AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 4

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- When no dimension is given, assume it is large relative to the other dimensions
- Homeworks will generally be due on Tuesday
- Do not submit the codes you used in calculations, but do make it clear which equations you have used
- I will post solutions that you may use to check your calculations
- ”Estimate” vs. ”Determine”
- Fixed problem 7 and 14

SCHEDULE

- 28 Jan - Review, Plastic Zone
- 2 Feb - Plastic Zone, Homework 1 Due, Homework 2 Assigned
- 4 Feb - Plastic Zone
- 9 Feb - Fracture Toughness, Homework 2 Due
- 11 Feb - Fracture Toughness, Homework 3 Assigned

OUTLINE

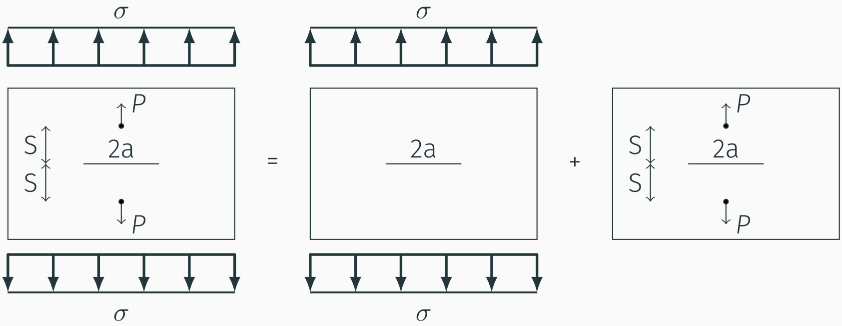
1. superposition
2. compounding
3. curved boundaries
4. plastic zone

SUPERPOSITION

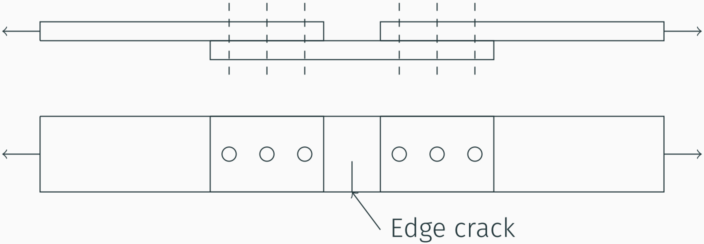
SUPERPOSITION

- Since the stress intensity factor is derived using Linear Elasticity, the principle of superposition applies
- Multiple applied loads can be superposed to find the effective stress intensity factor of the combined loading

SUPERPOSITION



For the splice shown, use superposition and suggest a method to estimate the stress intensity at the corner crack.



COMPOUNDING

- Different types of boundaries create different correction factors to the usual stress intensity factor
- We often use β to indicate the total correction factor
- When multiple boundaries are present, we can combine them into one effective correction factor
- There are two general methods we use to create a compound correction factor

- The first method uses linear superposition, and thus is restricted to cases where the effect of each boundary can be assumed to add linearly
- While in most cases this is not strictly true, it provides a reasonable approximation

$$K_r = \bar{K} + \sum_{i=1}^{N} (K_i - \bar{K}) \tag{4.1}$$

- Where N is the number of boundaries, $\bar{K}$ is the stress intensity factor with no boundaries present and K_i is the stress intensity factor associated with the i^{th} boundary.

- We can rewrite this equation as

$$K_r = \sigma\sqrt{\pi a}\beta_r = \sigma\sqrt{\pi a} + \sum_{i=1}^{N}(\sigma\sqrt{\pi a}\beta_i - \sigma\sqrt{\pi a}) \tag{4.2}$$

- Which leads to an expression for β_r as

$$\beta_r = 1 + \sum_{i=1}^{N}(\beta_i - 1) \tag{4.3}$$

- An alternative empirical method approximates the boundary effect as

$$\beta_r = \beta_1 \beta_2 ... \beta_N \tag{4.4}$$

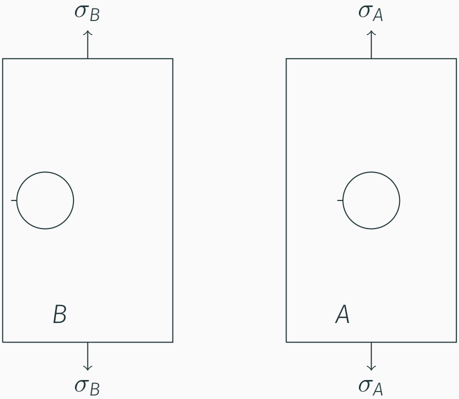
- If there is no interaction between the boundaries, method 1 and method 2 will give the same result

- Can use pp 71-73 for β estimates which are not given in previous equations
- For height effects, use the figure on p 50

CURVED BOUNDARIES

- For short cracks, we can use the *stress concentraction factor* on a curved boundary to determine the stress intensity factor
- Peterson's Stress Concentration Factors - good reference for various stress concentration factors
- pp 82-85 in text
- Supplemental chapter on Blackboard
- The stress intensity factor only gives the maximum stress at the curved boundary, thus the longer the crack is, the farther away from the curved boundary (and maximum stress) it is.

- Suppose we want to determine the stress intensity on a panel, panel B
- We find a similar panel with a known stress intensity factor, panel A
- We adjust the applied load on panel A such that $K_{I,A} = K_{I,B}$
- The magnitude of this load adjustment is determined using the *stress concentration factors* in panels B and A
- Note the notation: K_t for stress concentration factor, K_I for stress intensity factor


σ_B
σ_A
B
A
σ_B
σ_A

- Since A is a fictional panel, we set the applied stress, σ_A such that

$$\sigma_{max,B} = \sigma_{max,A}$$

- Substituting stress concentration factors

$$K_{t,B}\sigma_B = K_{t,A}\sigma_A$$

- Solving for σ_A

$$\sigma_A = \frac{K_{tB}}{K_t A}\sigma_B$$

- Since the crack is short and $\sigma_{max,A} = \sigma_{max,B}$ we can say

$$
\begin{aligned}
K_{I,B} &= K_{I,A} \\
&= \sigma_A \sqrt{\pi c}\,\beta_A \\
&= \frac{K_{t,B}}{K_{t,A}} \sigma_B \sqrt{\pi c}\,\beta_A
\end{aligned}
$$

Example 4 (p. 80)

- As a crack becomes very large, the effect of the curved boundary diminishes
- We find expressions for β_L (long crack) and β_S (short crack)
- We connect β_S to β_L using a straight line from β_S to a tangent intersection with β_L

COMPARING β_s AND β_l

- Many times we include other geometry in the crack length for a long crack, but we do not for a short crack
- To appropriately connect β_S and β_L, they both need to be functions of the same crack length, c
- If we refer to any extra geometry included in the long crack as e, then we have the following expressions

$$K_{I,S} = \sigma\sqrt{\pi c}\beta_S \tag{4.5a}$$

$$K_{I,L} = \sigma\sqrt{\pi(c+e)}\beta_L \tag{4.5b}$$

COMPARING β_S AND β_l

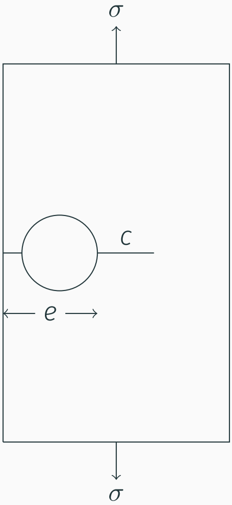
- For a more appropriate comparison, we desire to write $K_{I,L}$ as something multiplied by $\sigma\sqrt{\pi c}$
- If we do some clever factoring, we can write $K_{I,L}$ as

$$K_{I,L} = \sigma\sqrt{\pi c}\sqrt{\frac{c+e}{c}}\beta_L \tag{4.5c}$$

- So for our plot we compare β_S with $\sqrt{\frac{c+e}{c}}\beta_L$

- The equation for a tangent line, $t(x)$, of some function, $f(x)$ at a given point, x_1, is given by

$$t(x) = f(x_1) + f'(x_1)(x - x_1) \tag{4.6}$$


σ
c
e
σ

PLASTIC ZONE

- Previous developments assumed perfectly elastic materials
- Most common materials have some plasticity
- Any stress above the yield stress will undergo plastic deformation (no stress higher than σ_y will be present in the material)

- Plasticity helps retard crack propagation due to residual stresses
- After an overload, elastic regions will contract back to their undeformed shape
- The region which has undergone plastic deformation, however, holds its deformed shape
- This introduces a region of residual compressive stress near the crack tip
- Before the crack can propagate, a stress needs to overcome this residual stress

- We often simplify the full 3D elasticity equations for planar problems
- For very thin panels, we assume that all out-of-plane stresses are 0
- This is called plane stress

$$\sigma_z = \tau_{xz} = \tau_{zy} = 0 \tag{4.7a}$$

$$\epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} \tag{4.7b}$$

$$\epsilon_y = -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} \tag{4.7c}$$

$$\epsilon_z = -\nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} \tag{4.7d}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \tag{4.7e}$$

$$\gamma_{xz} = \gamma_{yz} = 0 \tag{4.7f}$$

- When instead a panel is very thick, we assume that any strains through the thickness are small relative to other strains
- $\epsilon_z = \gamma_{xz} = \gamma_{yz} = 0$
- This is known as plane strain

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \tag{4.8a}$$

$$\epsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \tag{4.8b}$$

$$0 = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E} \tag{4.8c}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \tag{4.8d}$$

$$\gamma_{xz} = \gamma_{yz} = 0 \tag{4.8e}$$

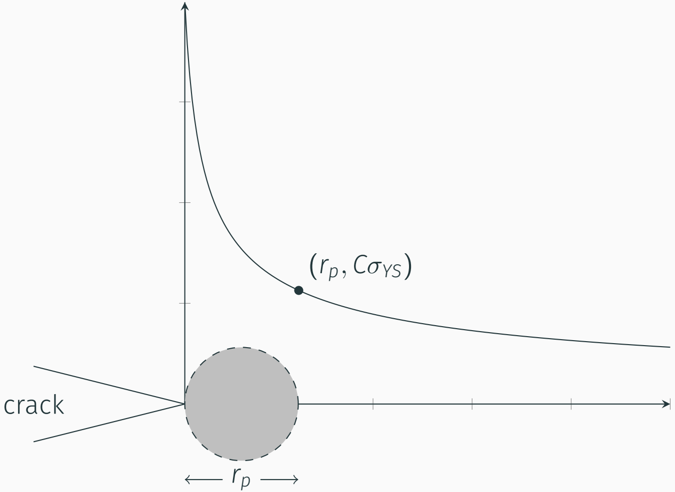
- If we recall the equation for opening stress (σ_y) near the crack tip

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{1.2}$$

- In the plane of the crack, when $\theta = 0$ we find

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}$$

IRWIN'S FIRST APPROXIMATION



IRWIN'S FIRST APPROXIMATION

- We use C "Plastic Constraint Factor" to convert between Plane Strain and Plane Stress solutions
- The plastic zone size can now be approximated by solving the equation $\sigma_{yy}(r = r_p) = C\sigma_{YS}$

$$\sigma_{yy}(r = r_p) = C\sigma_{YS} \tag{4.9a}$$

$$\frac{K_I}{\sqrt{2\pi r_p}} = C\sigma_{YS} \tag{4.9b}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{C\sigma_{YS}}\right)^2 \tag{4.9c}$$

IRWIN'S FIRST APPROXIMATION

- For plane stress (thin panels) we let $C = 1$ and find r_p as

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{4.10}$$

- And for plane strain (thick panels) we let $C = \sqrt{3}$ and find

$$r_p = \frac{1}{6\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{4.11}$$

- For panels which lie between plane strain and plane stress states, we use the following expression to estimate the plastic zone size

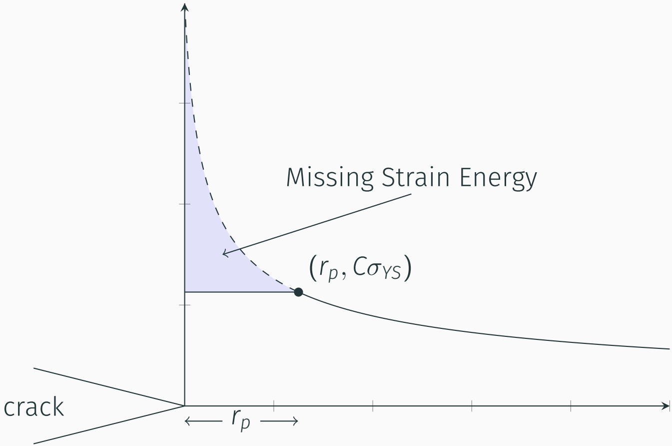
$$r_p = \frac{1}{I\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{4.12}$$

- Where I is defined as

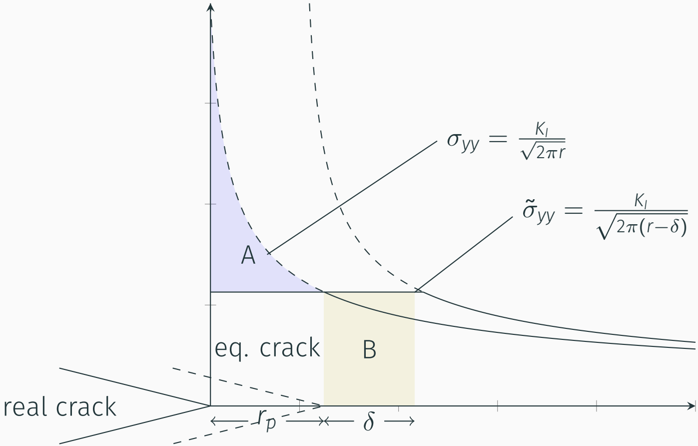
$$I = 6.7 - \frac{1.5}{t}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{4.13}$$

- And $2 \leq I \leq 6$

- If our material is perfectly elastic-plastic, no stresses above $C\sigma_{YS}$ will exist in the material
- This ignores the strain energy (represented by the area under the curve) in the plastic zone


Missing Strain Energy
$(r_p, C\sigma_{YS})$
crack
r_p

- To account for the additional strain energy, Irwin considered a plastic zone size increased by some δ
- He also needed to adjust the stress function, and considered an equivalent crack tip in these calculations


$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$
$\tilde{\sigma}_{yy} = \frac{K_I}{\sqrt{2\pi(r-\delta)}}$
A
B
eq. crack
real crack
r_p
δ

- We need $A = B$, so we set them equivalent and solve for δ.

$$A = \int_0^{r_p} \sigma_{yy} dr - r_p \sigma_{YS} \tag{4.14a}$$
$$= \int_0^{r_p} \frac{K_I}{\sqrt{2\pi r}} dr - r_p \sigma_{YS} \tag{4.14b}$$
$$= \frac{K_I}{\sqrt{2\pi}} \int_0^{r_p} r^{-1/2} dr - r_p \sigma_{YS} \tag{4.14c}$$
$$= \frac{2K_I\sqrt{r_p}}{\sqrt{2\pi}} - r_p \sigma_{YS} \tag{4.14d}$$

- We have already found r_p as

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{4.14e}$$

- If we solve this for K_I we find

$$K_I = \sqrt{2\pi r_p}\sigma_{YS} \tag{4.14f}$$

- We can now substitute back into the strain energy of A

$$A = \frac{2\sqrt{2\pi r_p}\sigma_{YS}\sqrt{r_p}}{\sqrt{2\pi}} - r_p\sigma_{YS} \tag{4.14g}$$

$$= 2\sigma_{YS} r_p - r_p\sigma_{YS} \tag{4.14h}$$

$$= r_p\sigma_{YS} \tag{4.14i}$$

- B is given simply as $B = \delta\sigma_{YS}$, so we equate A and B to find δ

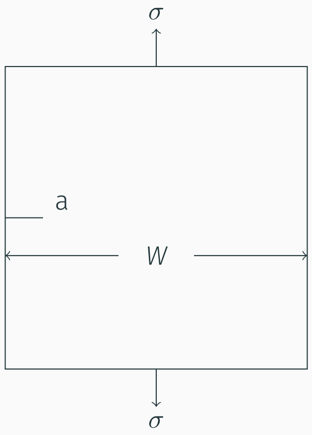
$$A = B \tag{4.14j}$$

$$r_p\sigma_{YS} = \delta\sigma_{YS} \tag{4.14k}$$

$$r_p = \delta \tag{4.14l}$$

- This means the plastic zone size is simply $2r_p$
- However, it also means that the effective crack length is $a + r_p$
- Since r_p depends on K_I, we must iterate a bit to find the ”real” r_p and K_I

EXAMPLE



EQUATIONS

$$\beta = \left[1.122 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.71\left(\frac{a}{W}\right)^3 + 30.82\left(\frac{a}{W}\right)^4\right] \tag{2.4a}$$

$$I = 6.7 - \frac{1.5}{t}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{4.13}$$

$$r_p = \frac{1}{I\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{4.12}$$