AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 16

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SCHEDULE

- 24 Mar - Stress based fatigue
- 29 Mar - Influence of notches on fatigue, Homework 7 assigned, Homework 6 due
- 31 Mar - Strain based fatigue, project abstract due
- 5 Apr - Strain based fatigue, Homework 7 due

EXTRA CREDIT

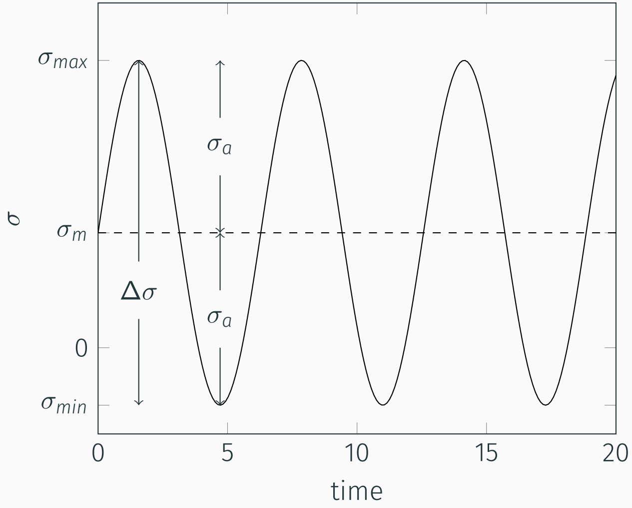
- The extra credit opportunity is still open
- There has been some confusion about the Google Sheets page
- The page numbers listed are already being worked on or have been completed
- If you want to digitize a plot, find a figure not listed on the Google Sheet and add it to the sheet
- Once you have e-mailed me the data, I will put an ”x” in the completed column and record your extra credit points
- For now I do not have a due date for this project, but I may eventually allow students who have already completed a chart to complete a second one

OUTLINE

1. fatigue review
2. modeling real loads
3. mean stress effects
4. scatter
5. general stress

FATIGUE REVIEW

- One of the simplest assumptions we can make is that a load cycles between a constant maximum and minimum stress value
- This is a good approximation for many cases (axles, for example) and can also be easily replicated experimentally
- This is referred to as constant amplitude stressing


σmax
σa
σ
σm
Δσ
σa
0
σmin
0
5
10
15
20
time

- $\Delta\sigma$ is known as the stress range, and is the difference between max and min stress
- σ_m is the mean stress, and can sometimes be zero, but this is not always the case
- σ_a is the stress amplitude, and is the variation about the mean
- We can express all of these in terms of the maximum and minimum stress

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \tag{16.1}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \tag{16.2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{16.3}$$

- It is important to distinguish between the nominal (global) stress and the local stress at some point of interest
- We use σ for the stress at a point (local stress)
- We use S as the nominal (global) stress
- In simple tension, $\sigma = S$
- For many cases (bending, notches), $\sigma \neq S$ in general
- We must also be careful to note σ_y, in some cases $S < \sigma_y$ but at some locations $\sigma > \sigma_y$

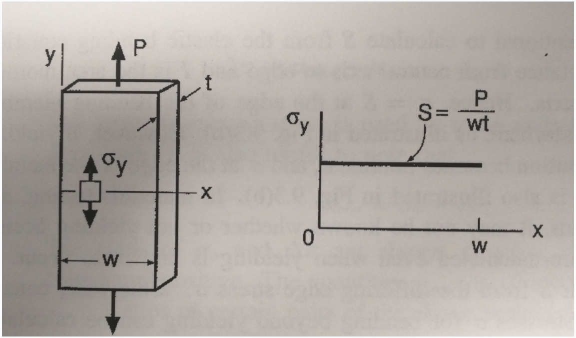


Figure 1: In this case $S = \sigma$

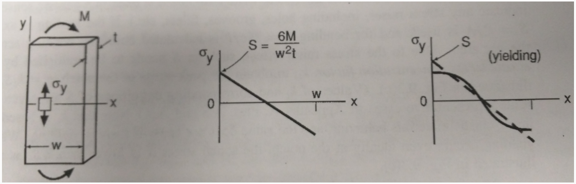


Figure 2: As long as $\sigma < \sigma_y$, σ varies linearly. If $\sigma > \sigma_y$ at any location, however, the relationship is non-linear

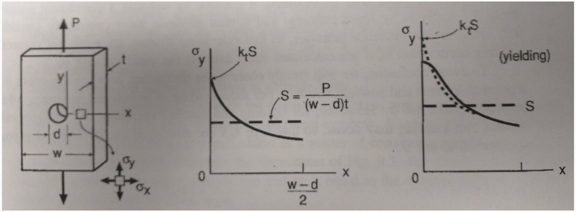
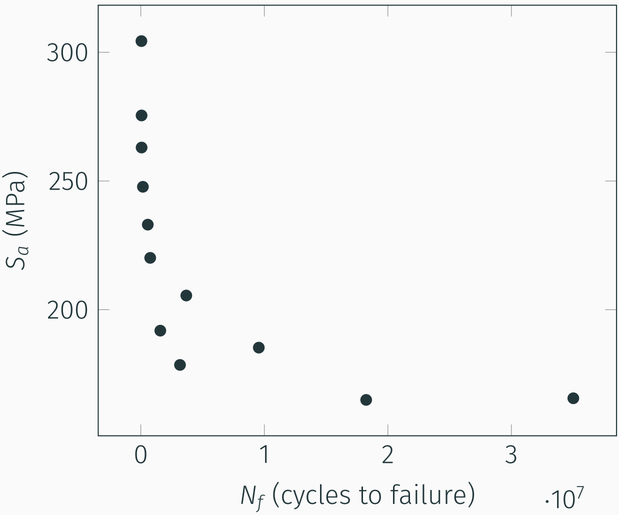
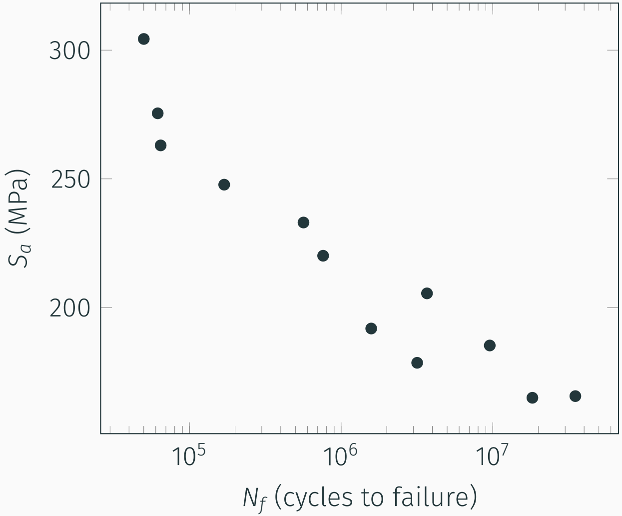


Figure 3: As long as $\sigma < \sigma_y$, σ varies linearly. If $\sigma > \sigma_y$ at any location, however, the relationship is non-linear

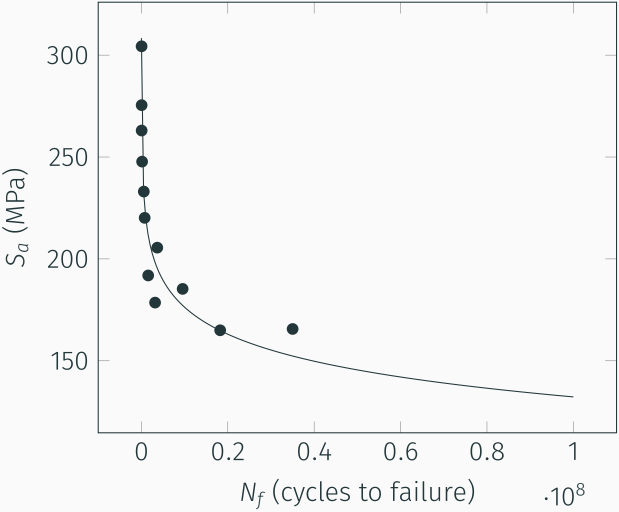
- The length of a fatigue test is determined by two factors
  1. How many cycles it takes for the specified load to cause failure
  2. The speed of the motor controlling the test
- Servohydraulic machines generally have a speed of 10 - 100 Hz.
- At a speed of 100 Hz, it would take 28 hours for 10^7 cycles, 12 days for 10^8 cycles, and nearly 4 months for 10^9 cycles
- While some machines can test at very high speeds, the inertia of the sample can interfere with results

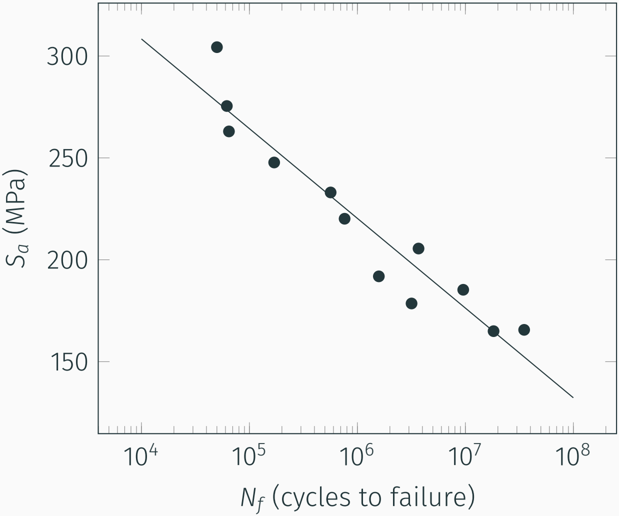

300
250
200
S_a (MPa)
0
1
2
3
N_f (cycles to failure)
$\cdot 10^7$

- On a linear scale, the data appear not to agree well with any standard fit
- It is also very difficult to differentiate between low-cycle fatigue failure stresses
- Instead S-N curves are often plotted on a semi-log or log-log scale, so pay attention to the axes


300
250
200
S_a (MPa)
10^5
10^6
10^7
N_f (cycles to failure)

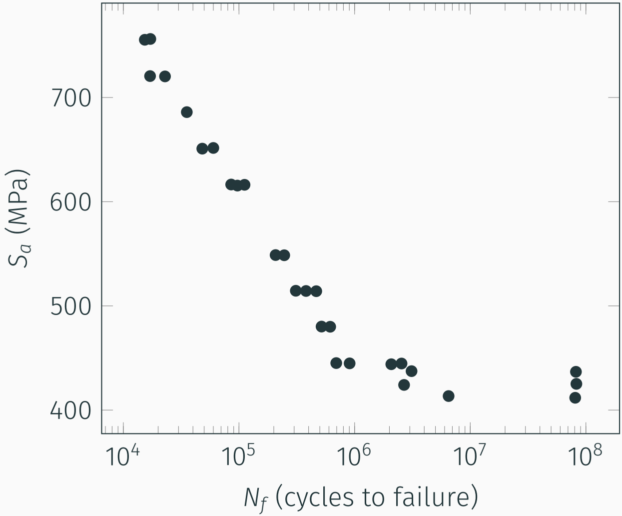
- If the curve is nearly linear on a log-linear plot, we use the following form to fit the data
- $$\sigma_a = C + D \log N_f \tag{16.4}$$
- When the data are instead linear on a log-log scale, the following form is generally used
- $$\sigma_a = \sigma_f' \left(2N_f\right)^b \tag{16.5}$$
- σ_f' and b are often considered material properties and can often be looked up on a table (p. 235)


300
250
200
150
S_a (MPa)
0
0.2
0.4
0.6
0.8
1
N_f (cycles to failure)
$\cdot 10^8$



S_a (MPa)
300
250
200
150
10^4
10^5
10^6
10^7
10^8
N_f (cycles to failure)

- The fatigue limit, or endurance limit, is a feature of some materials where below a certain stress, no fatigue failure is observed
- Below the fatigue limit, this material is considered to have infinite life
- This most notably occurs in plain-carbon and low-alloy steels
- In these materials, σ_e is considered to be a material property
- This phenomenon is not typical of aluminum or copper alloys, but is sometimes arbitrarily assigned using whatever the failure stress is at some large number of cycles (10^7 or 10^8)


S_a (MPa)
700
600
500
400
10^4
10^5
10^6
10^7
10^8
N_f (cycles to failure)

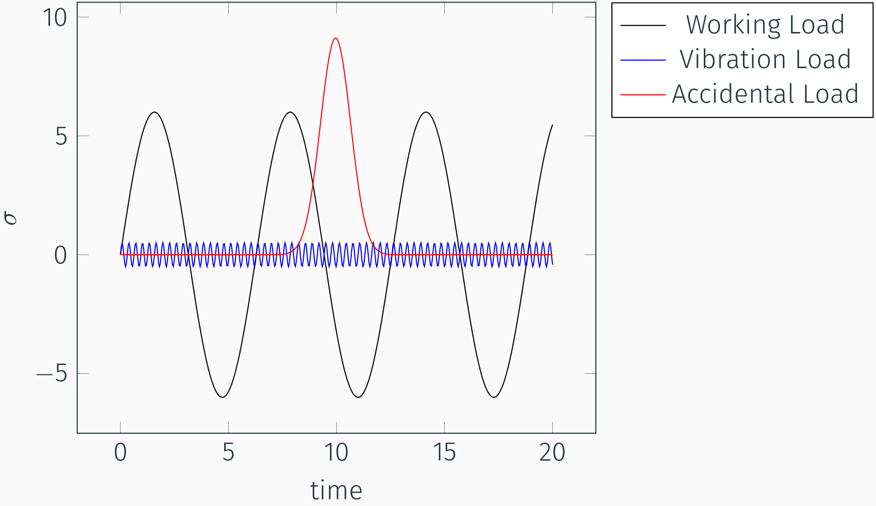
- Some other important terms are high cycle fatigue and low cycle fatigue
- ”High cycle fatigue” generally is considered anything above 10^3 cycles, but varies somewhat by material
- High cycle fatigue occurs when the stress is sufficiently low that yielding effects do not dominate behavior
- When yielding effects do dominate behavior, the strain-based approach is more appropriate

MODELING REAL LOADS

REAL LOADS

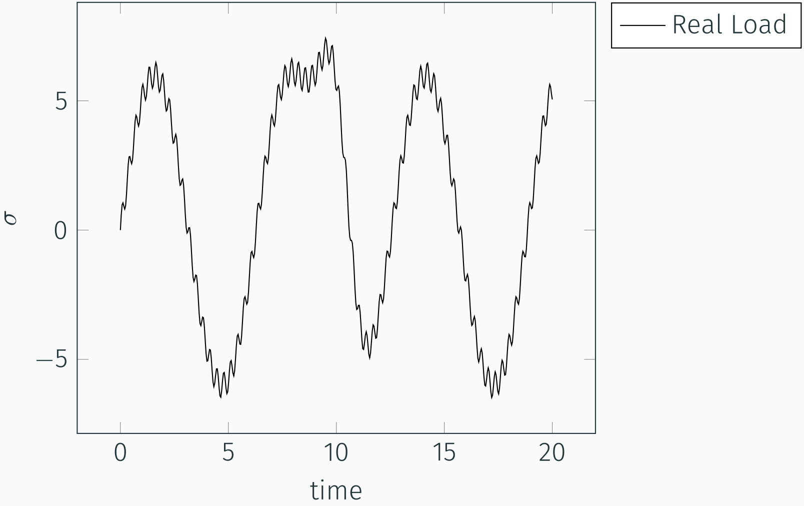
- Static loads are constant and do not vary. While they are not generally considered ”fatigue” loads, they can be present during fatigue loads, which will change the response.
- Working loads change with time as a function of the normal operation of a component
- Vibratory loads occur at a higher frequency than working loads and may be caused by the environment or secondary effects of normal operation.
- Accidental loads can occur at a much lower frequency than working loads

REAL LOADS



10
5
0
−5
σ
0
5
10
15
20
time
Working Load
Vibration Load
Accidental Load

REAL LOADS



- We know that variable loads can often occur in real scenarios, but how can we model the effect?
- Miner's Rule is often used to approximate the effect of variable amplitude load
- We consider each load amplitude (and the number of cycles at that amplitude) as having used up a percentage of a part's life

$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \ldots = \sum \frac{N_i}{N_{fi}} = 1 \tag{16.6}$$

- Often there are ”blocks” of variable amplitude loads which repeat
- A typical flight cycle is a good example of this
- A flight will have working loads, vibrations, as well as storms/turbulence, but each flight should have similar loading
- If we call the number of ”block” B then we have

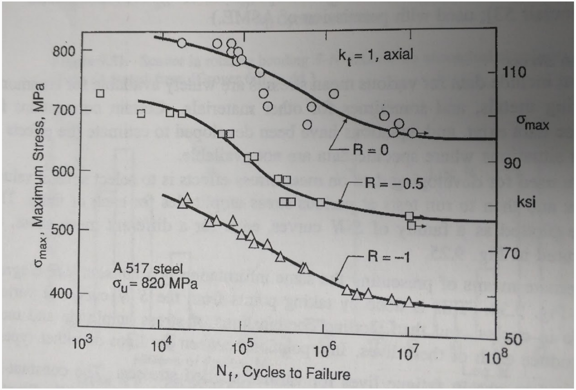
$$B\left[\sum \frac{N_i}{N_{if}}\right]_{rep} = 1 \tag{16.7}$$

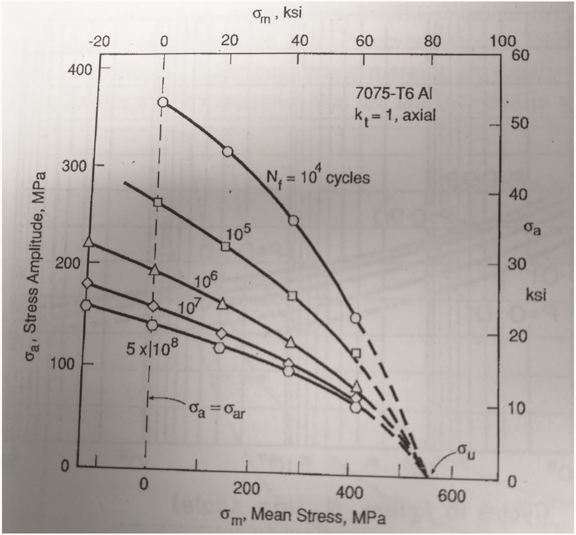
VARIABLE AMPLITUDE LOAD EXAMPLE

- It is possible for each variable load case to have a different mean stress
- This would mean generating a different S-N curve for each potential mean stress
- Much work has been done to instead convert a zero-mean stress curve to different mean stress amplitudes

MEAN STRESS EFFECTS

- Since mean stress has an effect on fatigue life, sometimes a family of S-N curves at varying mean stress values is created
- S-N curves for these are reported in different ways, but commonly σ_{max} replaces σ_a on the y-axis
- One useful way of representing these data, instead of many S-N curves, is a constant-life diagram
- It is created by taking points from the S-N curves and plotting a line through constant N_f values

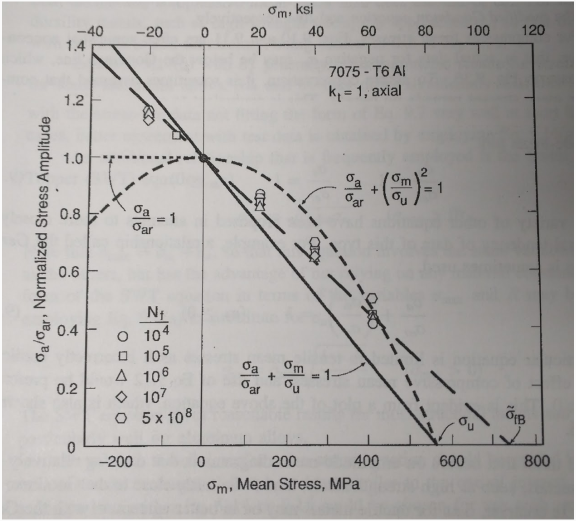

k_t = 1, axial
R = 0
R = −0.5
R = −1
A 517 steel
σ_u = 820 MPa
σ_{max}, Maximum Stress, MPa
800
700
600
500
400
10^3
10^4
10^5
10^6
10^7
10^8
N_f, Cycles to Failure
110
90
70
50
σ_{max}
ksi



σ_m , ksi
-20
0
20
40
60
80
100
60
400
7075-T6 Al
k_t = 1, axial
50
300
$N_f = 10^4$ cycles
40
σ_a
10^5
200
30
10^6
ksi
10^7
20
5×10^8
100
$\sigma_a = \sigma_{ar}$
10
σ_u
0
0
0
200
400
600
σ_m, Mean Stress, MPa
σ_a , Stress Amplitude, MPa

- One very useful way to plot this data is to normalize the amplitude by the zero-mean amplitude
- We call the zero-mean amplitude as σ_{ar}
- Plotting σ_a/σ_{ar} vs. σ_m provides a good way to group all the data together on one plot with the potential to fit a curve

NORMALIZED AMPLITUDE-MEAN DIAGRAM



- The first work on this problem was done by Goodman, who proposed the line

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1 \tag{16.8}$$

- This equation can also be used for fatigue limits, since they are just a point on the S-N curves

$$\frac{\sigma_e}{\sigma_{er}} + \frac{\sigma_m}{\sigma_u} = 1 \tag{16.9}$$

- While the Goodman line gives a good approximation to convert non-zero mean stress S-N curves, it is somewhat overly conservative at high mean stresses
- It is also non-conservative for negative mean stresses
- An alternative fit is known as the Gerber Parabola

$$\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1 \tag{16.10}$$

- In general, the Goodman line is a good fit for brittle materials (steels) while the Gerber parabola is a better fit for more ductile materials

- The Goodman line can also be improved by replacing σ_u with the corrected true fracture strength $\tilde{\sigma}_{fB}$ or the constant σ_f' from the S-N curve fit

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f'} = 1 \tag{16.11}$$

- This is known as the Morrow Equation
- For steels, $\sigma_f' \approx \tilde{\sigma}_{fB}$, but for aluminums these values can be significantly different, and better agreement is found using $\tilde{\sigma}_{fB}$.

- One more relationship that has shown particularly good results with aluminum alloys is the Smith, Watson, and Topper equations (SWT)

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a} \tag{16.12}$$

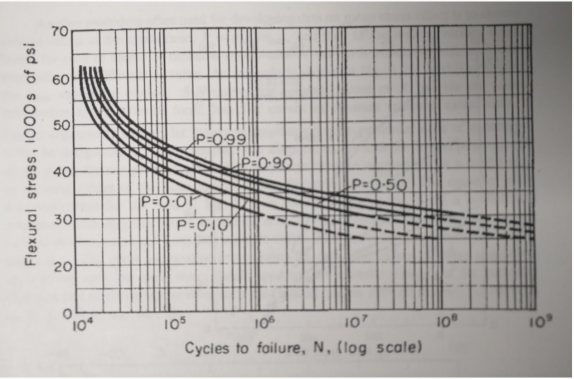
- In general, it is best to use a form that matches your data
- If data is lacking, the SWT (16.12) and Morrow (16.11) equations generally provide the best fit

VARIABLE AMPLITUDE LOAD WITH MEAN STRESS

SCATTER

- One of the challenges with fatigue is that there is generally considerable scatter in the data
- Quantifying this scatter requires many repetitions, which makes for time consuming tests
- In general, the scatter follows a lognormal distribution (or a normal distribution in $\log(N_f)$)

S-N-P CURVE


Flexural stress, 1000s of psi
70
60
50
40
30
20
0
P=0·99
P=0·90
P=0·50
P=0·01
P=0·10
10^4
10^5
10^6
10^7
10^8
10^9
Cycles to failure, N, (log scale)

GENERAL STRESS

- Often combined loads from different sources introduce stresses which are not uni-axial
- For ductile materials, good agreement has been found using an effective stress amplitude, similar to the octahedral shear yield criterion

$$\bar{\sigma}_a = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{xa}-\sigma_{ya})^2+(\sigma_{ya}-\sigma_{za})^2+(\sigma_{za}-\sigma_{xa})^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)} \tag{16.13}$$

- The effective mean stress is given by

$$\bar{\sigma}_m = \bar{\sigma}_{xm}+\bar{\sigma}_{ym}+\bar{\sigma}_{zm} \tag{16.14}$$

- This effective stress can be used in all other relationships, including mean stress relationships
- Note that mean shear stress has no effect on the effective mean stress
- This is surprising, but agrees well with experiments