AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 12

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SCHEDULE

- 1 Mar - Section 1 Review, Homework 5 Due
- 3 Mar - Section 1 Review, Homework 5 return
- 8 Mar - Exam 1
- 10 Mar - Exam return, Final Project discussion

OUTLINE

1. mixed mode fracture
2. exam
3. stress intensity
4. plastic zone

MIXED MODE FRACTURE

- Most cracks are primarily Mode I, but sometimes Mode II can also have an effect

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- We can look at the combined stress field for Mode I and Mode II
- Recall the stress field near the crack tip

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{12.1a}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{12.1b}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{12.1c}$$

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$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \tag{12.2a}$$

$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{12.2b}$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{12.2c}$$

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$$\sigma_r = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta \tag{12.3a}$$

$$\sigma_\theta = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta \tag{12.3b}$$

$$\tau_{r\theta} = -\sigma_x \sin\theta \cos\theta + \sigma_y \sin\theta \cos\theta + \tau_{xy}(\cos^\theta - \sin^2\theta) \tag{12.3c}$$

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$$\sigma_r = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right) \tag{12.4a}$$

$$\sigma_\theta = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right) \tag{12.4b}$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right) \tag{12.4c}$$

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- Thus fracture begins when

$$\sigma_\theta(\theta_P) = \sigma_\theta(\theta = 0, K_{II} = 0, K_I = K_{Ic}) = \frac{K_{IC}}{\sqrt{2\pi r}} \tag{12.5}$$

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$$4K_{IC} = K_I\left(3\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\right) - 3K_{II}\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right) \tag{12.7}$$

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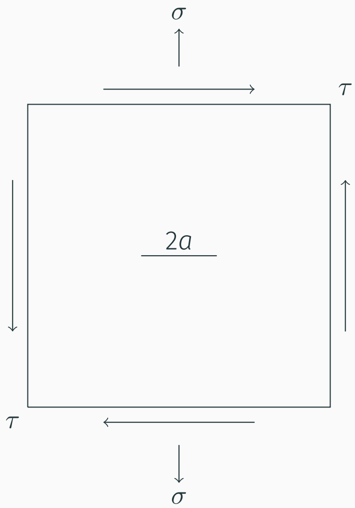
- The general form for a Mode II stress intensity factor is

$$K_{II} = \tau\sqrt{\pi a}\beta' \tag{12.8}$$

EXAMPLE

Assuming $|\sigma| = 4|\tau|$, $K_{IC} = 60$ ksi$\sqrt{\text{in}}$, and $2a = 1.5$ in.

Note: Assume $\beta = \beta' = 1$



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$$0 = \sigma_\theta dA - \sigma_x dA \sin^2\theta - \sigma_y dA \cos^2\theta + 2\tau_{xy} dA \cos\theta \sin\theta \tag{12.9a}$$

$$\sigma_\theta = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta \tag{12.9b}$$

$$\frac{\partial \sigma_\theta}{\partial \theta} = (\sigma_x - \sigma_y) \sin 2\theta_p - 2\tau_{xy} \cos 2\theta_P \tag{12.9c}$$

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{12.9d}$$

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- We then find the remote failure stress by

$$\sigma_c = \frac{K_{IC}}{C\sqrt{\pi a}\beta} \tag{12.11}$$

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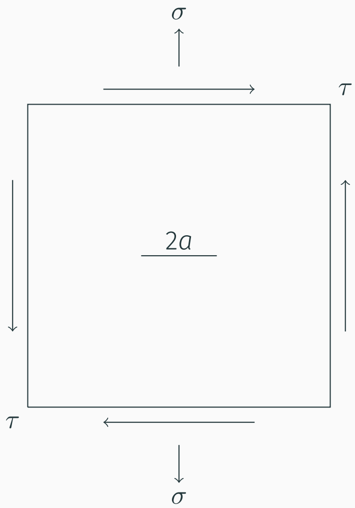

EXAM

EXAM FORMAT

- 4-5 questions
- Closed book, notes
- Equation sheet provided
- No T/F section, but the T/F questions in text can still be helpful
- Like homework, but simpler calculations
- In-class group review problems are also good practice for exam

- Stress Intensity

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    - Geometry-specific formulas
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    - Superposition
    - Cracks on curved boundary

- Plastic Zone

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    - Effective crack length
    - Plane strain vs. plane stress
    - Plastic zone shape

- Fracture Toughness

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    - Plane strain fracture toughness - material property
    - Plane stress fracture toughness - not
    - Crack resistance curve
    - General test/analysis methodology (ASTM)
    - Thickness effects

- Residual Strength

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- Stiffeners

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    - Panel, stiffener, rivet residual strength

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    - Panel, stiffener, rivet residual strength
    - Positive slope in residual strength curve
    - Analyze residual strength curves

- Multiple site damage

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    - Modified link-up equation
    - Ductile/tough vs. stiff/brittle materials

- Mixed-mode fracture

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    - Maximum Circumferential Stress

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    - Maximum Circumferential Stress
    - Principal Stress

- Mixed-mode fracture
    - Maximum Circumferential Stress
    - Principal Stress
    - Why is principal stress method bad?

STRESS INTENSITY

- Different types of boundaries create different correction factors to the usual stress intensity factor
- We often use β to indicate the total correction factor
- When multiple boundaries are present, we can combine them into one effective correction factor
- There are two general methods we use to create a compound correction factor

- The first method uses linear superposition, and thus is restricted to cases where the effect of each boundary can be assumed to add linearly
- While in most cases this is not strictly true, it provides a reasonable approximation

$$K_r = \bar{K} + \sum_{i=1}^{N}(K_i - \bar{K}) \tag{12.12}$$

- Where N is the number of boundaries, $\bar{K}$ is the stress intensity factor with no boundaries present and K_i is the stress intensity factor associated with the i^{th} boundary.

- We can rewrite this equation as

$$K_r = \sigma\sqrt{\pi a}\beta_r = \sigma\sqrt{\pi a} + \sum_{i=1}^{N}(\sigma\sqrt{\pi a}\beta_i - \sigma\sqrt{\pi a}) \tag{12.13}$$

- Which leads to an expression for β_r as

$$\beta_r = 1 + \sum_{i=1}^{N}(\beta_i - 1) \tag{12.14}$$

- An alternative empirical method approximates the boundary effect as

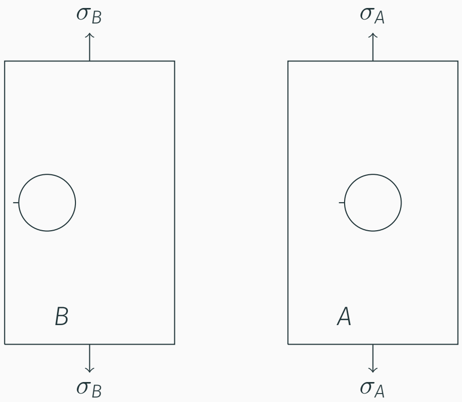
$$\beta_r = \beta_1 \beta_2 ... \beta_N \tag{12.15}$$

- If there is no interaction between the boundaries, method 1 and method 2 will give the same result

SHORT CRACKS ON CURVED BOUNDARIES

- For short cracks, we can use the *stress concentraction factor* on a curved boundary to determine the stress intensity factor
- The stress intensity factor only gives the maximum stress at the curved boundary, thus the longer the crack is, the farther away from the curved boundary (and maximum stress) it is.

- Suppose we want to determine the stress intensity on a panel, panel B
- We find a similar panel with a known stress intensity factor, panel A
- We adjust the applied load on panel A such that $K_{I,A} = K_{I,B}$
- The magnitude of this load adjustment is determined using the *stress concentration factors* in panels B and A
- Note the notation: K_t for stress concentration factor, K_I for stress intensity factor


σ_B
σ_A
B
A
σ_B
σ_A

- Since A is a fictional panel, we set the applied stress, σ_A such that

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- Solving for σ_A

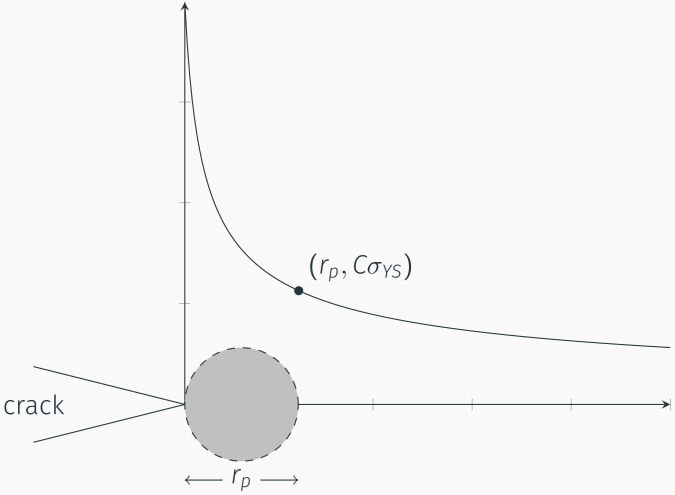
$$\sigma_A = \frac{K_{tB}}{K_tA}\sigma_B$$

- Since the crack is short and $\sigma_{max,A} = \sigma_{max,B}$ we can say

$$\begin{aligned} K_{I,B} &= K_{I,A} \\ &= \sigma_A \sqrt{\pi c} \beta_A \\ &= \frac{K_{t,B}}{K_{t,A}} \sigma_B \sqrt{\pi c} \beta_A \end{aligned}$$

ON-BOARD EXAMPLE

PLASTIC ZONE


$(r_p, C\sigma_{YS})$
crack
r_p

- We use C "Plastic Constraint Factor" to convert between Plane Strain and Plane Stress solutions
- The plastic zone size can now be approximated by solving the equation $\sigma_{yy}(r = r_p) = C\sigma_{YS}$

IRWIN'S FIRST APPROXIMATION

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- The plastic zone size can now be approximated by solving the equation $\sigma_{yy}(r = r_p) = C\sigma_{YS}$

$$\sigma_{yy}(r = r_p) = C\sigma_{YS} \tag{12.16a}$$

$$\frac{K_I}{\sqrt{2\pi r_p}} = C\sigma_{YS} \tag{12.16b}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{C\sigma_{YS}}\right)^2 \tag{12.16c}$$

- For plane stress (thin panels) we let $C = 1$ and find r_p as

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{12.17}$$

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- And for plane strain (thick panels) we let $C = \sqrt{3}$ and find

$$r_p = \frac{1}{6\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{12.18}$$

- For panels which lie between plane strain and plane stress states, we use the following expression to estimate the plastic zone size

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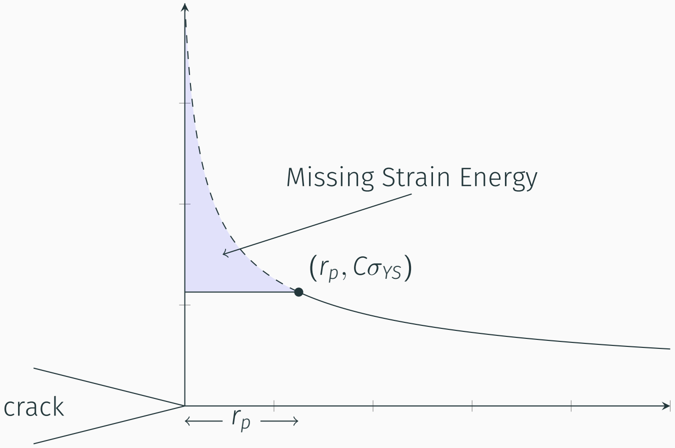
- Where I is defined as

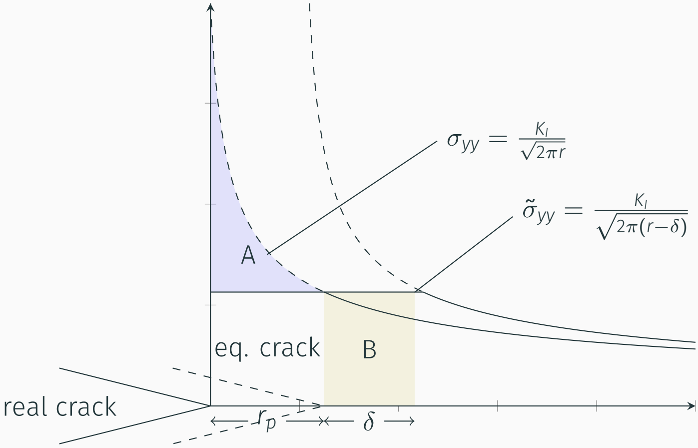
$$I = 6.7 - \frac{1.5}{t}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{12.20}$$

- And $2 \leq I \leq 6$

- If our material is perfectly elastic-plastic, no stresses above $C\sigma_{YS}$ will exist in the material
- This ignores the strain energy (represented by the area under the curve) in the plastic zone


Missing Strain Energy
$(r_p, C\sigma_{YS})$
crack
r_p

- To account for the additional strain energy, Irwin considered a plastic zone size increased by some δ
- He also needed to adjust the stress function, and considered an equivalent crack tip in these calculations


$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$
$\tilde{\sigma}_{yy} = \frac{K_I}{\sqrt{2\pi(r-\delta)}}$
A
B
eq. crack
real crack
r_p
δ

- We need $A = B$, so we set them equivalent and solve for δ.

$$A = \int_0^{r_p} \sigma_{yy} dr - r_p \sigma_{YS} \tag{12.21a}$$
$$= \int_0^{r_p} \frac{K_I}{\sqrt{2\pi r}} dr - r_p \sigma_{YS} \tag{12.21b}$$
$$= \frac{K_I}{\sqrt{2\pi}} \int_0^{r_p} r^{-1/2} dr - r_p \sigma_{YS} \tag{12.21c}$$
$$= \frac{2K_I \sqrt{r_p}}{\sqrt{2\pi}} - r_p \sigma_{YS} \tag{12.21d}$$

- We have already found r_p as

$$r_p = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_{YS}} \right)^2 \tag{12.21e}$$

- If we solve this for K_I we find

$$K_I = \sqrt{2\pi r_p} \sigma_{YS} \tag{12.21f}$$

- We can now substitute back into the strain energy of A

$$A = \frac{2\sqrt{2\pi r_p}\sigma_{YS}\sqrt{r_p}}{\sqrt{2\pi}} - r_p\sigma_{YS} \tag{12.21g}$$

$$= 2\sigma_{YS}r_p - r_p\sigma_{YS} \tag{12.21h}$$

$$= r_p\sigma_{YS} \tag{12.21i}$$

- B is given simply as $B = \delta\sigma_{YS}$, so we equate A and B to find δ

$$A = B \tag{12.21j}$$

$$r_p\sigma_{YS} = \delta\sigma_{YS} \tag{12.21k}$$

$$r_p = \delta \tag{12.21l}$$

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- However, it also means that the effective crack length is $a + r_p$

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- Since r_p depends on K_I, we must iterate a bit to find the ”real” r_p and K_I