AE 737 - Mechanics of Damage Tolerance

Lecture 6

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Last Updated: February 11, 2016 at 11:46am

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schedule

- 9 Feb - Fracture Toughness, Homework 2 Due, Homework 3 Assigned
- 11 Feb - Fracture Toughness
- 16 Feb - Residual Strength, Homework 3 Due, Homework 4 Assigned
- 18 Feb - Residual Strength
- 23 Feb - Multiple Site Damage, Homework 4 Due, Homework 5 Assigned
- 25 Feb - Mixed-mode Fracture

outline

1. fracture toughness
2. plane strain, brittle
3. plane stress, ductile

fracture toughness

- The critical load at which a cracked specimen fails produces a critical stress intensity factor
- The ”critical stress intensity factor” is known as K_c
- For Mode I, this is called K_{Ic}
- The critical stress intensity factor is also known as fracture toughness

$$K_{IC} = \sigma_c \sqrt{\pi a} \beta \tag{6.1}$$

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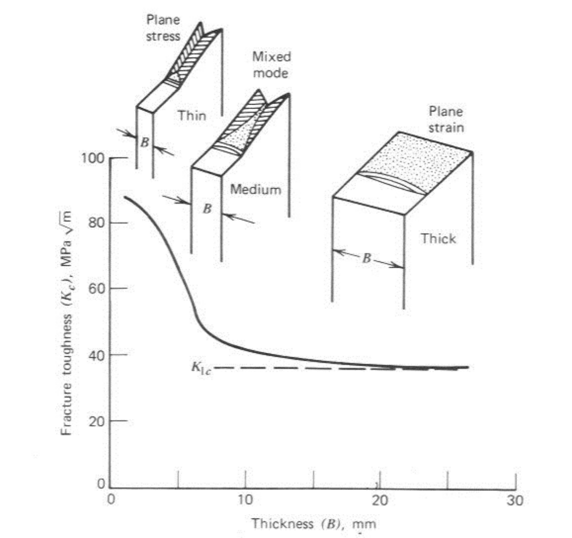
- NOTE: "Fracture Toughness" can also refer to G_{Ic}, which is analogous to K_{Ic}, but not the same

- Fracture toughness is a material property, but it is only well-defined in certain conditions
- Brittle materials
- Plane strain (smaller plastic zone)
- In these cases ASTM E399-12 is used.

fracture toughness



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- The load must be continually increased until a critical value causes unstable crack growth

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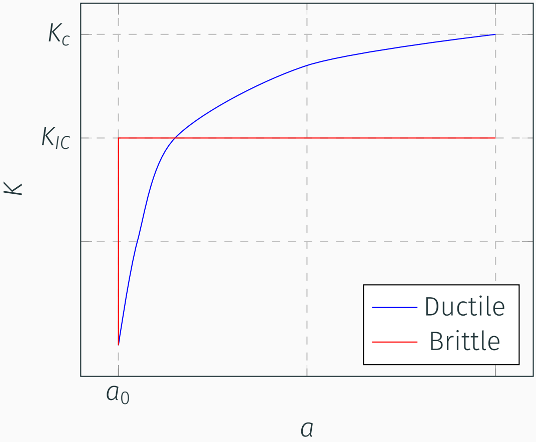
- We can calculate a unique stress intensity factor K_{Ii} at each of these points
- These are then used to create a ”K-curve”, plotting K_I vs. a

K-curve



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- Plane strain conditions and brittle materials will tend towards a ”flat” K-curve
- K_{IC} for brittle/plane strain is very well defined
- K_C for plane stress can refer to two things
- Either the maximum K_C during a test, or tangent point on K_R-curve (R-curve)

example - stable crack growth

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- The Double-Cantilever Beam (DCB) experiment to find G_{IC} illustrates this

$$C = \frac{\delta}{P} \tag{6.2a}$$

$$C = \frac{2a^3}{3EI} \tag{6.2b}$$

$$G = \frac{P^2}{2b}\frac{dC}{da} \tag{6.2c}$$

$$G = \frac{P^2 a^2}{bEI} \tag{6.2d}$$

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- For crack growth to be stable we need

$$\frac{dG}{da} \leq 0 \tag{6.3}$$

- Under fixed-load conditions, we find

$$\frac{dG}{da} = \frac{2P^2 a}{bEI} \tag{6.4}$$

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- Which is always stable, so for DCB tests, displacement control is generally used

plane strain, brittle

- For relatively brittle materials, we don't need to worry about the R-curve
- Specimens are made according to these specifications

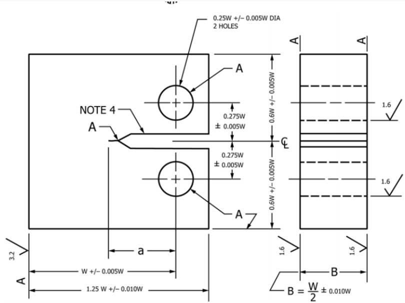
$$a \geq 2.5\left(\frac{K_{IC}}{\sigma_{YS}}\right)^2 \tag{6.6a}$$

$$b \geq 2.5\left(\frac{K_{IC}}{\sigma_{YS}}\right)^2 \tag{6.6b}$$

$$W \geq 5\left(\frac{K_{IC}}{\sigma_{YS}}\right)^2 \tag{6.6c}$$

1. Select specimen size (see (6.6))
2. Select specimen type (Compact Tension or Single Edge Notched Bend)

ASTM E399



NOTE 1—Surface finishes in µm.

NOTE 2—*A* surfaces shall be perpendicular and parallel to within 0.002 *W* TIR.

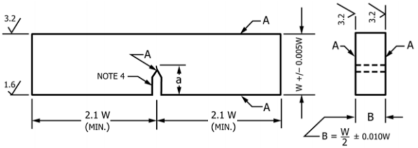
NOTE 3—The intersection of the crack starter notch tips with the two specimen surfaces shall be equally distant from the top and bottom edges of the specimen within 0.005 *W*.

NOTE 4—Integral or attachable knife edges for clip gage attachment to the crack mouth may be used (see Figs. 3 and 4).

NOTE 5—For starter notch and fatigue crack configuration see Fig. 5.

NOTE 6—1.6 µm = 63 µin., 3.2 µm = 125 µin.

FIG. A4.1 Compact C(T) Specimen—Standard Proportions and Tolerances



NOTE 1—Surface finishes in µm.
NOTE 2—*A* surfaces shall be perpendicular and parallel as applicable within 0.001 *W* TIR.
NOTE 3—Crack starter notch shall be perpendicular to specimen surfaces within 2°.
NOTE 4—Integral or attachable knife edges for clip gage attachment may be used (see Figs. 3 and 4)
NOTE 5—For starter notch and fatigue crack configuration see Fig. 5.
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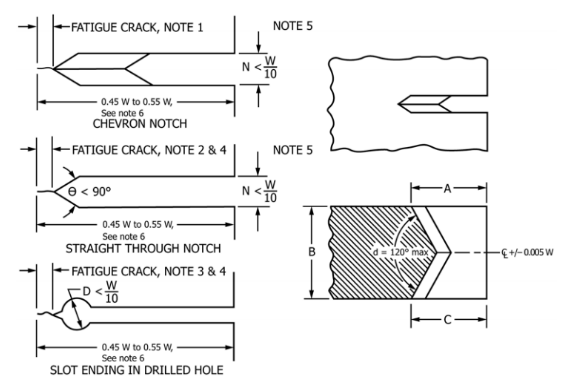
FIG. A3.1 Bend SE(B) Specimen—Standard Proportions and Tolerances

ASTM E399

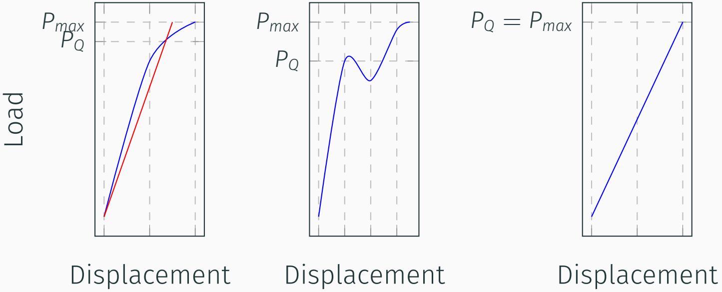


(*a*) Starter Notches and Fatigue Cracks

(*b*) Detail of Chevron Notch

3. Machine specimen
4. Fatigue crack specimen $K_f < 0.6 K_{IC}$
    - This is to ensure that the plastic zone size during fatigue is smaller than the plastic zone size during testing
    - If K_{IC} has not yet been determined, you may have to guess the first time

5. Mount specimen, attach gage
6. Load rate should ensure ”static” load conditions. (30 - 150 ksi$\sqrt{\text{in.}}$/min.)
7. Determine the ”provisional” value of K_{IC} (known as K_Q)



ASTM E399

7.1 If the load-displacement curve is like the first figure, with some non-linearity, we let P_Q be the point of intersection between the load-displacement curve and a line whose slope is 5% lower than the slope in the elastic region

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$$K_Q = \frac{P_Q}{BW^{1/2}} f\left(\frac{a}{W}\right) \qquad \text{Compact Tension} \tag{6.7}$$

$$K_Q = \frac{P_Q}{BW^{3/2}} g\left(\frac{a}{W}\right) \qquad \text{SENB} \tag{6.8}$$

8. Ensure that your specimen is still valid

$$a \geq 2.5 \left(\frac{K_Q}{\sigma_{YS}} \right)^2 \tag{6.9}$$

$$b \geq 2.5 \left(\frac{K_Q}{\sigma_{YS}} \right)^2 \tag{6.10}$$

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- Load-displacement should have an initial slope between 0.7 and 1.5

plane stress, ductile

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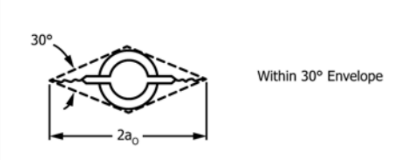
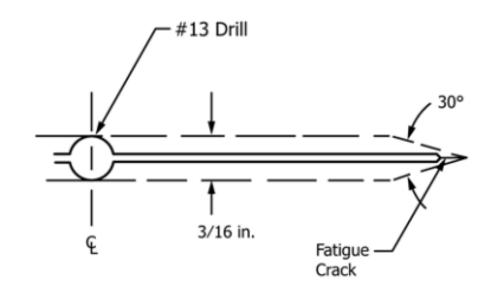
- The other specimen which is permitted is a middle-cracked tension specimen (M(T))
- M(T) specimens are preferred in many cases due to a more uniform stress distribution (particularly important for anisotropic materials)

ASTM E561





minimum sample dimensions

Table of Minimum M(T) Specimen Geometry for Given Conditions

K_{Rmax}/σ_{YS}		Width		$2a_o$		Length[A]	
$\sqrt{m}$	$\sqrt{in.}$	m	in.	m	in.	m	in.
0.08	0.5	0.076	3.0	0.025	1.0	0.229	9
0.16	1.0	0.152	6.0	0.051	2.0	0.457	18
0.24	1.5	0.305	12.0	0.102	4.0	0.914	36
0.32	2.0	0.508	20.0	0.170	6.7	0.762	30
0.48	3.0	1.219	48.0	0.406	16.0	1.829	72

Figure 1: M(T) minimum recommended dimensions

Table of Minimum C(T) Specimen Width W for Given Conditions, m (in.)

K_{Rmax}/σ_{YS}		Maximum a_p/W				
$\sqrt{m}$	$\sqrt{in.}$	0.4	0.5	0.6	0.7	0.8
0.10	0.6	0.02 (0.8)	0.03 (1.0)	0.03 (1.3)	0.04 (1.7)	0.06 (2.5)
0.20	1.3	0.08 (3.3)	0.10 (4.0)	0.13 (5.0)	0.17 (6.7)	0.25 (10.0)
0.30	1.9	0.19 (7.5)	0.23 (9.0)	0.29 (11.3)	0.38 (15.0)	0.57 (22.6)
0.40	2.5	0.34 (13.3)	0.40 (15.9)	0.51 (19.9)	0.67 (26.5)	1.01 (39.8)
0.50	3.1	0.53 (20.9)	0.64 (25.1)	0.80 (31.3)	1.06 (41.8)	1.59 (62.7)

Figure 2: C(T) minimum recommended dimensions

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    1. Measure the crack length visually and calculate r_p

effective crack length

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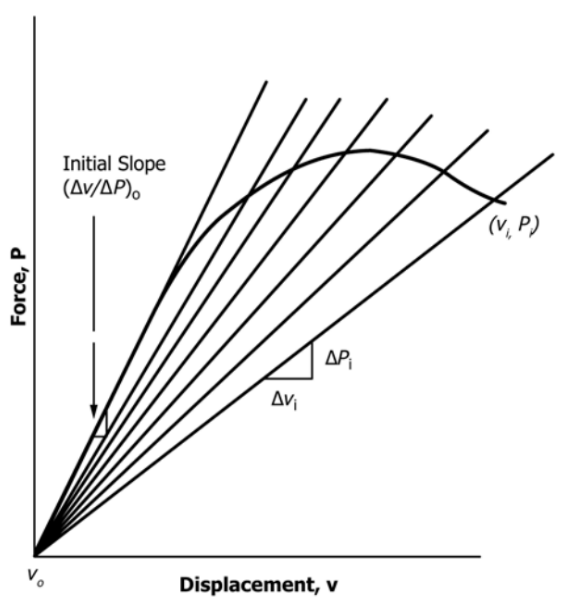
    1. Measure the crack length visually and calculate r_p
    2. Measure crack length using ”unloading compliance” and adding plastic zone size
    3. Measure the effective crack size directly using ”secant compliance”

secant compliance



secant compliance M(T)

- Using the slope data from our load-displacement curve, we can calculate the effective crack length using

$$EB\left(\frac{\Delta v}{\Delta P}\right) = \frac{2Y}{W}\sqrt{\frac{\pi a/W}{\sin(\pi a/W)}} \left[\frac{2W}{\pi Y}\cosh^{-1}\left(\frac{\cosh(\pi Y/W)}{\cos(\pi a/W)}\right) - \frac{1+\nu}{\sqrt{1+\left(\frac{\sin(\pi a/W)}{\sinh(\pi Y/W)}\right)^2}} + \nu\right] \tag{6.14}$$

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- Instead it is generally solved iteratively
- The following equations are used to give a good initial guess to use in iterations

$$X = 1 - \exp\left[\frac{-\sqrt{[EB(\Delta v/\Delta P)]^2 - (2Y/W)^2}}{2.141}\right] \tag{6.15}$$

$$\frac{2a}{W} = 1.2235X - 0.699032X^2 + 3.25584X^3 - 6.65042X^4 + 5.54X^5 - 1.66989X^6 \tag{6.16}$$

secant compliance M(T)

- In the above equations, the following are the definitions of parameters used

$$
\begin{aligned}
E &= \quad \text{Young's Modulus} \\
\Delta v/\Delta P &= \quad \text{specimen compliance} \\
B &= \quad \text{specimen thickness} \\
W &= \quad \text{specimen width} \\
Y &= \quad \text{half span of the displacement measurement points} \\
a &= \quad \text{effective crack length} \\
\nu &= \quad \text{Poisson's ratio}
\end{aligned}
$$

secant compliance C(T)

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$$EB\frac{\Delta v}{\Delta P} = A_0 + A_1\left(\frac{a}{W}\right) + A_2\left(\frac{a}{W}\right)^2 + A_3\left(\frac{a}{W}\right)^3 + A_4\left(\frac{a}{W}\right)^4 \quad (6.17)$$

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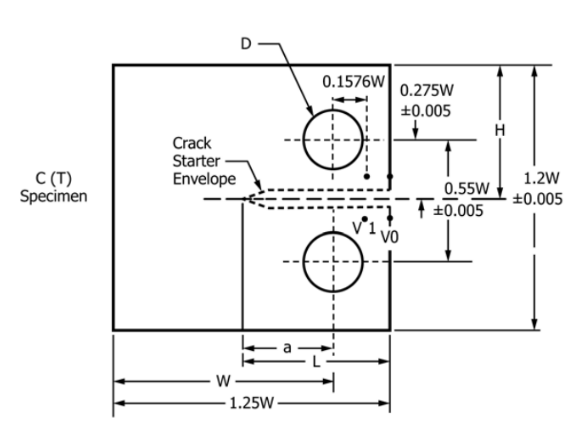
- The coefficients will differ based on where the displacement is measured from

secant compliance C(T)



secant compliance C(T)

location	A_0	A_1	A_2	A_3	A_4
V_0	120.7	-1065.3	4098.0	-6688.0	4450.5
V_1	103.8	-930.4	3610.0	-5930.5	3979.0

location	C_0	C_1	C_2	C_3	C_4	C_5
V_0	1.0010	-4.6695	18.460	-236.82	1214.90	-2143.6
V_1	1.0008	-4.4473	15.400	-180.55	870.92	-1411.3

- Where the initial guess for a is provided by

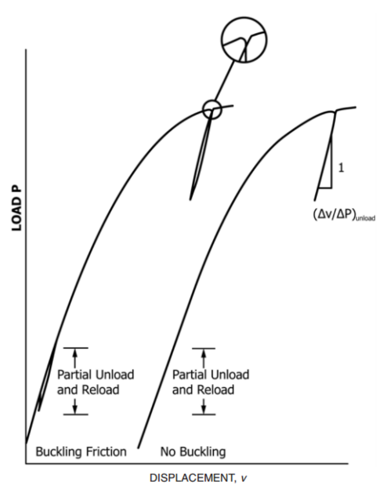
$$\frac{a}{W} = C_0 + C_1 U + C_2 U^2 + C_3 U^3 + C_4 U^4 + C_5 U^5 \tag{6.18}$$

- and U is given by

$$U = \frac{1}{1 + \sqrt{EB\frac{\Delta v}{\Delta P}}} \tag{6.19}$$

buckling

- If the test is stopped and re-started frequently (to measure crack length by hand or to use the compliance method of crack measurement) buckling can interfere with results



buckling

- If buckling is shown to be present in the test, supports can be used to prevent buckling
- These supports can introduce friction
- They should be well-lubricated for accurate test results

net section stress

- One final consideration when dealing with plane stress fracture mechanics is the net section stress
- For the test to be valid, failure must occur due to fracture, not general static failure
- Static failure will occur when $\sigma_N = \sigma_{YS}$

generate K_R curve

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- The K_R curve is quite simply a plot of K_{Ie} vs. a for the test performed (i.e. with varying stress and increasing crack length)
- When the test is performed correctly, the K_R curve is not a function of the initial crack length
- For this reason, we often plot K_{Ie} vs. Δa, to subtract the initial crack length

generate K_R curve

- Once the effective crack length and K_{Ie} has been determined for the test, we can generate the K_R curve
- The K_R curve is quite simply a plot of K_{Ie} vs. a for the test performed (i.e. with varying stress and increasing crack length)
- When the test is performed correctly, the K_R curve is not a function of the initial crack length
- For this reason, we often plot K_{Ie} vs. Δa, to subtract the initial crack length
- We can superpose constant-stress K-curves on this graph, the curve which intersects at a tangent point creates the most "standard" definition for K_C

example

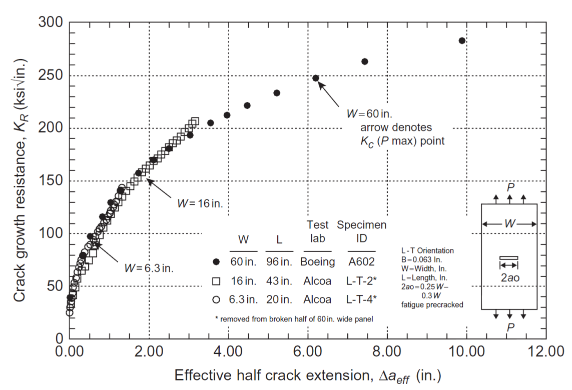

Figure 3: K_R Curve for C188-T3 aluminum for varying sample thickness (Boeing and Alcoa)

example

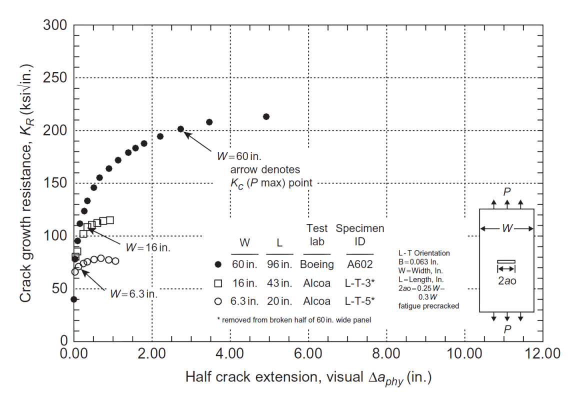

Figure 4: K_R curve for the same specimens, but without adjusting for the plastic zone size.