

## Lecture 4 - Curved Boundaries

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## schedule

- 10 Feb - Curved Boundaries, HW 1 Due
- 15 Feb - Plastic Zone
- 17 Feb - Plastic Zone, HW 2 Due, HW 1 Self-grade due
- 22 Feb - Fracture Toughness

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- compounding
- curved boundaries
- stress concentration factors

## supplemental material

- I was unable to find the source for all of Dr. Horn's formulas, but I have made an alternate set of equations (taken from the AFGROW user's manual) available on Blackboard under supplemental material.
- Can also be found here

## compounding

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### superposition vs. compounding

- In this course, we use *superposition* to combine loading conditions
- We use *compounding* to combine edge effects
- Both are very powerful tools and important concepts

- Different types of boundaries create different correction factors to the usual stress intensity factor
- We often use  $\beta$  to indicate the total correction factor
- When multiple boundaries are present, we can combine them into one effective correction factor
- There are two general methods we use to create a compound correction factor

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## method 1

- The first method uses linear superposition, and thus is restricted to cases where the effect of each boundary can be assumed to add linearly
- While in most cases this is not strictly true, it provides a reasonable approximation

$$K_r = \bar{K} + \sum_{i=1}^N (K_i - \bar{K})$$

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## method 1

- Where  $N$  is the number of boundaries,  $\bar{K}$  is the stress intensity factor with no boundaries present and  $K_i$  is the stress intensity factor associated with the  $i^{\text{th}}$  boundary.

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## method 1

- We can rewrite this equation as

$$K_r = \sigma\sqrt{\pi a}\beta_r = \sigma\sqrt{\pi a} + \sum_{i=1}^N(\sigma\sqrt{\pi a}\beta_i - \sigma\sqrt{\pi a})$$

- Which leads to an expression for  $\beta_r$  as

$$\beta_r = 1 + \sum_{i=1}^N(\beta_i - 1)$$

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- An alternative empirical method approximates the boundary effect as

$$\beta_r = \beta_1 \beta_2 \dots \beta_N$$

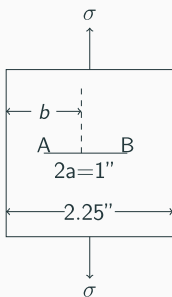
- If there is no interaction between the boundaries, method 1 and method 2 will give the same result

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## p. 68 - example 1

- A crack in a finite-width panel is centered between two stiffeners
- Assume the  $\beta$  correction factor for this stiffener configuration is  $\beta_s = 0.9$
- Assume the  $\beta$  correction factor for this finite-width panel is  $\beta_w = 1.075$
- Use both compounding methods to estimate the stress intensity
- How accurate do you expect this to be?

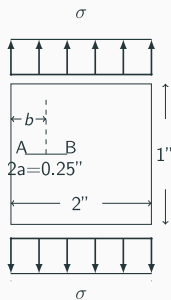
11



$$b = 1 \text{ inch}$$

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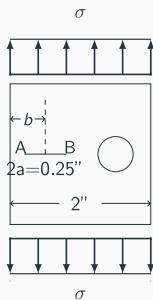
## group 1



$$b = 0.4 \text{ inches}$$

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## group 2

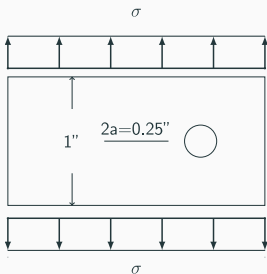


$b = 0.4$  inches

Hole diameter is 0.5 inches and spaced 0.5 inches away from the crack tip

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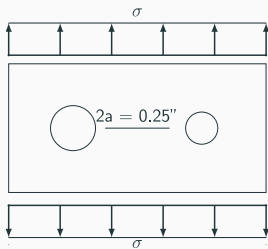
## group 3



Hole diameter is 0.5 inches and spaced 0.5 inches away from the crack tip

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The right crack tip is 0.5 inches away from a 0.5 inch diameter hole and the left crack tip is 0.25 inches away from a 1 inch diameter hole.

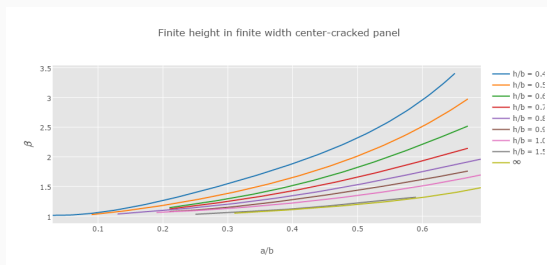
## errata and supplemental charts

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- on p. 64 there is a + missing between two terms, see Lecture 2 for the fix
- Also on p. 64, in equation 29 it is not clear, but use the  $f_w$  from a previous equation, on p. 56
- Some of the black and white figures can be difficult to use, we have scanned and re-created the plots online
- Interactive versions of compounding figures from p. 50, 71-73 can be found at [here](#)

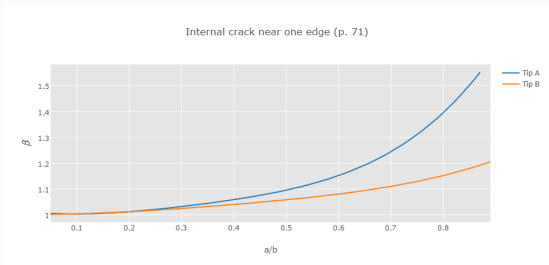
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## finite height - p. 50



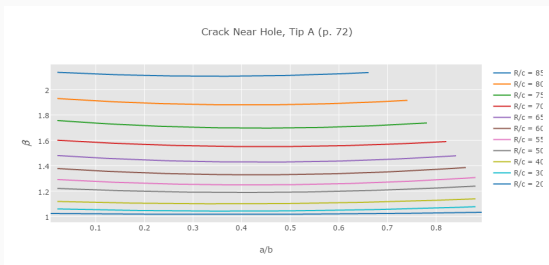
**Figure 1:** beta for finite height effects, see text p. 50 or interactive chart linked in previous slide

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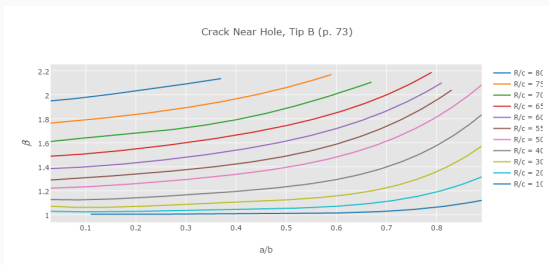
**Figure 2:** beta for offset internal crack, see text p. 71 or interactive chart linked previously

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**Figure 3:** beta for the crack tip farther away from a hole, see text p. 72 or interactive chart linked previously

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**Figure 4:** beta for the crack tip closer to a hole, see text p. 73 or [interactive chart linked previously](#)

## curved boundaries

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## short cracks on curved boundaries

- For short cracks, we can use the *stress concentration factor* on a curved boundary to determine the stress intensity factor
- The stress concentration factor only gives the maximum stress at the curved boundary, thus the longer the crack is, the farther away from the curved boundary (and maximum stress) it is.
- Stress concentration factors can be found: pp. 82-85 in the text
- Also see supplemental text on Blackboard or here

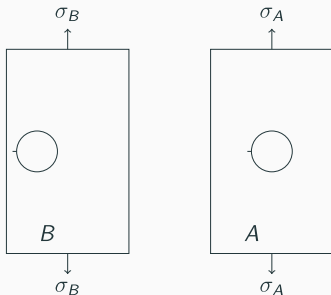
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## short cracks on curved boundaries

- Suppose we want to determine the stress intensity on a panel, panel B
- We find a similar panel with a known stress intensity factor, panel A
- We adjust the applied load on panel A such that  $K_{I,A} = K_{I,B}$
- The magnitude of this load adjustment is determined using the *stress concentration factors* in panels B and A Note: the notation:  $K_t$  for stress concentration factor,  $K_I$  for stress intensity factor

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## short cracks on curved boundaries



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## short cracks on curved boundaries

- Since A is a fictional panel, we set the applied stress,  $\sigma_A$  such that

$$\sigma_{max,B} = \sigma_{max,A}$$

- Substituting stress concentration factors

$$K_{t,B}\sigma_B = K_{t,A}\sigma_A$$

- Solving for  $\sigma_A$

$$\sigma_A = \frac{K_{t,B}}{K_{t,A}}\sigma_B$$

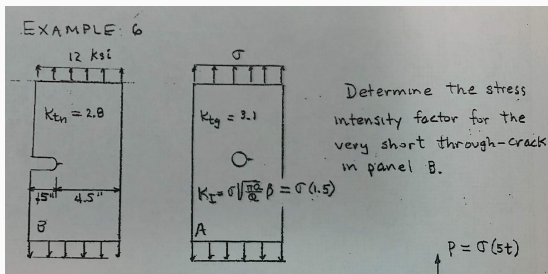
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- Since the crack is short and  $\sigma_{max,A} = \sigma_{max,B}$  we can say

$$\begin{aligned} K_{I,B} &= K_{I,A} \\ &= \sigma_A \sqrt{\pi C} \beta_A \\ &= \frac{K_{t,B}}{K_{t,A}} \sigma_B \sqrt{\pi C} \beta_A \end{aligned}$$

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## example 6 (p. 86)



**Figure 5:** See p. 86, there is a short through crack on the edge of a 0.5" deep notch on a 5 inch wide panel with a remote 12 ksi stress applied. The net section stress concentration factor is 2.8, while the global stress concentration factor for a similar panel with a hole is

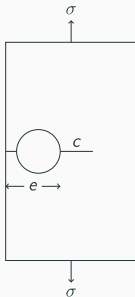
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## long cracks on curved boundaries

- As a crack becomes very large, the effect of the curved boundary diminishes
- We find expressions for  $\beta_L$  (long crack) and  $\beta_S$  (short crack)
- We connect  $\beta_S$  to  $\beta_L$  using a straight line from  $\beta_S$  to a tangent intersection with  $\beta_L$

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## long cracks on curved boundaries



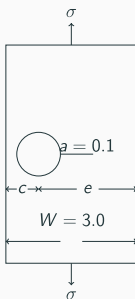
29



- Example here

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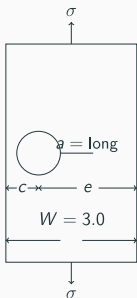
## group one



- $c = 0.75$ ,  $e = 2.25$ ,  $r = 0.5$
- assume  $a$  is short and calculate  $\beta$  for this case
- calculate in terms of  $\beta$  for known state

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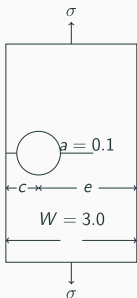
## group two



- $c = 0.75$ ,  $e = 2.25$ ,  $r = 0.5$
- assume  $a$  is long and calculate  $\beta$  for this case
- calculate in terms of  $\beta$  for known state

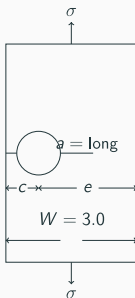
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## group three



- $c = 0.75$ ,  $e = 2.25$ ,  $r = 0.5$
- assume  $a$  is short and calculate  $\beta$  for this case
- calculate in terms of  $\beta$  for known state

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- $c = 0.75$ ,  $e = 2.25$ ,  $r = 0.5$
- assume  $a$  is long and calculate  $\beta$  for this case
- calculate in terms of  $\beta$  for known state

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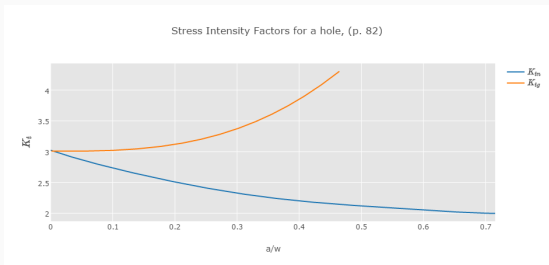
## discussion

Draw a sketch to show how we could use this method to find cracks of intermediate length near a curved boundary

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## stress concentration factors

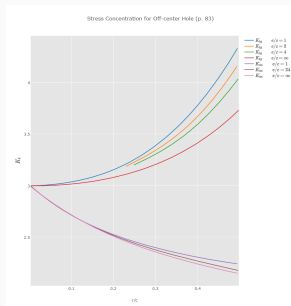
### centered hole tension - p. 82



**Figure 6:** A plot of stress concentration factors near a hole, see text p. 82 or the interactive plots linked in the last slide.

$K_{tg}$  uses stress for the cross-sectional area if no hole was present,  $K_{tn}$  uses stress at the net section (subtracting hole

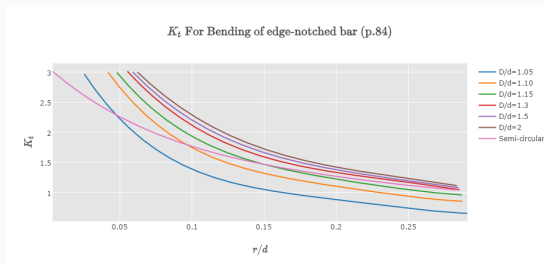
## off-center hole tension - p. 83



$K_{tg}$  uses stress for the cross-sectional area if no hole was present,  $K_{tn}$  uses stress at the net section (subtracting hole area).  $c$  is the distance from the closest edge to the center of

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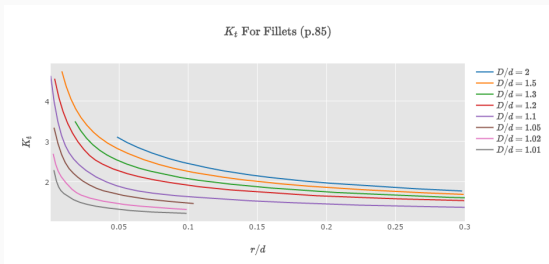
## bending of a bar with u-shaped notch - p. 84



**Figure 7:** A plot of stress concentration factors in a bar with a u-shaped notch, see text p. 84 or the interactive plots linked in the last slide.

Nominal stress used for  $K_t$  is given by  $\sigma_{nom} = 6M/hd^2$  where

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$D$  is the larger width (before the step),  $d$  is the width after the step. Nominal stress is  $\sigma_{nom} = P/hd$ , where  $h$  is specimen thickness.  $r$  is the fillet radius.

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## interactive page

- An interactive page with these plots can be accessed [here](#)

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