AE 737: Mechanics of Damage Tolerance

Lecture 6 - Plastic Zone

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schedule

- 3 Feb - Plastic Zone, HW 2 Due, HW 1 Self-grade due
- 8 Feb - Fracture Toughness
- 10 Feb - Fracture Toughness, HW3 Due, HW 2 Self-grade due
- 15 Feb - Residual Strength

outline

- plastic stress intensity ratio
- plastic zone shape
- group problems

plastic stress intensity ratio

plastic stress intensity ratio

- Engineers often use stress intensity to decide which material to use for a certain application
- The ratio of plastic stress intensity to elastic stress intensity, as a function of yield stress over applied stress, can help illustrate the effects of plasticity for different materials.

plastic stress intensity ratio

For an infinitely wide center-cracked panel, we can solve for KIe/KI symbolically, in plane stress

$$K_I = \sigma\sqrt{\pi a}$$

$$K_{Ie} = \sigma\sqrt{\pi(a + r_p)}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2$$

$$K_{Ie} = \sigma\sqrt{\pi\left(a + \frac{1}{2\pi}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2\right)}$$

stress intensity ratio

$$K_{Ie}^2 = \sigma^2 \pi \left(a + \frac{1}{2\pi} \left(\frac{K_{Ie}}{\sigma_{YS}} \right)^2 \right)$$

$$K_{Ie}^2 = \sigma^2 \pi a + \frac{\sigma^2}{2} \left(\frac{K_{Ie}}{\sigma_{YS}} \right)^2$$

$$K_{Ie}^2 - \frac{\sigma^2}{2} \left(\frac{K_{Ie}}{\sigma_{YS}} \right)^2 = \sigma^2 \pi a$$

$$K_{Ie}^2 \left(1 - \frac{\sigma^2}{2\sigma_{YS}^2} \right) = \sigma^2 \pi a$$

Note: square both sides

plastic stress intensity ratio

$$K_{Ie}^2 = \frac{\sigma^2 \pi a}{1 - \frac{\sigma^2}{2\sigma_{YS}^2}}$$

$$K_{Ie} = \frac{\sigma \sqrt{\pi a}}{\sqrt{1 - \frac{\sigma^2}{2\sigma_{YS}^2}}}$$

$$K_{Ie} = \frac{K_I}{\sqrt{1 - \frac{\sigma^2}{2\sigma_{YS}^2}}}$$

$$\frac{K_{Ie}}{K_I} = \frac{1}{\sqrt{1 - \frac{\sigma^2}{2\sigma_{YS}^2}}}$$

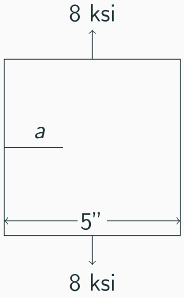
Note: We divide both sides by $\left(1 - \frac{\sigma^2}{2\sigma_{YS}^2} \right)$

plastic stress intensity ratio

- We can also find the plastic stress intensity ratio numerically for finite width panels
- Panel thickness, yield stress, panel width, crack length could all be variables in this case
- Different heat treatments of metal alloys can give a different yield stress, with most other properties remaining the same
- Typical crack lengths can vary based on inspection cycles

example

- You are asked to design an inspection cycle for a panel
- Consider the plastic stress intensity ratio, and the effect of varying crack lengths on it



example

online example here[1]

[1] ../examples/Plastic%20stress%20intensity%20ratio.html

plastic zone shape

plastic zone shape

- Although we drew a circle to give a rough idea of the plastic zone in Irwin's method, this solution was only 1D
- We considered $\theta = 0$.
- It is advantageous to model the plastic zone shape, we will do so using two different yield theories
- Von Mises and Tresca

principal stresses

- Principal stresses are often used in yield theories
- We can determine the principal stresses near the crack tip as

$$\begin{aligned}
\sigma_1 &= \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\right) \\
\sigma_2 &= \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right) \\
\sigma_3 &= 0 && \text{(plane stress)} \\
\sigma_3 &= \frac{2\nu K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} && \text{(plane strain)}
\end{aligned}$$

Von Mises yield theory

- The Von Mises yield theory is also known as the Distortion Energy Yield Theory
- In this yield theory, we assume that failure or yielding occurs when the strain energy exceeds some threshold
- It has been observed that hydrostatic pressure does not generally cause yielding
- We separate the strain energy into two parts, volumetric and distortional
- Only the distortional strain energy is used to determine the yield strength

Von Mises yield theory

- The distortional strain energy is given by

$$W_d = \frac{1}{12} G \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]$$

- Which for a uniaxially loaded point becomes

$$W_d = \frac{1}{6} G \sigma_{YS}^2$$

- We can equate the two cases and solve

$$\begin{aligned} \frac{1}{6} G \sigma_{YS}^2 &= \frac{1}{12} G \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \\ 2\sigma_{YS}^2 &= (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \end{aligned}$$

Von Mises yield theory

- We can find the plastic zone size, *rp* by substituting the principal stress relations into the distortional strain energy equation
- In plane stress we find

$$2\sigma_{YS}^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - 0)^2 + (0 - \sigma_1)^2$$

Von Mises yield theory

$$\begin{aligned} 2\sigma_{YS}^2 = & \left(\frac{K_I}{\sqrt{2\pi r_p}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right) - \right. \\ & \left. \frac{K_I}{\sqrt{2\pi r_p}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right)\right)^2 + \\ & \left(\frac{K_I}{\sqrt{2\pi r_p}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right) - 0\right)^2 + \\ & \left(0 - \frac{K_I}{\sqrt{2\pi r_p}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right)\right)^2 \end{aligned}$$

Von Mises yield theory

- After solving we find

$$r_p = \frac{K_I^2}{2\pi\sigma_{YS}^2}\cos^2\frac{\theta}{2}\left(1 + 3\sin^2\frac{\theta}{2}\right)$$

- We can similarly solve for *rp* in plane strain to find

$$r_p = \frac{K_I^2}{2\pi\sigma_{YS}^2}\cos^2\frac{\theta}{2}\left(1 - 4\nu + 4\nu^2 + 3\sin^2\frac{\theta}{2}\right)$$

Tresca yield theory

- Tresca yield theory assumes that yielding begins when the maximum shear stress reaches a critical value
- In uniaxial tension this gives

$$\tau_0 = \tau_{max} = \frac{1}{2}\left(\sigma_{max} - \sigma_{min}\right) = \frac{1}{2}\left(\sigma_{YS} - 0\right) = \frac{\sigma_{YS}}{2}$$

Tresca yield theory

- Using the results for principal stress we found previously, we see that

$$
\begin{aligned}
\sigma_{max} &= \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right) \\
\sigma_{min} &= 0
\end{aligned}
$$

- We can substitute and solve as before to find

$$
r_p = \frac{K_I^2}{2\pi\sigma_{YS}^2} \cos^2\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right)^2
$$

Tresca yield theory

- In plane strain, it is not clear whether σ_2 or σ_3 will be σ_{min}
- We can solve for when σ_2 will be σ_{min}

$$
\begin{aligned}
\sigma_2 &\quad lt;\sigma_3 \\
\frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right) &\quad lt;\frac{2\nu K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \\
1 - \sin\frac{\theta}{2} &\quad lt;2\nu \\
\theta_t gt; 2\sin^{-1}(1 - 2\nu)
\end{aligned}
$$

Tresca yield theory

- When $2\pi - \theta_t < \theta < \theta_t$, σ_2 is the minimum, otherwise σ_3 is the minimum
- Note: Error(s) in text p. 101
- Once we have chosen the appropriate minimum stress (σ_2 or σ_3), we can solve for *rp* as before

Tresca yield theory

$$
\begin{aligned}
r_p &= \frac{2K_I^2}{\pi\sigma_{YS}^2}\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2} && \theta_t lt; \theta \qquad lt; 2\pi - \theta_t \\
r_p &= \frac{K_I^2}{2\pi\sigma_{YS}^2}\cos^2\frac{\theta}{2}\left(1 - 2\nu + \sin\frac{\theta}{2}\right)^2 && \theta lt; \theta_t, \theta \quad gt; 2\pi - \theta_t
\end{aligned}
$$

3D plastic zone shape

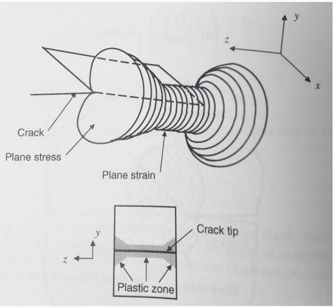


Figure 1: An image showing the 3D plastic zone shape, which looks a little bit like a dumbell. The plastic zone is much larger near the surface, where the material behaves as if in plane stress. In the

example

online example here[2]

[2] ../examples/Plastic%20Zone%20Shape.html

group problems

group one

- Calculate the plastic zone size for an infinitely wide, center-cracked panel
- Consider a crack-length of 4 cm, and a yield strength of $\sigma_{YS} = 55$ MPa, with an applied load of $\sigma = 20$ MPa
- Assume the panel is very thin

group two

- Calculate the plastic zone size for an infinitely wide, center-cracked panel
- Consider a crack-length of 4 cm, and a yield strength of $\sigma_{YS} = 55$ MPa, with an applied load of $\sigma = 20$ MPa
- Assume the panel is very thick

group three

- Calculate the plastic zone size for an infinitely wide, center-cracked panel
- Consider a crack-length of 4 cm, and a yield strength of $\sigma_{YS} = 55$ MPa, with an applied load of $\sigma = 20$ MPa
- The panel thickness is $t = 0.65$ cm

group four

- Find the plastic stress intensity ratio for an infinitely wide, center-cracked panel
- What factors will increase or decrease the plastic stress intensity ratio?