

Lecture 2 - Common Stress Intensity Factors

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schedule

- 20 Jan - Common Stress Intensity Factors
- 25 Jan - Superposition, Compounding
- 27 Jan - Curved Boundaries, HW 1 Due
- 1 Feb - Plastic Zone

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review

fracture mechanics

- In fracture mechanics we consider three different modes
- Mode I is known as the “opening mode”
- Mode II is known as the “sliding mode”
- Mode III is known as the “tearing mode”

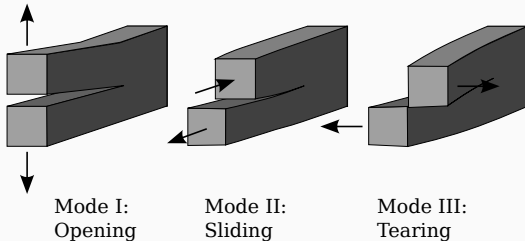


Figure 1: An image of the three fracture modes, with a representative crack in the xy plane. The first mode shows a crack opening vertically in the z -direction, like jaws opening. The second mode is known as the sliding mode, where one face moves into the

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stress intensity

- A key finding from Linear Elastic Fracture Mechanics (LEFM) is known as the *Stress Intensity Factor*
- The stress intensity factor is often written in this form

$$K = \sigma \sqrt{\pi a \beta}$$

- Where K is the stress intensity factor, σ is the applied stress, a is the crack length, and β is a dimensionless parameter depending on geometry

- Be careful that although the notation is similar, the *Stress Intensity Factor* is different from the *Stress Concentration Factor* from strength of materials
- We are usually most concerned with Mode I, but there will be a unique stress intensity factor for each mode, we label these K_I , K_{II} , and K_{III}
- If no subscript is given, assume Mode I

stress intensity

- For brittle materials (where “linear” fracture mechanics assumptions hold true) we can find the full stress field near the crack in terms of the stress intensity factor

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$

mode II

- Similarly for Mode II we find

$$\begin{aligned}\sigma_x &= \frac{-K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left(2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right) \\ \sigma_y &= \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \\ \tau_{xy} &= \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)\end{aligned}$$

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mode III

- And for Mode III

$$\begin{aligned}\tau_{xz} &= \frac{-K_{III}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \\ \tau_{yz} &= \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2}\end{aligned}$$

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common stress intensity factors

center crack, infinite width

$$K_I = \sigma \sqrt{\pi a}$$

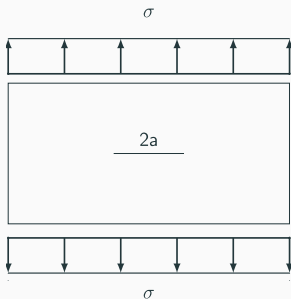


Figure 2: center crack, infinite width

center crack, finite width

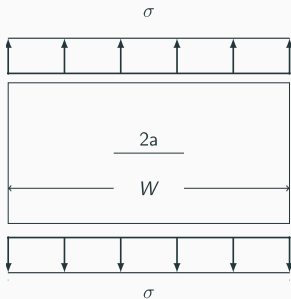


Figure 3: center crack, finite width

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center crack, finite width

$$K_I = \sigma \sqrt{\pi a} \sqrt{\sec(\pi a/W)}$$

- Accurate within 0.3% for $2a/W \leq 0.7$
- within 1.0% for $2a/W = .8$

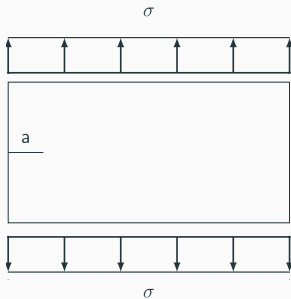
$$K_I = \sigma \sqrt{\pi a} \left[1.0 - 0.025 \left(\frac{2a}{W} \right)^2 + 0.06 \left(\frac{2a}{W} \right)^4 \right] \sqrt{\sec(\pi a/W)}$$

- Accurate within 0.1% for all crack lengths.

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edge crack, semi-infinite width

$$K_I = 1.122\sigma\sqrt{\pi a}$$



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Figure 4: edge crack, semi-infinite

edge crack, finite width

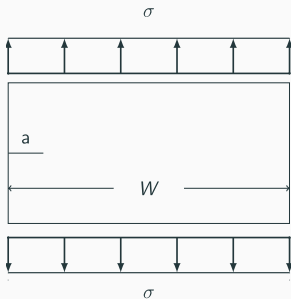


Figure 5: edge crack, finite width

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edge crack, finite width

$$\beta = \left[1.122 - 0.231 \frac{a}{W} + 10.55 \left(\frac{a}{W} \right)^2 - 21.71 \left(\frac{a}{W} \right)^3 + 30.82 \left(\frac{a}{W} \right)^4 \right]$$

- Within 0.5% accuracy for $\frac{a}{W} < 0.6$

$$\beta = \frac{0.752 + 2.02 \frac{a}{W} + 0.37 \left(1 - \sin \frac{\pi a}{2W} \right)^3}{\cos \frac{\pi a}{2W}} \sqrt{\frac{2W}{\pi a} \tan \frac{\pi a}{2W}}$$

- Within 0.5% accuracy for $0 < \frac{a}{W} < 1.0$

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edge crack, bending moment

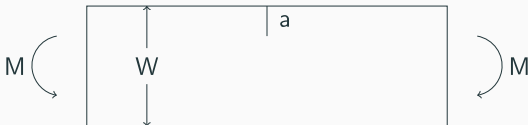


Figure 6: edge crack under bending

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edge crack, bending moment

- The usual form for stress intensity still applies

$$K_I = \sigma \sqrt{\pi a} \beta$$

- Where $\sigma = \frac{6M}{tW^2}$

$$\beta = 1.122 - 1.40 \left(\frac{a}{W} \right) + 7.33 \left(\frac{a}{W} \right)^2 - 13.08 \left(\frac{a}{W} \right)^3 + 14.0 \left(\frac{a}{W} \right)^4$$

- valid within 0.2% accuracy for $\frac{a}{W} \leq 0.6$

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edge crack, bending moment

$$\beta = \frac{0.923 + 0.199 \left(1 - \sin \frac{\pi a}{2W} \right)^4}{\cos \frac{\pi a}{2W}} \sqrt{\frac{2W}{\pi a} \tan \frac{\pi a}{2W}}$$

- valid within 0.5% for any $\frac{a}{W}$

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- The nominal bending stress is for rectangular cross-sections
- A more general form is given by $\sigma = \frac{Mc}{I}$
- Where for a rectangular cross-section, $c = W/2$ and $I = tW^3/12$ which simplifies as shown previously

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center crack, finite width, splitting forces

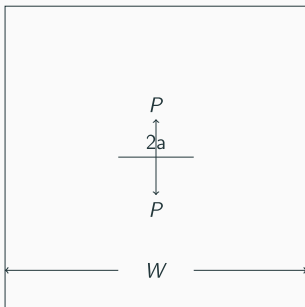


Figure 7: center crack, finite width, splitting forces

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- With an applied load we use a slightly modified form for the stress intensity factor $K_I = \frac{P}{t\sqrt{\pi a}}\beta$
- With β in this case given as

$$\beta = \frac{1 - 0.5 \left(\frac{a}{W}\right) + 0.975 \left(\frac{a}{W}\right)^2 - 0.16 \left(\frac{a}{W}\right)^3}{\sqrt{1 - \left(\frac{a}{W}\right)}}$$

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offset crack

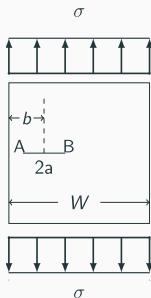


Figure 8: off-center crack

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$$K_{IA} = \sigma \sqrt{\pi a} \beta_c \beta_A \text{ and } K_{IB} = \sigma \sqrt{\pi a} \beta_c \beta_B$$

$$\beta_c = \sqrt{\sec \frac{\pi a}{W}}$$

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$$\beta_A = (1 - 0.025\lambda^2 + 0.6\lambda^4 - \gamma\lambda^{11})$$

$$\sqrt{\sec \left(\frac{\pi \lambda}{2} \right) \frac{\sin \left(2\lambda - 4 \frac{a}{W} \right)}{2\lambda - 4 \frac{a}{W}}}$$

$$\beta_B = (1 - 0.025\delta^2 + 0.06\delta^4 - \zeta\lambda^{30})$$

$$\left(1 + \frac{\sqrt{\sec \left(\frac{2\pi\lambda + 1.5\pi\delta}{7} \right) - 1}}{1 + 0.21 \sin \left(8 \tan^{-1} \left(\frac{\lambda - \delta}{\lambda + \delta} \right)^{0.9} \right)} \right)$$

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- The parameters λ , δ are given as

$$\lambda = \frac{a}{b}$$
$$\delta = \frac{a}{W - b}$$

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- And γ and ζ can be looked up on a table

$\frac{b}{W}$	γ	ζ
0.1	0.382	0.114
0.25	0.136	0.286
0.4	0.0	0.0
0.5	0.0	0.0

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non-uniform stress, infinite width

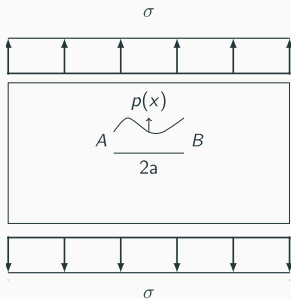


Figure 9: arbitrary pressure function loading along crack

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non-uniform stress, infinite width

- Stress intensity will be different at points A and B

$$K_{IA} = \int_{-a}^a \frac{p(x)}{\sqrt{\pi a}} \frac{\sqrt{a-x}}{\sqrt{a+x}} dx$$
$$K_{IB} = \int_{-a}^a \frac{p(x)}{\sqrt{\pi a}} \frac{\sqrt{a+x}}{\sqrt{a-x}} dx$$

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cracks around a hole

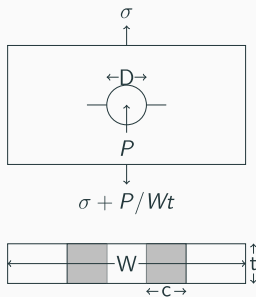


Figure 10: a crack around a hole under both remote stress and a local bearing load

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cracks around a hole

- For symmetric through cracks under uniform applied stress, we have

$$\beta = \beta_1 + \beta_2$$

$$\beta_1 = F_{c/R} F_w F_{ww}$$

$$\beta_2 = \frac{\sigma_{br}}{\sigma} F_3 F_w F_{ww}$$

$$F_{c/R} = \frac{3.404 + 3.8172 \frac{c}{R}}{1 + 3.9273 \frac{c}{R} - 0.00695 \left(\frac{c}{R} \right)^2}$$

$$F_w = \sqrt{\sec \frac{\pi R}{W} \sec \frac{\pi(R+c)}{W}}$$

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$$F_{ww} = 1 - \left(\left(1.32 \frac{W}{D} - 0.14 \right)^{-(.98 + (.1 \frac{W}{D})^{0.1})} - 0.02 \right) \left(\frac{2c}{W - D} \right)^N$$

$$F_3 = 0.098 + 0.3592e^{-3.5089 \frac{c}{R}} + 0.3817e^{-0.5515 \frac{c}{R}}$$

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- Note that

$$\sigma_{br} = \frac{P}{Dt}$$

$$N = \frac{W}{D} + 2.5 \quad \text{when} \quad \frac{W}{D} < 2$$

$$N = 4.5 \quad \text{otherwise}$$

- Also R is the radius, $R = \frac{D}{2}$

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cracks around a hole

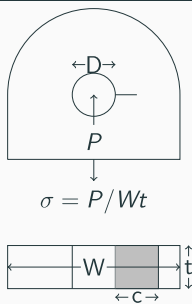


Figure 11: a crack around a hole under both remote stress and a local bearing load, but there is only a crack on one side

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cracks around a hole

$$\beta = \beta_1 + \beta_2$$

$$\beta_1 = \beta_3 F_w F_{ww}$$

$$\beta_2 = \frac{\sigma_{br}}{\sigma} F_4 F_w F_{ww}$$

$$\beta_3 = 0.7071 + 0.7548 \frac{R}{R+c} + 0.3415 \left(\frac{R}{R+C} \right)^2 +$$

$$0.6420 \left(\frac{R}{R+c} \right)^3 + 0.9196 \left(\frac{R}{R+c} \right)^4$$

$$F_4 = 0.9580 + 0.2561 \frac{c}{R} - 0.00193 \left(\frac{c}{R} \right)^{2.5} - 0.9804 \left(\frac{c}{R} \right)^{0.5}$$

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$$F_w = \sqrt{\sec \frac{\pi R}{W} \sec \frac{\pi(R + c/2)}{W - c}}$$
$$F_{ww} = 1 - N^{-\frac{W}{D}} \left(\frac{2c}{W - D} \right)^{\frac{W}{D} + 0.5}$$
$$N = 2.65 - 0.24 \left(2.75 - \frac{W}{D} \right)^2$$
$$N \geq 2.275 \quad (\text{if } N < 2.275, \text{ let } N = 2.275)$$

Also note that R indicates radius, $R = \frac{D}{2}$

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group problems

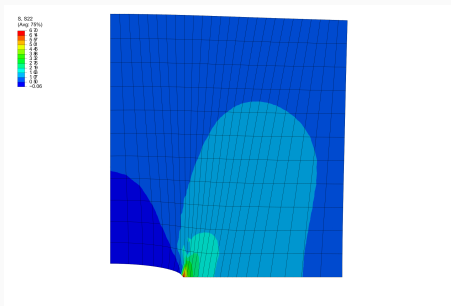
1. Find K_I for a center-cracked panel with $W/2a = 3$ and a uniformly applied remote stress, σ .
2. Find K_I for an edge-cracked panel with $W/a = 3$ and a uniformly applied remote stress, σ .
3. Find K_I for an edge-cracked panel with $W/a = 3$ and a remote bending moment, $M = tW^2\sigma/6$.

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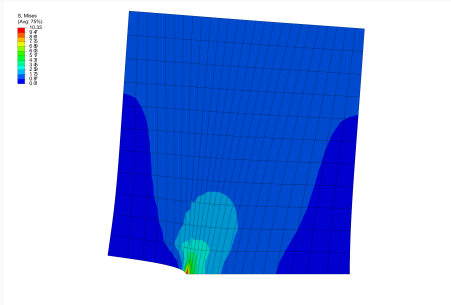
- Find K_I for a center-cracked panel with $W/2a = 3$ and a concentrated splitting force, $P = \sigma at$.
- What do you think causes the difference (if any) in stress intensity between these panels?

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example 1



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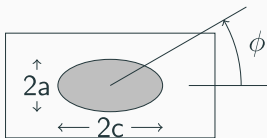
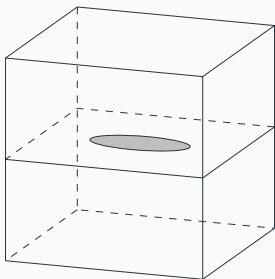
2D crack shapes

crack depth

- The previous stress intensity factors all assume a 2D problem (with a 1D crack)
- Through the thickness, it is assumed that the crack length is the same
- In many cases this is not an accurate assumption
- We will now consider 2D crack shapes and their effect on the stress intensity factor

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elliptical flaw, infinite solid



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- For an ellipse the stress intensity factor will vary with the angle, ϕ

$$K_I = \sigma \sqrt{\pi a} \beta$$

$$\beta = \sqrt{\frac{1}{Q}} f_\phi$$

$$Q = 1 + 1.464 \left(\frac{a}{c} \right)^{1.65} \quad \text{if } a/c \leq 1$$

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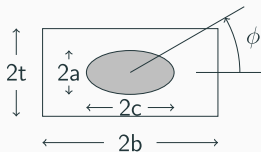
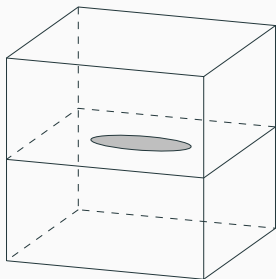
- For an ellipse the stress intensity factor will vary with the angle, ϕ

$$Q = 1 + 1.464 \left(\frac{c}{a} \right)^{1.65} \quad \text{if } a/c > 1$$

$$f_\phi = \left(\left(\frac{a}{c} \right)^2 \cos^2 \phi + \sin^2 \phi \right)^{1/4} \quad \text{if } a/c \leq 1$$

$$f_\phi = \left(\cos^2 \phi + \left(\frac{c}{a} \right)^2 \sin^2 \phi \right)^{1/4} \quad \text{if } a/c > 1$$

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finite solid

$$\beta = \sqrt{\frac{1}{Q}} F_e$$

$$F_e = \left(M_1 + M_2 \left(\frac{a}{t} \right)^2 + M_3 \left(\frac{a}{t} \right)^4 \right) g f_\phi f_w$$

$$f_w = \sqrt{\sec \left(\frac{\pi c}{2b} \sqrt{\frac{a}{t}} \right)}$$

$$g = 1 - \frac{\left(\frac{a}{t} \right)^4 \left(2.6 - 2 \frac{a}{t} \right)^{1/2}}{1 + 4 \frac{a}{c}} \cos \phi$$

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$$M_2 = \frac{0.05}{0.11 + \left(\frac{a}{c}\right)^{3/2}}$$

$$M_3 = \frac{0.29}{0.23 \left(\frac{a}{c}\right)^{3/2}}$$

- If $a/c \leq 1$

$$M_1 = 1$$

$$Q = 1 + 1.464 \left(\frac{a}{c}\right)^{1.65}$$

$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2 \phi + \sin^2 \phi \right)^{1/4}$$

- Otherwise ($a/c > 1$)

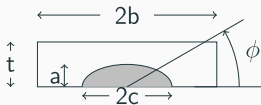
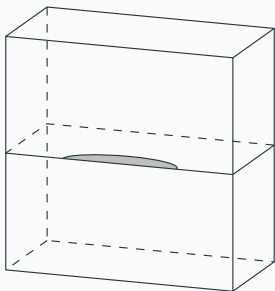
$$M_1 = \left(\frac{c}{a}\right)^{1/2}$$

$$Q = 1 + 1.464 \left(\frac{c}{a}\right)^{1.65}$$

$$f_\phi = \left(\cos^2 \phi + \left(\frac{c}{a}\right)^2 \sin^2 \phi \right)^{1/4}$$

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semi-elliptical surface flaw, finite body



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$$K_I = \sigma \sqrt{\pi a} \beta$$

$$\beta = \sqrt{\frac{1}{Q}} F_s$$

$$F_s = \left(M_1 + M_2 \left(\frac{a}{t} \right)^2 + M_3 \left(\frac{a}{t} \right)^4 \right) g f_\phi f_w$$

$$f_w = \sqrt{\sec \left(\frac{\pi c}{2b} \sqrt{\frac{a}{t}} \right)}$$

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surface flaw, $\frac{a}{c} \leq 1$

$$M_1 = 1.13 - 0.09 \left(\frac{a}{c} \right)$$

$$M_2 = -0.52 + \frac{0.89}{0.2 + \frac{a}{c}}$$

$$M_3 = 0.5 - \frac{1}{0.65 + \frac{a}{c}} + 14 \left(1 - \frac{a}{c} \right)^4$$

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surface flaw, $\frac{a}{c} \leq 1$

$$Q = 1 + 1.464 \left(\frac{a}{c} \right)^{1.65}$$

$$f_{\phi} = \left(\left(\frac{a}{c} \right)^2 \cos^2 \phi + \sin^2 \phi \right)^{1/4}$$

$$g = 1 + \left(0.1 + 0.35 \left(\frac{a}{t} \right)^2 \right) (1 - \sin \phi)^2$$

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surface flaw, $\frac{a}{c} > 1$

$$M_1 = \left(\frac{c}{a} \right)^{1/2} \left(1 + 0.04 \frac{c}{a} \right)$$

$$M_2 = 0.2 \left(\frac{c}{a} \right)^4$$

$$M_3 = -0.11 \left(\frac{c}{a} \right)^4$$

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surface flaw, $\frac{a}{c} > 1$

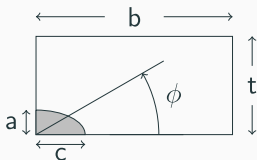
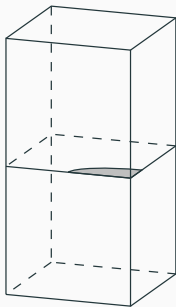
$$Q = 1 + 1.464 \left(\frac{c}{a} \right)^{1.65}$$

$$f_{\phi} = \left(\cos^2 \phi + \left(\frac{c}{a} \right)^2 \sin^2 \phi \right)^{1/4}$$

$$g = 1 + \left(0.1 + 0.35 \left(\frac{c}{a} \right) \left(\frac{a}{t} \right)^2 \right) (1 - \sin \phi)^2$$

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corner flaw, finite body



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$$K_I = \sigma \sqrt{\pi a} \beta$$

$$\beta = \sqrt{\frac{1}{Q}} F_c$$

$$F_c = \left(M_1 + M_2 \left(\frac{a}{t} \right)^2 + M_3 \left(\frac{a}{t} \right)^4 \right) g_1 g_2 f_\phi f_w$$

$$f_w = 1 - 0.2\lambda + 9.4\lambda^2 - 19.4\lambda^3 + 27.1\lambda^4$$

$$\lambda = \left(\frac{c}{b} \right) \left(\frac{a}{t} \right)^{1/2}$$

$$M_1 = 1.08 - 0.03 \left(\frac{a}{c} \right)$$

$$M_2 = -0.44 + \frac{1.06}{0.3 + \frac{a}{c}}$$

$$M_3 = -0.5 + 0.25 \frac{a}{c} + 14.8 \left(1 - \frac{a}{c} \right)^{1.5}$$

corner flaw, finite body, $\frac{a}{c} \leq 1$

$$Q = 1 + 1.464 \left(\frac{a}{c} \right)^{1.65}$$

$$f_{\phi} = \left(\left(\frac{a}{c} \right)^2 \cos^2 \phi + \sin^2 \phi \right)^{1/4}$$

$$g_1 = 1 + \left(0.08 + 0.4 \left(\frac{a}{t} \right)^2 \right) (1 - \sin \phi)^3$$

$$g_2 = 1 + \left(0.08 + 0.15 \left(\frac{a}{t} \right)^2 \right) (1 - \cos \phi)^3$$

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corner flaw, finite body, $\frac{a}{c} > 1$

$$M_1 = \left(\frac{c}{a} \right)^{1/2} \left(1.08 - 0.03 \frac{c}{a} \right)$$

$$M_2 = 0.375 \left(\frac{c}{a} \right)^4$$

$$M_3 = -0.25 \left(\frac{c}{a} \right)^2$$

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$$Q = 1 + 1.464 \left(\frac{c}{a} \right)^{1.65}$$

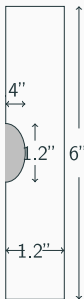
$$f_{\phi} = \left(\cos^2 \phi + \left(\frac{c}{a} \right)^2 \sin^2 \phi \right)^{1/4}$$

$$g_1 = 1 + \left(0.08 + 0.4 \left(\frac{c}{t} \right)^2 \right) (1 - \sin \phi)^3$$

$$g_2 = 1 + \left(0.08 + 0.15 \left(\frac{c}{t} \right)^2 \right) (1 - \cos \phi)^3$$

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example 2



- Find maximum value of K_I for semi-elliptical surface flaw
- $\sigma = 20$ kpsi (in opening direction)

Figure 12: A surface flaw shown with a major diameter of 1.2 inches and a minor radius (the semi-elliptical axis) of 0.4 inches

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- Here we will use the formula for a semi-elliptical surface flaw
- In the first step we find $a/c = 0.4/0.6 < 1$, so we use that set of formulae
- A worked python notebook of this example can be found here¹

¹<https://colab.research.google.com/drive/11i24jBHUGPautBloU1FgGGBJN-y2HqDJ?usp=sharing>

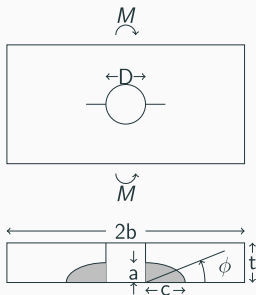
2D cracks at a hole

when to consider 2D crack shape

- When do we need to worry about 2D crack shape?
- The important factor is ratio of crack length to thickness
- When crack length is less than 5 times thickness, 2D shape effects are not negligible

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cracks around a hole



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$$K_I = \sigma \sqrt{\pi a} \beta$$

$$\beta = \sqrt{\frac{1}{Q}} F_{ch}$$

$$F_{ch} = \left(M_1 + M_2 \left(\frac{a}{t} \right)^2 + M_3 \left(\frac{a}{t} \right)^4 \right) g_1 g_2 g_3 g_4 f_\phi f_w$$

$$f_w = \sqrt{\sec \left(\frac{\pi r}{2b} \right) \sec \left(\frac{\pi(2r + nc)}{4(b - c) + 2nc} \sqrt{\frac{a}{t}} \right)}$$

$$g_2 = \frac{1 + 0.358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4}{1 + 0.13\lambda^2}$$

$$\lambda = \frac{1}{1 + (c/r) \cos(0.85\phi)}$$

Where n = number of cracks (1 or 2)

remote stress when $a/c \leq 1$

$$M_1 = 1.13 - 0.09(a/c)$$

$$M_2 = -0.54 + \frac{0.89}{0.2 + a/c}$$

$$M_3 = 0.5 - \frac{1}{0.65 + a/c} + 14(1 - a/c)^{24}$$

$$Q = 1 + 1.464(a/c)^{1.65}$$

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remote stress when $a/c \leq 1$

$$g_1 = 1 + (0.1 + 0.35(a/t)^2)(1 - \sin \phi)^2$$

$$g_3 = (1 + 0.04(a/c))(1 + 0.1(1 - \cos \phi)^2)(0.85 + 0.15(a/t)^{1/4})$$

$$g_4 = 1 - 0.7(1 - a/t)(a/c - 0.2)(1 - a/c)$$

$$f_\phi = ((a/c)^2 \cos^2 \phi + \sin^2 \phi)^{1/4}$$

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$$M_1 = \sqrt{c/a}(1 + 0.04(c/a))$$

$$M_2 = 0.2(c/a)^4$$

$$M_3 = -0.11(c/a)^4$$

$$Q = 1 + 1.464 \left(\frac{c}{a}\right)^{1.65}$$

$$g_1 = 1 + \left(0.1 + 0.35(c/a)(a/t)^2\right)(1 - \sin \phi)^2$$

$$g_3 = (1.13 - 0.09(c/a)) \left(1 + 0.1(1 - \cos \phi)^2\right) (0.85 + 0.15(a/t)^{1/4})$$

$$g_4 = 1$$

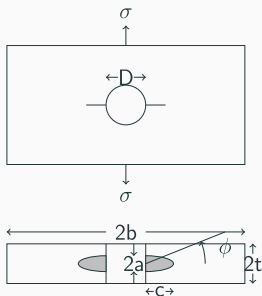
$$f_\phi = \left(\cos^2 \phi + \left(\frac{c}{a}\right)^2 \sin^2 \phi\right)^{1/4}$$

- The same formulas apply for both symmetric cracks ($n = 2$) and a single crack ($n = 1$) with one additional correction factor applied to the single crack case

$$K_{I,single} = \sqrt{\frac{4/\pi + ac/2tr}{4/\pi + ac/tr}} K_{I,symmetric}$$

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surface cracks around a hole



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$$K_I = \sigma \sqrt{\pi a} \beta$$

$$\beta = \sqrt{\frac{1}{Q}} F_{sh}$$

$$F_{sh} = \left(M_1 + M_2 \left(\frac{a}{t} \right)^2 + M_3 \left(\frac{a}{t} \right)^4 \right) g_1 g_2 g_3 f_\phi f_w$$

$$f_w = \sqrt{\sec \left(\frac{\pi r}{2b} \right) \sec \left(\frac{\pi(2r + nc)}{4(b - c) + 2nc} \sqrt{\frac{a}{t}} \right)}$$

$$M_2 = \frac{0.05}{0.11 + (a/c)^{3/2}}$$

$$M_3 = \frac{0.29}{0.23 + (a/c)^{3/2}}$$

Where n = number of cracks (1 or 2)

$$g_1 = 1 - \frac{(a/t)^4(2.6 - 2a/t)^{1/2}}{1 + 4a/c} \cos \phi$$

$$g_2 = \frac{1 + 0.358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4}{1 + 0.08\lambda^2}$$

$$\lambda = \frac{1}{1 + (c/r) \cos(0.9\phi)}$$

$$g_3 = 1 + 0.1(1 - \cos \phi)^2(1 - a/t)^{10}$$

$$Q = 1 + 1.464(a/c)^{1.65}$$

$$M_1 = 1$$

$$f_\phi = \left(\left(\frac{a}{c} \right)^2 \cos^2 \phi + \sin^2 \phi \right)^{1/4}$$

$$Q = 1 + 1.464(c/a)^{1.65}$$

$$M_1 = \sqrt{c/a}$$

$$f_\phi = \left(\cos^2 \phi + \left(\frac{c}{a} \right)^2 \sin^2 \phi \right)^{1/4}$$

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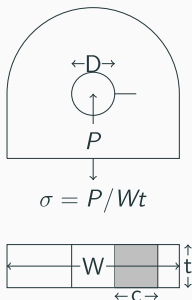
single-crack correction

- When the surface crack is only on one side of the hole, we use the same correction as for corner cracks

$$K_{I,single} = \sqrt{\frac{4/\pi + ac/2tr}{4/\pi + ac/tr}} K_{I,symmetric}$$

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edge crack on a lug



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edge crack on a lug

$$K_I = \sigma_{br} \sqrt{\pi C} \beta$$

$$\sigma_{br} = P/Dt$$

$$\beta = \left(\frac{G_0 D}{2W} + G_1 \right) G_w G_L G_2$$

$$z = \left(1 + \frac{2C}{D} \right)^{-1}$$

$$G_0 = 0.7071 + 0.7548z + 0.3415z^2 + 0.642z^3 + 0.9196z^4$$

$$G_1 = 0.078z + 0.7588z^2 - 0.4293z^3 + 0.0644z^4 + 0.651z^5$$

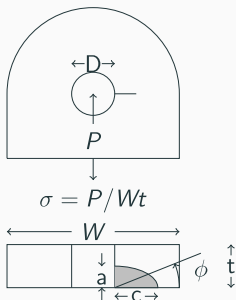
$$G_L = \left(\sec \left(\frac{\pi D}{2W} \right) \right)^{1/2}$$

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$$\begin{aligned}\lambda &= \frac{\pi}{2} \left(\frac{D+c}{W-c} \right) \\ G_w &= (\sec \lambda)^{1/2} \\ b &= \frac{W-D}{2} \\ A_1 &= 0.688 + 0.772 \frac{D}{W} + 0.613 \left(\frac{D}{W} \right)^2 \\ A_2 &= 4.948 - 17.318 \frac{D}{W} + 16.785 \left(\frac{D}{W} \right)^2\end{aligned}$$

$$\begin{aligned}A_3 &= -14.297 + 62.994 \frac{D}{W} - 69.818 \left(\frac{D}{W} \right)^2 \\ A_4 &= 12.35 - 58.644 \frac{D}{W} + 66.387 \left(\frac{D}{W} \right)^2 \\ G_2 &= A_1 + A_2 \frac{c}{b} + A_3 \left(\frac{c}{b} \right)^2 + A_4 \left(\frac{c}{b} \right)^3\end{aligned}$$

corner crack on a lug



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corner crack on a lug

$$\beta = \left(\frac{G_0 D}{2W} + G_1 \right) G_w$$

$$z = \left(1 + 2 \frac{c}{D} \cos(0.85\phi) \right)^{-1}$$

$$f_0(z) = 0.7071 + 0.7548z + 0.3415z^2 + 0.642z^3 + 0.9196z^4$$

$$f_1(z) = 0.078z + 0.7588z^2 - 0.4293z^3 + 0.0644z^4 + 0.651z^5$$

$$G_0 = \frac{f_0(z)}{d_0}$$

$$d_0 = 1 + 0.13z^2$$

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$$g_p = \left(\frac{W + D}{W - D} \right)^{1/2}$$

$$G_1 = f_1(z) \left(\frac{g_p}{d_0} \right)$$

$$G_w = M_0 g_1 g_3 g_4 f_\phi f_w f_x$$

$$v = \frac{a}{t}$$

$$\lambda = \frac{\pi}{2} \sqrt{v} \left(\frac{D + c}{W - c} \right)$$

$$f_w = \left(\sec \lambda \sec \frac{\pi D}{2W} \right)^{1/2}$$

$$x = \frac{a}{c}$$

corner crack on a lug $a/c \leq 1$

$$f_{\phi} = \left(\left(\frac{a}{c} \cos \phi \right)^2 + \sin^2 \phi \right)^{1/4}$$

$$f_x = \left(1 + 1.464 \left(\frac{a}{c} \right)^{1.65} \right)^{-1/2}$$

$$M_0 = (1.13 - 0.09x) + \left(-0.54 + \frac{0.89}{0.2 + x} \right) v^2 + \left(0.5 - \frac{1}{.65 - x} + 14(1 - x^{24}) \right) v^4$$

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corner crack on a lug $a/c \leq 1$

$$g_1 = 1 + (0.1 + 0.35v^2) (1 - \sin \phi)^2$$

$$g_3 = (1 + 0.04x) (1 + 0.1(1 - \cos \phi)^2) (0.85 + 0.15v^{1/4})$$

$$g_4 = 1 - 0.7(1 - v)(x - 0.2)(1 - x)$$

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corner crack on a lug $a/c > 1$

$$f_{\phi} = \left(\left(\frac{ac}{c} \sin \phi \right)^2 + \cos^2 \phi \right)^{1/4}$$

$$f_x = \left(1 + 1.464 \left(\frac{c}{a} \right)^{1.65} \right)^{-1/2}$$

$$M_0 = x^{-1/2} + 0.04x^{-3/2} + 0.2x^{-4}v^2 - 0.11x^{-4}v^4$$

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corner crack on a lug $a/c > 1$

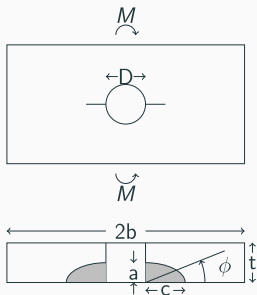
$$g_1 = 1 + \left(0.1 + \frac{0.35}{x} v^2 \right) (1 - \sin \phi)^2$$

$$g_3 = \left(1.13 + \frac{0.09}{x} \right) (1 + 0.1(1 - \cos \phi)^2) (0.85 + 0.15v^{1/4})$$

$$g_4 = 1$$

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symmetric corner cracks under bending



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corner cracks under bending

$$\sigma_b = \frac{Mt}{2I}$$

$$I = \frac{bt^3}{6}$$

$$\beta = H_{ch} \left(\frac{a}{cQ} \right)^{1/2} F_{ch}$$

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$$H_{ch} = H_1 + (H_2 - H_1) \sin^p \phi$$

$$H_1 = 1 + G_{11}(a/t) + G_{12}(a/t)^2 + G_{13}(a/t)^3$$

$$H_2 = 1 + G_{21}(a/t) + G_{22}(a/t)^2 + G_{23}(a/t)^3$$

$$F_{ch} = \left(M_1 + M_2(a/t)^2 + M_3(a/t)^4 \right) g_1 g_2 g_3 g_4 f_\phi f_w$$

$$\lambda = \frac{1}{1 + (c/r) \cos(0.85\phi)}$$

$$g_2 = \frac{1 + .358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4}{1 + 0.13\lambda^2}$$

corner cracks under bending $a/c \leq 1$

$$M_1 = 1.13 - 0.09(a/c)$$

$$M_2 = -0.54 + \frac{0.89}{0.2 + a/c}$$

$$M_3 = 0.5 - \frac{1}{0.65 + a/c} + 14(1 - a/c)^4$$

$$Q = 1 + 1.464(a/c)^{1.65}$$

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corner cracks under bending $a/c \leq 1$

$$g_1 = 1 + (0.1 + (a/t)v^2) (1 - \sin \phi)^2$$

$$g_3 = (1 + 0.04(a/c)) (1 + 0.1(1 - \cos \phi)^2) (0.85 + 0.15(a/t)^{1/4})$$

$$g_4 = 1 - 0.7(1 - a/t)(a/c - 0.2)(1 - a/c)$$

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$$f_{\phi} = \left(\left(\frac{a}{c} \cos \phi \right)^2 + \sin^2 \phi \right)^{1/4}$$

$$G_{11} = -0.43 - 0.74a/c - 0.84(a/c)^2$$

$$G_{12} = 1.25 - 1.19a/c + 4.39(a/c)^2$$

$$G_{13} = -1.94 + 4.22a/c - 5.51(a/c)^2$$

$$G_{21} = -1.5 - 0.04a/c - 1.73(a/c)^2$$

$$G_{22} = 1.71 - 3.17a/c + 6.84(a/c)^2$$

$$G_{23} = -1.28 + 2.71a/c - 5.22(a/c)^2$$

$$p = 0.1 + 1.3a/t + 1.1a/c - 0.7(a/c)(a/t)$$

corner cracks under bending $a/c > 1$

$$M_1 = (c/a)^{1/2}(1 + 0.04c/a)$$

$$M_2 = 0.2(c/a)^4$$

$$M_3 = -0.11(c/a)^4$$

$$Q = 1 + 1.464(c/a)^{1.65}$$

$$g_1 = 1 + (0.10.35(c/a)(a/t)^2)(1 - \sin \phi)^2$$

$$g_3 = (1.13 - 0.09(c/a))(1 + 0.1(1 - \cos \phi)^2)(0.85 + 0.15(a/t)^{1/4})$$

$$g_4 = 1$$

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corner cracks under bending

$$f_\phi = \left(\left(\cos^2 \phi + \frac{c}{a} \sin \phi \right)^2 \right)^{1/4}$$

$$G_{11} = -2.07 + 0.06c/a$$

$$G_{12} = 4.35 + 0.16c/a$$

$$G_{13} = -2.93 - 0.3c/a$$

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$$G_{21} = -3.64 + 0.37c/a$$

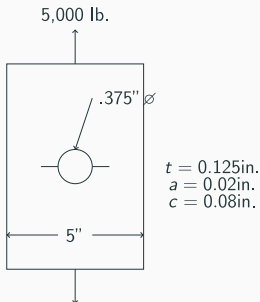
$$G_{22} = 5.87 - 0.49c/a$$

$$G_{23} = -4.32 + 0.53c/a$$

$$p = 0.2 + c/a + 0.6a/t$$

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example 3



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- Case 1 - symmetric through cracks
- Case 2 - single through crack
- Case 3 - symmetric corner cracks
- Case 4 - single corner crack
- Case 5 - symmetric surface cracks
- Case 6 - single surface crack
- Viewable here²

²<https://colab.research.google.com/drive/1fml1vs1Rpwn9BkXPz-8FV6-lqHnVvQu0?usp=sharing>