AE 737: Mechanics of Damage Tolerance

Lecture 9 - Residual Strength

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February 21, 2019

schedule

- 21 Feb - Residual Strength
- 26 Feb - Residual Strength, Homework 4 Due
- 28 Feb - Multiple Site Damage
- 5 Mar - Mixed-Mode Fracture, Homework 5 Due

outline

- fracture toughness
- R-curve
- thickness effects
- fracture toughness review
- residual strength
- fedderson approach
- proof testing
- residual strength review
- stiffeners

fracture toughness

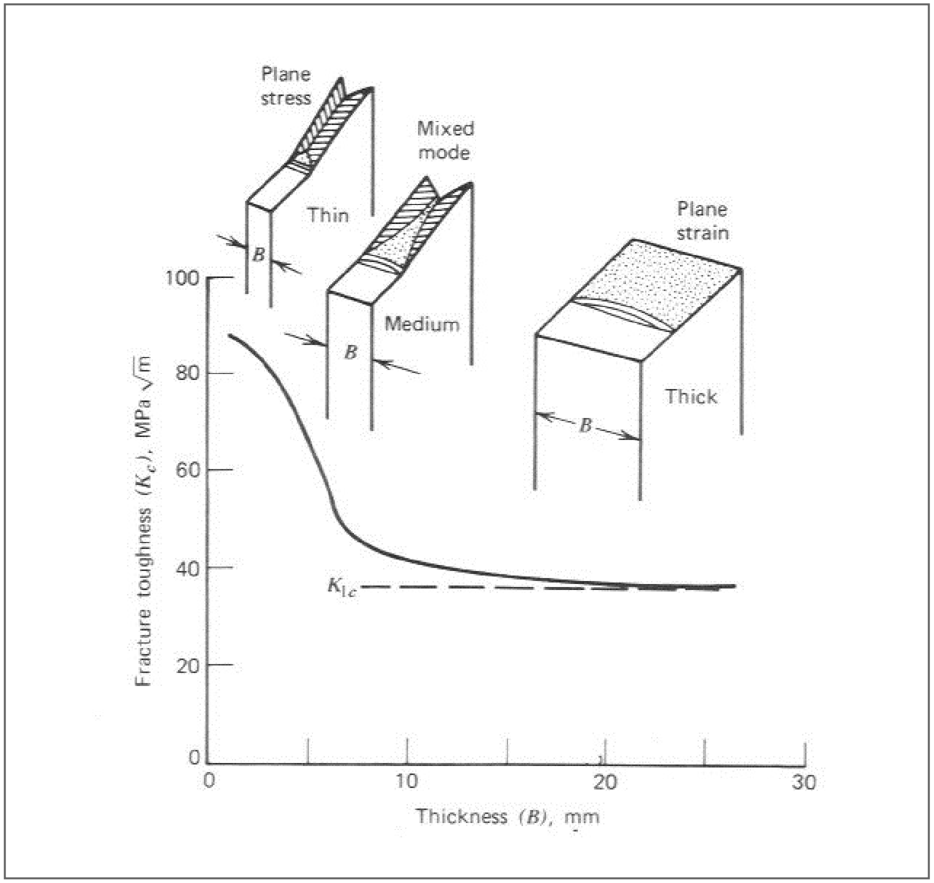
fracture toughness

- The critical load at which a cracked specimen fails produces a critical stress intensity factor
- The "critical stress intensity factor" is known as K_c
- For Mode I, this is called K_{Ic}
- The critical stress intensity factor is also known as fracture toughness $K_{IC} = \sigma_c \sqrt{\pi a} \beta$
- Note: "Fracture Toughness" can also refer to G_{Ic}, which is analogous to K_{Ic}, but not the same

fracture toughness

- Fracture toughness is a material property, but it is only well-defined in certain conditions
- Brittle materials
- Plane strain (smaller plastic zone)
- In these cases ASTM E399-12 is used.

fracture toughness



unstable cracks

- Stable crack growth means the crack extends only with increased load
- Unstable crack growth means the crack will continue to extend indefinitely under the same load
- For a perfectly brittle material, there is no stable crack growth, as soon as a critical load is reached, the crack will extend indefinitely

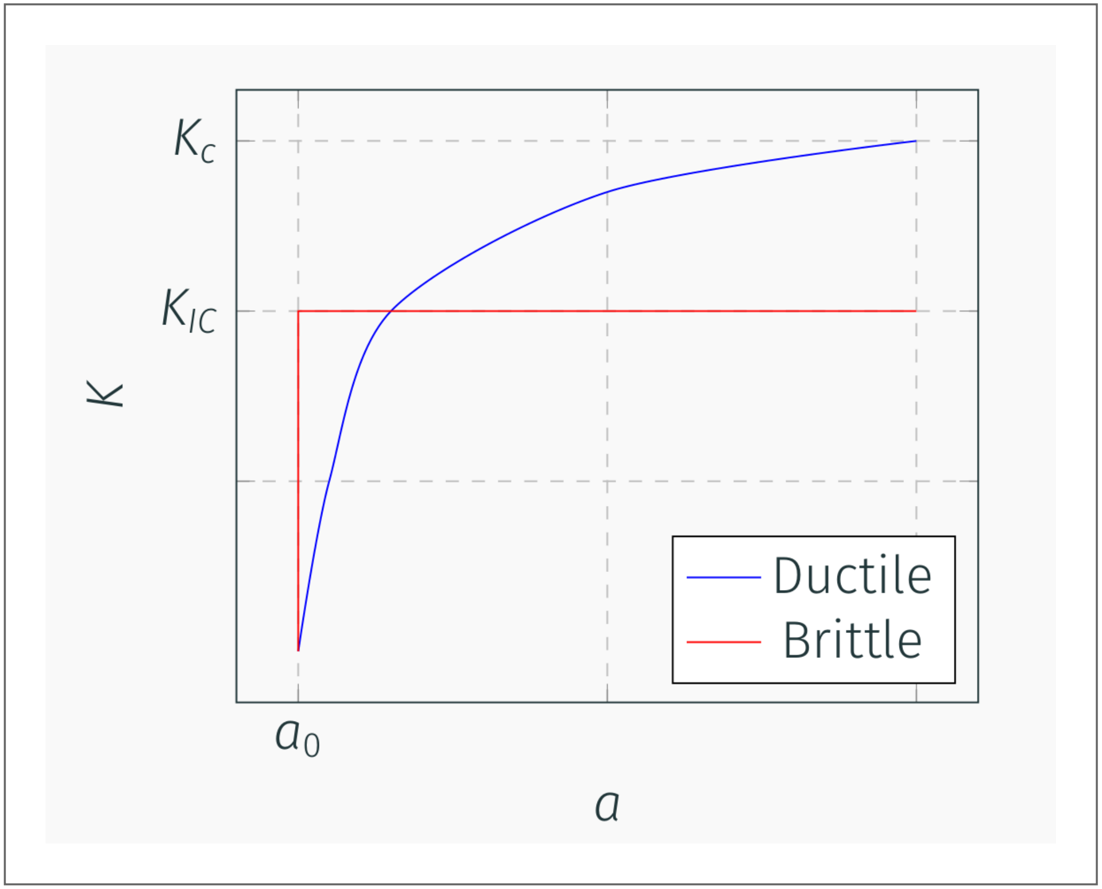
stable cracks

- For an elastic-plastic material, once the load is large enough to extend the crack, it will extend slightly
- The load must be continually increased until a critical value causes unstable crack growth

fracture toughness

- During an experiment, we will record the crack length and applied load (P_i, a_i) each time we increase the load
- We can calculate a unique stress intensity factor K_{Ii} at each of these points
- These are then used to create a "K-curve", plotting K_I vs. a

K-curve



K-curve

- Materials will generally not be as flat as the perfectly brittle example
- Plane strain conditions and brittle materials will tend towards a "flat" K-curve
- K_{IC} for brittle/plane strain is very well defined
- K_C for plane stress can refer to two things
- Either the maximum K_C during a test, or tangent point on K_R-curve (R-curve)

example

- In composites, and adhesives, some work is needed to ensure stable crack growth
- The Double-Cantilever Beam (DCB) experiment to find G_{IC} illustrates this

$$C = \frac{\delta}{P}$$

$$C = \frac{2a^3}{3EI}$$

$$G = \frac{P^2}{2b}\frac{dC}{da}$$

$$G = \frac{P^2 a^2}{bEI}$$

example

- For crack growth to be stable we need
$$\frac{dG}{da} \leq 0$$
- Under fixed-load conditions, we find
$$\frac{dG}{da} = \frac{2P^2a}{bEI}$$
- This is always positive, and thus results in unstable crack growth

example

- Under fixed-displacement conditions, we substitute for P
- We find
  $$\frac{dG}{da} = -\frac{9\delta^2 EI}{ba^3}$$
- Which is always stable, so for DCB tests, displacement control is generally used

R-curve

R-curve

- For materials with some plasticity, the K_R Curve, or R Curve, is very important
- Sometimes called a "resistance curve" it is generally dependent on
  - Thickness
  - Temperature
  - Strain rate

R-curve

- When done correctly, K_R curves are not dependent on initial crack size or the specimen type used
- ASTM E561 describes a general procedure

K_c

- While we can look up plane stress K_c for various materials, it is best if we have a K_R curve
- We may not know if the table uses K_c using the tangent intersection method, or maximum stress intensity
- Even if tangent intersection method is used, K_c will different somewhat based on initial crack length

R-curve

- There are two main methods for plotting the R-curve
- Crack size is measured directly (possibly with a drawn-on scale and camera)
- Effective crack size is calculated from the load-displacement data

physical crack

- When the physical crack size is measured, we need to calculate the effective crack length (and effective stress intensity factor) at each data point
- The effective crack length calculated from the load-displacement data already has the plastic zone effect built in

plane stress fracture toughness

- For a plane stress (or any thickness that is not plane strain) we can find K_c two ways
- One way is simply the maximum value of the K_R curve, but this does not account for unstable crack growth
- The more reliable way is to use the tangent intersection method

tangent intersection method

- Construct curves for K_I based on our specimen geometry with constant applied load and varying crack length
- Plot these curves on the same graph as K_R
- NOTE: K_R curve should be plotted vs. a_{eff}, not Δa or Δa_{eff}
- K_c is the point at which one of the K_I curves is the tangent intersection with the K_R curve

R-Curve examples

- **example**
- Excel Solver

thickness effects

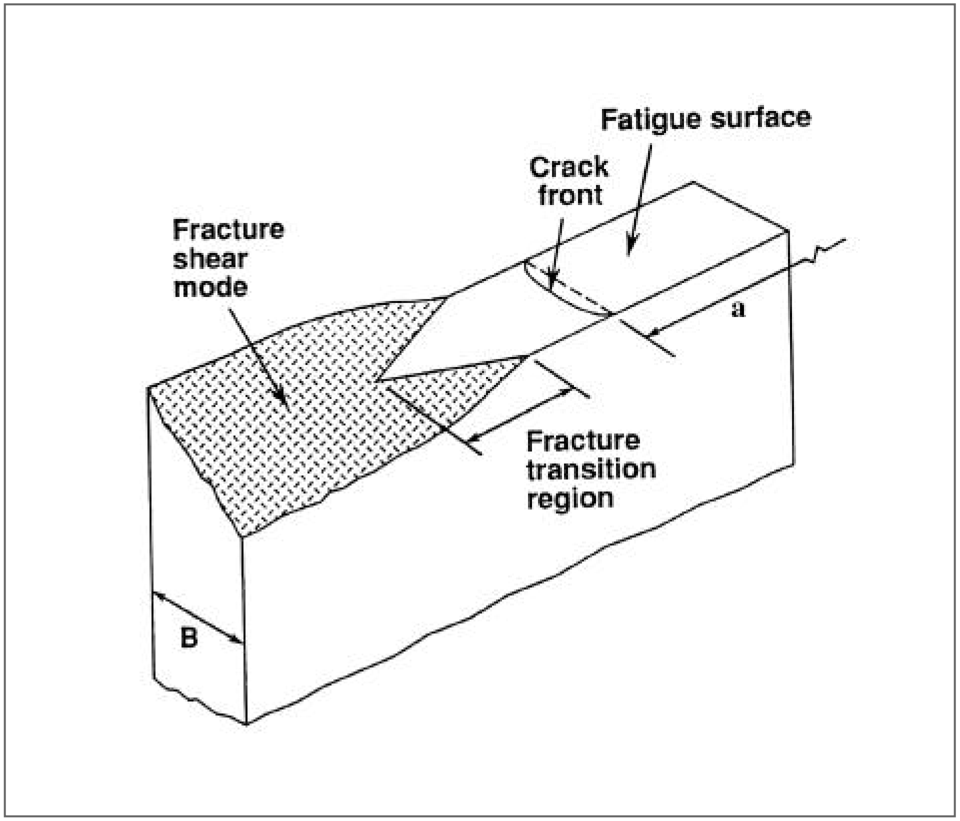
thickness effects

- We already know there is a difference between plane strain and plane stress fracture toughness
- As a material gets thicker and thicker, it converges to the plane strain solution
- Thinner specimens tend towards the plane stress solution
- When a specimen is thinner than some critical thickness, the material behavior is somewhat unknown

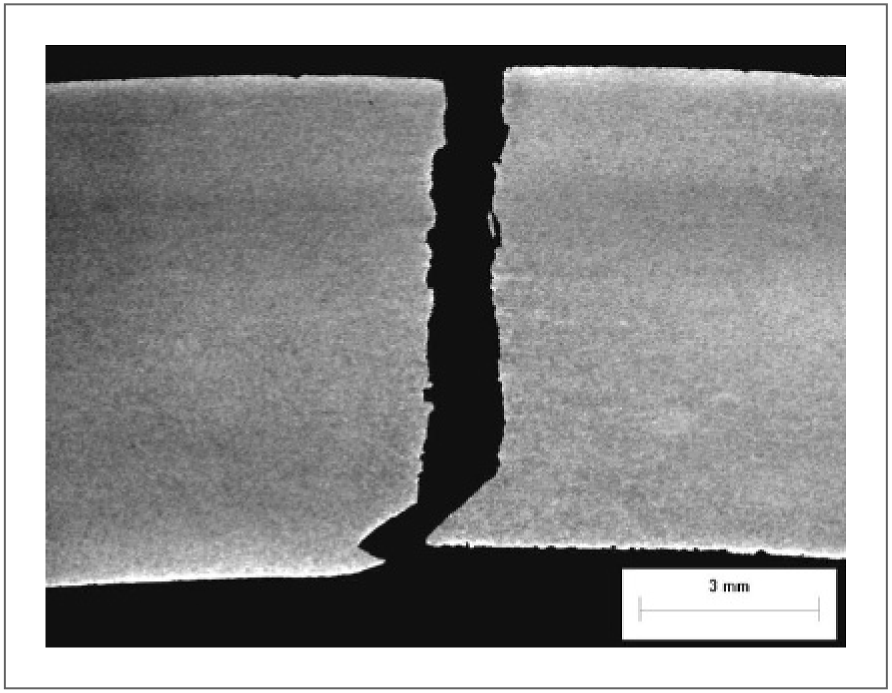
thickness effects

- There is also a difference in the fracture surface between thin and thick specimens
- Thin specimens (in plane stress region) fail due to slant fracture
- This actually indicates some mixed-mode conditions at failure
- Thick specimens fail due to square fracture (with a small shear tip near the edges)
- This is more consistent with pure Mode I

slant fracture



shear lip



fracture toughness review

group review

- Group 1 - Sketch K_R-curve (for ductile material), explain what it means, how to find K_c
- Group 2 - Sketch K_c vs. crack length, explain whatâ™s happening
- Group 3 - How can we determine whether a panel is in plane strain or plane stress?
- Group 4 - Sketch K_R-curves for ductile and brittle materials, what is the difference?

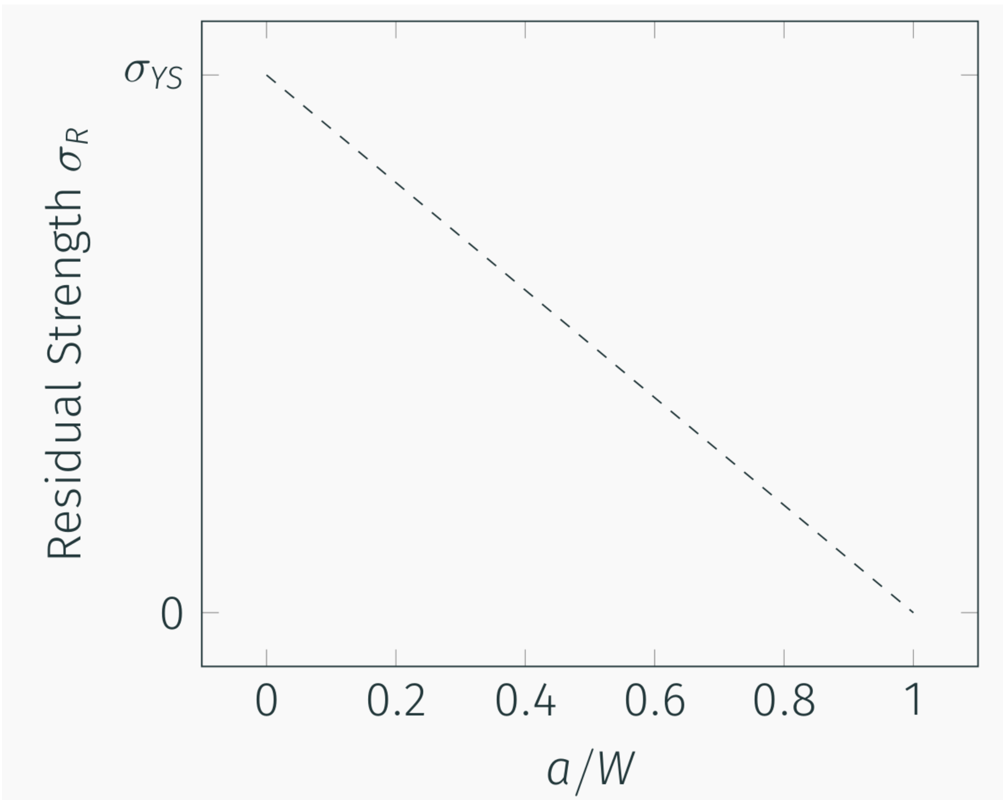
residual strength

residual strength

- As the crack grows, the area of the sample decreases, increasing the net section stress
- The residual strength, σ_R is given in terms of the gross area, so as the crack grows the residual strength due to yield decreases
- We can relate the net-section stress to σ_R by

$$\sigma_R = \sigma_{YS} \frac{A_{net}}{A_{gross}}$$

residual strength

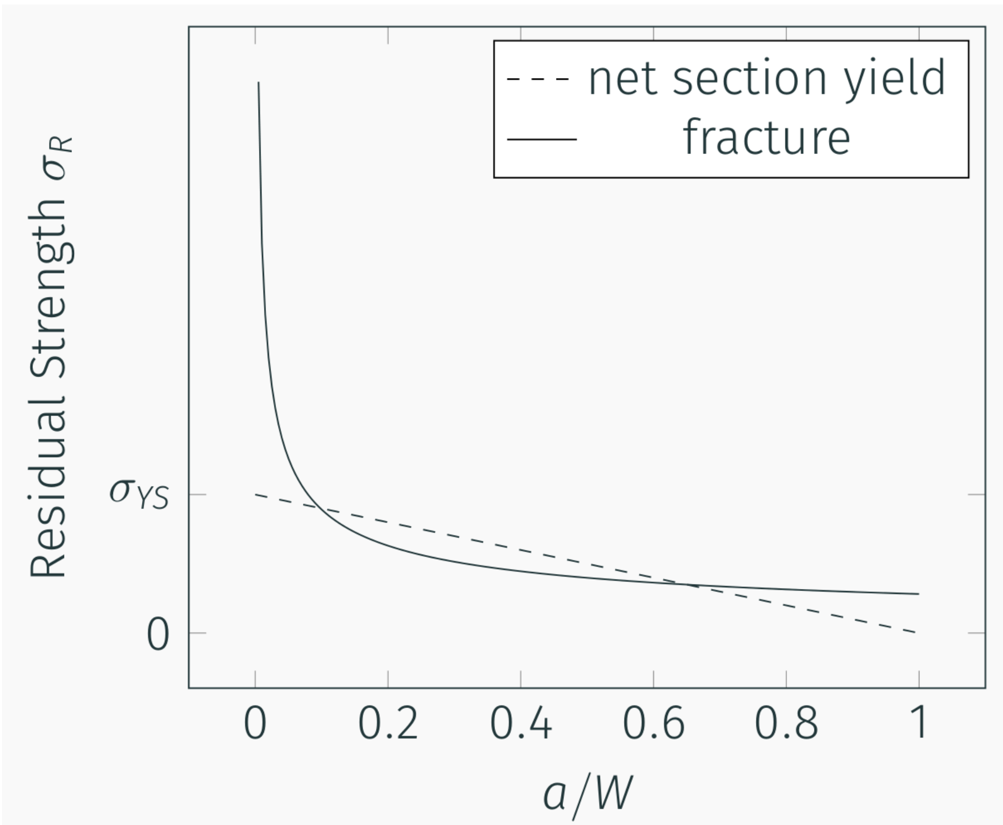


residual strength

- For brittle fracture to occur, we need to satisfy the condition

$$\sigma_R = \sigma_C = \frac{K_C}{\sqrt{\pi a}\,\beta}$$

residual strength



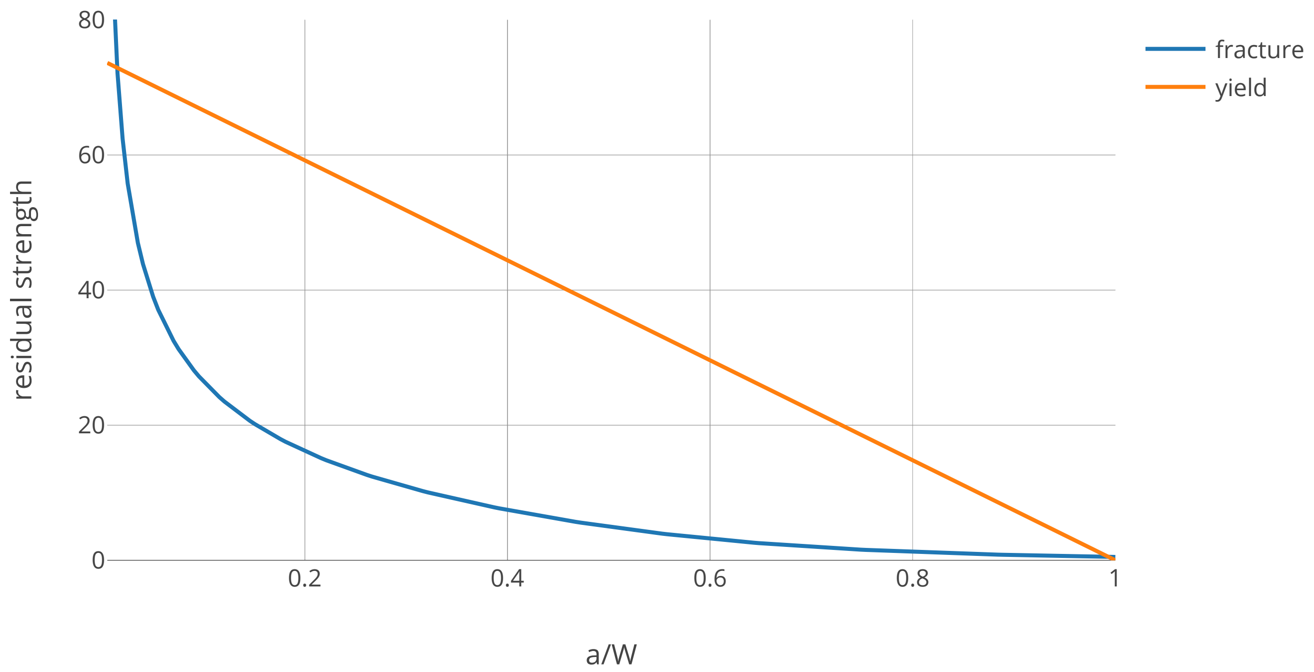
residual strength

- Within the same family of materials (i.e. Aluminum), there is generally a trade-off between yield stress and fracture toughness
- As we increase the yield strength, we decrease the fracture toughness (and vice versa)
- Consider a comparison of the following aluminum alloys
  1. 7178-T6, $K_C = 43 \text{ ksi}\sqrt{\text{in.}}$, $\sigma_{YS} = 74$ ksi
  2. 7075-T6, $K_C = 68 \text{ ksi}\sqrt{\text{in.}}$, $\sigma_{YS} = 63$ ksi
  3. 2024-T3, $K_C = 144 \text{ ksi}\sqrt{\text{in.}}$, $\sigma_{YS} = 42$ ksi

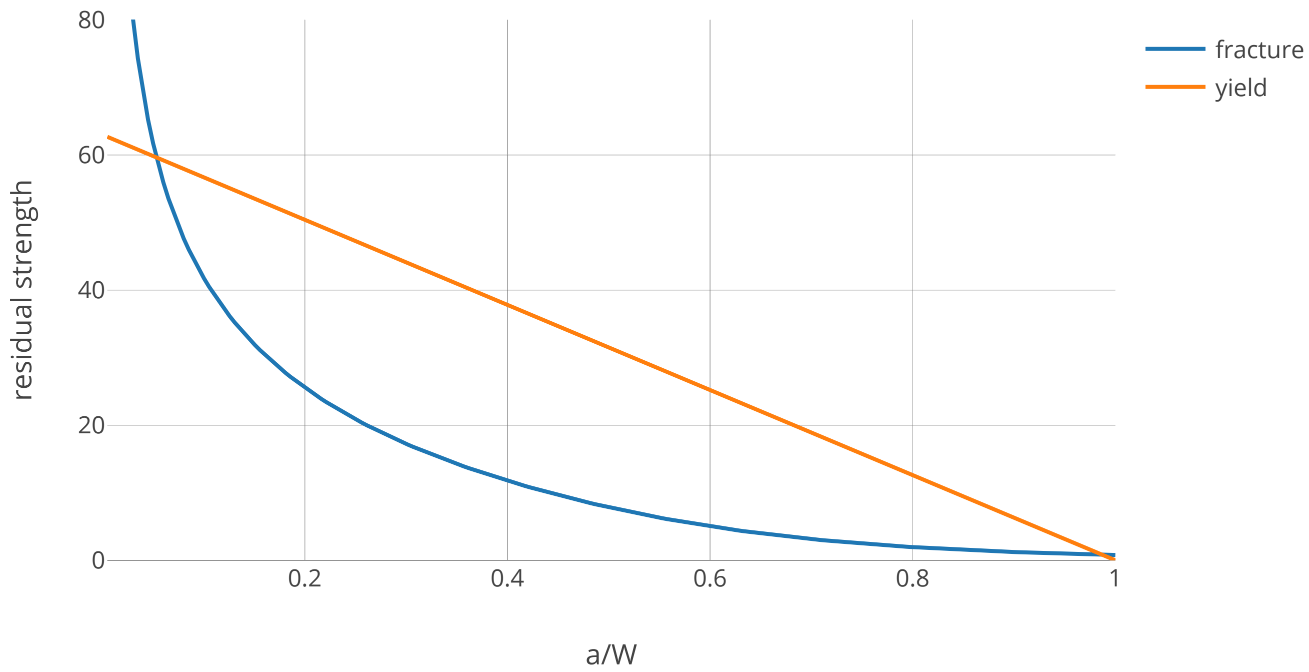
residual strength

- As an example let us consider an edge-cracked panel with $W = 6$ ” and $t = 0.1$ ”
- The net section yield condition will be given by
  $$\sigma_C = \sigma_{YS} \frac{W-a}{W} = \sigma_{YS} \frac{6-a}{6}$$
- And the fracture condition by
  $$\sigma_C = \frac{K_C}{\sqrt{\pi a}\beta}$$
  With
  $$\beta = 1.12 - 0.231\left(\frac{a}{W}\right) + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4$$

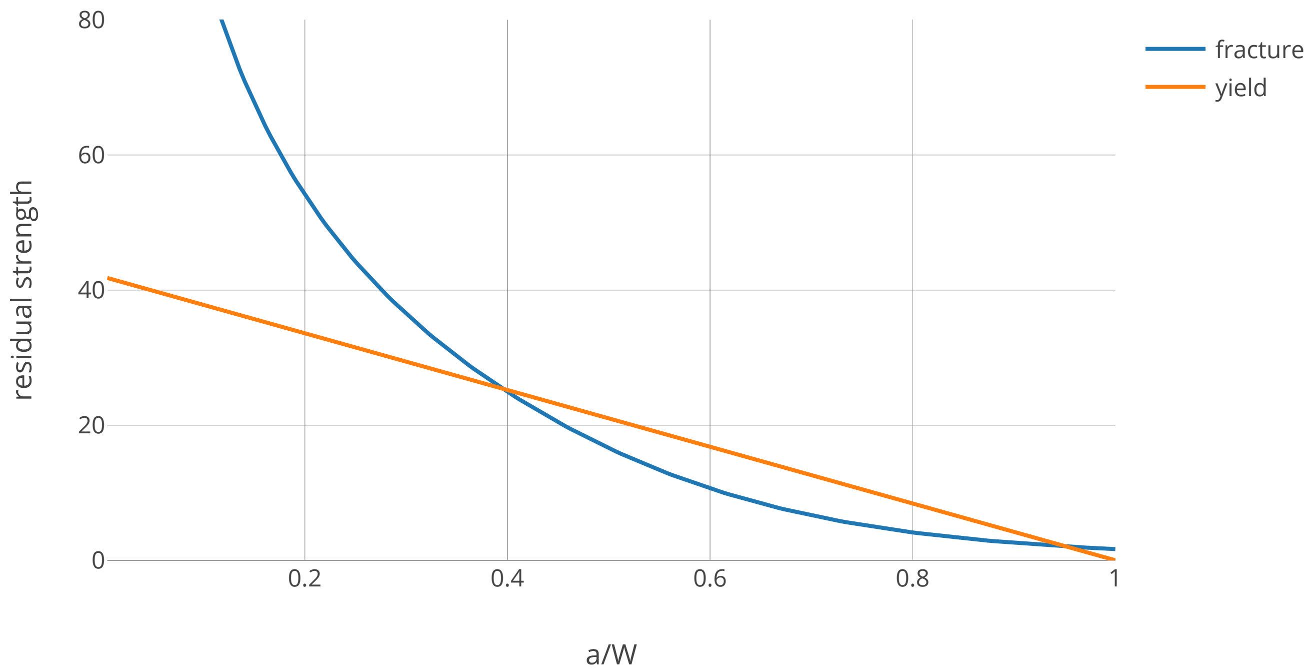
7178-T6



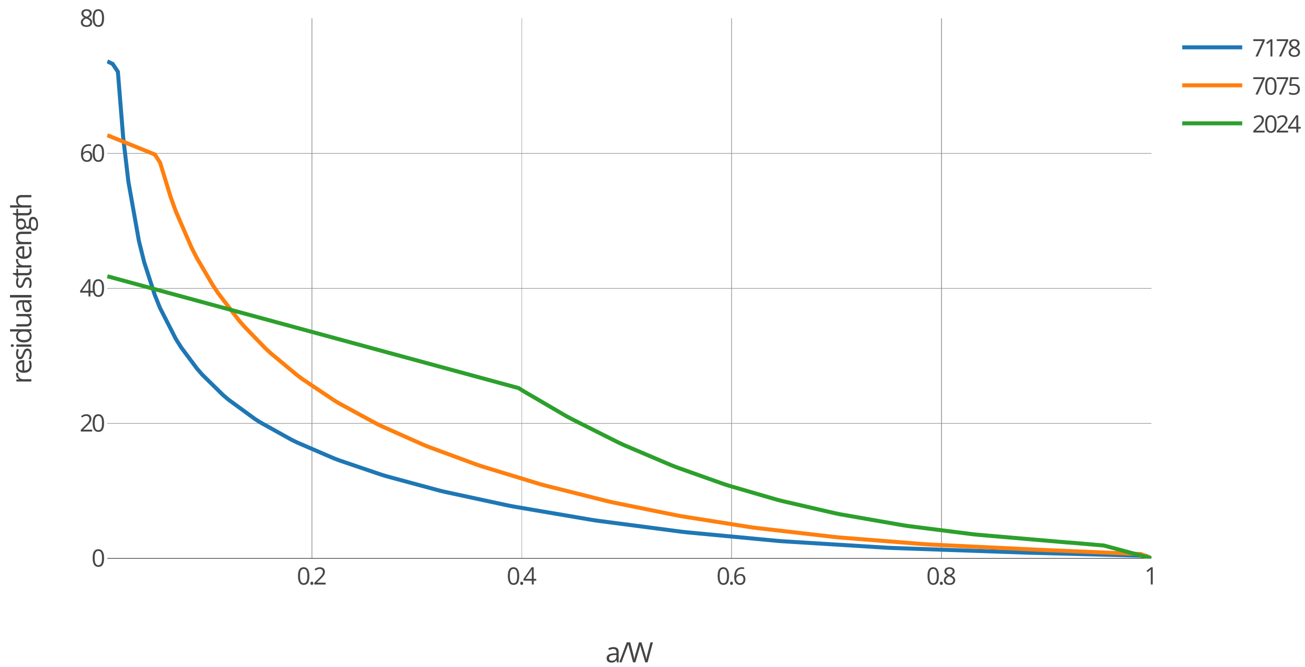
7075-T6



2024-T3



comparison



using MIL-handbook

- Uses a different grain nomenclature

K_C	σ_{YS}
L-T	L
T-L	L-T

- A-Basis vs. B-Basis values are reported (A = 99% of population will meet/exceed value, B = 90% of population)
- S-Basis - no statistical information available, standard value to be used

using MIL-handbook

- F_{tu} - ultimate tensile strength
- F_{ty} - tensile yield strength
- F_{cy} - compressive yield strength
- F_{su} - ultimate shear strength
- F_{bru} - ultimate bearing strength
- F_{bry} - bearing yield strength
- E - tensile Young's Modulus
- E_c - compressive Young's Modulus
- G - shear modulus
- μ - Poisson's ratio

data

- Fracture data is on pp. 111-121
- Tensile data is on pp. 138-143
- K_c charts are also available in interactive versions **here**

fedderson approach

Fedderson approach

- Unfortunately, the method we described above does not quite match experimental results
- Fedderson proposed an alternative, where we connect the net-section yield and brittle fracture curves with a tangent line
- This approach agrees very well with experimental data
- Note: We could do something similar when the crack is very long, but we are generally less concerned with this region (failure will have already occurred)

Fedderson example

worked example **here**

proof testing

proof testing

- Proof testing is a way to use the concept of residual strength to check the size of a defect from manufacturing
- Due to the fatigue life of a certain panel, and/or an inspection cycle that we have prescribed for that part, we determine an "acceptable" initial flaw size, a_0

proof testing

- We then determine a load which would cause failure at this crack length
- This is the "proof load"
- If the part does not fail in the proof test, we can assume that the largest flaw in the material is a_o

example

- Suppose we are concerned about edge cracks in a panel with $\sigma_{YS} = 65 \text{ ksi}, W = 5"$
- We have determined that the largest allowable crack is 0.4"
- The fracture toughness of this panel is $K_c = 140 \text{ ksi}\sqrt{\text{in.}}$

example

- We can find the proof load

$$
\begin{aligned}
\sigma_c &= \frac{K_c}{\sqrt{\pi a_0}\beta} \\
&= \frac{140}{\sqrt{\pi 0.4}(1.161)} \\
&= 107.6
\end{aligned}
$$

- So the proof load would need to induce a gross section stress of 107.6 ksi.

residual strength review

residual strength review

- Group 1 - Sketch a residual strength curve for a single material (include fracture and net-section yield)
- Group 2 - Sketch and describe the difference in residual strength between stiff/brittle materials and ductile/tough materials
- Group 3 - Find the proof load needed to ensure no center-cracks less than 0.01" are present in a material with $K_C = 120 \text{ ksi}\sqrt{\text{in.}}$

residual strength review

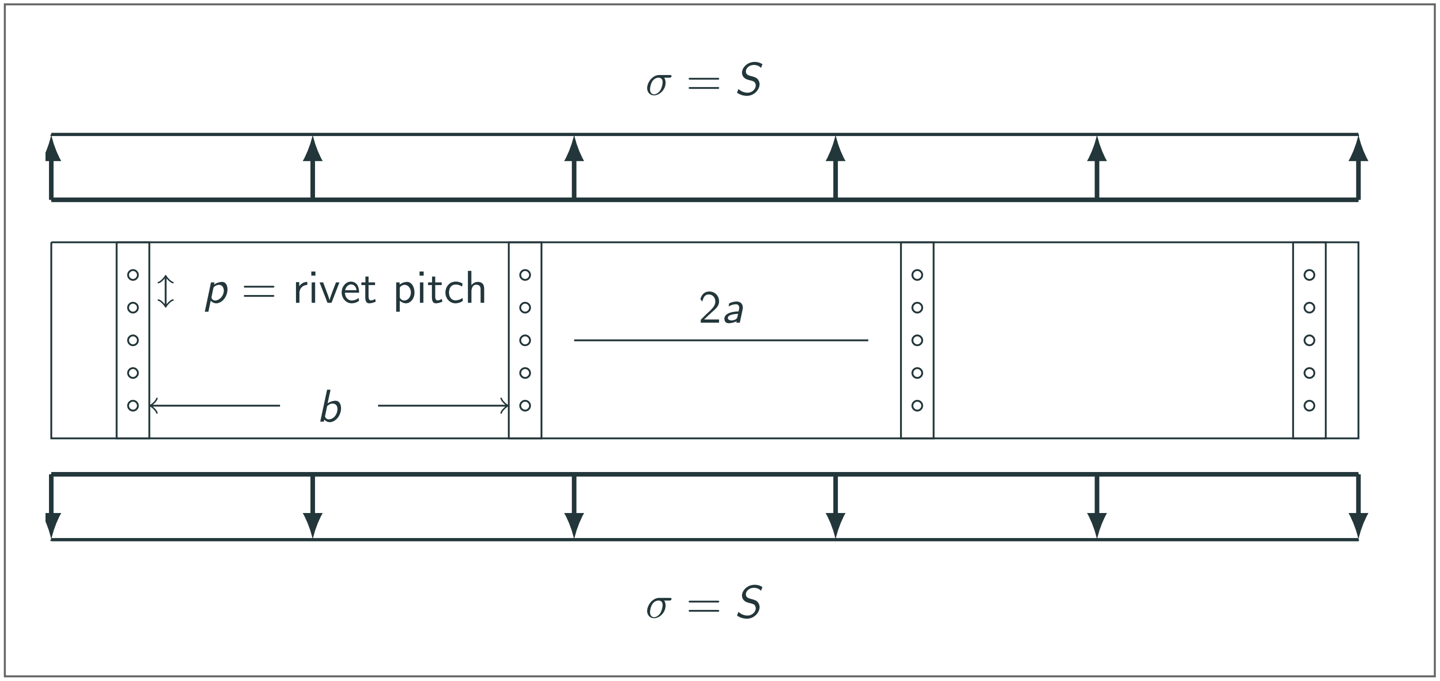
- Group 4 - Sketch the Fedderson approach to residual strength. How is this different from the traditional approach? Why is it beneficial?

stiffeners

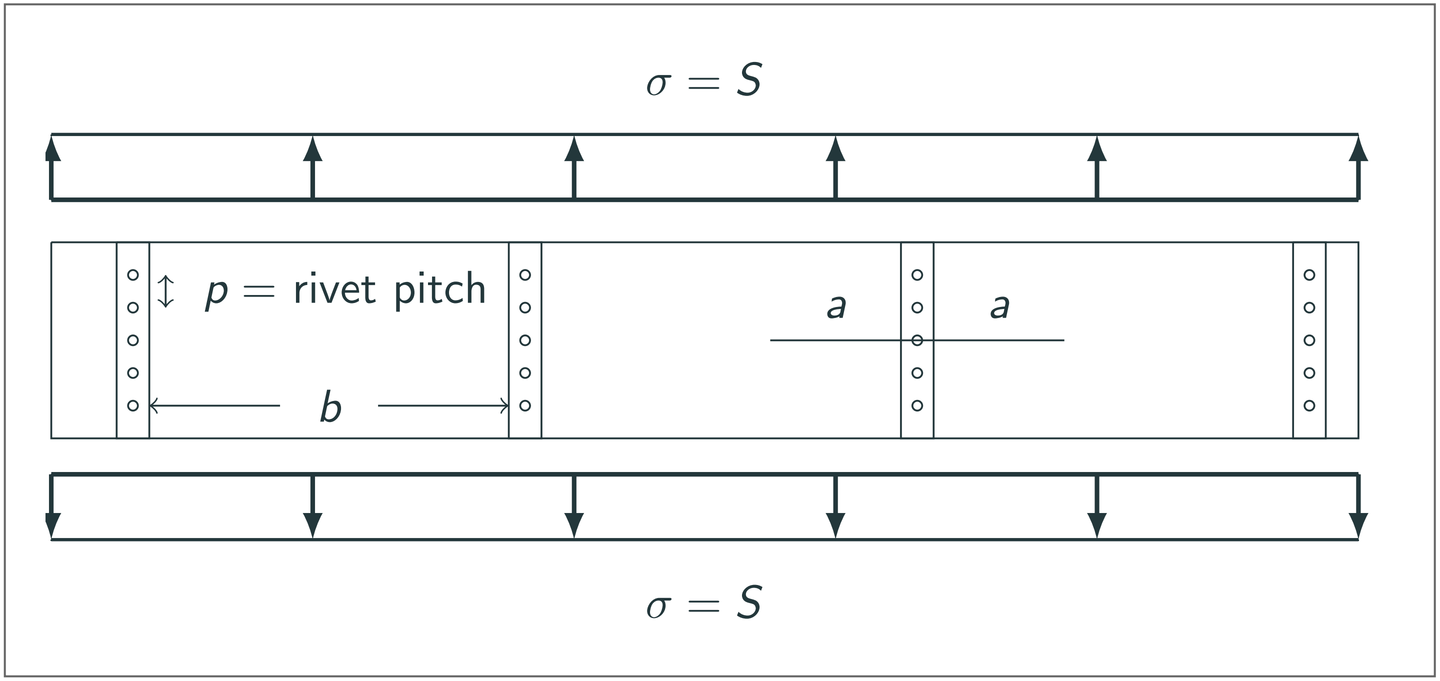
stiffened panels

- In aircraft the skin/stringer system provides many benefits (resistance to buckling)
- Stringers also act as stiffeners to resist crack propagation in the skin
- Panels in these configurations are generally very wide relative to expected crack dimensions
- Cracks are generally modeled either as centered between stiffeners or centered under a stiffener
- We need to consider the residual strength of the panel, the stiffener, and the rivets

centered between stiffeners



centered under stiffener



remote stress

- For equilibrium to be satisfied, we know that
$$\left(\frac{PL}{AE}\right)_{Skin} = \left(\frac{PL}{AE}\right)_{Stiffener}$$
- Since L is the same, we find
$$\frac{S}{E} = \frac{S_S}{E_S}$$
- Where the subscript $_S$ indicates stiffener values, we can express the remote stress in the stiffener as
$$S_S = S\frac{E_S}{E}$$

skin

- The critical stress in the skin is determined the same way as it was in the residual strength chapter
- The only exception is that the stiffener contributes to β

$$S_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

stiffener

- The maximum stress in a stiffener will be increased near a crack
- We represent the ratio of maximum force in stiffener to remote force with the Stiffener Load Factor, L

stiffener

$$
\begin{aligned}
L &= \frac{\text{max force in stiffener}}{\text{remote force applied to stiffener}} \\
&= \frac{S_{S,max} A_S}{S_S A_S} \\
&= \frac{S_{S,max}}{S \frac{E_S}{E}}
\end{aligned}
$$

$$
\begin{aligned}
LS\frac{E_S}{E} &= S_{S,max} \\
LS\frac{E_S}{E} &= \sigma_{YS} \\
S_C &= \frac{\sigma_{YS} E}{L E_S}
\end{aligned}
$$

rivet

- We can define a similar rivet load factor to relate maximum stress in the rivet to remote stress in the skin

$$L_R = \frac{\tau_{max} A_R}{Sbt}$$

$$L_R = \frac{\tau_{YS} A_R}{Sbt}$$

$$S_c = \frac{\tau_{YS} A_R}{L_R bt}$$

finite element analysis

- CC Poe found that panels could be related by a parameter he defines as μ
  $$\mu = \frac{A_S E_S}{A_S E_S + AE}$$
- Where A_S is the cross-sectional area of a stiffener, E_S is stiffener modulus
- A is the skin cross-sectional area (per stiffener) $A=bt$ and E is the modulus of the skin

finite element analysis

- pp 167 - 178 give β, L and L_R for various skin/stiffener configurations
- These values were determined using a finite element model

examples

- quantitative example (p. 179-180)
- qualitative notes on behavior (p. 181-182)
- **worked**