

## Lecture 7 - Fracture Toughness

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## schedule

- 8 Feb - Fracture Toughness
- 10 Feb - Fracture Toughness, HW3 Due, HW 2 Self-grade due
- 15 Feb - Residual Strength
- 17 Feb - Residual Strength, HW4 Due, HW 3 Self-grade due

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- fracture toughness
- plain strain
- plain stress

## fracture toughness

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## fracture toughness

- The critical load at which a cracked specimen fails produces a critical stress intensity factor
- The “critical stress intensity factor” is known as  $K_c$
- For Mode I, this is called  $K_{Ic}$
- The critical stress intensity factor is also known as fracture toughness

$$K_{Ic} = \sigma_c \sqrt{\pi a} \beta$$

- Note: “Fracture Toughness” can also refer to  $G_{Ic}$ , which is analogous to  $K_{Ic}$ , but not the same

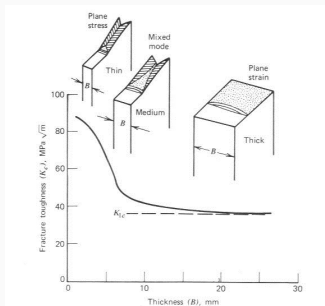
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## fracture toughness

- Fracture toughness is a material property, but it is only well-defined in certain conditions
- Brittle materials
- Plane strain (smaller plastic zone)
- In these cases ASTM E399-12 is used.

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# fracture toughness



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## unstable cracks

- Stable crack growth means the crack extends only with increased load
- Unstable crack growth means the crack will continue to extend indefinitely under the same load
- For a perfectly brittle material, there is no stable crack growth, as soon as a critical load is reached, the crack will extend indefinitely

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## stable cracks

- For an elastic-plastic material, once the load is large enough to extend the crack, it will extend slightly
- The load must be continually increased until a critical value causes unstable crack growth

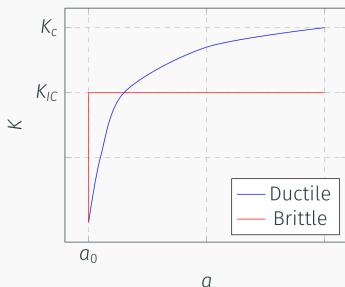
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## fracture toughness

- During an experiment, we will record the crack length and applied load ( $P_i$ ,  $a_i$ ) each time we increase the load
- We can calculate a unique stress intensity factor  $K_{Ii}$  at each of these points
- These are then used to create a “K-curve”, plotting  $K_I$  vs.  $a$

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## K-curve



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## K-curve

- Materials will generally not be as flat as the perfectly brittle example
- Plane strain conditions and brittle materials will tend towards a “flat” K-curve
- $K_{IC}$  for brittle/plane strain is very well defined
- $K_c$  for plane stress can refer to two things
- Either the maximum  $K_c$  during a test, or tangent point on  $K_R$ -curve (R-curve)

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## example

- In composites, and adhesives, some work is needed to ensure stable crack growth
- The Double-Cantilever Beam (DCB) experiment to find  $G_{Ic}$  illustrates this

$$C = \frac{\delta}{P}$$

$$C = \frac{2a^3}{3EI}$$

$$G = \frac{P^2}{2b} \frac{dC}{da}$$

$$G = \frac{P^2 a^2}{bEI}$$

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## example

- For crack growth to be stable we need

$$\frac{dG}{da} \leq 0$$

- Under fixed-load conditions, we find

$$\frac{dG}{da} = \frac{2P^2 a}{bEI}$$

- This is always positive, and thus results in unstable crack growth

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- Under fixed-displacement conditions, we substitute for  $P$
- We find

$$\frac{dG}{da} = -\frac{9\delta^2 EI}{ba^3}$$

- Which is always stable, so for DCB tests, displacement control is generally used

## plane strain, brittle

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- For relatively brittle materials, we don't need to worry about the R-curve
- Specimens are made according to these specifications

$$a \geq 2.5 \left( \frac{K_{IC}}{\sigma_{YS}} \right)^2$$

$$b \geq 2.5 \left( \frac{K_{IC}}{\sigma_{YS}} \right)^2$$

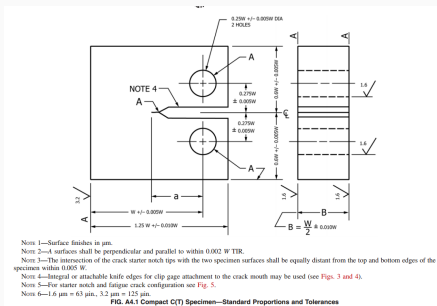
$$W \geq 5 \left( \frac{K_{IC}}{\sigma_{YS}} \right)^2$$

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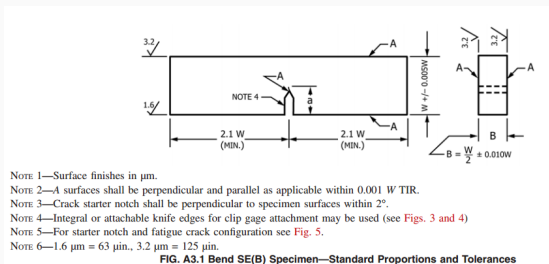
## ASTM E399

1. Select specimen size
2. Select specimen type (Compact Tension or Single Edge Notched Bend)

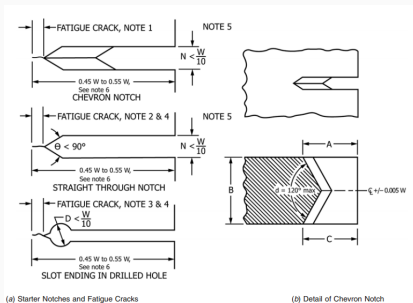
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(a) Starter Notches and Fatigue Cracks

(b) Detail of Chevron Notch

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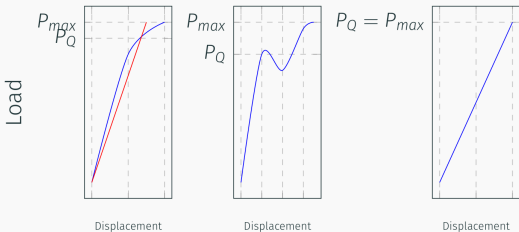
3. Machine specimen
4. Fatigue crack specimen  $K_r < 0.6K_{IC}$ 
  - This is to ensure that the plastic zone size during fatigue is smaller than the plastic zone size during testing
  - If  $K_{IC}$  has not yet been determined, you may have to guess the first time

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5. Mount specimen, attach gage
6. Load rate should ensure “static” load conditions. (30 - 150 ksi  $\sqrt{\text{in.}}$  /min.)
7. Determine the “provisional” value of  $K_{Ic}$  (known as  $K_Q$ )

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## ASTM E399



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- If the load-displacement curve is like the first figure, with some non-linearity, we let  $P_Q$  be the point of intersection between the load-displacement curve and a line whose slope is 5% lower than the slope in the elastic region
- “Pop-in” occurs when there is stable crack extension before the plasticity begins. We let  $P_Q$  be the point where stable crack extension begins.

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## ASTM E399

- For a perfectly linear material,  $P_Q = P_{max}$

$$K_Q = \frac{P_Q}{BW^{1/2}} f\left(\frac{a}{W}\right) \quad \text{Compact Tension}$$

$$K_Q = \frac{P_Q}{BW^{3/2}} g\left(\frac{a}{W}\right) \quad \text{SENB}$$

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8. Ensure that your specimen is still valid

$$a \geq 2.5 \left( \frac{K_Q}{\sigma_{YS}} \right)^2$$

$$b \geq 2.5 \left( \frac{K_Q}{\sigma_{YS}} \right)^2$$

$$W \geq 5 \left( \frac{K_Q}{\sigma_{YS}} \right)^2$$

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## ASTM E399

- For stable crack extension, check the  $P_{max}$

$$\frac{P_{max}}{P_Q} \leq 1.10$$

- Check for symmetric crack front,  $a_1$ ,  $a_2$ , and  $a_3$  must be within 5% of  $a$ .  $a_s$  must be within 10% of  $a$ .

$$\frac{a_1 + a_2 + a_3}{3} = a$$

- Load-displacement should have an initial slope between 0.7 and 1.5

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## plane stress, ductile

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### R-curve

- For materials with some plasticity, the  $K_R$  Curve, or R Curve, is very important
- Sometimes called a “resistance curve” it is generally dependent on
  - Thickness
  - Temperature
  - Strain rate

- When done correctly,  $K_R$  curves are not dependent on initial crack size or the specimen type used
- ASTM E561 describes a general procedure

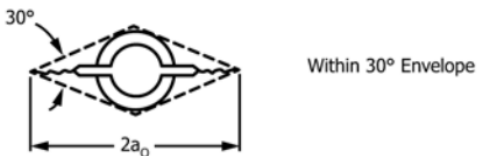
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## ASTM E561

- Compact Tension (CT or C(T)) specimens may be used for plane stress  $K_R$  curves
- The other specimen which is permitted is a middle-cracked tension specimen (M(T))
- M(T) specimens are preferred in many cases due to a more uniform stress distribution (particularly important for anisotropic materials)

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**Figure 1:** An image showing how long cracks are allowed to be relative to the center hole in middle-cracked tension specimens

#13 Drill

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## minimum sample dimensions

Table of Minimum M(T) Specimen Geometry for Given Conditions							
$K_{Rmax}/\sigma_{YS}$ $\sqrt{m}$	$\sqrt{in.}$	Width		$2a_0$		Length <sup>A</sup>	
		m	in.	m	in.	m	in.
0.08	0.5	0.076	3.0	0.025	1.0	0.229	9
0.16	1.0	0.152	6.0	0.051	2.0	0.457	18
0.24	1.5	0.305	12.0	0.102	4.0	0.914	36
0.32	2.0	0.508	20.0	0.170	6.7	0.762	30
0.48	3.0	1.219	48.0	0.406	16.0	1.829	72

**Figure 3:** A table of minimum recommended specimen dimensions for middle-cracked tension specimens.

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## minimum sample dimensions

Table of Minimum C(T) Specimen Width  $W$  for Given Conditions,  $m$  (in.)

$K_{Rmax}/\sigma_{YS}$		Maximum $a_p/W$				
$\sqrt{m}$	$\sqrt{in.}$	0.4	0.5	0.6	0.7	0.8
0.10	0.6	0.02 (0.8)	0.03 (1.0)	0.03 (1.3)	0.04 (1.7)	0.06 (2.5)
0.20	1.3	0.08 (3.3)	0.10 (4.0)	0.13 (5.0)	0.17 (6.7)	0.25 (10.0)
0.30	1.9	0.19 (7.5)	0.23 (9.0)	0.29 (11.3)	0.38 (15.0)	0.57 (22.6)
0.40	2.5	0.34 (13.3)	0.40 (15.9)	0.51 (19.9)	0.67 (26.5)	1.01 (39.8)
0.50	3.1	0.53 (20.9)	0.64 (25.1)	0.80 (31.3)	1.06 (41.8)	1.59 (62.7)

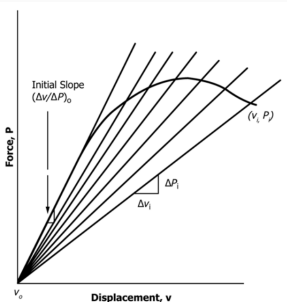
**Figure 4:** A table of minimum recommended specimen dimensions for compact tension specimens.

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## effective crack length

- ASTM E561 describes three ways to obtain the effective crack length during testing
  1. Measure the crack length visually and calculate  $r_p$
  2. Measure crack length using “unloading compliance” and adding plastic zone size
  3. Measure the effective crack size directly using “secant compliance”

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## secant compliance $M(T)$

- Using the slope data from our load-displacement curve, we can calculate the effective crack length using

$$EB \left( \frac{\Delta v}{\Delta P} \right) = \frac{2Y}{W} \sqrt{\frac{\pi a/W}{\sin(\pi a/W)}}$$

$$\left[ \frac{2W}{\pi Y} \cosh^{-1} \left( \frac{\cosh(\pi Y/W)}{\cos(\pi a/W)} \right) - \frac{1 + \nu}{\sqrt{1 + \left( \frac{\sin(\pi a/W)}{\sinh(\pi Y/W)} \right)^2}} + \nu \right]$$

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## secant compliance $M(T)$

- This equation is difficult to solve directly for  $a$  (for  $M(T)$  specimens)
- Instead it is generally solved iteratively
- The following equations are used to give a good initial guess to use in iterations

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## secant compliance $M(T)$

$$X = 1 - \exp \left[ \frac{-\sqrt{[EB(\Delta v / \Delta P)]^2 - (2Y/W)^2}}{2.141} \right]$$
$$\frac{2a}{W} = 1.2235X - 0.699032X^2 + 3.25584X^3 - 6.65042X^4 + 5.54X^5 - 1.66989X^6$$

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## secant compliance $M(T)$

In the above equations, the following are the definitions of parameters used

$E =$	Young's Modulus
$\Delta v / \Delta P =$	specimen compliance
$B =$	specimen thickness
$W =$	specimen width
$Y =$	half span
$a =$	effective crack length
$\nu =$	Poisson's ratio

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## secant compliance $C(T)$

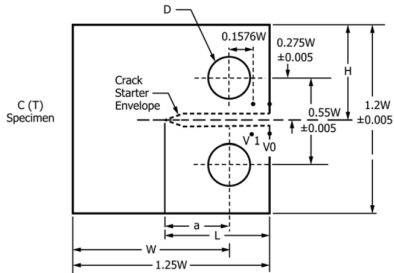
- For  $C(T)$  specimens, we use the following equations

$$EB \frac{\Delta v}{\Delta P} = A_0 + A_1 \left( \frac{a}{W} \right) + A_2 \left( \frac{a}{W} \right)^2 + A_3 \left( \frac{a}{W} \right)^3 + A_4 \left( \frac{a}{W} \right)^4$$

- The coefficients will differ based on where the displacement is measured from

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## secant compliance $C(T)$



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## secant compliance $C(T)$

loc	$A_0$	$A_1$	$A_2$	$A_3$	$A_4$
$V_0$	120.7	-1065.3	4098.0	-6688.0	4450.5
$V_1$	103.8	-930.4	3610.0	-5930.5	3979.0

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## secant compliance $C(T)$

loc	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
$V_0$	1.0010	-4.6695	18.460	-236.82	1214.90	-2143.6
$V_1$	1.0008	-4.4473	15.400	-180.55	870.92	-1411.3

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## secant compliance $C(T)$

- Where the initial guess for  $a$  is provided by

$$\frac{a}{W} = C_0 + C_1 U + C_2 U^2 + C_3 U^3 + C_4 U^4 + C_5 U^5$$

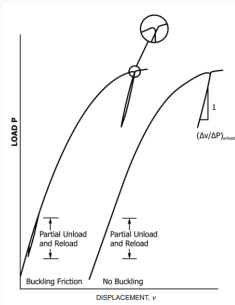
- and  $U$  is given by

$$U = \frac{1}{1 + \sqrt{EB \frac{\Delta v}{\Delta P}}}$$

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- If the test is stopped and re-started frequently (to measure crack length by hand or to use the compliance method of crack measurement) buckling can interfere with results

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- If buckling is shown to be present in the test, supports can be used to prevent buckling
- These supports can introduce friction
- They should be well-lubricated for accurate test results

## net section stress

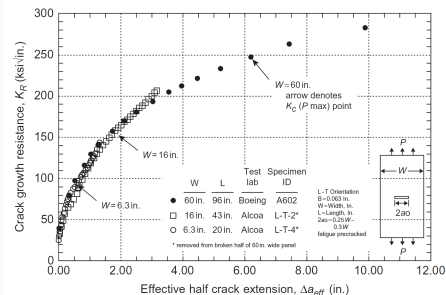
- One final consideration when dealing with plane stress fracture mechanics is the net section stress
- For the test to be valid, failure must occur due to fracture, not general static failure
- Static failure will occur when  $\sigma_N = \sigma_{YS}$

- Once the effective crack length and  $K_{Ie}$  has been determined for the test, we can generate the  $K_R$  curve
- The  $K_R$  curve is quite simply a plot of  $K_{Ie}$  vs.  $a$  for the test performed (i.e. with varying stress and increasing crack length)

## initial crack length

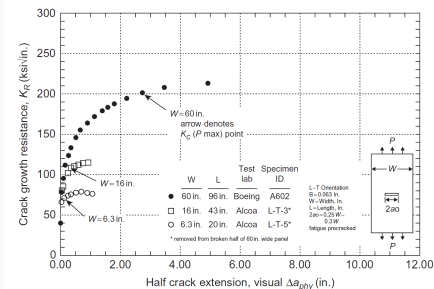
- When the test is performed correctly, the  $K_R$  curve is not a function of the initial crack length
- For this reason, we often plot  $K_{Ie}$  vs.  $\Delta a$ , to subtract the initial crack length
- We can superpose constant-stress  $K$ -curves on this graph, the curve which intersects at a tangent point creates the most “standard” definition for  $K_C$

## example



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## example



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