

# AE 737 - MECHANICS OF DAMAGE TOLERANCE

## LECTURE 22

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# SCHEDULE

- 14 Apr - Exam Review
- 19 Apr - Damage Tolerance, Homework 8 Due
- 21 Apr - Exam 2
- 26 Apr - Exam Solutions, Damage Tolerance
- 28 Apr - SPTE, AFGROW, Finite Elements

1. crack growth retardation
2. exam 2
3. stress based fatigue
4. strain based fatigue
5. fracture based fatigue

## CRACK GROWTH RETARDATION

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- This zone has residual compressive stresses, which slow crack growth until the crack grows beyond this over-sized plastic zone
- We will discuss three retardation models, but no model has been shown to be perfect in all cases
- The Wheeler method reduces  $da/dN$ , the Willenborg model reduces  $\Delta K$ , and the Closure model increases  $R$  (increases  $\sigma_{min}$ )



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- and the constant,  $m$  is to be determined experimentally



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- Willenborg assumes that the residual compressive stress in the plastic zone creates an effective,  $K_{max,eff}$ , where  $K_{max,eff} = K_{max} - K_{comp}$
- The effective stress intensity factor is given by

$$K_{max,eff} = K_{max,i} - \left[ K_{max,OL} \sqrt{1 - \frac{\Delta a_i}{r_{pol}}} - K_{max,i} \right] \quad (22.3)$$



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- And the correction factor,  $\phi_i$  is given by

$$\phi_i = \frac{1 - K_{TH}/K_{max,i}}{S_{ol} - 1} \quad (22.5)$$



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- Where  $C_{f0}$  is the value of the Closure Factor at  $R = 0$

- When using the closure model, we replace  $R$  with  $C_f$

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- If the model we are using is in terms of  $\Delta K$  we will also need to use  $\Delta K = (1 - C_f)K_{max}$



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  1. Compressive underloads are uncommon in airframes
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  3. Analysis is generally adjusted with experimental data, so acceleration can be built in to current model
  4. Structures with large compressive loads are not generally subject to crack propagation problems

## EXAM 2

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- Make sure you can integrate the Paris Law equation (see Homework 8 problems 1 and 2)
- No table look-ups (stress intensity factors, paris/walker law constants will be given in problem)

## STRESS BASED FATIGUE

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- Has the simplest analysis of any fatigue analysis

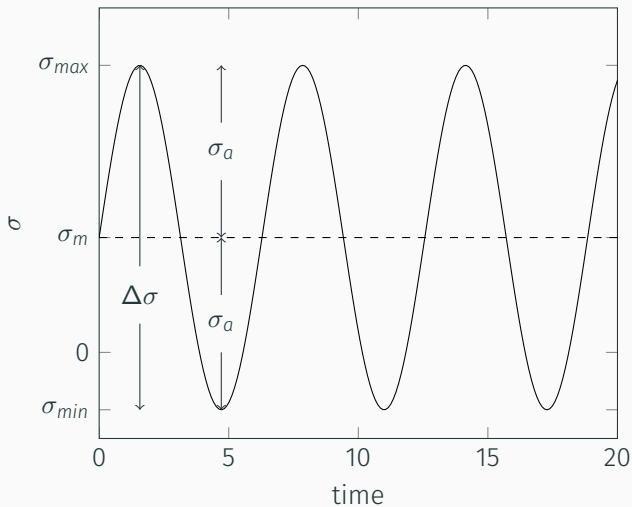
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- This point varies by material, but can be found using strain-based fatigue analysis

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- $\sigma_a$  is the stress amplitude, and is the variation about the mean
- We can express all of these in terms of the maximum and minimum stress

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \quad (22.8)$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \quad (22.9)$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \quad (22.10)$$

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- And the amplitude ratio,  $A$  is defined as

$$A = \frac{\sigma_a}{\sigma_m} \quad (22.12)$$

- There are some useful relationships between the above equations

$$\Delta\sigma = 2\sigma_a = \sigma_{max}(1 - R) \quad (22.13a)$$

$$\sigma_m = \frac{\sigma_{max}}{2}(1 + R) \quad (22.13b)$$

$$R = \frac{1 - A}{1 + A} \quad (22.13c)$$

$$A = \frac{1 - R}{1 + R} \quad (22.13d)$$



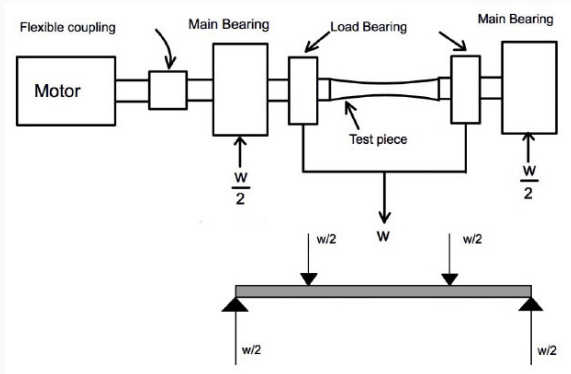
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- Rotating 4-point bend is one of the most common modern methods
- Tensile test machines can be used, but require much more time

## ROTATING FOUR-POINT BEND



**Figure 1:** Four-point bend gives uniform stress (along top and bottom surfaces)

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- In general, one set (or family) of S-N curves is generated using the same  $\sigma_m$
- Usually  $S_a$  (the nominal stress equivalent of  $\sigma_a$ ) is plotted versus  $N$  (the number of cycles)



- Each individual point on an S-N curve represents one fatigue experiment

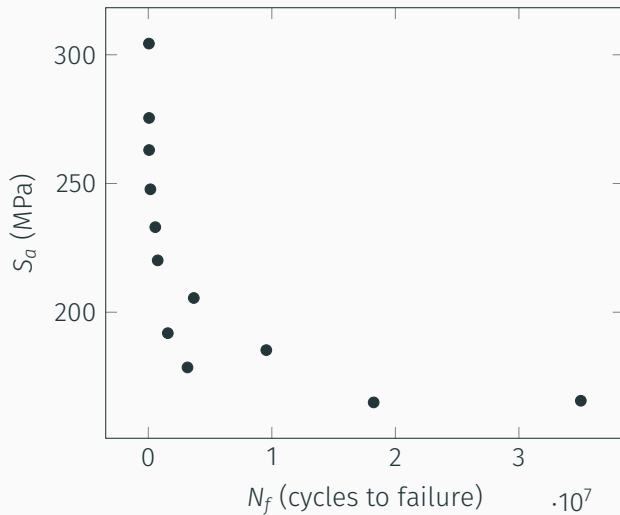
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- In the following plot, if only one test was performed for each point, the total number of cycles tested would be about  $7.3 \times 10^7$
- For a 100 Hz machine, this represents over 200 hours of consecutive testing
- Each repetition would further increase the test time required

## STRESS LIFE CURVES



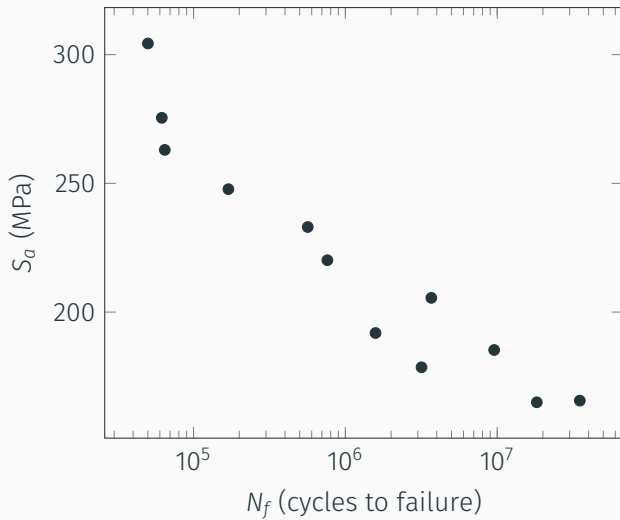
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- It is also very difficult to differentiate between low-cycle fatigue failure stresses
- Instead S-N curves are often plotted on a semi-log or log-log scale, so pay attention to the axes

## STRESS LIFE CURVES



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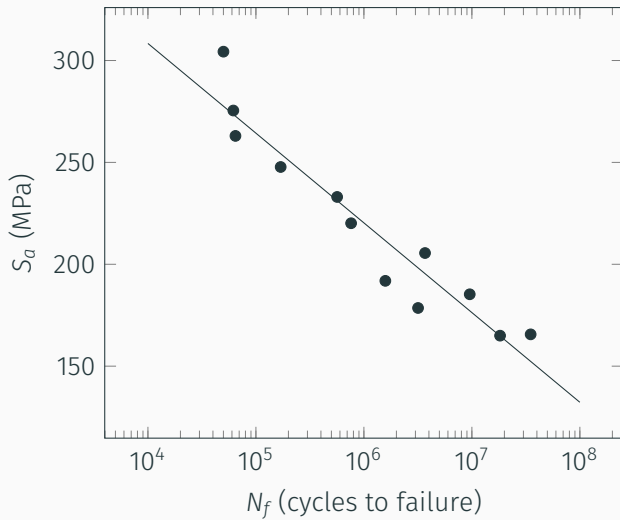
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$$\sigma_a = \sigma'_f (2N_f)^b \quad (22.15)$$

- $\sigma'_f$  and  $b$  are often considered material properties and can often be looked up on a table (p. 235)

## STRESS LIFE CURVES





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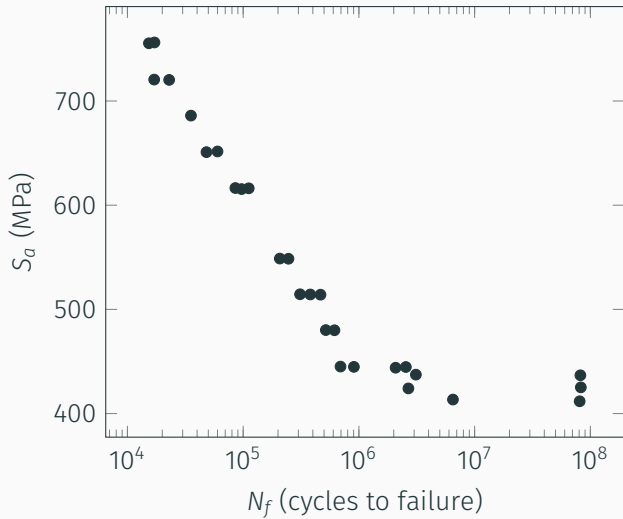
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- In these materials,  $\sigma_e$  is considered to be a material property
- This phenomenon is not typical of aluminum or copper alloys, but is sometimes arbitrarily assigned using whatever the failure stress is at some large number of cycles ( $10^7$  or  $10^8$ )

# FATIGUE LIMIT



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$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \dots = \sum \frac{N_i}{N_{fi}} = 1 \quad (22.16)$$

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$$B \left[ \sum \frac{N_i}{N_{if}} \right]_{rep} = 1 \quad (22.17)$$

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- One useful way of representing these data, instead of many S-N curves, is a constant-life diagram
- It is created by taking points from the S-N curves and plotting a line through constant  $N_f$  values

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- In general, the Goodman line is a good fit for brittle materials (steels) while the Gerber parabola is a better fit for more ductile materials

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- For steels,  $\sigma'_f \approx \tilde{\sigma}_{fB}$ , but for aluminums these values can be significantly different, and better agreement is found using  $\tilde{\sigma}_{fB}$ .



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- In general, it is best to use a form that matches your data
- If data is lacking, the SWT (22.22) and Morrow (22.21) equations generally provide the best fit

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$$\bar{\sigma}_m = \bar{\sigma}_{xm} + \bar{\sigma}_{ym} + \bar{\sigma}_{zm} \quad (22.24)$$

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**Table 1:** Table of  $\alpha$  values for Peterson notch sensitivity equation

Material	$\alpha$ (mm)	$\alpha$ (in)
Aluminum alloys	0.51	0.02
Annealed or low-carbon steels	0.25	0.01
Quenched and tempered steels	0.064	0.0025

- For high-strength steels, a more specific  $\alpha$  estimate can be found

$$\alpha = 0.025 \left( \frac{2070}{\sigma_u} \right)^{1.8} \quad \text{mm} \quad \sigma_u \geq 550 \text{ MPa} \quad (22.28)$$

$$\alpha = 0.001 \left( \frac{300}{\sigma_u} \right)^{1.8} \quad \text{in} \quad \sigma_u \geq 80 \text{ ksi} \quad (22.29)$$

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$$\alpha_{\text{torsion}} = 0.6\alpha \quad (22.30)$$

## STRAIN BASED FATIGUE

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- When the plastic strain is not significant, mean stress will exist
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$$\sigma_a = \sigma'_f \left[ \left( 1 - \frac{\sigma_m}{\sigma'_f} \right)^{\frac{1}{b}} (2N_f) \right]^b \quad (22.36)$$

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- From the zero-mean stress graph, find the point corresponding to your applied strain
- For a non zero mean stress, this point represents  $(\epsilon_a, N^*)$ , we can now solve for  $N_f$  using 22.37

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- There is no convenient solution method for this form, and it generally must be solved numerically, or plotted with many families of  $\sigma_m$

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- This method can also be solved graphically if a plot of  $\sigma_{max}\epsilon_a$  is made using zero-mean data. All we need to do is find the new  $\sigma_{max}\epsilon_a$  point to find a new  $N_f$

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- The SWT approach is very good for general use, but is non-conservative with a compressive mean stress

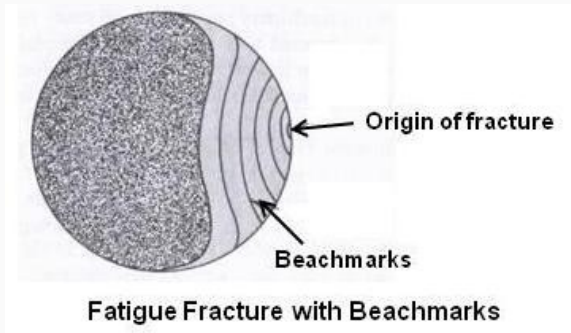
## FRACTURE BASED FATIGUE

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# FRACTURE SURFACE



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- It would be beneficial to predict at what rate a crack will extend



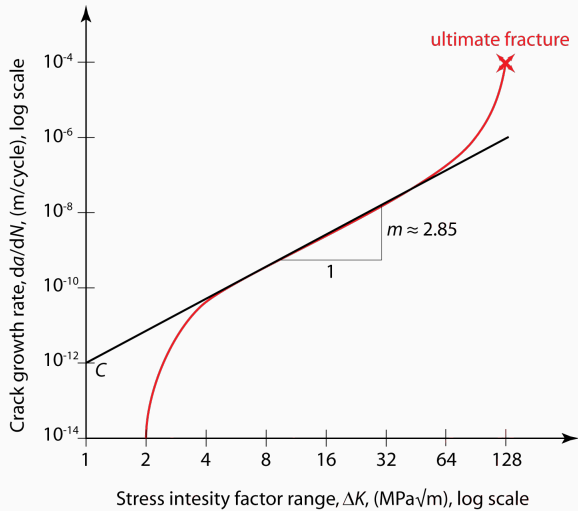
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- This chart is then commonly divided into three regions



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- In general,  $R$  dependence vanishes for  $R > 0.8$  or  $R < -0.3$ . This effect is known as the band width

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$$N = 10^4 \left( \frac{m_t}{Z\sigma_{max}} \right)^p \int_{L_0}^{L_f} \frac{dL}{\left( n \sqrt{\pi L / n \beta} \right)^p} \quad (22.45)$$

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- Using this method,  $G$  is typically looked up from a chart (such as on p. 369)

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- Which simplifies to

$$\sum_i (z\sigma_{max})_i^p N_i = (S)^p \quad (22.50)$$



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- ASTM E1049-85 "Standard Practices for Cycle Counting in Fatigue Analysis"

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5. When end of data is reached, count each range as 1/2-cycle



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3. If  $X \geq Y$ , count  $Y$  as 1 cycle and discard both points in  $Y$ , go to 1
4. Remaining cycles are counted backwards from end of history