AE 737: Mechanics of Damage Tolerance

Lecture 14 - Stress based fatigue

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

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schedule

- 22 Mar - Stress-based Fatigue
- 24 Mar - Stress-based Fatigue
- 26 Mar - Project Abstract Due
- 29 Mar - Strain-based Fatigue
- 31 Mar - Crack Growth
- 2 Apr - Homework 6 Due

outline

- fatigue
- nominal and local stress
- fatigue tests
- fatigue life analysis

fatigue

fatigue

- We refer to damage from repeated, or cyclic loads as fatigue damage
- Some of the earliest work on fatigue began in the 1800's
- Chains, railway axles, etc.
- An estimated 80% of failure expenses are due to fatigue

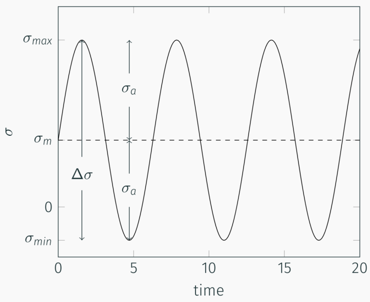
fatigue

- There are three main approaches to fatigue analysis
  - Stress based fatigue analysis
  - Strain based fatigue analysis
  - Fracture mechanics fatigue analysis

stress based fatigue

- One of the simplest assumptions we can make is that a load cycles between a constant maximum and minimum stress value
- This is a good approximation for many cases (axles, for example) and can also be easily replicated experimentally
- This is referred to as constant amplitude stressing

constant amplitude stressing



constant amplitude stressing

- $\Delta\sigma$ is known as the stress range, and is the difference between max and min stress
- σ_m is the mean stress, and can sometimes be zero, but this is not always the case
- σ_a is the stress amplitude, and is the variation about the mean

constant amplitude stressing

- We can express all of these in terms of the maximum and minimum stress

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$
$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

constant amplitude stressing

- It is also common to describe some ratios
- The stress ratio, R is defined as

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

- And the amplitude ratio, A is defined as

$$A = \frac{\sigma_a}{\sigma_m}$$

useful relations

- There are some useful relationships between the above equations

$$\Delta\sigma = 2\sigma_a = \sigma_{max}(1 - R)$$
$$\sigma_m = \frac{\sigma_{max}}{2}(1 + R)$$
$$R = \frac{1 - A}{1 + A}$$
$$A = \frac{1 - R}{1 + R}$$

nominal and local stress

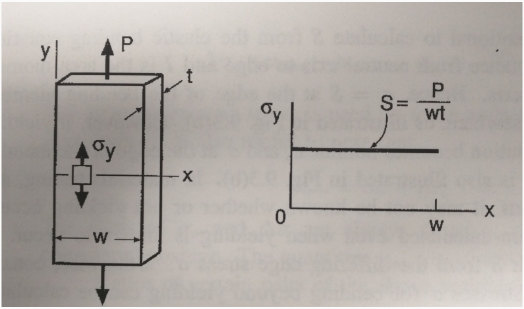
definition and notation

- It is important to distinguish between the nominal (global) stress and the local stress at some point of interest
- We use σ for the stress at a point (local stress)
- We use S as the nominal (global) stress
- In simple tension, $\sigma = S$

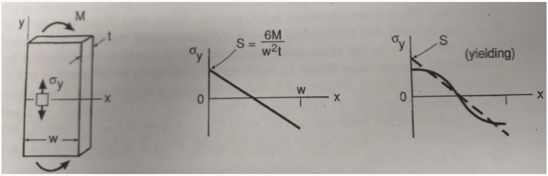
notation

- For many cases (bending, notches), $\sigma \neq S$ in general
- We must also be careful to note σ_y, in some cases $S < \sigma_y$ but at some locations $\sigma > \sigma_y$

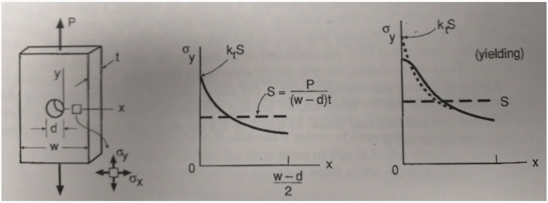
simple tension



bending



notches



fatigue life analysis

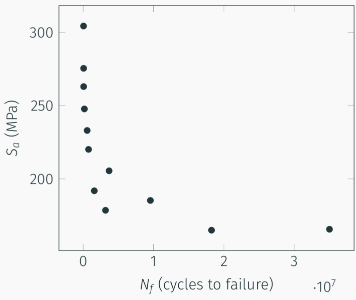
stress life curves

- Stress-life curves, or S-N curves, are generated from test data to predict the number of cycles to failure
- In general, one set (or family) of S-N curves is generated using the same σ_m
- Usually Sa (the nominal stress equivalent of σ_a) is plotted versus N (the number of cycles)

stress life curves

- Each individual point on an S-N curve represents one fatigue experiment
- To find enough data to form statistical significance, as well as to fit a curve across all levels of fatigue is very time-consuming
- In the following plot, if only one test was performed for each point, the total number of cycles tested would be about 7.3*x*107
- For a 100 Hz machine, this represents over 200 hours of consecutive testing

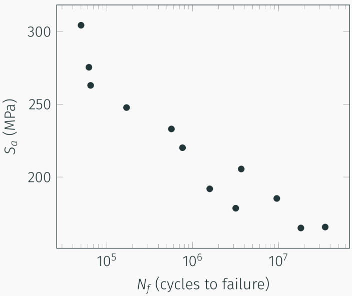
stress life curves



stress life curves

- On a linear scale, the data appear not to agree well with any standard fit
- It is also very difficult to differentiate between low-cycle fatigue failure stresses
- Instead S-N curves are often plotted on a semi-log or log-log scale, so pay attention to the axes

stress life curves



curve fits

- If the curve is nearly linear on a log-linear plot, we use the following form to fit the data

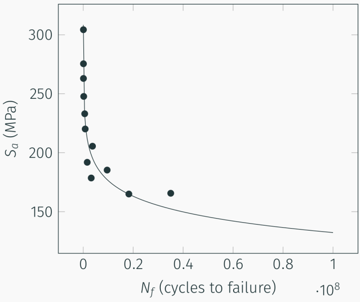
$$\sigma_a = C + D \log N_f$$

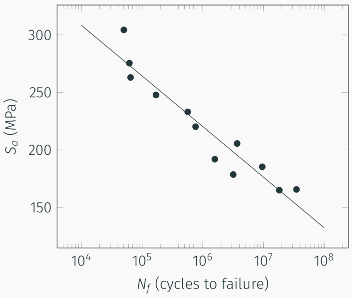
- When the data are instead linear on a log-log scale, the following form is generally used

$$\sigma_a = \sigma_f'(2N_f)^b$$

- σ_f' and b are often considered material properties and can often be looked up on a table (p. 235)

curve fit





fatigue limit

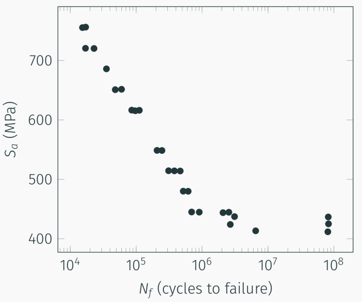
fatigue limit

- The fatigue limit, or endurance limit, is a feature of some materials where below a certain stress, no fatigue failure is observed
- Below the fatigue limit, this material is considered to have infinite life
- This most notably occurs in plain-carbon and low-alloy steels
- In these materials, σ_e is considered to be a material property

fatigue limit

- This phenomenon is not typical of aluminum or copper alloys
- It is sometimes arbitrarily assigned using whatever the failure stress is at some large number of cycles (107 or 108)

fatigue limit



high and low cycle fatigue

- Some other important terms are high cycle fatigue and low cycle fatigue
- "High cycle fatigue" generally is considered anything above 1000 cycles, but varies somewhat by material

high and low cycle fatigue

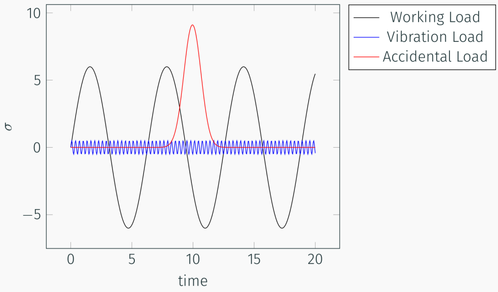
- High cycle fatigue occurs when the stress is sufficiently low that yielding effects do not dominate behavior

modeling real loads

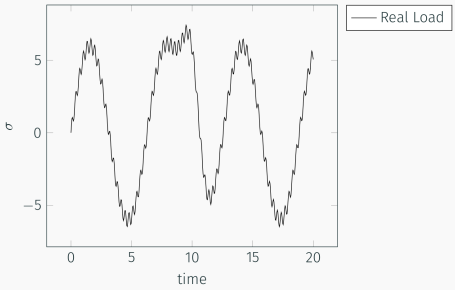
real loads

- Static loads are constant and do not vary. While they are not generally considered "fatigue" loads, they can be present during fatigue loads, which will change the response.
- Working loads change with time as a function of the normal operation of a component
- Vibratory loads occur at a higher frequency than working loads and may be caused by the environment or secondary effects of normal operation.
- Accidental loads can occur at a much lower frequency than working loads

real loads



real loads



simplified load sketch book p 239

effect of variable amplitude

- We know that variable loads can often occur in real scenarios, but how can we model the effect?
- Miner's Rule is often used to approximate the effect of variable amplitude load
- We consider each load amplitude (and the number of cycles at that amplitude) as having used up a percentage of a part's life

$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + ... = \sum \frac{N_i}{N_{fi}} = 1$$

effect of variable amplitude

- Often there are "blocks" of variable amplitude loads which repeat
- A typical flight cycle is a good example of this
- A flight will have working loads, vibrations, as well as storms/turbulence, but each flight should have similar loading
- If we call the number of "block" B then we have

$$B\left[\sum \frac{N_i}{N_{if}}\right]_{rep} = 1$$

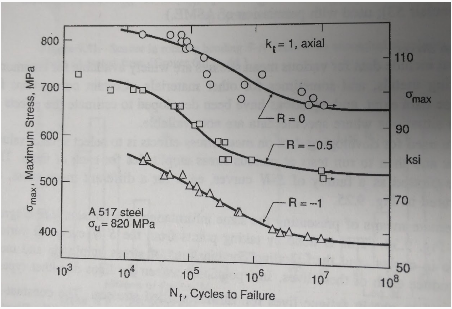
- It is possible for each variable load case to have a different mean stress
- This would mean generating a different S-N curve for each potential mean stress
- Much work has been done to instead convert a zero-mean stress curve to different mean stress amplitudes

mean stress effects

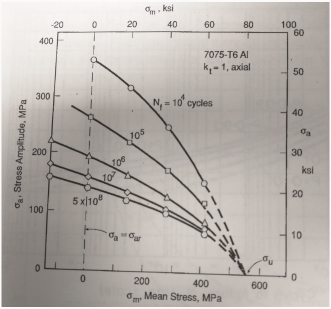
mean stress

- Since mean stress has an effect on fatigue life, sometimes a family of S-N curves at varying mean stress values is created
- S-N curves for these are reported in different ways, but commonly σ_{max} replaces σ_a on the y-axis
- One useful way of representing these data, instead of many S-N curves, is a constant-life diagram
- It is created by taking points from the S-N curves and plotting a line through constant Nf values

S-N curves at variable σ_m



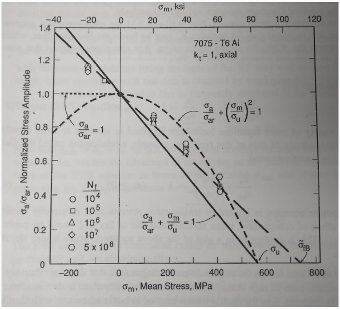
constant life diagram



normalizing

- One very useful way to plot this data is to normalize the amplitude by the zero-mean amplitude
- We call the zero-mean amplitude as σ_{ar}
- Plotting σ_a/σ_{ar} vs. σ_m provides a good way to group all the data together on one plot with the potential to fit a curve

normalized amplitude-mean diagram



Goodman line

- The first work on this problem was done by Goodman, who proposed the line

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$$

- This equation can also be used for fatigue limits, since they are just a point on the S-N curves

$$\frac{\sigma_e}{\sigma_{er}} + \frac{\sigma_m}{\sigma_u} = 1$$

modifications

- While the Goodman line gives a good approximation to convert non-zero mean stress S-N curves, it is somewhat overly conservative at high mean stresses
- It is also non-conservative for negative mean stresses
- An alternative fit is known as the Gerber Parabola

$$\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1$$

- In general, the Goodman line is a good fit for brittle materials (steels) while the Gerber parabola is a better fit for more ductile materials

modifications

- The Goodman line can also be improved by replacing σ_u with the corrected true fracture strength $\tilde{\sigma}_{fB}$ or the constant σ_f' from the S-N curve fit

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f'} = 1$$

- This is known as the Morrow Equation
- For steels, $\sigma_f' \approx \tilde{\sigma}_{fB}$, but for aluminums these values can be significantly different, and better agreement is found using $\tilde{\sigma}_{fB}$.

modifications

- One more relationship that has shown particularly good results with aluminum alloys is the Smith, Watson, and Topper equations (SWT)

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a}$$

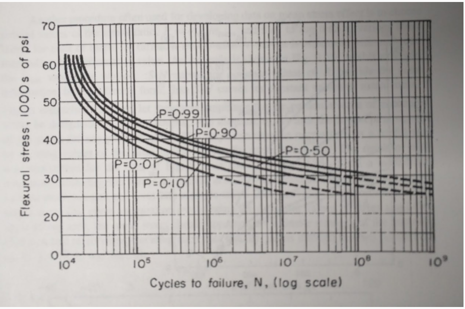
- In general, it is best to use a form that matches your data
- If data is lacking, the SWT and Morrow equations generally provide the best fit

scatter

fatigue scatter

- One of the challenges with fatigue is that there is generally considerable scatter in the data
- Quantifying this scatter requires many repetitions, which makes for time consuming tests
- In general, the scatter follows a lognormal distribution (or a normal distribution in $\log(Nf)$)

S-N-P Curve



general stress

general stress

- Often combined loads from different sources introduce stresses which are not uni-axial
- For ductile materials, good agreement has been found using an effective stress amplitude, similar to the octahedral shear yield criterion

$$\bar{\sigma}_a = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{xa} - \sigma_{ya})^2 + (\sigma_{ya} - \sigma_{za})^2 + (\sigma_{za} - \sigma_{xa})^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

- The effective mean stress is given by

$$\bar{\sigma}_m = \bar{\sigma}_{xm} + \bar{\sigma}_{ym} + \bar{\sigma}_{zm}$$

effective stress

- This effective stress can be used in all other relationships, including mean stress relationships
- Note that mean shear stress has no effect on the effective mean stress
- This is surprising, but agrees well with experiments
- When yielding effects do dominate behavior, the strain-based approach is more appropriate