AE 737: Mechanics of Damage Tolerance

Lecture 5 - Plastic Zone

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schedule

- 15 Feb - Plastic Zone
- 17 Feb - Plastic Zone, HW 2 Due, HW 1 Self-grade due
- 22 Feb - Fracture Toughness
- 24 Feb - Fracture Toughness, HW3 Due, HW 2 Self-grade due

- curved boundaries
- stress concentration factors
- plastic zone

curved boundaries

short cracks on curved boundaries

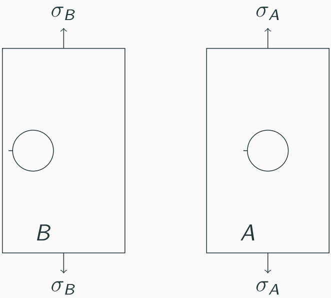
- For short cracks, we can use the *stress concentration factor* on a curved boundary to determine the stress intensity factor
- The stress concentration factor only gives the maximum stress at the curved boundary, thus the longer the crack is, the farther away from the curved boundary (and maximum stress) it is.
- Stress concentration factors can be found: pp. 82-85 in the text
- Also see supplemental text on Blackboard or here[1]

[1] ../classdocs/stress_concentrations.pdf

short cracks on curved boundaries

- Suppose we want to determine the stress intensity on a panel, panel B
- We find a similar panel with a known stress intensity factor, panel A
- We adjust the applied load on panel A such that $K_{I,A} = K_{I,B}$
- The magnitude of this load adjustment is determined using the *stress concentration factors* in panels B and A Note: the notation: K_t for stress concentration factor, K_I for stress intensity factor

short cracks on curved boundaries



short cracks on curved boundaries

- Since A is a fictional panel, we set the applied stress, σ_A such that

$$\sigma_{max,B} = \sigma_{max,A}$$

- Substituting stress concentration factors

$$K_{t,B}\sigma_B = K_{t,A}\sigma_A$$

- Solving for σ_A

$$\sigma_A = \frac{K_{t,B}}{K_{t,A}}\sigma_B$$

short cracks on curved boundaries

- Since the crack is short and $\sigma_{max,A} = \sigma_{max,B}$ we can say

$$
\begin{aligned}
K_{I,B} &= K_{I,A} \\
&= \sigma_A \sqrt{\pi c}\beta_A \\
&= \frac{K_{t,B}}{K_{t,A}} \sigma_B \sqrt{\pi c}\beta_A
\end{aligned}
$$

example 6 (p. 86)

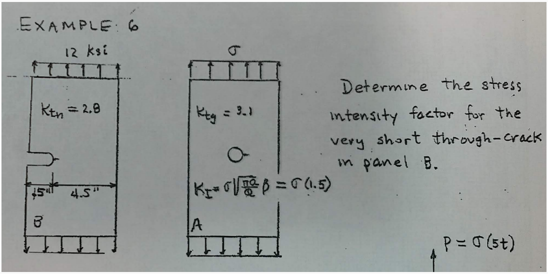
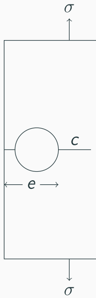


Figure 1: See p. 86, there is a short through crack on the edge of a 0.5" deep notch on a 5 inch wide panel with a remote 12 ksi stress applied. The net section stress concentration factor is 2.8, while the global stress concentration factor for a similar panel with a hole is 3.1

long cracks on curved boundaries

- As a crack becomes very large, the effect of the curved boundary diminishes
- We find expressions for β_L (long crack) and β_S (short crack)
- We connect β_S to β_L using a straight line from β_S to a tangent intersection with β_L

long cracks on curved boundaries

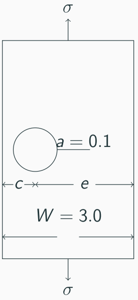


example

- Example here[2]

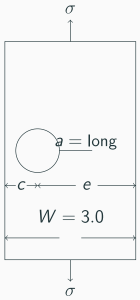
[2] ../examples/Long%20Cracks%20on%20Curved%20Boundaries.html

group one



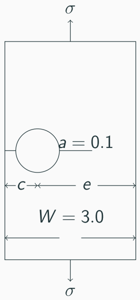
- $c = 0.75$, $e = 2.25$, $r = 0.5$
- assume a is short and calculate β for this case
- calculate in terms of β for known state

group two



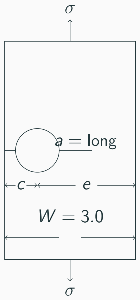
- $c = 0.75$, $e = 2.25$, $r = 0.5$
- assume a is long and calculate β for this case
- calculate in terms of β for known state

group three



- $c = 0.75$, $e = 2.25$, $r = 0.5$
- assume a is short and calculate β for this case
- calculate in terms of β for known state

group four



- $c = 0.75$, $e = 2.25$, $r = 0.5$
- assume a is long and calculate β for this case
- calculate in terms of β for known state

discussion

Draw a sketch to show how we could use this method to find cracks of intermediate length near a curved boundary

stress concentration factors

centered hole tension - p. 82

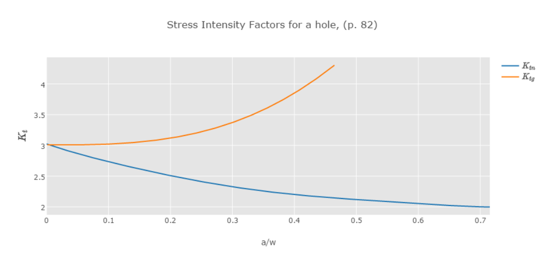
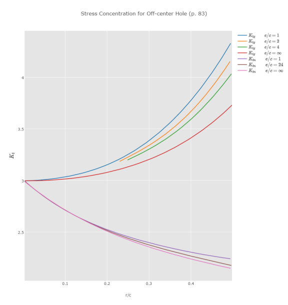


Figure 2: A plot of stress concentration factors near a hole, see text p. 82 or the interactive plots linked in the last slide.

K_{tg} uses stress for the cross-sectional area if no hole was present, K_{tn} uses stress at the net section (subtracting hole

off-center hole tension - p. 83



K_{tg} uses stress for the cross-sectional area if no hole was present, K_{tn} uses stress at the net section (subtracting hole area). c is the distance from the closest edge to the center of

bending of a bar with u-shaped notch - p. 84

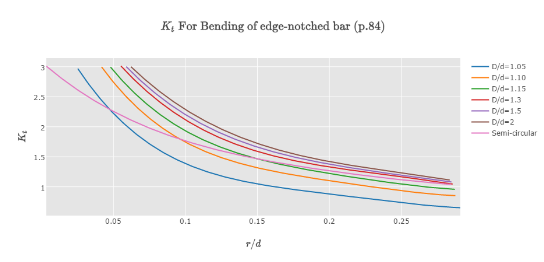
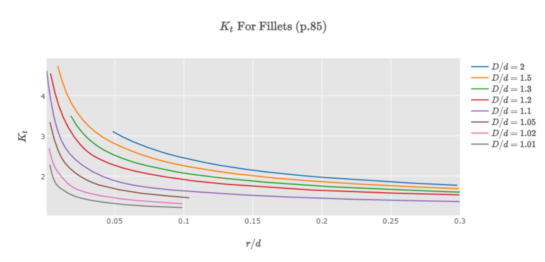


Figure 3: A plot of stress concentration factors in a bar with a u-shaped notch, see text p. 84 or the interactive plots linked in the last slide.

Nominal stress used for K_t is given by $\sigma_{nom} = 6M/hd^2$ where

tension of a stepped bar with shoulder fillets - p. 85



D is the larger width (before the step), *d* is the width after the step. Nominal stress is $\sigma_{nom} = P/hd$, where *h* is specimen thickness. *r* is the fillet radius.

interactive page

- An interactive page with these plots can be accessed here[3]

[3] ../examples/Stress%20Concentration%20Factors.html

plastic zone

plastic zone

- Previous developments assumed perfectly elastic materials
- Most common materials have some plasticity
- Any stress above the yield stress will undergo plastic deformation (no stress higher than σ_y will be present in the material)

plastic zone

- Plasticity helps retard crack propagation due to residual stresses
- After an overload, elastic regions will contract back to their undeformed shape
- The region which has undergone plastic deformation, however, holds its deformed shape
- This introduces a region of residual compressive stress near the crack tip
- Before the crack can propagate, a stress needs to overcome this residual stress

2D problems

- We often simplify the full 3D elasticity equations for planar problems
- For very thin panels, we assume that all out-of-plane stresses are 0
- This is called plane stress

plane stress

$$\sigma_z = \tau_{xz} = \tau_{zy} = 0$$
$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$
$$\epsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E}$$
$$\epsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$
$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$
$$\gamma_{xz} = \gamma_{yz} = 0$$

2D problems

- When instead a panel is very thick, we assume that any strains through the thickness are small relative to other strains
- $\epsilon_z = \gamma_{xz} = \gamma_{yz} = 0$
- This is known as plane strain

plane strain

$$
\begin{aligned}
\epsilon_x &= \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \\
\epsilon_y &= -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \\
0 &= -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E} \\
\gamma_{xy} &= \frac{\tau_{xy}}{G} \\
\gamma_{xz} &= \gamma_{yz} = 0
\end{aligned}
$$

Irwin's first approximation

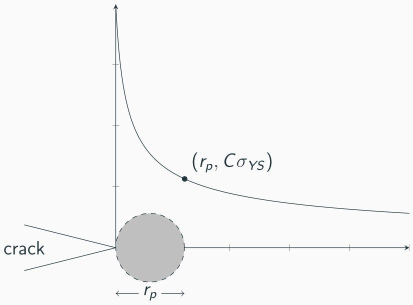
- If we recall the equation for opening stress (σ_y) near the crack tip

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{1.2}$$

- In the plane of the crack, when $\theta = 0$ we find

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}$$

Irwin's first approximation



Irwin's first approximation

- We use C, the *Plastic Constraint Factor* to convert between Plane Strain and Plane Stress solutions
- The plastic zone size can now be approximated

$$\sigma_{yy}(r = r_p) = C\sigma_{YS}$$

$$\frac{K_I}{\sqrt{2\pi r_p}} = C\sigma_{YS}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{C\sigma_{YS}}\right)^2$$

Irwin's first approximation

- For plane stress (thin panels) we let $C = 1$ and find rp as

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

- And for plane strain (thick panels) we let $C = \sqrt{3}$ and find

$$r_p = \frac{1}{6\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

Intermediate panels

- For panels which lie between plane strain and plane stress states, we use the following expression to estimate the plastic zone size

$$r_p = \frac{1}{I\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

- Where I is defined as

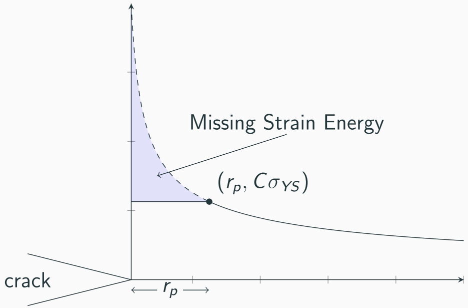
$$I = 6.7 - \frac{1.5}{t}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

- And $2 \leq I \leq 6$

Irwin's second approximation

- If our material is perfectly elastic-plastic, no stresses above $C\sigma_{ys}$ will exist in the material
- This ignores the strain energy (represented by the area under the curve) in the plastic zone

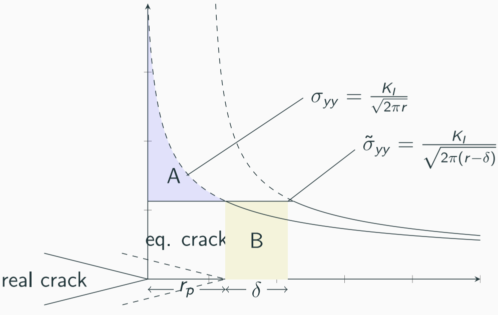
Irwin's second approximation



Irwin's second approximation

- To account for the additional strain energy, Irwin considered a plastic zone size increased by some δ
- He also needed to adjust the stress function, and considered an equivalent crack tip in these calculations

Irwin's second approximation



Irwin's second approximation

We need $A=B$, so we set them equivalent and solve for δ.

$$\begin{aligned} A &= \int_0^{r_p} \sigma_{yy} dr - r_p \sigma_{YS} \\ &= \int_0^{r_p} \frac{K_I}{\sqrt{2\pi r}} dr - r_p \sigma_{YS} \\ &= \frac{K_I}{\sqrt{2\pi}} \int_0^{r_p} r^{-1/2} dr - r_p \sigma_{YS} \\ &= \frac{2K_I\sqrt{r_p}}{\sqrt{2\pi}} - r_p \sigma_{YS} \end{aligned}$$

Irwin's second approximation

- We have already found *rp* as

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

- If we solve this for *KI* we find

$$K_I = \sqrt{2\pi r_p} \sigma_{YS}$$

Irwin's second approximation

- We can now substitute back into the strain energy of A

$$
\begin{aligned}
A &= \frac{2\sqrt{2\pi r_p}\sigma_{YS}\sqrt{r_p}}{\sqrt{2\pi}} - r_p\sigma_{YS} \\
&= 2\sigma_{YS} r_p - r_p\sigma_{YS} \\
&= r_p\sigma_{YS}
\end{aligned}
$$

Irwin's second approximation

- B is given simply as $B = \delta\sigma_{ys}$ so we equate A and B to find δ

$$
\begin{aligned}
A &= B \\
r_p\sigma_{YS} &= \delta\sigma_{YS} \\
r_p &= \delta
\end{aligned}
$$

Irwin's second approximation

- This means the plastic zone size is simply $2rp$
- However, it also means that the effective crack length is $a+rp$
- Since rp depends on KI, we must iterate a bit to find the "real" rp and KI

Example

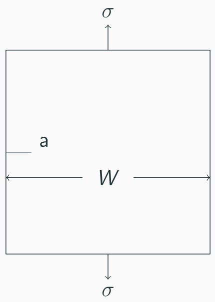


Figure 4: An edge crack of length a in a panel of width W is subjected to a remote load

equations

$$\beta = \left[1.122 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.71\left(\frac{a}{W}\right)^3 + 30.82\left(\frac{a}{W}\right)^4\right]$$

$$I = 6.7 - \frac{1.5}{t}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

$$r_p = \frac{1}{I\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$