AE 737: Mechanics of Damage Tolerance

Lecture 4 - Curved Boundaries

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

January 30, 2020

schedule

- 30 Jan - Curved Boundaries, Homework 1 Due
- 4 Feb - Plastic Zone
- 6 Feb - Plastic Zone, Homework 2 Due
- 11 Feb - Fracture Toughness

outline

- compounding
- curved boundaries
- stress concentration factors

supplemental material

- I was unable to find the source for all of Dr. Horn's formulas, but I have made an alternate set of equations (taken from the AFGROW user's manual) available on Blackboard under supplemental material.
- Can also be found here

compounding

superposition vs. compounding

- In this course, we use *superposition* to combine loading conditions
- We use *compounding* to combine edge effects
- Both are very powerful tools and important concepts

compounding

- Different types of boundaries create different correction factors to the usual stress intensity factor
- We often use β to indicate the total correction factor
- When multiple boundaries are present, we can combine them into one effective correction factor
- There are two general methods we use to create a compound correction factor

method 1

- The first method uses linear superposition, and thus is restricted to cases where the effect of each boundary can be assumed to add linearly
- While in most cases this is not strictly true, it provides a reasonable approximation

$$K_r = \bar{K} + \sum_{i=1}^{N} (K_i - \bar{K})$$

method 1

- Where N is the number of boundaries, $\bar{K}$ is the stress intensity factor with no boundaries present and K_i is the stress intensity factor associated with the i^{th} boundary.

method 1

- We can rewrite this equation as

$$K_r = \sigma\sqrt{\pi a}\beta_r = \sigma\sqrt{\pi a} + \sum_{i=1}^{N}(\sigma\sqrt{\pi a}\beta_i - \sigma\sqrt{\pi a})$$

- Which leads to an expression for β_r as

$$\beta_r = 1 + \sum_{i=1}^{N}(\beta_i - 1)$$

method 2

- An alternative empirical method approximates the boundary effect as

$$\beta_r = \beta_1 \beta_2 \ldots \beta_N$$

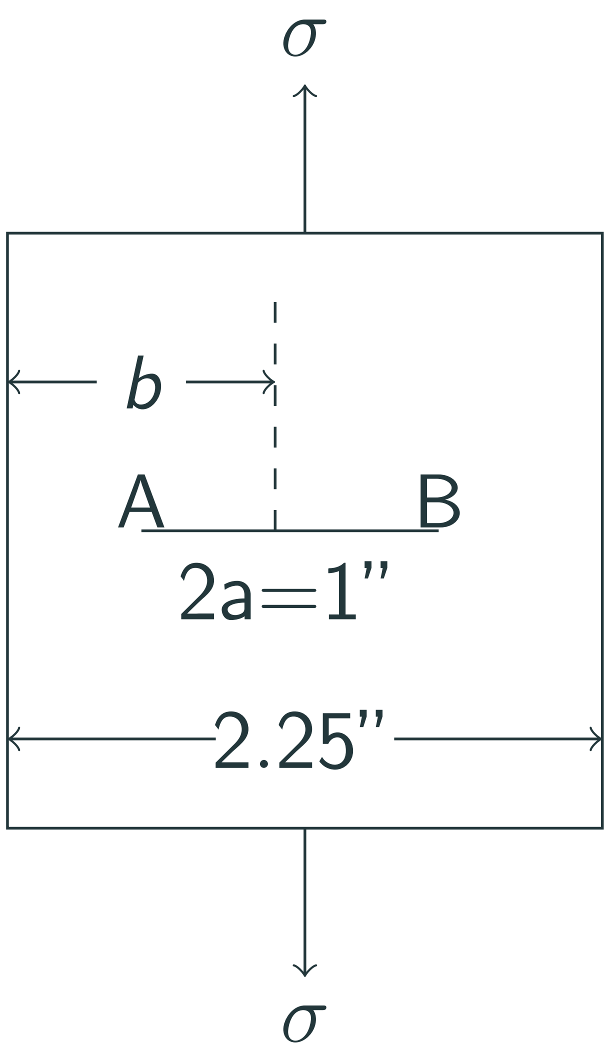
- If there is no interaction between the boundaries, method 1 and method 2 will give the same result

p. 68 - example 1

- A crack in a finite-width panel is centered between two stiffeners
- Assume the β correction factor for this stiffener configuration is β_s = 0.9
- Assume the β correction factor for this finite-width panel is β_w = 1.075
- Use both compounding methods to estimate the stress intensity
- How accurate do you expect this to be?

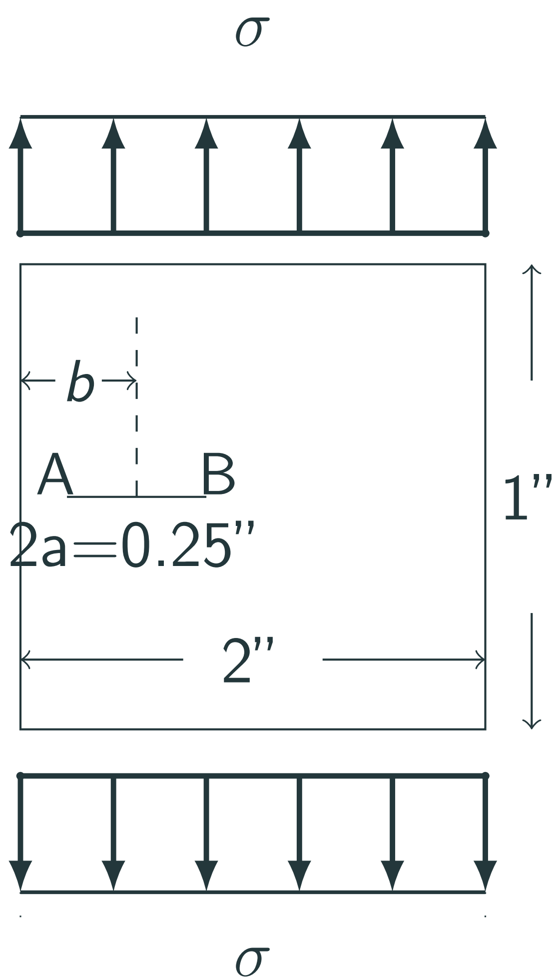
p. 69 - example 3

$b = 1$ inch

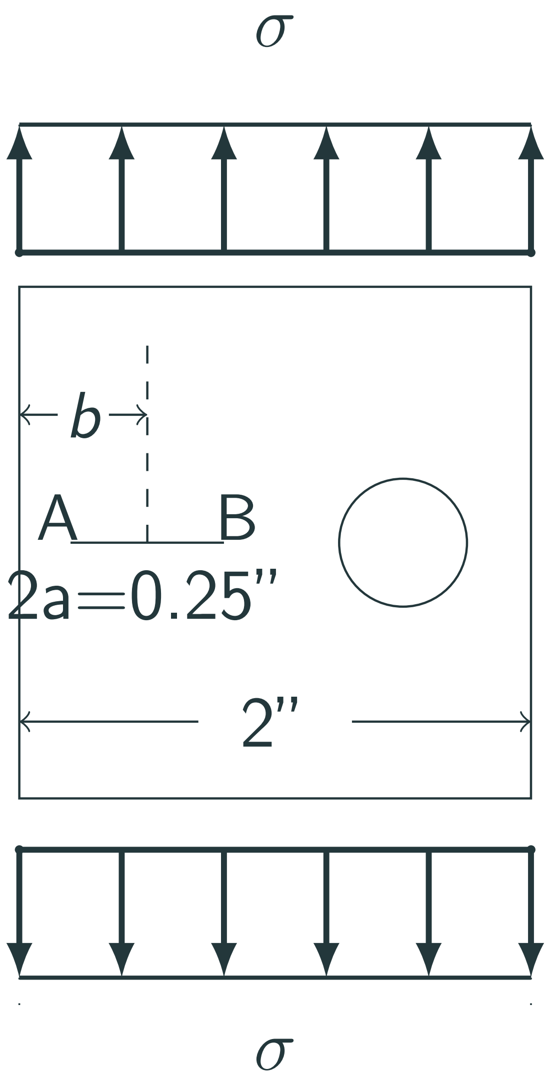


group 1

$$b = 0.4 \text{ inches}$$



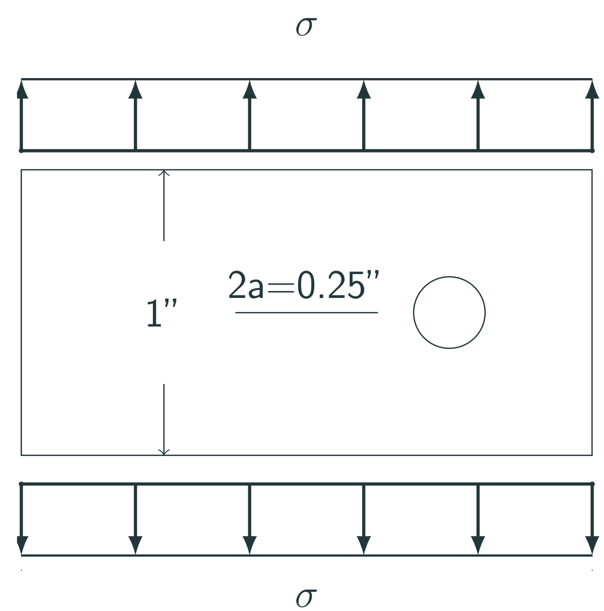
group 2



$b = 0.4$ inches

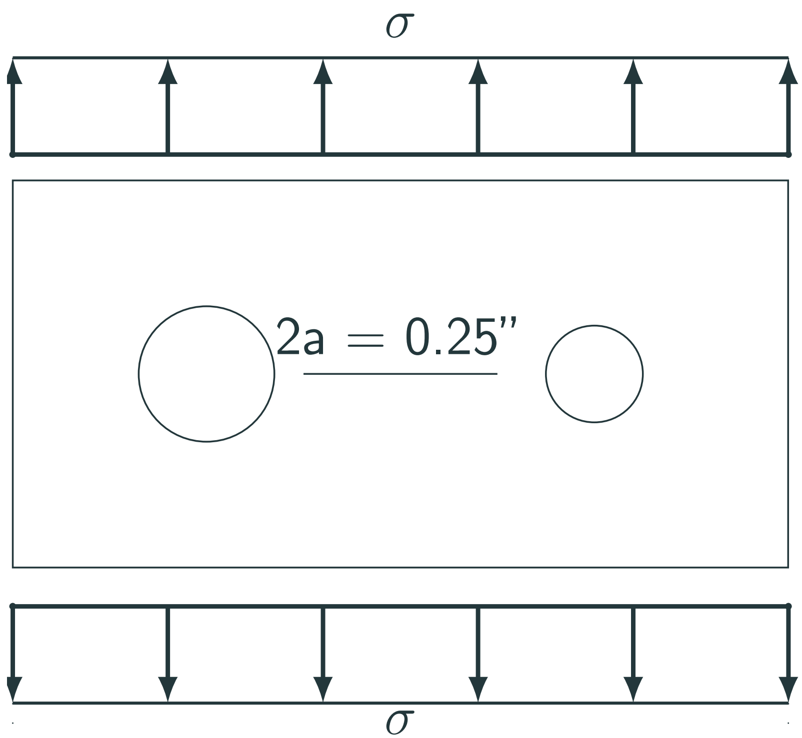
Hole diameter is 0.5 inches and spaced 0.5 inches away from the crack tip

group 3



Hole diameter is 0.5 inches and spaced 0.5 inches away from the crack tip

group 4



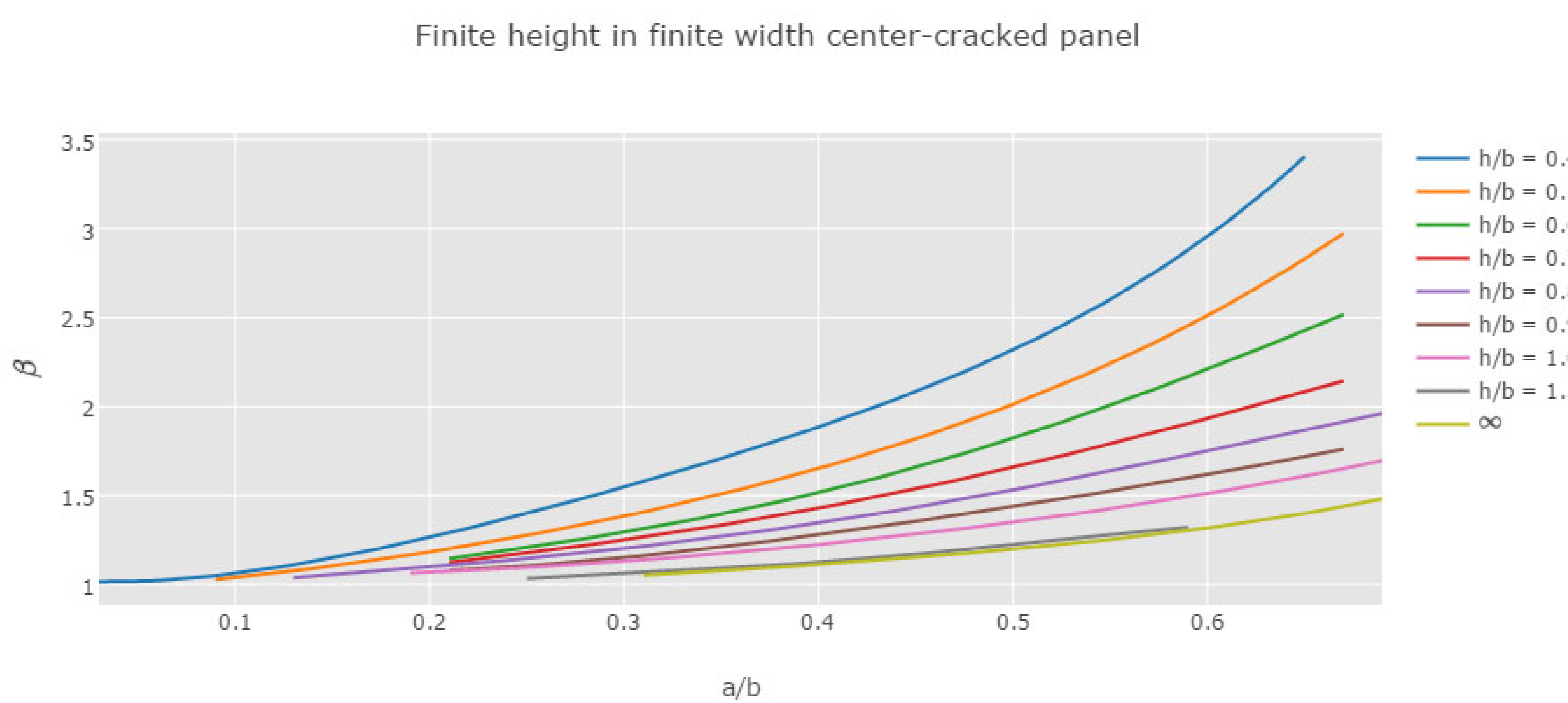
The right crack tip is 0.5 inches away from a 0.5 inch diameter hole and the left crack tip is 0.25 inches away from a 1 inch diameter hole.

errata and supplemental charts

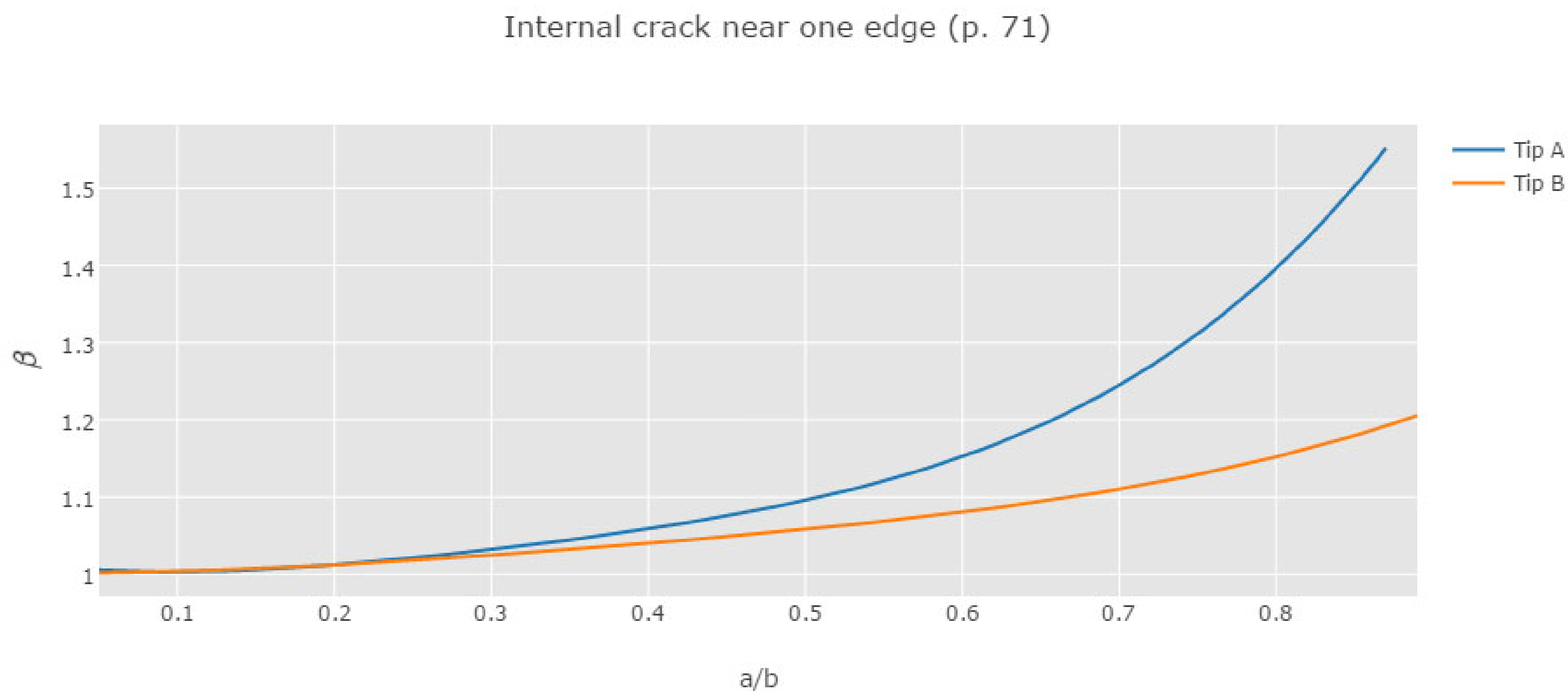
textbook notes

- on p. 64 there is a + missing between two terms, see Lecture 2 for the fix
- Also on p. 64, in equation 29 it is not clear, but use the f_w from a previous equation, on p. 56
- Some of the black and white figures can be difficult to use, we have scanned and re-created the plots online
- Interactive versions of compounding figures from p. 50, 71-73 can be found at here

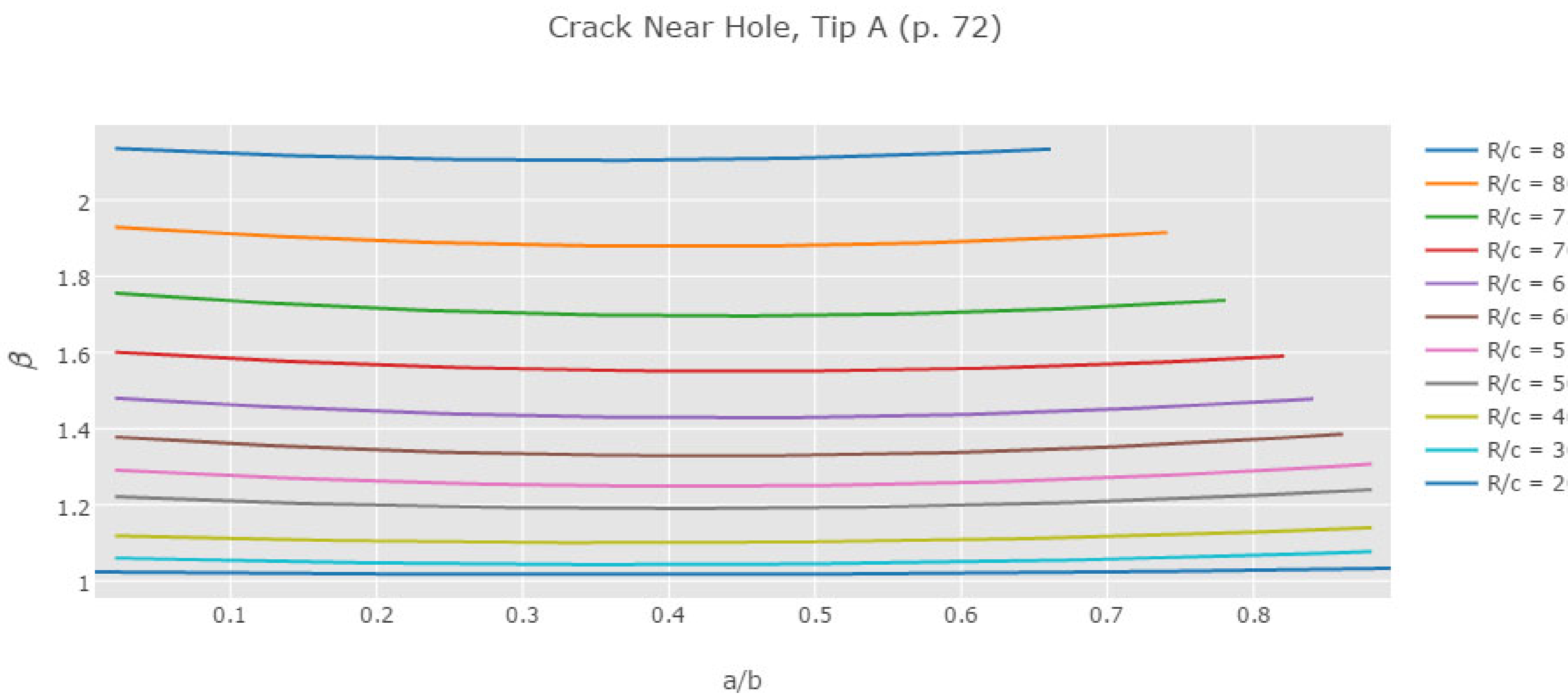
finite height - p. 50



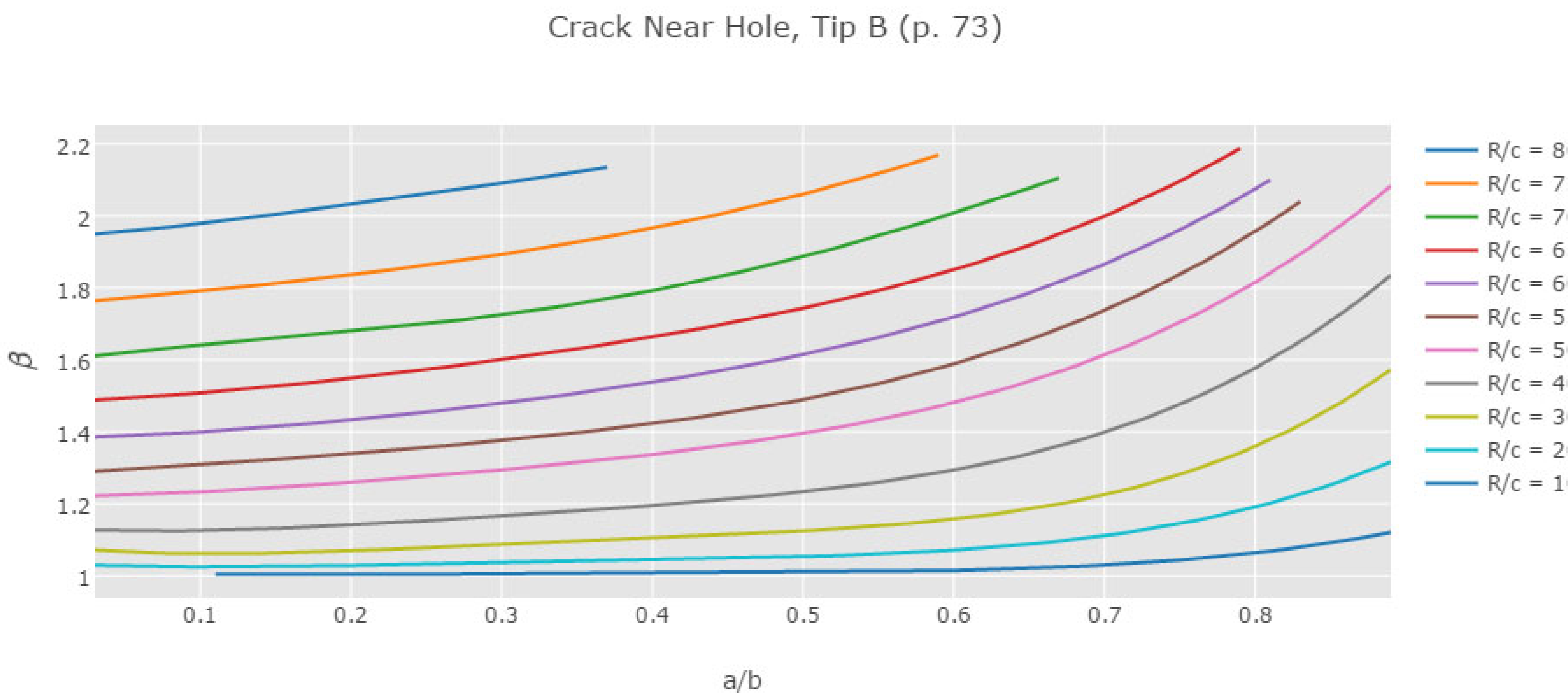
offset crack - p. 71



crack near hole - p. 72



crack near hole - p. 73



curved boundaries

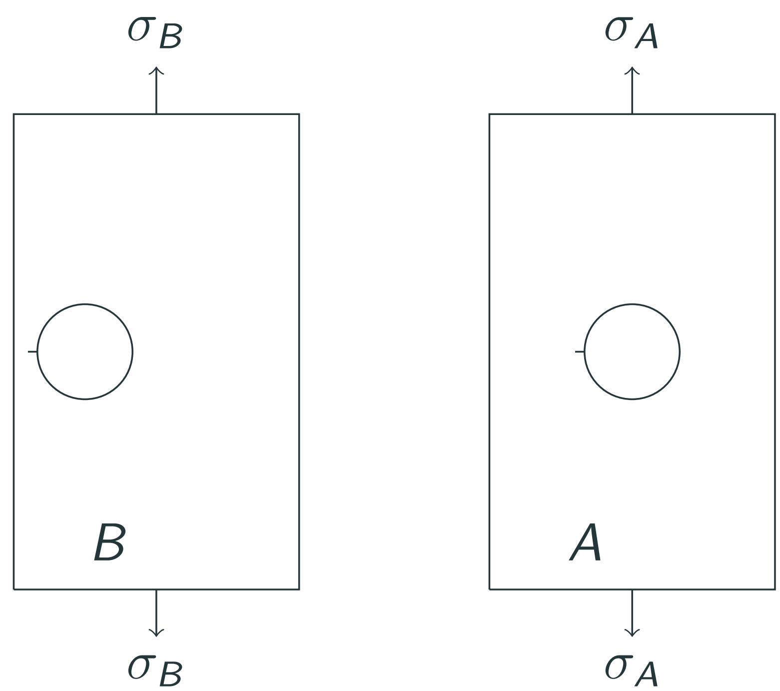
short cracks on curved boundaries

- For short cracks, we can use the *stress concentration factor* on a curved boundary to determine the stress intensity factor
- The stress concentration factor only gives the maximum stress at the curved boundary, thus the longer the crack is, the farther away from the curved boundary (and maximum stress) it is.
- Stress concentration factors can be found: pp. 82-85 in the text
- Also see supplemental text on Blackboard or here

short cracks on curved boundaries

- Suppose we want to determine the stress intensity on a panel, panel B
- We find a similar panel with a known stress intensity factor, panel A
- We adjust the applied load on panel A such that $K_{I,A}=K_{I,B}$
- The magnitude of this load adjustment is determined using the *stress concentration factors* in panels B and A

short cracks on curved boundaries



short cracks on curved boundaries

- Since A is a fictional panel, we set the applied stress, σ_A such that

$$\sigma_{max,B} = \sigma_{max,A}$$

- Substituting stress concentration factors

$$K_{t,B}\sigma_B = K_{t,A}\sigma_A$$

- Solving for σ_A

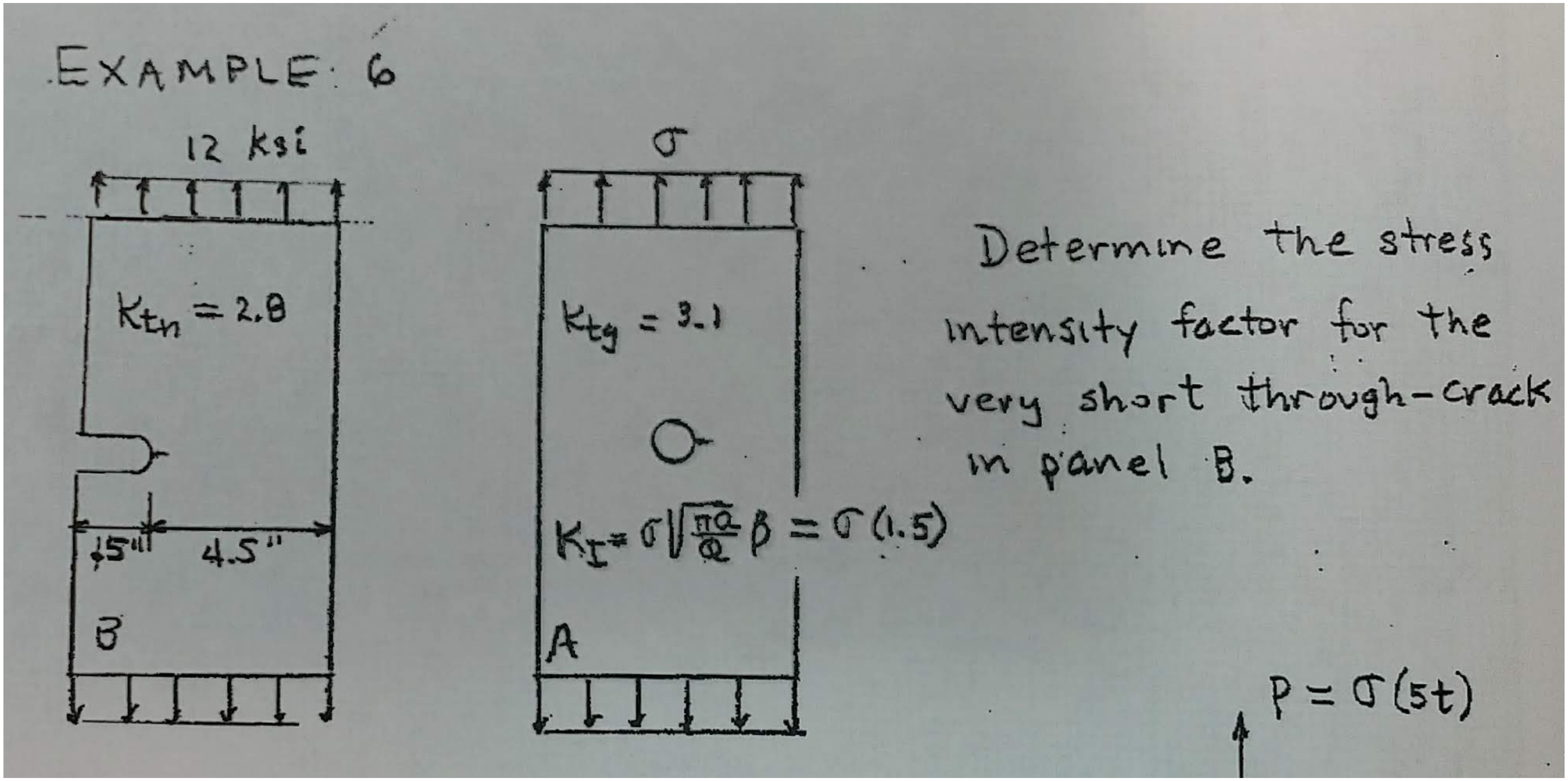
$$\sigma_A = \frac{K_{t,B}}{K_{t,A}}\sigma_B$$

short cracks on curved boundaries

- Since the crack is short and $\sigma_{max,A} = \sigma_{max,B}$ we can say

$$
\begin{aligned}
K_{I,B} &= K_{I,A} \\
&= \sigma_A \sqrt{\pi c} \beta_A \\
&= \frac{K_{t,B}}{K_{t,A}} \sigma_B \sqrt{\pi c} \beta_A
\end{aligned}
$$

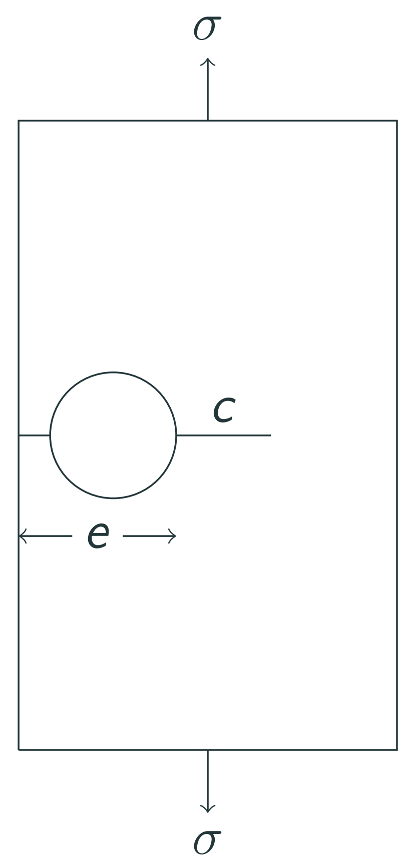
example 6 (p. 86)



long cracks on curved boundaries

- As a crack becomes very large, the effect of the curved boundary diminishes
- We find expressions for β_L (long crack) and β_S (short crack)
- We connect β_S to β_L using a straight line from β_S to a tangent intersection with β_L

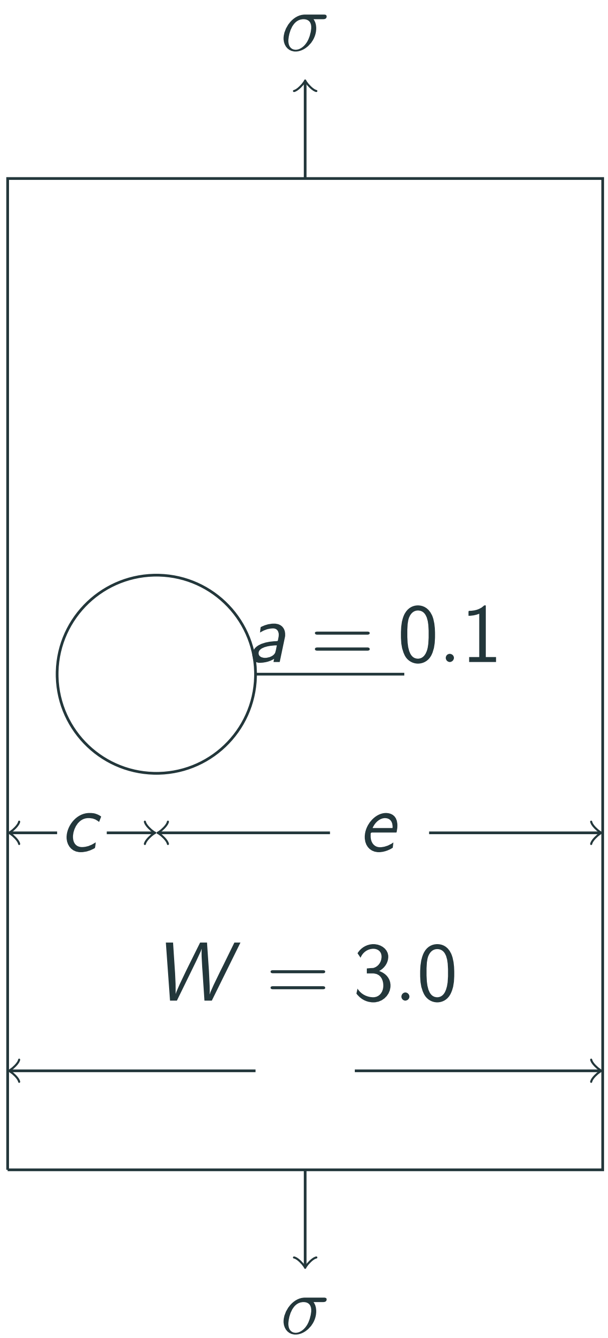
long cracks on curved boundaries



example

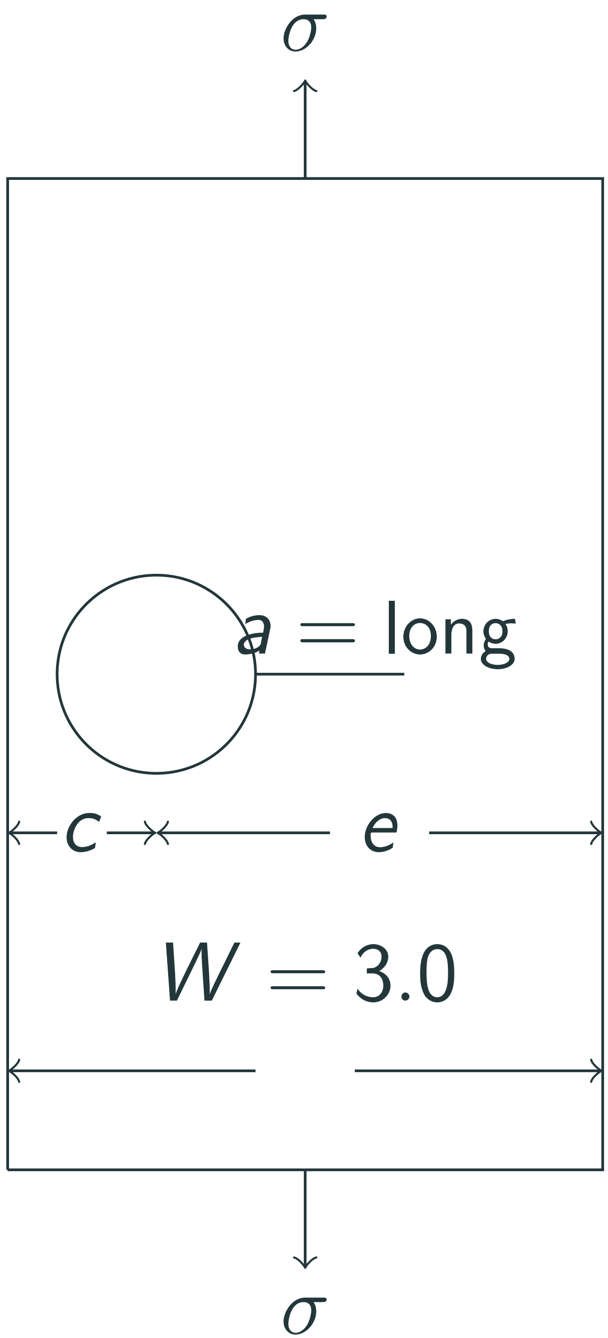
- Example here

group one



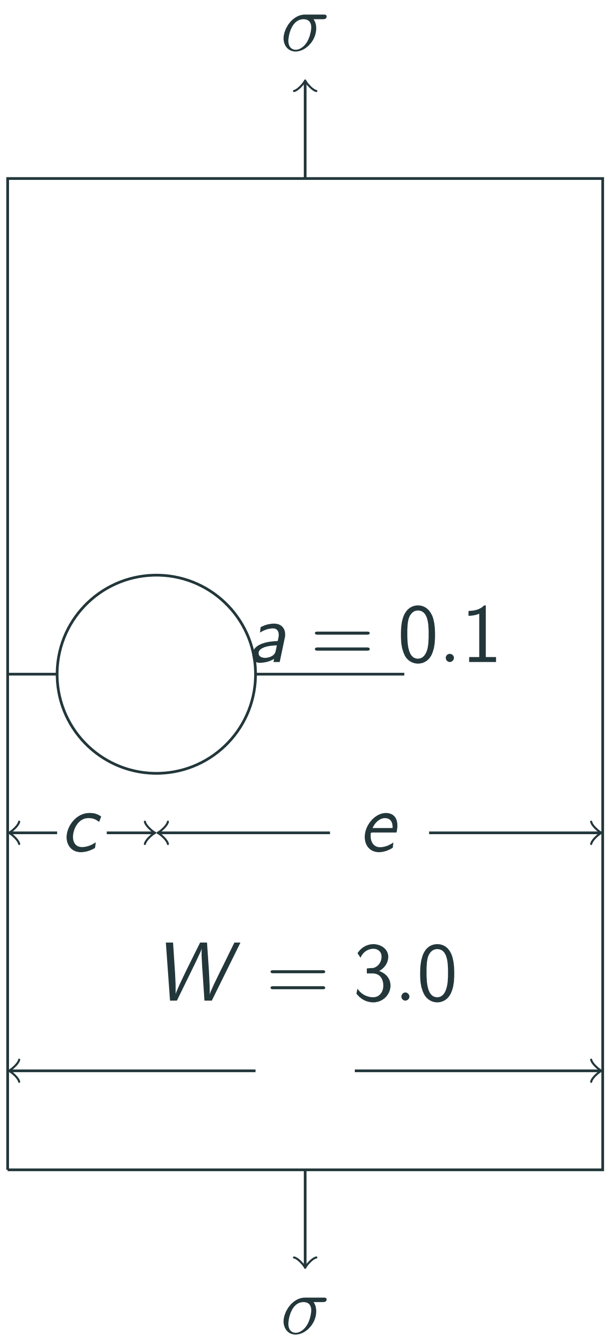
- $c = 0.75$, $e = 2.25$, $r = 0.5$
- assume a is short and calculate β for this case
- calculate in terms of β for known state

group two



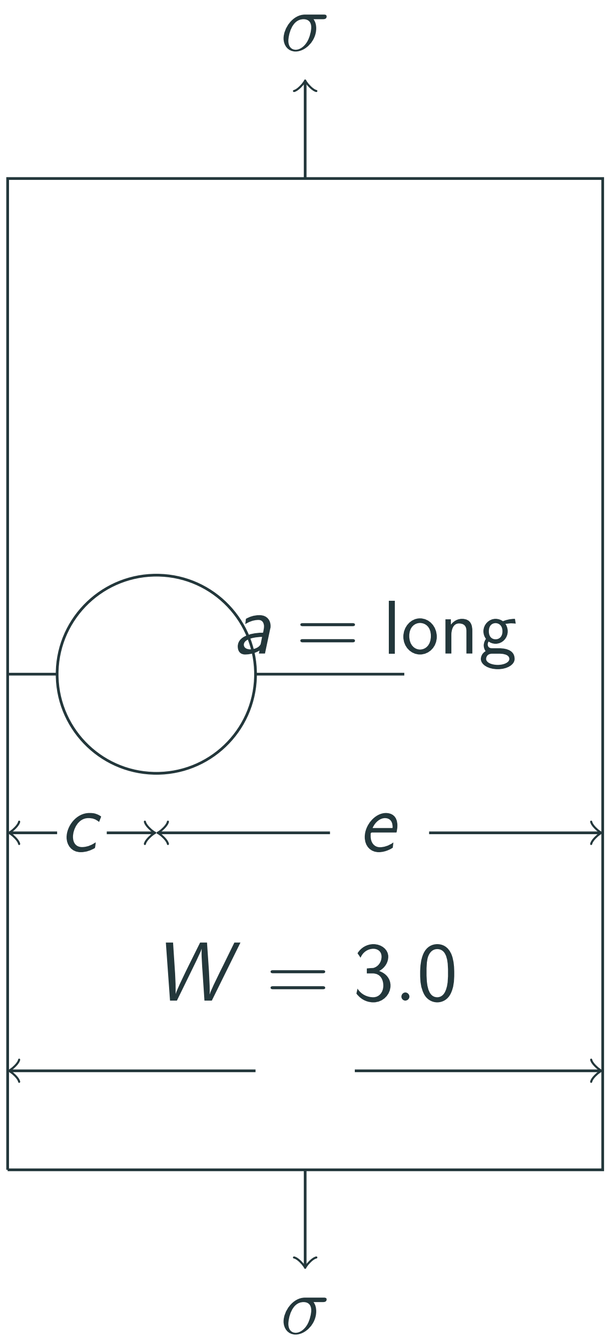
- $c = 0.75$, $e = 2.25$, $r = 0.5$
- assume a is long and calculate β for this case
- calculate in terms of β for known state

group three



- c = 0.75, e = 2.25, r = 0.5
- assume a is short and calculate β for this case
- calculate in terms of β for known state

group four



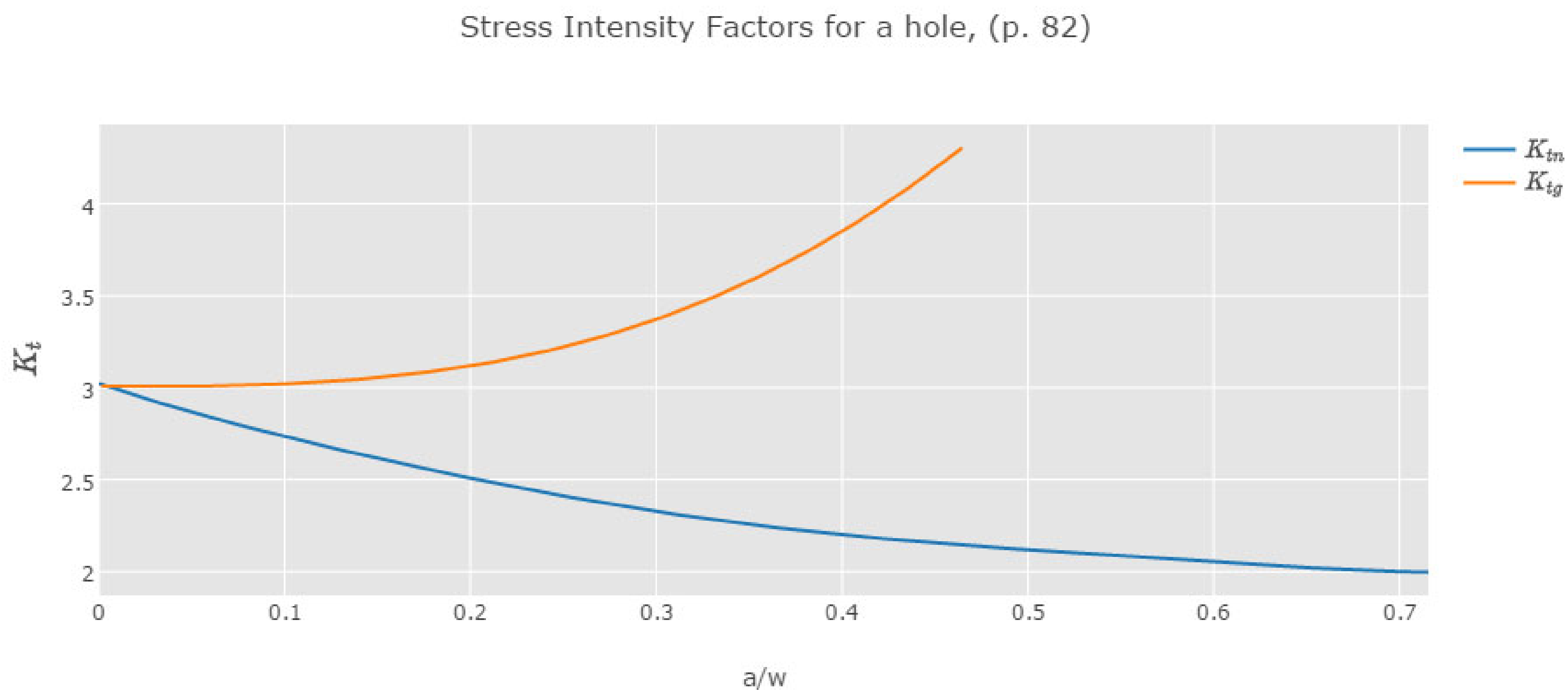
- $c = 0.75$, $e = 2.25$, $r = 0.5$
- assume a is long and calculate β for this case
- calculate in terms of β for known state

discussion

Draw a sketch to show how we could use this method to find cracks of intermediate length near a curved boundary

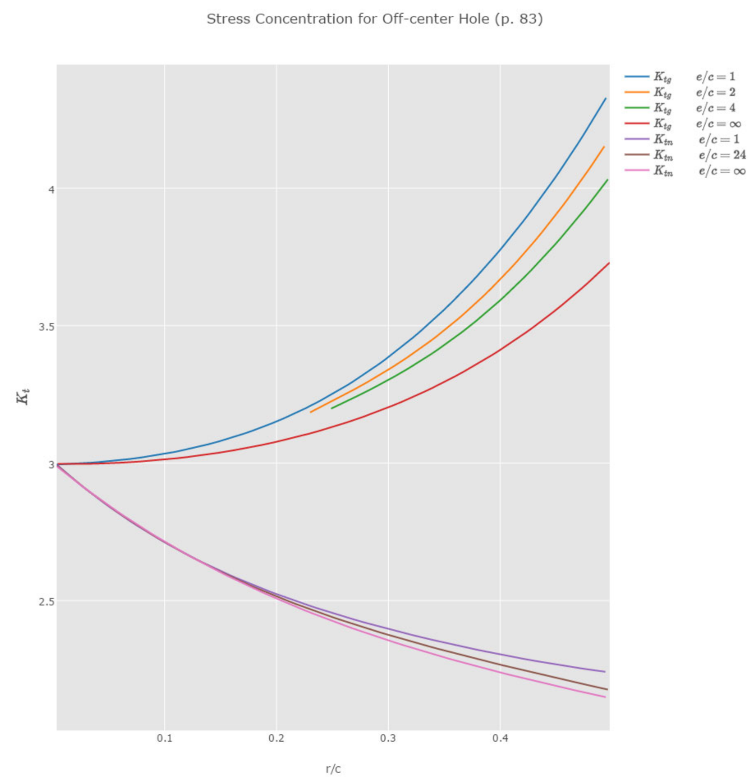
stress concentration factors

centered hole tension - p. 82



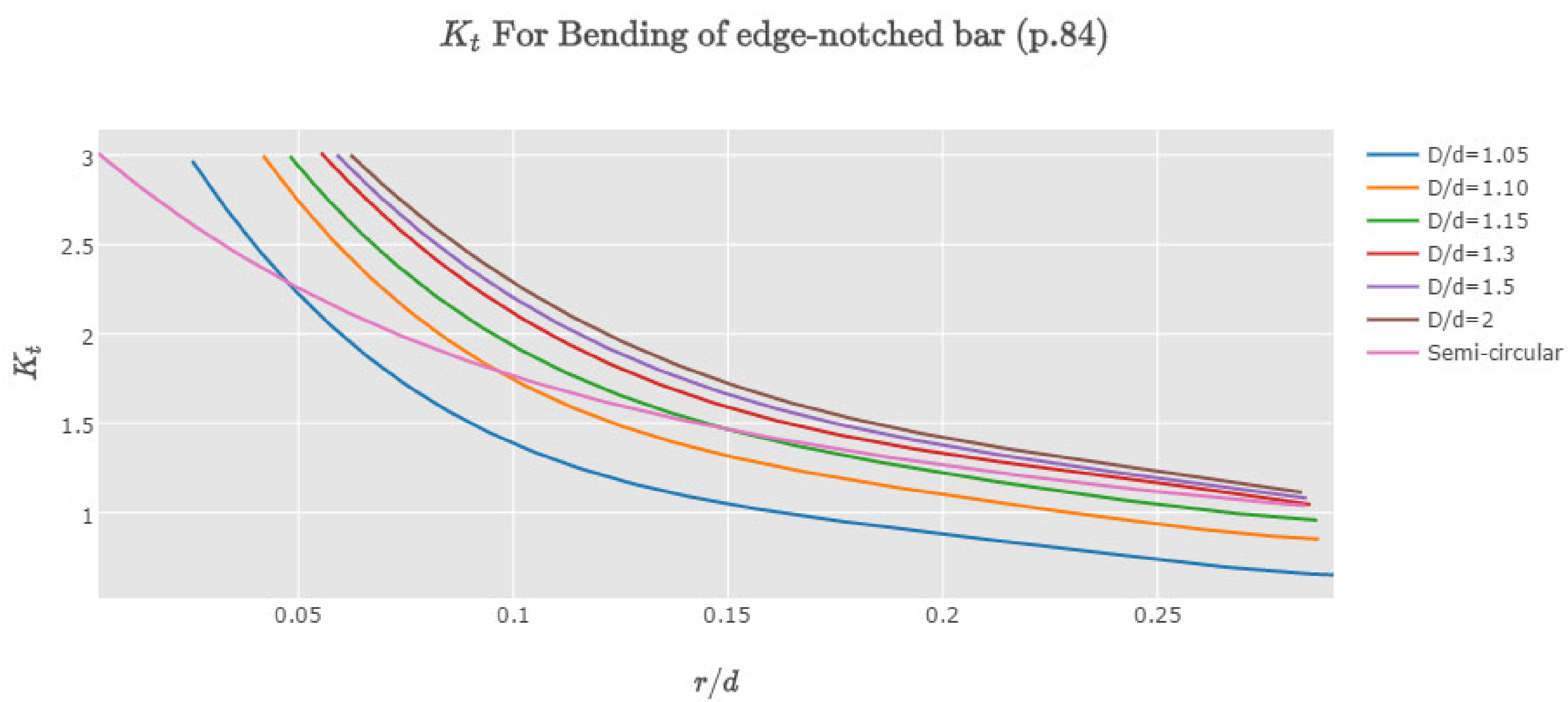
K_{tg} uses stress for the cross-sectional area if no hole was present, K_{tn} uses stress at the net section (subtracting hole area). a is the hole diameter, W is specimen width.

off-center hole tension - p. 83



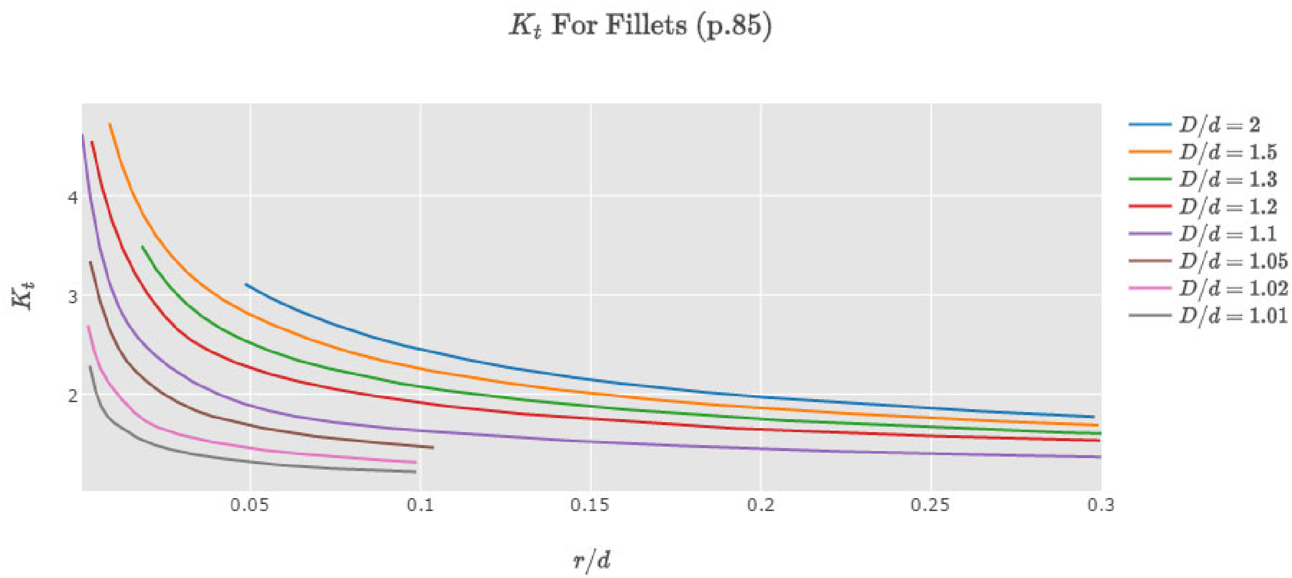
K_{tg} uses stress for the cross-sectional area if no hole was present, K_{tn} uses stress at the net section (subtracting hole area). c is the distance from the closest edge to the center of the hole, e is the distance from the farthest edge to the center of the hole, r is hole radius.

bending of a bar with u-shaped notch - p. 84



Nominal stress used for K_t is given by $\sigma_{nom} = 6M/hd^2$ where M is applied bending moment, h is thickness, d is the net-section height (height minus notch depth). D is the height of the panel without a notch and r is the notch radius.

tension of a stepped bar with shoulder fillets - p. 85



D is the larger width (before the step), d is the width after the step. Nominal stress is $\sigma_{nom} = P/hd$, where h is specimen thickness. r is the fillet radius.

interactive page

- An interactive page with these plots can be accessed here