AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 18

Dr. Nicholas Smith

Last Updated: March 31, 2016 at 1:50pm

Wichita State University, Department of Aerospace Engineering

SCHEDULE

- 31 Mar - Strain based fatigue, project abstract due
- 5 Apr - Crack Growth, Homework 7 due, Homework 8 assigned
- 7 Apr - Crack Growth, Stress Spectrum
- 12 Apr - Retardation, Boeing Commercial Method
- 14 Apr - Exam Review, Homework 8 Due
- 19 Apr - Exam 2
- 21 Apr - Exam Solutions, Damage Tolerance

OUTLINE

1. strain based fatigue
2. general trends
3. other factors affecting fatigue
4. mean stress effects
5. multiaxial loading

STRAIN BASED FATIGUE

STRAIN BASED FATIGUE

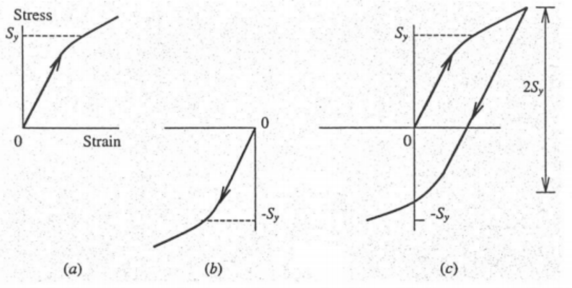
- The strain based fatigue method uses local stresses and strains (instead of global, nominal values)
- The strain-based method gives greater detail, and validity at lower cycles
- It is still valid for high cycle fatigue
- Does not include crack growth analysis or fracture mechanics

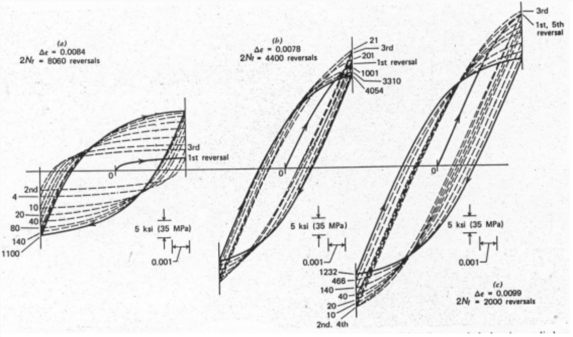
STRAIN LIFE CURVE

- Similar to the S-N curves in stress-based fatigue analysis, we can plot the cyclic strain amplitude vs. number of cycles to failure
- This is most commonly done using axial test machines (instead of rotating bending tests)
- The test is run in strain control (not load control)
- Generally plotted on log-log scale

- We can separate the total strain into elastic and plastic components

$$\epsilon_a = \epsilon_{ea} + \epsilon_{pa} \tag{18.1}$$

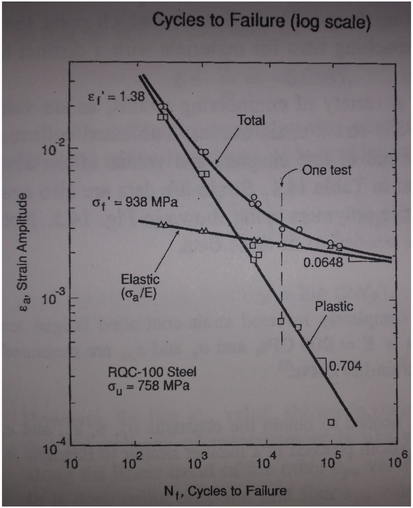

Stress
S_y
0
Strain
0
$-S_y$
S_y
0
$2S_y$
$-S_y$
(a)
(b)
(c)

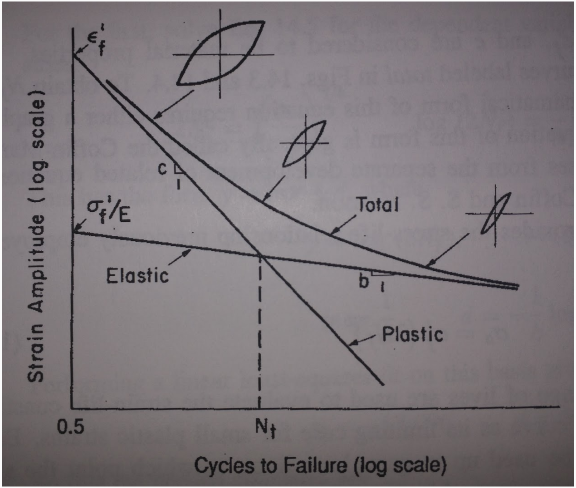

(a)
Δε = 0.0084
$2N_f$ = 8060 reversals
3rd
1st reversal
0
2nd
4
10
20
40
80
140
1100
5 ksi (35 MPa)
0.001
(b)
Δε = 0.0078
$2N_f$ = 4400 reversals
21
3rd
201
1st reversal
1001
3310
4054
0
5 ksi (35 MPa)
0.001
1232
466
140
40
20
10
2nd, 4th
3rd
1st, 5th reversal
0
5 ksi (35 MPa)
0.001
(c)
Δε = 0.0099
$2N_f$ = 2000 reversals

- While strain-life data will generally just report ϵ_a and ϵ_{pa}, some will also tabulate a form for the cyclic stress-strain curve

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} \tag{18.2}$$

- On strain life curves, the strain is often plotted three times per each experiment
- Once for total strain, once for plastic strain, and once for elastic strain
- Since plastic strain and elastic strain vary by the number of cycles, a hysteresis loop from half the fatigue life is generally used
- This is considered representative of stable behavior


Cycles to Failure (log scale)
$\varepsilon_f' = 1.38$
Total
One test
$\sigma_f' = 938$ MPa
0.0648
Elastic
(σ_a/E)
Plastic
0.704
RQC-100 Steel
$\sigma_u = 758$ MPa
ε_a, Strain Amplitude
10^{-2}
10^{-3}
10^{-4}
10
10^2
10^3
10^4
10^5
10^6
N_f, Cycles to Failure



ϵ_f'
σ_f'/E
c
1
Total
Elastic
b
1
Plastic
Strain Amplitude (log scale)
0.5
N_t
Cycles to Failure (log scale)

- We notice that the data for elastic and plastic strains are represented by straight lines, in the log-log scale
- If we recall the form used for a straight line in log-log plots for S-N curves:

$$\sigma_a = \sigma_f'(2N_f)^b \tag{18.3}$$

- We can convert this to find the elastic component of strain

$$\epsilon_{ea} = \frac{\sigma_f'}{E}(2N_f)^b \tag{18.4}$$

- We can use the same form with new constants for the plastic component of strain

$$\epsilon_{pa} = \epsilon_f'(2N_f)^c \tag{18.5}$$

- We can combine 18.4 with 18.5 to find the total strain-life curve

$$\epsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \epsilon_f'(2N_f)^c \tag{18.6}$$

EXAMPLE

Data from p. 270

- With the strain-based fatigue method we are better equipped to discuss the difference between high and low-cycle fatigue
- Low-cycle fatigue is dominated by plastic effects, while high-cycle fatigue has little plasticity
- We can find the intersection of the plastic strain and elastic strain lines
- This point is N_t, the transition fatigue life

$$N_t = \frac{1}{2}\left(\frac{\sigma'_f}{\epsilon'_f}\right)^{\frac{1}{c-b}} \tag{18.7}$$

- If we consider the equation for the cyclic stress train curve

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} \tag{18.8}$$

- We can consider the plastic portion and solve for σ_a

$$\sigma_a = H' \epsilon_{pa}^{n'} \tag{18.9}$$

- We can eliminate $2N_f$ from the plastic strain equation

$$\epsilon_{pa} = \epsilon_f'(2N_f)^c \tag{18.10}$$

- By solving the stress-life relationship for $2N_f$

$$\sigma_a = \sigma_f'(2N_f)^b \tag{18.11}$$

  and substituting that into the plastic strain
- We then compare with 18.9 and find

$$H' = \frac{\sigma_f'}{(\epsilon_f')^{b/c}} \tag{18.12a}$$

$$n' = \frac{b}{c} \tag{18.12b}$$

- However, in practice these constants are fit from different curves
- In some cases there can be large inconsistencies in these values
- One cause for this is data that do not lie on a straight line in the log-log domain
- For ductile materials at short lives, the true stresses and strains may differ significantly from engineering stress and strain

GENERAL TRENDS

TRUE FRACTURE STRENGTH

- We can consider a tensile test as a fatigue test with $N_f = 0.5$
- We would then expect the true fracture strength $\tilde{\sigma}_f \approx \sigma'_f$
- And similarly for strain $\tilde{\epsilon}_f \approx \epsilon'_f$

- Since ductile materials experience large strains before failure, we expect relatively large ϵ_f' and relatively small σ_f'
- This will cause a less steep slope in the plastic strain line
- In turn this intersects with the elastic strain line much later, resulting a longer transition life for ductile materials

- Brittle materials exhibit the opposite effect, with relatively low ϵ_f' and relatively high σ_f'
- This results in a steeper plastic strain line
- And shorter transition life

- Tough materials have intermediate values for both ϵ_f' and σ_f'
- This gives a transition life somewhere between brittle and ductile materials
- It is also noteworthy that strain-life for many metals pass through the point $\epsilon_a = 0.01$ and $N_f = 1000$ cycles
- Steels also follow a trend with Brinell Hardness, the higher they are on the HB scale, the lower their transition life

TYPICAL PROPERTY RANGES

- Most common engineering materials have $-0.8 < c < -0.5$, with most values being very close to $c = -0.6$
- The elastic strain slope generally has $b = -0.085$
- A ”steep” elastic slope is around $b = -0.12$, common in soft metals
- While ”shallow” slopes are around $b = -0.05$, common for hardened metals

OTHER FACTORS AFFECTING FATIGUE

FACTORS AFFECTING FATIGUE LIFE

- Factors other than the stress/strain can effect fatigue life
- At temperatures above one-half the melting temperature (absolute scale), creep-relaxation is significant
- This will cause the strain/stress-life curves to become rate dependent
- Occurs at room temperature for many materials (lead, tin, many polymers)
- At a sufficiently elevated temperature for any material

SURFACE FINISH

- S-N curves (stress-based method) are highly sensitive to surface finish, samples are generally polished
- Strain life curves are not very sensitive to surface finish or residual strength at short lives
- The plastic deformation tends to remove residual stresses
- In high-cycle fatigue, crack initiation is important (poor surface finish allows cracks to form earlier)
- When plastic deformation is present (low-cycle fatigue), cracks form relatively quickly regardless of surface finish

- Since low-cycle fatigue has little effect from surface finish, we could modify the strain life curve by altering only the elastic portion
- If we define the surface effect factor, m_s, we can find a new b_s to replace b in the strain-life equation

$$b_s = \frac{\log\left(m_s(2N_e)^b\right)}{\log(2N_e)} \tag{18.13}$$

SURFACE TREATMENTS

- Surfaces are often treated for cosmetic or corrosion purposes, these treatments can affect fatigue life
- Treatments which decrease fatigue life:
    - Electro-plating (chrome, +corrosion resistance, -fatigue life)
    - Grinding improves surface finish, but introduces surface tension, and heat generated can temper quench
    - Stamping introduces discontinuities and irregularities
    - Forging is generally good for refining grain structure and improving physical properties, but can cause decarburization in steels, which is harmful to fatigue life
    - Hot rolling can also cause decarburization

- Some treatments improve fatigue life:
  - Cold rolling improves surface finish, introduces residual compressive stress on surface (slows crack initiation on surface)
  - Shot peeing introduces many small divots on surface, which can be detrimental in corrosion, but it does cause a residual compressive stress on the surface

SIZE

- Size can also have effects on fatigue life
- Larger parts are more susceptible to damage/imperfections at the same stress level
- This is why composites are often made from very small fibers (glass fiber, carbon fiber, ceramic-matrix composites)
- The exact effect of size will depend on material, one study for low carbon steels found

$$m_d = \left(\frac{d}{25.4\text{mm}}\right)^{-0.093} \tag{18.14}$$

- Which is then used to re-calculate material constants

$$\sigma'_{fd} = m_d \sigma'_f, \qquad \epsilon'_{fd} = m_d \epsilon'_f \tag{18.15}$$

THERMAL FATIGUE

- Thermal loading can be introduced when two dissimilar parts are attached together, the coefficient of thermal expansion causes them to expand differently, introducing extra stresses due to the temperature change
- If the temperature is significantly different between two sides of a part thermal stresses can also be introduced
- Low temperatures generally cause a material to behave in a more brittle fashion, which alters the fatigue life
- High temperatures cause problems with creep-relaxation and can also affect the crystalline structure

MEAN STRESS EFFECTS

- In regions where plastic strain is significant, some applied mean stress is likely to be relaxed through cyclic plastic strain
- When the plastic strain is not significant, mean stress will exist
- Mean strain does not generally affect fatigue life

- Recall the Morrow approach for mean stress effects from the stress-based method

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f'} = 1 \tag{18.16}$$

- We can rearrange the equation such that

$$\sigma_a = \sigma_f' \left[\left(1 - \frac{\sigma_m}{\sigma_f'} \right)^{\frac{1}{b}} (2N_f) \right]^b \tag{18.17}$$

- If we compare to the stress-life equation ($\sigma_a = \sigma_f'(2N_f)^b$), we see that we can replace N_f with

$$N^* = N_f \left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{1}{b}} \tag{18.18}$$

- We can now substitute N^* for N_f in the strain-life equation to find

$$\epsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)(2N_f)^b + \epsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{c}{b}}(2N_f)^c \tag{18.19}$$

- Graphically, we can use the Morrow approach very easily using only the zero-mean stress graph
- From the zero-mean stress graph, find the point corresponding to your applied strain
- For a non zero mean stress, this point represents (ϵ_a, N^*), we can now solve for N_f using 18.18

- While the Morrow equation agrees very well with many data, some are better fit with a modification
- In the modified version, it is assumed that the mean stress has no effect on the plastic term

$$\epsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_f}{\sigma_f'}\right)(2N_f)^b + \epsilon_f'(2N_f)^c \tag{18.20}$$

- There is no convenient solution method for this form, and it generally must be solved numerically, or plotted with many families of σ_m

- The Smith, Watson, and Topper approach assumes that the life for any given state is dependent on the product $\sigma_m a x \epsilon_a$
- After some manipulation, this gives

$$\sigma_{max}\epsilon_a = \frac{\left(\sigma'_f\right)^2}{E}(2N_f)^{2b} + \sigma'_f\epsilon'_f(2N_f)^{b+c} \tag{18.21}$$

- This method can also be solved graphically if a plot of $\sigma_{max}\epsilon_a$ is made using zero-mean data. All we need to do is find the new $\sigma_{max}\epsilon_a$ point to find a new N_f

- All three methods discussed are in general use
- The Morrow method is very good for steel
- The modified Morrow method gives improved results in many materials
- The SWT approach is very good for general use, but is non-conservative with a compressive mean stress

MULTIAXIAL LOADING

- Multi-axial loading in strain-based fatigue analysis is still an active field of research
- We are currently only capable of handling proportional loads that are in-phase (i.e. have the same frequency)
- If we consider the principal directions where $\sigma_{2a} = \lambda\sigma_{1a}$, we find an expression for the strain-life as

$$\epsilon_{1a} = \frac{\frac{\sigma'_f}{E}(1-\nu\lambda)(2N_f)^b + \epsilon'_f(1-0.5\lambda)(2N_f)^c}{\sqrt{1-\lambda+\lambda^2}} \tag{18.22}$$

- Another approach is to consider the stress triaxiality factor

$$T = \frac{1+\lambda}{\sqrt{1-\lambda+\lambda^2}} \tag{18.23}$$

- Three notable cases of this are
    1. Pure planar shear: $\lambda = -1, T = 0$
    2. Uniaxial stress: $\lambda = 0, T = 1$
    3. Equal biaxial stress: $\lambda = 1, T = 2$
- Marloff suggests the following inclusion of stress triaxiality

$$\bar{\epsilon_a} = \frac{\sigma_f'}{E}(2N_f)^b + 2^{1-T}\epsilon_f'(2N_f)^c \tag{18.24}$$