AE 737: Mechanics of Damage Tolerance

Lecture 1 - Stress Intensity

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18 January 2022

schedule

- 18 Jan - Introduction, Stress Intensity
- 20 Jan - Common Stress Intensity Factors
- 25 Jan - Superposition, Compounding
- 27 Jan - Curved Boundaries, HW 1 Due

outline

- introduction
- syllabus
- overview
- fracture mechanics
- stress intensity
- making good plots

about me

education

- B.S. in Mechanical Engineering from Brigham Young University
  - Worked with ATK to develop tab-less gripping system for tensile testing composite tow specimens
  - Needed to align the specimen, as well as grip it without causing a stress concentration

education

- M.S. and Ph.D. from School of Aeronautics and Astronautics at Purdue University
  - Worked with Boeing to simulate mold flows
  - First ever mold simulation with anisotropic viscosity

research

Figure 1: picture of chopped carbon fiber prepreg

research

Figure 2: picture of lamborghini symbol made from compression molded chopped carbon fiber

research

- No simulation is currently able to predict fiber orientation from these processes
- Part of the challenge is that we only have information from initial state and final state
- I want to quantify intermediate stages using a transparent mold

research

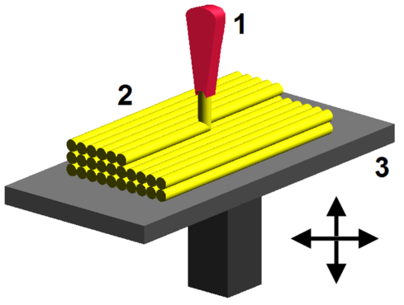

Figure 3: picture illustrating the fused deposition modeling 3D printing process, where plastic filament is melted and deposited next to other filament, and fuses together

introductions

- Name
- Student status (Undergrad, Masters, Ph.D)
- Full time or part time student?
- One interesting thing to remember you by

course textbook

- Printed notes from Dr. Bert L. Smith and Dr. Walter J. Horn
- Bring $30 cash or check to AE offices to pick up your copy
- Make checks out to Wichita State University
- Homework will be administered via Blackboard
- Supplemental textbooks are listed in the syllabus and in the text for further study

homework

- Homework will be posted and submitted via Blackboard assignments
- Homework will be self-graded: I will give you a score based only on completion
- One week (generally) after the Homework is due, a second Blackboard assignment will be due for the self-grade
- You need to go through the posted solutions and compare your work to the solutions
- This will also be graded only for completion

office hours

- Fill out the Doodle link I have sent to indicate your office hours preference
- You can also schedule an in-person or remote office visit via e-mail

tentative course outline

- Section 1 - fracture mechanics
- Stress intensity (18 Jan)
- Plastic zone (25 Jan)
- Fracture toughness (1 Feb)
- Residual strength (8 Feb)
- Multiple Site Damage (15 Feb)
- Mixed-Mode Fracture (17 Feb)
- Exam 1 (1 Mar)

tentative course outline

- Section 2 - fatigue and propagation
- Fatigue analysis (22 Feb)
- Crack growth (22 Mar)
- Exam 2 (12 Apr)

tentative course outline

- Section 3 - damage tolerance
  - Damage tolerance (12 Apr)
  - Test methods
  - Finite elements
  - Non Destructive Testing
  - Special topics
  - Final project (due 6 May)

grades

- Homework 10%
- Exam 1 30%
- Exam 2 30%
- Final Project 30%

A	A-	B+	B	B-	C+	C	C-	D+	D
93+	90-93	87-90	83-87	80-83	77-80	73-77	70-73	67-70	63

final project

- Perform residual strength, fatigue and damage tolerance analysis on a real part
- Examples: car axle, fuselage panel, wing panel, landing gear, bike pedal
- Individual project
- More discussion after Exam 1

class expectations

- Consider the cost (to you or others) of your being in class
- I ask that you refrain from distracting behaviors during class
- When you have something more important than class to take care of, please take care of it outside of class

software, in-class examples

- This class will involve a great deal of multi-step calculations
- While it is possible to do these by hand on paper, I STRONGLY recommend using some form of software
- Excel, MATLAB, Python, Maple, Mathematica, etc. can all be used
- Most of my in-class demos will use Python (and will be linked in notes)

damage

- In linear elasticity, we generally consider materials in their pristine state
- Realities of manufacturing, cyclic loads, and unforeseen loads result in a material which is something other than pristine
- In this course we will develop methods for predicting the strength of a material with some damage (residual strength)
- We will learn to predict the rate at which damage will grow (fatigue)

damage tolerance

- There are many ways to address the problem of damage in a material
    1. Infinite-life design
    2. Safe-life design
    3. Damage tolerant design
- To ensure damage tolerant design, we must ensure that crack growth is always stable
- Another important concept of damage tolerant design is to include multiple load paths, so failure in one part does not cause critical failure of the whole structure

damage tolerance

Figure 4: B17 with failed tail section

damage tolerance

Figure 5: close-up of failed tail section on B17

damage tolerance

- A B-17 collided with a Germain plane during WWII
- In spite of the damage, the B-17 was able to fly 90 minutes and land safely

damage tolerance

Figure 6: Image of Boeing 737 with top of fuselage missing (and passengers inside)

damage tolerance

- An example of multiple damaged sites occurred on a Boeing 737 in 1988
- Damage around multiple rivet holes connected and a full piece of the fuselage was blown off
- The plane was able to land safely
- This particular instance led to the study of “Multiple Site Damage”

fracture mechanics

- *Linear Elastic Fracture Mechanics* is the study of the propagation of cracks in materials
- There are some corrections we add to account for plasticity
- In this course we will not follow the full mathematical development of fracture mechanics (there is a separate course dedicated to that)
- Instead we will take some results and apply them

fracture mechanics

- In fracture mechanics we consider three different modes
- Mode I is known as the "opening mode"
- Mode II is known as the "sliding mode"
- Mode III is known as the "tearing mode"

fracture mechanics

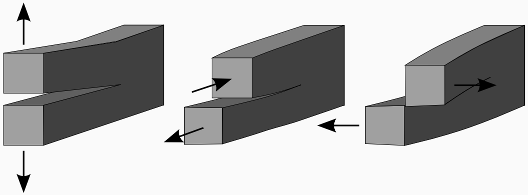

Figure 7: An image of the three fracture modes, with a representative crack in the xy plane. The first mode showns a crack opening vertically in the z-direction, like jaws opening. The second mode is known as the sliding mode, where one face moves into the

stress intensity

- A key finding from Linear Elastic Fracture Mechanics (LEFM) is known as the *Stress Intensity Factor*
- The stress intensity factor is often written in this form

$$K = \sigma\sqrt{\pi a}\beta$$

- Where K is the stress intensity factor, σ is the applied stress, a is the crack length, and β is a dimensionless parameter depending on geometry

stress intensity

- Be careful that although the notation is similar, the *Stress Intensity Factor* is different from the *Stress Concentration Factor* from strength of materials
- We are usually most concerned with Mode I, but there will be a unique stress intensity factor for each mode, we label these K_I, K_{II}, and K_{III}
- If no subscript is given, assume Mode I

stress intensity

- For brittle materials (where "linear" fracture mechanics assumptions hold true) we can find the full stress field near the crack in terms of the stress intensity factor

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

mode II

- Similarly for Mode II we find

$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$
$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

mode III

- And for Mode III

$$\tau_{xz} = \frac{-K_{III}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}$$
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}$$

plotting

- Plotting is an important part of graduate work, and this course
- There are many software programs which can generate good scientific plots
- Microsoft Excel
- MATLAB
- Maple
- Mathematica
- Python
- R
- Plot.ly

plotting

- You are welcome to use whatever software you desire, I will use Python for a quick demonstration
- This demo is accessible here[1]

[1] https://colab.research.google.com/drive/1GaAMFmEkYn7M8dNxCbT9jGGOdM8qSapK?usp=sharing

plotting

- To make a good scientific plot, we must first decide what to plot, and which plot style will best illustrate our data
- Let us consider the Mode I stress near a crack tip

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

plotting

- One interesting plot could be to examine stress magnitudes along the crack propagation direction as we get farther away from the crack
- In this case we would have $\theta = 0$.
- Since θ is a constant, it is not ideal to use a polar plot, instead we will use a standard rectangular plot

plotting

- Since we are looking at stresses near the crack tip, it is convenient to normalize the distance by the crack length
- If substitute for θ and K_I we have

$$\sigma_x = \frac{\sigma\sqrt{\pi a}\beta}{\sqrt{2\pi r}}$$
$$\sigma_y = \frac{\sigma\sqrt{\pi a}\beta}{\sqrt{2\pi r}}$$
$$\tau_{xy} = 0$$

plotting

- Since σ_x and σ_y are identical for this case, we consider only one, and normalize by the applied stress. After simplification

$$\frac{\sigma_x}{\sigma\beta} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{(r/a)}}$$

plotting

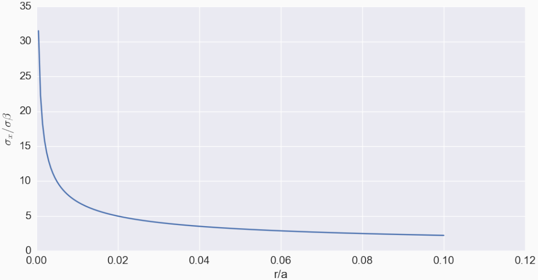

Figure 8: stress in the x direction plotted vs normalized distance from crack tip, r/a

plotting

- Since we found $\sigma_x = \sigma_y$ for $\theta = 0$, we decide it might be better to look at a polar plot using θ as a variable
- To make a polar plot in this style, we need a function such that $r = f(\theta)$
- To do this we consider a constant stress value, we will solve for and plot the distance, r at which the stress is equal to the same constant value for each of the three stress terms

plotting

$$
\begin{aligned}
\sigma_x = C &= \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \\
\sigma_y = C &= \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \\
\tau_{xy} = C &= \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}
\end{aligned}
$$

plotting

- After solving for r we find

$$
\begin{aligned}
r &= \frac{K_I^2}{2C^2\pi}\cos^2\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)^2 \\
r &= \frac{K_I^2}{2C^2\pi}\cos^2\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)^2 \\
r &= \frac{K_I^2}{2C^2\pi}\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\cos^2\frac{3\theta}{2}
\end{aligned}
$$

plotting