AE 737: Mechanics of Damage Tolerance

Lecture 7 - Fracture Toughness

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schedule

- 12 Feb - Fracture Toughness
- 14 Feb - Fracture Toughness, Homework 3 Due
- 19 Feb - Residual Strength
- 21 Feb - Residual Strength, Homework 4 Due

outline

- plastic zone review
- fracture toughness
- plain strain
- plain stress

plastic zone review

Irwin's first approximation

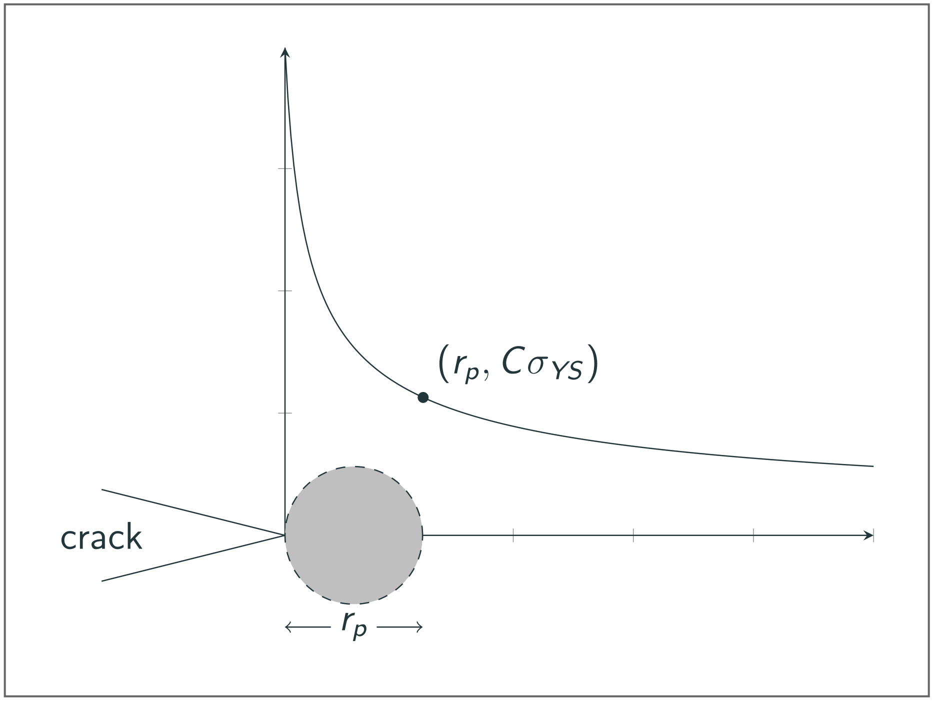
- If we recall the equation for opening stress (σ_y) near the crack tip

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{1.2}$$

- In the plane of the crack, when $\theta = 0$ we find

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}$$

Irwin's first approximation



Irwin's first approximation

- We use C, the *Plastic Constraint Factor* to convert between Plane Strain and Plane Stress solutions
- The plastic zone size can now be approximated

$$\sigma_{yy}(r = r_p) = C\sigma_{YS}$$

$$\frac{K_I}{\sqrt{2\pi r_p}} = C\sigma_{YS}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{C\sigma_{YS}}\right)^2$$

Irwin's first approximation

- For plane stress (thin panels) we let $C = 1$ and find r_p as
  $$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$
- And for plane strain (thick panels) we let $C = \sqrt{3}$ and find
  $$r_p = \frac{1}{6\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

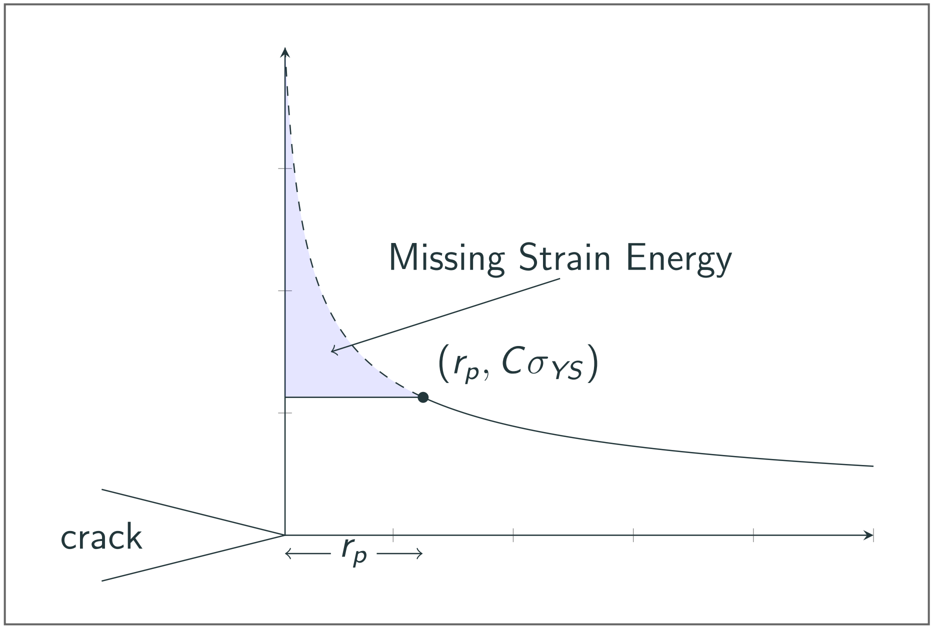
Intermediate panels

- For panels which lie between plane strain and plane stress states, we use the following expression to estimate the plastic zone size
$$r_p = \frac{1}{I\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$
- Where I is defined as
$$I = 6.7 - \frac{1.5}{t}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$
- And $2 \leq I \leq 6$

Irwin's second approximation

- If our material is perfectly elastic-plastic, no stresses above $C\sigma_{ys}$ will exist in the material
- This ignores the strain energy (represented by the area under the curve) in the plastic zone

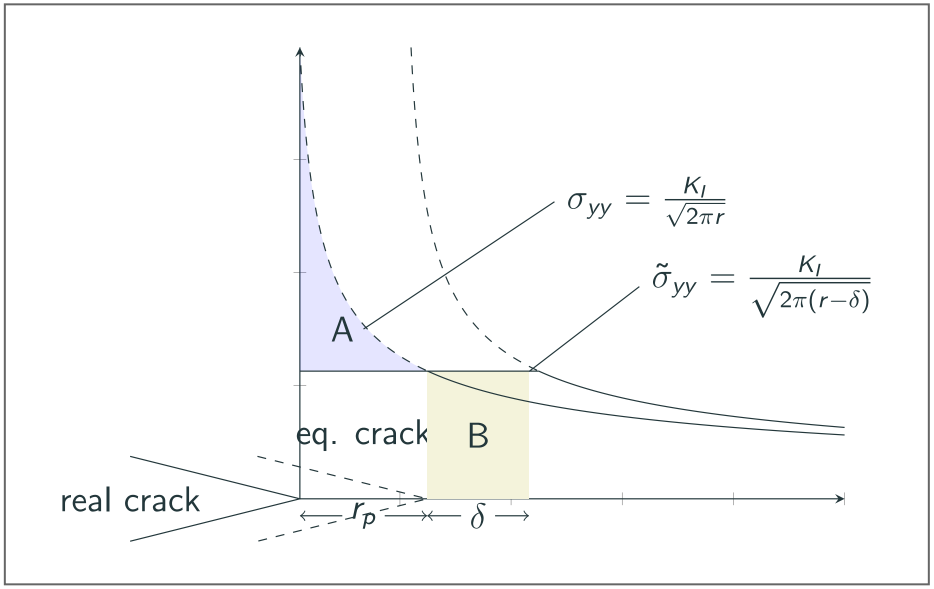
Irwin's second approximation



Irwin's second approximation

- To account for the additional strain energy, Irwin considered a plastic zone size increased by some δ
- He also needed to adjust the stress function, and considered an equivalent crack tip in these calculations

Irwin's second approximation



Irwin's second approximation

- This means the plastic zone size is simply $2r_p$
- However, it also means that the effective crack length is $a+r_p$
- Since r_p depends on K_I, we must iterate a bit to find the "real" r_p and K_I

plastic stress intensity ratio

For an infinitely wide center-cracked panel, we can solve for K_{Ie}/K_I symbolically, in plane stress

$$\begin{aligned} K_I &= \sigma\sqrt{\pi a} \\ K_{Ie} &= \sigma\sqrt{\pi(a+r_p)} \\ r_p &= \frac{1}{2\pi}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2 \\ K_{Ie} &= \sigma\sqrt{\pi\left(a+\frac{1}{2\pi}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2\right)} \end{aligned}$$

stress intensity ratio

$$K_{Ie}^2 = \sigma^2\pi\left(a + \frac{1}{2\pi}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2\right)$$

$$K_{Ie}^2 = \sigma^2\pi a + \frac{\sigma^2}{2}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2$$

$$K_{Ie}^2 - \frac{\sigma^2}{2}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2 = \sigma^2\pi a$$

$$K_{Ie}^2\left(1 - \frac{\sigma^2}{2\sigma_{YS}^2}\right) = \sigma^2\pi a$$

plastic stress intensity ratio

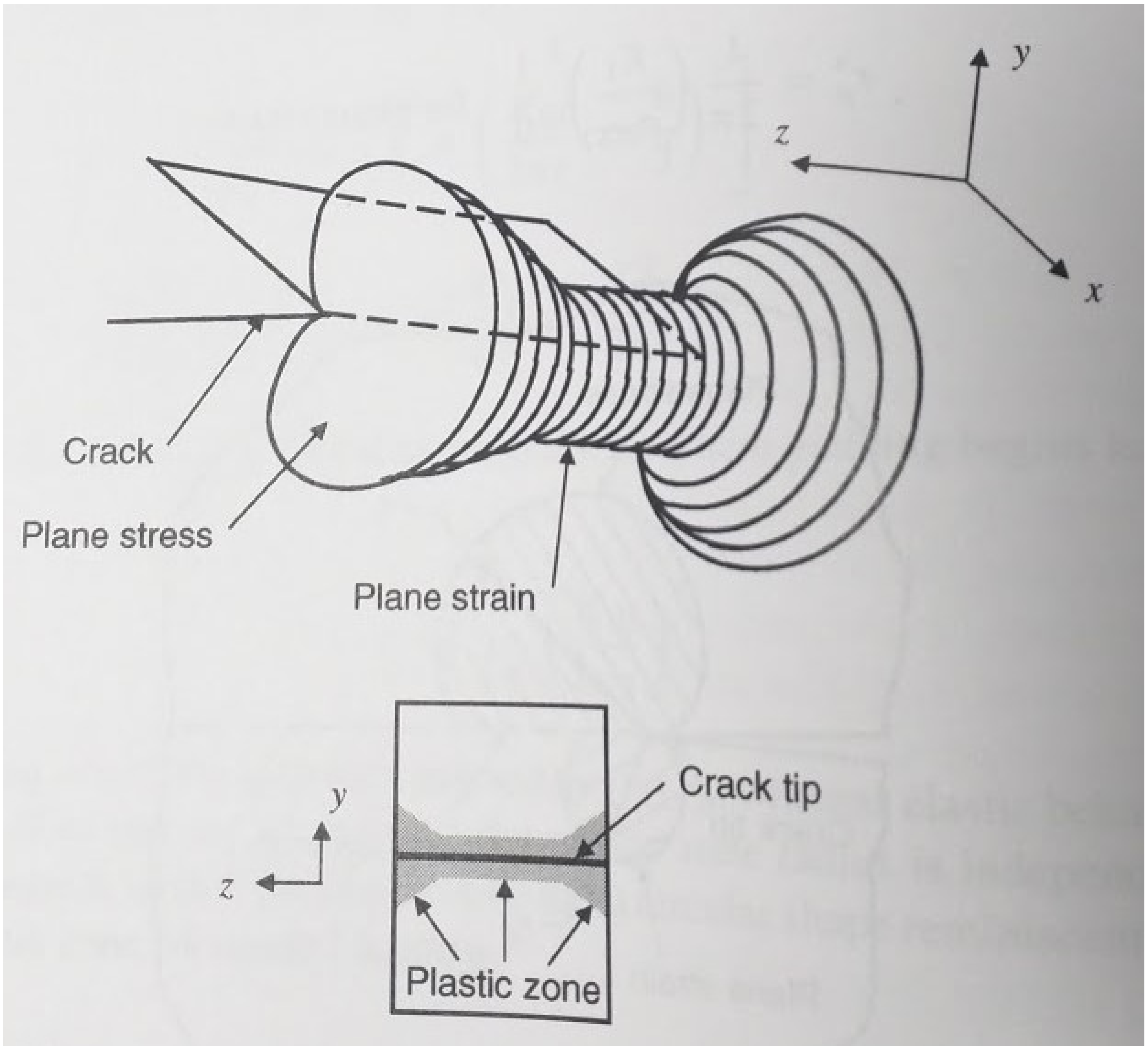
$$K_{Ie}^2 = \frac{\sigma^2 \pi a}{1 - \frac{\sigma^2}{2\sigma_{YS}^2}}$$

$$K_{Ie} = \frac{\sigma\sqrt{\pi a}}{\sqrt{1 - \frac{\sigma^2}{2\sigma_{YS}^2}}}$$

$$K_{Ie} = \frac{K_I}{\sqrt{1 - \frac{\sigma^2}{2\sigma_{YS}^2}}}$$

$$\frac{K_{Ie}}{K_I} = \frac{1}{\sqrt{1 - \frac{\sigma^2}{2\sigma_{YS}^2}}}$$

3D plastic zone shape



group problems

group one

- Calculate the plastic zone size for an infinitely wide, center-cracked panel
- Consider a crack-length of 4 cm, and a yield strength of $\sigma_{YS} = 55$ MPa, with an applied load of $\sigma = 20$ MPa
- Assume the panel is very thin

group two

- Calculate the plastic zone size for an infinitely wide, center-cracked panel
- Consider a crack-length of 4 cm, and a yield strength of $\sigma_{YS} = 55$ MPa, with an applied load of $\sigma = 20$ MPa
- Assume the panel is very thick

group three

- Calculate the plastic zone size for an infinitely wide, center-cracked panel
- Consider a crack-length of 4 cm, and a yield strength of $\sigma_{YS} = 55$ MPa, with an applied load of $\sigma = 20$ MPa
- The panel thickness is t = 0.65 cm

group four

- Find the plastic stress intensity ratio for an infinitely wide, center-cracked panel
- What factors will increase or decrease the plastic stress intensity ratio?

fracture toughness

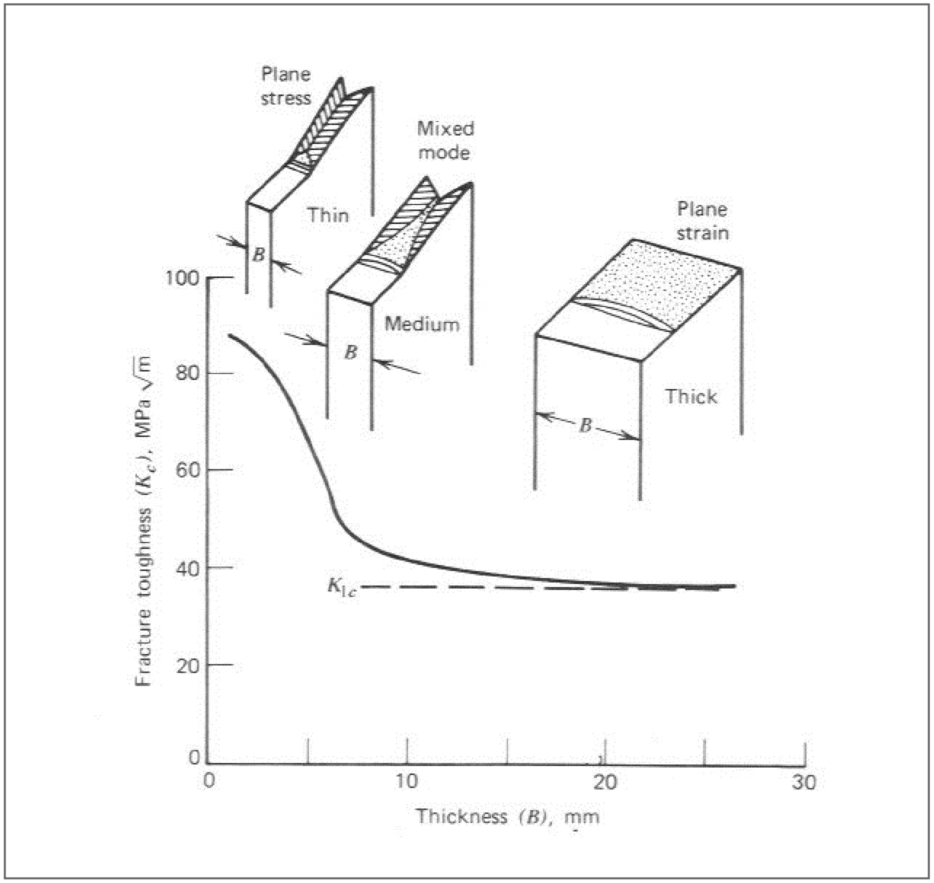
fracture toughness

- The critical load at which a cracked specimen fails produces a critical stress intensity factor
- The "critical stress intensity factor" is known as K_c
- For Mode I, this is called K_{Ic}
- The critical stress intensity factor is also known as fracture toughness $K_{IC} = \sigma_c \sqrt{\pi a} \beta$
- Note: "Fracture Toughness" can also refer to G_{Ic}, which is analogous to K_{Ic}, but not the same

fracture toughness

- Fracture toughness is a material property, but it is only well-defined in certain conditions
- Brittle materials
- Plane strain (smaller plastic zone)
- In these cases ASTM E399-12 is used.

fracture toughness



unstable cracks

- Stable crack growth means the crack extends only with increased load
- Unstable crack growth means the crack will continue to extend indefinitely under the same load
- For a perfectly brittle material, there is no stable crack growth, as soon as a critical load is reached, the crack will extend indefinitely

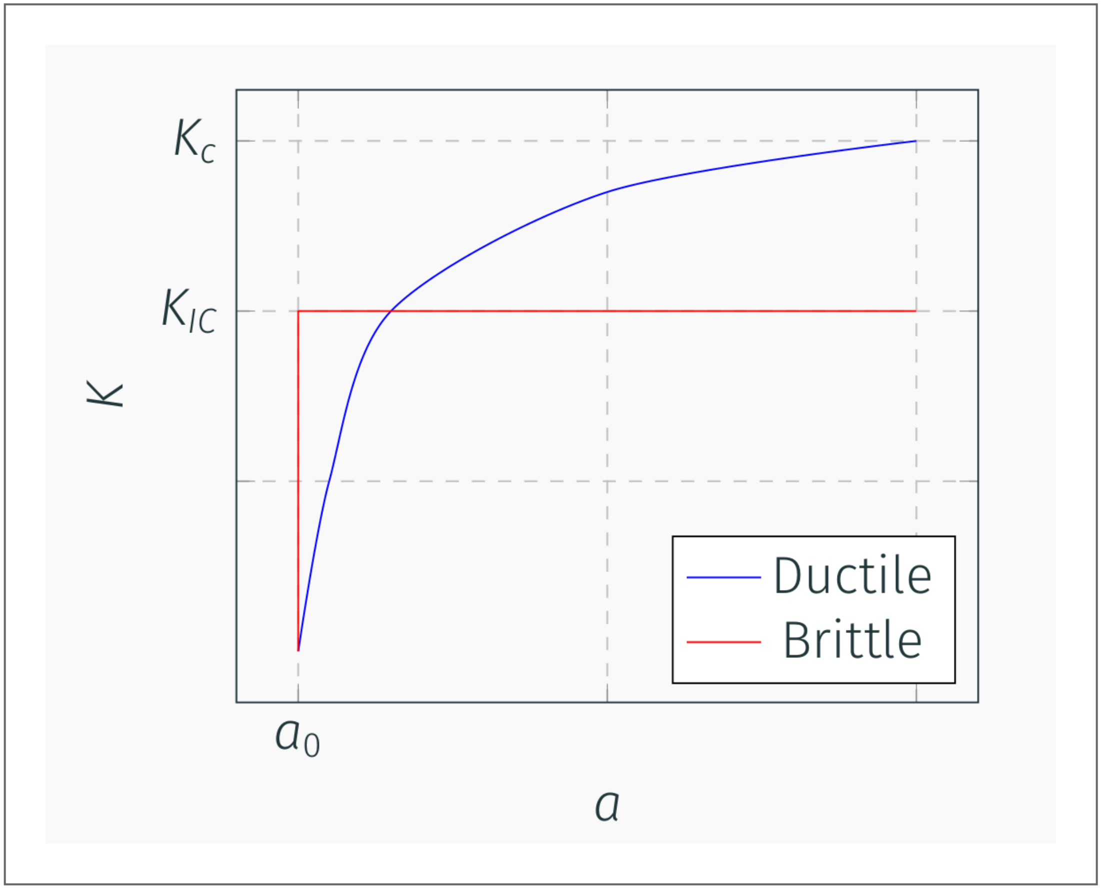
stable cracks

- For an elastic-plastic material, once the load is large enough to extend the crack, it will extend slightly
- The load must be continually increased until a critical value causes unstable crack growth

fracture toughness

- During an experiment, we will record the crack length and applied load (P_i, a_i) each time we increase the load
- We can calculate a unique stress intensity factor K_{Ii} at each of these points
- These are then used to create a "K-curve", plotting K_I vs. a

K-curve



K-curve

- Materials will generally not be as flat as the perfectly brittle example
- Plane strain conditions and brittle materials will tend towards a "flat" K-curve
- K_{IC} for brittle/plane strain is very well defined
- K_C for plane stress can refer to two things
- Either the maximum K_C during a test, or tangent point on K_R-curve (R-curve)

example

- In composites, and adhesives, some work is needed to ensure stable crack growth
- The Double-Cantilever Beam (DCB) experiment to find G_{IC} illustrates this

$$C = \frac{\delta}{P}$$

$$C = \frac{2a^3}{3EI}$$

$$G = \frac{P^2}{2b}\frac{dC}{da}$$

$$G = \frac{P^2 a^2}{bEI}$$

example

- For crack growth to be stable we need
$$\frac{dG}{da} \leq 0$$
- Under fixed-load conditions, we find
$$\frac{dG}{da} = \frac{2P^2 a}{bEI}$$
- This is always positive, and thus results in unstable crack growth

example

- Under fixed-displacement conditions, we substitute for P
- We find
  $$\frac{dG}{da} = -\frac{9\delta^2 EI}{ba^3}$$
- Which is always stable, so for DCB tests, displacement control is generally used

plane strain, brittle

plane strain, brittle

- For relatively brittle materials, we donâ™t need to worry about the R-curve
- Specimens are made according to these specifications

$$a \geq 2.5\left(\frac{K_{IC}}{\sigma_{YS}}\right)^2$$

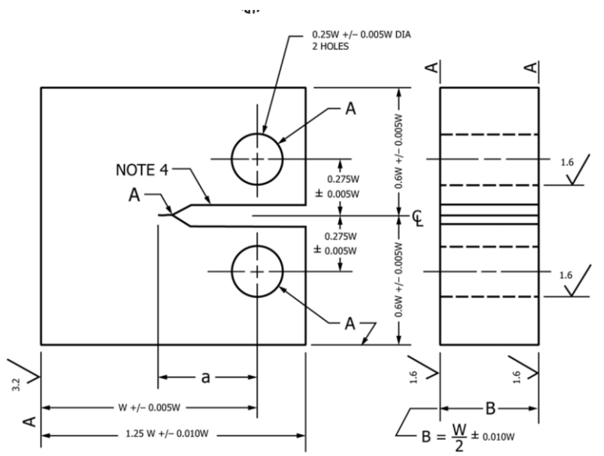
$$b \geq 2.5\left(\frac{K_{IC}}{\sigma_{YS}}\right)^2$$

$$W \geq 5\left(\frac{K_{IC}}{\sigma_{YS}}\right)^2$$

ASTM E399

1. Select specimen size
2. Select specimen type (Compact Tension or Single Edge Notched Bend)

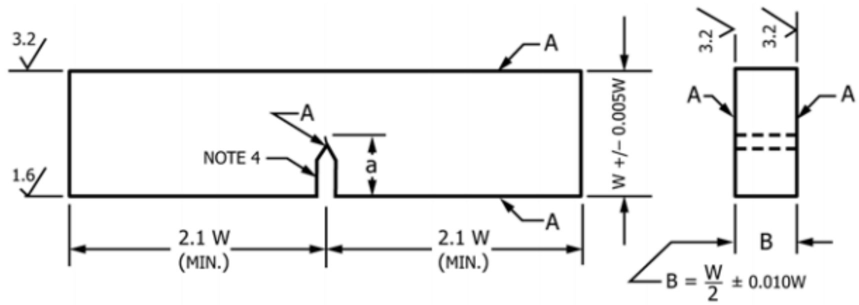
ASTM E399



NOTE 1—Surface finishes in μm.
NOTE 2—*A* surfaces shall be perpendicular and parallel to within 0.002 *W* TIR.
NOTE 3—The intersection of the crack starter notch tips with the two specimen surfaces shall be equally distant from the top and bottom edges of the specimen within 0.005 *W*.
NOTE 4—Integral or attachable knife edges for clip gage attachment to the crack mouth may be used (see Figs. 3 and 4).
NOTE 5—For starter notch and fatigue crack configuration see Fig. 5.
NOTE 6—1.6 μm = 63 μin., 3.2 μm = 125 μin.

FIG. A4.1 Compact C(T) Specimen—Standard Proportions and Tolerances

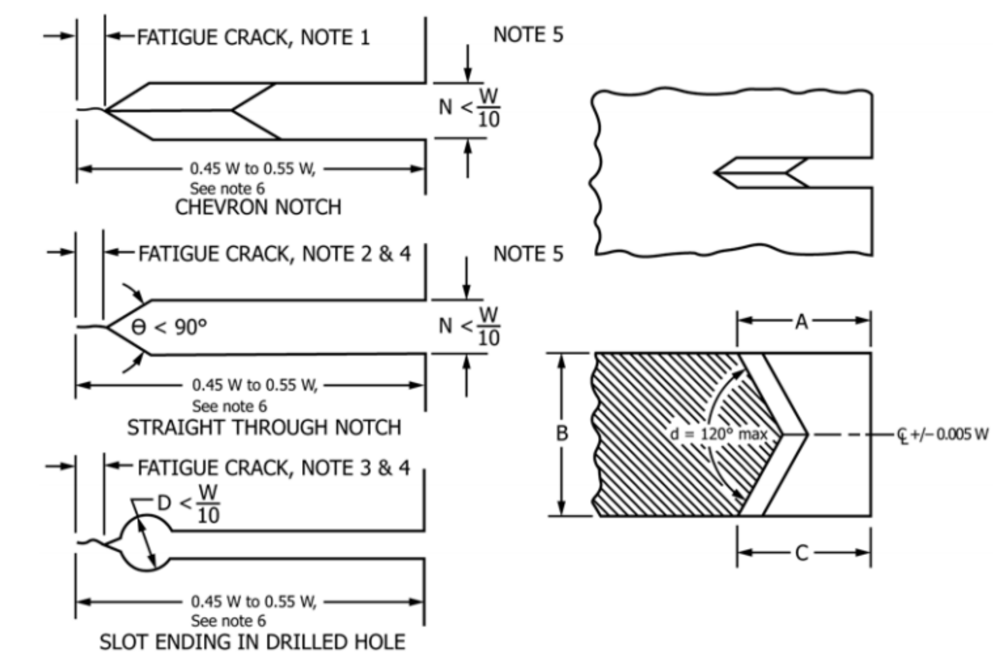
ASTM E399



NOTE 1—Surface finishes in µm.
NOTE 2—*A* surfaces shall be perpendicular and parallel as applicable within 0.001 *W* TIR.
NOTE 3—Crack starter notch shall be perpendicular to specimen surfaces within 2°.
NOTE 4—Integral or attachable knife edges for clip gage attachment may be used (see Figs. 3 and 4)
NOTE 5—For starter notch and fatigue crack configuration see Fig. 5.
NOTE 6—1.6 µm = 63 µin., 3.2 µm = 125 µin.

FIG. A3.1 Bend SE(B) Specimen—Standard Proportions and Tolerances

ASTM E399



(*a*) Starter Notches and Fatigue Cracks (*b*) Detail of Chevron Notch

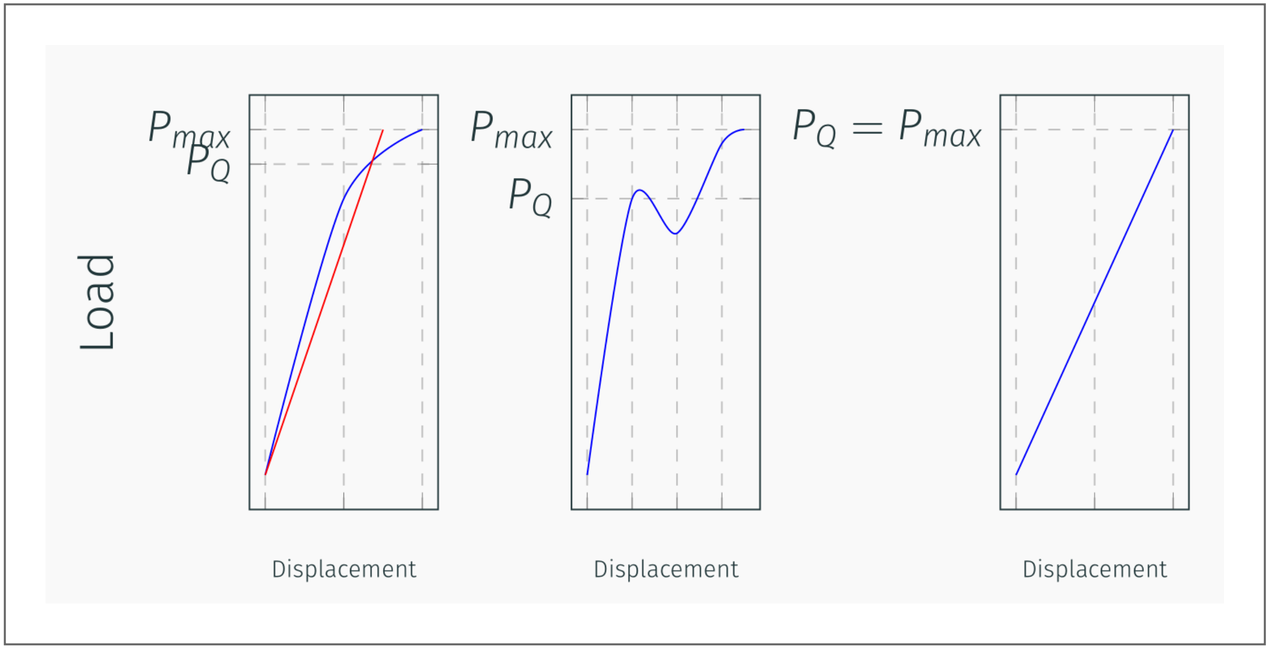
ASTM E399

3. Machine specimen
4. Fatigue crack specimen $K_f < 0.6K_{IC}$
    - This is to ensure that the plastic zone size during fatigue is smaller than the plastic zone size during testing
    - If K_{IC} has not yet been determined, you may have to guess the first time

ASTM E399

5. Mount specimen, attach gage
6. Load rate should ensure "static" load conditions. (30 - 150 ksi $\sqrt{\text{in.}}$/min.)
7. Determine the "provisional" value of K_{IC} (known as K_Q)

ASTM E399


Load
P_{max}
P_Q
Displacement
P_{max}
P_Q
Displacement
$P_Q = P_{max}$
Displacement

ASTM E399

- If the load-displacement curve is like the first figure, with some non-linearity, we let P_Q be the point of intersection between the load-displacement curve and a line whose slope is 5% lower than the slope in the elastic region
- "Pop-in" occurs when there is stable crack extension before the plasticity begins. We let P_Q bet the point where stable crack extension begins.

ASTM E399

- For a perfectly linear material, $P_Q = P_{max}$.

$$K_Q = \frac{P_Q}{BW^{1/2}} f\left(\frac{a}{W}\right) \qquad \text{Compact Tension}$$

$$K_Q = \frac{P_Q}{BW^{3/2}} g\left(\frac{a}{W}\right) \qquad \text{SENB}$$

ASTM E399

8. Ensure that your specimen is still valid

$$a \geq 2.5\left(\frac{K_Q}{\sigma_{YS}}\right)^2$$

$$b \geq 2.5\left(\frac{K_Q}{\sigma_{YS}}\right)^2$$

$$W \geq 5\left(\frac{K_Q}{\sigma_{YS}}\right)^2$$

ASTM E399

- For stable crack extension, check the P_{max}

$$\frac{P_{max}}{P_Q} \leq 1.10$$

- Check for symmetric crack front, a_1, a_2, and a_3 must be within 5% of a. a_s must be within 10% of a.

$$\frac{a_1 + a_2 + a_3}{3} = a$$

- Load-displacement should have an initial slope between 0.7 and 1.5

plane stress, ductile

R-curve

- For materials with some plasticity, the K_R Curve, or R Curve, is very important
- Sometimes called a "resistance curve" it is generally dependent on
  - Thickness
  - Temperature
  - Strain rate

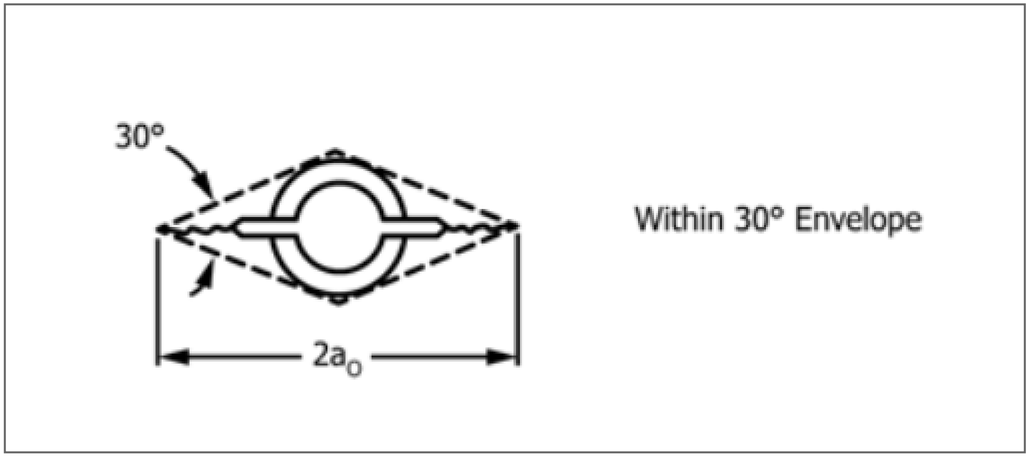
R-curve

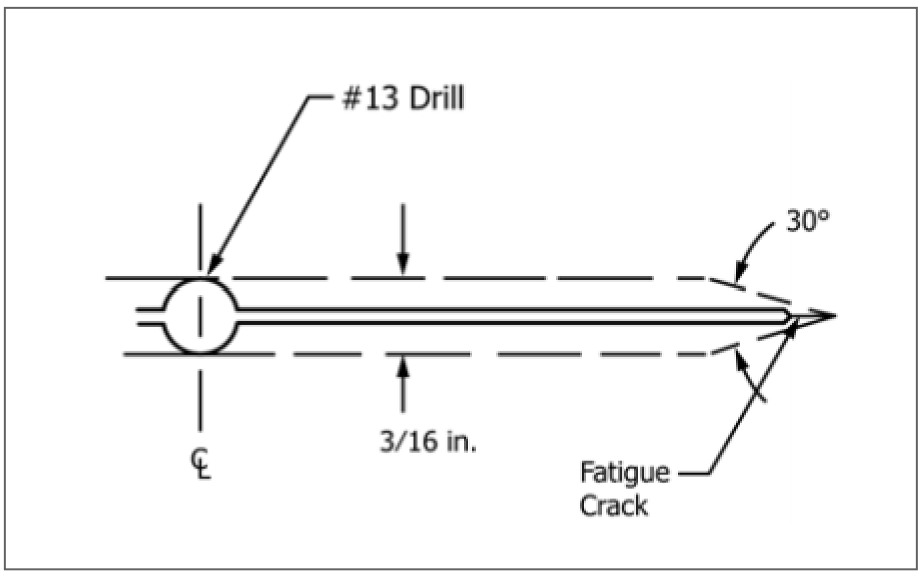
- When done correctly, K_R curves are not dependent on initial crack size or the specimen type used
- ASTM E561 describes a general procedure

ASTM E561

- Compact Tension (CT or C(T)) specimens may be used for plane stress K_R curves
- The other specimen which is permitted is a middle-cracked tension specimen (M(T))
- M(T) specimens are preferred in many cases due to a more uniform stress distribution (particularly important for anisotropic materials)

ASTM E561





minimum sample dimensions

Table of Minimum M(T) Specimen Geometry for Given Conditions

K_{Rmax}/σ_{YS}		Width		$2a_o$		Length[A]	
$\sqrt{m}$	$\sqrt{in.}$	m	in.	m	in.	m	in.
0.08	0.5	0.076	3.0	0.025	1.0	0.229	9
0.16	1.0	0.152	6.0	0.051	2.0	0.457	18
0.24	1.5	0.305	12.0	0.102	4.0	0.914	36
0.32	2.0	0.508	20.0	0.170	6.7	0.762	30
0.48	3.0	1.219	48.0	0.406	16.0	1.829	72

minimum sample dimensions

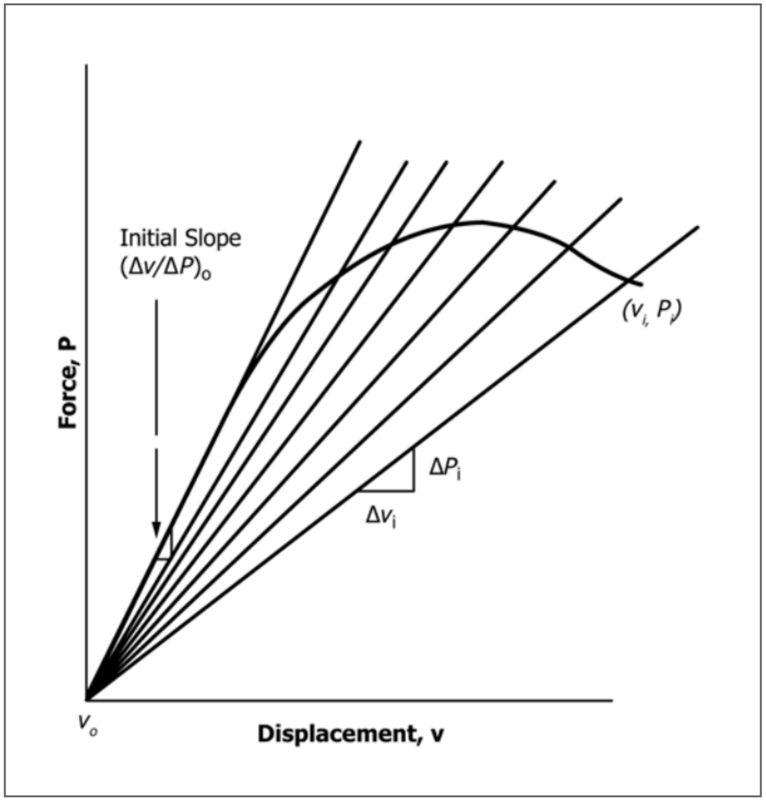
Table of Minimum C(T) Specimen Width W for Given Conditions, m (in.)

K_{Rmax}/σ_{YS} $\sqrt{m}$	K_{Rmax}/σ_{YS} $\sqrt{in.}$	Maximum a_p/W 0.4	0.5	0.6	0.7	0.8
0.10	0.6	0.02 (0.8)	0.03 (1.0)	0.03 (1.3)	0.04 (1.7)	0.06 (2.5)
0.20	1.3	0.08 (3.3)	0.10 (4.0)	0.13 (5.0)	0.17 (6.7)	0.25 (10.0)
0.30	1.9	0.19 (7.5)	0.23 (9.0)	0.29 (11.3)	0.38 (15.0)	0.57 (22.6)
0.40	2.5	0.34 (13.3)	0.40 (15.9)	0.51 (19.9)	0.67 (26.5)	1.01 (39.8)
0.50	3.1	0.53 (20.9)	0.64 (25.1)	0.80 (31.3)	1.06 (41.8)	1.59 (62.7)

effective crack length

- ASTM E561 describes three ways to obtain the effective crack length during testing
  1. Measure the crack length visually and calculate r_p
  2. Measure crack length using "unloading compliance" and adding plastic zone size
  3. Measure the effective crack size directly using "secant compliance"

secant compliance



secant compliance M(T)

- Using the slope data from our load-displacement curve, we can calculate the effective crack length using

$$EB\left(\frac{\Delta v}{\Delta P}\right) = \frac{2Y}{W}\sqrt{\frac{\pi a/W}{\sin(\pi a/W)}}$$

$$\left[\frac{2W}{\pi Y}\cosh^{-1}\left(\frac{\cosh(\pi Y/W)}{\cos(\pi a/W)}\right) - \frac{1+\nu}{\sqrt{1+\left(\frac{\sin(\pi a/W)}{\sinh(\pi Y/W)}\right)^2}} + \nu\right]$$

secant compliance M(T)

- This equation is difficult to solve directly for a (for M(T) specimens)
- Instead it is generally solved iteratively
- The following equations are used to give a good initial guess to use in iterations

secant compliance M(T)

$$X = 1 - \exp\left[\frac{-\sqrt{[EB(\Delta v/\Delta P)]^2 - (2Y/W)^2}}{2.141}\right]$$

$$\frac{2a}{W} = 1.2235X - 0.699032X^2 + 3.25584X^3 - 6.65042X^4 + 5.54X^5 - 1.66989X^6$$

secant compliance M(T)

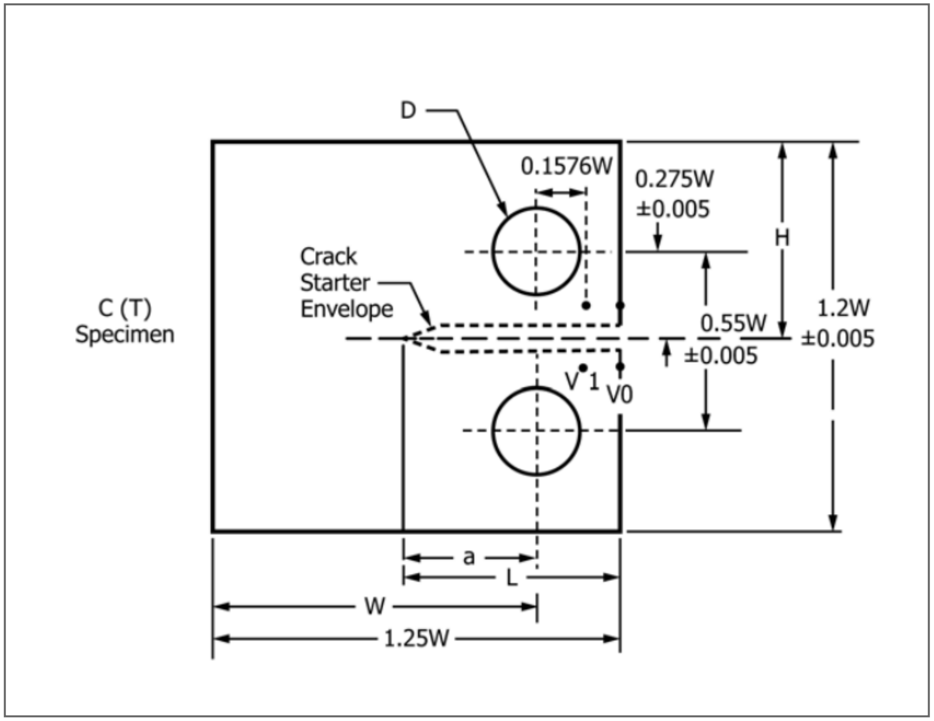
In the above equations, the following are the definitions of parameters used

$$\begin{aligned} E &= \text{Young's Modulus} \\ \Delta v/\Delta P &= \text{specimen complianc} \\ B &= \text{specimen thickness} \\ W &= \text{specimen width} \\ Y &= \text{half span} \\ a &= \text{effective crack lengt} \\ \nu &= \text{Poisson's ratio} \end{aligned}$$

secant compliance C(T)

- For C(T) specimens, we use the following equations
$$EB\frac{\Delta v}{\Delta P} = A_0 + A_1\left(\frac{a}{W}\right) + A_2\left(\frac{a}{W}\right)^2 + A_3\left(\frac{a}{W}\right)^3 + A_4\left(\frac{a}{W}\right)^4$$
- The coefficients will differ based on where the displacement is measured from

secant compliance C(T)



secant compliance C(T)

loc	A_0	A_1	A_2	A_3	A_4
V_0	120.7	-1065.3	4098.0	-6688.0	4450.5
V_1	103.8	-930.4	3610.0	-5930.5	3979.0

secant compliance C(T)

loc	C_0	C_1	C_2	C_3	C_4	C_5
V_0	1.0010	-4.6695	18.460	-236.82	1214.90	-2143.6
V_1	1.0008	-4.4473	15.400	-180.55	870.92	-1411.3

secant compliance C(T)

- Where the initial guess for a is provided by

$$\frac{a}{W} = C_0 + C_1 U + C_2 U^2 + C_3 U^3 + C_4 U^4 + C_5 U^5$$

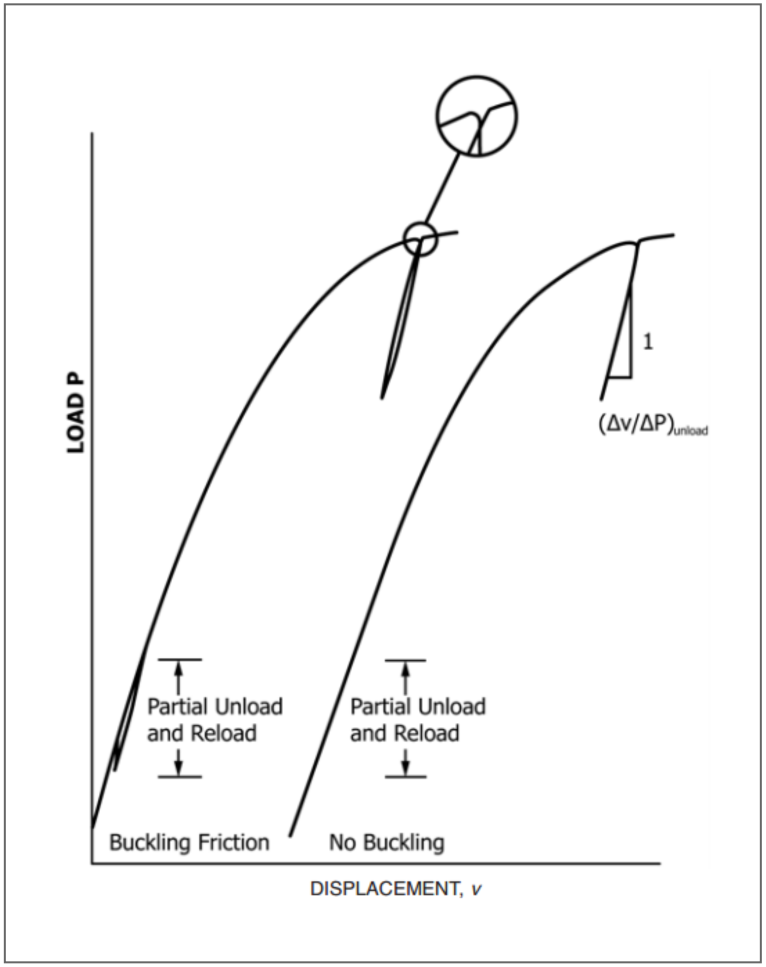
- and U is given by

$$U = \frac{1}{1 + \sqrt{EB\frac{\Delta v}{\Delta P}}}$$

buckling

- If the test is stopped and re-started frequently (to measure crack length by hand or to use the compliance method of crack measurement) buckling can interfere with results

buckling


LOAD P
1
(Δv/ΔP)unload
Partial Unload
and Reload
Partial Unload
and Reload
Buckling Friction
No Buckling
DISPLACEMENT, v

buckling

- If buckling is shown to be present in the test, supports can be used to prevent buckling
- These supports can introduce friction
- They should be well-lubricated for accurate test results

net section stress

- One final consideration when dealing with plane stress fracture mechanics is the net section stress
- For the test to be valid, failure must occur due to fracture, not general static failure
- Static failure will occur when $\sigma_N = \sigma_{YS}$

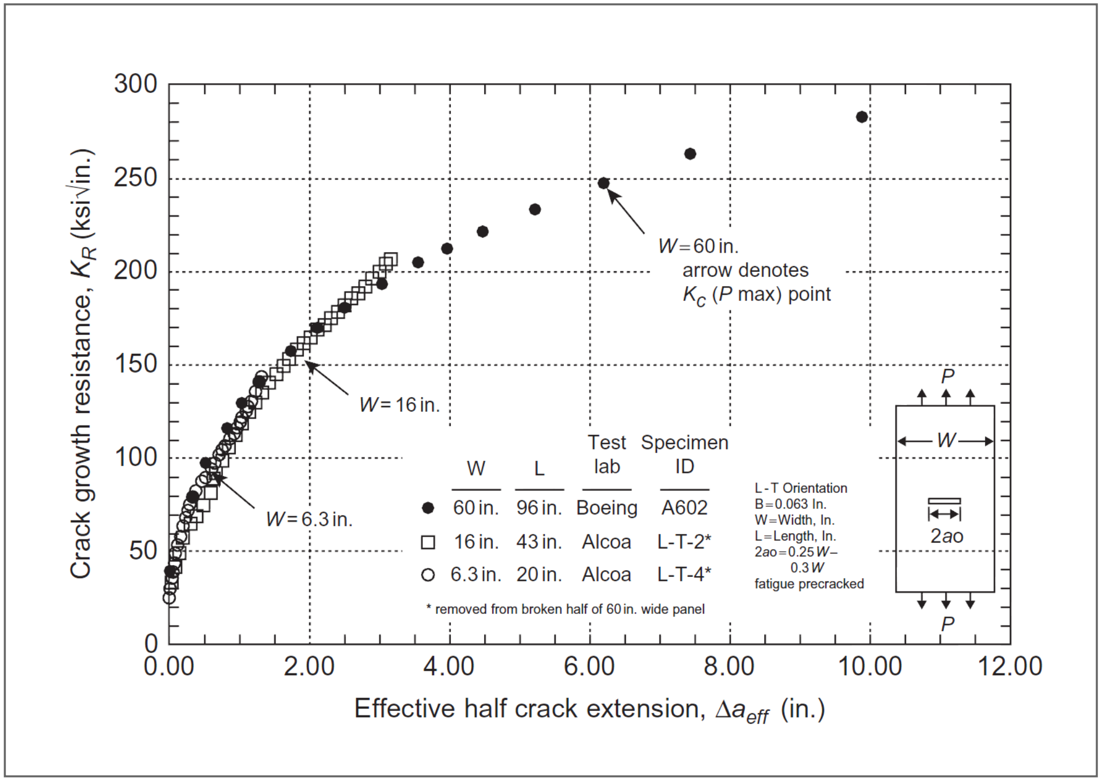
generate K_R curve

- Once the effective crack length and K_{Ie} has been determined for the test, we can generate the K_R curve
- The K_R curve is quite simply a plot of K_{Ie} vs. a for the test performed (i.e. with varying stress and increasing crack length)

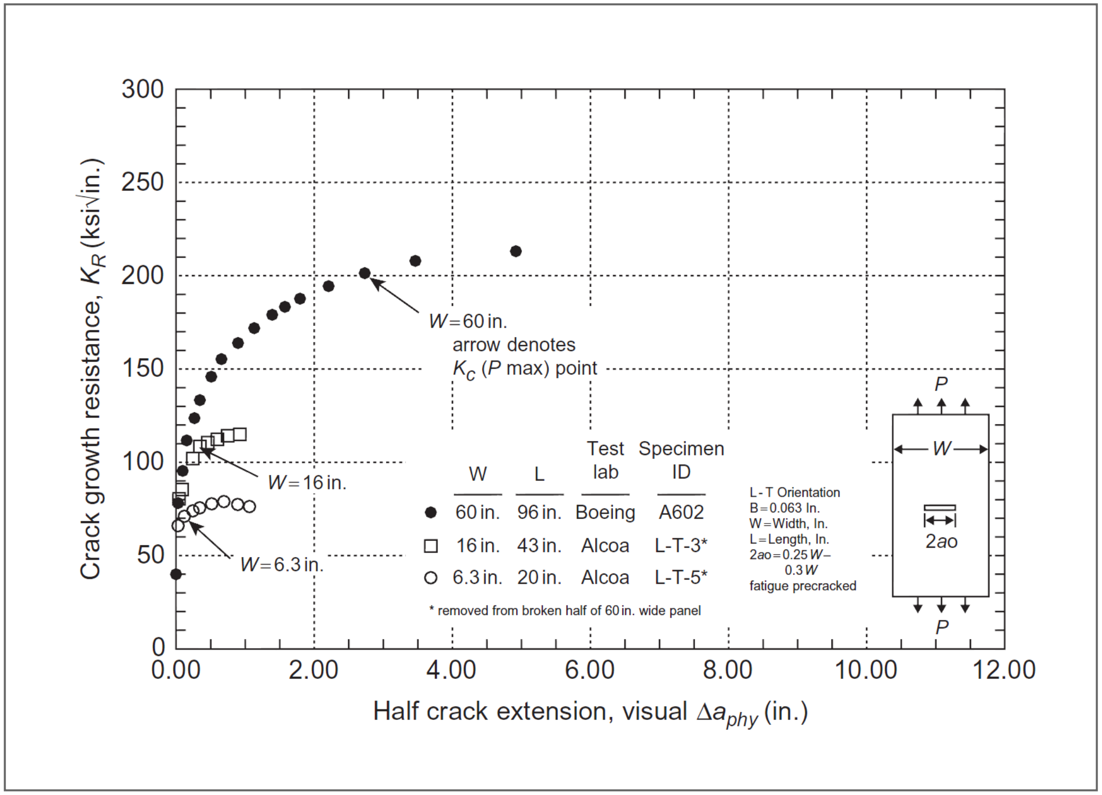
initial crack length

- When the test is performed correctly, the K_R curve is not a function of the initial crack length
- For this reason, we often plot K_{Ie} vs. Δa, to subtract the initial crack length
- We can superpose constant-stress K-curves on this graph, the curve which intersects at a tangent point creates the most "standard" definition for K_C

example


300
250
200
150
100
50
0
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Crack growth resistance, K_R (ksi√in.)
Effective half crack extension, Δa_{eff} (in.)
W = 60 in.
arrow denotes
K_c (P max) point
W = 16 in.
W = 6.3 in.
W L Test lab Specimen ID
60 in. 96 in. Boeing A602
16 in. 43 in. Alcoa L-T-2*
6.3 in. 20 in. Alcoa L-T-4*
* removed from broken half of 60 in. wide panel
L-T Orientation
B = 0.063 In.
W = Width, In.
L = Length, In.
2ao = 0.25 W– 0.3 W
fatigue precracked
P
W
2ao
P

example


300
250
200
150
100
50
0
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Crack growth resistance, K_R (ksi√in.)
Half crack extension, visual Δa_{phy} (in.)
W = 60 in.
arrow denotes
K_c (P max) point
W = 16 in.
W = 6.3 in.
W
L
Test lab
Specimen ID
60 in. 96 in. Boeing A602
16 in. 43 in. Alcoa L-T-3*
6.3 in. 20 in. Alcoa L-T-5*
* removed from broken half of 60 in. wide panel
L - T Orientation
B = 0.063 In.
W = Width, In.
L = Length, In.
2ao = 0.25 W – 0.3 W
fatigue precracked
P
W
2ao
P