

Lecture 10 - Residual Strength

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schedule

- 17 Feb - Residual Strength, HW4 Due, HW 3 Self-grade due
- 22 Feb - Multiple Site Damage, Mixed-Mode
- 24 Feb - Exam 1 Review, HW5 Due, HW 4 Self-grade due
- 1 Mar - Exam 1
- 3 Mar - Stress-based Fatigue

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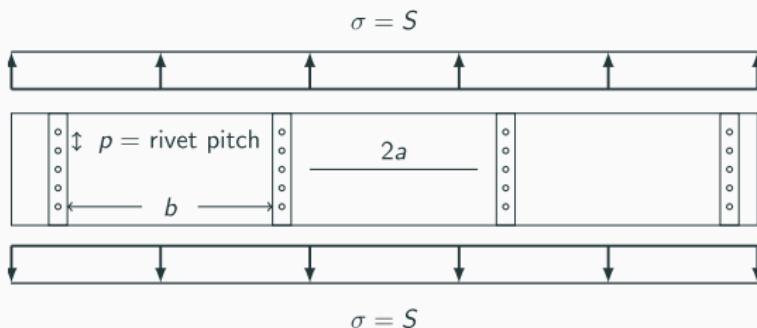
- stiffeners
- severed stiffeners
- crack stoppers

stiffeners

- In aircraft the skin/stringer system provides many benefits (resistance to buckling)
- Stringers also act as stiffeners to resist crack propagation in the skin
- Panels in these configurations are generally very wide relative to expected crack dimensions
- Cracks are generally modeled either as centered between stiffeners or centered under a stiffener
- We need to consider the residual strength of the panel, the stiffener, and the rivets

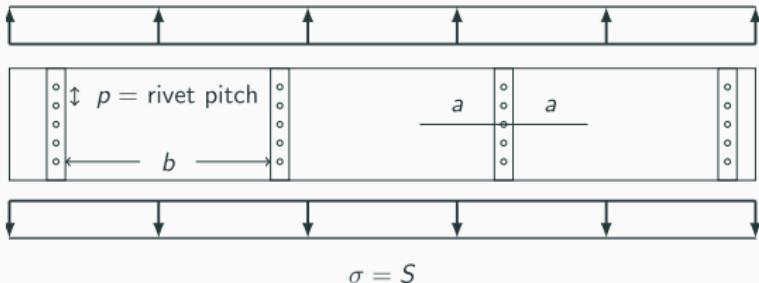
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centered between stiffeners



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$$\sigma = S$$



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remote stress

- For displacement continuity, we know that

$$\left(\frac{PL}{AE}\right)_{Skin} = \left(\frac{PL}{AE}\right)_{Stiffener}$$

- Since L is the same, we find

$$\frac{S}{E} = \frac{S_S}{E_S}$$

- Where the subscript S indicates stiffener values, we can express the remote stress in the stiffener as

$$S_S = S \frac{E_S}{E}$$

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- The critical stress in the skin is determined the same way as it was in the residual strength chapter
- The only exception is that the stiffener contributes to β

$$S_c = \frac{K_c}{\sqrt{\pi a} \beta}$$

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stiffener

- The maximum stress in a stiffener will be increased near a crack
- We represent the ratio of maximum force in stiffener to remote force with the Stiffener Load Factor, L

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$$L = \frac{\text{max force in stiffener}}{\text{remote force applied to stiffener}}$$

$$= \frac{S_{S,\max} A_S}{S_S A_S}$$

$$= \frac{S_{S,\max}}{S \frac{E_S}{E}}$$

$$LS \frac{E_S}{E} = S_{S,\max}$$

$$LS \frac{E_S}{E} = \sigma_{YS}$$

$$S_c = \frac{\sigma_{YS} E}{L E_S}$$

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rivet

- We can define a similar rivet load factor to relate maximum stress in the rivet to remote stress in the skin

$$L_R = \frac{\tau_{max} A_R}{S_{bt}}$$

$$L_R = \frac{\tau_{YS} A_R}{S_{bt}}$$

$$S_c = \frac{\tau_{YS} A_R}{L_R b t}$$

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- CC Poe found that panels could be related by a parameter he defines as μ

$$\mu = \frac{A_s E_s}{A_s E_s + A E}$$

- Where A_s is the cross-sectional area of a stiffener, E_s is stiffener modulus
- A is the skin cross-sectional area (per stiffener) $A = bt$ and E is the modulus of the skin

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- pp 167 - 178 give β , L and L_R for various skin/stiffener configurations
- These values were determined using a finite element model

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- quantitative example (p. 179-180)
- qualitative notes on behavior (p. 181-182)
- worked¹

¹https://colab.research.google.com/drive/1QKnp580F_LD8ak7PALeYiOrBXX31nsP9?usp=sharing

severed stiffeners

- Sometimes the stiffeners fail before the panel
- T. Swift conducted some parametric studies on panels with a severed stiffener
- When the crack is short (and near the severed stiffener) the residual strength is lower due to the broken stiffener
- As the crack nears the next stiffener, residual strength is very similar to a panel with all stiffeners intact

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failure in stiffener

- Swift considers the difference in stress at different points in the stiffener
- Instead of one general load factor (L), he uses $SCFO$ and $SCFI$
- We can find the critical value of remote stress at the outer flange as

$$\sigma_c = \frac{\sigma_u}{SCFO}$$

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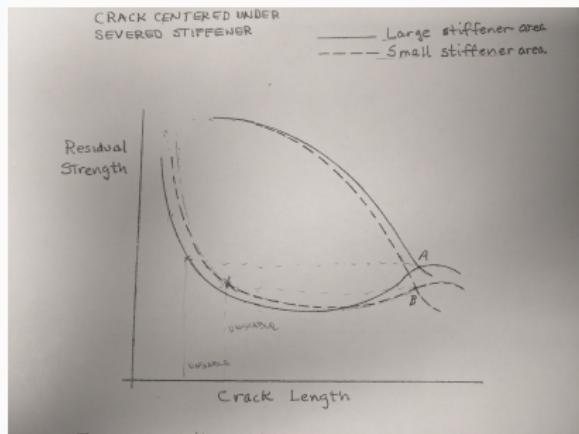
- And similarly at the inner flange

$$\sigma_c = \frac{\sigma_u}{SCFI}$$

- Swift's parametric study did not consider rivet failure

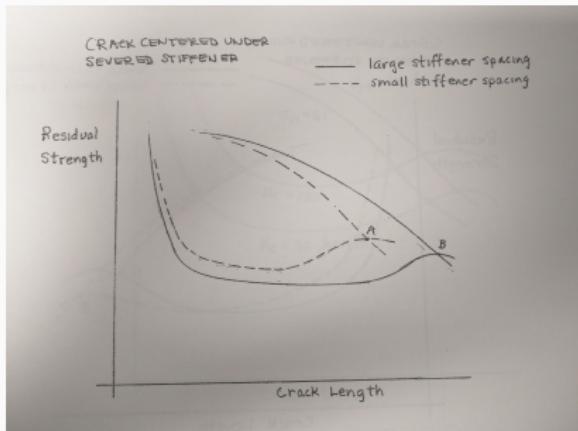
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stiffener area



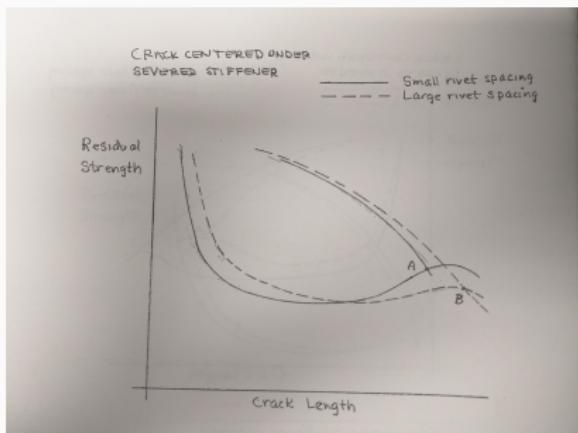
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stiffener spacing

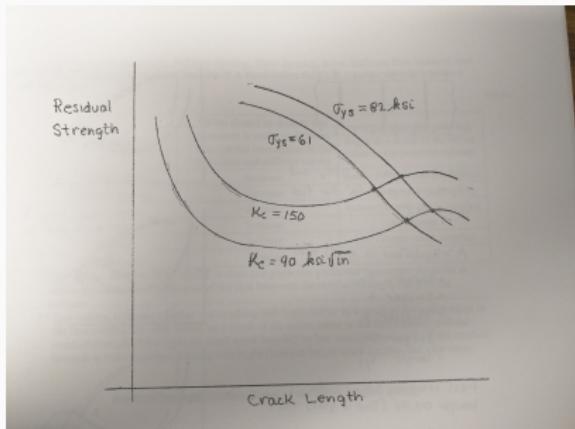


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rivet spacing



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example

- If we consider the case from Swift's data most similar to our previous example:

$$P = 1.0 \text{ in}$$

$$A_{st} = 0.2538 \text{ in}^2$$

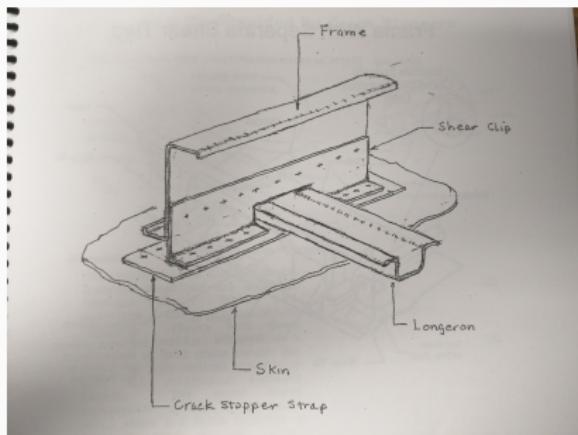
$$b = 10.0 \text{ in}$$

- So we use the tables for Case 10

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crack stoppers

crack stopper



- Swift found that the ideal crack stopper has a cross-sectional area approximately equal to 1/4 the stiffener area
- The ideal material was titanium (as opposed to steel or aluminum).
- Aluminum did not transfer enough load to the stiffeners, steel transferred too much

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example

- Compare cases 1, 3, and 5

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