AE 737: Mechanics of Damage Tolerance

Lecture 8 - Fracture Toughness

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schedule

- 10 Feb - Fracture Toughness, HW3 Due, HW 2 Self-grade due
- 15 Feb - Residual Strength
- 17 Feb - Residual Strength, HW4 Due, HW 3 Self-grade due
- 22 Feb - Multiple Site Damage

outline

- fracture toughness
- R-curve
- thickness effects
- fracture toughness review
- residual strength

fracture toughness

fracture toughness

- The critical load at which a cracked specimen fails produces a critical stress intensity factor
- The "critical stress intensity factor" is known as K_C
- For Mode I, this is called K_{Ic}
- The critical stress intensity factor is also known as fracture toughness

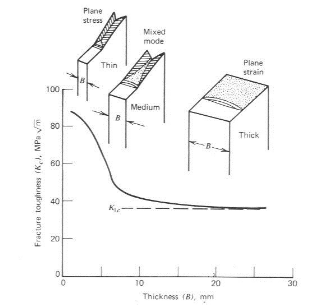
$$K_{IC} = \sigma_c \sqrt{\pi a} \beta$$

- Note: "Fracture Toughness" can also refer to G_{Ic} which is analogous to K_{Ic} but not the same

fracture toughness

- Fracture toughness is a material property, but it is only well-defined in certain conditions
- Brittle materials
- Plane strain (smaller plastic zone)
- In these cases ASTM E399-12 is used.

fracture toughness



unstable cracks

- Stable crack growth means the crack extends only with increased load
- Unstable crack growth means the crack will continue to extend indefinitely under the same load
- For a perfectly brittle material, there is no stable crack growth, as soon as a critical load is reached, the crack will extend indefinitely

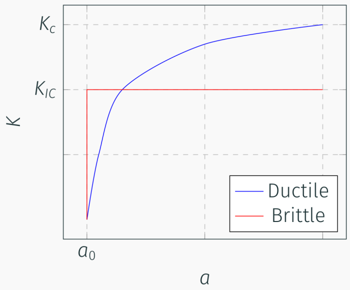
stable cracks

- For an elastic-plastic material, once the load is large enough to extend the crack, it will extend slightly
- The load must be continually increased until a critical value causes unstable crack growth

fracture toughness

- During an experiment, we will record the crack length and applied load (P_i, a_i) each time we increase the load
- We can calculate a unique stress intensity factor K_{Ii} at each of these points
- These are then used to create a "K-curve", plotting K_I vs. a

K-curve



K-curve

- Materials will generally not be as flat as the perfectly brittle example
- Plane strain conditions and brittle materials will tend towards a "flat" K-curve
- K_{IC} for brittle/plane strain is very well defined
- K_C for plane stress can refer to two things
- Either the maximum K_C during a test, or tangent point on K_R-curve (R-curve)

example

- In composites, and adhesives, some work is needed to ensure stable crack growth
- The Double-Cantilever Beam (DCB) experiment to find G_{Ic} illustrates this

$$C = \frac{\delta}{P}$$

$$C = \frac{2a^3}{3EI}$$

$$G = \frac{P^2}{2b}\frac{dC}{da}$$

$$G = \frac{P^2 a^2}{bEI}$$

example

- For crack growth to be stable we need

$$\frac{dG}{da} \leq 0$$

- Under fixed-load conditions, we find

$$\frac{dG}{da} = \frac{2P^2 a}{bEI}$$

- This is always positive, and thus results in unstable crack growth

- Under fixed-displacement conditions, we substitute for P
- We find

$$\frac{dG}{da} = -\frac{9\delta^2 EI}{ba^3}$$

- Which is always stable, so for DCB tests, displacement control is generally used

R-curve

R-curve

- For materials with some plasticity, the K_R Curve, or R Curve, is very important
- Sometimes called a "resistance curve" it is generally dependent on
  - Thickness
  - Temperature
  - Strain rate

R-curve

- When done correctly, K_R curves are not dependent on initial crack size or the specimen type used
- ASTM E561 describes a general procedure

K_c

- While we can look up plane stress K_c for various materials, it is best if we have a K_R curve
- We may not know if the table uses K_c using the tangent intersection method, or maximum stress intensity
- Even if tangent intersection method is used, K_c will different somewhat based on initial crack length

R-curve

- There are two main methods for plotting the R-curve
- Crack size is measured directly (possibly with a drawn-on scale and camera)
- Effective crack size is calculated from the load-displacement data

physical crack

- When the physical crack size is measured, we need to calculate the effective crack length (and effective stress intensity factor) at each data point
- The effective crack length calculated from the load-displacement data already has the plastic zone effect built in

plane stress fracture toughness

- For a plane stress (or any thickness that is not plane strain) we can find K_c two ways
- One way is simply the maximum value of the K_R curve, but this does not account for unstable crack growth
- The more reliable way is to use the tangent intersection method

tangent intersection method

- Construct curves for K_I based on our specimen geometry with constant applied load and varying crack length
- Plot these curves on the same graph as K_R
- NOTE: K_R curve should be plotted vs. a_{eff}, not Δa or Δa_{eff}
- K_c is the point at which one of the K_I curves is the tangent intersection with the K_R curve

R-Curve examples

- example[1]
- Excel Solver

[1] https://colab.research.google.com/drive/1TIGuadrMRM5xSGic8soVFglPDyWuDaeP?usp=sharing

thickness effects

thickness effects

- We already know there is a difference between plane strain and plane stress fracture toughness
- As a material gets thicker and thicker, it converges to the plane strain solution
- Thinner specimens tend towards the plane stress solution
- When a specimen is thinner than some critical thickness, the material behavior is somewhat unknown

thickness effects

- There is also a difference in the fracture surface between thin and thick specimens
- Thin specimens (in plane stress region) fail due to slant fracture
- This actually indicates some mixed-mode conditions at failure
- Thick specimens fail due to square fracture (with a small shear tip near the edges)
- This is more consistent with pure Mode I

slant fracture

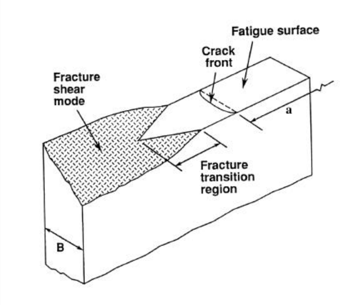


Figure 1: A slant fracture, where the failure plane rotates 45 degrees from the crack plane, considered a shear mode

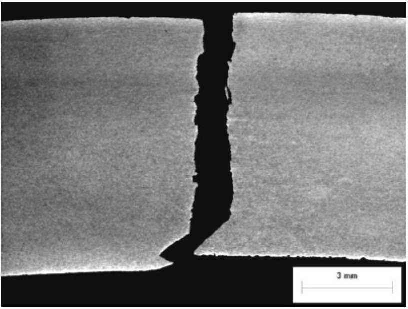


Figure 2: In this shear lip, there is a long crack that near the end rotates away by 45 degrees creating a shear lip near the surface

fracture toughness review

group review

- Group 1 - Sketch K_R-curve (for ductile material), explain what it means, how to find K_c
- Group 2 - Sketch K_c vs. crack length, explain what's happening
- Group 3 - How can we determine whether a panel is in plane strain or plane stress?
- Group 4 - Sketch K_R-curves for ductile and brittle materials, what is the difference?

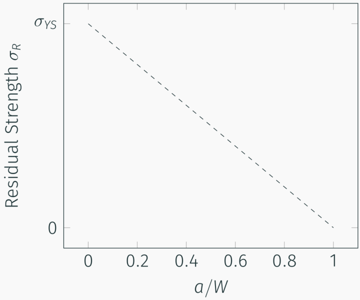
residual strength

residual strength

- As the crack grows, the area of the sample decreases, increasing the net section stress
- The residual strength, σ_R is given in terms of the gross area, so as the crack grows the residual strength due to yield decreases
- We can relate the net-section stress to σ_R by

$$\sigma_R = \sigma_{YS} \frac{A_{net}}{A_{gross}}$$

residual strength

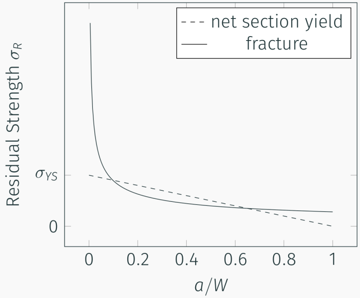


residual strength

- For brittle fracture to occur, we need to satisfy the condition

$$\sigma_R = \sigma_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

residual strength



residual strength

- Within the same family of materials (i.e. Aluminum), there is generally a trade-off between yield stress and fracture toughness
- As we increase the yield strength, we decrease the fracture toughness (and vice versa)
- Consider a comparison of the following aluminum alloys
  1. 7178-T6, $K_C = 43$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 74$ ksi
  2. 7075-T6, $K_C = 68$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 63$ ksi
  3. 2024-T3, $K_C = 144$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 42$ ksi

residual strength

- As an example let us consider an edge-cracked panel with $W = 6$ and $t = 0.1$ inches
- The net section yield condition will be given by

$$\sigma_C = \sigma_{YS}\frac{W-a}{W} = \sigma_{YS}\frac{6-a}{6}$$

- And the fracture condition by

$$\sigma_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

With

$$\{\beta = 1.12 - 0.231\left(\frac{a}{W}\right) + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4\}$$

7178-T6

:::

7075-T6

:::

2024-T3

:::

comparison

:::

using MIL-handbook

- Uses a different grain nomenclature

K_C	σ_{YS}
L-T	L
T-L	L-T

- A-Basis vs. B-Basis values are reported (A = 99% of population will meet/exceed value, B = 90% of population)
- S-Basis - no statistical information available, standard value to be used

using MIL-handbook

- F_{tu} - ultimate tensile strength
- F_{ty} - tensile yield strength
- F_{cy} - compressive yield strength
- F_{su} - ultimate shear strength
- F_{bru} - ultimate bearing strength
- F_{bry} - bearing yield strength
- E - tensile Young's Modulus
- E_c - compressive Young's Modulus
- G - shear modulus
- μ- Poisson's ratio

data

- Fracture data is on pp. 111-121
- Tensile data is on pp. 138-143
- K_c charts are also available in interactive versions here[2]

[2] http://ndaman.github.io/damagetolerance/examples/Fracture%20Toughness%20Figures.html