AE 737: Mechanics of Damage Tolerance

Lecture 9 - Residual Strength

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

15 February 2022

schedule

- 15 Feb - Residual Strength
- 17 Feb - Residual Strength, HW4 Due, HW 3 Self-grade due
- 22 Feb - Multiple Site Damage, Mixed-Mode
- 24 Feb - Exam 1 Review

outline

- residual strength
- fedderson approach
- proof testing
- residual strength review

R-curve

R-curve

- For materials with some plasticity, the K_R Curve, or R Curve, is very important
- Sometimes called a "resistance curve" it is generally dependent on
  - Thickness
  - Temperature
  - Strain rate

R-curve

- When done correctly, K_R curves are not dependent on initial crack size or the specimen type used
- ASTM E561 describes a general procedure

K_c

- While we can look up plane stress K_c for various materials, it is best if we have a K_R curve
- We may not know if the table uses K_c using the tangent intersection method, or maximum stress intensity
- Even if tangent intersection method is used, K_c will different somewhat based on initial crack length

R-curve

- There are two main methods for plotting the R-curve
- Crack size is measured directly (possibly with a drawn-on scale and camera)
- Effective crack size is calculated from the load-displacement data

physical crack

- When the physical crack size is measured, we need to calculate the effective crack length (and effective stress intensity factor) at each data point
- The effective crack length calculated from the load-displacement data already has the plastic zone effect built in

plane stress fracture toughness

- For a plane stress (or any thickness that is not plane strain) we can find K_c two ways
- One way is simply the maximum value of the K_R curve, but this does not account for unstable crack growth
- The more reliable way is to use the tangent intersection method

tangent intersection method

- Construct curves for K_I based on our specimen geometry with constant applied load and varying crack length
- Plot these curves on the same graph as K_R
- NOTE: K_R curve should be plotted vs. a_{eff}, not Δa or Δa_{eff}
- K_c is the point at which one of the K_I curves is the tangent intersection with the K_R curve

R-Curve examples

- example[1]
- Excel Solver

[1] https://colab.research.google.com/drive/1TlGuadrMRM5xSGic8soVFglPDyWuDaeP?usp=sharing

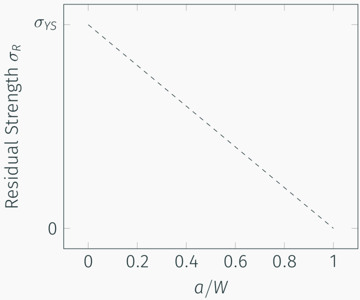
residual strength

residual strength

- As the crack grows, the area of the sample decreases, increasing the net section stress
- The residual strength, σ_R is given in terms of the gross area, so as the crack grows the residual strength due to yield decreases
- We can relate the net-section stress to σ_R by

$$\sigma_R = \sigma_{YS} \frac{A_{net}}{A_{gross}}$$

residual strength

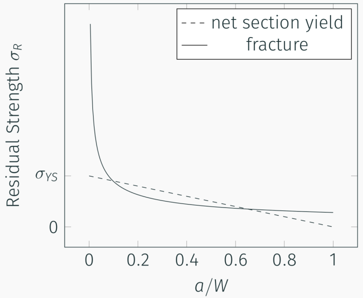


residual strength

- For brittle fracture to occur, we need to satisfy the condition

$$\sigma_R = \sigma_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

residual strength



residual strength

- Within the same family of materials (i.e. Aluminum), there is generally a trade-off between yield stress and fracture toughness
- As we increase the yield strength, we decrease the fracture toughness (and vice versa)
- Consider a comparison of the following aluminum alloys
    1. 7178-T6, $K_C = 43$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 74$ ksi
    2. 7075-T6, $K_C = 68$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 63$ ksi
    3. 2024-T3, $K_C = 144$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 42$ ksi

residual strength

- As an example let us consider an edge-cracked panel with W=6" and t=0.1"
- The net section yield condition will be given by

$$\sigma_C = \sigma_{YS}\frac{W-a}{W} = \sigma_{YS}\frac{6-a}{6}$$

- And the fracture condition by

$$\sigma_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

With

$$\{\beta = 1.12 - 0.231\left(\frac{a}{\ }\right) + 10.55\left(\frac{a}{\ }\right)^2 - 21.72\left(\frac{a}{\ }\right)^3 + 30.39\left(\frac{a}{\ }\right)^4\}$$

7178-T6

:::

7075-T6

:::

2024-T3

:::

comparison

:::

using MIL-handbook

- Uses a different grain nomenclature

K_c	σ_{YS}
L-T	L
T-L	L-T

- A-Basis vs. B-Basis values are reported (A = 99% of population will meet/exceed value, B = 90% of population)
- S-Basis - no statistical information available, standard value to be used

using MIL-handbook

- F_{tu} - ultimate tensile strength
- F_{ty} - tensile yield strength
- F_{cy} - compressive yield strength
- F_{su} - ultimate shear strength
- F_{bru} - ultimate bearing strength
- F_{bry} - bearing yield strength
- E - tensile Young's Modulus
- E_c - compressive Young's Modulus
- G - shear modulus
- μ - Poisson's ratio

data

- Fracture data is on pp. 111-121
- Tensile data is on pp. 138-143
- K_c charts are also available in interactive versions here[2]

[2] http://ndaman.github.io/damagetolerance/examples/Fracture%20Toughness%20Figures.html

fedderson approach

Fedderson approach

- Unfortunately, the method we described above does not quite match experimental results
- Fedderson proposed an alternative, where we connect the net-section yield and brittle fracture curves with a tangent line
- This approach agrees very well with experimental data
- Note: We could do something similar when the crack is very long, but we are generally less concerned with this region (failure will have already occurred)

Fedderson example

worked example here[3]

[3] https://colab.research.google.com/drive/1qPFJt95_5DGR7wDuKX0ntGOoPwQXcGFU?usp=sharing

proof testing

proof testing

- Proof testing is a way to use the concept of residual strength to check the size of a defect from manufacturing
- Due to the fatigue life of a certain panel, and/or an inspection cycle that we have prescribed for that part, we determine an "acceptable" initial flaw size, a_0

proof testing

- We then determine a load which would cause failure at this crack length
- This is the "proof load"
- If the part does not fail in the proof test, we can assume that the largest flaw in the material is a_0

example

- Suppose we are concerned about edge cracks in a panel with $\sigma_{YS} = 65$ ksi, $W = 5$"
- We have determined that the largest allowable crack is 0.4"
- The fracture toughness of this panel is $K_c = 140$ ksi$\sqrt{\text{in.}}$

example

- We can find the proof load

$$\begin{aligned}\sigma_c &= \frac{K_c}{\sqrt{\pi a_0}\beta} \\ &= \frac{140}{\sqrt{\pi 0.4}(1.161)} \\ &= 107.6\end{aligned}$$

- So the proof load would need to induce a gross section stress of 107.6 ksi.

residual strength review

residual strength review

- Group 1 - Sketch a residual strength curve for a single material (include fracture and net-section yield)
- Group 2 - Sketch and describe the difference in residual strength between stiff/brittle materials and ductile/tough materials
- Group 3 - Find the proof load needed to ensure no center-cracks less than 0.01" are present in a material with $K_C = 120$ ksi$\sqrt{\text{in.}}$

residual strength review

- Group 4 - Sketch the Fedderson approach to residual strength. How is this different from the traditional approach? Why is it beneficial?