AE 737: Mechanics of Damage Tolerance

Lecture 9 - Residual Strength

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schedule

- 18 Feb - Residual Strength
- 20 Feb - Residual Strength, Homework 4 Due
- 25 Feb - Multiple Site Damage
- 27 Mar - Mixed-Mode Fracture, Homework 5 Due

outline

- residual strength
- fedderson approach
- proof testing
- residual strength review
- stiffeners

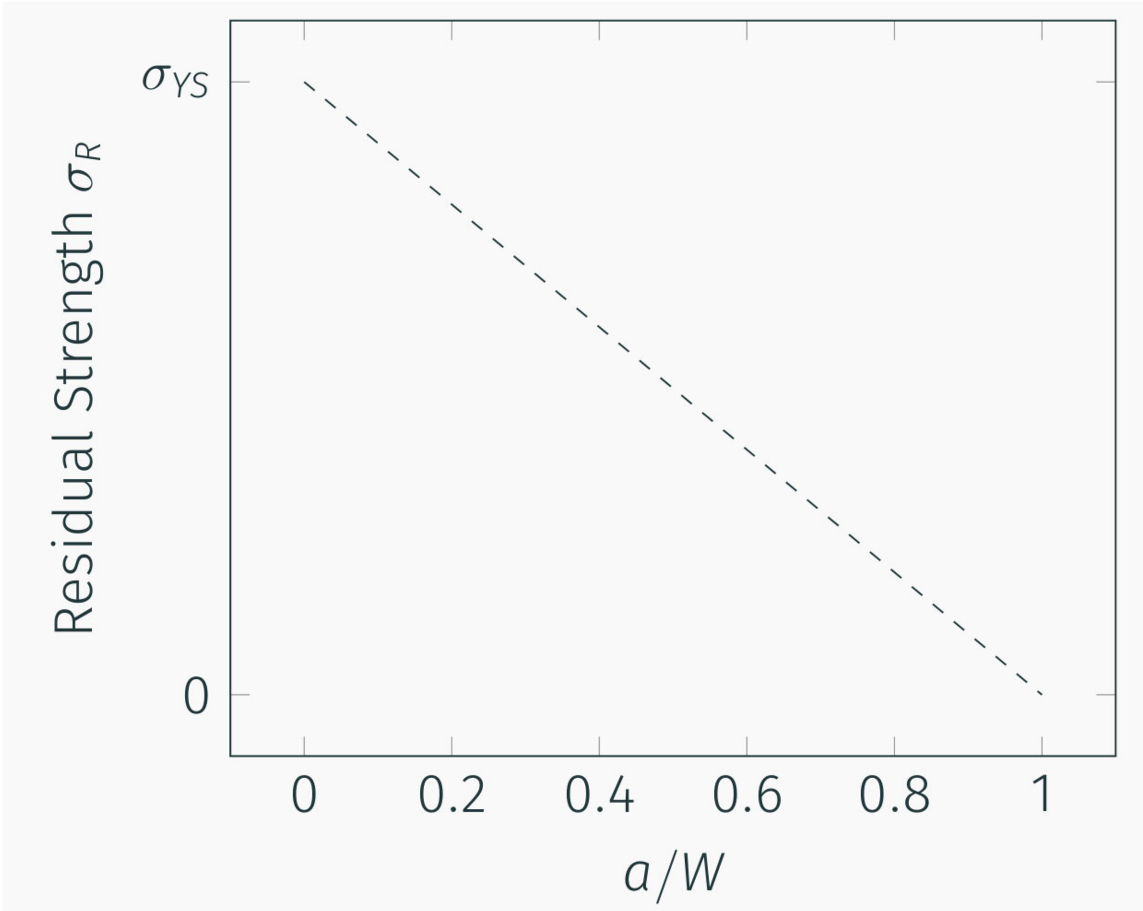
residual strength

residual strength

- As the crack grows, the area of the sample decreases, increasing the net section stress
- The residual strength, σ_R is given in terms of the gross area, so as the crack grows the residual strength due to yield decreases
- We can relate the net-section stress to σ_R by

$$\sigma_R = \sigma_{YS}\frac{A_{net}}{A_{gross}}$$

residual strength

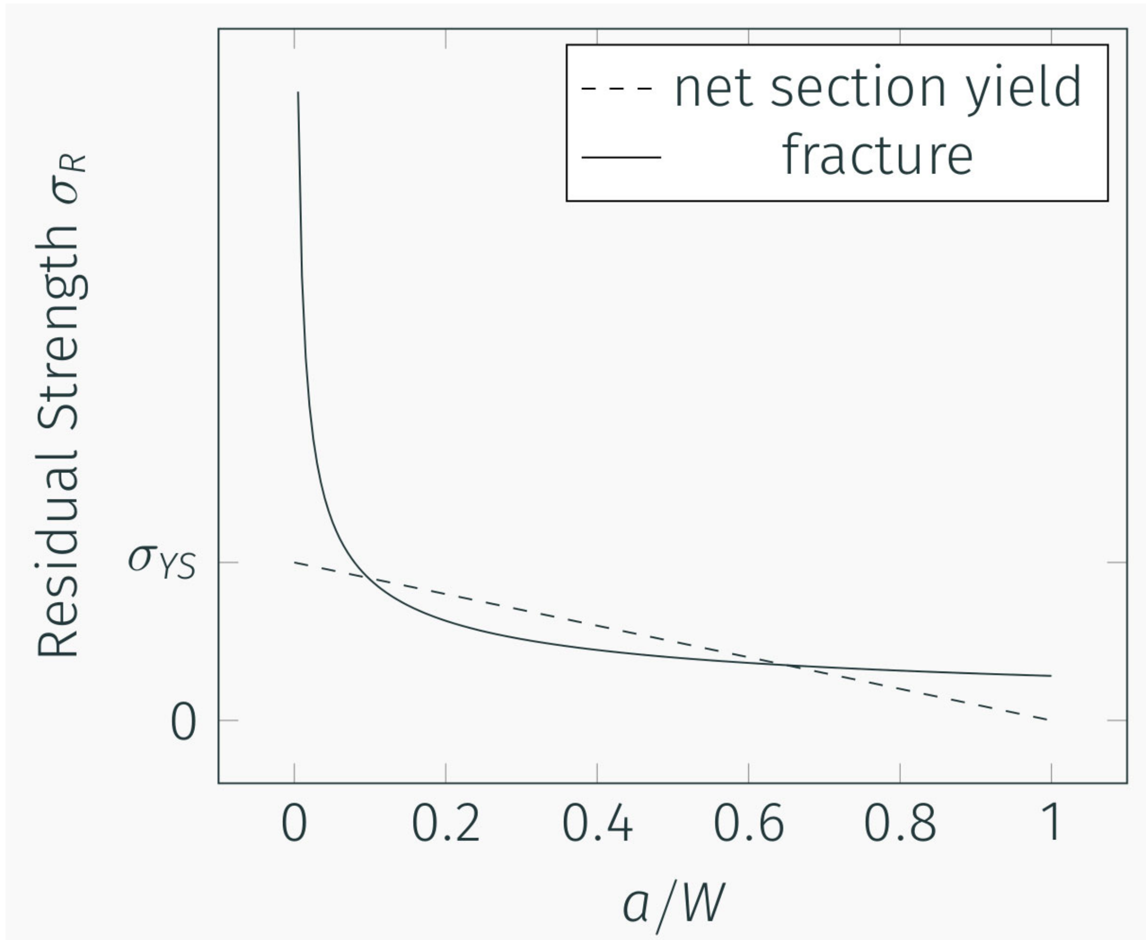


residual strength

- For brittle fracture to occur, we need to satisfy the condition

$$\sigma_R = \sigma_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

residual strength



residual strength

- Within the same family of materials (i.e. Aluminum), there is generally a trade-off between yield stress and fracture toughness
- As we increase the yield strength, we decrease the fracture toughness (and vice versa)
- Consider a comparison of the following aluminum alloys
    1. 7178-T6, $K_C = 43 \text{ ksi}\sqrt{\text{in.}}$, $\sigma_{YS} = 74 \text{ ksi}$
    2. 7075-T6, $K_C = 68 \text{ ksi}\sqrt{\text{in.}}$, $\sigma_{YS} = 63 \text{ ksi}$
    3. 2024-T3, $K_C = 144 \text{ ksi}\sqrt{\text{in.}}$, $\sigma_{YS} = 42 \text{ ksi}$

residual strength

- As an example let us consider an edge-cracked panel with W = 6” and t = 0.1”
- The net section yield condition will be given by

$$\sigma_C = \sigma_{YS}\frac{W-a}{W} = \sigma_{YS}\frac{6-a}{6}$$

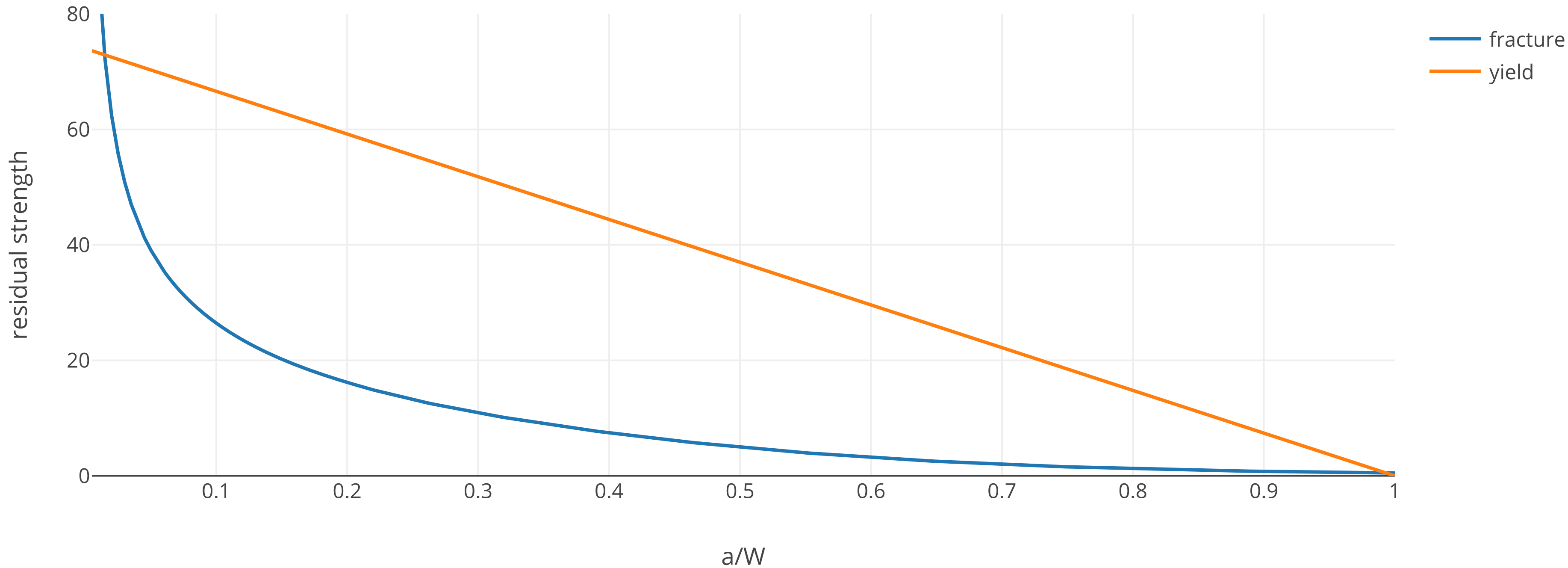
- And the fracture condition by

$$\sigma_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

With

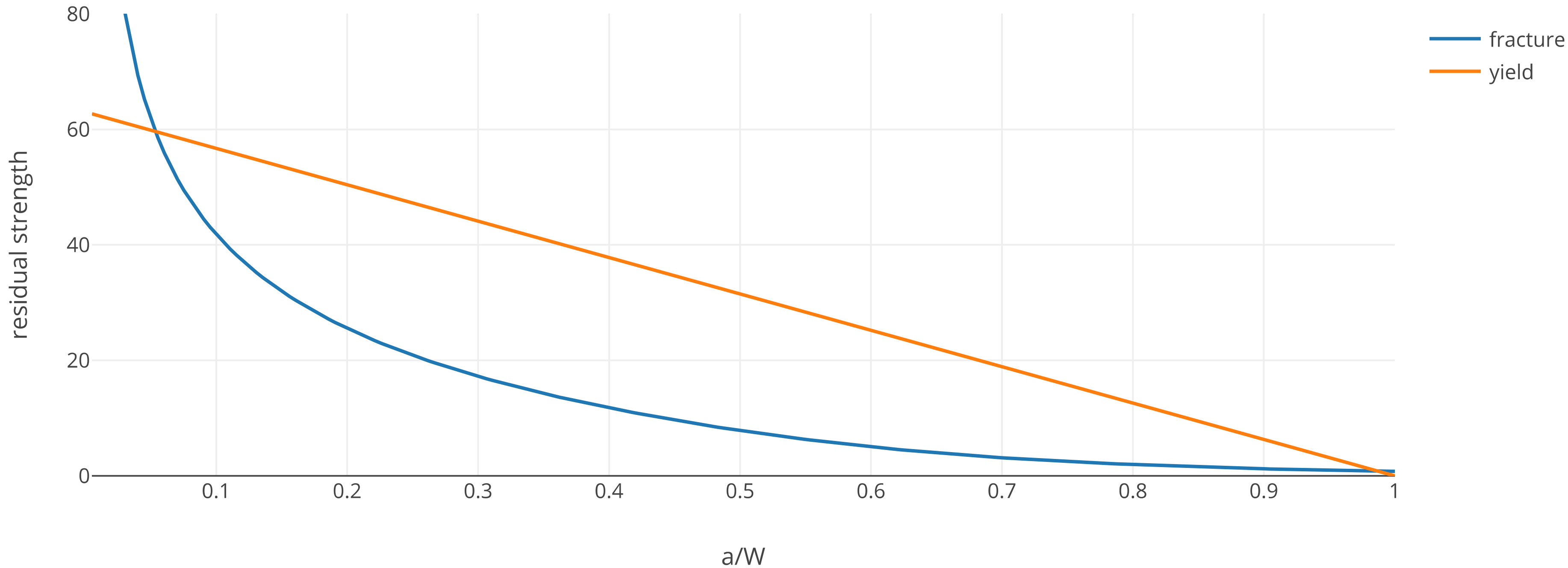
$$\beta = 1.12 - 0.231\left(\frac{a}{W}\right) + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4$$

7178-T6



fracture
yield
residual strength
a/W
0
20
40
60
80
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

7075-T6



fracture
yield
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a/W
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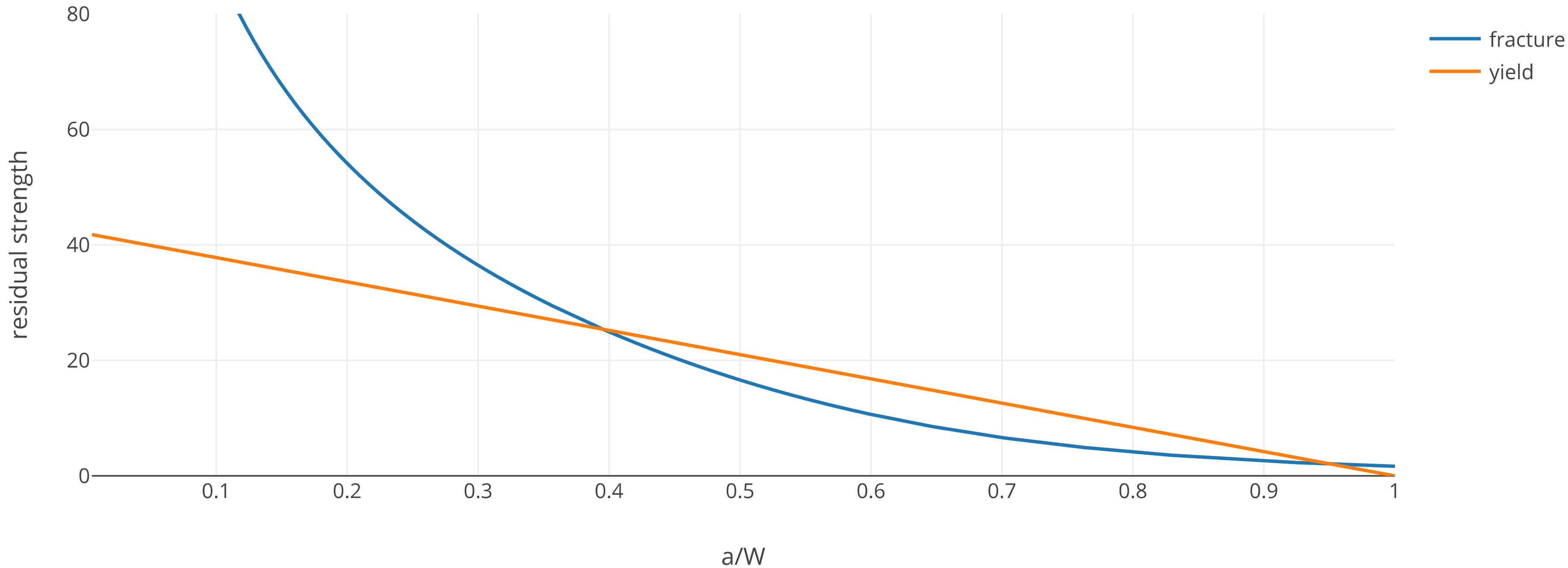
1

2024-T3



fracture
yield
residual strength
a/W
0
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40
60
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0.5
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1

comparison

using MIL-handbook

- Uses a different grain nomenclature

K_C	σ_{YS}
L-T	L
T-L	L-T

- A-Basis vs. B-Basis values are reported (A = 99% of population will meet/exceed value, B = 90% of population)
- S-Basis - no statistical information available, standard value to be used

using MIL-handbook

- F_{tu} - ultimate tensile strength
- F_{ty} - tensile yield strength
- F_{cy} - compressive yield strength
- F_{su} - ultimate shear strength
- F_{bru} - ultimate bearing strength
- F_{bry} - bearing yield strength
- E - tensile Young's Modulus
- E_c - compressive Young's Modulus
- G - shear modulus
- μ - Poisson's ratio

data

- Fracture data is on pp. 111-121
- Tensile data is on pp. 138-143
- K_c charts are also available in interactive versions here (http://nbviewer.jupyter.org/github/ndaman/damagetolerance/blob/master/examples/Fracture%20Toughness%20Figures.ipynb)

fedderson approach

Fedderson approach

- Unfortunately, the method we described above does not quite match experimental results
- Fedderson proposed an alternative, where we connect the net-section yield and brittle fracture curves with a tangent line
- This approach agrees very well with experimental data
- Note: We could do something similar when the crack is very long, but we are generally less concerned with this region (failure will have already occurred)

Fedderson example

worked example here (http://nbviewer.jupyter.org/github/ndaman/damagetolerance/blob/master/examples/Fedderson%20Approach.ipynb)

proof testing

proof testing

- Proof testing is a way to use the concept of residual strength to check the size of a defect from manufacturing
- Due to the fatigue life of a certain panel, and/or an inspection cycle that we have prescribed for that part, we determine an “acceptable” initial flaw size, a_o

proof testing

- We then determine a load which would cause failure at this crack length
- This is the “proof load”
- If the part does not fail in the proof test, we can assume that the largest flaw in the material is a_0

example

- Suppose we are concerned about edge cracks in a panel with $\sigma_{YS} = 65$ ksi, $W = 5$”
- We have determined that the largest allowable crack is 0.4”
- The fracture toughness of this panel is $K_c = 140\ \text{ksi}\sqrt{\text{in.}}$

example

- We can find the proof load

$$
\begin{aligned}
\sigma_c &= \frac{K_c}{\sqrt{\pi a_0}\beta} \\
&= \frac{140}{\sqrt{\pi 0.4}(1.161)} \\
&= 107.6
\end{aligned}
$$

- So the proof load would need to induce a gross section stress of 107.6 ksi.

residual strength review

residual strength review

- Group 1 - Sketch a residual strength curve for a single material (include fracture and net-section yield)
- Group 2 - Sketch and describe the difference in residual strength between stiff/brittle materials and ductile/tough materials
- Group 3 - Find the proof load needed to ensure no center-cracks less than 0.01" are present in a material with $K_C = 120 \text{ ksi}\sqrt{\text{in.}}$

residual strength review

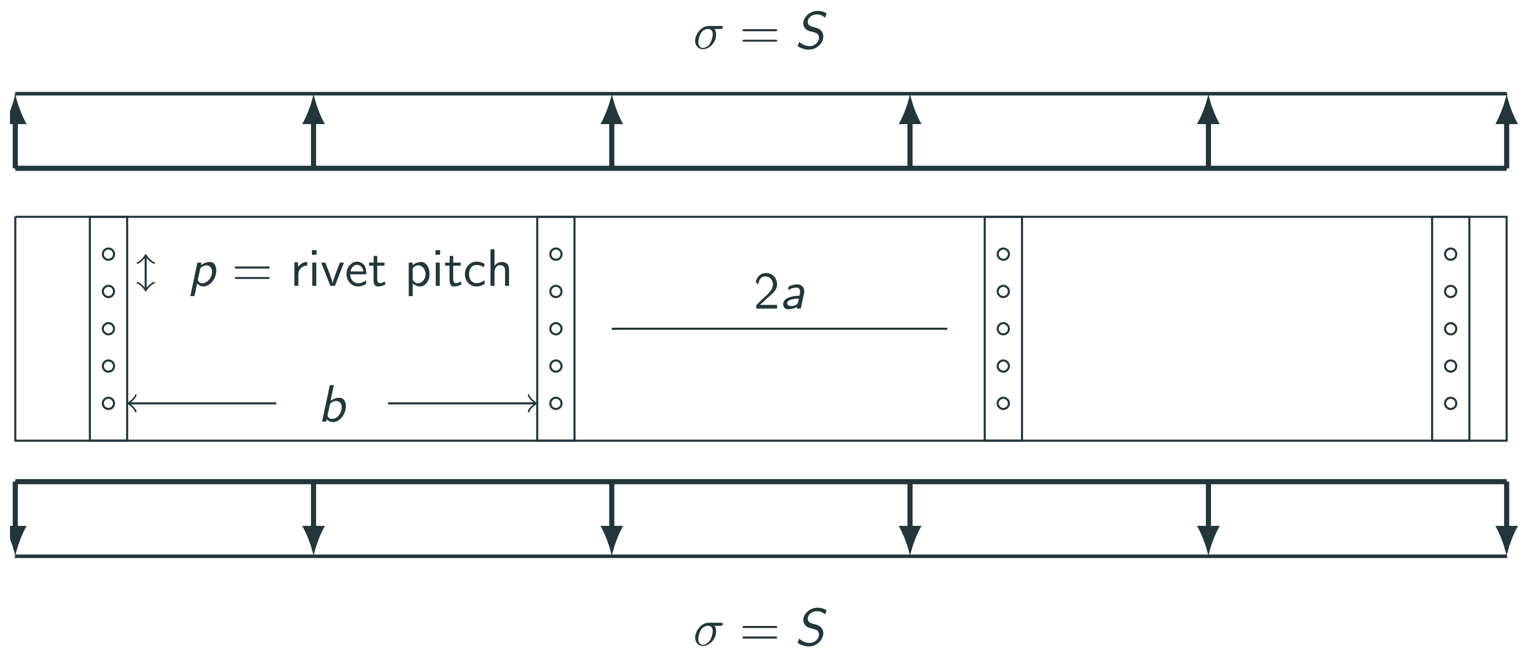
- Group 4 - Sketch the Fedderson approach to residual strength. How is this different from the traditional approach? Why is it beneficial?

stiffeners

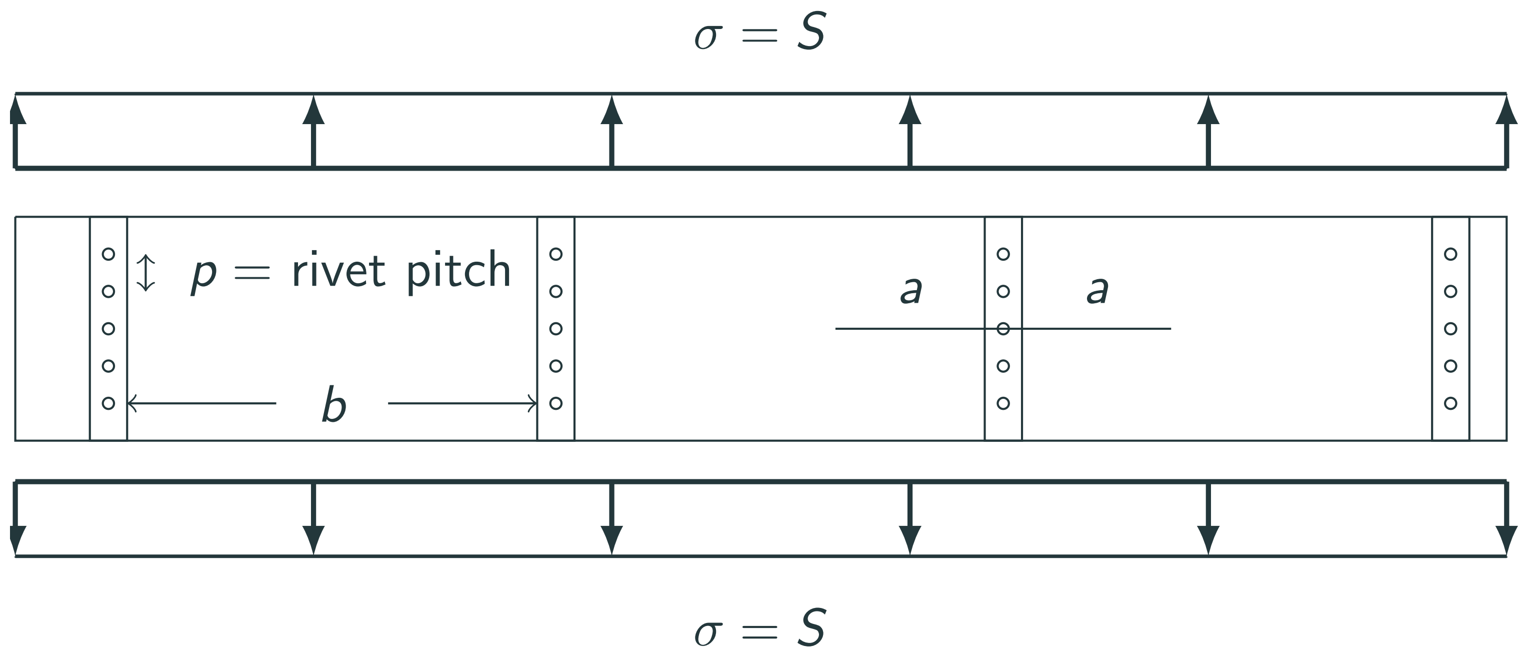
stiffened panels

- In aircraft the skin/stringer system provides many benefits (resistance to buckling)
- Stringers also act as stiffeners to resist crack propagation in the skin
- Panels in these configurations are generally very wide relative to expected crack dimensions
- Cracks are generally modeled either as centered between stiffeners or centered under a stiffener
- We need to consider the residual strength of the panel, the stiffener, and the rivets

centered between stiffeners



centered under stiffener



remote stress

- For displacement continuity, we know that

$$\left(\frac{PL}{AE}\right)_{Skin} = \left(\frac{PL}{AE}\right)_{Stiffener}$$

- Since L is the same, we find

$$\frac{S}{E} = \frac{S_S}{E_S}$$

- Where the subscript S indicates stiffener values, we can express the remote stress in the stiffener as

$$S_S = S\frac{E_S}{E}$$

skin

- The critical stress in the skin is determined the same way as it was in the residual strength chapter
- The only exception is that the stiffener contributes to β

$$S_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

stiffener

- The maximum stress in a stiffener will be increased near a crack
- We represent the ratio of maximum force in stiffener to remote force with the Stiffener Load Factor, L

stiffener

$$L = \frac{\text{max force in stiffener}}{\text{remote force applied to stiffener}}$$

$$= \frac{S_{S,max} A_S}{S_S A_S}$$

$$= \frac{S_{S,max}}{S \frac{E_S}{E}}$$

$$LS \frac{E_S}{E} = S_{S,max}$$

$$LS \frac{E_S}{E} = \sigma_{YS}$$

$$S_C = \frac{\sigma_{YS} E}{L E_S}$$

rivet

- We can define a similar rivet load factor to relate maximum stress in the rivet to remote stress in the skin

$$L_R = \frac{\tau_{max} A_R}{Sbt}$$

$$L_R = \frac{\tau_{YS} A_R}{Sbt}$$

$$S_c = \frac{\tau_{YS} A_R}{L_R bt}$$

finite element analysis

- CC Poe found that panels could be related by a parameter he defines as μ

$$\mu = \frac{A_S E_S}{A_S E_S + AE}$$

- Where A_S is the cross-sectional area of a stiffener, E_S is stiffener modulus
- A is the skin cross-sectional area (per stiffener) $A = bt$ and E is the modulus of the skin

finite element analysis

- pp 167 - 178 give β, L and L_R for various skin/stiffener configurations
- These values were determined using a finite element model

examples

- quantitative example (p. 179-180)
- qualitative notes on behavior (p. 181-182)
- worked (http://nbviewer.jupyter.org/github/ndaman/damagetolerance/blob/master/examples/stiffener%20example.ipynb)