AE 737 - Mechanics of Damage Tolerance

Lecture 2

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outline

office hours

- I will e-mail everyone in the course a Doodle link we can use to find the optimal office hours
- Let me know if you do not receive the e-mail, you may need to update your information in Blackboard
- Take advantage of office hours, this is time that I have already set aside for you
- If the regular office hours do not work for your schedule, send me an e-mail and we can work out a time to meet
- While in person visits are often the most helpful, I will always try to answer questions as best as I can via e-mail

Fracture Mechanics

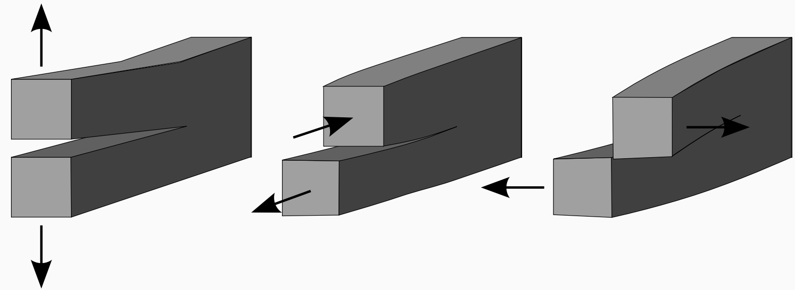
fracture mechanics

- *Linear Elastic Fracture Mechanics* is the study of the propagation of cracks in materials
- There are some corrections we add to account for plasticity
- In this course we will not follow the full mathematical development of fracture mechanics (there is a separate course dedicated to that)
- Instead we will take some results and apply them

fracture mechanics

- In fracture mechanics we consider three different modes
- Mode I is known as the ”opening mode”
- Mode II is known as the ”sliding mode”
- Mode III is known as the ”tearing mode”

fracture mechanics



Mode I: Opening   Mode II: Sliding   Mode III: Tearing

Stress Intensity

stress intensity

- A key finding from Linear Elastic Fracture Mechanics (LEFM) is known as the *Stress Intensity Factor*
- The stress intensity factor is often written in this form

$$K = \sigma\sqrt{\pi a}\beta$$

- Where K is the stress intensity factor, σ is the applied stress, a is the crack length, and β is a dimensionless parameter depending on geometry
- Be careful that although the notation is similar, the *Stress Intensity Factor* is different from the *Stress Concentration Factor* from strength of materials
- We are usually most concerned with Mode I, but there will be a unique stress intensity factor for each mode, we label these K_I, K_{II}, and K_{III}
- If no subscript is given, assume Mode I

stress intensity

- For brittle materials (where ”linear” fracture mechanics assumptions hold true) we can find the full stress field near the crack in terms of the stress intensity factor

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

stress intensity

- Similarly for Mode II we find

$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$
$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

stress intensity

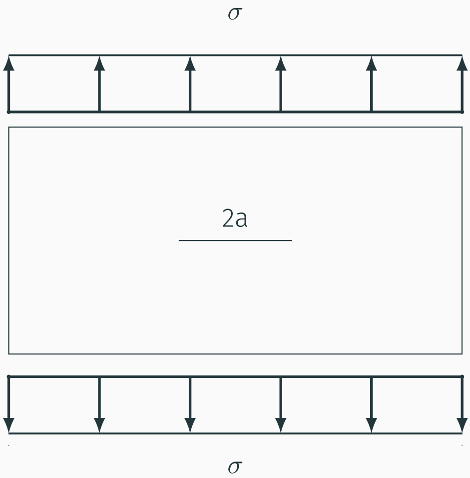
- And for Mode III

$$\tau_{xz} = \frac{-K_{III}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}$$
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}$$

Common stress intensity factors

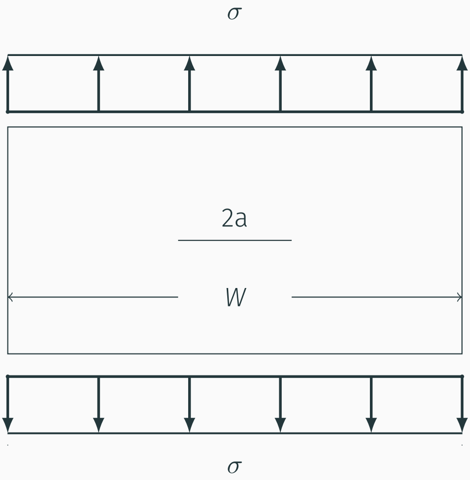
center crack, infinite width

$$K_I = \sigma\sqrt{\pi a} \tag{1}$$



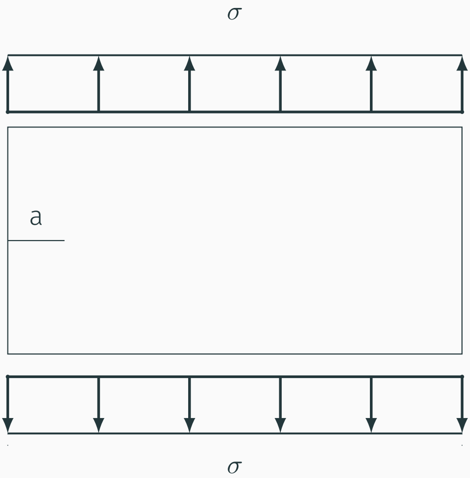
center crack, finite width

$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)} \tag{2}$$



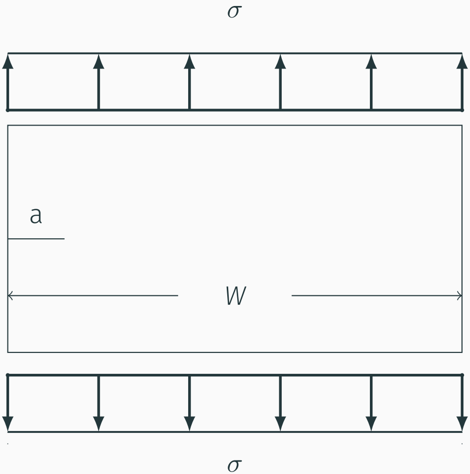
edge crack, semi-infinite width

$$K_I = 1.12\sigma\sqrt{\pi a} \tag{3}$$

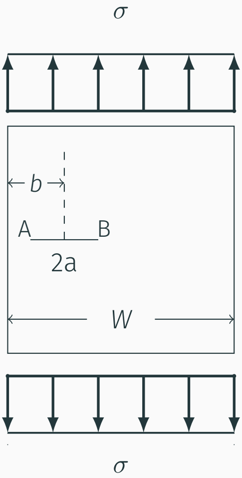


edge crack, finite width

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4\right] \tag{4}$$



offset crack



offset crack

$$K_{IA} = \sigma\sqrt{\pi a}\beta_c\beta_A \text{ and } K_{IB} = \sigma\sqrt{\pi a}\beta_c\beta_B \tag{5}$$

$$\beta_c = \sqrt{\sec\frac{\pi a}{W}} \tag{6a}$$

$$\beta_A = (1 - 0.025\lambda^2 + 0.6\lambda^4 - \gamma\lambda^{11}) \sqrt{\sec\left(\frac{\pi\lambda}{2}\right)\frac{\sin\left(2\lambda - 4\frac{a}{W}\right)}{2\lambda - 4\frac{a}{W}}} \tag{6b}$$

$$\beta_B = (1 - 0.025\delta^2 + 0.06\delta^4 - \zeta\lambda^{30}) \left(1 + \frac{\sqrt{\sec\left(\frac{2\pi\lambda+1.5\pi\delta}{7}\right) - 1}}{1 + 0.21\sin\left(8\tan^{-1}\left(\frac{\lambda-\delta}{\lambda+\delta}\right)^{0.9}\right)}\right) \tag{6c}$$

offset crack

- The parameters λ, δ are given as

$$\lambda = \frac{a}{b} \tag{6d}$$

$$\delta = \frac{a}{W - b} \tag{6e}$$

- And γ and ζ can be looked up on a table

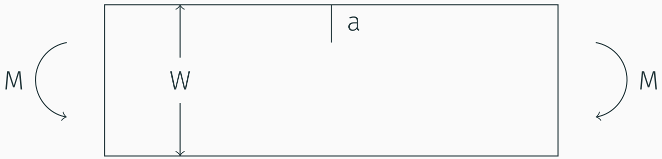
Table 1: Parameters for offset crack

$\frac{b}{W}$	γ	ζ
0.1	0.382	0.114
0.25	0.136	0.286
0.4	0.0	0.0
0.5	0.0	0.0

edge crack, bending moment

$$K_I = \frac{6M}{(W-a)^{3/2}} f(a/W) \tag{7}$$

a/W	.05	.1	.2	.3	.4	.5	.6 +
$f(a/W)$	.36	.49	.60	.66	.69	.72	.73

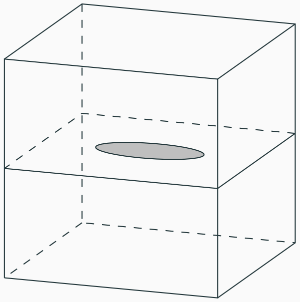


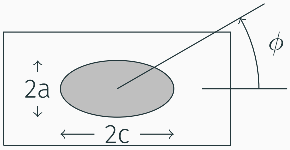
3D crack shapes

crack depth

- The previous stress intensity factors all assume a 2D problem
- Through the thickness, it is assumed that the crack length is the same
- In many cases this is not an accurate assumption
- We will now consider 3D crack shapes and their effect on the stress intensity factor

elliptical flaw





elliptical flaw

$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4} \tag{8}$$

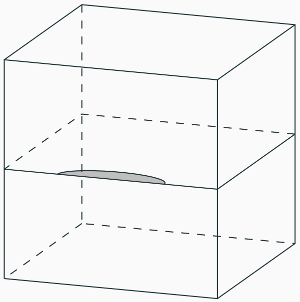
$$Q = \Phi^2 - 0.212\left(\frac{\sigma}{\sigma_y}\right)^2 \text{(2nd term usually ignored)}$$

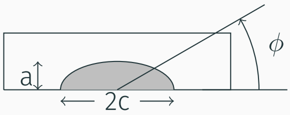
$$\Phi = \int_0^{\pi/2}\left[1 - \left(\frac{c^2 - a^2}{c^2}\sin^2\phi\right)\right]^2 d\phi$$

$$\approx \frac{\pi}{2}\left[1 - \frac{1}{4}\frac{c^2 - a^2}{c^2} - \frac{3}{64}\left(\frac{c^2 - a^2}{c^2}\right)^2 - ...\right]$$

$$\Phi \approx \frac{\pi}{2}\left[1 - \frac{1}{4}\frac{c^2 - a^2}{c^2}\right] \text{(sufficient for most cases)}$$

semi-elliptical surface flaw



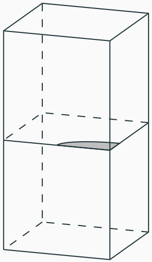


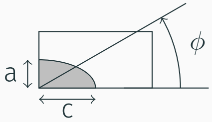
$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4} \quad (1.1)$$

(9)

Note: $1.1 \approx \sqrt{1.2}$, which is known as the front surface correction factor. A more accurate correction factor is given as

$$1 + .12\left(1 - \frac{a}{c}\right)$$



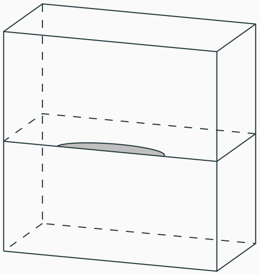


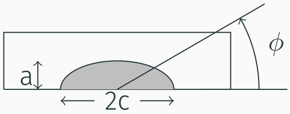
$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4} \quad (1.1)(1.1) \tag{10}$$

Note: $1.1 \approx \sqrt{1.2}$, which is known as the front surface correction factor. A more accurate correction factor is given as

$$1 + .12\left(1 - \frac{a}{c}\right)$$

semi-elliptical surface flaw in finite body



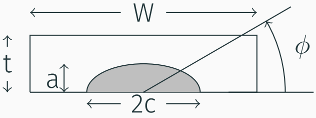


$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4} (1.1) M_K \tag{11}$$

Front surface correction same as for the infinite body.

M_K is the back surface correction, and can be looked up on a chart in your textbook.

finite thickness correction



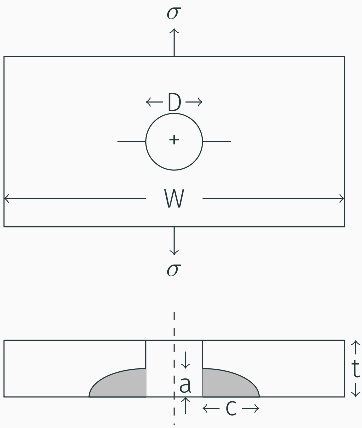
- An alternative method for calculating the finite thickness and finite width correction factors uses the following formula

$$K_I = \sigma\sqrt{\pi c}\sqrt{\frac{a}{cQ'}}\left[M_1 + \left(\sqrt{\frac{Q'c}{a}} - M_1\right)\left(\frac{a}{t}\right)^P\right]\sqrt{\sec\left(\frac{\pi c}{W}\sqrt{\frac{a}{t}}\right)} \tag{12}$$

finite thickness correction

$$
\begin{aligned}
M_1 &= 1.2 - 0.1\frac{a}{c} && \text{for} \qquad 0.02 \leq \frac{a}{c} \leq 1 \\
&= \sqrt{\frac{c}{a}}\left(1 + 0.1\frac{a}{c}\right) && \text{for} \qquad \frac{a}{c} > 1 \\
Q' &= 1 + 1.47\left(\frac{a}{c}\right)^{1.64} && \text{for} \qquad \frac{a}{c} \leq 1 \\
&= 1 + 1.47\left(\frac{c}{a}\right)^{1.64} && \text{for} \qquad \frac{a}{c} > 1 \\
P &= 2 + 8\left(\frac{a}{c}\right)^{3}
\end{aligned}
$$

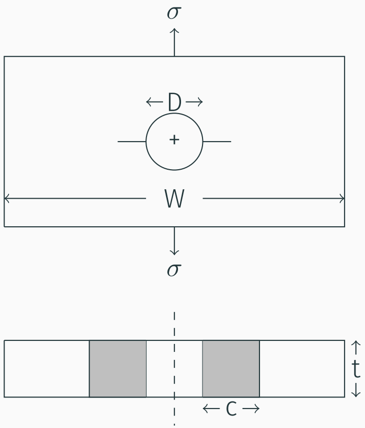
cracks around a hole



- For corner cracks under uniform applied stress, we have

$$K_I = \sigma\sqrt{\pi c}\sqrt{\frac{a}{cQ'}}\left[M_1 + \left(\sqrt{\frac{Q'c}{a}} - M_1\right)\left(\frac{a}{t}\right)^P\right] \sqrt{\sec\left(\frac{\pi}{2}\frac{D+bc}{W-2C+bc}\sqrt{\frac{a}{t}}\right)} f_b\sqrt{\sec\frac{\pi D}{2W}} \tag{13}$$

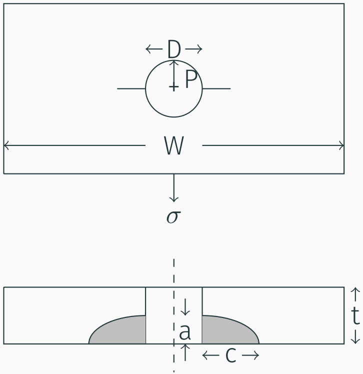
cracks around a hole



- For through cracks under uniform applied stress, we have

$$K_I = \sigma\sqrt{\pi c}\sqrt{\sec\left(\frac{\pi}{2}\frac{D+bc}{W-2C+bc}\right)}f_b\sqrt{\sec\frac{\pi D}{2W}} \tag{14}$$

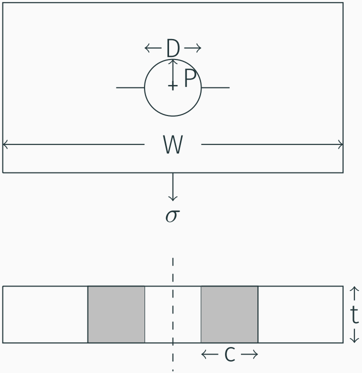
cracks around a hole



- For corner cracks under fastener load, we have

$$K_I = \frac{P}{Wt}\sqrt{\pi c}\sqrt{\frac{a}{cQ'}}\left[M_1 + \left(\sqrt{\frac{Q'c}{a}} - M_1\right)\left(\frac{a}{t}\right)^P\right]$$
$$\sqrt{\sec\left(\frac{\pi}{2}\frac{D+bc}{W-2C+bc}\sqrt{\frac{a}{t}}\right)} f_b \sqrt{\sec\frac{\pi D}{2W}} G_b \tag{15}$$

cracks around a hole



- For through cracks under fastener load, we have

$$K_I = \frac{P}{Wt}\sqrt{\pi c}\sqrt{\sec\left(\frac{\pi}{2}\frac{D+bc}{W-2C+bc}\right)}f_b\sqrt{\sec\frac{\pi D}{2W}}G_b \tag{16}$$

cracks around a hole

$$P = 2 + 8\left(\frac{a}{c}\right)^3 \qquad \lambda = \frac{1}{1 + 2C/D}$$

$$\begin{aligned}
M_1 &= 1.2 - 0.1\left(\frac{a}{c}\right) && \text{for} \quad .02 \le \frac{a}{c} \le 1 \\
&= \sqrt{\frac{c}{a}}\left[1 + 0.1\left(\frac{a}{c}\right)\right] && \text{for} \quad \frac{a}{c} > 1 \\
f_b = f_1 &= .707 - .18\lambda + 6.55\lambda^2 - 10.54\lambda^3 + 6.85\lambda^4 && \text{for} \quad b = 1 \\
f_b = f_2 &= 1 - .15\lambda + 3.46\lambda^2 - 4.47\lambda^3 + 3.52\lambda^4 && \text{for} \quad b = 2 \\
G_1 &= \frac{1}{2} + \frac{W}{\pi(D + c)}\sqrt{\frac{D}{D + 2c}} && \text{for} \quad b = 1 \\
G_2 &= \frac{1}{2} + \frac{W}{\pi(D + 2c)} && \text{for} \quad b = 2
\end{aligned}$$

examples

1.
    1.1 Determine the value of K_I for a center-cracked panel with $W/2a = 3$ and a uniformly applied remote stress, σ.
    1.2 Determine the value of K_I for an edge-cracked panel with $W/a = 3$ and a uniformly applied remote stress, σ.
    1.3 Compare these two results. Note that in both cases the panel width to crack length ratio is the same.

- Based on the ratio of crack length to width, we choose (2) over (1)

$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)}$$

- This gives $K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi/6)}$
- If we normalize by the infinite width solution, we find $K_{I,f}/K_{I,i} \approx 1.075$

example 1

- Once again, based on the ratio of crack length to width, we choose (4) over (3)

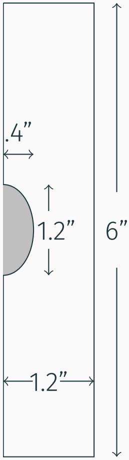
$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4\right]$$

- This gives $K_I \approx 1.786\sigma\sqrt{\pi a}$
- If we normalize by the infinite width solution, we find $K_{I,f}/K_{I,i} \approx 1.595$

example 1

- Comparing the two cases, we see that the finite width effects are much more significant for the edge-crack specimen
- The edge-crack specimen is also overall more effected by a crack of that relative length.
- Why are they not the same?

example 2



- Find maximum value of K_I for semi-elliptical surface flaw
- $\sigma = 20$kpsi (in opening direction)

example 2

- Here we will compare three different equations
- (8), which is for infinite width and thickness

$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4}(1.1) \tag{8}$$

- (10), which is for infinite width, but finite thickness

$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4}(1.1)M_K \tag{10}$$

- And (11), which accounts for finite width and thickness

$$K_I = \sigma\sqrt{\pi c}\sqrt{\frac{a}{cQ'}}\left[M_1 + \left(\sqrt{\frac{Q'c}{a}} - M_1\right)\left(\frac{a}{t}\right)^P\right]\sqrt{\sec\left(\frac{\pi c}{W}\sqrt{\frac{a}{t}}\right)} \tag{11}$$