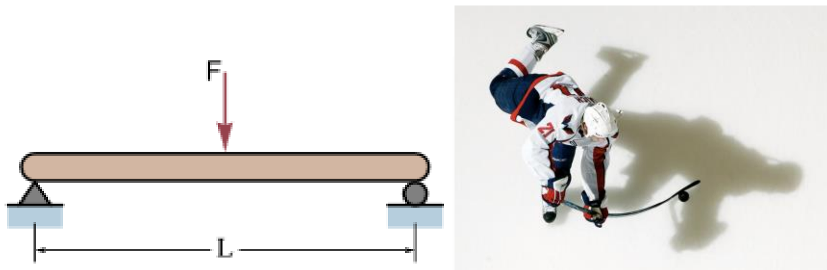
Damage Effects on Hockey Stick Shaft due to Bending

During a typical game of ice hockey, the hockey sticks are subjected to a variety of loading through different types of shots and passes. It is important to design and manufacture the sticks so they can bend and flex properly while still maintaining structural integrity. The manner in which hockey sticks are subjected to loading can be modeled by a simply supported beam as shown in Figures 1 and 2. The loading is applied from the bottom hand and the opposing reaction forces come from the resistance of the top hand and the ice.



Figures 1,2: Hockey Stick Loading Diagram and Stick Bend

Hockey sticks usually break near the center of the shaft since that is the location of the largest bending moment. The area to be tested can therefore be narrowed down to the shaft, which is hollow. Shafts are created using materials with high tensile strength and varying stiffness in order to create ideal flexural properties when used in a game. For the experimental material, 2024-T3 Bare Aluminum was chosen since it is more ductile than steel and has actually been used to make hockey sticks in the past. The geometry of the stick shaft and dimensions can be seen in Figure 3, below.

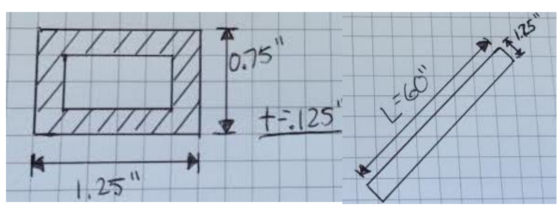


Figure 3: Hockey Shaft Dimensions

Three types of possible crack scenarios are examined along the face of the shaft: Center crack in a finite width panel, edge crack with a remote bending moment, and semi-elliptical surface crack in a finite width panel.

Stress Intensity Factor Estimation

The bending of the stick's shaft will cause mode I fracture to occur. Since the shaft of the hockey stick is hollow and rectangular, the moment of inertia was calculated to account for cross-sectional geometry effects. The stress from the bending moment was determined using Eq. 1. To calculate K_I for the three crack scenarios, Eq. 2 was used with different β correction factors to account for the edge effects on the crack tips.

$$\text{Eq. 1: } \sigma = \frac{My}{I}$$

$$\text{Eq. 2: } K_I = \sigma\sqrt{\pi a}\beta$$

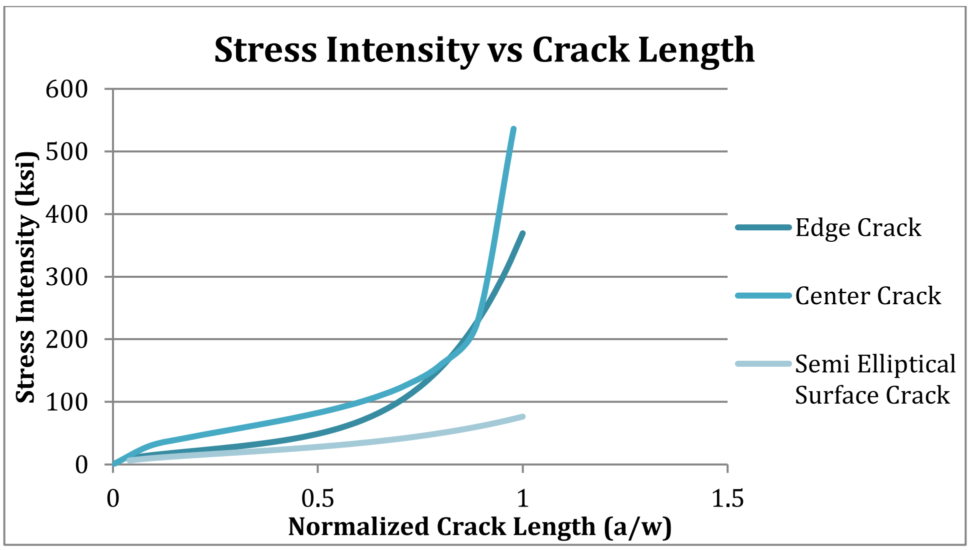


Figure 4: Plot of Stress Intensity vs Crack Length for different crack types

A comparison of stress intensities for the different crack scenarios is shown above in Figure 4. The load applied for each was 400 lbf directly in the center of the shaft which results in a stress of σ = 73.92 ksi. It is seen that the stress intensities for the edge crack and center crack are similar and higher than the semi-elliptical crack. The difference can be attributed to the measurement of the semi-elliptical crack length being only in the minor axis direction and until the entire thickness is reached. If the comparison were done in the direction of the major axis, the crack growth would be similar to the center crack's growth after the minor axis growth reached the entire thickness. In general, the K_I values grow at a high rate as they reach the edges of a surface since the edge effects grow with increasing crack length.

Residual Strength Estimate

In order to create a residual strength curve, fracture toughness needs to be determined. Typically, ductile and thin materials are in a state of plane stress, while brittle and thick materials are in a state of plane strain. Using Eq. 3, the value for I when a stress corresponding to a load of 400 lbf is applied comes out to be 6.056 for an initial crack length of .005 in. This corresponds to a state of plane strain and therefore, a K_{IC} value of 32 ksi$\sqrt{\text{in}}$ is to be used for the fracture toughness of Aluminum 2024-T3.

$$\text{Eq. 3: } I = 6.7 - \frac{1.5}{t}\left[\frac{K_I}{\sigma_{YS}}\right]^2$$

If the initial stress or crack length were much larger, the I value would correspond to a state of plane stress and K_I would have to be iterated a few times with the plastic zone correction factor, r_p, for whatever crack length is being analyzed. The state of plane strain for this loading is due to the ratio of initial crack length to material thickness being small. Charts for the residual strengths based on plane strain fracture toughness are shown in Figures 5 and 6. Critical stress values, σ_c, for brittle fracture and net section yield were calculated using Eq.'s 4 and 5.

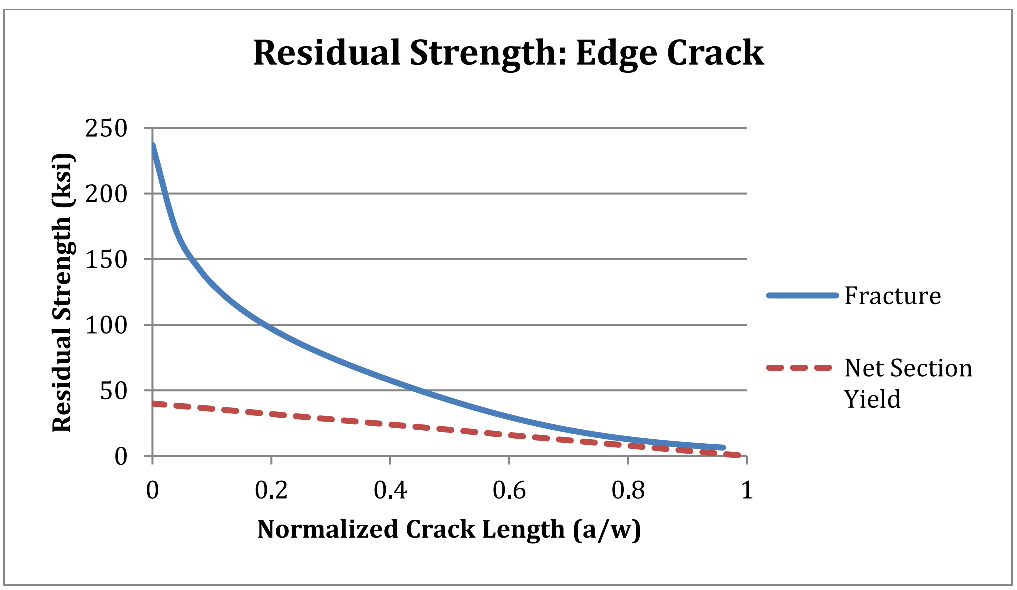


Figure 5: Residual Strength Curve for Edge Crack

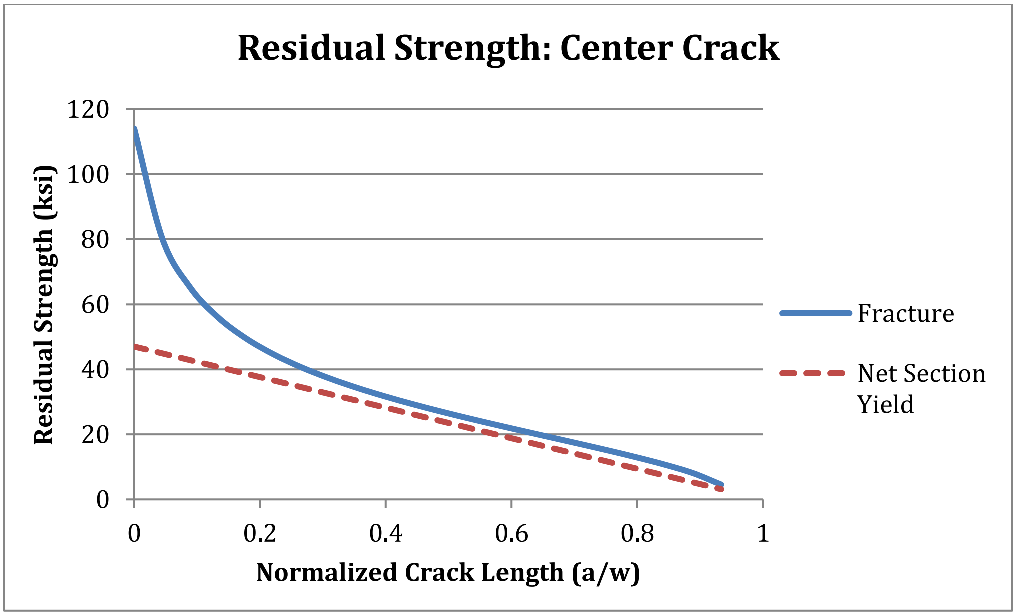


Figure 6: Residual Strength Curve for Center Crack

$$\text{Eq. 4: } \sigma_c = \frac{K_{IC}}{\sqrt{\pi a}\beta}$$

$$\text{Eq. 5: } \sigma_c = \sigma_{YS}\frac{A_{net}}{A_{gross}}$$

The residual strength curves above show critical values of stress that will cause the material to fail based on fracture and net section yield at normalized crack lengths. Any values of stress applied above the curve values will cause unstable crack growth and fracture. Curves of brittle fracture stress based on crack length cannot actually approach infinity as the crack length approaches zero. To fix this, the Feddersen approach is implemented which draws a line tangent from the brittle fracture stress curve to the yield stress at crack length zero. In the cases for the hockey stick shaft, the edge cracked Feddersen line would be in the same place as the net section yield line, and the center cracked Feddersen line would be non-existent since there does not appear to be a tangent point that faces the yield stress at a/w=0.

Fatigue Life Estimate

When a specimen such as a hockey stick shaft is subjected to large cyclic stresses, it will eventually develop fatigue cracks, which lead to failure. For this experiment, the shaft is subjected to loadings from 10 – 500 lbf and the resulting cycles to failure are plotted in Figure 7, below.

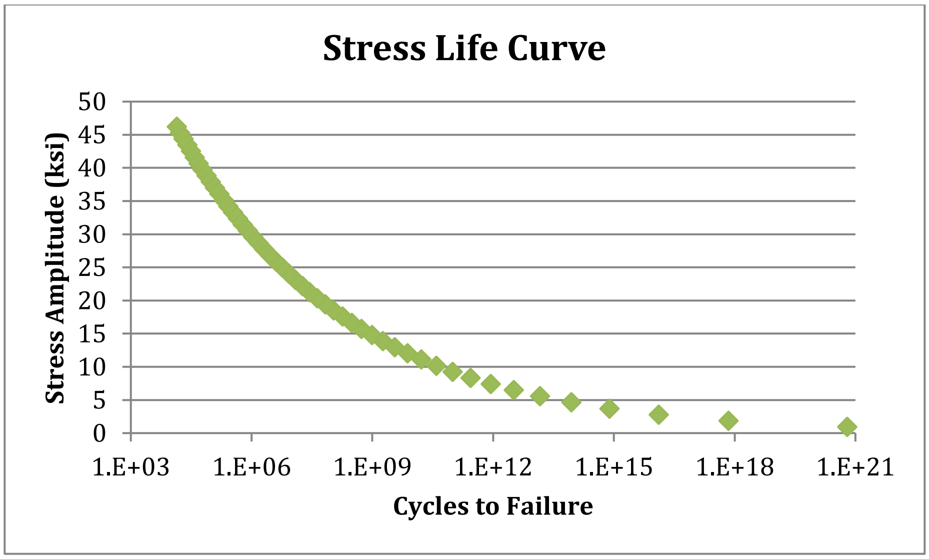


Figure 7: Stress Life Curve for Hockey Stick Shaft

The stress amplitudes correspond to the magnitude of the stresses resulting from the loadings divided by two. The number of cycles to failure (N_f) are determined using Eq. 6, shown below.

$$\text{Eq. 6: } \sigma_a = \sigma'_f \left(2N_f\right)^b$$

The values for σ'_f and b are material properties found for Aluminum 2024-T3. A high number of cycles are required to reach the shaft's fatigue life, even at very high stresses. Therefore, stress based fatigue analysis is the ideal method as opposed to strain based which is best for dealing with a low number of cycles and plastic deformation.

The fatigue life was determined for a realistic loading cycle of a hockey stick shaft during one hockey game using Eq. 7, below.

$$\text{Eq. 7: } B_f [\Sigma \frac{N}{N_f}] = 1$$

This equation takes a variable amplitude loading for a number of cycles and combines the weighted influence of each individual cycle on fatigue life into a block.

The result gives the number of repetitions of one block that the material can endure until failure. The variable amplitude loading and result is shown in Table 1, below.

Table 1: Variable Amplitude Loading Values

Loading	N	σ_{min}	σ_{max}	σ_a	σ_m	Nf	B
Slapshot	2	0	37	18.5	18.5	24269242.5	
Hard Pass	12	0	5.5	2.75	2.75	1.1456E+16	
Stick Contact	15	0	3	1.5	1.5	4.7986E+18	
Wristshot	5	0	18.5	9.25	9.25	4.7072E+10	
							12119000.5

One block has a very small effect on the fatigue in this case since it predicts that the life to failure is 12.12 million blocks. This number seems quite high if applied in real life, and this may be due to a low estimate of stress on the shaft and a low estimate of the amount of use it incurs during one hockey game.

Crack Propagation Estimate

To begin the crack propagation estimate, a loading cycle was combined into one weighted stress amplitude using the Boeing-Walker method. The cycle that was analyzed can be seen below in Figure 8.

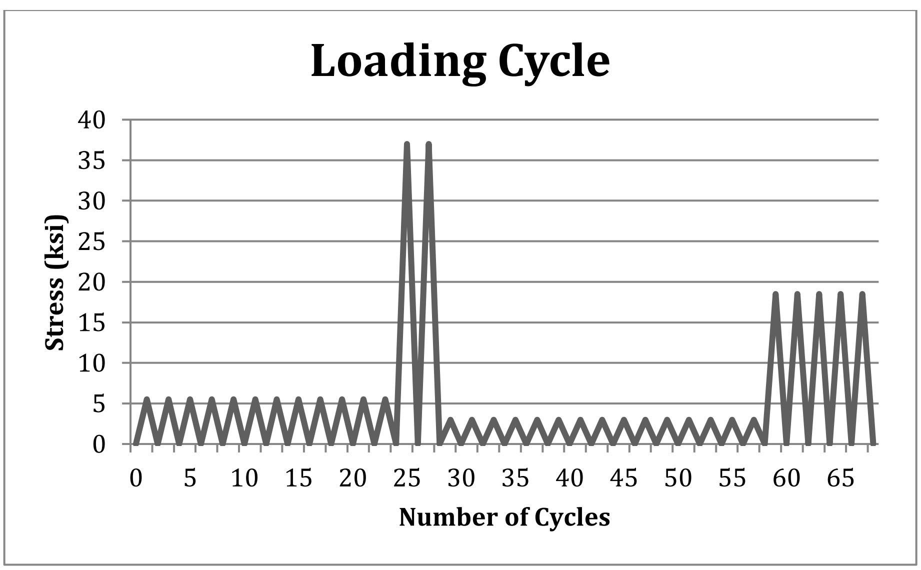


Figure 8: Plot of Cycles of Various Stress Amplitudes

$$\text{Eq. 8: } S^p = \Sigma\left[N\left(\frac{1}{z\sigma_{max}}\right)^{-p}\right]$$

Using Eq. 8, the combined stress amplitude, S, was calculated to be 23.37 ksi. Variable p is a growth-rate slope for aluminum, and z equals 1 since the minimum stress amplitudes are all equal to zero. Taking that result and plugging in S for the max stress of K_{max}in Eq. 9, a crack growth propagation curve could be made for a center-cracked shaft. Variables m_T and q are material growth rate constants for aluminum.

$$\text{Eq. 9: } \frac{da}{dN} = 10^{-4}\left(\frac{1}{m_T}\right)^p [K_{max}(1-R)^q]^p$$

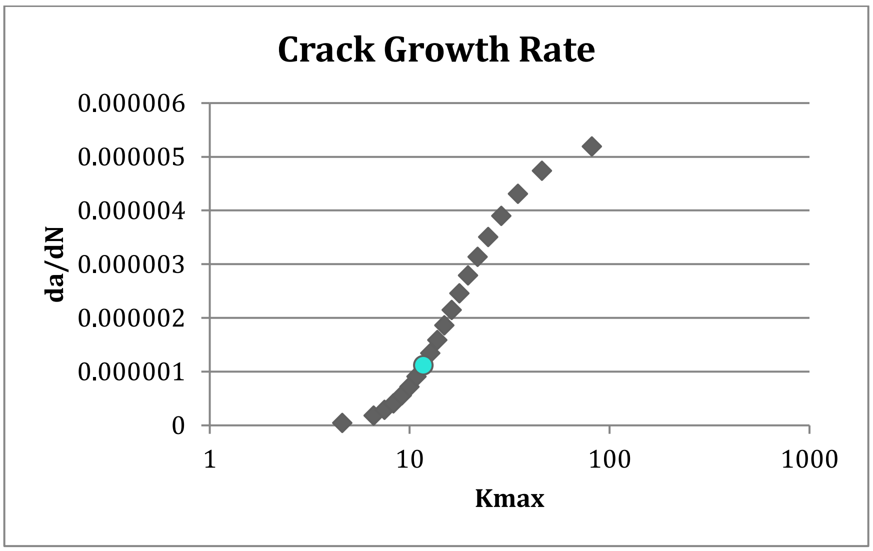


Figure 9: Crack Growth Rate Curve

Inspection Cycle

Calculating the crack length per number of cycles and number of cycles until failure gives reference for when a material or part should be inspected or replaced. For the hockey stick shaft, the residual strength curve of Figure 6 was used with S = 23.27 ksi to determine the crack length at failure to be .3696 in. From there, Eq. 9 was integrated to give the final number of cycles to failure to be 21,239. Confirming that the crack length is stable and in region II, the value is marked as a light blue circle in the crack growth rate curve on Figure 9. To be safe, the shaft could be inspected at 65% of the number of cycles to failure, which is 13805 cycles.

Damage Tolerant Improvement

Some choices for improving damage tolerance for the shaft are switching to a stiffer metal, adding stiffeners along the length, or increasing the thickness of the aluminum. The ideal option would be to keep the same material and increase its thickness since minimal manufacturing change would be necessary. Increasing the thickness of the shaft would increase the moment of inertia and therefore decrease the stress that the material is subjected to from the loading. In doing so, the stress intensity factors would be lower and crack growth rate would decrease.

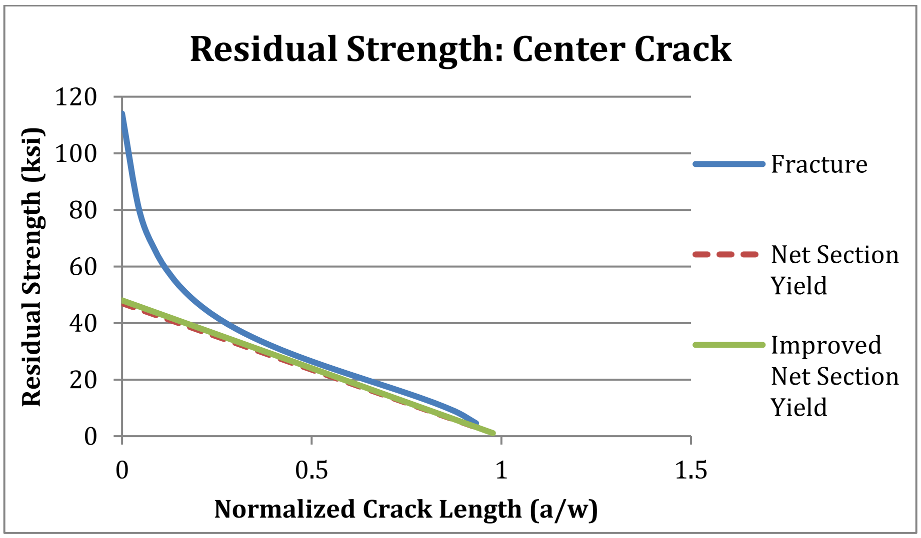


Figure 10: Residual Strength Curve with Improved Net Section Yield

Looking at the residual strength curve for a specimen twice as thick does not change the brittle fracture curve whatsoever, and only increases the yield strength by 1 ksi. The improvement is that the stresses used with this curve are much lower with a greater thickness, and correspond to failure on the plot at longer crack lengths. Doubling the thickness of the shaft also increases the fatigue life from 21,239 cycles to 228,244 cycles, which is a big improvement. This is because the new combined stress, S, turned out to be 14.64 ksi compared to the original 23.27 ksi. Making the material two times as thick is most likely overkill, but it proves that the shaft will survive for a longer duration as thickness increases.

Conclusion

It is hard to judge the results of the analysis of the aluminum hockey stick shaft since there are no numbers for comparison, and the estimations made for loads and therefore stresses could be inaccurate. With that being said, the results from each section can be compared for the different crack scenarios. Looking at the stress intensity factors, the edge and center cracks grow similarly and at a larger rate than the semi-elliptical surface crack. The residual strength curves for center and edge cracks show that the shaft will fail due to net section yield for each scenario, but the center-cracked scenario has a much lower critical brittle fracture stress. This is due to the edge effects of the center crack being smaller than those of the edge crack. The stress fatigue analysis result for cycles to failure was higher than the crack propagation fatigue analysis because the stress analysis is solely based on material properties and does not take any geometric properties of the crack or correction factors into account. To make the shaft more damage tolerant, the ideal solution is to increase the thickness until the stresses are low enough to meet the minimum cycles to failure requirements. The downfall of increasing thickness is that you lose flexibility, which is crucial to a powerful hockey shot. Luckily, 21,239 cycles to failure is much greater than the amount of games someone could use a hockey stick for.