AE 737: Mechanics of Damage Tolerance

Lecture 11 - Multiple Site Damage, Mixed-Mode Fracture

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schedule

- 24 Feb - Multiple Site Damage, Mixed-Mode, HW5 Due, HW 4 Self-grade due
- 1 Mar - Exam 1 Review
- 3 Mar - Exam 1
- 8 Mar - Stress-based Fatigue

outline

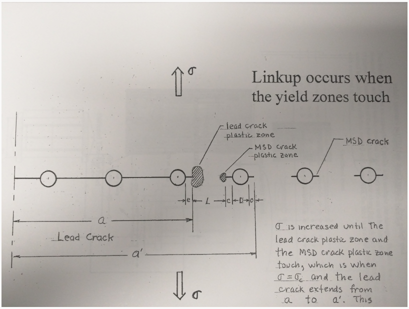
- multiple site damage
- mixed mode fracture

multiple site damage

multiple site damage

- Often damage can accumulate among multiple sources
- This is very common when there are a series of holes, each can develop cracks with a potential to link up
- "link up" occurs when the plastic zones between two adjacent cracks touch

linkup



linkup equation

- We know that

$$R_p = \frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2$$

- Where we define the stress intensity factors at a and L as

$$K_{Ia} = \sigma\sqrt{\pi a}\beta_a$$

$$K_{Il} = \sigma\sqrt{\pi l}\beta_l$$

linkup equation

- Since fast cracking occurs when $R_p + r_p = L$ we solve for the condition where $R_p + r_p < L$

$$\frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2 + \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2 < L$$

$$\frac{1}{2\pi\sigma_{YS}^2}\left[K_{Ia}^2 + K_{Il}^2\right] < L$$

linkup equation

$$\frac{1}{2\pi\sigma_{YS}^2}\left[\sigma^2\pi a\beta_a^2 + \sigma^2\pi l\beta_l^2\right] < L$$

$$\frac{\sigma^2}{2\sigma_{YS}^2}\left[a\beta_a^2 + l\beta_l^2\right] < L$$

$$\sigma_c = \sigma_{YS}\sqrt{\frac{2L}{a\beta_a^2 + l\beta_l^2}}$$

example

worked link-up example here[1]

[1] https://colab.research.google.com/drive/11Nt8oT53jTECs1JZO2FFhCpphy1gwXqi?usp=sharing

modified linkup equations

- We see that for a brittle material (with a small plastic zone) we predict no effect of “link-up”
- This does not agree with test data
- Even the 2024 predictions don’t agree well with test data
- WSU used some empirical parameters to modify the linkup equations and better predict residual strength when multiple site damage is present

modified 2024

- For 2024-T3 we use the following procedure
- First find σ_c from the unmodified equation

$$\sigma_{c,mod} = \frac{\sigma_c}{A_1 \ln(L) + A_2}$$

- Where $A_1 = 0.3065$ and $A_2 = 1.3123$ for A-basis yield strength and $A_1 = 0.3054$ and $A_2 = 1.3502$ for B-basis yield strength
- The same equation can also be used for 2524 with $A_1 = 0.1905$, $A_2 = 0.9683$ for A-basis yield and $A_1 = 0.2024$, $A_2 = 1.0719$ for B-basis yield

modified 7075

- A similar modification was made for 7075

$$\sigma_{c,mod} = \frac{\sigma_c}{B_1 + B_2 L}$$

- Where $B_1 = 1.377$, $B_2 = 1.042$ for A-basis yield strength and $B_1 = 1.417$, $B_2 = 1.073$ for B-basis yield strength

modified 7075

- However, since general fracture had a closer prediction to real failure than the linkup equation, it may make more sense to modify the brittle fracture equation

$$\sigma_{c,mod} = \frac{K_c}{\sqrt{\pi a}(0.856 - 0.496 \ln(L))}$$

mixed mode fracture

mixed-mode fracture

- Most cracks are primarily Mode I, but sometimes Mode II can also have an effect
- We can look at the combined stress field for Mode I and Mode II
- Recall the stress field near the crack tip

stress field

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

mixed-mode fracture

- For Mode II we have

$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$
$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

polar coordinates

- In mixed-mode fracture problems, the crack will generally propagate in a different direction from the initial crack plane
- It is more convenient to handle this scenario in Polar Coordinates
- We can convert stress from Cartesian coordinates to Polar Coordinates using the stress transformation equations

polar coordinates

$$\sigma_r = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$\sigma_\theta = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta$$
$$\tau_{r\theta} = -\sigma_x \sin\theta\cos\theta + \sigma_y \sin\theta\cos\theta + \tau_{xy}(\cos^\theta - \sin^2\theta)$$

combined stress field

- When we convert the stress fields from Mode I and Mode II into polar coordinates and combine them, we find

$$
\begin{aligned}
\sigma_r &= \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right)\\
\sigma_\theta &= \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right)\\
\tau_{r\theta} &= \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right)
\end{aligned}
$$

max circumferential stress

- The Maximum Circumferential Stress Criterion assumes that a crack will propagate in the principal direction
- In this direction, the shear stress is 0
- The fracture toughness is determined by the Mode I fracture toughness of the material

max circumferential stress

- **Note:** In this discussion, we will use K_{IC} to differentiate Mode I fracture toughness from Mode II fracture toughness. This does NOT necessarily mean we are referring to plane strain fracture toughness
- Thus fracture begins when

$$\sigma_\theta(\theta_P) = \sigma_\theta(\theta = 0, K_{II} = 0, K_I = K_{Ic}) = \frac{K_{IC}}{\sqrt{2\pi r}}$$

max circumferential stress

- Following the above assumptions, we can solve these equations to find θ_p
- Note: This assumes that we know both K_I and K_{II}, in this class we have not discussed any Mode II stress intensity factors, so they will be given.

max circumferential stress

- In this case it simplifies to

$$K_I \sin \theta_p + K_{II}(3 \cos \theta_p - 1) = 0$$

- and

$$4K_{IC} = K_I \left(3 \cos \frac{\theta}{2} + \cos \frac{3\theta}{2}\right) - 3K_{II} \left(\sin \frac{\theta}{2} + \sin \frac{3\theta}{2}\right)$$

maximum circumferential stress criterion

- The general form for a Mode II stress intensity factor is

$$K_{II} = \tau \sqrt{\pi a} \beta'$$

example

Assuming $\sigma = 4\tau$, $K_{IC} = 60$ ksi$\sqrt{\text{in}}$, and $2a = 1.5$ in.

Note: Assume $\beta = \beta' = 1$

principal stress

- In the maximum circumferential stress criterion, we found the principal stress direction near the crack tip in polar coordinates
- We can also find the principal direction (if there were no crack) in Cartesian coordinates
- **Note:** This is not mathematically rigorous, but much easier to calculate and sometimes it's close enough

principal stress

- If we make a free body cut along some angle θ we find, from equilibrium

$$
\begin{aligned}
0 &= \sigma_\theta dA - \sigma_x dA \sin^2\theta - \sigma_y dA \cos^2\theta + 2\tau_{xy} dA \cos\theta \sin\theta \\
\sigma_\theta &= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta \\
\frac{\partial \sigma_\theta}{\partial \theta} &= (\sigma_x - \sigma_y) \sin 2\theta_p - 2\tau_{xy} \cos 2\theta_P \\
\tan 2\theta_P &= \frac{2\tau_{xy}}{\sigma_x - \sigma_y}
\end{aligned}
$$

principal stress

- As before, we consider crack propagation purely due to Mode I
- In the principal stress criterion, we find the maximum Mode I stress as a function of the remote applied stress

$$\sigma_{P1} = C\sigma$$

- We then find the remote failure stress by

$$\sigma_c = \frac{K_{IC}}{C\sqrt{\pi a}\beta}$$

example

Assuming $\sigma = 4\tau$, $K_{IC} = 60$ ksi$\sqrt{\text{in}}$, and $2a = 1.5$ in.

Note: Assume $\beta = \beta' = 1$

example

worked mixed-mode fracture example here[2]

[2] https://colab.research.google.com/drive/15vHyABV7Q98UF9ZRZhQoDnn2858nNE0F?usp=sharing