AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 2

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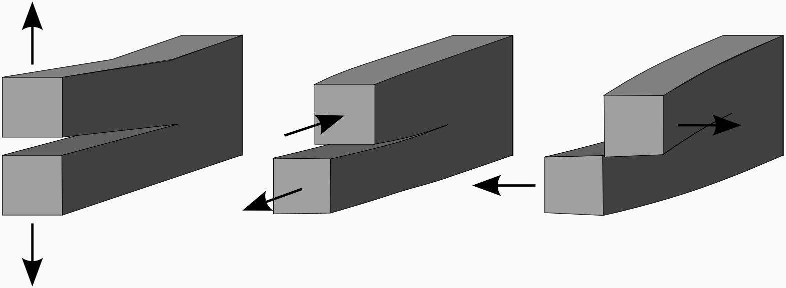
OFFICE HOURS

- So far 8/30 students have participated in Doodle, looks like Friday Afternoon
- Let me know if you do not receive the e-mail, you may need to update your information in Blackboard
- Take advantage of office hours, this is time that I have already set aside for you
- If the regular office hours do not work for your schedule, send me an e-mail and we can work out a time to meet
- While in person visits are often the most helpful, I will always try to answer questions as best as I can via e-mail

FRACTURE MECHANICS

- *Linear Elastic Fracture Mechanics* is the study of the propagation of cracks in materials
- There are some corrections we add to account for plasticity
- In this course we will not follow the full mathematical development of fracture mechanics (there is a separate course dedicated to that)
- Instead we will take some results and apply them

- In fracture mechanics we consider three different modes
- Mode I is known as the ”opening mode”
- Mode II is known as the ”sliding mode”
- Mode III is known as the ”tearing mode”



Mode I: Opening Mode II: Sliding Mode III: Tearing

STRESS INTENSITY

- A key finding from Linear Elastic Fracture Mechanics (LEFM) is known as the *Stress Intensity Factor*
- The stress intensity factor is often written in this form

$$K = \sigma\sqrt{\pi a}\beta$$

- Where K is the stress intensity factor, σ is the applied stress, a is the crack length, and β is a dimensionless parameter depending on geometry
- Be careful that although the notation is similar, the *Stress Intensity Factor* is different from the *Stress Concentration Factor* from strength of materials
- We are usually most concerned with Mode I, but there will be a unique stress intensity factor for each mode, we label these K_I, K_{II}, and K_{III}
- If no subscript is given, assume Mode I

- For brittle materials (where ”linear” fracture mechanics assumptions hold true) we can find the full stress field near the crack in terms of the stress intensity factor

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

- Similarly for Mode II we find

$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$
$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

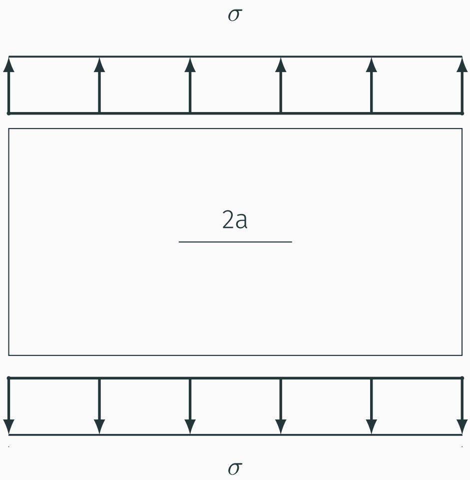
- And for Mode III

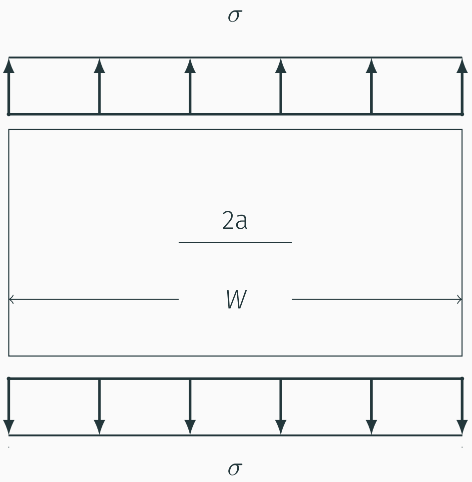
$$\tau_{xz} = \frac{-K_{III}}{\sqrt{2\pi r}} \sin \frac{\theta}{2}$$
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2}$$

COMMON STRESS INTENSITY FACTORS

CENTER CRACK, INFINITE WIDTH

$$K_I = \sigma\sqrt{\pi a} \qquad (2.1)$$



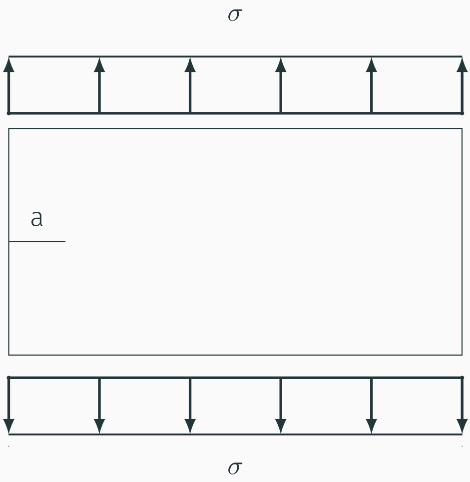


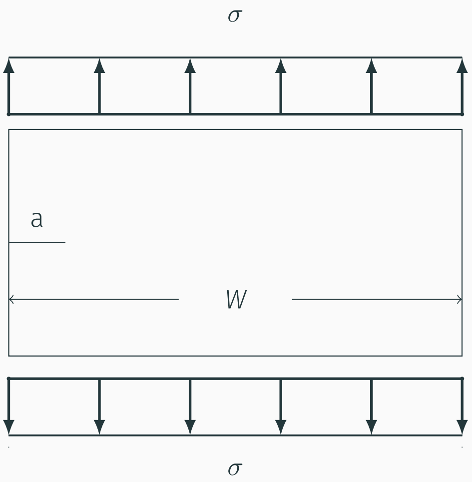
σ
2a
W
σ

-
$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)} \tag{2.2a}$$
- Accurate within 0.3% for $2a/W \leq 0.7$
- within 1.0% for $2a/W = -.8$
-
$$K_I = \sigma\sqrt{\pi a}\left[1.0 - 0.025\left(\frac{2a}{W}\right)^2 + 0.06\left(\frac{2a}{W}\right)^4\right]\sqrt{\sec(\pi a/W)} \tag{2.2b}$$
- Accurate within 0.1% for all crack lengths.

EDGE CRACK, SEMI-INFINITE WIDTH

$$K_I = 1.122\sigma\sqrt{\pi a} \tag{2.3}$$




σ
a
W
σ

EDGE CRACK, FINITE WIDTH

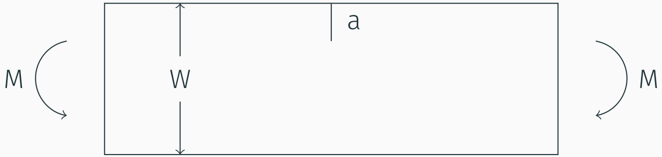
$$\beta = \left[1.122 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.71\left(\frac{a}{W}\right)^3 + 30.82\left(\frac{a}{W}\right)^4\right] \tag{2.4a}$$

- Within 0.5% accuracy for $\frac{a}{W} < 0.6$

$$\beta = \frac{0.752 + 2.02\frac{a}{W} + 0.37\left(1 - \sin\frac{\pi a}{2W}\right)^3}{\cos\frac{\pi a}{2W}} \sqrt{\frac{2W}{\pi a} \tan\frac{\pi a}{2W}} \tag{2.4b}$$

- Within 0.5% accuracy for $0 < \frac{a}{W} < 1.0$

EDGE CRACK, BENDING MOMENT



- The usual form for stress intensity still applies

$$K_I = \sigma\sqrt{\pi a}\beta$$

- Where

$$\sigma = \frac{6M}{tW^2}$$

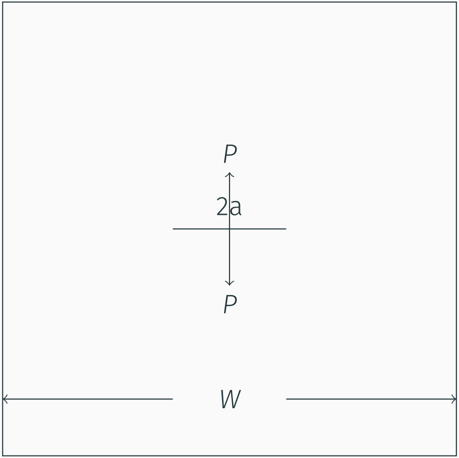
$$\beta = 1.122 - 1.40\left(\frac{a}{W}\right) + 7.33\left(\frac{a}{W}\right)^2 - 13.08\left(\frac{a}{W}\right)^3 + 14.0\left(\frac{a}{W}\right)^4 \tag{2.5a}$$

- valid within 0.2% accuracy for $\frac{a}{W} \leq 0.6$

$$\beta = \frac{0.923 + 0.199\left(1 - \sin\frac{\pi a}{2W}\right)^4}{\cos\frac{\pi a}{2W}}\sqrt{\frac{2W}{\pi a}\tan\frac{\pi a}{2W}} \tag{2.5b}$$

- valid within 0.5% for any $\frac{a}{W}$

CENTER CRACK, FINITE WIDTH, SPLITTING FORCES



CENTER CRACK, FINITE WIDTH, SPLITTING FORCES

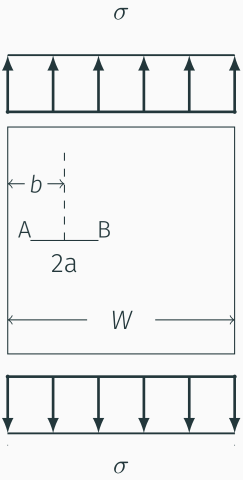
- With an applied load we use a slightly modified form for the stress intensity factor

$$K_I = \frac{P}{t\sqrt{\pi a}}\beta \tag{2.6}$$

- With β in this case given as

$$\beta = \frac{1 - 0.5\left(\frac{a}{W}\right) + 0.975\left(\frac{a}{W}\right)^2 - 0.16\left(\frac{a}{W}\right)^3}{\sqrt{1 - \left(\frac{a}{W}\right)}} \tag{2.7}$$

OFFSET CRACK



$$K_{IA} = \sigma\sqrt{\pi a}\beta_c\beta_A \text{ and } K_{IB} = \sigma\sqrt{\pi a}\beta_c\beta_B \tag{2.8}$$

$$\beta_c = \sqrt{\sec\frac{\pi a}{W}} \tag{2.9a}$$

$$\beta_A = (1 - 0.025\lambda^2 + 0.6\lambda^4 - \gamma\lambda^{11})\sqrt{\sec\left(\frac{\pi\lambda}{2}\right)\frac{\sin\left(2\lambda - 4\frac{a}{W}\right)}{2\lambda - 4\frac{a}{W}}} \tag{2.9b}$$

$$\beta_B = (1 - 0.025\delta^2 + 0.06\delta^4 - \zeta\lambda^{30})\left(1 + \frac{\sqrt{\sec\left(\frac{2\pi\lambda+1.5\pi\delta}{7}\right) - 1}}{1 + 0.21\sin\left(8\tan^{-1}\left(\frac{\lambda-\delta}{\lambda+\delta}\right)^{0.9}\right)}\right) \tag{2.9c}$$

- The parameters λ, δ are given as

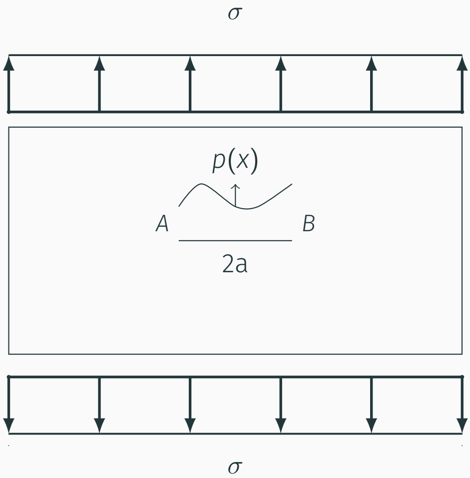
$$\lambda = \frac{a}{b} \tag{2.9d}$$

$$\delta = \frac{a}{W - b} \tag{2.9e}$$

- And γ and ζ can be looked up on a table

Table 1: Parameters for offset crack

$\frac{b}{W}$	γ	ζ
0.1	0.382	0.114
0.25	0.136	0.286
0.4	0.0	0.0
0.5	0.0	0.0

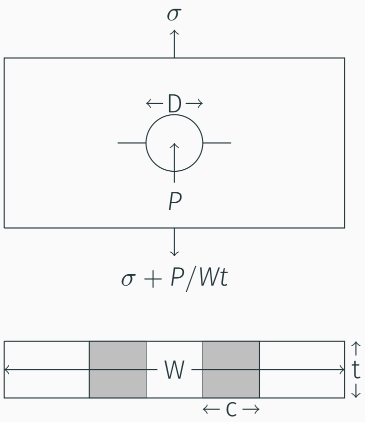

σ
p(x)
A
B
2a
σ

- Stress intensity will be different at points *A* and *B*

$$K_{IA} = \int_{-a}^{a} \frac{p(x)}{\sqrt{\pi a}} \frac{\sqrt{a-x}}{\sqrt{a+x}} dx \tag{2.10a}$$

$$K_{IB} = \int_{-a}^{a} \frac{p(x)}{\sqrt{\pi a}} \frac{\sqrt{a+x}}{\sqrt{a-x}} dx \tag{2.10b}$$

CRACKS AROUND A HOLE



- For symmetric through cracks under uniform applied stress, we have

$$\beta = \beta_1 + \beta_2 \tag{2.11a}$$

$$\beta_1 = F_{c/R} F_w F_{ww} \tag{2.11b}$$

$$\beta_2 = \frac{\sigma_{br}}{\sigma} F_3 F_w F_{ww} \tag{2.11c}$$

$$F_{c/R} = \frac{3.404 + 3.1872\frac{c}{R}}{1 + 3.9273\frac{c}{R} - 0.00695\left(\frac{c}{R}\right)^2} \tag{2.11d}$$

$$F_w = \sqrt{\sec\frac{\pi R}{W}\sec\frac{\pi(R+c)}{W}} \tag{2.11e}$$

$$F_{ww} = 1 - \left(\left(1.32\frac{W}{D} - 0.14\right)^{-(.98+\left(0.1\frac{W}{D}\right)^{0.1})} - 0.02\right)\left(\frac{2c}{W-D}\right)^N \tag{2.11f}$$

$$F_3 = 0.098 + 0.3592e^{-3.5089\frac{c}{R}} + 0.3817e^{-0.5515\frac{c}{R}} \tag{2.11g}$$

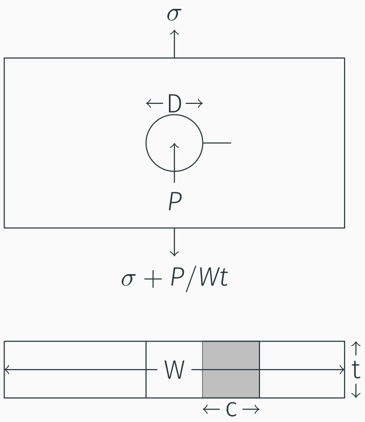
- Note that

$$\sigma_{br} = \frac{P}{Dt} \tag{2.11h}$$

$$N = \frac{W}{D} + 2.5 \qquad \text{when} \qquad \frac{W}{D} < 2 \tag{2.11i}$$

$$N = 4.5 \qquad \text{otherwise} \tag{2.11j}$$

- Also R is the radius, $R = \frac{D}{2}$



σ
D
P
$\sigma + P/Wt$
W
t
c

- For one through crack under uniform applied stress, we have

$$\beta = \beta_1 + \beta_2 \tag{2.12a}$$

$$\beta_1 = \beta_3 F_w F_{ww} \tag{2.12b}$$

$$\beta_2 = \frac{\sigma_{br}}{\sigma} F_4 F_w F_{ww} \tag{2.12c}$$

$$\begin{split}\beta_3 = 0.7071 + 0.7548 \frac{R}{R+c} + 0.3415 \left(\frac{R}{R+C}\right)^2 + \\ 0.6420 \left(\frac{R}{R+c}\right)^3 + 0.9196 \left(\frac{R}{R+c}\right)^4\end{split} \tag{2.12d}$$

$$F_4 = 0.9580 + 0.2561 \frac{c}{R} - 0.00193 \left(\frac{c}{R}\right)^{2.5} - 0.9804 \left(\frac{c}{R}\right)^{0.5} \tag{2.12e}$$

$$F_w = \sqrt{\sec\frac{\pi R}{W}\sec\frac{\pi(R+c/2)}{W-c}} \tag{2.12f}$$

$$F_{ww} = 1 - N^{-\frac{W}{D}}\left(\frac{2c}{W-D}\right)^{\frac{W}{D}+0.5} \tag{2.12g}$$

$$N = 2.65 - 0.24\left(2.75 - \frac{W}{D}\right)^2 \tag{2.12h}$$

$$N \geq 2.275 \qquad (\text{if } N < 2.275, \text{ let } N = 2.275) \tag{2.12i}$$

Also note that R indicates radius, $R = \frac{D}{2}$

1. 1.1 Determine the value of K_I for a center-cracked panel with $W/2a = 3$ and a uniformly applied remote stress, σ.
   1.2 Determine the value of K_I for an edge-cracked panel with $W/a = 3$ and a uniformly applied remote stress, σ.
   1.3 Compare these two results. Note that in both cases the panel width to crack length ratio is the same.

- Based on the ratio of crack length to width, we choose (2.2a) over (2.1) and (2.2b)

$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)}$$

- This gives $K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi/6)}$
- If we normalize by the infinite width solution, we find $K_{I,f}/K_{I,i} \approx 1.075$

- Once again, based on the ratio of crack length to width, we choose (2.4a) over (2.3) and (2.4b)

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4\right]$$

- This gives $K_I \approx 1.786\sigma\sqrt{\pi a}$
- If we normalize by the infinite width solution, we find $K_{I,f}/K_{I,i} \approx 1.595$

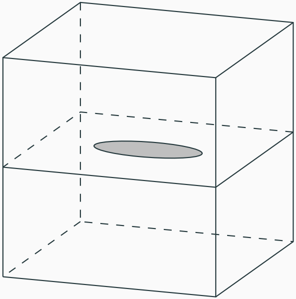
- Comparing the two cases, we see that the finite width effects are much more significant for the edge-crack specimen
- The edge-crack specimen is also overall more effected by a crack of that relative length.
- Why are they not the same?

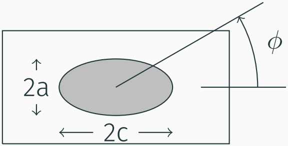
2D crack shapes

CRACK DEPTH

- The previous stress intensity factors all assume a 2D problem (with a 1D crack)
- Through the thickness, it is assumed that the crack length is the same
- In many cases this is not an accurate assumption
- We will now consider 2D crack shapes and their effect on the stress intensity factor

ELLIPTICAL FLAW, INFINITE SOLID





- For an ellipse the stress intensity factor will vary with the angle, ϕ

$$K_I = \sigma\sqrt{\pi a}\beta \tag{2.13a}$$

$$\beta = \sqrt{\frac{1}{Q}f_\phi} \tag{2.13b}$$

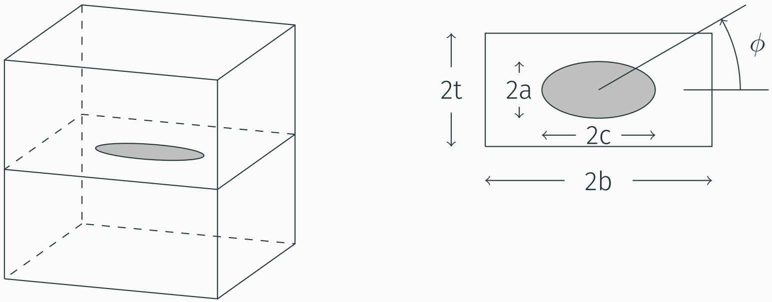
$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \qquad \text{if } a/c \leq 1 \tag{2.13c}$$

$$Q = 1 + 1.464\left(\frac{c}{a}\right)^{1.65} \qquad \text{if } a/c > 1 \tag{2.13d}$$

$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4} \qquad \text{if } a/c \leq 1 \tag{2.13e}$$

$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2 \sin^2\phi\right)^{1/4} \qquad \text{if } a/c > 1 \tag{2.13f}$$

ELLIPTICAL FLAW, FINITE SOLID



ELLIPTICAL FLAW, FINITE SOLID

$$K_I = \sigma\sqrt{\pi a}\beta \tag{2.14a}$$

$$\beta = \sqrt{\frac{1}{Q}}F_e \tag{2.14b}$$

$$F_e = \left(M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right) g f_\phi f_w \tag{2.14c}$$

$$f_w = \sqrt{\sec\left(\frac{\pi c}{2b}\sqrt{\frac{a}{t}}\right)} \tag{2.14d}$$

$$g = 1 - \frac{\left(\frac{a}{t}\right)^4 \left(2.6 - 2\frac{a}{t}\right)^{1/2}}{1 + 4\frac{a}{c}} \cos\phi \tag{2.14e}$$

$$M_2 = \frac{0.05}{0.11 + \left(\frac{a}{c}\right)^{3/2}} \tag{2.14f}$$

$$M_3 = \frac{0.29}{0.23\left(\frac{a}{c}\right)^{3/2}} \tag{2.14g}$$

- If $a/c \leq 1$

$$M_1 = 1 \tag{2.14h}$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \tag{2.14i}$$

$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4} \tag{2.14j}$$

$$\tag{2.14k}$$

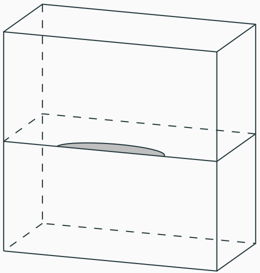
- Otherwise ($a/c > 1$)

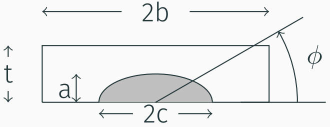
$$M_1 = \left(\frac{c}{a}\right)^{1/2} \tag{2.14l}$$

$$Q = 1 + 1.464\left(\frac{c}{a}\right)^{1.65} \tag{2.14m}$$

$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2 \sin^2\phi\right)^{1/4} \tag{2.14n}$$

SEMI-ELLIPTICAL SURFACE FLAW, FINITE BODY





$$K_I = \sigma\sqrt{\pi a}\beta \tag{2.15a}$$

$$\beta = \sqrt{\frac{1}{Q}}F_s \tag{2.15b}$$

$$F_s = \left(M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right) g f_\phi f_w \tag{2.15c}$$

$$f_w = \sqrt{\sec\left(\frac{\pi c}{2b}\sqrt{\frac{a}{t}}\right)} \tag{2.15d}$$

For $\frac{a}{c} \leq 1$

$$M_1 = 1.13 - 0.09\left(\frac{a}{c}\right) \tag{2.15e}$$

$$M_2 = -0.52 + \frac{0.89}{0.2 + \frac{a}{c}} \tag{2.15f}$$

$$M_3 = 0.5 - \frac{1}{0.65 + \frac{a}{c}} + 14\left(1 - \frac{a}{c}\right)^4 \tag{2.15g}$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \tag{2.15h}$$

$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4} \tag{2.15i}$$

$$g = 1 + \left(0.1 + 0.35\left(\frac{a}{t}\right)^2\right)(1 - \sin\phi)^2 \tag{2.15j}$$

For $\frac{a}{c} > 1$

$$M_1 = \left(\frac{c}{a}\right)^{1/2}\left(1 + 0.04\frac{c}{a}\right) \tag{2.15k}$$

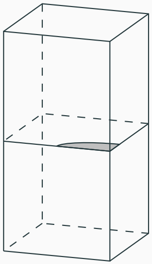
$$M_2 = 0.2\left(\frac{c}{a}\right)^4 \tag{2.15l}$$

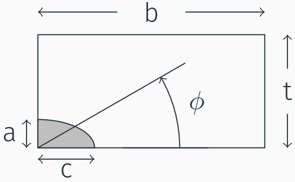
$$M_3 = -0.11\left(\frac{c}{a}\right)^4 \tag{2.15m}$$

$$Q = 1 + 1.464\left(\frac{c}{a}\right)^{1.65} \tag{2.15n}$$

$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2\sin^2\phi\right)^{1/4} \tag{2.15o}$$

$$g = 1 + \left(0.1 + 0.35\left(\frac{c}{a}\right)\left(\frac{a}{t}\right)^2\right)(1 - \sin\phi)^2 \tag{2.15p}$$




b
t
ϕ
a
c

$$K_I = \sigma\sqrt{\pi a}\beta \tag{2.16a}$$

$$\beta = \sqrt{\frac{1}{Q}}F_c \tag{2.16b}$$

$$F_c = \left(M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right) g_1 g_2 f_\phi f_w \tag{2.16c}$$

$$f_w = 1 - 0.2\lambda + 9.4\lambda^2 - 19.4\lambda^3 + 27.1\lambda^4 \tag{2.16d}$$

$$\lambda = \left(\frac{c}{b}\right)\left(\frac{a}{t}\right)^{1/2} \tag{2.16e}$$

For $\frac{a}{c} \leq 1$

$$M_1 = 1.08 - 0.03\left(\frac{a}{c}\right) \tag{2.16f}$$

$$M_2 = -0.44 + \frac{1.06}{0.3 + \frac{a}{c}} \tag{2.16g}$$

$$M_3 = -0.5 + 0.25\frac{a}{c} + 14.8\left(1 - \frac{a}{c}\right)^{1.5} \tag{2.16h}$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \tag{2.16i}$$

$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4} \tag{2.16j}$$

$$g_1 = 1 + \left(0.08 + 0.4\left(\frac{a}{t}\right)^2\right)(1 - \sin\phi)^3 \tag{2.16k}$$

$$g_2 = 1 + \left(0.08 + 0.15\left(\frac{a}{t}\right)^2\right)(1 - \cos\phi)^3 \tag{2.16l}$$

For $\frac{a}{c} > 1$

$$M_1 = \left(\frac{c}{a}\right)^{1/2} \left(1.08 - 0.03\frac{c}{a}\right) \tag{2.16m}$$

$$M_2 = 0.375 \left(\frac{c}{a}\right)^4 \tag{2.16n}$$

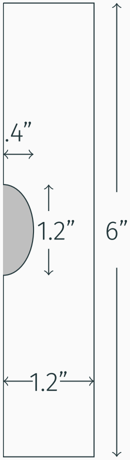
$$M_3 = -0.25 \left(\frac{c}{a}\right)^2 \tag{2.16o}$$

$$Q = 1 + 1.464 \left(\frac{c}{a}\right)^{1.65} \tag{2.16p}$$

$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2 \sin^2\phi\right)^{1/4} \tag{2.16q}$$

$$g_1 = 1 + \left(0.08 + 0.4\left(\frac{c}{t}\right)^2\right)(1 - \sin\phi)^3 \tag{2.16r}$$

$$g_2 = 1 + \left(0.08 + 0.15\left(\frac{c}{t}\right)^2\right)(1 - \cos\phi)^3 \tag{2.16s}$$



- Find maximum value of K_I for semi-elliptical surface flaw
- $\sigma = 20$kpsi (in opening direction)

EXAMPLE 2

- Here we will use (2.15)
- The first step we find $a/c = 0.4/0.6 < 1$, so we use (2.15e)-(2.15j)