# AE 737: Mechanics of Damage Tolerance

## Lecture 17 - Crack Propagation

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# schedule

- 2 Apr - Crack growth, HW6 Due
- 4 Apr - Crack growth
- 9 Apr - Boeing Method, HW7 Due
- 11 Apr - Retardation

# outline

- other factors affecting fatigue
- crack growth rate
- crack growth rate equations
- factors affecting crack propagation
- numerical algorithm

# other factors affecting fatigue

# factors affecting fatigue life

- At temperatures above one-half the melting temperature (absolute scale), creep-relaxation is significant
- This will cause the strain/stress-life curves to become rate dependent
- Occurs at room temperature for many materials (lead, tin, many polymers)
- At a sufficiently elevated temperature for any material

## surface finish

- High cycle fatigue is sensitive to surface finish, samples are generally polished
- Low cycle fatigue is not sensitive to surface finish or residual stress
- The plastic deformation tends to remove residual stresses
- In high-cycle fatigue, crack initiation is important (poor surface finish allows cracks to form earlier)
- When plastic deformation is present (low-cycle fatigue), cracks form relatively quickly regardless of surface finish

# surface finish

- Since low-cycle fatigue has little effect from surface finish, we could modify the strain life curve by altering only the elastic portion
- If we define the surface effect factor, $m_s$, we can find a new $b_s$ to replace $b$ in the strain-life equation

$$b_s = \frac{\log\left(m_s(2N_e)^b\right)}{\log(2N_e)}$$

# surface treatments

- Treatments which decrease fatigue life:
  - Electro-plating (chrome, +corrosion resistance, -fatigue life)
  - Grinding improves surface finish, but introduces surface tension, and heat generated can temper quench
  - Stamping introduces discontinuities and irregularities
  - Forging can refine grain structure and improve physical properties, but can cause decarburization in steels, which hurts fatigue life
  - Hot rolling can also cause decarburization

## surface treatments

- Some treatments improve fatigue life:
    - Cold rolling improves surface finish, introduces residual compressive stress on surface (slows crack initiation on surface)
    - Shot peeing introduces many small divots on surface, which can be detrimental in corrosion, but it does cause a residual compressive stress on the surface

# size

- Size can also have effects on fatigue life
- Larger parts are more susceptible to damage/imperfections at the same stress level
- This is why composites are often made from very small fibers (glass fiber, carbon fiber, ceramic-matrix composites)

# size

- The exact effect of size will depend on material, one study for low carbon steels found

$$m_d = \left(\frac{d}{25.4\text{mm}}\right)^{-0.093}$$

- Which is then used to re-calculate material constants $\sigma_{fd}{}' = m_d \sigma_f{}', \qquad \epsilon_{fd}{}' = m_d \epsilon_f{}'$

# thermal fatigue

- Thermal loading can be introduced when two dissimilar parts are attached together, the coefficient of thermal expansion causes them to expand differently, introducing extra stresses due to the temperature change
- If the temperature is significantly different between two sides of a part thermal stresses can also be introduced

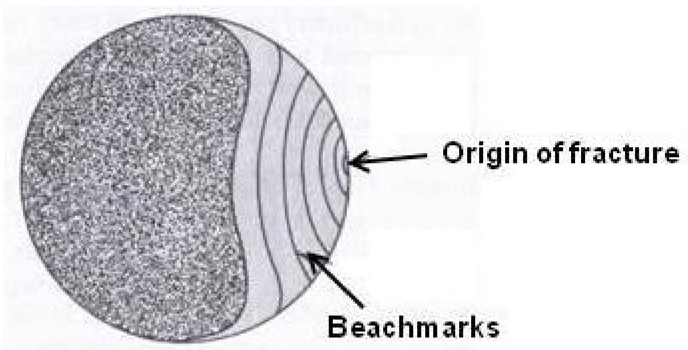
# thermal fatigue

- Low temperatures generally cause a material to behave in a more brittle fashion, which alters the fatigue life
- High temperatures cause problems with creep-relaxation and can also affect the crystalline structure

# crack growth rate

# fracture surface

# fracture surface




Fatigue Fracture with Beachmarks

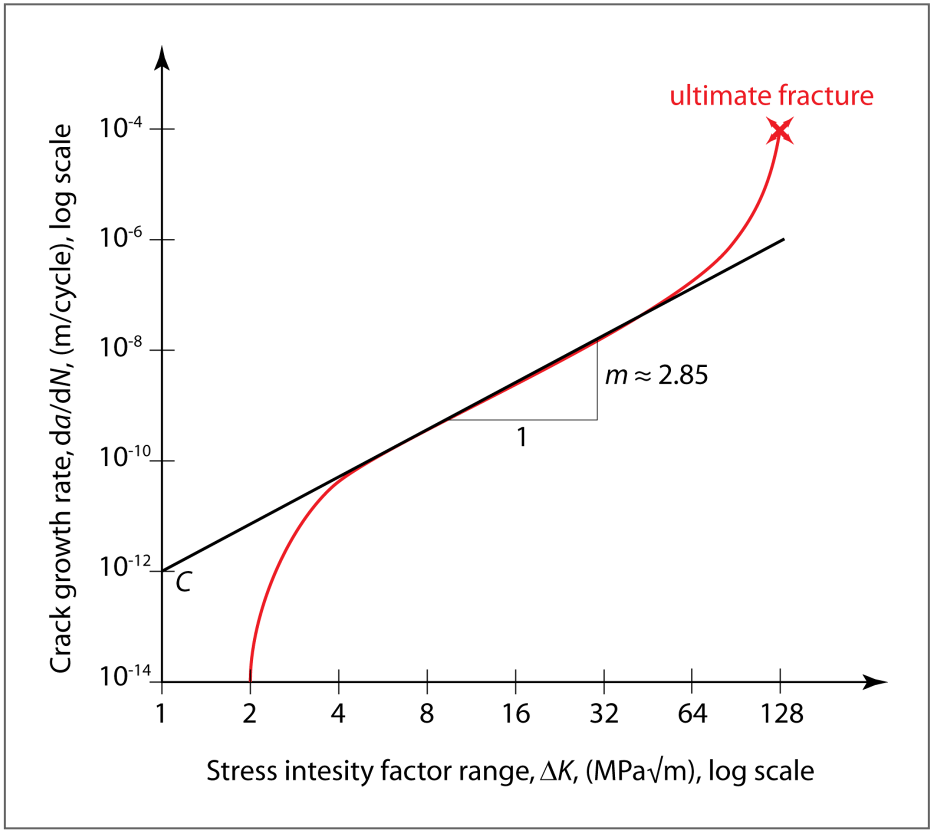
# crack growth rate

- We can observe that fatigue damage occurs through crack propagation
- “cracks” and fracture mechanics have been omitted from all our fatigue discussion thus far
- It would be beneficial to predict at what rate a crack will extend

# crack growth rate

- Crack growth rate can be measured experimentally
- Using a center-crack specimen, a fatigue load is applied
- The crack length is measured and plotted vs. the number of cycles
- The slope of this curve ($\frac{da}{dN}$) is then plotted vs. either $K_{Imax}$ or $\Delta K_I$ on a log-log scale
- This chart is then commonly divided into three regions

# da-dN vs K


ultimate fracture
10-4
10-6
10-8
10-10
10-12
10-14
C
m ≈ 2.85
1
1
2
4
8
16
32
64
128
Crack growth rate, da/dN, (m/cycle), log scale
Stress intesity factor range, ΔK, (MPa√m), log scale

# region I

- In Region I crack growth is very slow and/or difficult to measure
- In many cases, da/dN corresponds to the spacing between atoms!
- The point at which the da/dN curve intersects the x-axis (usually with a relatively vertical slope) is called the fatigue threshold
- Typically 3-15 $\text{ksi}\sqrt{\text{in}}$ for steel
- 3-6 $\text{ksi}\sqrt{\text{in}}$ for aluminum

# region II

- Most important region for general engineering analysis
- Once a crack is present, most of the growth and life occurs in Region II
- Generally linear in the log-log scale

# region III

- Unstable crack growth
- Usually neglected (we expect failure before Region III fully develops in actual parts)
- Can be significant for parts where we expect high stress and relatively short life

# crack growth rate curve

- The crack growth rate curve is considered a material property
- The same considerations for thickness apply as with fracture toughness ($K_c$ vs. $K_{Ic}$)
- Is also a function of the load ratio, $R = \sigma_{min}/\sigma_{max}$

# R effects

- While the x-axis can be either $\Delta K$ or $K_{max}$, the shape of the data is the same
- When we look at the effects of load ratio, $R$, the axis causes some differences on the plot
- With $\Delta K$ on the x-axis, increasing $R$ will shift the curve up and to the left, shifting the fatigue threshold and fracture toughness on the graph as well

# R effects

- With $K_{max}$ on the x-axis, increasing $R$ shifts the curve down and to the right, but fatigue threshold and fracture toughness keep same values
- In general, $R$ dependence vanishes for $R > 0.8$ or $R < -0.3$. This effect is known as the band width

# crack growth rate equations

# crack growth rate equations

- There are many crack growth rate equations of varying complexity
- The “best” form to use will depend on design needs

# growth equations

- The important features in curve-fit equations are
    1. Region II curve fit (linear on log-log scale)
    2. Region I curve fit (fatigue threshold)
    3. Region III curve fit (critical stress intensity)
    4. Stress ratio effects
    5. Band width of R-curves

# paris law

- The original
- Fits the linear portion (Region II)
- Does not fit Region I, Region III, or have R-dependence

$$\frac{da}{dN} = C(\Delta K)^n$$

- Note: this assumes the x-axis is $\Delta K$, but $\Delta K = (1 - R)K_{max}$, so we can easily convert

# walker

- Region II is usually all that is needed for engineering, but R-dependence is often an important effect to capture
- Walker modified the Paris law to account for R-dependence

$$\frac{da}{dN} = C[(1-R)^m K_{max}]^n$$

- Gives a good fit for Region II with R-dependence and band width

# forman

- The Forman equation was developed to capture the effects of Region II and Region III
- Also includes the effects of $R$, but does not control the band width of R effects

$$\frac{da}{dN} = \frac{C\left[(1-R)K_{max}\right]^n}{(1-R)K_c-(1-R)K_{max}}$$

# modified forman

- The Forman equation can be modified to include the effect of band width

$$\frac{da}{dN} = \frac{C[(1-R)^m K_{max}]^n}{[(1-R)^m K_c - (1-R)^m K_{max}]^L}$$

# collipriest

- The Collipriest equation fits Regions I, II and III, but has no R-dependence

$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1}\left[\frac{\log\left(\frac{K_{max}^2}{K_o K_c}\right)}{\log(K_c/K_o)}\right]$$

# modified collipriest

- Following the same methods as before, we can modify the Collipriest equation for R-dependence and band width control

$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1}\left[\frac{\log\left(\frac{(1-R)^m K_{max}^2}{K_o K_c}\right)}{\log(K_c/K_o)}\right]$$

- For a cleaner graph, experimental data at different R-values is sometimes plotted vs. $K_{eff}$ $K_{eff} = (1 - R)^m K_{max}$

# nasgrow growth rate equation

- A very complicated curve fit is provided in the NASGROW growth rate equation

$$\frac{da}{dN} = C\left[\frac{1-f}{1-R}\Delta K\right]^n \frac{\left[1-\frac{\Delta K_{th}}{\Delta K}\right]}{\left[1-\frac{K_{max}}{K_{crit}}\right]}$$

- The curve fit parameters can be found in p. 307 of your text (or the NASGROLW/AFGROW documentation)

# boeing-walker growth rate equation

- The Boeing-Walker growth equation is given as (for $R \geq 0$ )

$$\frac{da}{dN} = 10^{-4}\left(\frac{1}{mT}\right)^{p}\left[K_{max}(1-R)^{q}\right]^{p}$$

# conversion of constants

- Much of the data available to us is from Boeing, and given in terms of the Boeing-Walker equation
- We can re-write some other equations to more easily convert parameters between the various equations
- Walker-Boeing:

$$\frac{da}{dN} = 10^{-4}\left(\frac{1}{mT}\right)^p \left[\Delta K(1-R)^{q-1}\right]^p$$

- Walker-AFGROW:

$$\frac{da}{dN} = C_w\left[\Delta K(1-R)^{m-1}\right]^{n_w}$$

- Forman:

$$\frac{da}{dN} = \frac{C_F}{(1-R)K_c - \Delta K}(\Delta K)^{n_f}$$

# conversion of constants

| Walker-Boeing | Walker-AFGROW | Forman |
|---|---|---|
| $10^{-4}\left(\frac{1}{mT}\right)^p$ | $C_w = 10^{-4}\left(\frac{1}{mT}\right)^p$ | $C_F = (K_c - 1)10^{-4}\left(\frac{1}{mT}\right)^p$ |
| q | $m = q$ | |
| p | $n_w = p$ | $n_f = p$ |

# paris example

- A wide center-cracked panel with $C = 6.75 \times 10^{-10}$ and $n = 3.89$ (with units in ksi and inches)
- If the crack is initially 1 inch long, find the crack length after 50,000 cycles of 15 ksi loading
- What if the load cycles varied from 5 ksi to 15 ksi? ($m = 0.6$)

# factors affecting crack propagation

# factors affecting crack propagation

- thickness
- stress ratio
- temperature
- environment
- frequency
- crack orientation
- manufacturer
- heat treatment

# thickness

- We already discussed the effects of thickness on fracture toughness
- The same effects are important in crack propagation
- In thin (plane stress) plates, cracks can be treated as through cracks
- In thick plates (plain strain), we generally need to consider the crack shape

# thickness

- Cyclic life is primarily a function of $K_i/K_c$ where $K_i$ is the stress intensity factor in the first cycle
- Other experiments indicate a relationship between $\frac{d(a/Q)}{dN}$ and $K_{max}$
- $Q$ is a shape parameter for elliptical flaws

## temperature

- In general (for most aluminum alloys) cracks propagate more slowly with a decrease in temperature
- This trend is exactly opposite the trend for $K_c$
- The effect varies in different materials
- Most materials benefit from slightly lower temperatures, but as temperatures are further decreased the crack growth rate increases again

# temperature

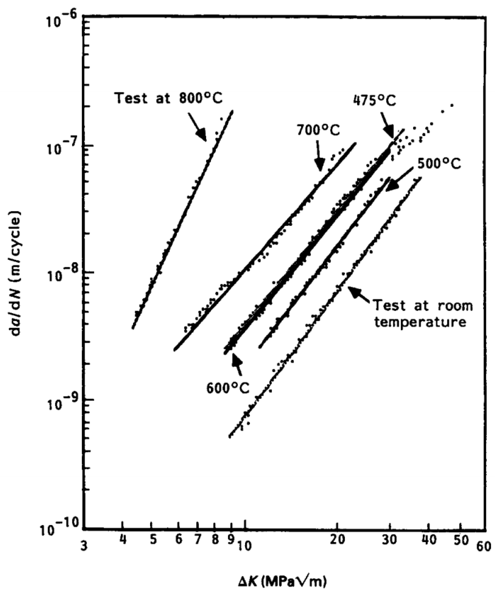



Fig. 2 Mid-range fatigue crack-growth rates with alternating stress intensity factor for 18%Cr–Nb ferritic stainless steel at room temperature, 475, 500, 600, 700 and 800°C; $R = 0.1$, $f = 15$ Hz

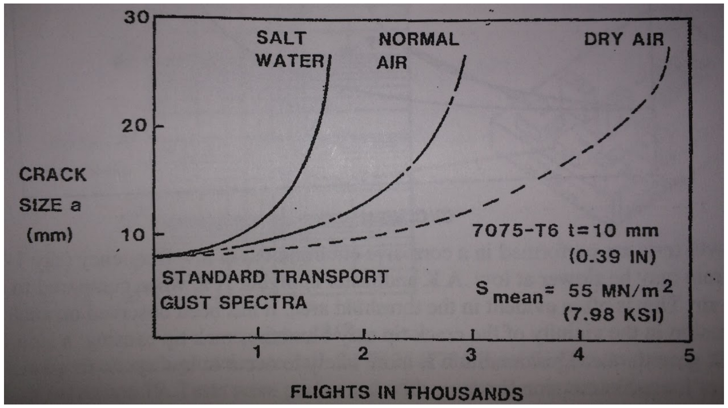
# temperature

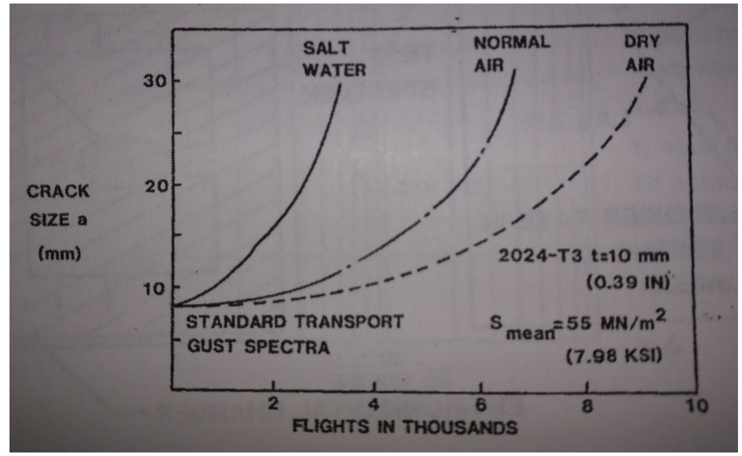
- In general, temperature effects can not be predicted well
- Instead, materials should be tested at a range of temperatures to establish a range of operating temperatures with corresponding crack growth data

# environment

- There are many conditions in the environment that can affect crack growth
- Moisture greatly increases the crack growth rate
- Salt water increases crack growth rate even further
- These effects have varying strength depending on the material used

# environment



# environment

- Further, the shape of the applied load curve has a significant effect when combined with adverse environments
- Crack growth is faster when the load increases slowly and decreases rapidly
- Crack growth is slower when the load increases rapidly and decreases slowly

# environment

- When the environment is corrosive, the test frequency is of particular importance
- At low frequencies, a corrosive environment increases the threshold, $K^{th}$
- However in Region II, crack growth is faster
- This effect can be explained by the corrosive environment blunting the crack tip

# frequency

- There is conflicting information about the effect of frequency in the absence of a corrosive environment
- Some experiments have found a frequency dependence, while others have not
- Many claim that the frequency dependence is due to small amounts of water in air during frequency dependence experiment

# crack orientation

- For rolled plates, a crack will generally propagate faster parallel to the rolling direction
- In many materials, however, the difference between orientations is not significant when compared to scatter, and it is often neglected
- Some materials behave very differently with different crack orientations (i.e. the slope of the paris law curve is different), so care should be taken based on the material used

## manufacturer

- Different manufacturers of the same material can produce different crack growth rates
- Some reasons for this may be
  - Slight variation in composition
  - Site cleanliness (inclusions)
  - Heat treatment/cold rolling variations

# heat and surface treatments

- Different heat and surface treatments are often applied
- They provide various benefits (corrosion resistance, residual stress, residual stress relief)
- But they will also affect the crack growth rate

# numerical algorithm

# numerical algorith

- While the Paris Law can be integrated directly (for simple load cases), many of the other formulas cannot
- A simple numerical algorithm for determining incremental crack growth is

$$a_{i+1} = a_i + \left(\frac{da}{dN}\right)_i (\Delta N)_i$$

- This method is quite tedious by hand (need many $a_i$ values for this to be accurate)
- But is simple to do in Excel, MATLAB, Python, or many other codes
- For most accurate results, use $\Delta N = 1$, but this is often unnecessary
- When trying to use large $\Delta N$, check convergence by using larger and smaller $\Delta$**$N$ values

# boeing-walker example

- Use the Boeing-Walker equation to find the crack length after 20000 cycles of 15 ksi load on a large, center-cracked sheet of bare 2024-T3 in dry air, with an initial crack of 0.5"
- Use the numerical algorithm with $\Delta N$ = 1000

## convergence example

- compare the results from the previous example with $\Delta N = 10, 100$, 10000 and direct integration

# variable load cases

- In practice variable loads are often seen
- The most basic way to handle these is to simply calculate the crack length after each block of loading
- We will discuss an alternate method, which is more convenient for flight “blocks” next class
- We will also discuss “retardation” models next class

# variable load example

- For the same material as above (2024-T3, center-cracked, dry air), consider 20000 cycles with 15 ksi load followed by 10000 cycles of 5 - 20 ksi.