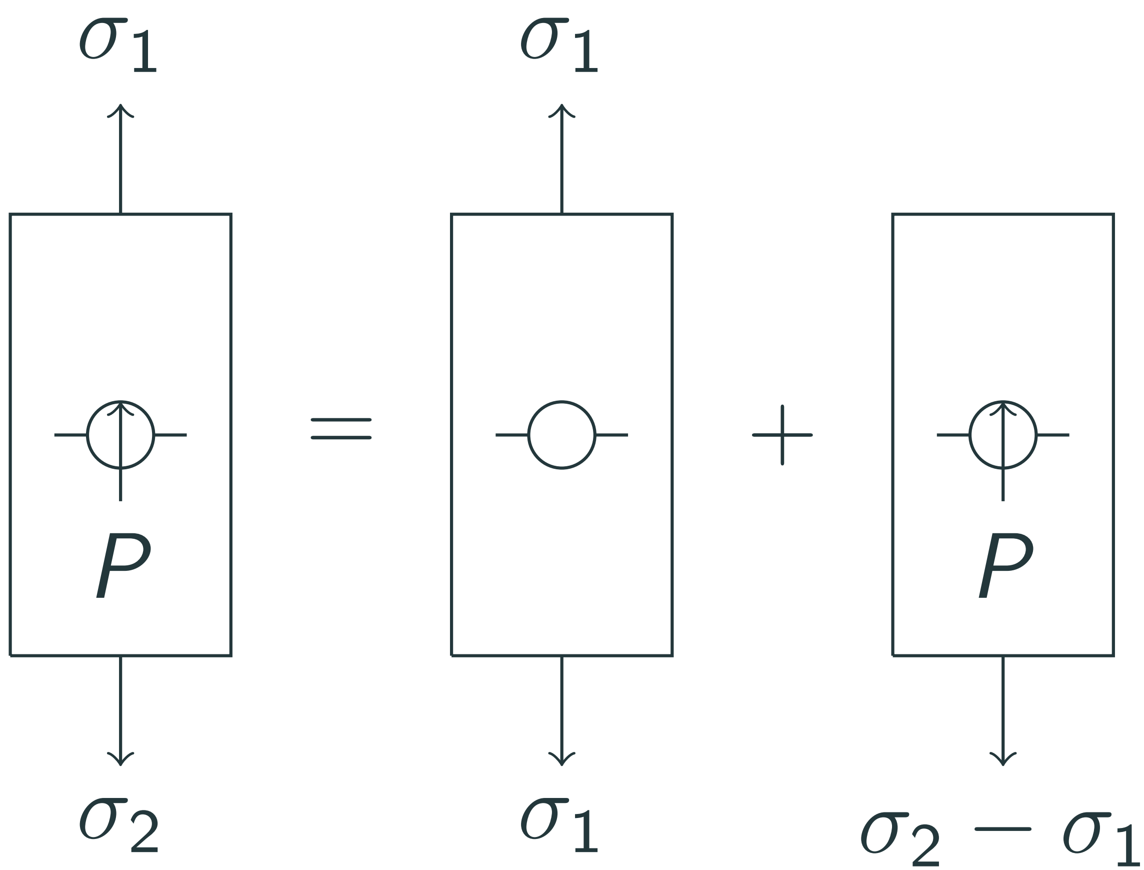
$$K_I = K_{I(\sigma)} + K_{I(P)}$$

$$K_I = \sigma\sqrt{\pi a} + \frac{P}{t\sqrt{\pi a}} \frac{1 - 0.5\left(\frac{a}{W}\right) + 0.975\left(\frac{a}{W}\right)^2 - 0.16\left(\frac{a}{W}\right)^3}{\sqrt{1 - \left(\frac{a}{W}\right)}}$$

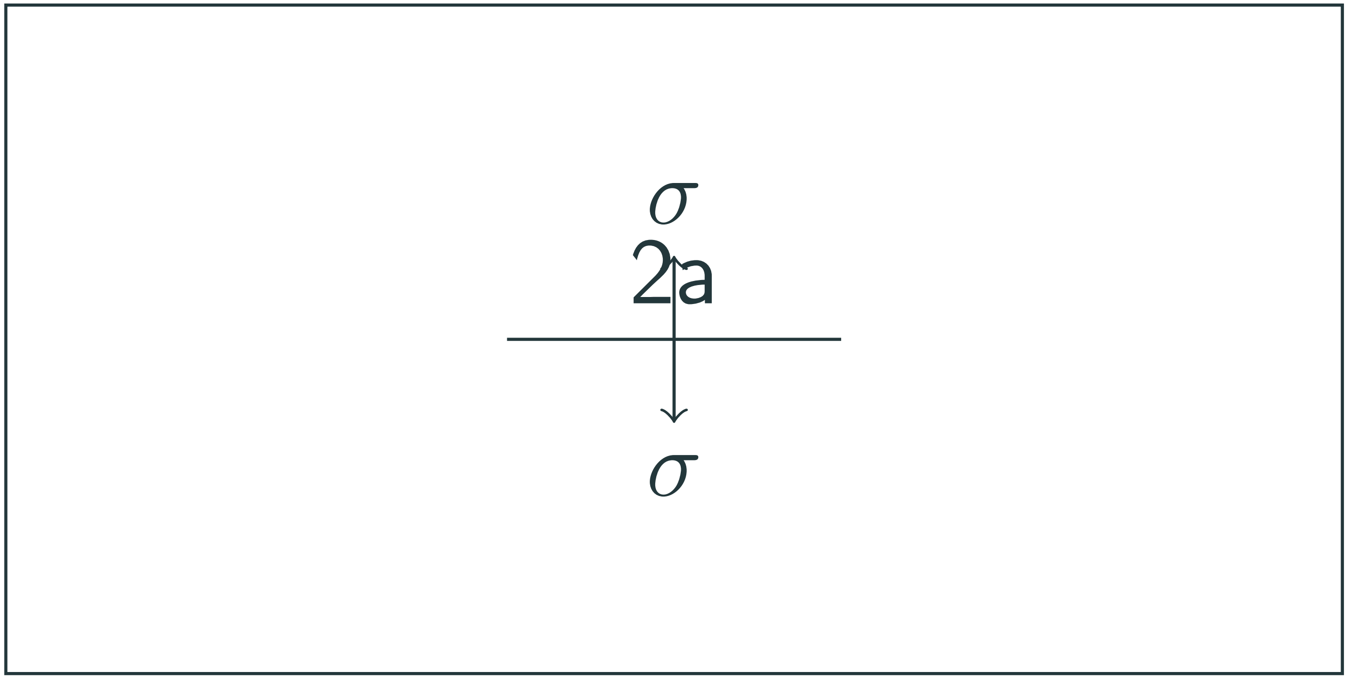
superposition

- Sometimes, the superposition needed to solve a problem is not obvious
- It can be helpful to subtract a known solution from the problem
- Note: Every super-posed solution must satisfy equilibrium.

superposition



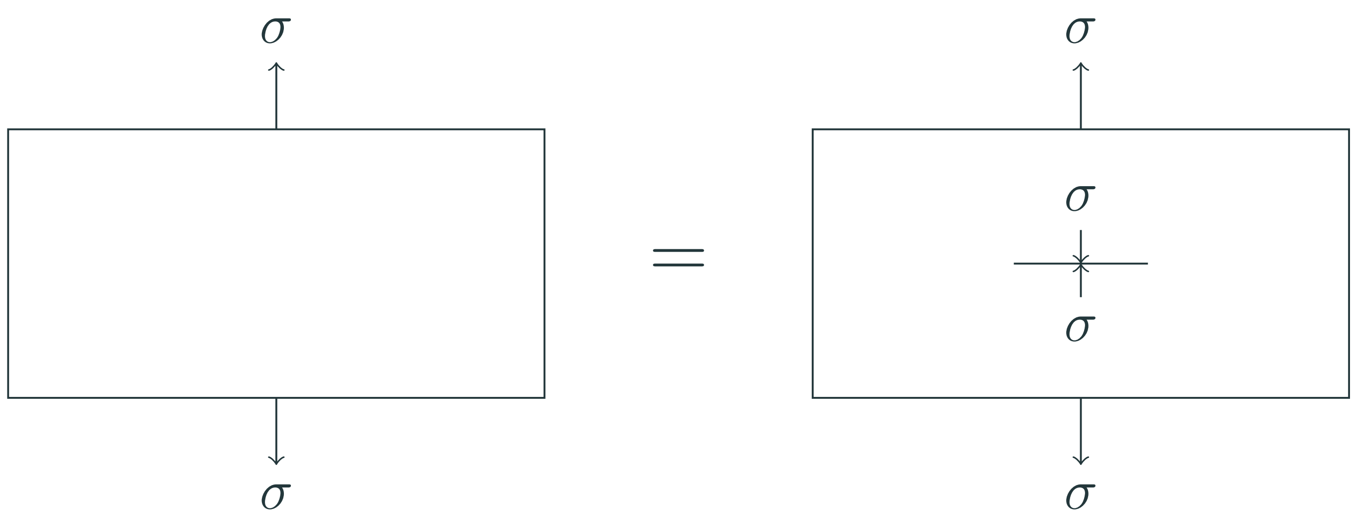
example - pressurized crack



example - pressurized crack

- We can find the stress intensity for a pressurized crack using a non-obvious superposition
- An un-cracked panel with remote stress would be equal to a cracked panel under remote stress with a negative pressure applied to the crack

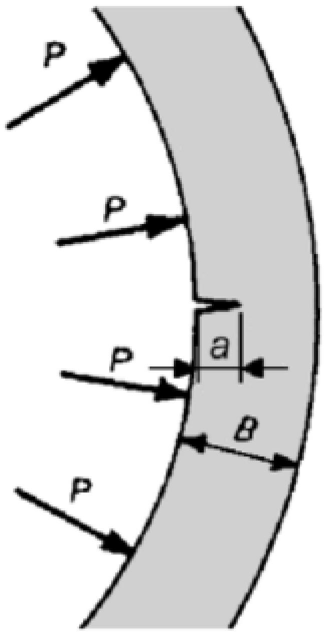
example - pressurized crack



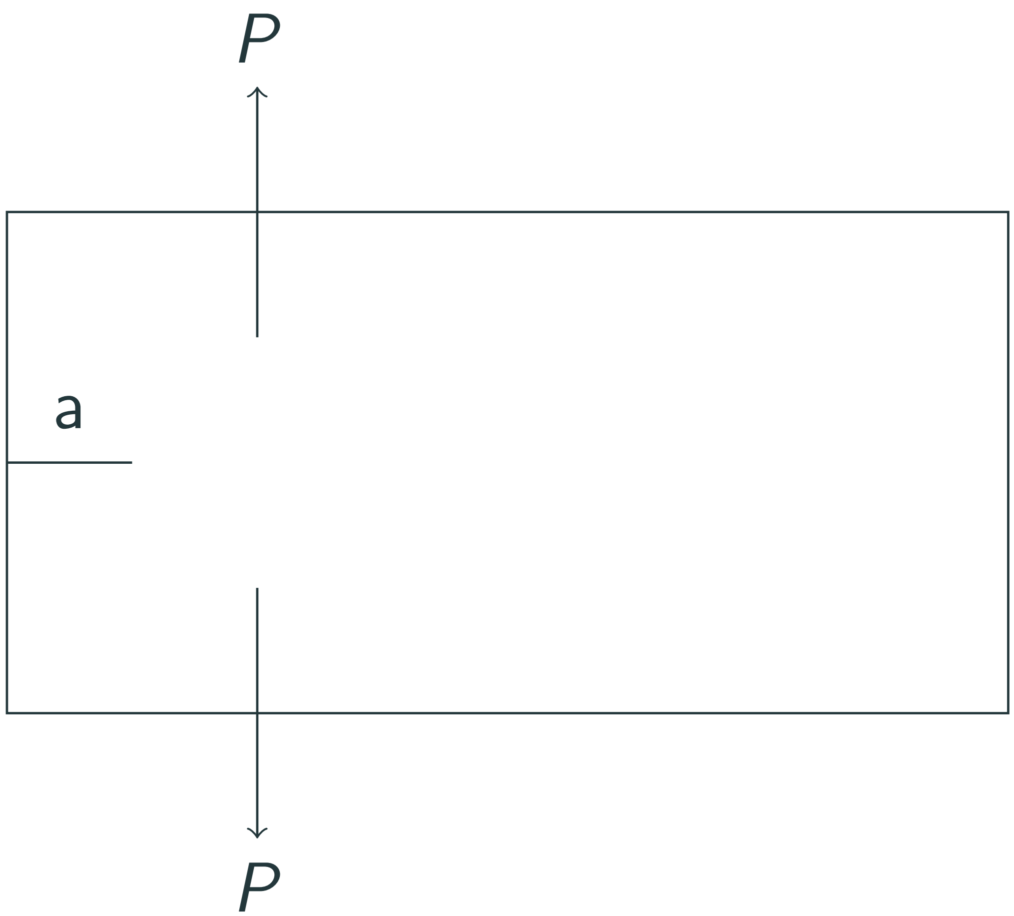
group problems

- Purpose of group problems is not just to solve a problem
- By teaching or explaining concepts to other members of your group, you also reinforce the concept yourself
- When problems are discussed as a group, you will find questions and problems you might not have otherwise found

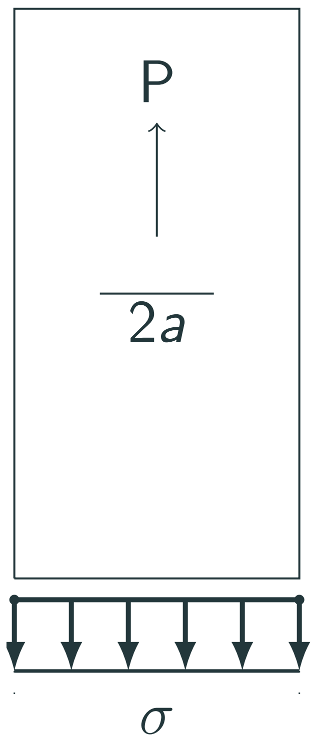
group 1



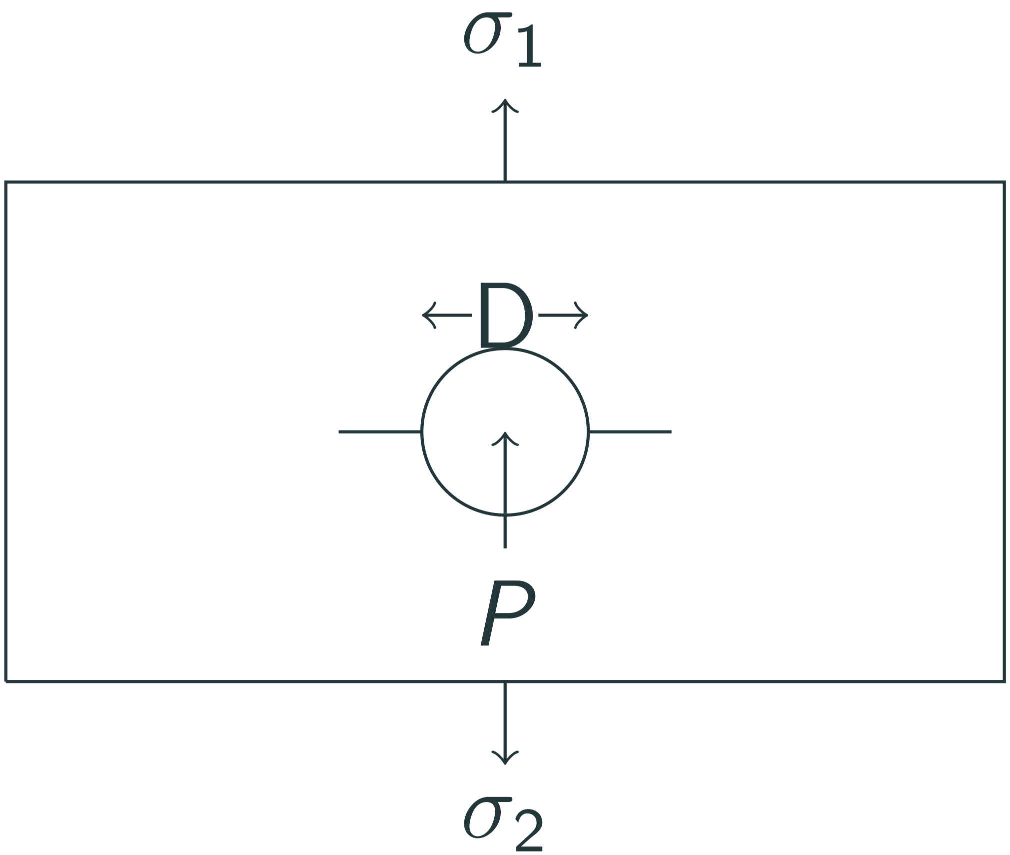
group 2



group 3



group 4



compounding

superposition vs. compounding

- In this course, we use *superposition* to combine loading conditions
- We use *compounding* to combine edge effects
- Both are very powerful tools and important concepts

compounding

- Different types of boundaries create different correction factors to the usual stress intensity factor
- We often use β to indicate the total correction factor
- When multiple boundaries are present, we can combine them into one effective correction factor
- There are two general methods we use to create a compound correction factor

method 1

- The first method uses linear superposition, and thus is restricted to cases where the effect of each boundary can be assumed to add linearly
- While in most cases this is not strictly true, it provides a reasonable approximation

$$K_r = \bar{K} + \sum_{i=1}^{N} (K_i - \bar{K})$$

method 1

- Where N is the number of boundaries, $\bar{K}$ is the stress intensity factor with no boundaries present and K_i is the stress intensity factor associated with the i^{th} boundary.

method 1

- We can rewrite this equation as

$$K_r = \sigma\sqrt{\pi a}\beta_r = \sigma\sqrt{\pi a} + \sum_{i=1}^{N}(\sigma\sqrt{\pi a}\beta_i - \sigma\sqrt{\pi a})$$

- Which leads to an expression for β_r as

$$\beta_r = 1 + \sum_{i=1}^{N}(\beta_i - 1)$$

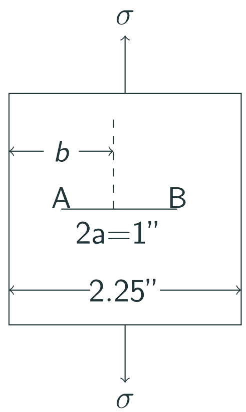
method 2

- An alternative empirical method approximates the boundary effect as $\beta_r = \beta_1 \beta_2 \ldots \beta_N$
- If there is no interaction between the boundaries, method 1 and method 2 will give the same result

p. 68 - example 1

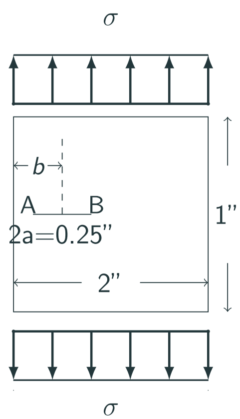
- A crack in a finite-width panel is centered between two stiffeners
- Assume the β correction factor for this stiffener configuration is β_s = 0.9
- Assume the β correction factor for this finite-width panel is β_w = 1.075
- Use both compounding methods to estimate the stress intensity
- How accurate do you expect this to be?

p. 69 - example 3



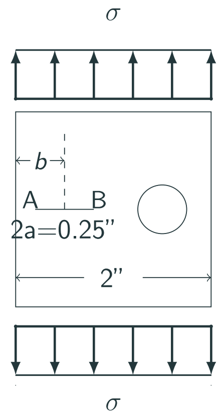
$$b = 1 \text{ inch}$$

group 1



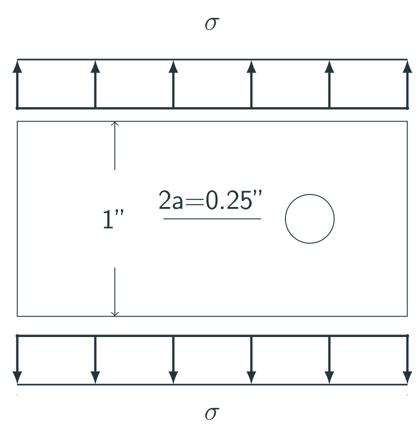
$$b = 0.4 \text{ inches}$$

group 2



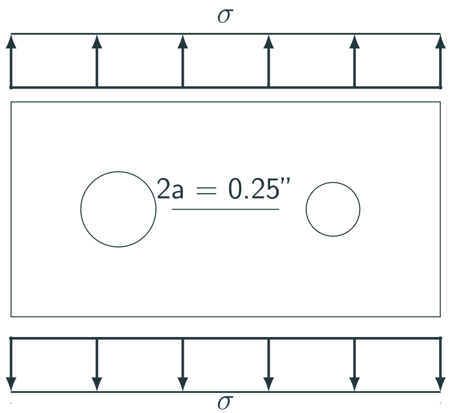
$b = 0.4$ inches
Hole diameter is 0.5 inches and spaced 0.5 inches away from the crack tip

group 3



Hole diameter is 0.5 inches and spaced 0.5 inches away from the crack tip

group 4



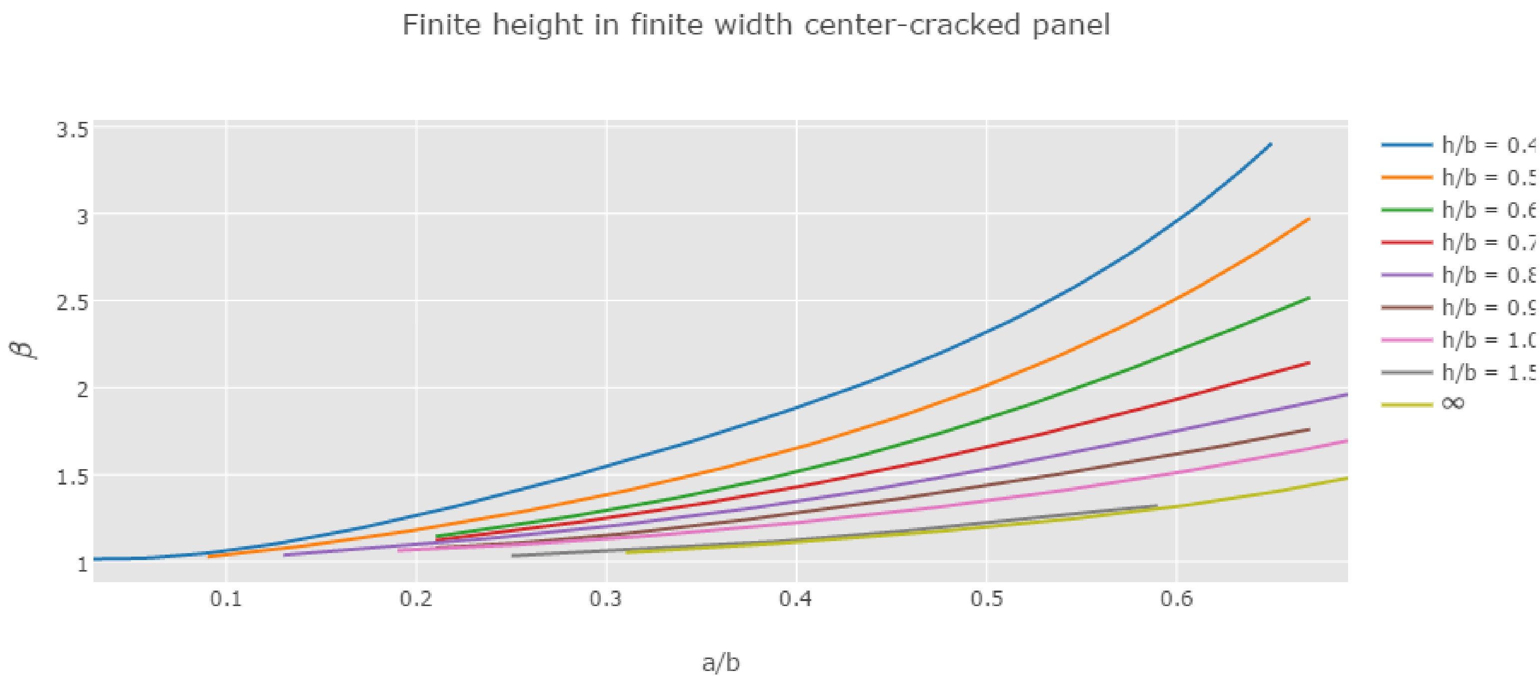
The right crack tip is 0.5 inches away from a 0.5 inch diameter hole and the left crack tip is 0.25 inches away from a 1 inch diameter hole.

errata and supplemental charts

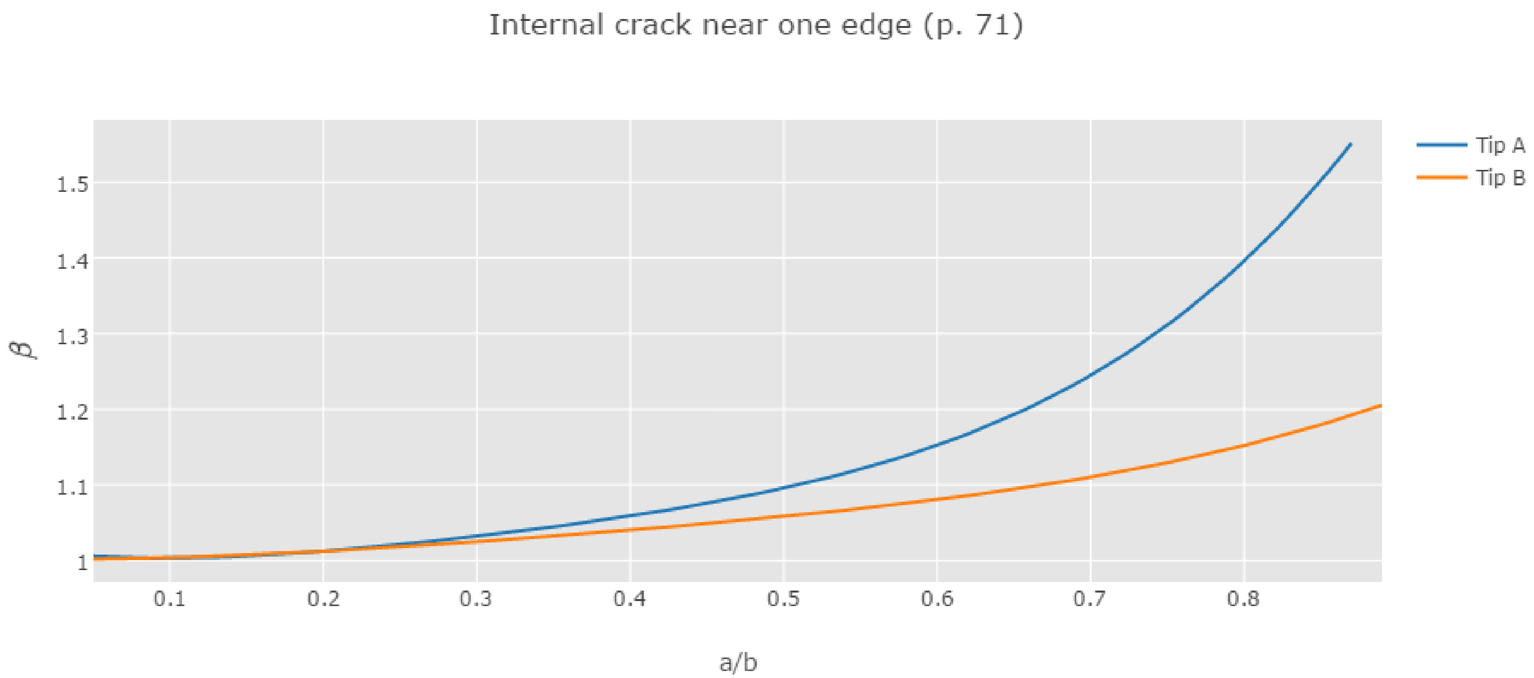
textbook notes

- on p. 64 there is a + missing between two terms, see Lecture 2 for the fix
- Also on p. 64, in equation 29 it is not clear, but use the f_w from a previous equation, on p. 56
- Some of the black and white figures can be difficult to use, we have scanned and re-created the plots online
- Interactive versions of compounding figures from p. 50, 71-73 can be found at **here**

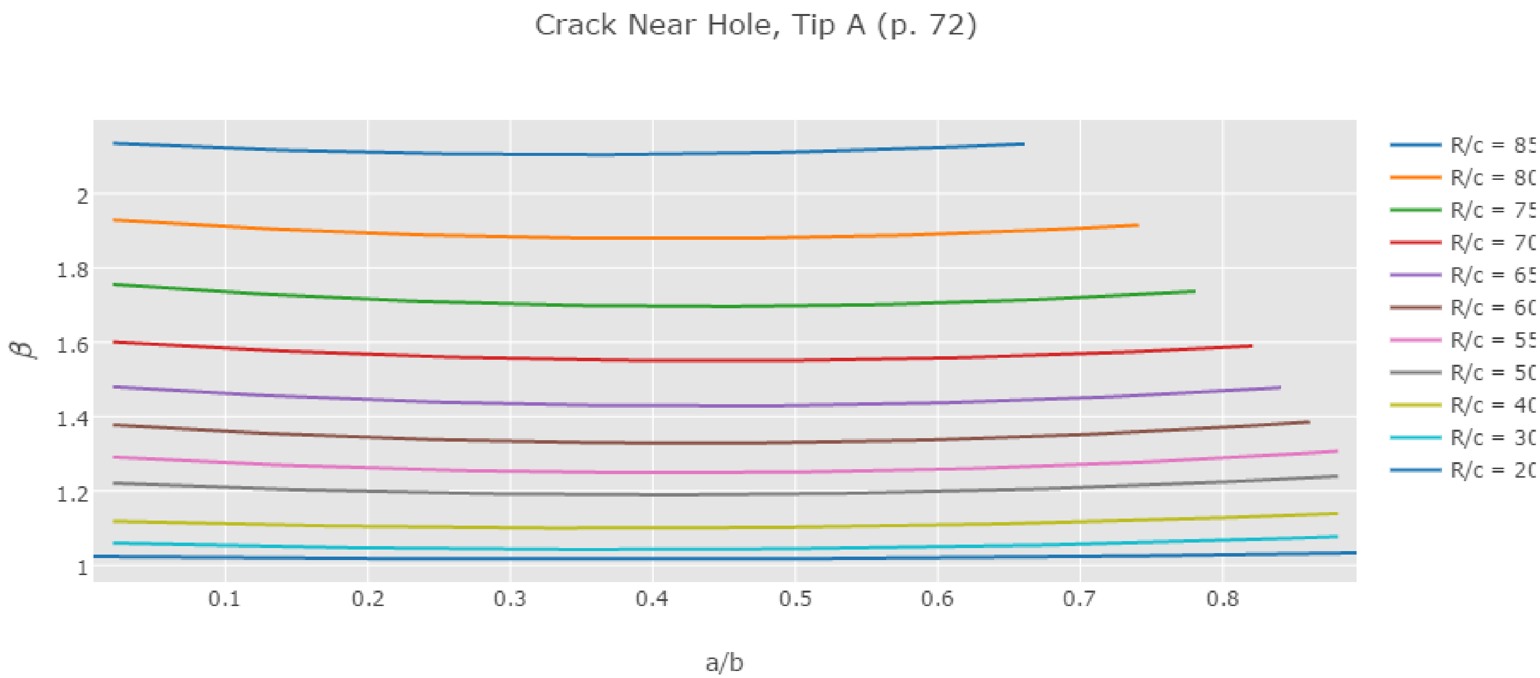
finite height - p. 50



offset crack - p. 71



crack near hole - p. 72



crack near hole - p. 73