AE 737: Mechanics of Damage Tolerance

Lecture 2 - Common Stress Intensity Factors

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schedule

- 20 Jan - Common Stress Intensity Factors
- 25 Jan - Superposition, Compounding
- 27 Jan - Curved Boundaries, HW 1 Due
- 1 Feb - Plastic Zone

review

fracture mechanics

- In fracture mechanics we consider three different modes
- Mode I is known as the “opening mode”
- Mode II is known as the “sliding mode”
- Mode III is known as the “tearing mode”

fracture mechanics

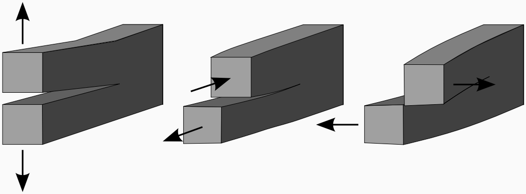


Figure 1: An image of the three fracture modes, with a representative crack in the xy plane. The first mode showns a crack opening vertically in the z-direction, like jaws opening. The second mode is known as the sliding mode, where one face moves into the

stress intensity

- A key finding from Linear Elastic Fracture Mechanics (LEFM) is known as the *Stress Intensity Factor*
- The stress intensity factor is often written in this form

$$K = \sigma\sqrt{\pi a}\beta$$

- Where K is the stress intensity factor, σ is the applied stress, a is the crack length, and β is a dimensionless parameter depending on geometry

stress Intensity

- Be careful that although the notation is similar, the *Stress Intensity Factor* is different from the *Stress Concentration Factor* from strength of materials
- We are usually most concerned with Mode I, but there will be a unique stress intensity factor for each mode, we label these K_I, K_{II}, and K_{III}
- If no subscript is given, assume Mode I

stress intensity

- For brittle materials (where "linear" fracture mechanics assumptions hold true) we can find the full stress field near the crack in terms of the stress intensity factor

$$
\begin{aligned}
\sigma_x &= \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \\
\sigma_y &= \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \\
\tau_{xy} &= \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}
\end{aligned}
$$

mode II

- Similarly for Mode II we find

$$
\begin{aligned}
\sigma_x &= \frac{-K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\\
\sigma_y &= \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\\
\tau_{xy} &= \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)
\end{aligned}
$$

mode III

- And for Mode III

$$
\begin{aligned}
\tau_{xz} &= \frac{-K_{III}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\\
\tau_{yz} &= \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}
\end{aligned}
$$

common stress intensity factors

center crack, infinite width

$$K_I = \sigma\sqrt{\pi a}$$

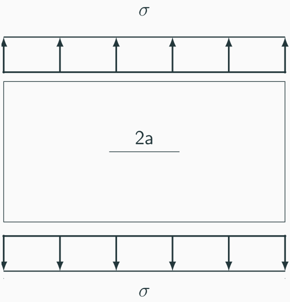


Figure 2: center crack, infinite width

center crack, finite width

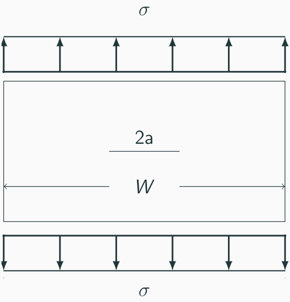


Figure 3: center crack, finite width

center crack, finite width

$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)}$$

- Accurate within 0.3% for $2a/W \leq 0.7$
- within 1.0% for $2a/W = -.8$

$$K_I = \sigma\sqrt{\pi a}\left[1.0 - 0.025\left(\frac{2a}{W}\right)^2 + 0.06\left(\frac{2a}{W}\right)^4\right]\sqrt{\sec(\pi a/W)}$$

- Accurate within 0.1% for all crack lengths.

edge crack, semi-infinite width

$$K_I = 1.122\sigma\sqrt{\pi a}$$

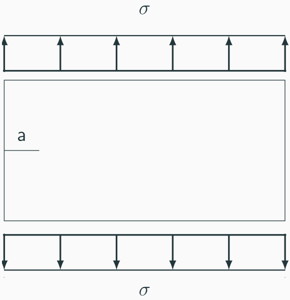


Figure 4: edge crack, semi-infinite

edge crack, finite width

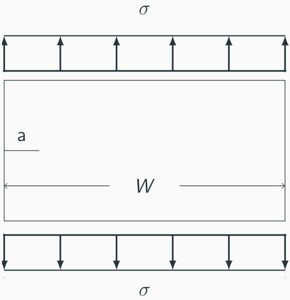


Figure 5: edge crack, finite width

edge crack, finite width

$$\beta = \left[1.122 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.71\left(\frac{a}{W}\right)^3 + 30.82\left(\frac{a}{W}\right)^4\right]$$

- Within 0.5% accuracy for $\frac{a}{W} < 0.6$

$$\beta = \frac{0.752 + 2.02\frac{a}{W} + 0.37\left(1 - \sin\frac{\pi a}{2W}\right)^3}{\cos\frac{\pi a}{2W}}\sqrt{\frac{2W}{\pi a}\tan\frac{\pi a}{2W}}$$

- Within 0.5% accuracy for $0 < \frac{a}{W} < 1.0$

edge crack, bending moment

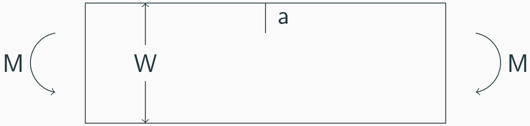


Figure 6: edge crack under bending

edge crack, bending moment

- The usual form for stress intensity still applies

$$K_I = \sigma\sqrt{\pi a}\beta$$

- Where $\sigma = \frac{6M}{tW^2}$

$$\beta = 1.122 - 1.40\left(\frac{a}{W}\right) + 7.33\left(\frac{a}{W}\right)^2 - 13.08\left(\frac{a}{W}\right)^3 + 14.0\left(\frac{a}{W}\right)^4$$

- valid within 0.2% accuracy for $\frac{a}{W} \leq 0.6$

edge crack, bending moment

$$\beta = \frac{0.923 + 0.199\left(1 - \sin\frac{\pi a}{2W}\right)^4}{\cos\frac{\pi a}{2W}}\sqrt{\frac{2W}{\pi a}\tan\frac{\pi a}{2W}}$$

- valid within 0.5% for any $\frac{a}{W}$

nominal bending stress

- The nominal bending stress is for rectangular cross-sections
- A more general form is given by $\sigma = \frac{Mc}{I}$
- Where for a rectangular cross-section, $c = W/2$ and $I = tW^3/12$ which simplifies as shown previously

center crack, finite width, splitting forces

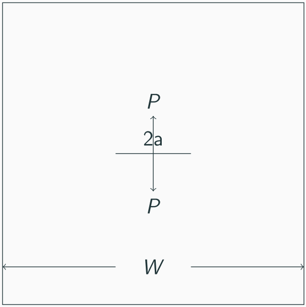


Figure 7: center crack, finite widte, splitting forces

center crack, finite width, splitting forces

- With an applied load we use a slightly modified form for the stress intensity factor $K_I = \frac{P}{t\sqrt{\pi a}}\beta$
- With β in this case given as

$$\beta = \frac{1 - 0.5\left(\frac{a}{W}\right) + 0.975\left(\frac{a}{W}\right)^2 - 0.16\left(\frac{a}{W}\right)^3}{\sqrt{1-\left(\frac{a}{W}\right)}}$$

offset crack

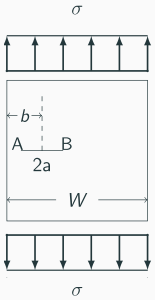


Figure 8: off-center crack

offset crack

$$K_{IA} = \sigma\sqrt{\pi a}\beta_c\beta_A \text{ and } K_{IB} = \sigma\sqrt{\pi a}\beta_c\beta_B$$

$$\beta_c = \sqrt{\sec\frac{\pi a}{W}}$$

offset crack

$$\beta_A = (1 - 0.025\lambda^2 + 0.6\lambda^4 - \gamma\lambda^{11})\sqrt{\sec\left(\frac{\pi\lambda}{2}\right)\frac{\sin\left(2\lambda - 4\frac{a}{W}\right)}{2\lambda - 4\frac{a}{W}}}$$

$$\beta_B = (1 - 0.025\delta^2 + 0.06\delta^4 - \zeta\lambda^{30})\left(1 + \frac{\sqrt{\sec\left(\frac{2\pi\lambda+1.5\pi\delta}{7}\right) - 1}}{1 + 0.21\sin\left(8\tan^{-1}\left(\frac{\lambda-\delta}{\lambda+\delta}\right)^{0.9}\right)}\right)$$

'

- The parameters λ, δ are given as

$$\lambda = \frac{a}{b}$$
$$\delta = \frac{a}{W - b}$$

- And γ and ζ can be looked up on a table

$\frac{b}{W}$	γ	ζ
0.1	0.382	0.114
0.25	0.136	0.286
0.4	0.0	0.0
0.5	0.0	0.0

non-uniform stress, infinite width

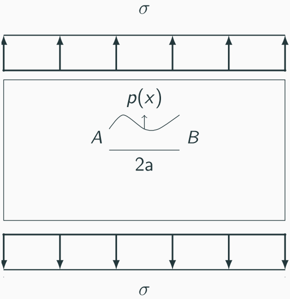


Figure 9: arbitrary pressure function loading along crack

non-uniform stress, infinite width

- Stress intensity will be different at points A and B

$$K_{IA} = \int_{-a}^{a} \frac{p(x)}{\sqrt{\pi a}} \frac{\sqrt{a-x}}{\sqrt{a+x}} dx$$
$$K_{IB} = \int_{-a}^{a} \frac{p(x)}{\sqrt{\pi a}} \frac{\sqrt{a+x}}{\sqrt{a-x}} dx$$

cracks around a hole

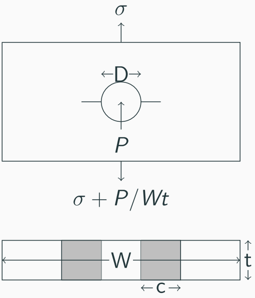


Figure 10: a crack around a hole under both remote stress and a local bearing load

cracks around a hole

- For symmetric through cracks under uniform applied stress, we have

$$
\begin{aligned}
\beta &= \beta_1 + \beta_2 \\
\beta_1 &= F_{c/R} F_w F_{ww} \\
\beta_2 &= \frac{\sigma_{br}}{\sigma} F_3 F_w F_{ww} \\
F_{c/R} &= \frac{3.404 + 3.8172 \frac{c}{R}}{1 + 3.9273 \frac{c}{R} - 0.00695 \left(\frac{c}{R}\right)^2} \\
F_w &= \sqrt{\sec \frac{\pi R}{W} \sec \frac{\pi (R + c)}{W}}
\end{aligned}
$$

cracks around a hole

$$F_{ww} = 1 - \left(\left(1.32 \frac{W}{D} - 0.14 \right)^{-(.98 + \left(0.1 \frac{W}{D}\right)^{0.1})} - 0.02 \right)$$

$$\left(\frac{2c}{W - D} \right)^{N}$$

$$F_3 = 0.098 + 0.3592 e^{-3.5089 \frac{c}{R}} + 0.3817 e^{-0.5515 \frac{c}{R}}$$

cracks around a hole

- Note that

$$\sigma_{br} = \frac{P}{Dt}$$

$$N = \frac{W}{D} + 2.5 \qquad \text{when} \qquad \frac{W}{D} < 2$$

$$N = 4.5 \qquad \text{otherwise}$$

- Also R is the radius, $R = \frac{D}{2}$

cracks around a hole

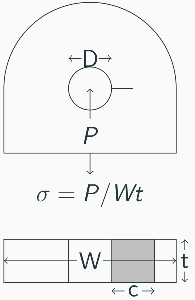


Figure 11: a crack around a hole under both remote stress and a local bearing load, but there is only a crack on one side

cracks around a hole

$$\beta = \beta_1 + \beta_2$$
$$\beta_1 = \beta_3 F_w F_{ww}$$
$$\beta_2 = \frac{\sigma_{br}}{\sigma} F_4 F_w F_{ww}$$
$$\beta_3 = 0.7071 + 0.7548 \frac{R}{R+c} + 0.3415 \left(\frac{R}{R+C}\right)^2 + 0.6420 \left(\frac{R}{R+c}\right)^3 + 0.9196 \left(\frac{R}{R+c}\right)^4$$
$$F_4 = 0.9580 + 0.2561 \frac{c}{R} - 0.00193 \left(\frac{c}{R}\right)^{2.5} - 0.9804 \left(\frac{c}{R}\right)^{0.5}$$

cracks around a hole

$$F_w = \sqrt{\sec\frac{\pi R}{W}\sec\frac{\pi(R+c/2)}{W-c}}$$

$$F_{ww} = 1 - N^{-\frac{W}{D}}\left(\frac{2c}{W-D}\right)^{\frac{W}{D}+0.5}$$

$$N = 2.65 - 0.24\left(2.75 - \frac{W}{D}\right)^2$$

$$N \geq 2.275 \qquad (\text{if } N < 2.275, \text{ let } N = 2.275)$$

Also note that R indicates radius, $R = \frac{D}{2}$

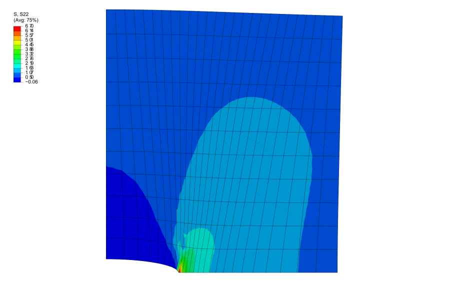
group problems

1. Find K_I for a center-cracked panel with $W/2a = 3$ and a uniformly applied remote stress, σ.
2. Find K_I for an edge-cracked panel with $W/a = 3$ and a uniformly applied remote stress, σ.
3. Find K_I for an edge-cracked panel with $W/a = 3$ and a remote bending moment, $M = tW^2\sigma/6$.

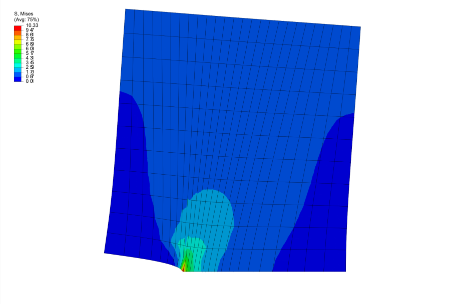
group problems

4. Find K_I for a center-cracked panel with $W/2a = 3$ and a concentrated splitting force, $P = \sigma a t$.
5. What do you think causes the difference (if any) in stress intensity between these panels?

example 1



example 1

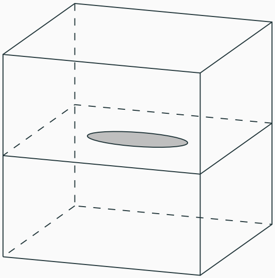


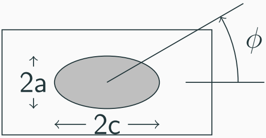
2D crack shapes

crack depth

- The previous stress intensity factors all assume a 2D problem (with a 1D crack)
- Through the thickness, it is assumed that the crack length is the same
- In many cases this is not an accurate assumption
- We will now consider 2D crack shapes and their effect on the stress intensity factor

elliptical flaw, infinite solid





elliptical flaw, infinite solid

- For an ellipse the stress intensity factor will vary with the angle, ϕ

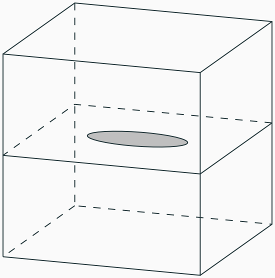
$$K_I = \sigma\sqrt{\pi a}\beta$$
$$\beta = \sqrt{\frac{1}{Q}f_\phi}$$
$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \qquad \text{if} a/c \leq 1$$

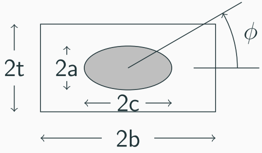
elliptical flaw, infinite solid

- For an ellipse the stress intensity factor will vary with the angle, ϕ

$$Q = 1 + 1.464\left(\frac{c}{a}\right)^{1.65} \qquad \text{if} a/c > 1$$
$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4} \qquad \text{if} a/c \leq 1$$
$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2 \sin^2\phi\right)^{1/4} \qquad \text{if} a/c > 1$$

elliptical flaw, finite solid





finite solid

$$\beta = \sqrt{\frac{1}{Q}} F_e$$

$$F_e = \left(M_1 + M_2 \left(\frac{a}{t} \right)^2 + M_3 \left(\frac{a}{t} \right)^4 \right) g f_\phi f_w$$

$$f_w = \sqrt{\sec \left(\frac{\pi c}{2b} \sqrt{\frac{a}{t}} \right)}$$

$$g = 1 - \frac{\left(\frac{a}{t} \right)^4 \left(2.6 - 2\frac{a}{t} \right)^{1/2}}{1 + 4\frac{a}{c}} \cos \phi$$

elliptical flaw, finite solid

$$M_2 = \frac{0.05}{0.11 + \left(\frac{a}{c}\right)^{3/2}}$$

$$M_3 = \frac{0.29}{0.23\left(\frac{a}{c}\right)^{3/2}}$$

elliptical flaw, finite solid

- If $a/c \leq 1$

$$M_1 = 1$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

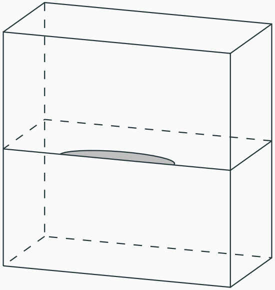
$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4}$$

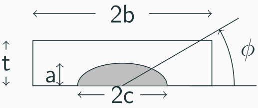
elliptical flaw, finite solid

- Otherwise ($a/c > 1$)

$$M_1 = \left(\frac{c}{a}\right)^{1/2}$$
$$Q = 1 + 1.464\left(\frac{c}{a}\right)^{1.65}$$
$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2 \sin^2\phi\right)^{1/4}$$

semi-elliptical surface flaw, finite body





semi-elliptical surface flaw, finite body

$$K_I = \sigma\sqrt{\pi a}\beta$$

$$\beta = \sqrt{\frac{1}{Q}}F_s$$

$$F_s = \left(M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right) g f_\phi f_w$$

$$f_w = \sqrt{\sec\left(\frac{\pi c}{2b}\sqrt{\frac{a}{t}}\right)}$$

surface flaw, $\frac{a}{c} \leq 1$

$$M_1 = 1.13 - 0.09\left(\frac{a}{c}\right)$$

$$M_2 = -0.52 + \frac{0.89}{0.2 + \frac{a}{c}}$$

$$M_3 = 0.5 - \frac{1}{0.65 + \frac{a}{c}} + 14\left(1 - \frac{a}{c}\right)^4$$

surface flaw, $\frac{a}{c} \leq 1$

$$Q = 1 + 1.464 \left(\frac{a}{c}\right)^{1.65}$$

$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4}$$

$$g = 1 + \left(0.1 + 0.35\left(\frac{a}{t}\right)^2\right)(1 - \sin\phi)^2$$

surface flaw, $\frac{a}{c} > 1$

$$M_1 = \left(\frac{c}{a}\right)^{1/2}\left(1 + 0.04\frac{c}{a}\right)$$

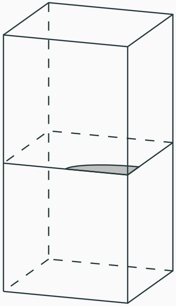
$$M_2 = 0.2\left(\frac{c}{a}\right)^4$$

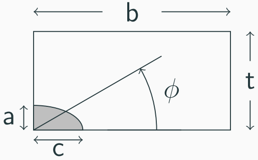
$$M_3 = -0.11\left(\frac{c}{a}\right)^4$$

surface flaw, $\frac{a}{c} > 1$

$$
\begin{aligned}
Q &= 1 + 1.464 \left(\frac{c}{a}\right)^{1.65} \\
f_\phi &= \left(\cos^2\phi + \left(\frac{c}{a}\right)^2 \sin^2\phi\right)^{1/4} \\
g &= 1 + \left(0.1 + 0.35 \left(\frac{c}{a}\right)\left(\frac{a}{t}\right)^2\right)(1 - \sin\phi)^2
\end{aligned}
$$

corner flaw, finite body





corner flaw, finite body

$$K_I = \sigma\sqrt{\pi a}\beta$$
$$\beta = \sqrt{\frac{1}{Q}}F_c$$
$$F_c = \left(M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right)g_1 g_2 f_\phi f_w$$
$$f_w = 1 - 0.2\lambda + 9.4\lambda^2 - 19.4\lambda^3 + 27.1\lambda^4$$
$$\lambda = \left(\frac{c}{b}\right)\left(\frac{a}{t}\right)^{1/2}$$

corner flaw, finite body, $\frac{a}{c} \leq 1$

$$M_1 = 1.08 - 0.03\left(\frac{a}{c}\right)$$
$$M_2 = -0.44 + \frac{1.06}{0.3 + \frac{a}{c}}$$
$$M_3 = -0.5 + 0.25\frac{a}{c} + 14.8\left(1 - \frac{a}{c}\right)^{1.5}$$

corner flaw, finite body, $\frac{a}{c} \leq 1$

$$Q = 1 + 1.464 \left(\frac{a}{c}\right)^{1.65}$$

$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4}$$

$$g_1 = 1 + \left(0.08 + 0.4\left(\frac{a}{t}\right)^2\right)(1 - \sin\phi)^3$$

$$g_2 = 1 + \left(0.08 + 0.15\left(\frac{a}{t}\right)^2\right)(1 - \cos\phi)^3$$

corner flaw, finite body, $\frac{a}{c} > 1$

$$M_1 = \left(\frac{c}{a}\right)^{1/2}\left(1.08 - 0.03\frac{c}{a}\right)$$

$$M_2 = 0.375\left(\frac{c}{a}\right)^4$$

$$M_3 = -0.25\left(\frac{c}{a}\right)^2$$

corner flaw, finite body, $\frac{a}{c} > 1$

$$Q = 1 + 1.464 \left(\frac{c}{a}\right)^{1.65}$$

$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2 \sin^2\phi\right)^{1/4}$$

$$g_1 = 1 + \left(0.08 + 0.4\left(\frac{c}{t}\right)^2\right)(1 - \sin\phi)^3$$

$$g_2 = 1 + \left(0.08 + 0.15\left(\frac{c}{t}\right)^2\right)(1 - \cos\phi)^3$$

example 2

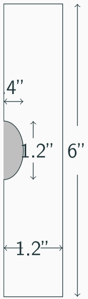


Figure 12: A surface flaw shown with a major diameter of 1.2 inches and a minor radius (the semi-elliptical axis) of 0.4 inches

- Find maximum value of K_I for semi-elliptical surface flaw
- $\sigma = 20$ kpsi (in opening direction)

example 2

- Here we will use the formula for a semi-elliptical surface flaw
- In the first step we find $a/c = 0.4/0.6 < 1$, so we use that set of formulae
- A worked python notebook of this example can be found here[1]

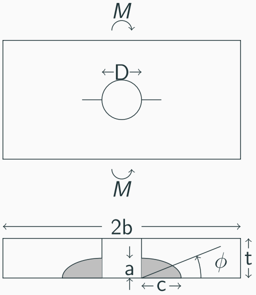
[1] https://colab.research.google.com/drive/11i24jBHuGPautBloU1FgGGBJN-y2HqDJ?usp=sharing

2D cracks at a hole

when to consider 2D crack shape

- When do we need to worry about 2D crack shape?
- The important factor is ratio of crack length to thickness
- When crack length is less than 5 times thickness, 2D shape effects are not negligible

cracks around a hole



remote stress

$$K_I = \sigma\sqrt{\pi a}\beta$$
$$\beta = \sqrt{\frac{1}{Q}}F_{ch}$$
$$F_{ch} = \left(M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right)g_1 g_2 g_3 g_4 f_\phi f_w$$
$$f_w = \sqrt{\sec\left(\frac{\pi r}{2b}\right)\sec\left(\frac{\pi(2r+nc)}{4(b-c)+2nc}\sqrt{\frac{a}{t}}\right)}$$

remote stress

$$g_2 = \frac{1 + 0.358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4}{1 + 0.13\lambda^2}$$
$$\lambda = \frac{1}{1 + (c/r)\cos(0.85\phi)}$$

Where $n =$ number of cracks (1 or 2)

remote stress when $a/c \leq 1$

$$M_1 = 1.13 - 0.09\,(a/c)$$
$$M_2 = -0.54 + \frac{0.89}{0.2 + a/c}$$
$$M_3 = 0.5 - \frac{1}{0.65 + a/c} + 14\,(1 - a/c)^{24}$$
$$Q = 1 + 1.464\,(a/c)^{1.65}$$

remote stress when $a/c \leq 1$

$$g_1 = 1 + \left(0.1 + 0.35\,(a/t)^2\right)(1 - \sin\phi)^2$$
$$g_3 = (1 + 0.04(a/c))\left(1 + 0.1(1 - \cos\phi)^2\right)\left(0.85 + 0.15(a/t)^{1/4}\right)$$
$$g_4 = 1 - 0.7(1 - a/t)(a/c - 0.2)(1 - a/c)$$
$$f_\phi = \left((a/c)^2 \cos^2\phi + \sin^2\phi\right)^{1/4}$$

remote stress when $a/c > 1$

$$M_1 = \sqrt{c/a}(1 + 0.04(c/a))$$
$$M_2 = 0.2(c/a)^4$$
$$M_3 = -0.11(c/a)^4$$
$$Q = 1 + 1.464\left(\frac{c}{a}\right)^{1.65}$$

remote stress when $a/c > 1$

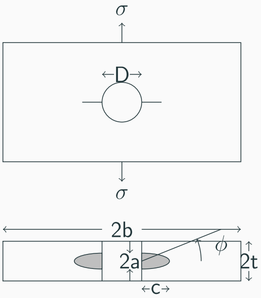
$$g_1 = 1 + \left(0.1 + 0.35(c/a)\left(a/t\right)^2\right)(1 - \sin\phi)^2$$
$$g_3 = (1.13 - 0.09(c/a))\left(1 + 0.1(1 - \cos\phi)^2\right)\left(0.85 + 0.15(a/t)^{1/4}\right)$$
$$g_4 = 1$$
$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2 \sin^2\phi\right)^{1/4}$$

remote stress

- The same formulas apply for both symmetric cracks ($n = 2$) and a single crack ($n = 1$) with one additional correction factor applied to the single crack case

$$K_{I,single} = \sqrt{\frac{4/\pi + ac/2tr}{4/\pi + ac/tr}} K_{I,symmetric}$$

surface cracks around a hole



remote stress

$$K_I = \sigma\sqrt{\pi a}\beta$$

$$\beta = \sqrt{\frac{1}{Q}}F_{sh}$$

$$F_{sh} = \left(M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right) g_1 g_2 g_3 f_\phi f_w$$

$$f_w = \sqrt{\sec\left(\frac{\pi r}{2b}\right)\sec\left(\frac{\pi(2r+nc)}{4(b-c)+2nc}\sqrt{\frac{a}{t}}\right)}$$

remote stress

$$M_2 = \frac{0.05}{0.11 + (a/c)^{3/2}}$$

$$M_3 = \frac{0.29}{0.23 + (a/c)^{3/2}}$$

Where $n =$ number of cracks (1 or 2)

remote stress

$$
\begin{aligned}
g_1 &= 1 - \frac{(a/t)^4(2.6 - 2a/t)^{1/2}}{1 + 4a/c}\cos\phi \\
g_2 &= \frac{1 + 0.358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4}{1 + 0.08\lambda^2} \\
\lambda &= \frac{1}{1 + (c/r)\cos(0.9\phi)} \\
g_3 &= 1 + 0.1(1 - \cos\phi)^2(1 - a/t)^{10}
\end{aligned}
$$

remote stress $a/c \leq 1$

$$
\begin{aligned}
Q &= 1 + 1.464(a/c)^{1.65} \\
M_1 &= 1 \\
f_\phi &= \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4}
\end{aligned}
$$

remote stress $a/c > 1$

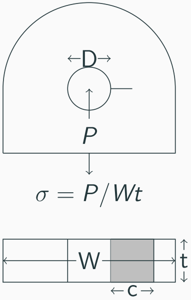
$$Q = 1 + 1.464(c/a)^{1.65}$$
$$M_1 = \sqrt{c/a}$$
$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2 \sin^2\phi\right)^{1/4}$$

single-crack correction

- When the surface crack is only on one side of the hole, we use the same correction as for corner cracks

$$K_{I,single} = \sqrt{\frac{4/\pi + ac/2tr}{4/\pi + ac/tr}} K_{I,symmetric}$$

edge crack on a lug



edge crack on a lug

$$K_I = \sigma_{br}\sqrt{\pi c}\beta$$
$$\sigma_{br} = P/Dt$$
$$\beta = \left(\frac{G_0 D}{2W} + G_1\right) G_w G_L G_2$$
$$z = \left(1 + \frac{2C}{D}\right)^{-1}$$
$$G_0 = 0.7071 + 0.7548z + 0.3415z^2 + 0.642z^3 + 0.9196z^4$$
$$G_1 = 0.078z + 0.7588z^2 - 0.4293z^3 + 0.0644z^4 + 0.651z^5$$
$$G_L = \left(\sec\left(\frac{\pi D}{2W}\right)\right)^{1/2}$$

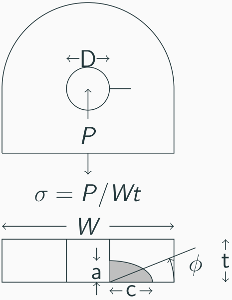
edge crack on a lug

$$\lambda = \frac{\pi}{2}\left(\frac{D+c}{W-c}\right)$$
$$G_w = (\sec\lambda)^{1/2}$$
$$b = \frac{W-D}{2}$$
$$A_1 = 0.688 + 0.772\frac{D}{W} + 0.613\left(\frac{D}{W}\right)^2$$
$$A_2 = 4.948 - 17.318\frac{D}{W} + 16.785\left(\frac{D}{W}\right)^2$$

edge crack on a lug

$$A_3 = -14.297 + 62.994\frac{D}{W} - 69.818\left(\frac{D}{W}\right)^2$$
$$A_4 = 12.35 - 58.644\frac{D}{W} + 66.387\left(\frac{D}{W}\right)^2$$
$$G_2 = A_1 + A_2\frac{c}{b} + A_3\left(\frac{c}{b}\right)^2 + A_4\left(\frac{c}{b}\right)^3$$

corner crack on a lug



corner crack on a lug

$$
\begin{aligned}
\beta &= \left(\frac{G_0 D}{2W} + G_1\right) G_w \\
z &= \left(1 + 2\frac{c}{D}\cos(0.85\phi)\right)^{-1} \\
f_0(z) &= 0.7071 + 0.7548z + 0.3415z^2 + 0.642z^3 + 0.9196z^4 \\
f_1(z) &= 0.078z + 0.7588z^2 - 0.4293z^3 + 0.0644z^4 + 0.651z^5 \\
G_0 &= \frac{f_0(z)}{d_0} \\
d_0 &= 1 + 0.13z^2
\end{aligned}
$$

corner crack on a lug

$$
\begin{aligned}
g_p &= \left(\frac{W+D}{W-D}\right)^{1/2} \\
G_1 &= f_1(z)\left(\frac{g_p}{d_0}\right) \\
G_w &= M_0 g_1 g_3 g_4 f_\phi f_w f_x \\
v &= \frac{a}{t}
\end{aligned}
$$

corner crack on a lug

$$
\begin{aligned}
\lambda &= \frac{\pi}{2}\sqrt{v}\left(\frac{D+c}{W-c}\right) \\
f_w &= \left(\sec\lambda \sec\frac{\pi D}{2W}\right)^{1/2} \\
x &= \frac{a}{c}
\end{aligned}
$$

corner crack on a lug $a/c \leq 1$

$$f_\phi = \left(\left(\frac{a}{c} \cos \phi \right)^2 + \sin^2 \phi \right)^{1/4}$$

$$f_x = \left(1 + 1.464 \left(\frac{a}{c} \right)^{1.65} \right)^{-1/2}$$

$$M_0 = (1.13 - 0.09x) + \left(-0.54 + \frac{0.89}{0.2 + x} \right) v^2 + \left(0.5 - \frac{1}{.65 - x} + 14(1 - x^{24}) \right) v^4$$

corner crack on a lug $a/c \leq 1$

$$g_1 = 1 + \left(0.1 + 0.35v^2 \right) (1 - \sin \phi)^2$$

$$g_3 = (1 + 0.04x) \left(1 + 0.1 (1 - \cos \phi)^2 \right) \left(0.85 + 0.15v^{1/4} \right)$$

$$g_4 = 1 - 0.7 (1 - v) (x - 0.2) (1 - x)$$

corner crack on a lug $a/c > 1$

$$f_\phi = \left(\left(\frac{ac}{c} \sin\phi \right)^2 + \cos^2\phi \right)^{1/4}$$

$$f_x = \left(1 + 1.464 \left(\frac{c}{a} \right)^{1.65} \right)^{-1/2}$$

$$M_0 = x^{-1/2} + 0.04x^{-3/2} + 0.2x^{-4}v^2 - 0.11x^{-4}v^4$$

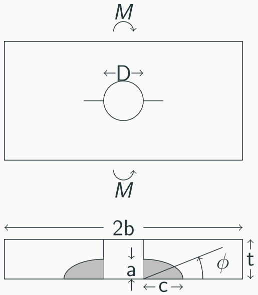
corner crack on a lug $a/c > 1$

$$g_1 = 1 + \left(0.1 + \frac{0.35}{x} v^2 \right) (1 - \sin\phi)^2$$

$$g_3 = \left(1.13 + \frac{0.09}{x} \right) \left(1 + 0.1(1 - \cos\phi)^2 \right) \left(0.85 + 0.15v^{1/4} \right)$$

$$g_4 = 1$$

symmetric corner cracks under bending



corner cracks under bending

$$\sigma_b = \frac{Mt}{2I}$$
$$I = \frac{bt^3}{6}$$
$$\beta = H_{ch}\left(\frac{a}{cQ}\right)^{1/2} F_{ch}$$

corner cracks under bending

$$
\begin{aligned}
H_{ch} &= H_1 + (H_2 - H_1)\sin^p \phi \\
H_1 &= 1 + G_{11}(a/t) + G_{12}(a/t)^2 + G_{13}(a/t)^3 \\
H_2 &= 1 + G_21(a/t) + G_{22}(a/t)^2 + G_{23}(a/t)^3
\end{aligned}
$$

corner cracks under bending

$$
\begin{aligned}
F_{ch} &= \left(M_1 + M_2(a/t)^2 + M_3(a/t)^4\right) g_1 g_2 g_3 g_4 f_\phi f_w \\
\lambda &= \frac{1}{1 + (c/r)\cos(0.85\phi)} \\
g_2 &= \frac{1 + .358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4}{1 + 0.13\lambda^2}
\end{aligned}
$$

corner cracks under bending $a/c \leq 1$

$$
\begin{aligned}
M_1 &= 1.13 - 0.09(a/c) \\
M_2 &= -0.54 + \frac{0.89}{0.2 + a/c} \\
M_3 &= 0.5 - \frac{1}{0.65 + a/c} + 14(1 - a/c)^4 \\
Q &= 1 + 1.464(a/c)1.65
\end{aligned}
$$

corner cracks under bending $a/c \leq 1$

$$
\begin{aligned}
g_1 &= 1 + \left(0.1 + (a/t)v^2\right)(1 - \sin\phi)^2 \\
g_3 &= (1 + 0.04(a/c))\left(1 + 0.1\,(1 - \cos\phi)^2\right)\left(0.85 + 0.15(a/t)^{1/4}\right) \\
g_4 &= 1 - 0.7\,(1 - a/t)\,(a/c - 0.2)\,(1 - a/c)
\end{aligned}
$$

corner cracks under bending

$$f_\phi = \left(\left(\frac{a}{c}\cos\phi\right)^2 + \sin^2\phi\right)^{1/4}$$

$$G_{11} = -0.43 - 0.74a/c - 0.84(a/c)^2$$

$$G_{12} = 1.25 - 1.19a/c + 4.39(a/c)^2$$

$$G_{13} = -1.94 + 4.22a/c - 5.51(a/c)^2$$

corner cracks under bending

$$G_{21} = -1.5 - 0.04a/c - 1.73(a/c)^2$$

$$G_{22} = 1.71 - 3.17a/c + 6.84(a/c)^2$$

$$G_{23} = -1.28 + 2.71a/c - 5.22(a/c)^2$$

$$p = 0.1 + 1.3a/t + 1.1a/c - 0.7(a/c)(a/t)$$

corner cracks under bending $a/c > 1$

$$
\begin{aligned}
M_1 &= (c/a)^{1/2}(1 + 0.04c/a)\\
M_2 &= 0.2(c/a)^4\\
M_3 &= -0.11(c/a)^4\\
Q &= 1 + 1.464(c/a)^{1.65}\\
g_1 &= 1 + \left(0.10.35(c/a)(a/t)^2\right)(1 - \sin\phi)^2\\
g_3 &= (1.13 - 0.09(c/a))\left(1 + 0.1(1 - \cos\phi)^2\right)\left(0.85 + 0.15(a/t)^{1/4}\right)\\
g_4 &= 1
\end{aligned}
$$

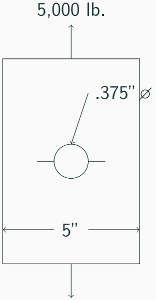
corner cracks under bending

$$
\begin{aligned}
f_\phi &= \left(\left(\cos^2\phi + \frac{c}{a}\sin\phi\right)^2\right)^{1/4}\\
G_{11} &= -2.07 + 0.06c/a\\
G_{12} &= 4.35 + 0.16c/a\\
G_{13} &= -2.93 - 0.3c/a
\end{aligned}
$$

corner cracks under bending

$$
\begin{aligned}
G_{21} &= -3.64 + 0.37c/a \\
G_{22} &= 5.87 - 0.49c/a \\
G_{23} &= -4.32 + 0.53c/a \\
p &= 0.2 + c/a + 0.6a/t
\end{aligned}
$$

example 3



$t = 0.125$in.
$a = 0.02$in.
$c = 0.08$in.

example

- Case 1 - symmetric through cracks
- Case 2 - single through crack
- Case 3 - symmetric corner cracks
- Case 4 - single corner crack
- Case 5 - symmetric surface cracks
- Case 6 - single surface crack
- Viewable here[2]

[2] https://colab.research.google.com/drive/1fmI1vs1Rpwn9BkXPz-8FV6-IqHnVvQu0?usp=sharing