AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 3

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26 January 2016

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- Official office hours will be Fridays from 3:00 - 5:00
- Homeworks will generally be due on Tuesday
- I can make time to meet outside of office hours, just send an e-mail to make an appointment

SCHEDULE

- 26 Jan - Superposition, Compounding
- 28 Jan - Plastic Zone,
- 2 Feb - Plastic Zone, Homework 1 Due, Homework 2 Assigned
- 4 Feb - Plastic Zone
- 9 Feb - Fracture Toughness, Homework 2 Due

OUTLINE

1. review
2. 2D cracks at a hole
3. superposition
4. compounding
5. curved boundaries

REVIEW

EXAMPLE 1

1.
    1.1 Determine the value of K_I for a center-cracked panel with $W/2a = 3$ and a uniformly applied remote stress, σ.
    1.2 Determine the value of K_I for an edge-cracked panel with $W/a = 3$ and a uniformly applied remote stress, σ.
    1.3 Compare these two results. Note that in both cases the panel width to crack length ratio is the same.

EXAMPLE 1

- Comparing the two cases, we see that the finite width effects are much more significant for the edge-crack specimen
- The edge-crack specimen is also overall more effected by a crack of that relative length.
- Why are they not the same?

EXAMPLE 1

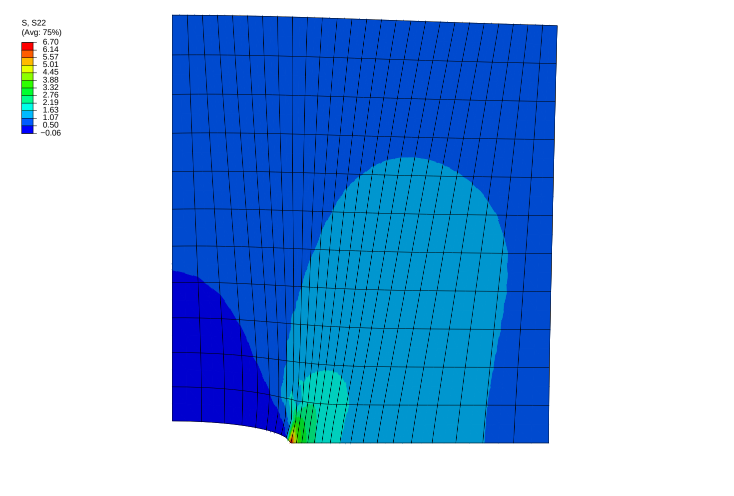


Figure 1: center-crack finite element simulation

EXAMPLE 1

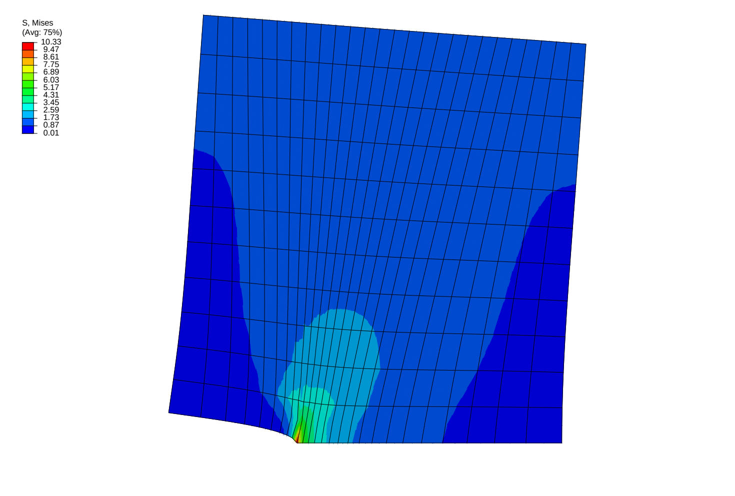
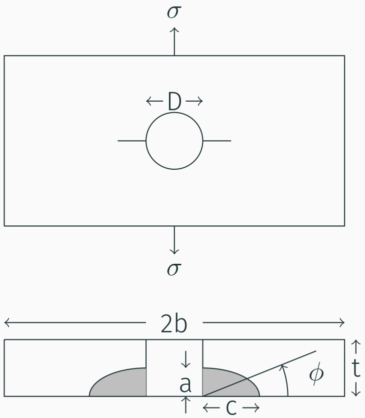


Figure 2: edge-crack finite element simulation

2D cracks at a hole

when to consider 2D crack shape

- When do we need to worry about 2D crack shape?
- The important factor is ratio of crack length to thickness
- When crack length is less than 5 times thickness, 2D shape effects are not negligible


σ
D
σ
2b
a
c
ϕ
t

$$K_I = \sigma\sqrt{\pi a}\beta \tag{3.1a}$$

$$\beta = \sqrt{\frac{1}{Q}}F_{ch} \tag{3.1b}$$

$$F_{ch} = \left(M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right)g_1 g_2 g_3 g_4 f_\phi f_w \tag{3.1c}$$

$$f_w = \sqrt{\sec\left(\frac{\pi r}{2b}\right)\sec\left(\frac{\pi(2r+nc)}{4(b-c)+2nc}\sqrt{\frac{a}{t}}\right)} \tag{3.1d}$$

$$g_2 = \frac{1 + 0.358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4}{1 + 0.13\lambda^2} \tag{3.1e}$$

$$\lambda = \frac{1}{1 + (c/r)\cos(0.85\phi)} \tag{3.1f}$$

Where n = number of cracks (1 or 2)

For $a/c \leq 1$

$$M_1 = 1.13 - 0.09\left(\frac{a}{c}\right) \tag{3.1g}$$

$$M_2 = -0.54 + \frac{0.89}{0.2 + \frac{a}{c}} \tag{3.1h}$$

$$M_3 = 0.5 - \frac{1}{0.65 + \frac{a}{c}} + 14\left(1 - \frac{a}{c}\right)^{24} \tag{3.1i}$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \tag{3.1j}$$

$$g_1 = 1 + \left(0.1 + 0.35\,(a/t)^2\right)(1 - \sin\phi)^2 \tag{3.1k}$$

$$g_3 = (1 + 0.04(a/c))\left(1 + 0.1(1 - \cos\phi)^2\right)\left(0.85 + 0.15(a/t)^{1/4}\right) \tag{3.1l}$$

$$g_4 = 1 - 0.7(1 - a/t)(a/c - 0.2)(1 - a/c) \tag{3.1m}$$

$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4} \tag{3.1n}$$

For $a/c > 1$

$$M_1 = \sqrt{c/a}(1 + 0.04(c/a)) \tag{3.1o}$$

$$M_2 = 0.2(c/a)^4 \tag{3.1p}$$

$$M_3 = -0.11(c/a)^4 \tag{3.1q}$$

$$Q = 1 + 1.464\left(\frac{c}{a}\right)^{1.65} \tag{3.1r}$$

$$g_1 = 1 + \left(0.1 + 0.35(c/a)\,(a/t)^2\right)(1 - \sin\phi)^2 \tag{3.1s}$$

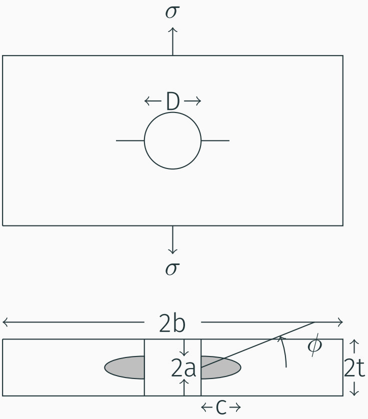
$$g_3 = (1.13 - 0.09(c/a))\left(1 + 0.1(1 - \cos\phi)^2\right)\left(0.85 + 0.15(a/t)^{1/4}\right) \tag{3.1t}$$

$$g_4 = 1 \tag{3.1u}$$

$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2\sin^2\phi\right)^{1/4} \tag{3.1v}$$

- The same formulas apply for both symmetric cracks ($n = 2$) and a single crack ($n = 1$) with one additional correction factor applied to the single crack case

$$K_{I,single} = \sqrt{\frac{4/\pi + ac/2tr}{4/\pi + ac/tr}} K_{I,symmetric} \tag{3.2}$$


σ
D
σ
2b
ϕ
2a
2t
c

$$K_I = \sigma\sqrt{\pi a}\beta \tag{3.3a}$$

$$\beta = \sqrt{\frac{1}{Q}}F_{sh} \tag{3.3b}$$

$$F_{sh} = \left(M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right) g_1 g_2 g_3 f_\phi f_w \tag{3.3c}$$

$$f_w = \sqrt{\sec\left(\frac{\pi r}{2b}\right)\sec\left(\frac{\pi(2r+nc)}{4(b-c)+2nc}\sqrt{\frac{a}{t}}\right)} \tag{3.3d}$$

$$M_2 = \frac{0.05}{0.11 + (a/c)^{3/2}} \tag{3.3e}$$

$$M_3 = \frac{0.29}{0.23 + (a/c)^{3/2}} \tag{3.3f}$$

Where $n =$ number of cracks (1 or 2)

$$g_1 = 1 - \frac{(a/t)^4(2.6 - 2a/t)^{1/2}}{1 + 4a/c} \cos\phi \tag{3.3g}$$

$$g_2 = \frac{1 + 0.358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4}{1 + 0.08\lambda^2} \tag{3.3h}$$

$$\lambda = \frac{1}{1 + (c/r)\cos(0.9\phi)} \tag{3.3i}$$

$$g_3 = 1 + 0.1(1 - \cos\phi)^2(1 - a/t)^{10} \tag{3.3j}$$

If $a/c \leq 1$

$$Q = 1 + 1.464(a/c)^{1.65} \tag{3.3k}$$

$$M_1 = 1 \tag{3.3l}$$

$$f_\phi = \left(\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right)^{1/4} \tag{3.3m}$$

If $a/c > 1$

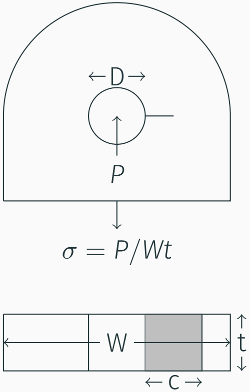
$$Q = 1 + 1.464(c/a)^{1.65} \tag{3.3n}$$

$$M_1 = \sqrt{c/a} \tag{3.3o}$$

$$f_\phi = \left(\cos^2\phi + \left(\frac{c}{a}\right)^2 \sin^2\phi\right)^{1/4} \tag{3.3p}$$

- When the surface crack is only on one side of the hole, we use the same correction as for corner cracks

$$K_{I,single} = \sqrt{\frac{4/\pi + ac/2tr}{4/\pi + ac/tr}} K_{I,symmetric} \tag{3.4}$$



←D→
P
$\sigma = P/Wt$
W
t
←c→

$$K_I = \sigma_{br}\sqrt{\pi c}\beta \tag{3.5a}$$

$$\beta = \left(\frac{G_0 D}{2W} + G_1\right) G_W G_L G_2 \tag{3.5b}$$

$$z = \left(1 + \frac{2C}{D}\right)^{-1} \tag{3.5c}$$

$$G_0 = 0.7071 + 0.7548z + 0.3415z^2 + 0.642z^3 + 0.9196z^4 \tag{3.5d}$$

$$G_1 = 0.078z + 0.7588z^2 - 0.4293z^3 + 0.0644z^4 + 0.651z^5 \tag{3.5e}$$

$$G_L = \left(\sec\left(\frac{\pi D}{2W}\right)\right)^{1/2} \tag{3.5f}$$

$$\lambda = \frac{\pi}{2}\left(\frac{D + c}{W - c}\right) \tag{3.5g}$$

$$G_W = (\sec\lambda)^{1/2} \tag{3.5h}$$

$$b = \frac{W - D}{2} \tag{3.5i}$$

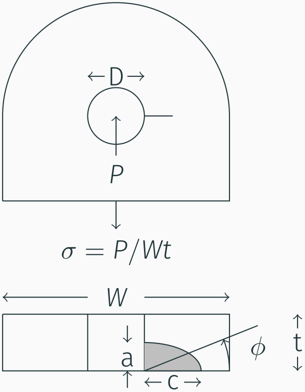
$$A_1 = 0.688 + 0.772\frac{D}{W} + 0.613\left(\frac{D}{W}\right)^2 \tag{3.5j}$$

$$A_2 = 4.948 - 17.318\frac{D}{W} + 16.785\left(\frac{D}{W}\right)^2 \tag{3.5k}$$

$$A_3 = -14.297 + 62.994\frac{D}{W} - 69.818\left(\frac{D}{W}\right)^2 \tag{3.5l}$$

$$A_4 = 12.35 - 58.644\frac{D}{W} + 66.387\left(\frac{D}{W}\right)^2 \tag{3.5m}$$

$$G_2 = A_1 + A_2\frac{c}{b} + A_3\left(\frac{c}{b}\right)^2 + A_4\left(\frac{c}{b}\right)^3 \tag{3.5n}$$



←D→
P
σ = P/Wt
W
a
c
ϕ
t

CORNER CRACK ON A LUG

$$\beta = \left(\frac{G_0 D}{2W} + G_1\right) G_w \tag{3.6a}$$

$$z = \left(1 + 2\frac{c}{D}\cos(0.85\phi)\right)^{-1} \tag{3.6b}$$

$$f_0(z) = 0.7071 + 0.7548z + 0.3415z^2 + 0.642z^3 + 0.9196z^4 \tag{3.6c}$$

$$f_1(z) = 0.078z + 0.7588z^2 - 0.4293z^3 + 0.0644z^4 + 0.651z^5 \tag{3.6d}$$

$$G_0 = \frac{f_0(z)}{d_0} \tag{3.6e}$$

$$d_0 = 1 + 0.13z^2 \tag{3.6f}$$

$$g_p = \left(\frac{W + D}{W - D}\right)^{1/2} \tag{3.6g}$$

$$G_1 = f_1(z)\left(\frac{g_p}{d_0}\right) \tag{3.6h}$$

$$G_w = M_0 g_1 g_3 g_4 f_\phi f_w f_x \tag{3.6i}$$

$$v = \frac{a}{t} \tag{3.6j}$$

$$\lambda = \frac{pi}{2}\sqrt{v}\left(\frac{D+c}{W-c}\right) \tag{3.6k}$$

$$f_w = \left(\sec\lambda \sec\frac{\pi D}{2W}\right)^{1/2} \tag{3.6l}$$

$$x = \frac{a}{c} \tag{3.6m}$$

For $a/c \leq 1$

$$f_\phi = \left(\left(\frac{a}{c}\cos\phi\right)^2 + \sin^2\phi\right)^{1/4} \tag{3.6n}$$

$$f_x = \left(1 + 1.464\left(\frac{a}{c}\right)^{1.65}\right)^{-1/2} \tag{3.6o}$$

$$\begin{aligned} M_0 = {} & (1.13 - 0.09x) + \left(-0.54 + \frac{0.89}{0.2 + x}\right) v^2 \boxed{+} \\ & \left(0.5 - \frac{1}{.65 - x} + 14(1 - x^{24})\right) v^4 \end{aligned} \tag{3.6p}$$

$$g_1 = 1 + \left(0.1 + 0.35v^2\right)(1 - \sin\phi)^2 \tag{3.6q}$$

$$g_3 = (1 + 0.04x)\left(1 + 0.1(1 - \cos\phi)^2\right)\left(0.85 + 0.15v^{1/4}\right) \tag{3.6r}$$

$$g_4 = 1 - 0.7(1 - v)(x - 0.2)(1 - x) \tag{3.6s}$$

For $a/c > 1$

$$f_\phi = \left(\left(\frac{ac}{c} \sin\phi \right)^2 + \cos^2\phi \right)^{1/4} \tag{3.6t}$$

$$f_x = \left(1 + 1.464 \left(\frac{c}{a} \right)^{1.65} \right)^{-1/2} \tag{3.6u}$$

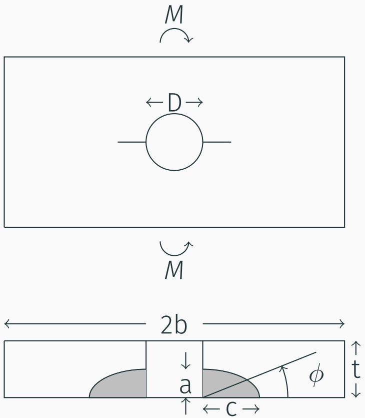
$$M_0 = x^{-1/2} + 0.04x^{-3/2} + 0.2x^{-4}v^2 - 0.11x^{-4}v^4 \tag{3.6v}$$

$$g_1 = 1 + \left(0.1 + \frac{0.35}{x} v^2 \right) (1 - \sin\phi)^2 \tag{3.6w}$$

$$g_3 = \left(1.13 + \frac{0.09}{x} \right) \left(1 + 0.1(1 - \cos\phi)^2 \right) \left(0.85 + 0.15v^{1/4} \right) \tag{3.6x}$$

$$g_4 = 1 \tag{3.6y}$$

SYMMETRIC CORNER CRACKS UNDER BENDING



$$\sigma_b = \frac{Mt}{2I} \tag{3.7a}$$

$$I = \frac{bt^3}{6} \tag{3.7b}$$

$$\beta = H_{ch} \left(\frac{a}{cQ} \right)^{1/2} F_{ch} \tag{3.7c}$$

$$H_{ch} = H_1 + (H_2 - H_1) \sin^p \phi \tag{3.7d}$$

$$H_1 = 1 + G_{11}(a/t) + G_{12}(a/t)^2 + G_{13}(a/t)^3 \tag{3.7e}$$

$$H_2 = 1 + G_2 1(a/t) + G_{22}(a/t)^2 + G_{23}(a/t)^3 \tag{3.7f}$$

$$F_{ch} = \left(M_1 + M_2(a/t)^2 + M_3(a/t)^4 \right) g_1 g_2 g_3 g_4 f_\phi f_w \tag{3.7g}$$

$$\lambda = \frac{1}{1 + (c/r) \cos(0.85\phi)} \tag{3.7h}$$

$$g_2 = \frac{1 + .358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4}{1 + 0.13\lambda^2} \tag{3.7i}$$

For $a/c \leq 1$

$$M_1 = 1.13 - 0.09(a/c) \tag{3.7j}$$

$$M_2 = -0.54 + \frac{0.89}{0.2 + a/c} \tag{3.7k}$$

$$M_3 = 0.5 - \frac{1}{0.65 + a/c} + 14(1 - a/c)^4 \tag{3.7l}$$

$$Q = 1 + 1.464(a/c)1.65 \tag{3.7m}$$

$$g_1 = 1 + \left(0.1 + (a/t)v^2\right)(1 - \sin\phi)^2 \tag{3.7n}$$

$$g_3 = (1 + 0.04(a/c))\left(1 + 0.1\,(1 - \cos\phi)^2\right)\left(0.85 + 0.15(a/t)^{1/4}\right) \tag{3.7o}$$

$$g_4 = 1 - 0.7\,(1 - a/t)\,(a/c - 0.2)\,(1 - a/c) \tag{3.7p}$$

$$f_\phi = \left(\left(\frac{a}{c}\cos\phi\right)^2 + \sin^2\phi\right)^{1/4} \tag{3.7q}$$

$$G_{11} = -0.43 - 0.74a/c - 0.84(a/c)^2 \tag{3.7r}$$

$$G_{12} = 1.25 - 1.19a/c + 4.39(a/c)^2 \tag{3.7s}$$

$$G_{13} = -1.94 + 4.22a/c - 5.51(a/c)^2 \tag{3.7t}$$

$$G_{21} = -1.5 - 0.04a/c - 1.73(a/c)^2 \tag{3.7u}$$

$$G_{22} = 1.71 - 3.17a/c + 6.84(a/c)^2 \tag{3.7v}$$

$$G_{23} = -1.28 + 2.71a/c - 5.22(a/c)^2 \tag{3.7w}$$

$$p = 0.1 + 1.3a/t + 1.1a/c - 0.7(a/c)(a/t) \tag{3.7x}$$

For $a/c > 1$

$$M_1 = (c/a)^{1/2}(1 + 0.04c/a) \tag{3.7y}$$

$$M_2 = 0.2(c/a)^4 \tag{3.7z}$$

$$M_3 = -0.11(c/a)^4 \tag{3.7aa}$$

$$Q = 1 + 1.464(c/a)^{1.65} \tag{3.7ab}$$

$$g_1 = 1 + \left(0.10.35(c/a)(a/t)^2\right)(1 - \sin\phi)^2 \tag{3.7ac}$$

$$g_3 = (1.13 - 0.09(c/a))\left(1 + 0.1(1 - \cos\phi)^2\right)\left(0.85 + 0.15(a/t)^{1/4}\right) \tag{3.7ad}$$

$$g_4 = 1 \tag{3.7ae}$$

$$f_\phi = \left(\left(\cos^2\phi + \frac{c}{a}\sin\phi\right)^2\right)^{1/4} \tag{3.7af}$$

$$G_{11} = -2.07 + 0.06c/a \tag{3.7ag}$$

$$G_{12} = 4.35 + 0.16c/a \tag{3.7ah}$$

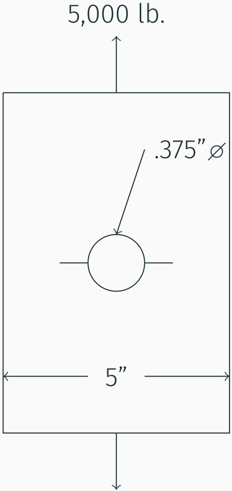
$$G_{13} = -2.93 - 0.3c/a \tag{3.7ai}$$

$$G_{21} = -3.64 + 0.37c/a \tag{3.7aj}$$

$$G_{22} = 5.87 - 0.49c/a \tag{3.7ak}$$

$$G_{23} = -4.32 + 0.53c/a \tag{3.7al}$$

$$p = 0.2 + c/a + 0.6a/t \tag{3.7am}$$

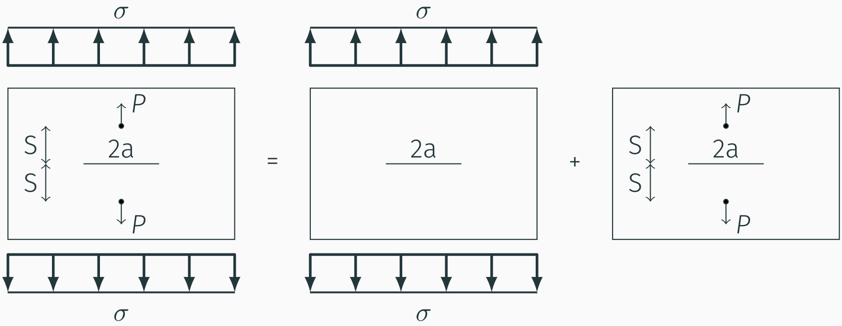


$t = 0.125\text{in.}$
$a = 0.02\text{in.}$
$c = 0.08\text{in.}$

- Case 1 - symmetric through cracks
- Case 2 - single through crack
- Case 3 - symmetric corner cracks
- Case 4 - single corner crack
- Case 5 - symmetric surface cracks
- Case 6 - single surface crack

SUPERPOSITION

- Since the stress intensity factor is derived using Linear Elasticity, the principle of superposition applies
- Multiple applied loads can be superposed to find the effective stress intensity factor of the combined loading

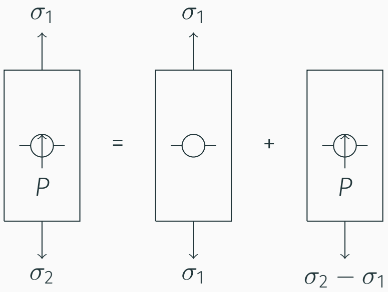

σ
P
S
2a
S
P
σ
=
σ
2a
σ
+
P
S
2a
S
P

$$K_I = K_{I(\sigma)} + K_{I(P)}$$

$$K_I = \sigma\sqrt{\pi a} + \frac{P}{t\sqrt{\pi a}} \frac{1 - 0.5\left(\frac{a}{W}\right) + 0.975\left(\frac{a}{W}\right)^2 - 0.16\left(\frac{a}{W}\right)^3}{\sqrt{1 - \left(\frac{a}{W}\right)}}$$

- Sometimes, the superposition needed to solve a problem is not obvious
- It can be helpful to subtract a known solution from the problem

SUPERPOSITION



COMPOUNDING

- Different types of boundaries create different correction factors to the usual stress intensity factor
- We often use β to indicate the total correction factor
- When multiple boundaries are present, we can combine them into one effective correction factor
- There are two general methods we use to create a compound correction factor

- The first method uses linear superposition, and thus is restricted to cases where the effect of each boundary can be assumed to add linearly
- While in most cases this is not strictly true, it provides a reasonable approximation

$$K_r = \bar{K} + \sum_{i=1}^{N} (K_i - \bar{K}) \tag{3.8}$$

- Where N is the number of boundaries, $\bar{K}$ is the stress intensity factor with no boundaries present and K_i is the stress intensity factor associated with the i^{th} boundary.

- We can rewrite this equation as

$$K_r = \sigma\sqrt{\pi a}\beta_r = \sigma\sqrt{\pi a} + \sum_{i=1}^{N}(\sigma\sqrt{\pi a}\beta_i - \sigma\sqrt{\pi a}) \tag{3.9}$$

- Which leads to an expression for β_r as

$$\beta_r = 1 + \sum_{i=1}^{N}(\beta_i - 1) \tag{3.10}$$

COMPOUNDING METHOD 2

- An alternative empirical method approximates the boundary effect as

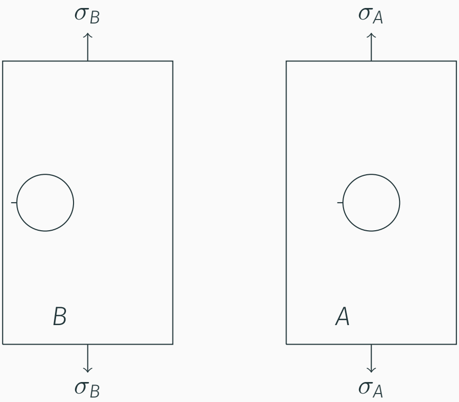
$$\beta_r = \beta_1\beta_2...\beta_N \tag{3.11}$$

- If there is no interaction between the boundaries, method 1 and method 2 will give the same result

CURVED BOUNDARIES

- For short cracks, we can use the *stress concentraction factor* on a curved boundary to determine the stress intensity factor
- The stress intensity factor only gives the maximum stress at the curved boundary, thus the longer the crack is, the farther away from the curved boundary (and maximum stress) it is.

- Suppose we want to determine the stress intensity on a panel, panel B
- We find a similar panel with a known stress intensity factor, panel A
- We adjust the applied load on panel A such that $K_{I,A} = K_{I,B}$
- The magnitude of this load adjustment is determined using the *stress concentration factors* in panels B and A
- Note the notation: K_t for stress concentration factor, K_I for stress intensity factor


σ_B
σ_A
B
A
σ_B
σ_A

- Since A is a fictional panel, we set the applied stress, σ_A such that

$$\sigma_{max,B} = \sigma_{max,A}$$

- Substituting stress concentration factors

$$K_{t,B}\sigma_B = K_{t,A}\sigma_A$$

- Solving for σ_A

$$\sigma_A = \frac{K_{tB}}{K_t A}\sigma_B$$

- Since the crack is short and $\sigma_{max,A} = \sigma_{max,B}$ we can say

$$
\begin{aligned}
K_{I,B} &= K_{I,A} \\
&= \sigma_A \sqrt{\pi c} \beta_A \\
&= \frac{K_{t,B}}{K_{t,A}} \sigma_B \sqrt{\pi c} \beta_A
\end{aligned}
$$

on board

- As a crack becomes very large, the effect of the curved boundary diminishes
- We find expressions for β_L (long crack) and β_S (short crack)
- We connect β_S to β_L using a straight line from β_S to a tangent intersection with β_L

LONG CRACKS ON CURVED BOUNDARIES