AE 737: Mechanics of Damage Tolerance

Lecture 15 - Stress based fatigue

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schedule

- 12 Mar - Stress-based fatigue, Project Abstract Due
- 17 Mar - Strain-based fatigue
- 19 Mar - Crack growth, HW6 Due
- 23-27 Mar - Spring Break

outline

- fatigue review
- influence of notches
- strain based fatigue
- variable amplitude strains
- general trends

fatigue review

group 1

- A part from AISI 4340 in a typical “block” undergoes 100,000 cycles with σ_{min} = 0 ksi and σ_{max} = 100 ksi and an additional 10 cycles with σ_{min} = 50 ksi and σ_{max} = 200 ksi
- How many “blocks” can this part support before failure?

group 2

- Use the S-N-P chart on p. 245 for 7075-T6 Aluminum
- What is the probability of failure for 30 ksi at 10^6 cycles?
- To ensure that 99% of parts do not fail, after how many cycles should a fully reversed load of 35 ksi be inspected?
- How many cycles could the same part sustain if only 50% of parts are needed?

group 3

- The fatigue limit for AISI 4142 steel is 58 ksi for completely reversed fatigue loads.
- What is the fatigue limit for fatigue loads with $\sigma_m = 10, 20, 30$ ksi?

group 4

- A material made of 2024-T4 Aluminum undergoes the following load cycle
    - $\sigma_{x,\,\mathrm{min}} = 10$, $\sigma_{x,\,\mathrm{max}} = 50$
    - $\sigma_{y,\,\mathrm{min}} = -20$, $\sigma_{y,\,\mathrm{max}} = 20$
    - $\tau_{x^{**}y,\,\mathrm{min}} = 0$, $\tau_{xy,\,\mathrm{max}} = 30$
- How many cycles can it support before failure?

influence of notches

notch effects

- In this discussion, we use “notch” to refer to any geometric feature that increases the local stress (such as holes, fillets, grooves, etc.)
- We discussed notches and stress concentration factors in terms of stress concentration factors
- In our fatigue notation, $\sigma_{\max} = K_t S$
- This relates local stress to the average, nominal stress
- The stress intensity factor can be used to characterize the “strength” of a notch

notch effects

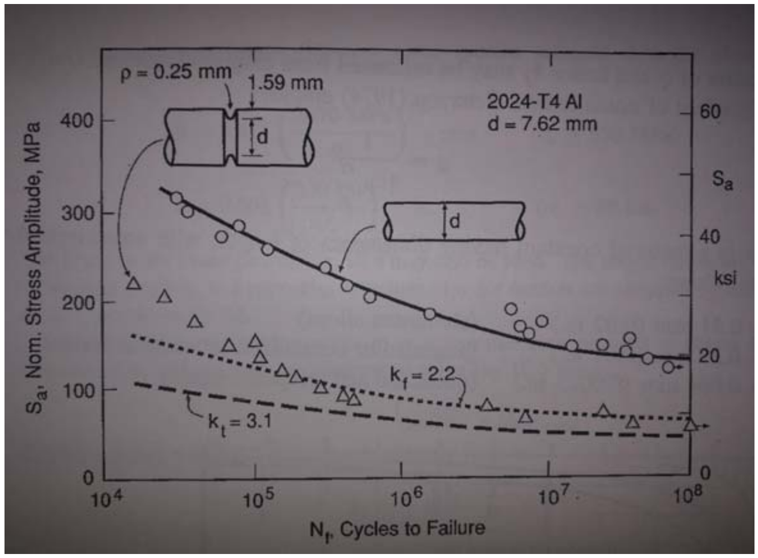
- We might expect the fatigue life of a notched specimen to be similar to a pristine specimen with $S_{a,\text{ pristine}} = \sigma_{\text{max, notched}}$
- If we look at actual test data, however, this estimate would be overly conservative
- Even when the stress is adjusted for some fatigue notch factor, k_f, it is only valid at longer cycles ($N_f > 10^6$)

notch effects

$$k_f = \frac{\sigma_{ar}}{S_{ar}}$$

- Notches will have different effects, largely depending on their radius.
- The maximum possible fatigue notch factor is $k_f = k_t$

notch effects



notch sensitivity factor

- To avoid generating fatigue data for every possible notch configuration, some empirical relationships have been developed
- A useful concept in these methods is the notch sensitivity factor

$$q = \frac{k_f - 1}{k_t - 1}$$

- When $k_f = 1$, $q = 0$, in which case the notch has no effect
- When $k_f = k_t$, $q = 1$, in which case the notch has its maximum effect

peterson notch sensitivity

- Peterson developed the following relationship

$$q = \frac{1}{1 + \frac{\alpha}{\rho}}$$

- Where ρ is the radius of the notch
- α is a material property

peterson notch sensitivity

Material	α (mm)	α (in)
Aluminum alloys	0.51	0.02
Annealed or low-carbon steels	0.25	0.01
Quenched and tempered steels	0.064	0.0025

peterson notch sensitivity

- For high-strength steels, a more specific α estimate can be found

$$\alpha = 0.025\left(\frac{2070}{\sigma_u}\right)^{1.8} \qquad \text{mm} \qquad \sigma_u \geq 550 \text{ MPa}$$

$$\alpha = 0.001\left(\frac{300}{\sigma_u}\right)^{1.8} \qquad \text{in} \qquad \sigma_u \geq 80 \text{ ksi}$$

peterson notch sensitivity

- α predictions are valid for bending and axial fatigue
- For torsion fatigue, a good estimate can be found
- $\alpha_{\text{torsion}} = 0.6\alpha$

alternative

- An alternative formulation for q was developed by Neuber

$$q = \frac{1}{1 + \sqrt{\frac{\beta}{\rho}}}$$

- Where the material property β for steels is given by

$$\log \beta = -\frac{\sigma_u - 134}{586} \qquad \text{mm} \qquad \sigma_u \leq 1520 \text{ MPa}$$
$$\log \beta = -\frac{\sigma_u + 100}{85} \qquad \text{in} \qquad \sigma_u \leq 220 \text{ ksi}$$

alternative

- For aluminum use the chart MPa (ksi) and mm (in.)

S_u	150 (22)	300 (43)	600 (87)
β	2 (0.08)	0.6 (0.025)	0.5 (0.015)

notch sensitivity factors

- While the above methods are useful, they should be regarded as estimates only
- Physical complexities are not fully modeled by these methods
- All of these have been developed for relatively “mild” notches
- For sharp notches, best results are found by treating the notch as a crack

example

- Find the notch sensitivity factor for the following scenario

$$\begin{aligned} \rho &= 0.25 \text{ in.} \\ \sigma_m &= 0 \text{ ksi} \\ K_t &= 3.0 \\ \sigma_u &= 84 \text{ ksi} \end{aligned}$$

strain based fatigue

strain based fatigue

- The strain based fatigue method uses local stresses and strains (instead of global, nominal values)
- The strain-based method gives greater detail, and validity at lower cycles
- It is still valid for high cycle fatigue (but gives same result as stress-based fatigue)
- Does not include crack growth analysis or fracture mechanics

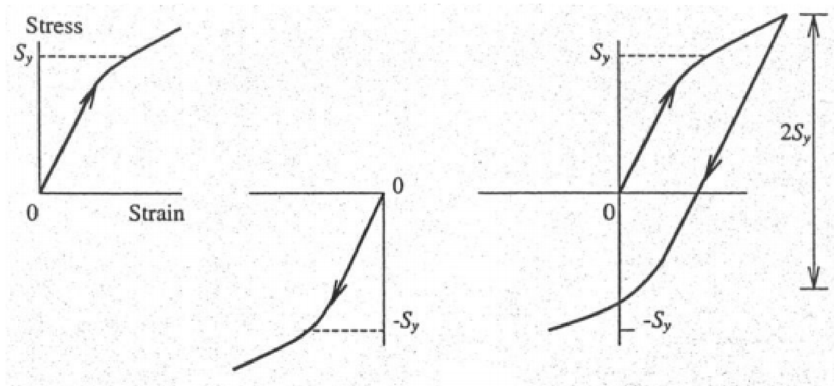
strain life curve

- Similar to the S-N curves in stress-based fatigue analysis, we can plot the cyclic strain amplitude vs. number of cycles to failure
- This is most commonly done using axial test machines (instead of rotating bending tests)
- The test is run in strain control (not load control)
- Generally plotted on log-log scale

plastic and elastic strain

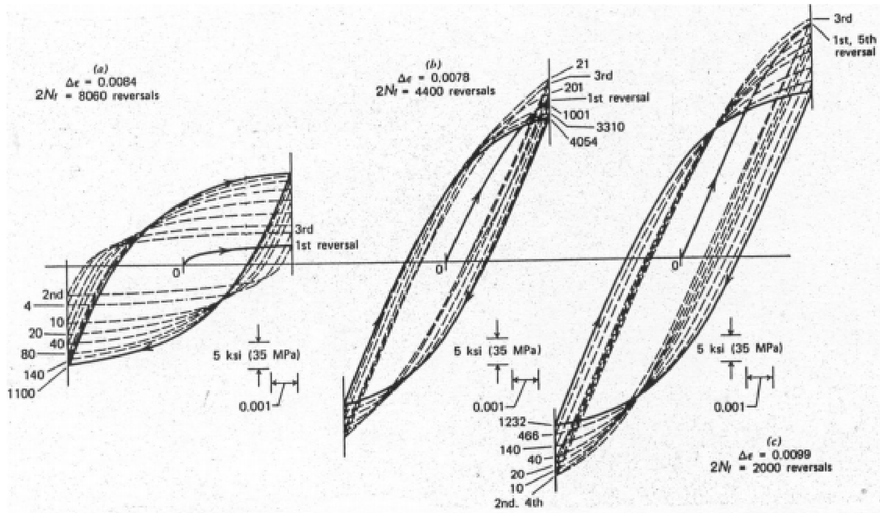
- We can separate the total strain into elastic and plastic components
- $\epsilon_a = \epsilon_{ea} + \epsilon_{pa}$

plastic strain



(a) (b) (c)

hysteresis loops



cyclic stress strain curve

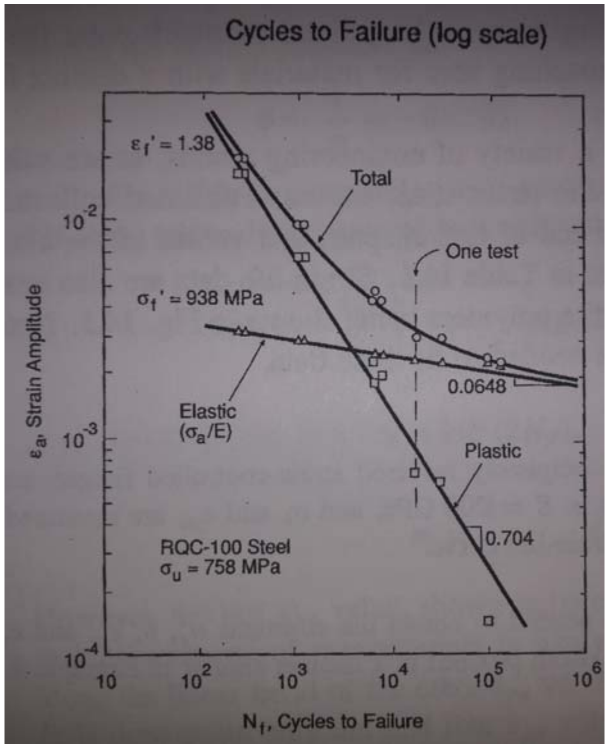
- While strain-life data will generally just report ϵ_a and ϵ_{pa}, some will also tabulate a form for the cyclic stress-strain curve

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

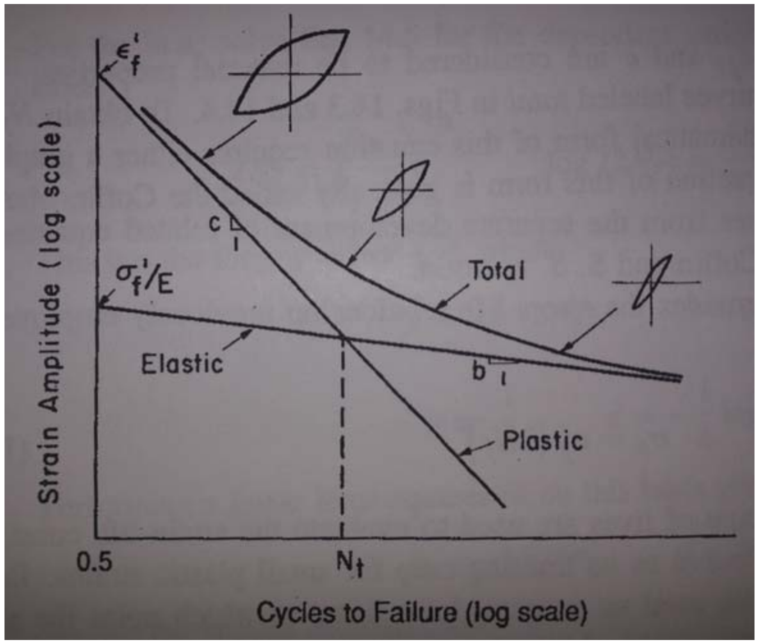
plastic and elastic strain

- On strain life curves, the strain is often plotted three times per each experiment
- Once for total strain, once for plastic strain, and once for elastic strain
- Since plastic strain and elastic strain vary by the number of cycles, a hysteresis loop from half the fatigue life is generally used
- This is considered representative of stable behavior

experimental data



trends



ϵ_f'
σ_f'/E
c
1
Total
Elastic
b
1
Plastic
Strain Amplitude (log scale)
0.5
N_t
Cycles to Failure (log scale)

lines

- We notice that the data for elastic and plastic strains are represented by straight lines, in the log-log scale
- If we recall the form used for a straight line in log-log plots for S-N curves:
- $\sigma_a = \sigma_f'(2N_f)^b$
- We can convert this to find the elastic component of strain

$$\epsilon_{ea} = \frac{\sigma'_f}{E}(2N_f)^b$$

lines

- We can use the same form with new constants for the plastic component of strain
- $\epsilon_{pa} = \epsilon_f^{'}(2N_f)^c$
- We can combine the elastic and plastic portions to find the total strain-life curve

$$\epsilon_a = \frac{\sigma'_f}{E}(2N_f)^b + \epsilon'_f(2N_f)^c$$

example

ϵ_a	σ_a (MPa)	$\epsilon_{p^{**}a}$	N_f
0.0202	631	0.01695	227
0.0100	574	0.00705	1030
0.0045	505	0.00193	6450
0.0030	472	0.00064	22250
0.0023	455	(0.00010)	110000

transition life

- With the strain-based fatigue method we are better equipped to discuss the difference between high and low-cycle fatigue
- Low-cycle fatigue is dominated by plastic effects, while high-cycle fatigue has little plasticity
- We can find the intersection of the plastic strain and elastic strain lines
- This point is N_t, the transition fatigue life

$$N_t = \frac{1}{2}\left(\frac{\sigma'_f}{\epsilon'_f}\right)^{\frac{1}{c-b}}$$

inconsistencies in constants

- If we consider the equation for the cyclic stress train curve

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

- We can consider the plastic portion and solve for σ_a
- $\sigma_a = H^{'} {\epsilon_{pa}}^{n^{'}}$

inconsistencies in constants

- We can eliminate $2N_f$ from the plastic strain equation
- $\epsilon_{pa} = \epsilon_f'(2N_f)^c$
- By solving the stress-life relationship for $2N_f$
- $\sigma_a = \sigma_f'(2N_f)^b$
- and substituting that into the plastic strain

inconsistencies in constants

- We then compare with stress-life equations and find

$$H' = \frac{\sigma'_f}{(\epsilon'_f)^{b/c}}$$

$$n' = \frac{b}{c}$$

inconsistencies in constants

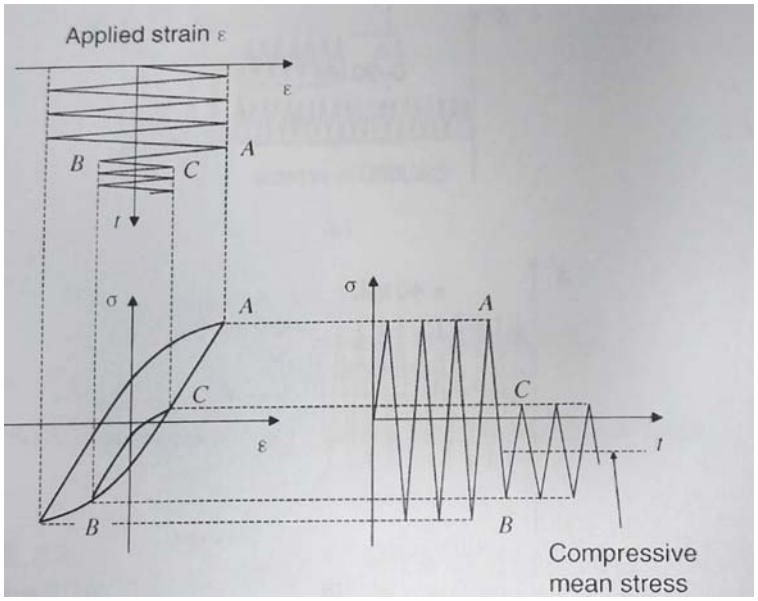
- However, in practice these constants are fit from different curves
- In some cases there can be large inconsistencies in these values
- One cause for this is data that do not lie on a straight line in the log-log domain
- For ductile materials at short lives, the true stresses and strains may differ significantly from engineering stress and strain

variable amplitude strains

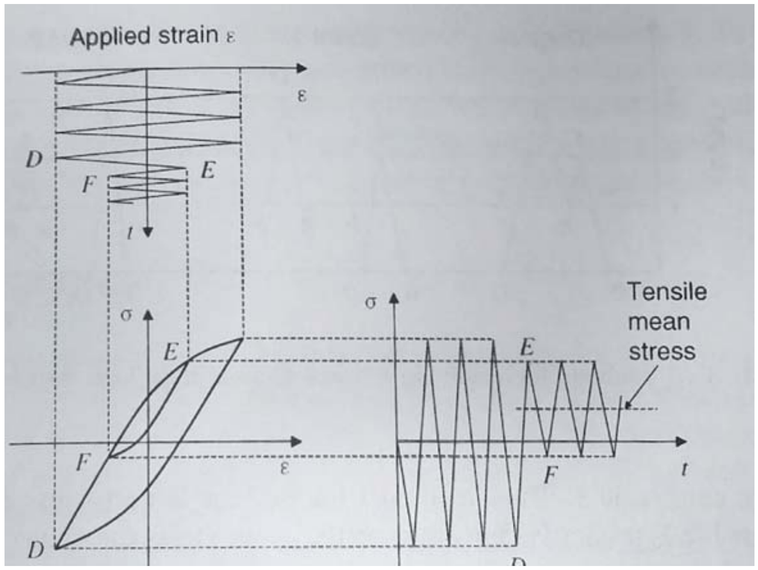
variable amplitude strains

- As with stresses, we can apply variable amplitude strains
- However, when the change is made will affect whether there is a tensile or compressive mean stress

compressive mean



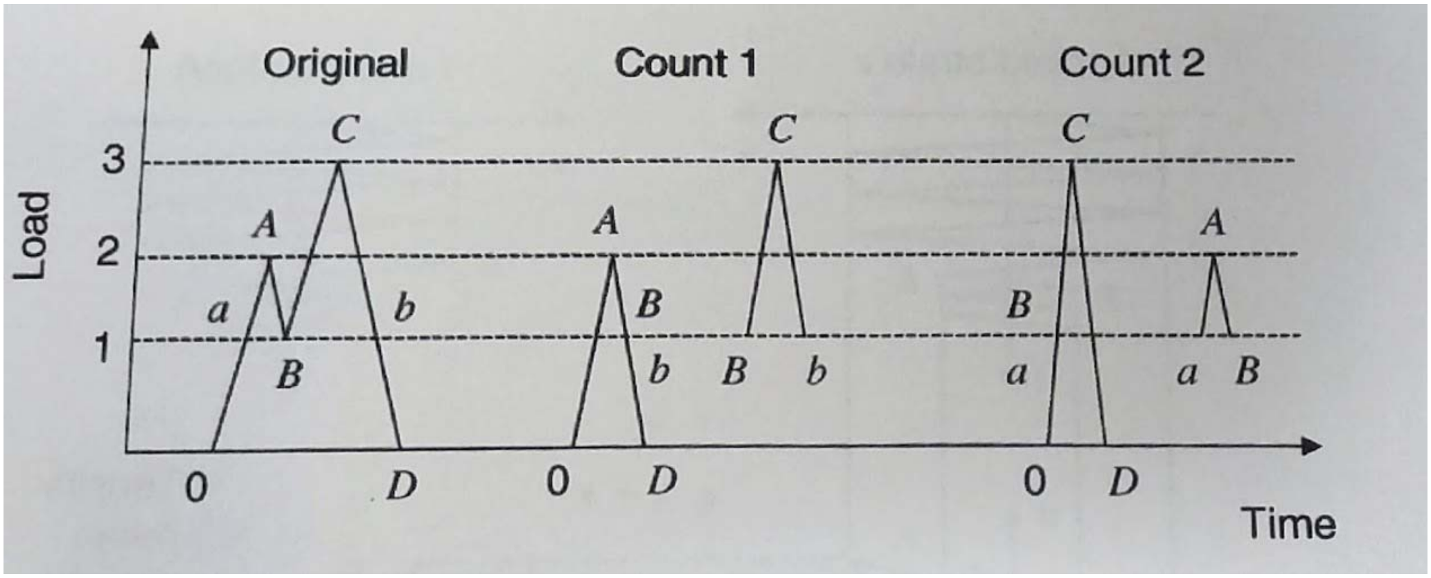
tensile mean



cycle counting

- In all fatigue methods (stress, strain, and crack propagation) the way we count load cycles can have an effect on our results
- To avoid being non-conservative, we need to always count the largest amplitudes first
- We will discuss some specific cycle-counting algorithms during crack propagation

cycle counting



general trends

true fracture strength

- We can consider a tensile test as a fatigue test with $N_f = 0.5$
- We would then expect the true fracture strength $\tilde{\sigma}_f \approx \sigma'_f$
- And similarly for strain $\tilde{\epsilon}_f \approx \epsilon'_f$

ductile materials

- Since ductile materials experience large strains before failure, we expect relatively large $\epsilon_f^{'}$ and relatively small $\sigma_f^{'}$
- This will cause a less steep slope in the plastic strain line
- In turn this intersects with the elastic strain line much later, resulting a longer transition life for ductile materials

brittle materials

- Brittle materials exhibit the opposite effect, with relatively low $\epsilon_f^{'}$ and relatively high $\sigma_f^{'}$
- This results in a steeper plastic strain line
- And shorter transition life

tough materials

- Tough materials have intermediate values for both $\epsilon_f^{'}$ and $\sigma_f^{'}$
- This gives a transition life somewhere between brittle and ductile materials
- It is also noteworthy that strain-life for many metals pass through the point ϵ_a = 0.01 and N_f = 1000 cycles
- Steels also follow a trend with Brinell Hardness, the higher they are on the HB scale, the lower their transition life

typical property ranges

- Most common engineering materials have $-0.8 < c < -0.5$, with most values being very close to $c = -0.6$
- The elastic strain slope generally has $b = -0.085$
- A “steep” elastic slope is around $b = -0.12$, common in soft metals
- While “shallow” slopes are around $b = -0.05$, common for hardened metals