

Lecture 23 - Finite Elements

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

April 30, 2020

## schedule

- 30 Apr - Finite Elements
- 5 May - Special Topics
- 7 May - Special Topics
- 14 May - Final Projects Due

## finite element techniques

---

### finite element methods in fracture

- Direct method (use near-tip stress field)
  - Requires very fine mesh near the tip to be accurate
  - Can be made feasible with specialty elements

- Crack closure method
  - An energy based method
  - Calculate energy to close crack one element away from crack tip
  - Can have a courser mesh than direct method

- J-integral method
  - Many FE codes give a convenient method to calculate the J-integral
  - Learn about this in 837, but gives a mesh-independent way to calculate stress intensity

- Cohesive elements
  - Specialty elements act like an adhesive between two materials
  - Used to model crack propagation when crack path (and material behavior) are known

- XFEM
  - eXtended Finite Element Method
  - Can predict crack growth in any direction
  - Adds extra physics model inside an element (fine mesh not necessarily required)

## direct method

- We already know that the stress field near the crack tip is

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi x}}$$

- We can solve this for  $K_I$  and we should (in theory) be able to calculate  $K_I$
- We will get a unique  $K_I$  value for every point ( $x$ ) along crack plane

8

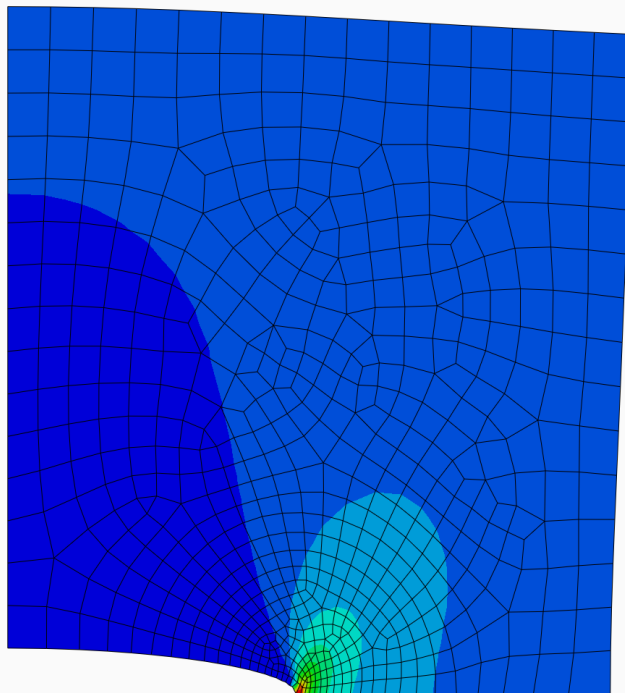
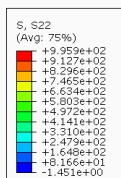
## direct method

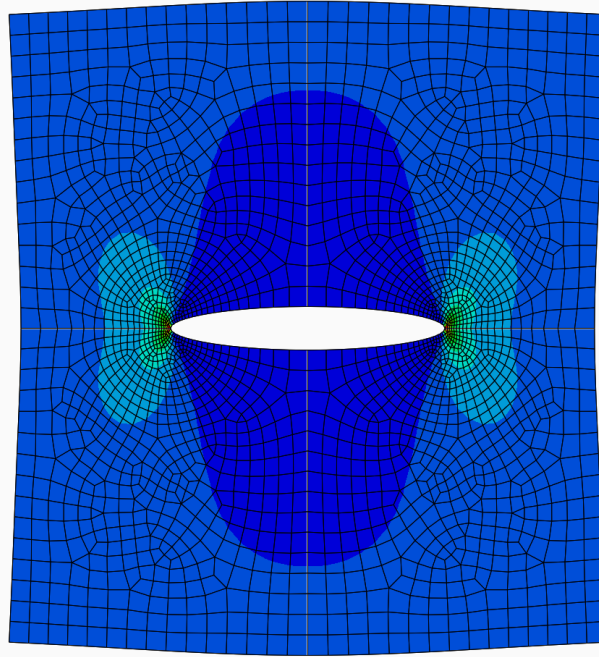
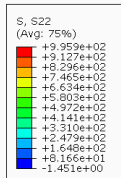
- For this method to be accurate, we need to capture the singularity at crack tip
- This requires a very fine mesh (computationally expensive)
- Alternatively, many FE packages include “singularity” elements which allow a coarse(r) mesh

9

- Use symmetry in your model to reduce node count
- Center-crack can be modeled using on 1/4 of the model
- Use biased node seeding (more nodes near tip)

## symmetry





12

## analyzing results

- If our results are accurate, we should be able to calculate the same  $K_I$  at any point
- To ensure convergence, we plot the calculated  $K_I$  vs.  $x$  (distance from crack tip)
- In the region where this plot is a horizontal line, we consider a converged  $K_I$

13

## analyzing results

- It is also possible to consider the crack opening displacement

$$u_y = \frac{K_I(\kappa + 1)}{4G\pi} \sqrt{-2\pi x}$$

- Where  $\kappa$  is to easily differentiate between plane stress and plane strain

$$\kappa = 3 - 4\nu \quad (\text{plane strain})$$

$$\kappa = \frac{3 - \nu}{1 + \nu} \quad (\text{plane stress})$$

14

## stress results

15



