1

Stress Intensity - measure of the magnitude of the local stress field. Gives a way to compare singular stress fields near a crack tip.

2

For a center-cracked panel with uniform remote stress, we must decide whether to use the Finite Width or Infinite Width formula. Since Width is given as a parameter, we will use the Finite Width equation.

$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)} \tag{2.2a}$$

For $W/2a = 4$ we can substitute to find

$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi/8)}$$

In [8]:
```
import numpy as np
K_I_center = np.sqrt(1/np.cos(np.pi/8))
K_I_center
```

Out[8]: 1.040380795811031

Thus

$$K_I = 1.040\sigma\sqrt{\pi a}$$

For an edge-cracked panel, since $a/w < 0.6$ we use the simpler finite-width formula:

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.71\left(\frac{a}{W}\right)^3 + 30.82\left(\frac{a}{W}\right)^4\right] \tag{2.4a}$$

Substituting $W/a = 4$ gives

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.231/4 + 10.55/16 - 21.71/4^3 + 30.82/4^4\right]$$

In [9]:
```
K_I_edge = 1.12 - 0.231/4 + 10.55/16 - 21.71/(4**3) + 30.82/(4**4)
K_I_edge
```

Out[9]: 1.5027968750000003

Thus

$$K_I = 1.50\sigma\sqrt{\pi a}$$

We can compare the two results, $K_{I,edge}/K_{I,center}$ to see the different effect the crack has for center and edge cracks

In [10]:
```
K_I_edge/K_I_center
```

Out[10]:
```
1.444468103458688
```

Thus a crack at the edge (for this given width) has a 44% stronger effect on the local stress than a center crack.

If we look at half of the center-crack specimen (so that it looks like an edge crack), there is an additional constraint (zero slope in y-deformation), this works to decrease the opening stress somewhat relative to the edge-cracked case.

3

For a through crack on one side of the hole, we use 2.12

$$K_I = \sigma\sqrt{\pi c}\beta$$

$$\beta = \beta_1 + \beta_2 \tag{2.12a}$$

$$\beta_1 = \beta_3 F_w F_{ww} \tag{2.12b}$$

$$\beta_2 = \frac{\sigma_{br}}{\sigma} F_4 F_w F_{ww} \tag{2.12c}$$

$$\beta_3 = 0.7071 + 0.7548\frac{R}{R+c} + 0.3415\left(\frac{R}{R+c}\right)^2 + 0.6420\left(\frac{R}{R+c}\right)^3 + 0.9196\left(\frac{R}{R+c}\right)^4 \tag{2.12d}$$

$$F_4 = 0.9580 + 0.2561\frac{c}{R} - 0.00193\left(\frac{c}{R}\right)^{2.5} - 0.9804\left(\frac{c}{R}\right)^{0.5} \tag{2.12e}$$

$$F_w = \sqrt{\sec\frac{\pi R}{W}\sec\frac{\pi(R+c/2)}{W-c}} \tag{2.12f}$$

$$F_{ww} = 1 - N^{-\frac{W}{D}}\left(\frac{2c}{W-D}\right)^{\frac{W}{D}+0.5} \tag{2.12g}$$

Where

$$\sigma_{br} = \frac{P}{Dt} = 0$$

We also check $2.65 - 0.24\left(2.75 - \frac{W}{D}\right)^2$ to find N

In [12]:
```
c = .125
D = .25
W = 7.0
R = D/2
print(2.65 - 0.24*(2.75-W/D)**2)
```

```
-150.36499999999998
```

Since this is less than 2.275, we use $N = 2.275$

In [14]:
```
N = 2.275
F_ww = 1-N**(-W/D)*(2*c/(W-D))**(W/D+0.5)
F_w = np.sqrt(1/np.cos(np.pi*R/W)/np.cos(np.pi*(R+c/2)/(W-c)))
F_4 = 0.958+0.2561*c/R-0.00193*(c/R)**2.5-0.9804*(c/R)**0.5
beta_3 = 0.7071 + 0.7548*R/(R+c) + 0.3415*(R/(R+c))**2 + \
    0.6420*(R/(R+c))**3 + + 0.9196*(R/(R+c))**4
beta_2 = 0
beta_1 = beta_3*F_w*F_ww
beta = beta_1 + beta_2
s = 9000/(7*.157)
K_I = s*np.sqrt(np.pi*c)*beta
print(K_I)
```

```
6728.055613427787
```

Which gives $K_I = 6.73 \text{ ksi}\sqrt{\text{in.}}$

For symmetric through cracks around a hole we use the formula

$$K_I = \sigma\sqrt{\pi c}\beta$$

$$\beta = \beta_1 + \beta_2 \tag{2.11a}$$

$$\beta_1 = F_{c/R}F_wF_{ww} \tag{2.11b}$$

$$\beta_2 = \frac{\sigma_{br}}{\sigma}F_3F_wF_{ww} \tag{2.11c}$$

$$F_{c/R} = \frac{3.404 + 3.8172\frac{c}{R}}{1 + 3.9273\frac{c}{R} - 0.00695\left(\frac{c}{R}\right)^2} \tag{2.11d}$$

$$F_w = \sqrt{\sec\frac{\pi R}{W}\sec\frac{\pi(R+c)}{W}} \tag{2.11e}$$

$$F_{ww} = 1 - \left(\left(1.32\frac{W}{D} - 0.14\right)^{-(.98+\left(0.1\frac{W}{D}\right)^{0.1})} - 0.02\right)\left(\frac{2c}{W-D}\right)^N \tag{2.11f}$$

$$F_3 = 0.098 + 0.3592e^{-3.5089\frac{c}{R}} + 0.3817e^{-0.5515\frac{c}{R}} \tag{2.11g}$$

Where

$$\sigma_{br} = \frac{P}{Dt} = 0$$

We also check $\frac{W}{D} + 2.5$

In [15]:
```
c = .125
D = .25
W = 7.0
R = D/2
print(W/D + 2.5)
```

```
30.5
```

So we use $N = 4.5$ (2.11j)

In [16]:
```
N = 4.5
s_br = 0
s = 9000/(7*.157)
F_cr = (3.404+3.1872*c/R)/(1+3.9273*c/R - 0.00695*(c/R)**2)
F_w = np.sqrt(1/np.cos(np.pi*R/W)/np.cos(np.pi*(R+c)/W))
F_ww = 1-((1.32*W/D-0.14)**(-(0.98+(0.1*W/D)**0.1))-.02)*(2*c/(W-D))**N
F_3 = 0.098 + 0.3592*np.exp(-3.5089*c/R)+0.3817*np.exp(-0.5515*c/R)
beta_1 = F_cr*F_w*F_ww
beta_2 = 0
beta = beta_1+beta_2
K_I = s*np.sqrt(np.pi*c)*beta
K_I
```

Out[16]:
```
6901.680091659546
```

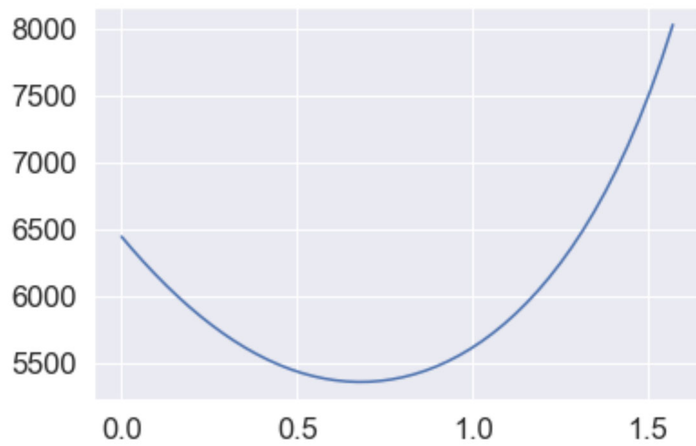
Thus $K_I = 6.90 \text{ ksi}\sqrt{\text{in.}}$

For a quarter circular crack we use (3.1) with $a = c$. With no minor axis, it is not obvious which direction in which direction the maximum stress intensity factor will be, so we vary ϕ and plot to find the maximum.

In [18]:
```
import matplotlib.pyplot as plt
import seaborn as sb
sb.set(font_scale=1.5)
%matplotlib inline

a = c
phi = np.linspace(0,np.pi/2)
n = 1
b = W/2
r = R
t=0.157
fw = np.sqrt(1/np.cos(np.pi*r/(2*b))/np.cos((
            np.pi*(2*r+n*c)/(4*(b-c)+2*n*c)*np.sqrt(a/t))))
lam = 1/(1+c/r*np.cos(0.85*phi))
g2 = (1+0.358*lam+1.425*lam**2-1.578*lam**3+2.156*lam**4)/(1+0.13*lam**2)
M1 = 1.13 - 0.09*a/c
M2 = -0.54 + 0.89/(0.2+a/c)
M3 = 0.5 - 1/(0.65+a/c) + 14*(1-a/c)**24
Q = 1+1.464*(a/c)**1.65
g1 = 1+(0.1+0.35*(a/t)**2)*(1-np.sin(phi))**2
g3 = (1+0.04*(a/c))*(1+0.1*(1-np.cos(phi))**2)*(0.85+0.15*(a/t)**.25)
g4 = 1 - 0.7*(1-a/t)*(a/c-0.2)*(1-a/c)
f_phi = ((a/c)**2*np.cos(phi)**2+np.sin(phi)**2)**.25
Fch = (M1 +M2*(a/t)**2 + M3*(a/t)**4)*g1*g2*g3*g4*f_phi*fw
B = np.sqrt(1/Q)*Fch
KI_double = s*np.sqrt(np.pi*a)*B
KI_b = np.sqrt((4/np.pi + a*c/(2*t*r))/(4/np.pi+a*c/(t*r)))*KI_double
plt.figure()
plt.plot(phi,KI_b)
print(max(KI_b))
```

```
8025.639659236196
```



This gives a maximum stress intensity at $\phi = \pi/2$ (i.e. in the direction of the hole) of $K_I = 8.03 \text{ ksi}\sqrt{\text{in.}}$

4

For a semi-elliptical surface flaw, we use 2.15, with $b \to \infty$ and consider the maximum stress intensity factor when $\phi = 90^\circ$

In [19]:
```
a = 0.2
c = 0.4 #2c = 0.8
t = 0.56
phi = np.pi/2
fw = 1.0
M1 = 1.13 - 0.09*a/c
M2 = -0.52 + 0.89/(.2+a/c)
M3 = 0.5-1.0/(.65+a/c)+14.0*(1.0-a/c)**4.0
Q = 1 + 1.464*(a/c)**1.65
f_phi = ((a/c)**2*np.cos(phi)**2+np.sin(phi)**2)**.25
g = 1+ (0.1+.35*(a/c)**2)*(1-np.sin(phi))**2
Fs = (M1 + M2*(a/t)**2+M3*(a/t)**4)*g*f_phi*fw
B = np.sqrt(1/Q)*Fs
s = 20
KI = s*np.sqrt(np.pi*a)*B
print(KI)
print(B)
```

```
15.566371214687482
0.9819004365404114
```

So we find $K_I = 15.6 \text{ ksi}\sqrt{\text{in.}}$

5

While we have no direct formula for an I-Beam, we can estimate the stress intensity factor using the edge-crack bending formula (2.5a) and replacing σ with an effective stress using an I-Beam cross-section. i.e.

$$\sigma = \frac{6M}{tW^2} = \frac{My}{I}$$

We can calculate the inertia of the I-beam cross-section using the Parallel Axis Theorem

$$I = \sum \bar{I}_i + Ad_i^2$$

Since this I-beam is symmetric, we only need to find the inertia of the middle section, I_1 and the top section, I_2

In [23]:
```
I_1 = 0.248*(8.14-2*.378)**3/12
I_2 = 5.268*0.378**3/12
```

We can now apply the parallel axis theorem, we need to include the top and bottom sections, which are equal so we multiply the top section by 2

In [24]:
```
I = I_1 + 2*(I_2+5.268*0.378*(8.14/2-.378/2)**2)
print(I)
```

```
68.35452859082667
```

So the inertia of this segmernt is 68.4 in.^4

Next we calculate the bending moment for this beam using a free-body diagram, we find the moment is a function of the distance from the applied load, $M = 3000x$ (ft-lb.), or in consistent units (in-lb.) $M = 3000(12)x = 36000x$ (in-lb.)

This is slightly different from the given formula (where M is constant), but we assume, as an estimate, that the bending at the crack should give a similar result, and take $x = 4$ feet.

We use this to calculate the effective stress as $\sigma = \frac{My}{I}$ with $y = W/2 = 4.07$

In [25]:
```
M = 3000*4*12
s = M*4.07/I
s
```

Out[25]:
```
8574.121014107222
```

The crack will first need to propagate through the flange before it can affect the web, which means the crack tip stress will actually vary through the thickness at this point of the problem, we will compare the beam bending solution to an edge crack under tension using the maximum tensile stress in the beam.

In [28]:
```
aw = 0.375/8.14
beta = 1.122  - 1.4*aw + 7.33*aw**2 - 13.08*aw**3 +14.0*aw**4
KI = s*np.sqrt(np.pi*0.375)*beta
print(KI)
```

```
9974.979100054854
```

Thus $K_I = 9.97\text{ksi}\sqrt{\text{in.}}$

For an edge crack under tension, we take the nominal stress found previously (8.57 ksi) and apply it to an edge crack with finite width, using 0.3 as the crack length and 5.268 as the width

In [30]:
```
aw = .3/5.268
B = 1.122 - 0.231*aw + 10.55*(aw**2) - 21.71*(aw**3) + 30.82*(aw**4)
KI = s*np.sqrt(np.pi*.3)*B
print(KI)
```

```
9483.9964145753
```

Which gives $K_I = 9.48\text{ksi}\sqrt{\text{in.}}$, and we would expect the actual stress intensity to lie between these two values.

6

There are many different geometric configurations we could use to make this comparison. Here I will use two different configurations: one in which $R = L = a/2$ and one in which I hold R constant, making $L = a - R$. In both cases I assume the panel is very wide.

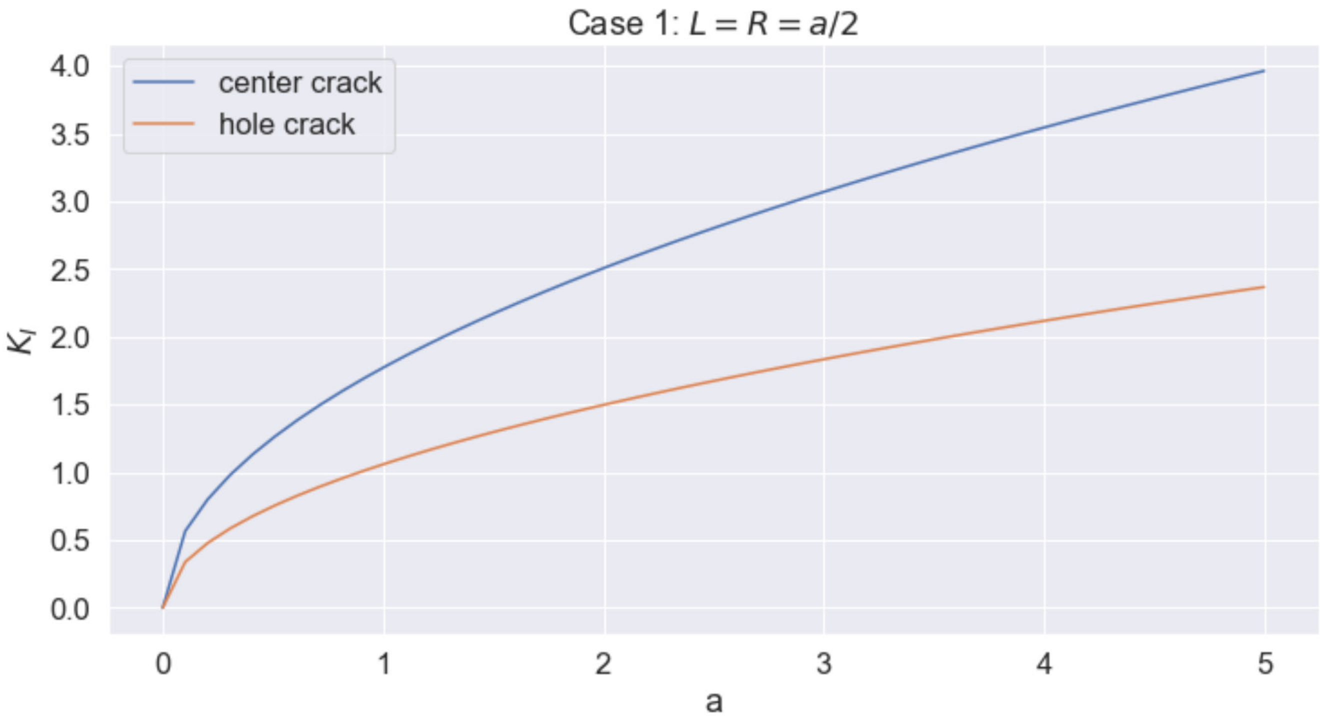
In [31]:

```
#case 1, r = L = a/2
a = np.linspace(0,5)
r = a/2
l = a/2
K_I1 = np.sqrt(np.pi*a)

#beta2 = 0, c/r = l/r = 1, w/d = infinity, r/w = 0
Fcr = (.3404+3.8172)/(1+3.9273-0.00695)
Fw = 1
Fww = 1
B = Fcr*Fw*Fww
K_I2 = np.sqrt(np.pi*l)*B

plt.figure(figsize=(12,6))
plt.plot(a,K_I1,label='center crack')
plt.plot(a,K_I2,label='hole crack')
plt.legend(loc='best')
plt.xlabel('a')
plt.ylabel('$K_I$')
plt.title('Case 1: $L = R = a/2$')
```

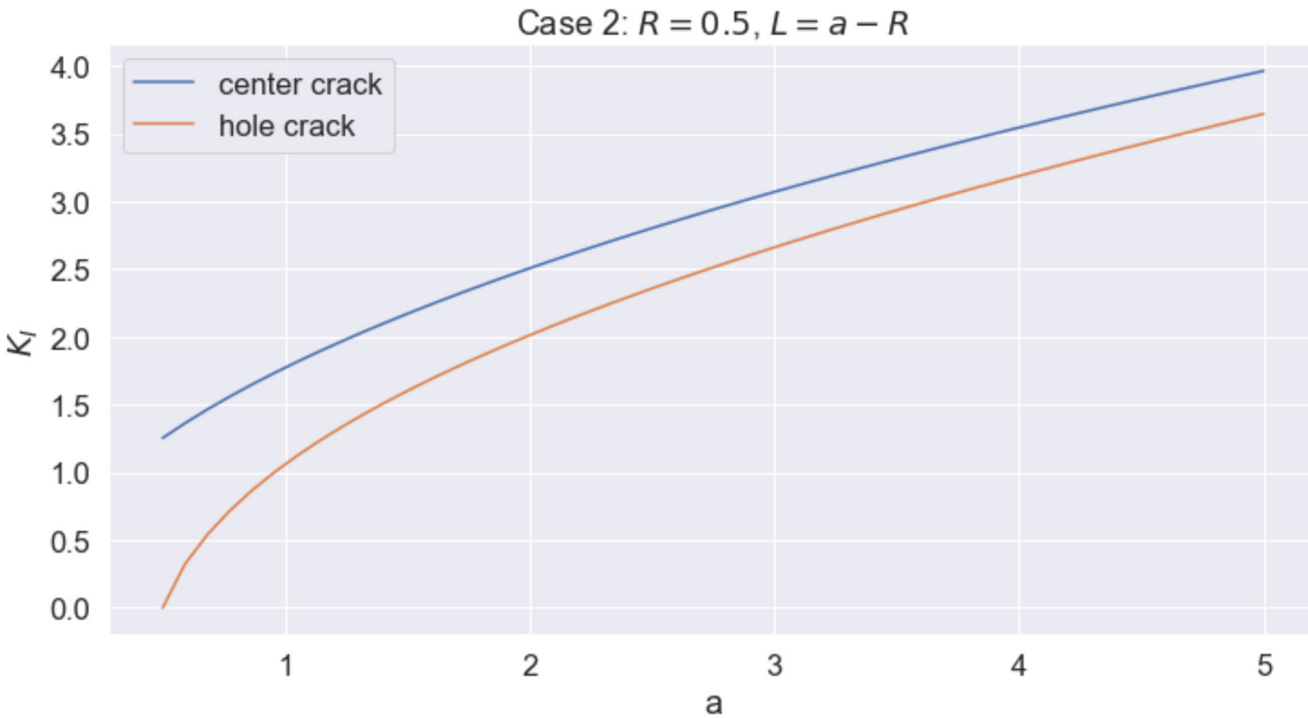
```
Out[31]: Text(0.5, 1.0, 'Case 1: $L = R = a/2$')
```



In [32]:
```
#case 2, r = const, L = a - R
r = 0.5
a = np.linspace(r,5)
l = a - r
K_I1 = np.sqrt(np.pi*a)
#beta2 = 0, c/r = l/r, w/d = infinity, r/w = 0
cr = l/r
Fcr = (.3404+3.8172*cr)/(1+3.9273*cr-0.00695*cr**2)
Fw = 1
Fww = 1
B = Fcr*Fw*Fww
K_I2 = np.sqrt(np.pi*l)*B

plt.figure(figsize=(12,6))
plt.plot(a,K_I1,label='center crack')
plt.plot(a,K_I2,label='hole crack')
plt.legend(loc='best')
plt.xlabel('a')
plt.ylabel('$K_I$')
plt.title('Case 2: $R = 0.5$, $L = a - R$')
```

Out[32]: Text(0.5, 1.0, 'Case 2: $R = 0.5$, $L = a - R$')

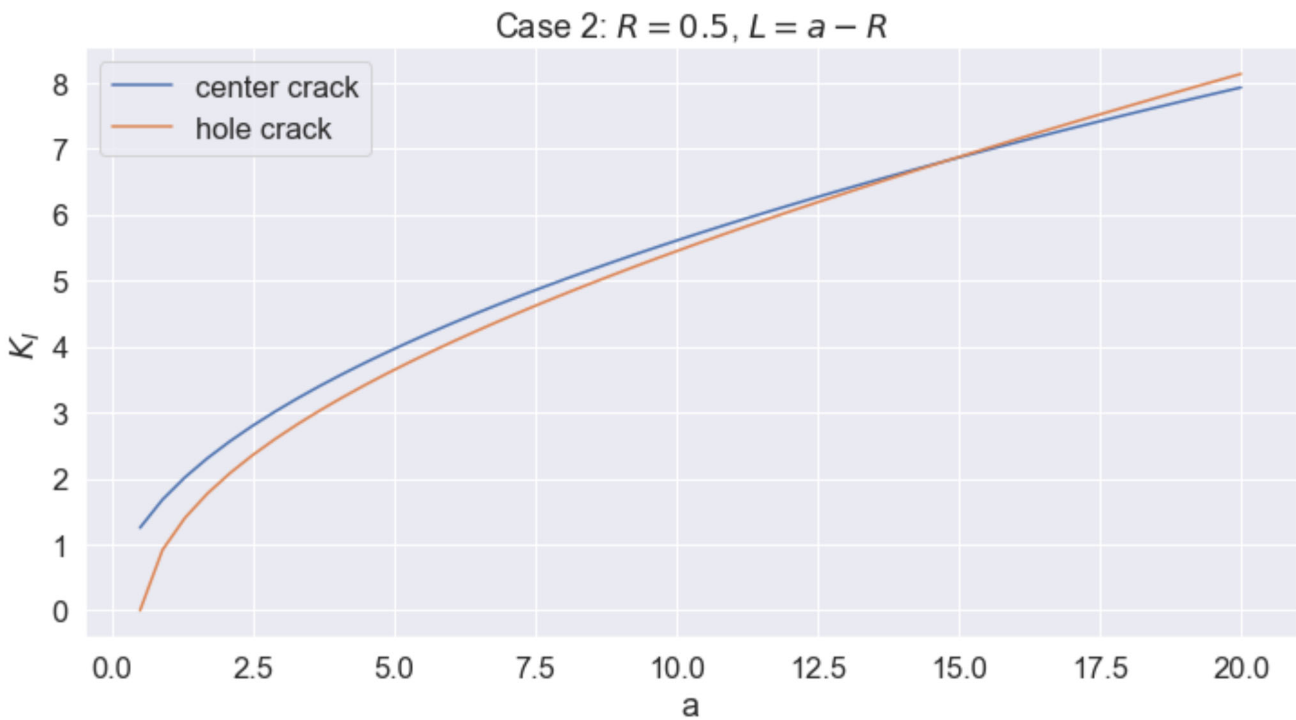


In both cases, the center crack has a larger stress intensity than the hole crack, although for a constant hole size, the longer the crack is the more the solutions appear to converge.

In [33]:
```
#case 2, r = const, L = a - R
r = 0.5
a = np.linspace(r,20)
l = a - r
K_I1 = np.sqrt(np.pi*a)
#beta2 = 0, c/r = l/r, w/d = infinity, r/w = 0
cr = l/r
Fcr = (.3404+3.8172*cr)/(1+3.9273*cr-0.00695*cr**2)
Fw = 1
Fww = 1
B = Fcr*Fw*Fww
K_I2 = np.sqrt(np.pi*l)*B

plt.figure(figsize=(12,6))
plt.plot(a,K_I1,label='center crack')
plt.plot(a,K_I2,label='hole crack')
plt.legend(loc='best')
plt.xlabel('a')
plt.ylabel('$K_I$')
plt.title('Case 2: $R = 0.5$, $L = a - R$')
```

```
Out[33]: Text(0.5, 1.0, 'Case 2: $R = 0.5$, $L = a - R$')
```



Indeed, when we allow the crack to get longer, we find that the hole crack begins to have a larger stress intensity than the center crack.

7

In this problem we have the same configuration as in (2.11) (Case 11 on p 53 of text). We "plug and chug."

In [34]:
```
c = 1.
t = 0.25
P = 9000.*t
D = 0.375
W = 6.
r = D/2.
cr = c/r
Fcr = (.3404+3.8172*cr)/(1+3.9273*cr-0.00695*cr**2)
Fw = np.sqrt(1/np.cos(np.pi*r/W)/np.cos(np.pi*(r+c)/W))
N = W/D + 2.5
if N > 4.5:
    N = 4.5
Fww = 1-((1.32*W/D-0.14)**-(.98+(0.1*W/D)**.1) - 0.02)*(2*c/(W-D))**N
F3 = 0.098 + 0.3592*np.exp(-3.5089*cr) + 0.3817*np.exp(-0.5515*cr)
s = 6000.
s_br = P/(D*t)
B1 = Fcr*Fw*Fww
B2 = s_br/s*F3*Fw*Fww
B = B1 + B2
KI = s*np.sqrt(np.pi*c)*B
print(KI)
```

```
16844.822211230163
```

So the stress intensity factor for this case is $K_I = 16.8\text{ksi}\sqrt{\text{in.}}$

8

For part a we have

In [35]:
```
a = 6.0
s = 10.0
w = 15.0
KIa = 1.12*s*np.sqrt(np.pi*a)
print(KIa)
```

```
48.626004306315586
```

Which gives $K_I = 48.6\text{ksi}\sqrt{\text{in.}}$

In part b we use (2.4a) (case 5 p 51)

In [36]:
```
aw = a/w
B = 1.122 - 0.231*aw + 10.55*aw**2 - 21.71*aw**3 + 30.82*aw**4
KIb = s*np.sqrt(np.pi*a)*B
print(KIb)
```

```
91.91843059743272
```

Which gives $K_I = 91.9 \text{ksi} \sqrt{\text{in.}}$

Here the difference is signficant. The assumptions that $\beta = 1.12$ is only appropriate when a/W is very small, as in the following where we consider $a = 0.5$

In [37]:
```
a = 0.5
s = 10.0
w = 15.0
KIa = 1.12*s*np.sqrt(np.pi*a)
print(KIa)

aw = a/w
B = 1.122 - 0.231*aw + 10.55*aw**2 - 21.71*aw**3 + 30.82*aw**4
KIb = s*np.sqrt(np.pi*a)*B
print(KIb)
```

```
14.037118337933602
14.102995004673152
```

9

In this problem we compare (2.12) (case 12 p 53) to 3.1 (case 18 p. 56). For a through crack we have

In [38]:
```
d = 0.375
r = d/2
c = .075
w = 0.96
t = 0.15
s = 20
rrc = r/(r+c)
cr = c/r
#no bearing stress, B2 = 0
B3 = 0.7071 + 0.7548*rrc + 0.3415*rrc**2 + .6420*rrc**3 + 0.9196*rrc**4
Fw = np.sqrt(1/np.cos(np.pi*r/w)/np.cos(np.pi*(r+c/2)/(w-c)))
N = 2.65 - 0.24*(2.75-w/d)**2
if N < 2.275:
    N = 2.275
Fww = 1-N**(-w/d)*(2*c/(w-d))**(w/d+0.5)
B = B3*Fw*Fww
K_Ia = s*np.sqrt(np.pi*c)*B
print(K_Ia)
```

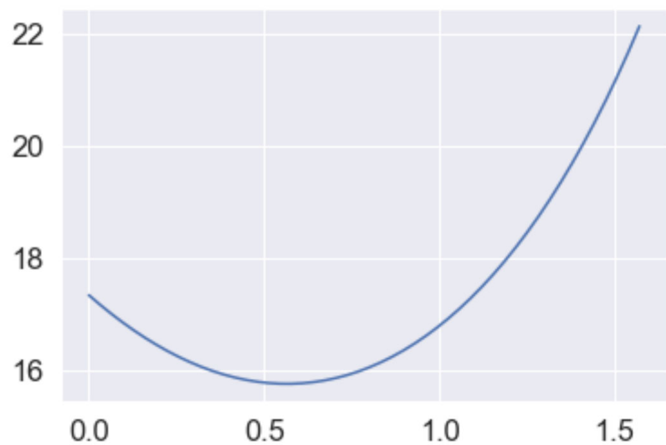
```
24.31271834172049
```

Which gives a stress intensity of $K_I = 24.3\text{ksi}\sqrt{\text{in.}}$

For a quarter circular crack, we use 3.1 (with $a = c$).

In [39]:
```
a = c
phi = np.linspace(0,np.pi/2)
n = 1
b = w/2
fw = np.sqrt(1/np.cos(np.pi*r/(2*b))/np.cos((
            np.pi*(2*r+n*c)/(4*(b-c)+2*n*c)*np.sqrt(a/t))))
lam = 1/(1+c/r*np.cos(0.85*phi))
g2 = (1+0.358*lam+1.425*lam**2-1.578*lam**3+2.156*lam**4)/(1+0.13*lam**2)
M1 = 1.13 - 0.09*a/c
M2 = -0.54 + 0.89/(0.2+a/c)
M3 = 0.5 - 1/(0.65+a/c) + 14*(1-a/c)**24
Q = 1+1.464*(a/c)**1.65
g1 = 1+(0.1+0.35*(a/t)**2)*(1-np.sin(phi))**2
g3 = (1+0.04*(a/c))*(1+0.1*(1-np.cos(phi))**2)*(0.85+0.15*(a/t)**.25)
g4 = 1 - 0.7*(1-a/t)*(a/c-0.2)*(1-a/c)
f_phi = ((a/c)**2*np.cos(phi)**2+np.sin(phi)**2)**.25
Fch = (M1 +M2*(a/t)**2 + M3*(a/t)**4)*g1*g2*g3*g4*f_phi*fw
B = np.sqrt(1/Q)*Fch
KI_double = s*np.sqrt(np.pi*a)*B
KI_b = np.sqrt((4/np.pi + a*c/(2*t*r))/(4/np.pi+a*c/(t*r)))*KI_double
plt.figure()
plt.plot(phi,KI_b)
print(max(KI_b))
```

```
22.12304612182012
```



Thus the stress intensity factor for a quarter circular crack is only $K_I = 22.1\text{ksi}\sqrt{\text{in.}}$

In [ ]:

```
```