AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 22

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SCHEDULE

- 14 Apr - Exam Review
- 19 Apr - Damage Tolerance, Homework 8 Due
- 21 Apr - Exam 2
- 26 Apr - Exam Solutions, Damage Tolerance
- 28 Apr - SPTE, AFGROW, Finite Elements

OUTLINE

1. crack growth retardation
2. exam 2
3. stress based fatigue
4. strain based fatigue
5. fracture based fatigue

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- The Wheeler method reduces da/dN, the Willenborg model reduces ΔK, and the Closure model increases R (increases σ_{min})

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- and the constant, m is to be determined experimentally

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- The effective stress intensity factor is given by

$$K_{max,eff} = K_{max,i} - \left[K_{max,OL} \sqrt{1 - \frac{\Delta a_i}{r_{pol}}} - K_{max,i} \right] \tag{22.3}$$

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- And the correction factor, ϕ_i is given by

$$\phi_i = \frac{1 - K_{TH}/K_{max,i}}{s_{ol} - 1} \tag{22.5}$$

WILLENBORG EXAMPLE

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- Where C_{f0} is the value of the Closure Factor at $R = 0$

- When using the closure model, we replace R with C_f

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- If the model we are using is in terms of ΔK we will also need to use $\Delta K = (1 - C_f)K_{max}$

CLOSURE EXAMPLE

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    1. Compressive underloads are uncommon in airframes
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    4. Structures with large compressive loads are not generally subject to crack propagation problems

EXAM 2

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- Make sure you can integrate the Paris Law equation (see Homework 8 problems 1 and 2)
- No table look-ups (stress intensity factors, paris/walker law constants will be given in problem)

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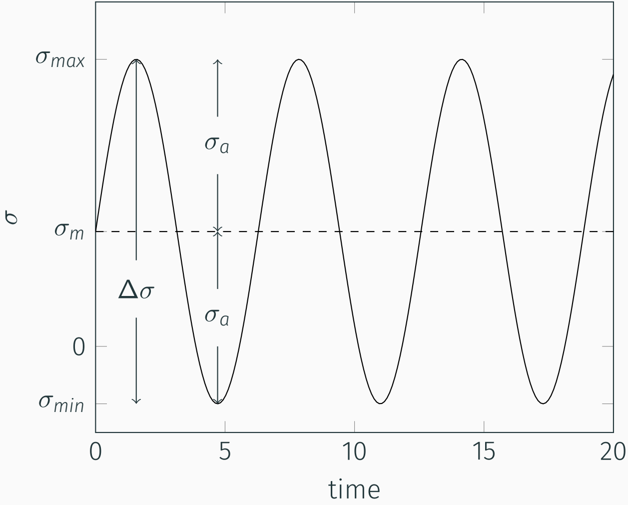
- This point varies by material, but can be found using strain-based fatigue analysis


σ_{max}
σ_a
σ
σ_m
$\Delta\sigma$
σ_a
0
σ_{min}
0
5
10
15
20
time

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- σ_m is the mean stress, and can sometimes be zero, but this is not always the case
- σ_a is the stress amplitude, and is the variation about the mean
- We can express all of these in terms of the maximum and minimum stress

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \tag{22.8}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \tag{22.9}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{22.10}$$

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- And the amplitude ratio, A is defined as

$$A = \frac{\sigma_a}{\sigma_m} \tag{22.12}$$

- There are some useful relationships between the above equations

$$\Delta\sigma = 2\sigma_a = \sigma_{max}(1 - R) \tag{22.13a}$$

$$\sigma_m = \frac{\sigma_{max}}{2}(1 + R) \tag{22.13b}$$

$$R = \frac{1 - A}{1 + A} \tag{22.13c}$$

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- Rotating 4-point bend is one of the most common modern methods
- Tensile test machines can be used, but require much more time

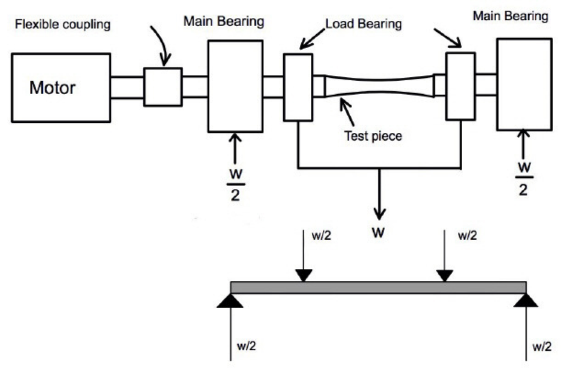

Figure 1: Four-point bend gives uniform stress (along top and bottom surfaces)

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- Usually S_a (the nominal stress equivalent of σ_a) is plotted versus N (the number of cycles)

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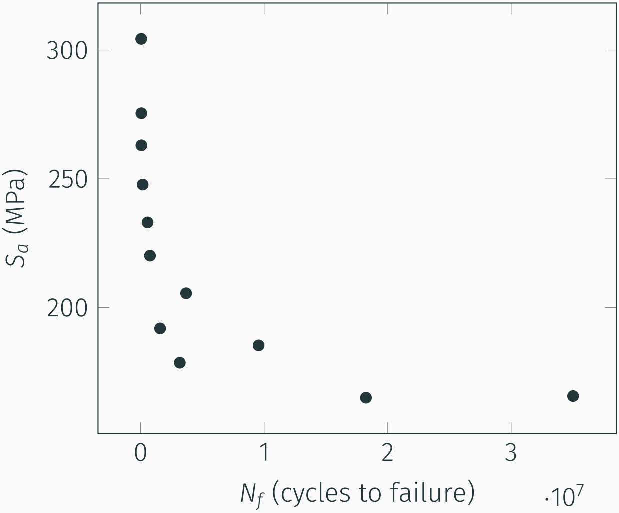
- In the following plot, if only one test was performed for each point, the total number of cycles tested would be about $7.3x10^7$
- For a 100 Hz machine, this represents over 200 hours of consecutive testing
- Each repetition would further increase the test time required

STRESS LIFE CURVES



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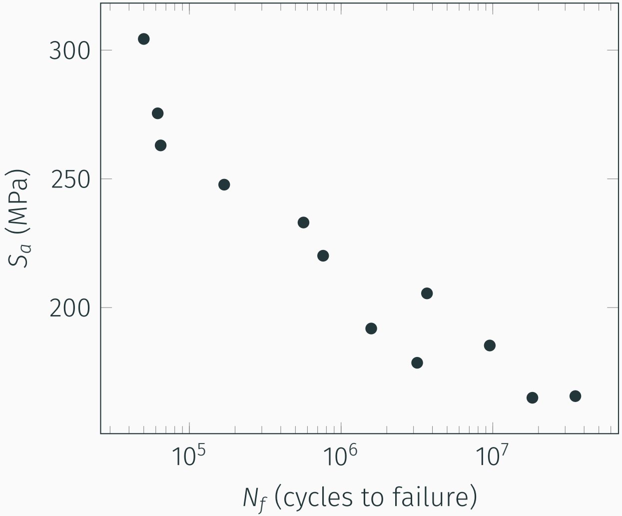
- It is also very difficult to differentiate between low-cycle fatigue failure stresses
- Instead S-N curves are often plotted on a semi-log or log-log scale, so pay attention to the axes


300
250
200
S_a (MPa)
10^5
10^6
10^7
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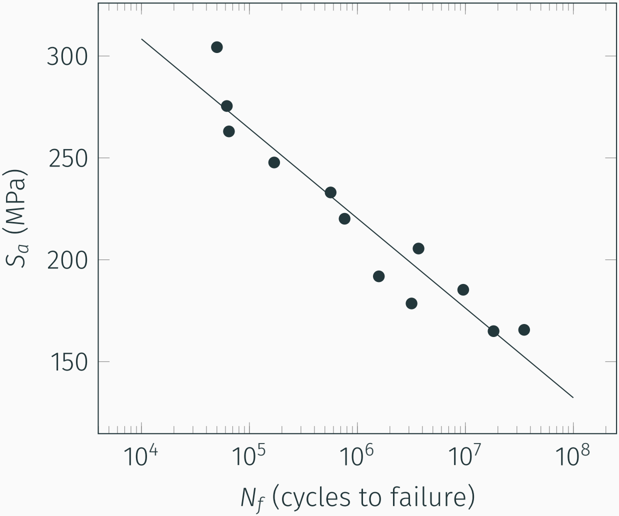
- $$\sigma_a = \sigma_f' \left(2N_f\right)^b \tag{22.15}$$
- σ_f' and b are often considered material properties and can often be looked up on a table (p. 235)



300
250
200
150
S_a (MPa)
10^4
10^5
10^6
10^7
10^8
N_f (cycles to failure)

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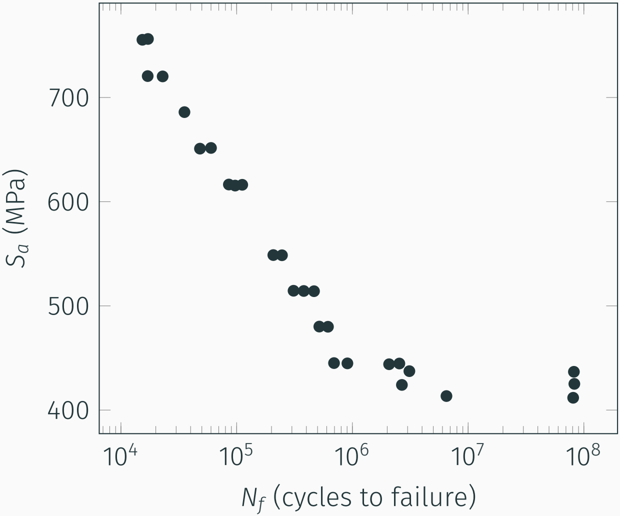
- In these materials, σ_e is considered to be a material property
- This phenomenon is not typical of aluminum or copper alloys, but is sometimes arbitrarily assigned using whatever the failure stress is at some large number of cycles (10^7 or 10^8)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$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \ldots = \sum \frac{N_i}{N_{fi}} = 1 \qquad (22.16)$$

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$$B\left[\sum \frac{N_i}{N_{if}}\right]_{rep} = 1 \tag{22.17}$$

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- S-N curves for these are reported in different ways, but commonly σ_{max} replaces σ_a on the y-axis
- One useful way of representing these data, instead of many S-N curves, is a constant-life diagram
- It is created by taking points from the S-N curves and plotting a line through constant N_f values

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- In general, the Goodman line is a good fit for brittle materials (steels) while the Gerber parabola is a better fit for more ductile materials

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- This is known as the Morrow Equation
- For steels, $\sigma_f' \approx \tilde{\sigma}_{fB}$, but for aluminums these values can be significantly different, and better agreement is found using $\tilde{\sigma}_{fB}$.

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- In general, it is best to use a form that matches your data
- If data is lacking, the SWT (22.22) and Morrow (22.21) equations generally provide the best fit

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GENERAL STRESS

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- Often combined loads from different sources introduce stresses which are not uni-axial
- For ductile materials, good agreement has been found using an effective stress amplitude, similar to the octahedral shear yield criterion

$$\bar{\sigma}_a = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{xa} - \sigma_{ya})^2 + (\sigma_{ya} - \sigma_{za})^2 + (\sigma_{za} - \sigma_{xa})^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} \tag{22.23}$$

- The effective mean stress is given by

$$\bar{\sigma}_m = \bar{\sigma}_{xm} + \bar{\sigma}_{ym} + \bar{\sigma}_{zm} \tag{22.24}$$

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Table 1: Table of α values for Peterson notch sensitivity equation

Material	α (mm)	α (in)
Aluminum alloys	0.51	0.02
Annealed or low-carbon steels	0.25	0.01
Quenched and tempered steels	0.064	0.0025

- For high-strength steels, a more specific α estimate can be found

$$\alpha = 0.025\left(\frac{2070}{\sigma_u}\right)^{1.8} \qquad \text{mm} \qquad \sigma_u \geq 550 \text{ MPa} \tag{22.28}$$

$$\alpha = 0.001\left(\frac{300}{\sigma_u}\right)^{1.8} \qquad \text{in} \qquad \sigma_u \geq 80 \text{ ksi} \tag{22.29}$$

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$$\alpha_{torsion} = 0.6\alpha \tag{22.30}$$

STRAIN BASED FATIGUE

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- For a non zero mean stress, this point represents (ϵ_a, N^*), we can now solve for N_f using 22.37

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- There is no convenient solution method for this form, and it generally must be solved numerically, or plotted with many families of σ_m

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- This method can also be solved graphically if a plot of $\sigma_{max}\epsilon_a$ is made using zero-mean data. All we need to do is find the new $\sigma_{max}\epsilon_a$ point to find a new N_f

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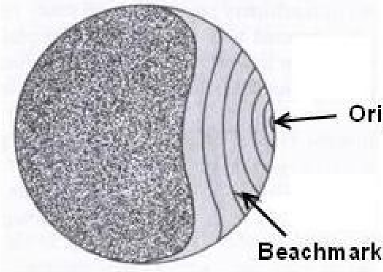
- The SWT approach is very good for general use, but is non-conservative with a compressive mean stress

FRACTURE BASED FATIGUE

FRACTURE SURFACE



Fatigue Fracture with Beachmarks

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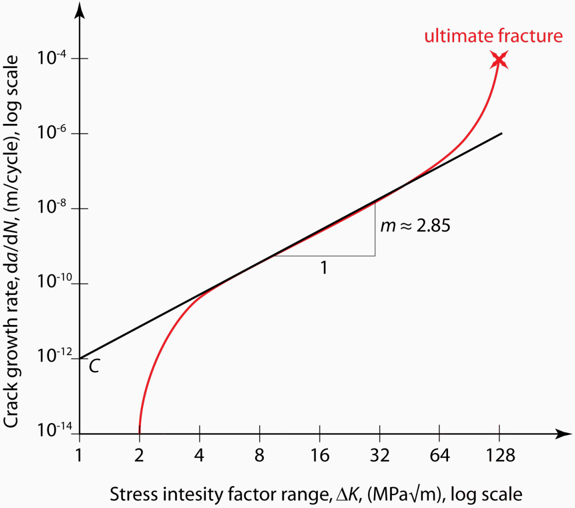
- This chart is then commonly divided into three regions

DA-DN VS K



ultimate fracture
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
$m \approx 2.85$
1
C
Crack growth rate, d*a*/d*N*, (m/cycle), log scale
1
2
4
8
16
32
64
128
Stress intesity factor range, Δ*K*, (MPa√m), log scale

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- The same considerations for thickness apply as with fracture toughness (K_c vs. K_{Ic})
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R EFFECTS

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- In general, R dependence vanishes for $R > 0.8$ or $R < -0.3$. This effect is known as the band width

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$$N = 10^4\left(\frac{m_t}{z\sigma_{max}}\right)^p \int_{L_0}^{L_f}\frac{dL}{\left(n\sqrt{\pi L/n}\beta\right)^p} \tag{22.45}$$

- In this form, the term $10^4 \left(\frac{m_t}{z\sigma_{max}}\right)^p$ is strictly from the applied load and material, while $\int_{L_0}^{L_f} \frac{dL}{\left(n\sqrt{\pi L/n\beta}\right)^p}$ is from geometry

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$$G = \left[\int_{L_0}^{L_f} \frac{dL}{\left(n\sqrt{\pi L/n}\beta\right)^p}\right]^{-1/p} \tag{22.46}$$

- In this form, the term $10^4\left(\frac{m_t}{z\sigma_{max}}\right)^p$ is strictly from the applied load and material, while $\int_{L_0}^{L_f}\frac{dL}{\left(n\sqrt{\pi L/n}\beta\right)^p}$ is from geometry
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- Using this method, G is typically looked up from a chart (such as on p. 369)

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- Which simplifies to

$$\sum_i\left(z\sigma_{max}\right)_i^p N_i = (S)^p \tag{22.50}$$

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- ASTM E1049-85 ”Standard Practices for Cycle Counting in Fatigue Analysis”

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5. When end of data is reached, count each range as 1/2-cycle

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4. Remaining cycles are counted backwards from end of history