

# AE 737 - MECHANICS OF DAMAGE TOLERANCE

## LECTURE 10

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## SCHEDULE

- 23 Feb - Residual Strength, Multiple Site Damage
- 25 Feb - Multiple Site Damage, Mixed-mode Fracture, Homework 4 Due, Homework 5 Assigned
- 1 Mar - Section 1 Review, Homework 5 Due
- 3 Mar - Section 1 Review, Homework 5 return
- 8 Mar - Exam 1
- 10 Mar - Exam return, Final Project discussion

# OUTLINE

1. residual strength review
2. stiffeners
3. severed stiffeners
4. crack stoppers
5. multiple site damage

## RESIDUAL STRENGTH REVIEW

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- Group 1 - Sketch a residual strength curve for a single material (include fracture and net-section yield)
- Group 2 - Sketch and describe the difference in residual strength between stiff/brittle materials and ductile/tough materials
- Group 3 - Find the proof load needed to ensure no center-cracks less than 0.01" are present in a material with  $K_C = 120 \text{ ksi}\sqrt{\text{in.}}$ .
- Group 4 - Sketch the Fedderson approach to residual strength. How is this different from the traditional approach? Why is it beneficial?

## STIFFENERS

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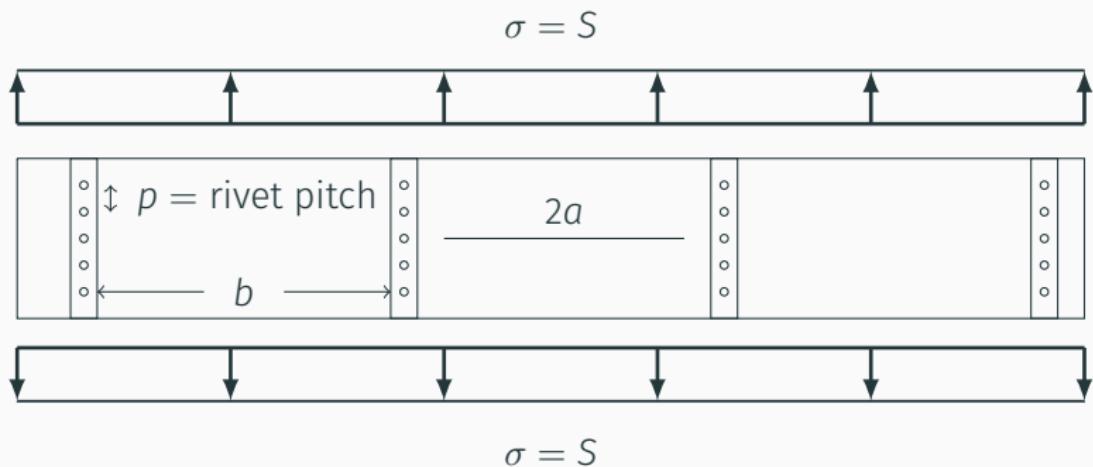
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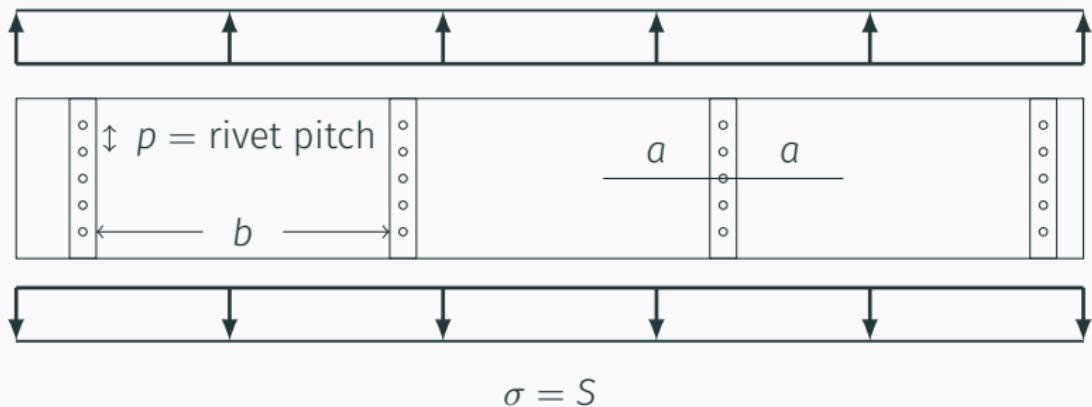
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- Panels in these configurations are generally very wide relative to expected crack dimensions
- Cracks are generally modeled either as centered between stiffeners or centered under a stiffener
- We need to consider the residual strength of the panel, the stiffener, and the rivets

## CENTERED BETWEEN STIFFENERS



# CENTERED UNDER STIFFENER

$$\sigma = S$$



## REMOTE STRESS IN STIFFENER

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- Where the subscript  $S$  indicates stiffener values, we can express the remote stress in the stiffener as

$$S_S = S \frac{E_S}{E} \quad (10.1)$$

# SKIN

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- The only exception is that the stiffener contributes to  $\beta$

$$S_c = \frac{K_c}{\sqrt{\pi a} \beta} \quad (10.2)$$

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- The maximum stress in a stiffener will be increased near a crack
- We represent the ratio of maximum force in stiffener to remote force with the Stiffener Load Factor,  $L$

$$L = \frac{\text{max force in stiffener}}{\text{remote force applied to stiffener}} \quad (10.3a)$$

$$= \frac{S_{S,max}A_S}{S_S A_S} \quad (10.3b)$$

$$= \frac{S_{S,max}}{S \frac{E_S}{E}} \quad (10.3c)$$

$$LS \frac{E_S}{E} = S_{S,max} \quad (10.3d)$$

$$LS \frac{E_S}{E} = \sigma_{YS} \quad (10.3e)$$

$$S_C = \frac{\sigma_{YS} E}{L E_S} \quad (10.3f)$$

- We can define a similar rivet load factor to relate maximum stress in the rivet to remote stress in the skin

$$L_R = \frac{\tau_{max} A_R}{Sbt} \quad (10.4a)$$

$$L_R = \frac{\tau_{YS} A_R}{Sbt} \quad (10.4b)$$

$$S_c = \frac{\tau_{YS} A_R}{L_R bt} \quad (10.4c)$$

# FINITE ELEMENT ANALYSIS

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- pp 167 - 178 give  $\beta$ ,  $L$  and  $L_R$  for various skin/stiffener configurations
- These values were determined using a finite element model

## EXAMPLES

- quantitative example (p. 179-180)
- qualitative notes on behavior (p. 181-182)

## SEVERED STIFFENERS

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- When the crack is short (and near the severed stiffener) the residual strength is lower due to the broken stiffener
- As the crack nears the next stiffener, residual strength is very similar to a panel with all stiffeners intact

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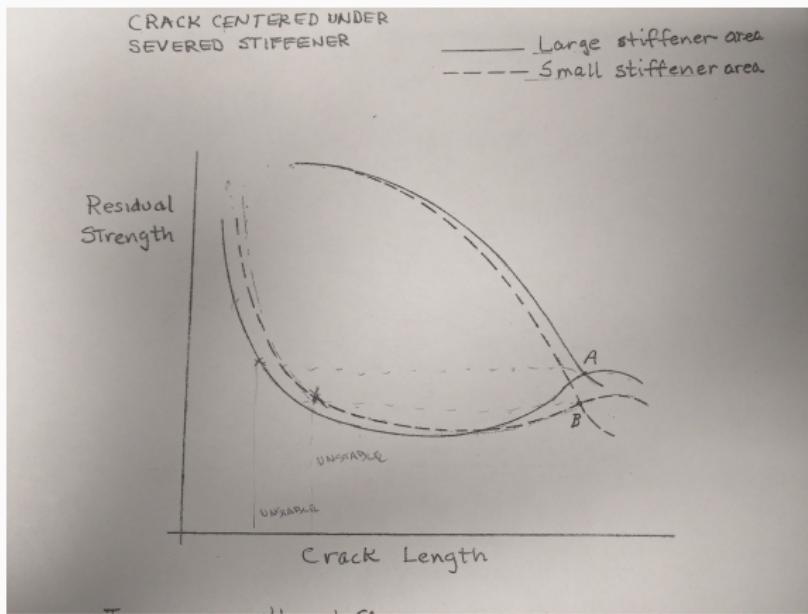
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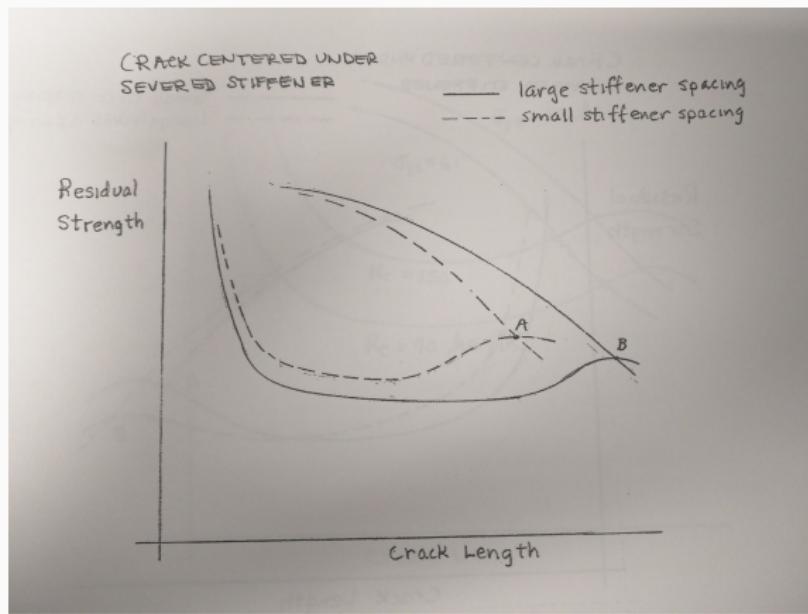
$$\sigma_c = \frac{\sigma_u}{SCFI} \quad (10.7)$$

- Swift's parametric study did not consider rivet failure

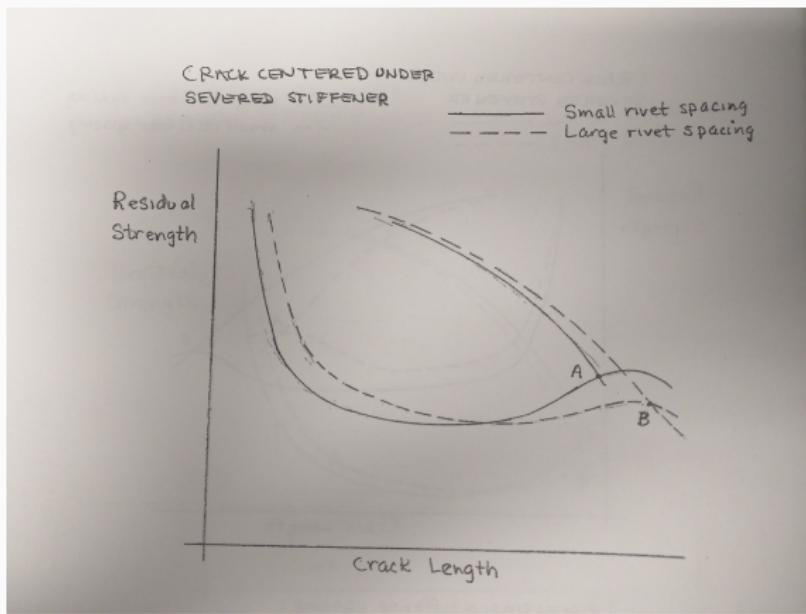
# STIFFENER AREA



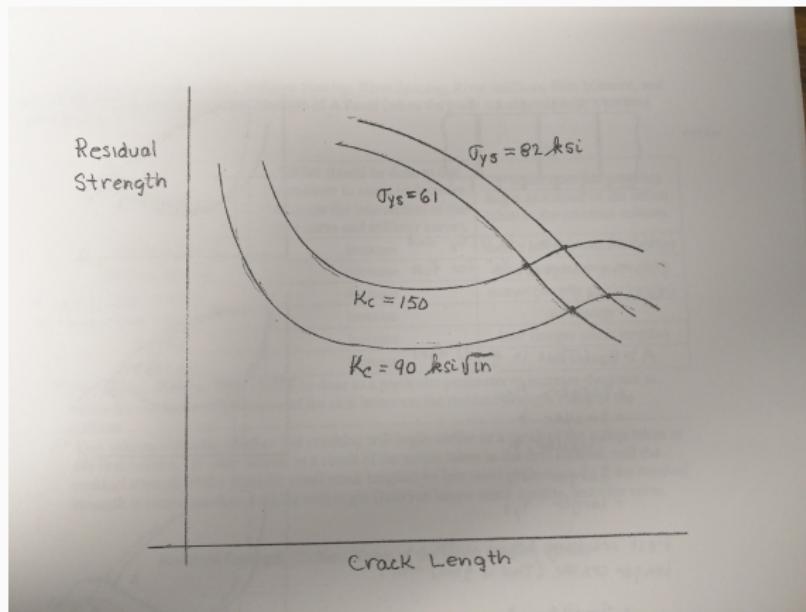
# STIFFENER SPACING



# RIVET SPACING



# STRENGTH AND TOUGHNESS INCREASE



## EXAMPLE

- If we consider the case from Swift's data most similar to our previous example:

$$P = 1.0 \text{ in}$$

$$A_{st} = 0.2538 \text{ in}^2$$

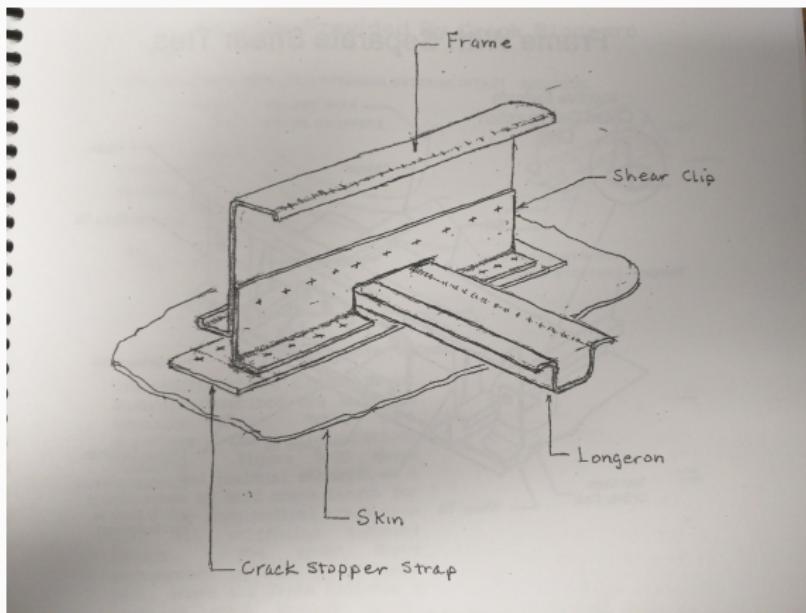
$$b = 10.0 \text{ in}$$

- So we use the tables for Case 10

## CRACK STOPPERS

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- The ideal material was titanium (as opposed to steel or aluminum).
- Aluminum did not transfer enough load to the stiffeners, steel transferred too much

## EXAMPLE

- Compare cases 1, 3, and 5

## MULTIPLE SITE DAMAGE

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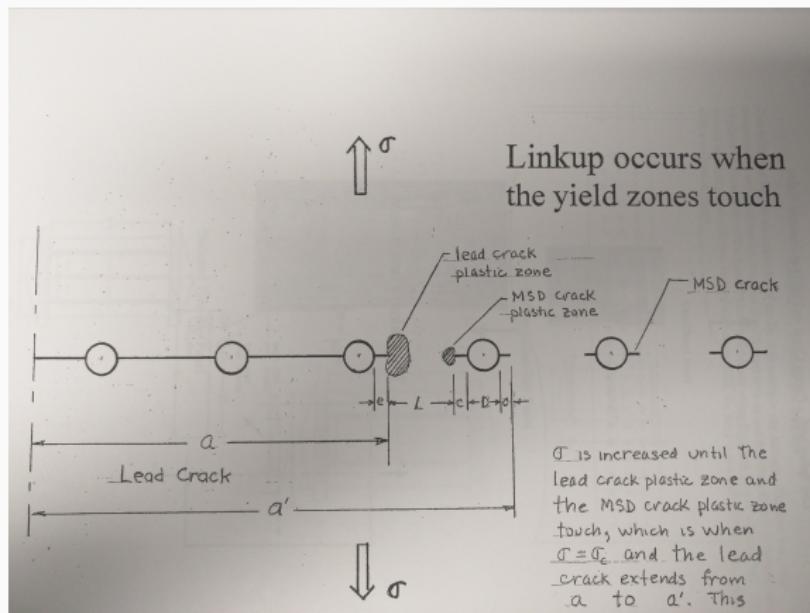
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- This is very common when there are a series of holes, each can develop cracks with a potential to link up
- "link up" occurs when the plastic zones between two adjacent cracks touch



## LINKUP EQUATION

- We know that

$$R_p = \frac{1}{2\pi} \left( \frac{K_{Ia}}{\sigma_{YS}} \right)^2 \quad (10.8)$$

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- Where we define the stress intensity factors at a and L as

$$K_{Ia} = \sigma \sqrt{\pi a} \beta_a \quad (10.10)$$

$$K_{IL} = \sigma \sqrt{\pi l} \beta_l \quad (10.11)$$

## LINKUP EQUATION

- Since fast cracking occurs when  $R_p + r_p = L$ , we solve for the condition where  $R_p + r_p < L$

$$\frac{1}{2\pi} \left( \frac{K_{la}}{\sigma_{YS}} \right)^2 + \frac{1}{2\pi} \left( \frac{K_{ll}}{\sigma_{YS}} \right)^2 < L \quad (10.12a)$$

$$\frac{1}{2\pi\sigma_{YS}^2} [K_{la}^2 + K_{ll}^2] < L \quad (10.12b)$$

$$\frac{1}{2\pi\sigma_{YS}^2} [\sigma^2 \pi a \beta_a^2 + \sigma^2 \pi l \beta_l^2] < L \quad (10.12c)$$

$$\frac{\sigma^2}{2\sigma_{YS}^2} [a \beta_a^2 + l \beta_l^2] < L \quad (10.12d)$$

$$\frac{\sigma_c^2}{2\sigma_{YS}^2} [a \beta_a^2 + l \beta_l^2] = L \quad (10.12e)$$

$$\sigma_c = \sigma_{YS} \sqrt{\frac{2L}{a \beta_a^2 + l \beta_l^2}} \quad (10.12f)$$