AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 15

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SCHEDULE

- 22 Mar - Stress based fatigue, Homework 6 assigned
- 24 Mar - Stress based fatigue
- 29 Mar - Influence of notches on fatigue, Homework 7 assigned, Homework 6 due
- 31 Mar - Strain based fatigue, project abstract due

ARCHES

1. fatigue
2. nominal and local stress
3. fatigue tests
4. fatigue life analysis
5. fatigue limit

FATIGUE

damage as function of tool speed

FATIGUE

- We refer to damage from repeated, or cyclic loads as fatigue damage
- Some of the earliest work on fatigue began in the 1800's
- Chains, railway axles, etc.
- An estimated 80% of failure expenses are due to fatigue

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- Fracture mechanics fatigue analysis

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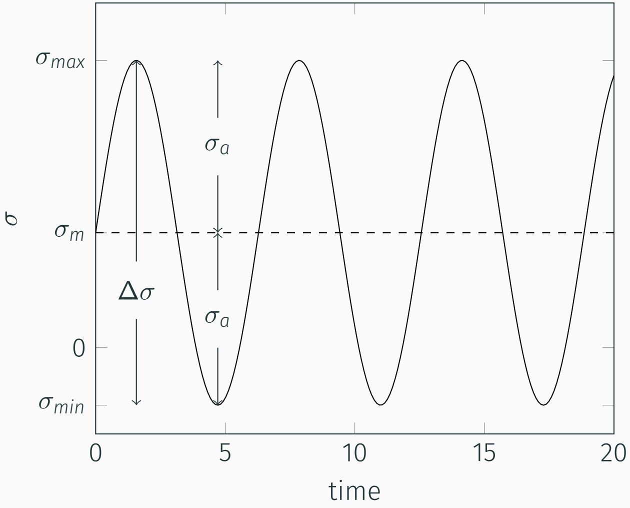
- This is referred to as constant amplitude stressing

CONSTANT AMPLITUDE STRESSING



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- σ_m is the mean stress, and can sometimes be zero, but this is not always the case
- σ_a is the stress amplitude, and is the variation about the mean
- We can express all of these in terms of the maximum and minimum stress

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \tag{15.1}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \tag{15.2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{15.3}$$

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- And the amplitude ratio, A is defined as

$$A = \frac{\sigma_a}{\sigma_m} \tag{15.5}$$

- There are some useful relationships between the above equations

$$\Delta\sigma = 2\sigma_a = \sigma_{max}(1 - R) \tag{15.6a}$$

$$\sigma_m = \frac{\sigma_{max}}{2}(1 + R) \tag{15.6b}$$

$$R = \frac{1 - A}{1 + A} \tag{15.6c}$$

$$A = \frac{1 - R}{1 + R} \tag{15.6d}$$

NOMINAL AND LOCAL STRESS

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DEFINITION AND NOTATION

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- We must also be careful to note σ_y, in some cases $S < \sigma_y$ but at some locations $\sigma > \sigma_y$

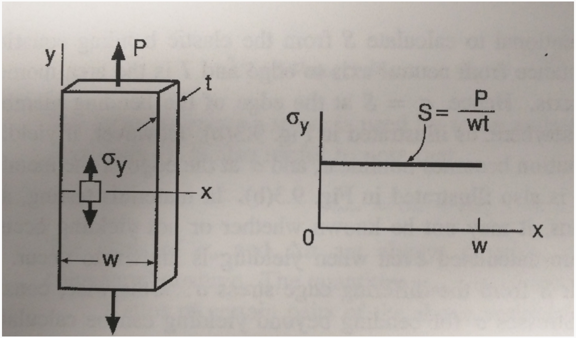


Figure 1: In this case $S = \sigma$

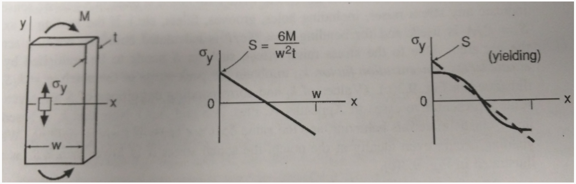
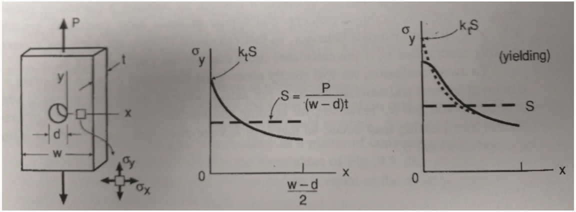


Figure 2: As long as $\sigma < \sigma_y$, σ varies linearly. If $\sigma > \sigma_y$ at any location, however, the relationship is non-linear



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FATIGUE TESTS

ROTATING CANTILEVER BEAM

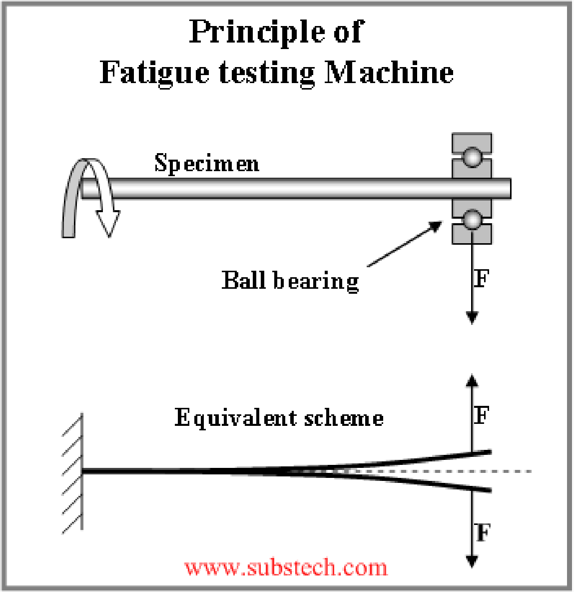

Figure 4: Cantilever beam produces non-uniform stress state

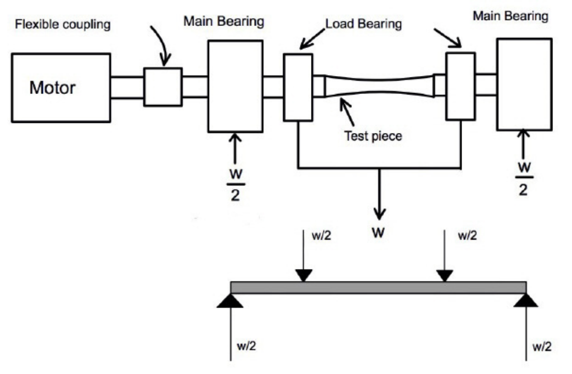

Figure 5: Four-point bend gives uniform stress (along top and bottom surfaces)

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- a reciprocating bend test can be used for non-zero mean stress

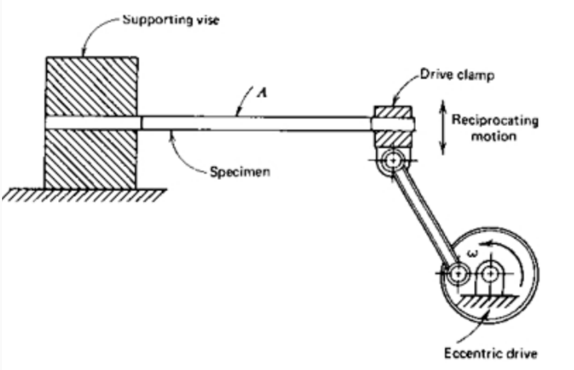

Figure 6: A reciprocating cantilever test allows for non-zero mean stress

AXIAL FATIGUE TEST

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- Servohydraulic machines generally have a speed of 10 - 100 Hz.
- At a speed of 100 Hz, it would take 28 hours for 10^7 cycles, 12 days for 10^8 cycles, and nearly 4 months for 10^9 cycles
- While some machines can test at very high speeds, the inertia of the sample can interfere with results

FATIGUE LIFE ANALYSIS

STRESS LIFE CURVES

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- In general, one set (or family) of S-N curves is generated using the same σ_m
- Usually S_a (the nominal stress equivalent of σ_a) is plotted versus N (the number of cycles)

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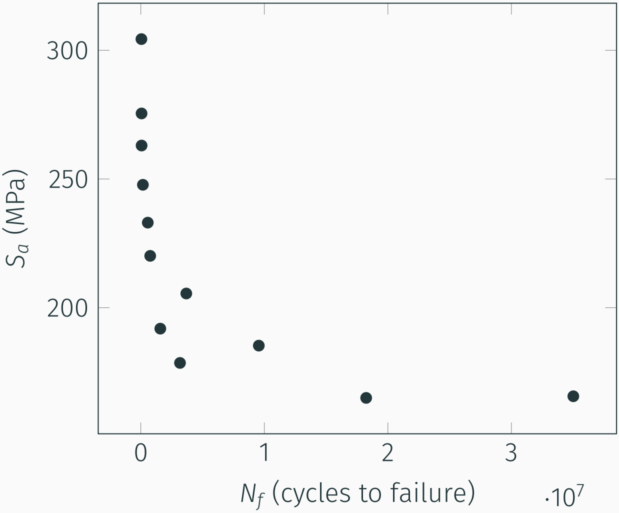
- In the following plot, if only one test was performed for each point, the total number of cycles tested would be about $7.3x10^7$
- For a 100 Hz machine, this represents over 200 hours of consecutive testing
- Each repetition would further increase the test time required


300
250
200
S_a (MPa)
0
1
2
3
N_f (cycles to failure)
$\cdot 10^7$

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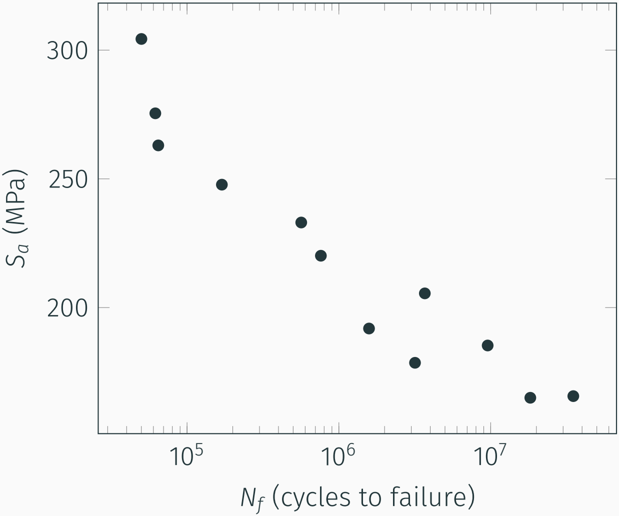
- It is also very difficult to differentiate between low-cycle fatigue failure stresses
- Instead S-N curves are often plotted on a semi-log or log-log scale, so pay attention to the axes


300
250
200
S_a (MPa)
10^5
10^6
10^7
N_f (cycles to failure)

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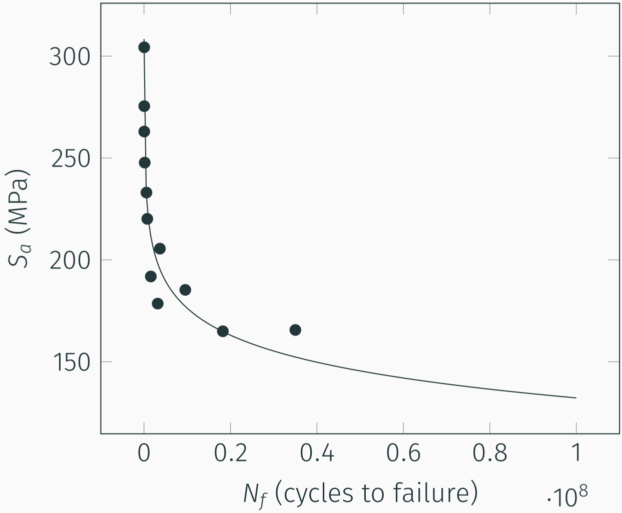
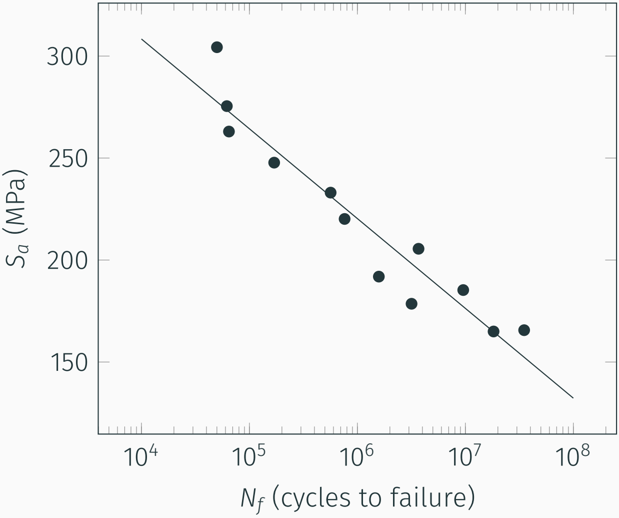
- $$\sigma_a = \sigma_f' \left(2N_f\right)^b \tag{15.8}$$
- σ_f' and b are often considered material properties and can often be looked up on a table (p. 235)


300
250
200
150
S_a (MPa)
0
0.2
0.4
0.6
0.8
1
N_f (cycles to failure)
$\cdot 10^8$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250
200
150
S_a (MPa)
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10^5
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- This most notably occurs in plain-carbon and low-alloy steels
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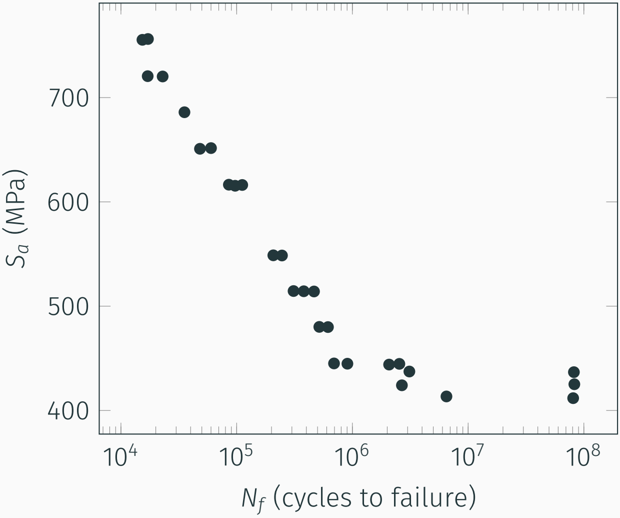
- This phenomenon is not typical of aluminum or copper alloys, but is sometimes arbitrarily assigned using whatever the failure stress is at some large number of cycles (10^7 or 10^8)



700
600
500
400
S_a (MPa)
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- ”High cycle fatigue” generally is considered anything above 10^3 cycles, but varies somewhat by material
- High cycle fatigue occurs when the stress is sufficiently low that yielding effects do not dominate behavior
- When yielding effects do dominate behavior, the strain-based approach is more appropriate