AE 737: Mechanics of Damage Tolerance

Lecture 11 - Multiple Site Damage, Mixed-Mode Fracture

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

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schedule

- 28 Feb - Multiple Site Damage, Mixed-Mode Fracture
- 5 Mar - Exam Review, Homework 5 Due
- 7 Mar - Exam 1
- 11-14 Mar - Spring Break

outline

- stiffeners
- multiple site damage
- mixed mode fracture

stiffeners

stiffened panels

- In aircraft the skin/stringer system provides many benefits (resistance to buckling)
- Stringers also act as stiffeners to resist crack propagation in the skin
- Panels in these configurations are generally very wide relative to expected crack dimensions
- Cracks are generally modeled either as centered between stiffeners or centered under a stiffener
- We need to consider the residual strength of the panel, the stiffener, and the rivets

remote stress in stiffener

- For displacement continuity at the skin-stiffener interface, we know that
$$\left(\frac{PL}{AE}\right)_{Skin} = \left(\frac{PL}{AE}\right)_{Stiffener}$$
- Since L is the same, we find
$$\frac{S}{E} = \frac{S_S}{E_S}$$
- Where the subscript $_S$ indicates stiffener values, we can express the remote stress in the stiffener as
$$S_S = S\frac{E_S}{E}$$

skin

- The critical stress in the skin is determined the same way as it was in the residual strength chapter
- The only exception is that the stiffener contributes to β

$$S_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

stiffener

- The maximum stress in a stiffener will be increased near a crack
- We represent the ratio of maximum force in stiffener to remote force with the Stiffener Load Factor, L

$$
\begin{aligned}
L &= \frac{\text{max force in stiffener}}{\text{remote force applied to stiffener}} \\
&= \frac{S_{S,max} A_S}{S_S A_S} \\
&= \frac{S_{S,max}}{S \frac{E_S}{E}}
\end{aligned}
$$

$$
\begin{aligned}
LS \frac{E_S}{E} &= S_{S,max} = \sigma_{YS} \\
S_C &= \frac{\sigma_{YS} E}{L E_S}
\end{aligned}
$$

rivet

- We can define a similar rivet load factor to relate maximum stress in the rivet to remote stress in the skin

$$L_R = \frac{\tau_{max} A_R}{Sbt}$$

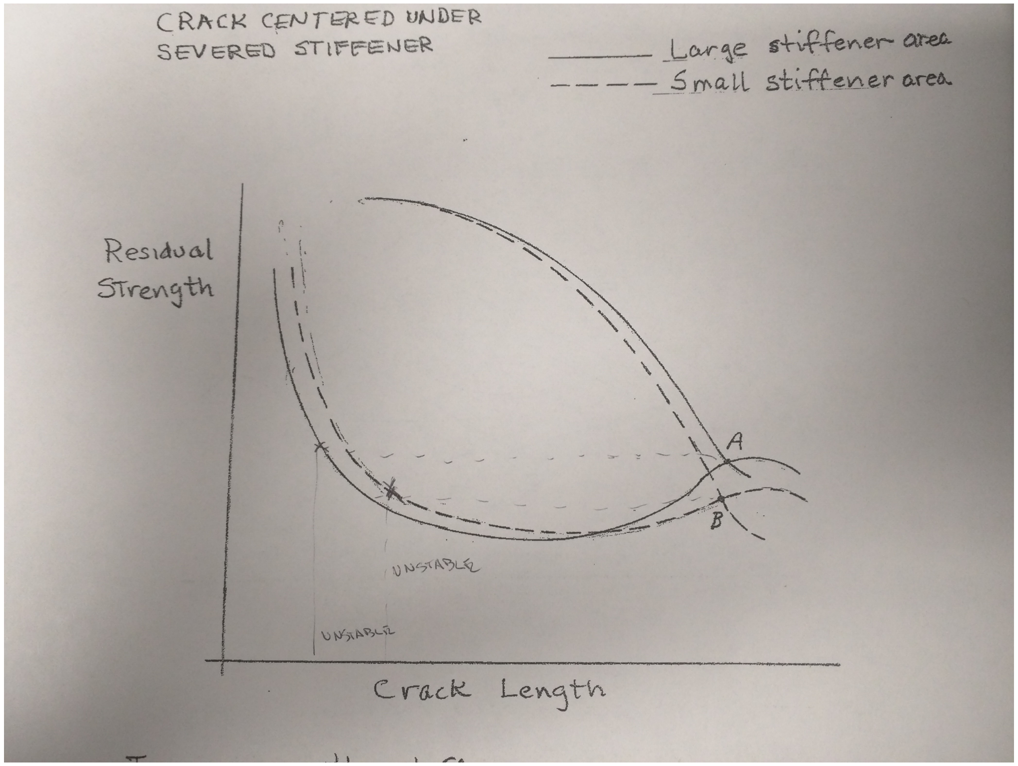
$$L_R = \frac{\tau_{YS} A_R}{Sbt}$$

$$S_c = \frac{\tau_{YS} A_R}{L_R bt}$$

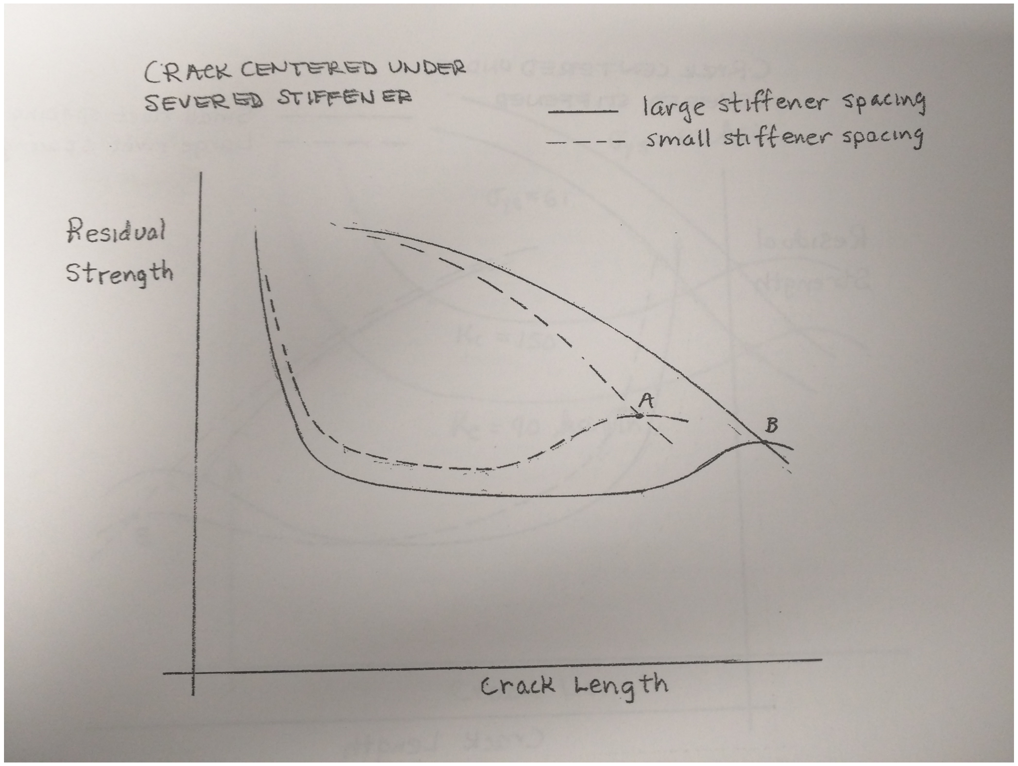
failure in stiffener

- Swift considers the difference in stress at different points in the stiffener
- Instead of one general load factor (L), he uses $SCFO$ and $SCFI$
- We can find the critical value of remote stress at the outer flange as
  $$\sigma_C = \frac{\sigma_U}{SCFO}$$
- And similarly at the inner flange
  $$\sigma_C = \frac{\sigma_U}{SCFI}$$
- Swift's parametric study did not consider rivet failure

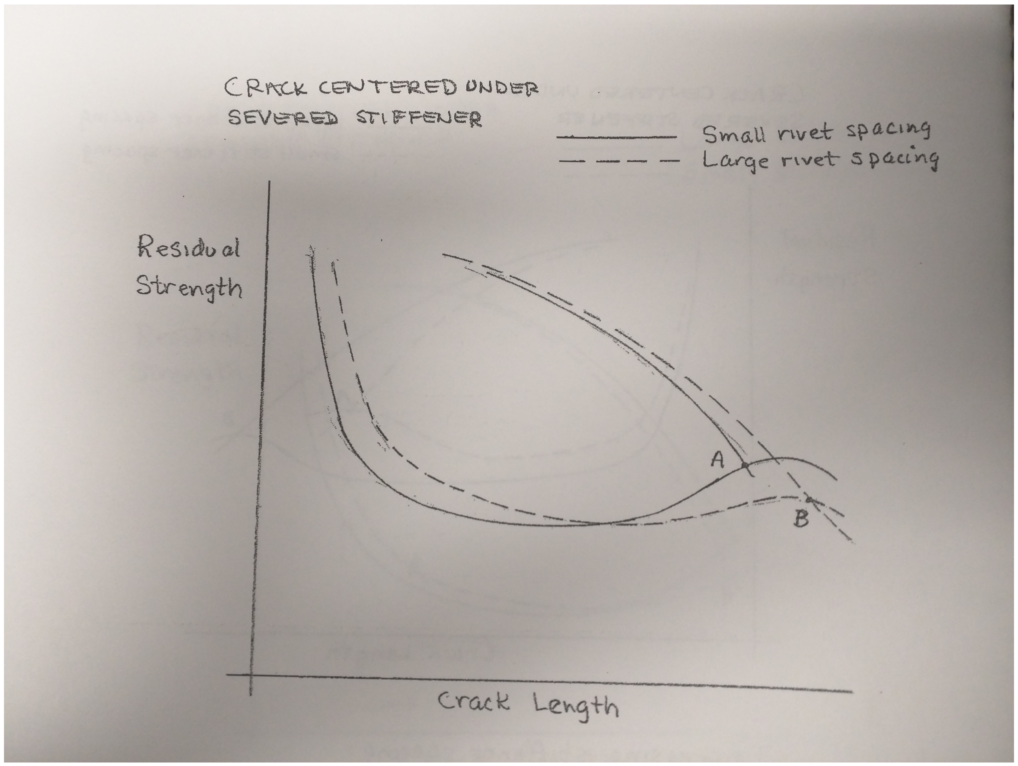
stiffener area



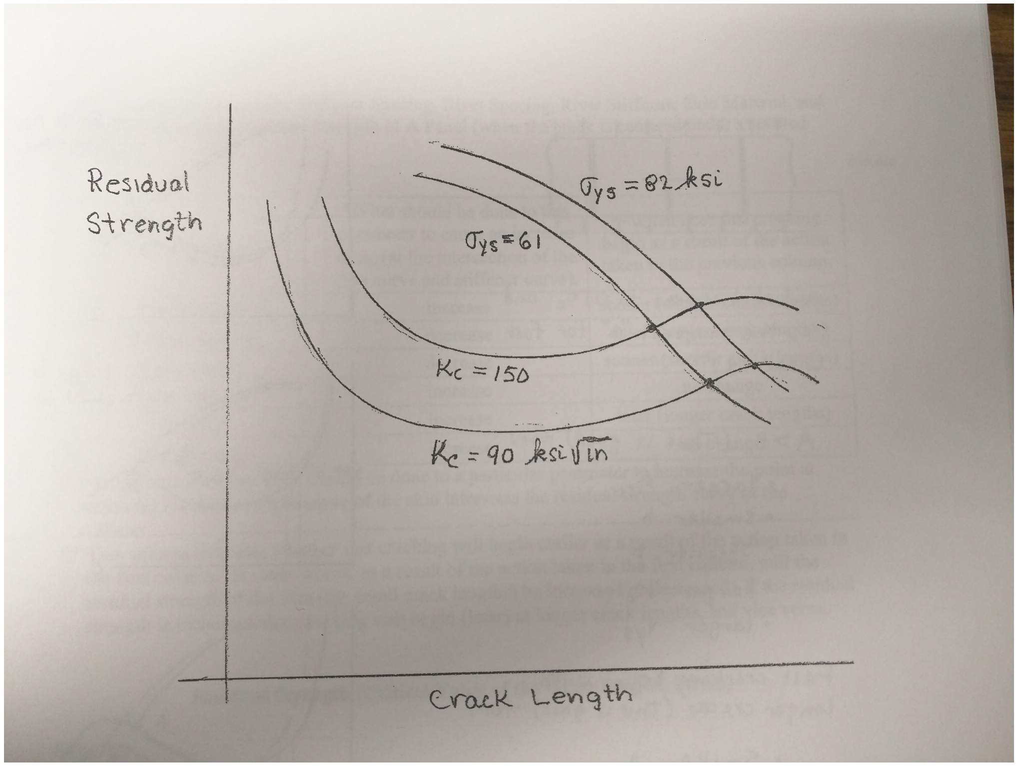
stiffener spacing



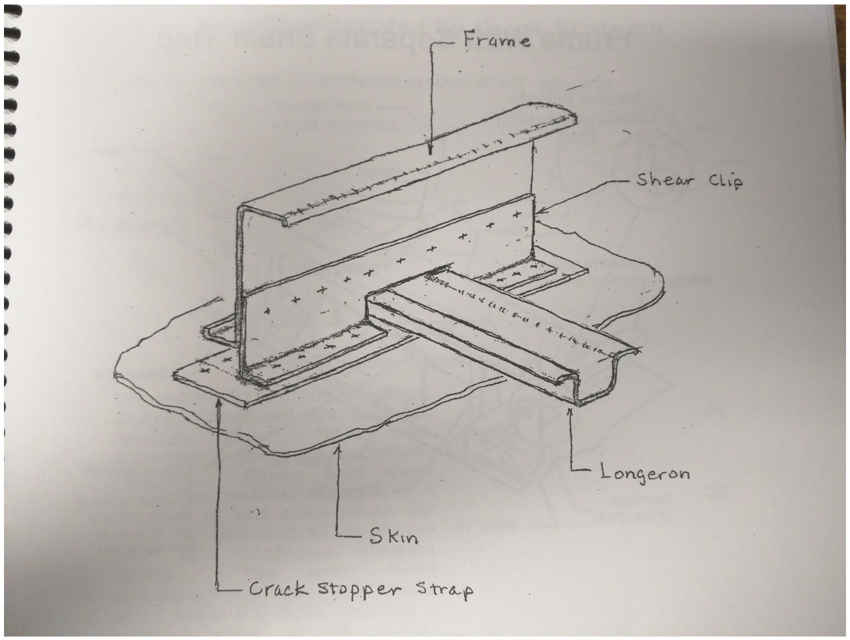
rivet spacing



strength and toughness increase



crack stopper



optimal crack stopper

- Swift found that the ideal crack stopper has a cross-sectional area approximately equal to 1/4 the stiffener area
- The ideal material was titanium (as opposed to steel or aluminum).
- Aluminum did not transfer enough load to the stiffeners, steel transferred too much

stiffeners

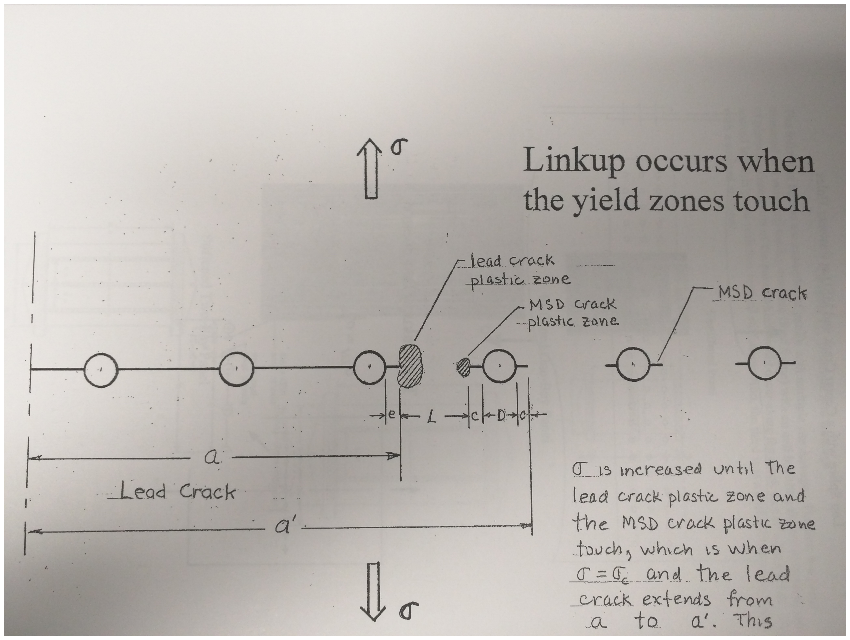
- Stiffener charts were made using physical crack length (not effective crack length)
- As cracks get long, the relative difference between a and a_{eff} is minor
- An active field of research is to integrate failsafes and crack stoppers in one part
- Manufacturing methods for composites are very different than for metals and damage tolerant designs need to adjust

multiple site damage

multiple site damage

- Often damage can accumulate among multiple sources
- This is very common when there are a series of holes, each can develop cracks with a potential to link up
- "link up" occurs when the plastic zones between two adjacent cracks touch

linkup


Linkup occurs when the yield zones touch
σ
lead crack plastic zone
MSD crack plastic zone
MSD crack
e
L
c
D
c
a
Lead Crack
a'
σ is increased until the lead crack plastic zone and the MSD crack plastic zone touch, which is when σ = σc and the lead crack extends from a to a'. This
σ

linkup equation

- We know that

$$R_p = \frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2$$

- Where we define the stress intensity factors at a and L as

$$K_{Ia} = \sigma\sqrt{\pi a}\beta_a$$

$$K_{Il} = \sigma\sqrt{\pi l}\beta_l$$

linkup equation

- Since fast cracking occurs when R_p+r_p=L, we solve for the condition where R_p+r_p<L

$$\frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2 + \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2 < L$$

$$\frac{1}{2\pi\sigma_{YS}^2}\left[K_{Ia}^2 + K_{Il}^2\right] < L$$

linkup equation

$$\frac{1}{2\pi\sigma_{YS}^2}\left[\sigma^2\pi a\beta_a^2 + \sigma^2\pi l\beta_l^2\right] < L$$

$$\frac{\sigma^2}{2\sigma_{YS}^2}\left[a\beta_a^2 + l\beta_l^2\right] < L$$

$$\sigma_c = \sigma_{YS}\sqrt{\frac{2L}{a\beta_a^2 + l\beta_l^2}}$$

example

worked link-up example **here**

modfied linkup equations

- We see that for a brittle material (with a small plastic zone) we predict no effect of "link-up"
- This does not agree with test data
- Even the 2024 predictions don't agree well with test data
- WSU used some empirical parameters to modify the linkup equations and better predict residual strength when multiple site damage is present

modified 2024

- For 2024-T3 we use the following procedure
- First find σ_c from the unmodified equation

$$\sigma_{c,mod} = \frac{\sigma_c}{A_1 \ln(L) + A_2}$$

- Where A_1 = 0.3065 and A_2 = 1.3123 for A-basis yield strength and A_1 = 0.3054 and A_2 = 1.3502 for B-basis yield strength
- The same equation can also be used for 2524 with A_1 = 0.1905, A_2 = 0.9683 for A-basis yield and A_1 = 0.2024, A_2 = 1.0719 for B-basis yield

modified 7075

- A similar modification was made for 7075
  $$\sigma_{c,mod} = \frac{\sigma_c}{B_1 + B_2 L}$$
- Where B_1 = 1.377, B_2 = 1.042 for A-basis yield strength and B_1 = 1.417, B_2 = 1.073 for B-basis yield strength

modified 7075

- However, since general fracture had a closer prediction to real failure than the linkup equation, it may make more sense to modify the brittle fracture equation

$$\sigma_{c,mod} = \frac{K_c}{\sqrt{\pi a}(0.856 - 0.496\ln(L))}$$

mixed mode fracture

mixed-mode fracture

- Most cracks are primarily Mode I, but sometimes Mode II can also have an effect
- We can look at the combined stress field for Mode I and Mode II
- Recall the stress field near the crack tip

stress field

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

mixed-mode fracture

- For Mode II we have

$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$
$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

polar coordinates

- In mixed-mode fracture problems, the crack will generally propagate in a different direction from the initial crack plane
- It is more convenient to handle this scenario in Polar Coordinates
- We can convert stress from Cartesian coordinates to Polar Coordinates using the stress transformation equations

polar coordinates

$$
\begin{aligned}
\sigma_r &= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \\
\sigma_\theta &= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta \\
\tau_{r\theta} &= -\sigma_x \sin\theta\cos\theta + \sigma_y \sin\theta\cos\theta + \tau_{xy}(\cos^\theta - \sin^2\theta)
\end{aligned}
$$

combined stress field

- When we convert the stress fields from Mode I and Mode II into polar coordinates and combine them, we find

$$\begin{aligned}
\sigma_r &= \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right) \\
\sigma_\theta &= \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right) \\
\tau_{r\theta} &= \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right)
\end{aligned}$$

max circumferential stress

- The Maximum Circumferential Stress Criterion assumes that a crack will propagate in the principal direction
- In this direction, the shear stress is 0
- The fracture toughness is determined by the Mode I fracture toughness of the material

max circumferential stress

- **Note:** In this discussion, we will use K_{IC} to differentiate Mode I fracture toughness from Mode II fracture toughness. This does NOT necessarily mean we are referring to plane strain fracture toughness
- Thus fracture begins when

$$\sigma_\theta(\theta_P) = \sigma_\theta(\theta = 0, K_{II} = 0, K_I = K_{Ic}) = \frac{K_{IC}}{\sqrt{2\pi r}}$$

max circumferential stress

- Following the above assumptions, we can solve these equations to find θ_p
- Note: This assumes that we know both K_I and K_{II}, in this class we have not discussed any Mode II stress intensity factors, so they will be given.

max circumferential stress

- In this case it simplifies to
  $K_I \sin \theta_p + K_{II}(3 \cos \theta_p - 1) = 0$
- and
  $$4K_{IC} = K_I \left(3 \cos \frac{\theta}{2} + \cos \frac{3\theta}{2}\right) - 3K_{II} \left(\sin \frac{\theta}{2} + \sin \frac{3\theta}{2}\right)$$

maximum circumferential stress criterion

- The general form for a Mode II stress intensity factor is $K_{II} = \tau\sqrt{\pi a}\beta'$

example

Assuming $\sigma = 4\tau$, $K_{IC} = 60 \text{ ksi}\sqrt{\text{in}}$, and $2a$ = 1.5 in.

Note: Assume $\beta = \beta' = 1$

principal stress

- In the maximum circumferential stress criterion, we found the principal direction in polar coordinates
- We can also find the principal direction in Cartesian coordinates

principal stress

- If we make a free body cut along some angle θ we find, from equilibrium

$$\begin{aligned} 0 &= \sigma_\theta dA - \sigma_x dA \sin^2\theta - \sigma_y dA \cos^2\theta + 2\tau_{xy} dA \cos\theta \sin\theta \\ \sigma_\theta &= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta \\ \frac{\partial \sigma_\theta}{\partial\theta} &= (\sigma_x - \sigma_y) \sin 2\theta_p - 2\tau_{xy} \cos 2\theta_P \\ \tan 2\theta_P &= \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \end{aligned}$$

principal stress

- As before, we consider crack propagation purely due to Mode I
- In the principal stress criterion, we find the maximum Mode I stress as a function of the remote applied stress
  $\sigma_{P1} = C\sigma$
- We then find the remote failure stress by
  $$\sigma_c = \frac{K_{IC}}{C\sqrt{\pi a}\beta}$$

example

Assuming $\sigma = 4\tau$, $K_{IC} = 60 \text{ ksi}\sqrt{\text{in}}$, and $2a$ = 1.5 in.

Note: Assume $\beta = \beta' = 1$

example

worked mixed-mode fracture example **here**