

AE 737: Mechanics of Damage Tolerance

Lecture 1 - Stress Intensity

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schedule

- 22 Jan - Introduction, Stress Intensity
- 24 Jan - Common Stress Intensity Factors
- 29 Jan - Superposition, Compounding
- 31 Jan - Curved Boundaries

outline

- introduction
- syllabus
- overview
- fracture mechanics
- stress intensity
- making good plots

about me



education

- B.S. in Mechanical Engineering from Brigham Young University
 - Worked with ATK to develop tab-less gripping system for tensile testing composite tow specimens
 - Needed to align the specimen, as well as grip it without causing a stress concentration

education

- M.S. and Ph.D. from School of Aeronautics and Astronautics at Purdue University
 - Worked with Boeing to simulate mold flows
 - First ever mold simulation with anisotropic viscosity

research



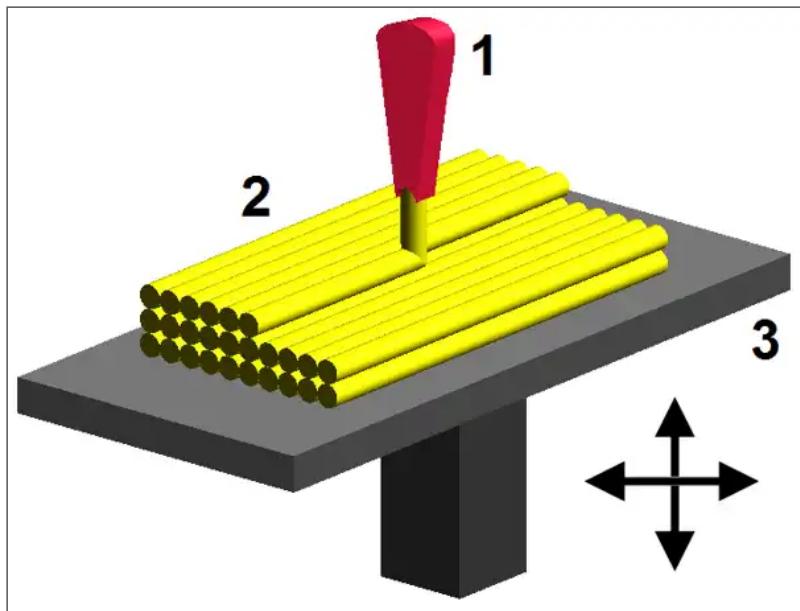
research



research

- No simulation is currently able to predict fiber orientation from these processes
- Part of the challenge is that we only have information from initial state and final state
- I want to quantify intermediate stages using a transparent mold

research



- Composites are being used in 3D printing now
- Printing patterns are optimized for isotropic materials
- Sometimes composites hurt more than they help when not utilized properly

introductions

- Name
- Student status (Undergrad, Masters, Ph.D)
- Full time or part time student?
- One interesting thing to remember you by

course textbook

- Printed notes from Dr. Bert L. Smith and Dr. Walter J. Horn
- Bring \$20 cash or check to AE offices to pick up your copy
- Make checks out to Wichita State University
- Homework will be given in handouts (via Blackboard/course website)
- Supplemental textbooks are listed in the syllabus and in the text for further study

office hours

- I will e-mail everyone in the course a Doodle link we can use to find the optimal office hours
- Let me know if you do not receive the e-mail, you may need to update your information in Blackboard
- Take advantage of office hours, this is time that I have already set aside for you
- If the regular office hours do not work for your schedule, send me an e-mail and we can work out a time to meet

tentative course outline

- Section 1 - fracture mechanics
 - Stress intensity (22 - 31 Jan)
 - Plastic zone (5 - 12 Feb)
 - Fracture toughness (12 - 19 Feb)
 - Residual strength (19 - 26 Feb)
 - Exam 1 (7 March)

tentative course outline

- Section 2 - fatigue and propagation
 - Fatigue analysis (19 - 28 Mar)
 - Crack propagation (28 Mar - 4 Apr)
 - Exam 2 (11 April)

tentative course outline

- Section 3 - damage tolerance
 - Damage tolerance (16-18 Apr)
 - Test methods
 - Finite elements
 - Non Destructive Testing
 - Special topics
 - Final project (due 16 May)

grades

Grade breakdown

- Homework 15%
- Exam 1 30%
- Exam 2 30%
- Final Project 25%

A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
93-	90-	87-	83-	80-	77-	73-	70-	67-	63-	60-	0-
100	93	90	87	83	80	77	73	70	67	63	60

final project

- Perform residual strength, fatigue and damage tolerance analysis on a real part
- Examples: car axle, fuselage panel, wing panel, landing gear, bike pedal
- Individual project
- More discussion after Exam 1

class expectations

- Consider the cost (to you or others) of your being in class
- I ask that you refrain from distracting behaviors during class
- When you have something more important than class to take care of, please take care of it outside of class

software, in-class examples

- This class will involve a great deal of multi-step calculations
- While it is possible to do these by hand on paper, I STRONGLY recommend using some form of software
- Excel, MATLAB, Python, Maple, Mathematica, etc. can all be used
- Most of my in-class demos will use Python (and will be linked in notes)

damage

- In linear elasticity, we generally consider materials in their pristine state
- Realities of manufacturing, cyclic loads, and unforeseen loads result in a material which is something other than pristine
- In this course we will develop methods for predicting the strength of a material with some damage (residual strength)
- We will learn to predict the rate at which damage will grow (fatigue)

damage tolerance

- There are many ways to address the problem of damage in a material
 1. Infinite-life design
 2. Safe-life design
 3. Damage tolerant design
- To ensure damage tolerant design, we must ensure that crack growth is always stable
- Another important concept of damage tolerant design is to include multiple load paths, so failure in one part does not cause critical failure of the whole structure

damage tolerance



damage tolerance



damage tolerance

- A B-17 collided with a German plane during WWII
- In spite of the damage, the B-17 was able to fly 90 minutes and land safely

damage tolerance



damage tolerance

- An example of multiple damaged sites occurred on a Boeing 737 in 1988
- Damage around multiple rivet holes connected and a full piece of the fuselage was blown off
- The plane was able to land safely
- This particular instance led to the study of "Multiple Site Damage"

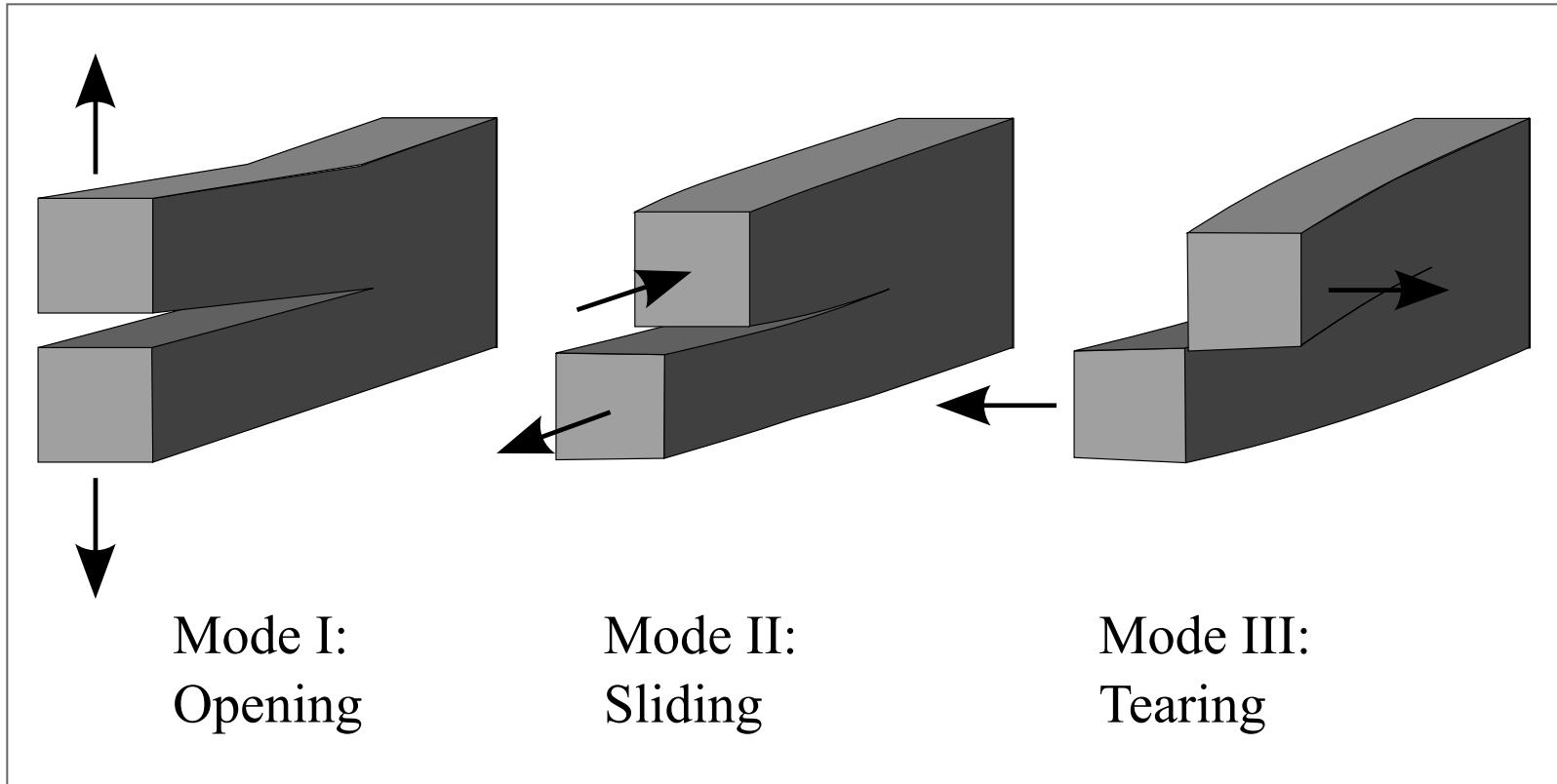
fracture mechanics

- *Linear Elastic Fracture Mechanics* is the study of the propagation of cracks in materials
- There are some corrections we add to account for plasticity
- In this course we will not follow the full mathematical development of fracture mechanics (there is a separate course dedicated to that)
- Instead we will take some results and apply them

fracture mechanics

- In fracture mechanics we consider three different modes
- Mode I is known as the "opening mode"
- Mode II is known as the "sliding mode"
- Mode III is known as the "tearing mode"

fracture mechanics



stress intensity

- A key finding from Linear Elastic Fracture Mechanics (LEFM) is known as the *Stress Intensity Factor*
- The stress intensity factor is often written in this form

$$K = \sigma \sqrt{\pi a} \beta$$

- Where K is the stress intensity factor, σ is the applied stress, a is the crack length, and β is a dimensionless parameter depending on geometry

stress Intensity

- Be careful that although the notation is similar, the *Stress Intensity Factor* is different from the *Stress Concentration Factor* from strength of materials
- We are usually most concerned with Mode I, but there will be a unique stress intensity factor for each mode, we label these K_I , K_{II} , and K_{III}
- If no subscript is given, assume Mode I

stress intensity

- For brittle materials (where "linear" fracture mechanics assumptions hold true) we can find the full stress field near the crack in terms of the stress intensity factor

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$

mode II

- Similarly for Mode II we find

$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left(2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right)$$

$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

mode III

- And for Mode III

$$\tau_{xz} = \frac{-K_{III}}{\sqrt{2\pi r}} \sin \frac{\theta}{2}$$

$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2}$$

plotting

- Plotting is an important part of graduate work, and this course
- There are many software programs which can generate good scientific plots
 - Microsoft Excel
 - MATLAB
 - Maple
 - Mathematica
 - Python
 - R
 - Plot.ly

plotting

- You are welcome to use whatever software you desire, I will use Python for a quick demonstration
- This demo is accessible [**here**](#)

plotting

- To make a good scientific plot, we must first decide what to plot, and which plot style will best illustrate our data
- Let us consider the Mode I stress near a crack tip

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$

plotting

- One interesting plot could be to examine stress magnitudes along the crack propagation direction as we get farther away from the crack
- In this case we would have $\theta = 0$.
- Since θ is a constant, it is not ideal to use a polar plot, instead we will use a standard rectangular plot

plotting

- Since we are looking at stresses near the crack tip, it is convenient to normalize the distance by the crack length
- If substitute for θ and K_I we have

$$\sigma_x = \frac{\sigma\sqrt{\pi a}\beta}{\sqrt{2\pi r}}$$

$$\sigma_y = \frac{\sigma\sqrt{\pi a}\beta}{\sqrt{2\pi r}}$$

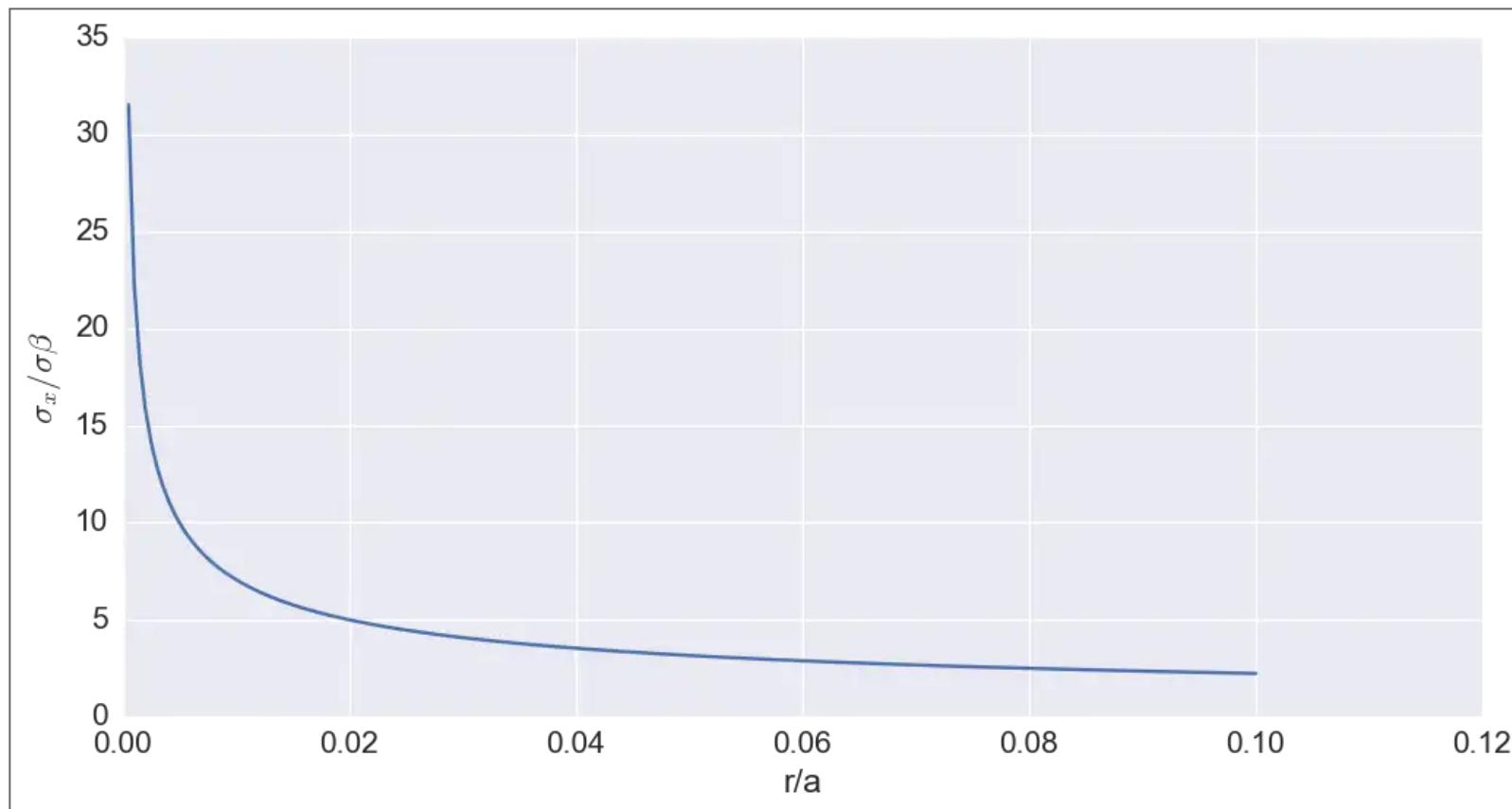
$$\tau_{xy} = 0$$

plotting

- Since σ_x and σ_y are identical for this case, we consider only one, and normalize by the applied stress. After simplification

$$\frac{\sigma_x}{\sigma\beta} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{(r/a)}}$$

plotting



plotting

- Since we found $\sigma_x = \sigma_y$ for $\theta = 0$, we decide it might be better to look at a polar plot using θ as a variable
- To make a polar plot in this style, we need a function such that $r = f(\theta)$
- To do this we consider a constant stress value, we will solve for and plot the distance, r at which the stress is equal to the same constant value for each of the three stress terms

plotting

$$\sigma_x = C = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_y = C = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = C = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$

plotting

- After solving for r we find

$$r = \frac{K_I^2}{2C^2\pi} \cos^2 \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)^2$$

$$r = \frac{K_I^2}{2C^2\pi} \cos^2 \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)^2$$

$$r = \frac{K_I^2}{2C^2\pi} \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2} \cos^2 \frac{3\theta}{2}$$

plotting

