



Designation: E1049 – 85 (Reapproved 2017)

Standard Practices for Cycle Counting in Fatigue Analysis¹

This standard is issued under the fixed designation E1049; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 These practices are a compilation of acceptable procedures for cycle-counting methods employed in fatigue analysis. This standard does not intend to recommend a particular method.

1.2 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

1.3 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*²

E912 Definitions of Terms Relating to Fatigue Loading;
Replaced by E 1150 (Withdrawn 1988)³

3. Terminology

3.1 *Definitions:*

3.1.1 *constant amplitude loading—in fatigue loading*, a loading in which all of the peak loads are equal and all of the valley loads are equal.

3.1.2 *cycle—in fatigue loading*, under constant amplitude loading, the load variation from the minimum to the maximum and then to the minimum load.

NOTE 1—In spectrum loading, definition of cycle varies with the counting method used.

¹ These practices are under the jurisdiction of ASTM Committee E08 on Fatigue and Fracture and are the direct responsibility of Subcommittee E08.04 on Structural Applications.

Current edition approved June 1, 2017. Published June 2017. Originally approved in 1985. Last previous edition approved in 2011 as E1049–85(2011)^{ε1}. DOI: 10.1520/E1049-85R17.

² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ The last approved version of this historical standard is referenced on www.astm.org.

3.1.3 *mean crossings—in fatigue loading*, the number of times that the load-time history crosses the mean-load level with a positive slope (or a negative slope, or both, as specified) during a given length of the history (see Fig. 1).

3.1.3.1 *Discussion*—For purposes related to cycle counting, a mean crossing may be defined as a crossing of the reference load level.

3.1.4 *mean load, P_m —in fatigue loading*, the algebraic average of the maximum and minimum loads in constant amplitude loading, or of individual cycles in spectrum loading,

$$P_m = (P_{\max} + P_{\min})/2 \quad (1)$$

or the integral average of the instantaneous load values or the algebraic average of the peak and valley loads of a spectrum loading history.

3.1.5 *peak—in fatigue loading*, the point at which the first derivative of the load-time history changes from a positive to a negative sign; the point of maximum load in constant amplitude loading (see Fig. 1).

3.1.6 *range—in fatigue loading*, the algebraic difference between successive valley and peak loads (positive range or increasing load range), or between successive peak and valley loads (negative range or decreasing load range); see Fig. 1.

NOTE 2—In spectrum loading, range may have a different definition, depending on the counting method used; for example, “overall range” is defined by the algebraic difference between the largest peak and the smallest valley of a given load-time history.

3.1.6.1 *Discussion*—In cycle counting by various methods, it is common to employ ranges between valley and peak loads, or between peak and valley loads, which are not necessarily successive events. In these practices, the definition of the word “range” is broadened so that events of this type are also included.

3.1.7 *reversal—in fatigue loading*, the point at which the first derivative of the load-time history changes sign (see Fig. 1).

NOTE 3—In constant amplitude loading, a cycle is equal to two reversals.

3.1.8 *spectrum loading—in fatigue loading*, a loading in which all of the peak loads are not equal or all of the valley loads are not equal, or both. (Also known as variable amplitude loading or irregular loading.)

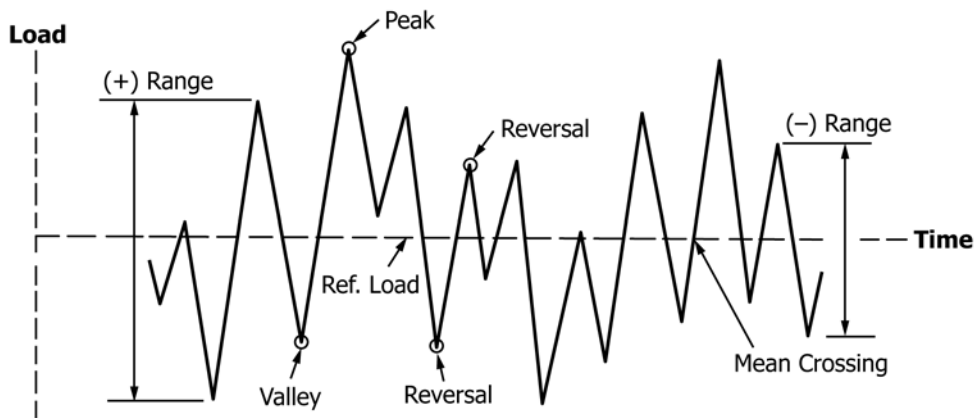


FIG. 1 Basic Fatigue Loading Parameters

3.1.9 *valley*—in fatigue loading, the point at which the first derivative of the load-time history changes from a negative to a positive sign (also known as trough); the point of minimum load in constant amplitude loading (see Fig. 1).

3.2 Definitions of Terms Specific to This Standard:

3.2.1 *load*—used in these practices to denote force, stress, strain, torque, acceleration, deflection, or other parameters of interest.

3.2.2 *reference load*—for spectrum loading, used in these practices to denote the loading level that represents a steady-state condition upon which load variations are superimposed. The reference load may be identical to the mean load of the history, but this is not required.

3.3 For other definitions of terms used in these practices refer to Definitions E912.

4. Significance and Use

4.1 Cycle counting is used to summarize (often lengthy) irregular load-versus-time histories by providing the number of times cycles of various sizes occur. The definition of a cycle varies with the method of cycle counting. These practices cover the procedures used to obtain cycle counts by various methods, including level-crossing counting, peak counting, simple-range counting, range-pair counting, and rainflow counting. Cycle counts can be made for time histories of force, stress, strain, torque, acceleration, deflection, or other loading parameters of interest.

5. Procedures for Cycle Counting

5.1 Level-Crossing Counting:

5.1.1 Results of a level-crossing count are shown in Fig. 2(a). One count is recorded each time the positive sloped portion of the load exceeds a preset level above the reference load, and each time the negative sloped portion of the load exceeds a preset level below the reference load. Reference load crossings are counted on the positive sloped portion of the loading history. It makes no difference whether positive or negative slope crossings are counted. The distinction is made only to reduce the total number of events by a factor of two.

5.1.2 In practice, restrictions on the level-crossing counts are often specified to eliminate small amplitude variations which can give rise to a large number of counts. This may be accomplished by filtering small load excursions prior to cycle counting. A second method is to make no counts at the reference load and to specify that only one count be made between successive crossings of a secondary lower level associated with each level above the reference load, or a secondary higher level associated with each level below the reference load. Fig. 2(b) illustrates this second method. A variation of the second method is to use the same secondary level for all counting levels above the reference load, and another for all levels below the reference load. In this case the levels are generally not evenly spaced.

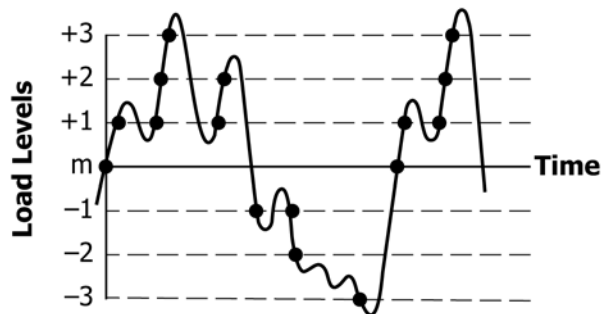
5.1.3 The most damaging cycle count for fatigue analysis is derived from the level-crossing count by first constructing the largest possible cycle, followed by the second largest, etc., until all level crossings are used. Reversal points are assumed to occur halfway between levels. This process is illustrated by Fig. 2(c). Note that once this most damaging cycle count is obtained, the cycles could be applied in any desired order, and this order could have a secondary effect on the amount of damage. Other methods of deriving a cycle count from the level-crossings count could be used.

5.2 Peak Counting:

5.2.1 Peak counting identifies the occurrence of a relative maximum or minimum load value. Peaks above the reference load level are counted, and valleys below the reference load level are counted, as shown in Fig. 3(a). Results for peaks and valleys are usually reported separately. A variation of this method is to count all peaks and valleys without regard to the reference load.

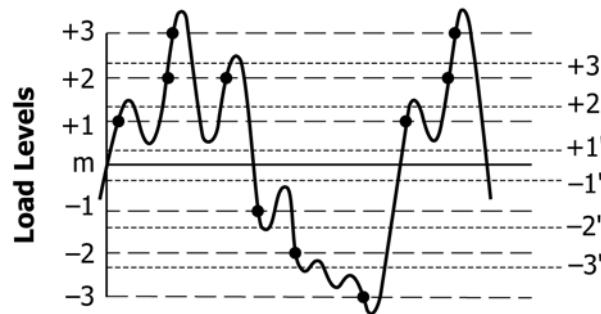
5.2.2 To eliminate small amplitude loadings, mean-crossing peak counting is often used. Instead of counting all peaks and valleys, only the largest peak or valley between two successive mean crossings is counted as shown in Fig. 3(b).

5.2.3 The most damaging cycle count for fatigue analysis is derived from the peak count by first constructing the largest possible cycle, using the highest peak and lowest valley, followed by the second largest cycle, etc., until all peak counts



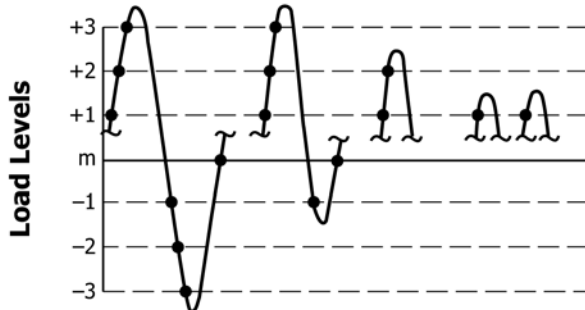
Level	Counts
+3	2
+2	3
+1	5
0	2
-1	2
-2	1
-3	1

(a)—Level Crossing Counting



Level	Counts
+3	2
+2	3
+1	2
0	0
-1	1
-2	1
-3	1

(b)—Restricted Level Crossing Counting



(c)—Cycles Derived from Level Crossing Count of (a)

Range (levels)	Cycle Counts
7	1
6	0
5	1
4	0
3	0
2	1
1	2

FIG. 2 Level-Crossing Counting Example

are used. This process is illustrated by Fig. 3(c). Note that once this most damaging cycle count is obtained, the cycles could be applied in any desired order, and this order could have a secondary effect on the amount of damage. Alternate methods of deriving a cycle count, such as randomly selecting pairs of peaks and valleys, are sometimes used.

5.3 Simple-Range Counting:

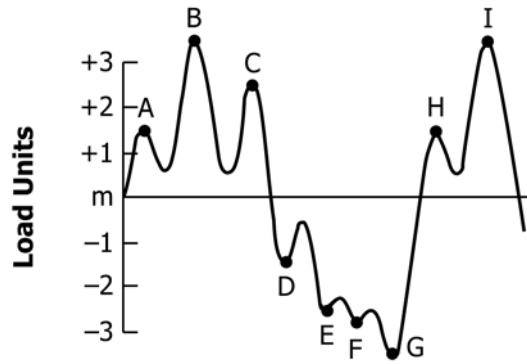
5.3.1 For this method, a range is defined as the difference between two successive reversals, the range being positive when a valley is followed by a peak and negative when a peak is followed by a valley. The method is illustrated in Fig. 4. Positive ranges, negative ranges, or both, may be counted with this method. If only positive or only negative ranges are

counted, then each is counted as one cycle. If both positive and negative ranges are counted, then each is counted as one-half cycle. Ranges smaller than a chosen value are usually eliminated before counting.

5.3.2 When the mean value of each range is also counted, the method is called simple range-mean counting. For the example of Fig. 4, the result of a simple range-mean count is given in X1.1 in the form of a range-mean matrix.

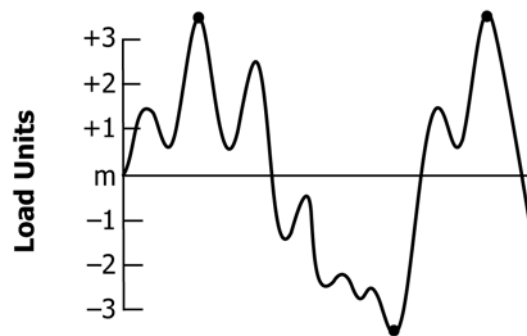
5.4 Rainflow Counting and Related Methods:

5.4.1 A number of different terms have been employed in the literature to designate cycle-counting methods which are similar to the rainflow method. These include range-pair



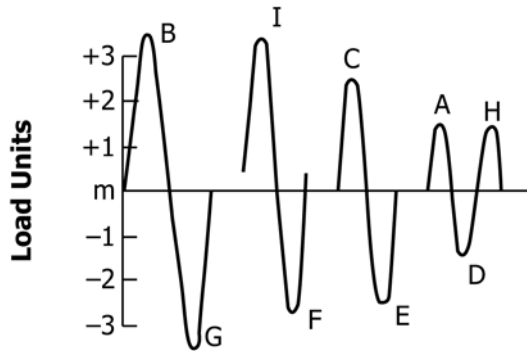
(a)—Peak Counting

Peak	Counts
+3.5	2
+2.5	1
+1.5	2
-1.5	1
-2.5	1
-2.7	1
-3.5	1



(b)—Mean Crossing Peak Counting

Peak	Counts
+3.5	2
-3.5	1



(c)—Cycles Derived from Peak Count of (a)

Range (units)	Cycle Counts
7	1
6.2	1
5	1
3	1.5

FIG. 3 Peak Counting Example

counting (1, 2),⁴ the Hayes method (3), the original rainflow method (4-6), range-pair-range counting (7), ordered overall range counting (8), racetrack counting (9), and hysteresis loop counting (10). If the load history begins and ends with its maximum peak, or with its minimum valley, all of these give identical counts. In other cases, the counts are similar, but not generally identical. Three methods in this class are defined here: range-pair counting, rainflow counting, and a simplified method for repeating histories.

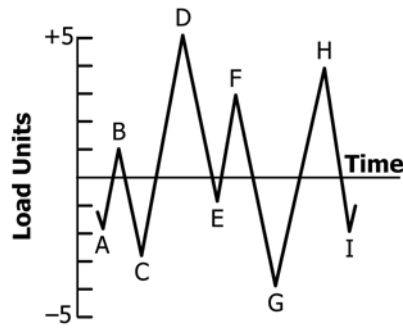
⁴ The boldface numbers in parentheses refer to the list of references appended to these practices.

5.4.2 The various methods similar to the rainflow method may be used to obtain cycles and the mean value of each cycle; they are referred to as two-parameter methods. When the mean value is ignored, they are one-parameter methods, as are simple-range counting, peak counting, etc.

5.4.3 *Range-Pair Counting*—The range-paired method counts a range as a cycle if it can be paired with a subsequent loading in the opposite direction. Rules for this method are as follows:

5.4.3.1 Let *X* denote range under consideration; and *Y*, previous range adjacent to *X*.

(1) Read next peak or valley. If out of data, go to Step 5.



Range (units)	Cycle Counts	Events
10	0	
9	0	
8	1.0	C-D, G-H
7	0.5	F-G
6	1.0	D-E, H-I
5	0	
4	1.0	B-C, E-F
3	0.5	A-B
2	0	
1	0	

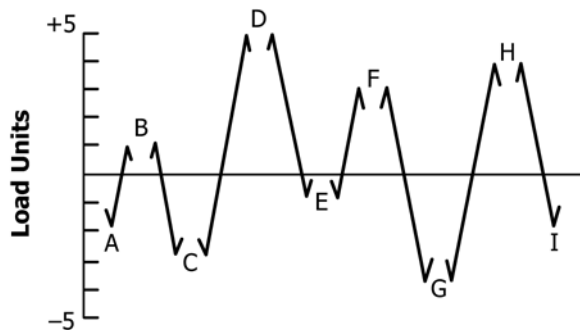


FIG. 4 Simple Range Counting Example—Both Positive and Negative Ranges Counted

(2) If there are less than three points, go to Step 1. Form ranges X and Y using the three most recent peaks and valleys that have not been discarded.

(3) Compare the absolute values of ranges X and Y .

(a) If $X < Y$, go to Step 1.

(b) If $X \geq Y$, go to Step 4.

(4) Count range Y as one cycle and discard the peak and valley of Y ; go to Step 2.

(5) The remaining cycles, if any, are counted by starting at the end of the sequence and counting backwards. If a single range remains, it may be counted as a half or full cycle.

5.4.3.2 The load history in Fig. 4 is replotted as Fig. 5(a) and is used to illustrate the process. Details of the cycle counting are as follows:

(1) $Y = |A-B|$; $X = |B-C|$; and $X > Y$. Count $|A-B|$ as one cycle and discard points A and B . (See Fig. 5(b).) Note that a cycle is formed by pairing range $A-B$ and a portion of range $B-C$.)

(2) $Y = |C-D|$; $X = |D-E|$; and $X < Y$.

(3) $Y = |D-E|$; $X = |E-F|$; and $X < Y$.

(4) $Y = |E-F|$; $X = |F-G|$; and $X > Y$. Count $|E-F|$ as one cycle and discard points E and F . (See Fig. 5(c).)

(5) $Y = |C-D|$; $X = |D-G|$; and $X > Y$. Count $|C-D|$ as one cycle and discard points C and D . (See Fig. 5(d).)

(6) $Y = |G-H|$; $X = |H-I|$; and $X < Y$. Go to the end and count backwards.

(7) $Y = |H-I|$; $X = |G-H|$; and $X > Y$. Count $|H-I|$ as one cycle and discard points H and I . (See Fig. 5(e).)

(8) End of counting. See the table in Fig. 5 for a summary of the cycles counted in this example, and see Appendix X1.2 for this cycle count in the form of a range-mean matrix.

5.4.4 Rainflow Counting:

5.4.4.1 Rules for this method are as follows: let X denote range under consideration; Y , previous range adjacent to X ; and S , starting point in the history.

(1) Read next peak or valley. If out of data, go to Step 6.

(2) If there are less than three points, go to Step 1. Form ranges X and Y using the three most recent peaks and valleys that have not been discarded.

(3) Compare the absolute values of ranges X and Y .

(a) If $X < Y$, go to Step 1.

(b) If $X \geq Y$, go to Step 4.

(4) If range Y contains the starting point S , go to Step 5; otherwise, count range Y as one cycle; discard the peak and valley of Y ; and go to Step 2.

(5) Count range Y as one-half cycle; discard the first point (peak or valley) in range Y ; move the starting point to the second point in range Y ; and go to Step 2.

(6) Count each range that has not been previously counted as one-half cycle.

5.4.4.2 The load history of Fig. 4 is replotted as Fig. 6(a) and is used to illustrate the process. Details of the cycle counting are as follows:

(1) $S = A$; $Y = |A-B|$; $X = |B-C|$; $X > Y$. Y contains S , that is, point A . Count $|A-B|$ as one-half cycle and discard point A ; $S = B$. (See Fig. 6(b).)

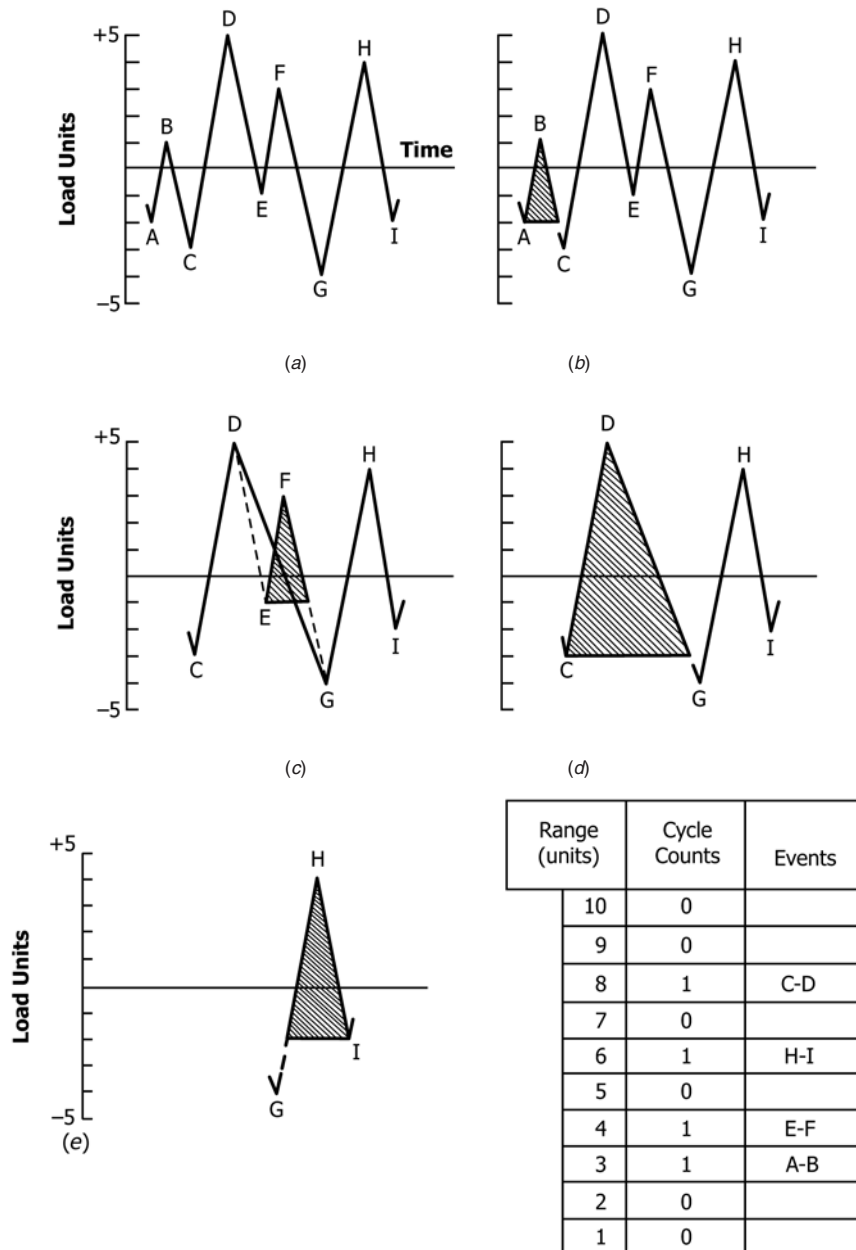


FIG. 5 Range-Pair Counting Example

(2) $Y = |B-C|$; $X = |C-D|$; $X > Y$. Y contains S , that is, point B . Count $|B-C|$ as one-half cycle and discard point B ; $S = C$. (See Fig. 6(c).)

(3) $Y = |C-D|$; $X = |D-E|$; $X < Y$.

(4) $Y = |D-E|$; $X = |E-F|$; $X < Y$.

(5) $Y = |E-F|$; $X = |F-G|$; $X > Y$. Count $|E-F|$ as one cycle and discard points E and F . (See Fig. 6(d). Note that a cycle is formed by pairing range $E-F$ and a portion of range $F-G$.)

(6) $Y = |C-D|$; $X = |D-G|$; $X > Y$; Y contains S , that is, point C . Count $|C-D|$ as one-half cycle and discard point C . $S = D$. (See Fig. 6(e).)

(7) $Y = |D-G|$; $X = |G-H|$; $X < Y$.

(8) $Y = |G-H|$; $X = |H-I|$; $X < Y$. End of data.

(9) Count $|D-G|$ as one-half cycle, $|G-H|$ as one-half cycle, and $|H-I|$ as one-half cycle. (See Fig. 6(f).)

(10) End of counting. See the table in Fig. 6 for a summary of the cycles counted in this example, and see Appendix X1.3 for this cycle count in the form of a range-mean matrix.

5.4.5 Simplified Rainflow Counting for Repeating Histories:

5.4.5.1 It may be desirable to assume that a typical segment of a load history is repeatedly applied. Here, once either the maximum peak or minimum valley is reached for the first time, the range-pair count is identical for each subsequent repetition of the history. The rainflow count is also identical for each subsequent repetition of the history, and for these subsequent repetitions, the rainflow count is the same as the range-pair count. Such a repeating history count contains no half cycles, only full cycles, and each cycle can be associated with a closed stress-strain hysteresis loop (4, 10-12). Rules for obtaining

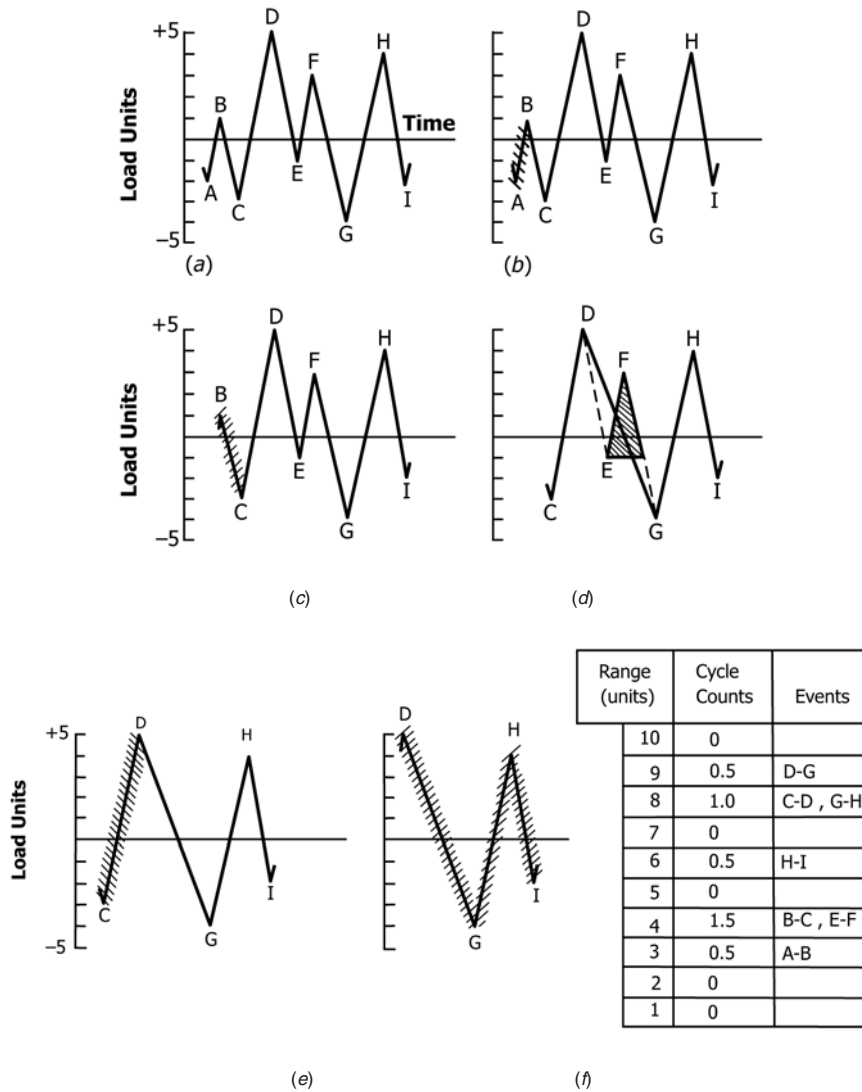


FIG. 6 Rainflow Counting Example

such a repeating history cycle count, called “simplified rainflow counting for repeating histories” are as follows:

5.4.5.2 Let X denote range under consideration; and Y , previous range adjacent to X .

(1) Arrange the history to start with either the maximum peak or the minimum valley. (More complex procedures are available that eliminate this requirement; see (Ref 12).

(2) Read the next peak or valley. If out of data, STOP.

(3) If there are less than three points, go to Step 2. Form ranges X and Y using the three most recent peaks and valleys that have not been discarded.

(4) Compare the absolute values of ranges X and Y .

(a) If $X < Y$, go to Step 2.

(b) If $X \geq Y$, go to Step 5.

(5) Count range Y as one cycle; discard the peak and valley of Y ; and go to Step 3.

5.4.5.3 The loading history of Fig. 4 is plotted as a repeating load history in Fig. 7(a) and is used to illustrate the process. Rearranging the history to start with the maximum peak gives

Fig. 7(b), Reversal Points A , B , and C being moved to the end of the history. Details of the cycle counting are as follows:

(1) $Y = |D-E|$; $X = |E-F|$; $X < Y$.

(2) $Y = |E-F|$; $X = |F-G|$; $X > Y$. Count $|E-F|$ as one cycle and discard points E and F . (See Fig. 7(c).) Note that a cycle is formed by pairing range $E-F$ and a portion of range $F-G$.

(3) $Y = |D-G|$; $X = |G-H|$; $X < Y$.

(4) $Y = |G-H|$; $X = |H-A|$; $X < Y$.

(5) $Y = |H-A|$; $X = |A-B|$; $X < Y$.

(6) $Y = |A-B|$; $X = |B-C|$; $X > Y$. Count $|A-B|$ as one cycle and discard points A and B . (See Fig. 7(d).)

(7) $Y = |G-H|$; $X = |H-C|$; $X < Y$.

(8) $Y = |H-C|$; $X = |C-D|$; $X > Y$. Count $|H-C|$ as one cycle and discard points H and C . (See Fig. 7(e).)

(9) $Y = |D-G|$; $X = |G-D|$; $X = Y$. Count $|D-G|$ as one cycle and discard points D and G . (See Fig. 7(f).)

(10) End of counting. See the table in Fig. 7 for a summary of the cycles counted in this example, and see Appendix X1.4 for this cycle count in the form of a range-mean matrix.

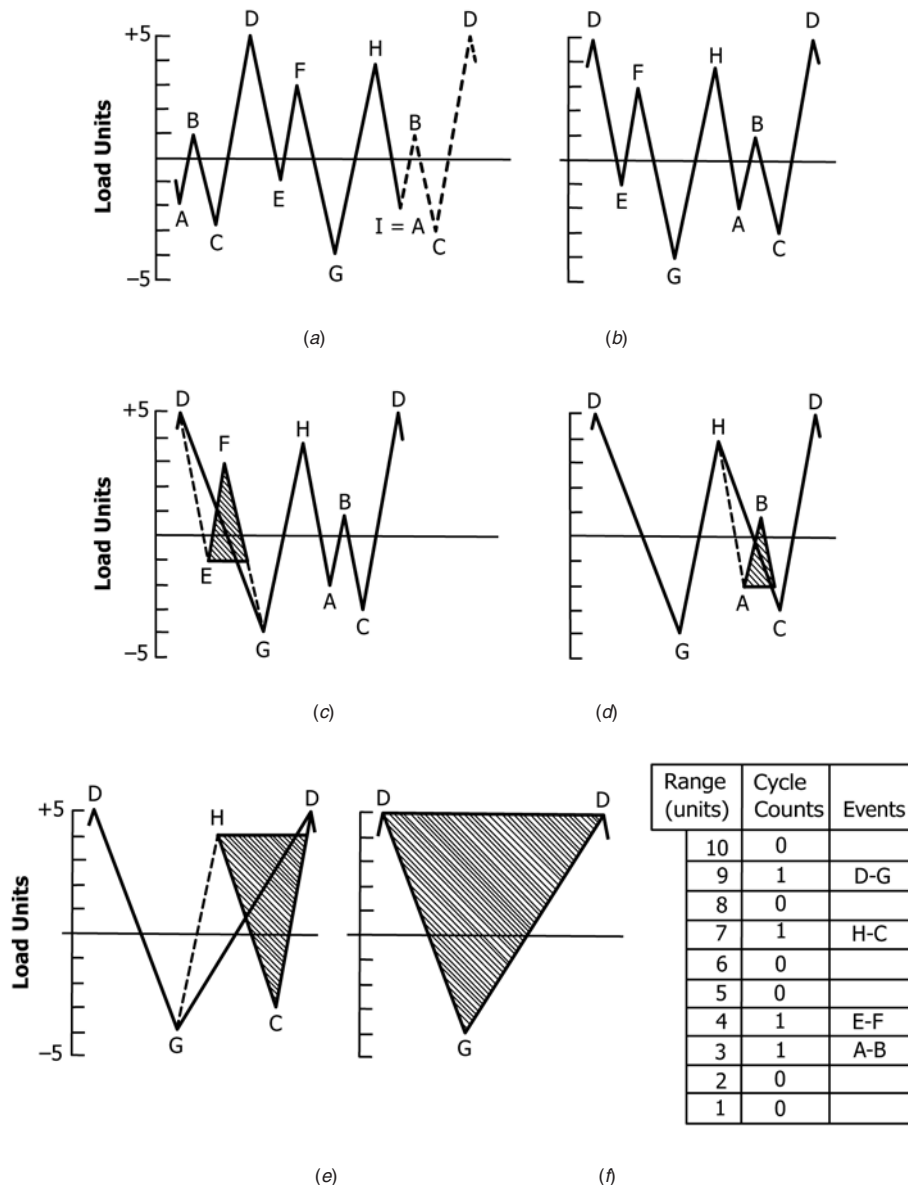


FIG. 7 Example of Simplified Rainflow Counting for a Repeating History

APPENDIX

(Nonmandatory Information)

X1. RANGE-MEAN MATRIXES FOR CYCLE COUNTING EXAMPLES

X1.1 The Tables X1.1-X1.4 which follow correspond to the cycle-counting examples of Figs. 4-7. In each case, the table is a matrix giving the number of cycles counted at the indicated combinations of range and mean. Note that these examples are

the ones illustrating (1) simple-range counting, (2) range-pair counting, (3) rainflow counting, and (4) simplified rainflow counting for repeating histories, which are the methods that can be used as two-parameter methods.

TABLE X1.1 Simple Range-Mean Counting (Fig. 4)

Range Units	Mean Units								
	–2.0	–1.5	–1.0	–0.5	0	+ 0.5	+ 1.0	+ 1.5	+ 2.0
10
9
8	0.5	...	0.5
7	0.5
6	0.5	...	0.5
5
4	0.5	0.5
3	0.5
2
1

TABLE X1.2 Range-Pair Counting (Fig. 5)

Range (units)	Mean (units)								
	–2.0	–1.5	–1.0	–0.5	0	+ 0.5	+ 1.0	+ 1.5	+ 2.0
10
9
8	1
7
6	1
5
4	1
3	1
2
1

TABLE X1.3 Rainflow Counting (Fig. 6)

Range (units)	Mean (units)								
	–2.0	–1.5	–1.0	–0.5	0	+ 0.5	+ 1.0	+ 1.5	+ 2.0
10
9	0.5
8	0.5	...	0.5
7
6	0.5
5
4	0.5	1.0
3	0.5
2
1

TABLE X1.4 Simplified Rainflow Counting for Repeating Histories (Fig. 7)

Range Units	Mean Units								
	–2.0	–1.5	–1.0	–0.5	0	+ 0.5	+ 1.0	+ 1.5	+ 2.0
10
9	1
8
7	1
6
5
4	1
3	1
2
1



REFERENCES

- (1) Anonymous, “*The Strain Range Counter*,” VTO/M/416, Vickers-Armstrongs Ltd. (now British Aircraft Corporation Ltd.), Technical Office, Weybridge, Surrey, England, April 1955.
- (2) Burns, A., “Fatigue Loadings in Flight: Loads in the Tailpipe and Fin of a Varsity,” *Technical Report C.P. 256*, Aeronautical Research Council, London, 1956.
- (3) Hayes, J. E., “Fatigue Analysis and Fail-Safe Design,” *Analysis and Design of Flight Vehicle Structures*, E. F. Bruhn, ed., Tristate Offset Co., Cincinnati, OH, 1965, pp. C13-1 - C13-42.
- (4) Matsuishi, M., and Endo, T., “Fatigue of Metals Subjected to Varying Stress,” *Paper presented to Japan Society of Mechanical Engineers*, Fukuoka, Japan, March 1968. See also Endo, T., et al., “Damage Evaluation of Metals for Random or Varying Loading,” *Proceedings of the 1974 Symposium on Mechanical Behavior of Materials*, Vol 1, The Society of Materials Science, Japan, 1974, pp. 371–380.
- (5) Anzai, H., and Endo, T., “On-Site Indication of Fatigue Damage Under Complex Loading,” *International Journal of Fatigue*, Vol 1, No. 1, 1979, pp. 49–57.
- (6) Endo, T., and Anzai, H., “Redefined Rainflow Algorithm: P/V Difference Method,” *Japan Society of Materials Science*, Japan, Vol 30, No. 328, 1981, pp. 89–93.
- (7) van Dijk, G. M., “Statistical Load Data Processing,” *Paper presented at Sixth ICAF Symposium*, Miami, FL, May 1971. See also NLR MP 71007U, National Aerospace Laboratory, Amsterdam, 1971.
- (8) Fuchs, H. O., et al., “Shortcuts in Cumulative Damage Analysis,” *Fatigue Under Complex Loading: Analyses and Experiments*, Vol AE-6, R. M. Wetzel, ed., The Society of Automotive Engineers, 1977, pp. 145–162.
- (9) Nelson, D. V., and Fuchs, H. O., “Predictions of Cumulative Fatigue Damage Using Condensed Load Histories,” *Fatigue Under Complex Loading: Analyses and Experiments*, Vol AE-6, R. M. Wetzel, ed., The Society of Automotive Engineers, 1977, pp. 163–187.
- (10) Richards, F. D., LaPointe, N. R., and Wetzel, R. M., “A Cycle Counting Algorithm for Fatigue Damage Analysis,” *Automotive Engineering Congress*, Paper No. 740278, Society of Automotive Engineers, Detroit, MI, February 1974.
- (11) Dowling, N. E., “Fatigue Failure Predictions for Complicated Stress-Strain Histories,” *Journal of Materials*, ASTM, Vol 7, No. 1, March 1972, pp. 71–87.
- (12) Downing, S. D., and Socie, D. F., “Simple Rainflow Counting Algorithms,” *International Journal of Fatigue*, Vol 4, No. 1, January 1982, pp. 31–40.

ASTM International takes no position respecting the validity of any patent rights asserted in connection with any item mentioned in this standard. Users of this standard are expressly advised that determination of the validity of any such patent rights, and the risk of infringement of such rights, are entirely their own responsibility.

This standard is subject to revision at any time by the responsible technical committee and must be reviewed every five years and if not revised, either reapproved or withdrawn. Your comments are invited either for revision of this standard or for additional standards and should be addressed to ASTM International Headquarters. Your comments will receive careful consideration at a meeting of the responsible technical committee, which you may attend. If you feel that your comments have not received a fair hearing you should make your views known to the ASTM Committee on Standards, at the address shown below.

This standard is copyrighted by ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959, United States. Individual reprints (single or multiple copies) of this standard may be obtained by contacting ASTM at the above address or at 610-832-9585 (phone), 610-832-9555 (fax), or service@astm.org (e-mail); or through the ASTM website (www.astm.org). Permission rights to photocopy the standard may also be secured from the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, Tel: (978) 646-2600; <http://www.copyright.com/>