# Homework 7

April 15, 2019

## 0.1 1

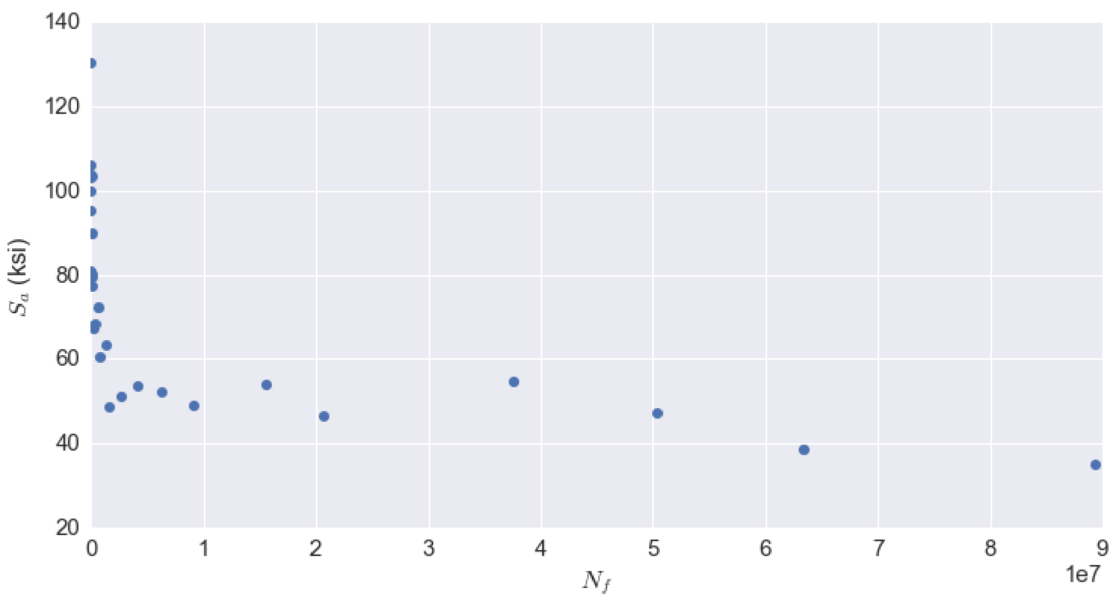
First we load and plot the data

In [2]:
```
#load libraries
import numpy as np
from matplotlib import pyplot as plt
import seaborn as sb #optional library
sb.set(font_scale=1.5) #make fonts bigger
%matplotlib inline

data = np.loadtxt('../hw7_data.txt')
```

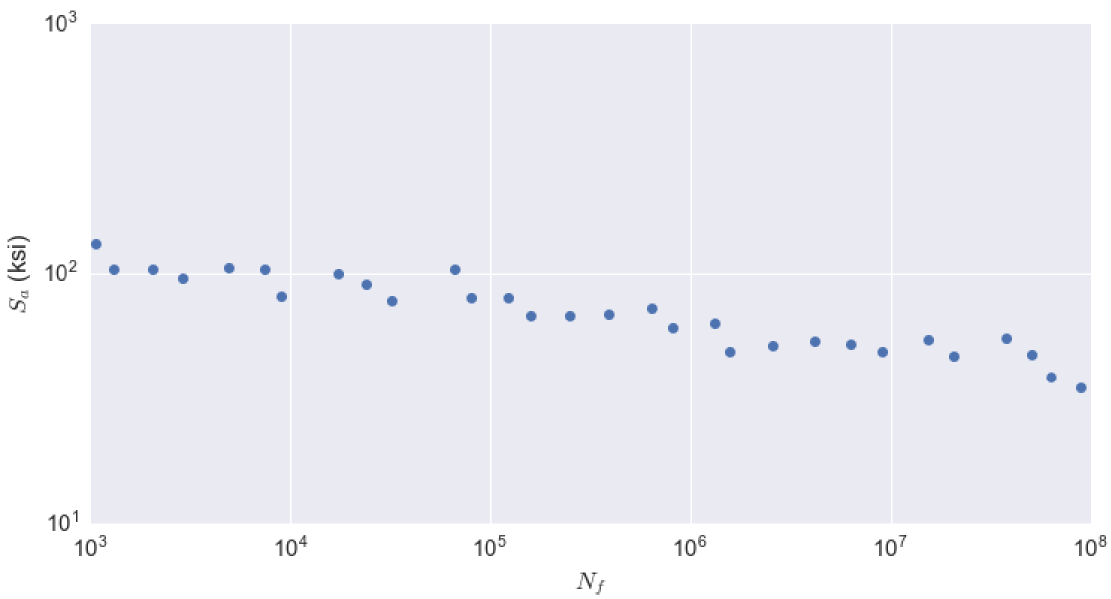
In [3]:
```
# plot data
plt.figure(figsize=(12,6))
plt.plot(data[:,0],data[:,1],'o')
plt.xlabel('$N_f$')
plt.ylabel('$S_a$ (ksi)')
```

Out[3]:
```
<matplotlib.text.Text at 0xa5e8518>
```



```
In [4]: #plot data on log-log scale
        plt.figure(figsize=(12,6))
        plt.loglog(data[:,0],data[:,1],'o')
        plt.xlabel('$N_f$')
        plt.ylabel('$S_a$ (ksi)')
```

```
Out[4]: <matplotlib.text.Text at 0xa842eb8>
```

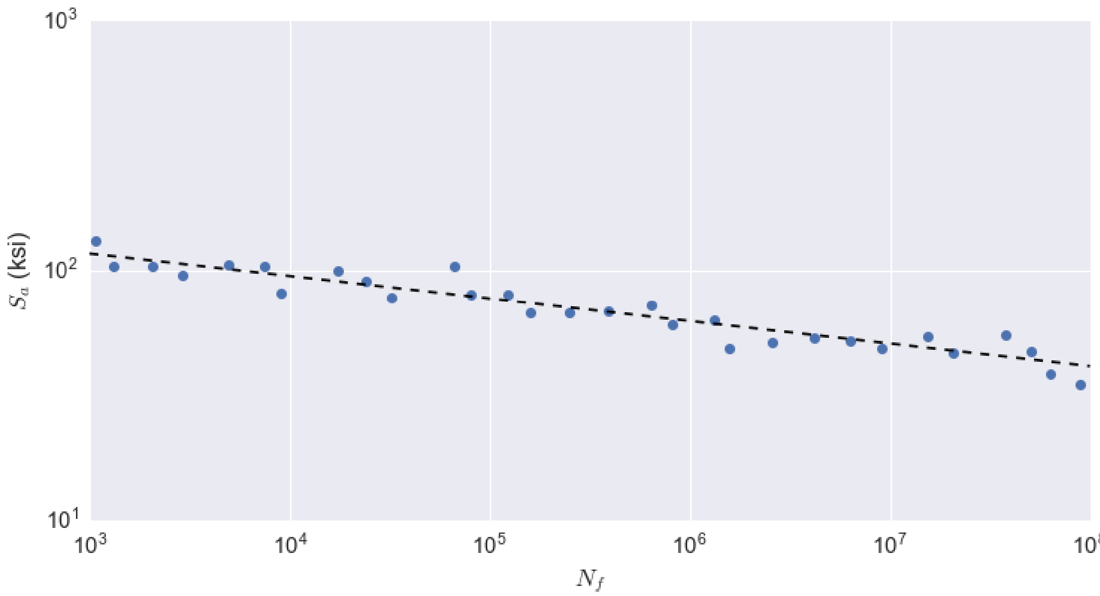



We can now use (15.8) to fit a line (linear in log-log) and find the material properties $\sigma_f'$ and $b$.

```
In [5]: from scipy.optimize import curve_fit #custom curve-fitting library
        def fitfunc(Nf, sf, b):
            #eqn 15.8
            return sf*(2*Nf)**b
        x = data[:,0]
        y = data[:,1]
        popt, pcov = curve_fit(fitfunc,x, y)
```

```
In [6]: x_fit = np.logspace(3,8)
        plt.figure(figsize=(12,6))
        plt.loglog(data[:,0],data[:,1],'o')
        plt.loglog(x_fit,fitfunc(x_fit, popt[0],popt[1]),'k--')
        plt.xlabel('$N_f$')
        plt.ylabel('$S_a$ (ksi)')
```

```
Out[6]: <matplotlib.text.Text at 0xbe91898>
```




We can see that this is provides a very good fit for the data, so we identify the material parameters

```
In [7]: s_f = popt[0]
        b = popt[1]
        print 's_f = %.1f, b=%.3f' % (popt[0],popt[1])
```

```
s_f = 231.0, b=-0.090
```

Thus $\sigma_f' = 231.0$ ksi and $b = -0.090$

### 0.2 2

To estimate the S-N curve for a non-zero mean stress, we use a conversion equation, such as the Goodman equation, the Morrow equation, or the Smith, Watson, and Topper equation. Solving the various equations for $\sigma_{ar}$ we find:

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}} \qquad \text{Goodman}$$

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_f'}} \qquad \text{Morrow}$$

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a} \qquad \text{SWT}$$

For this material we have $\sigma_f' = 231.0$ and $\sigma_m = 30$ ksi. Since we do not know $\sigma_u$, and since the Goodman equation gives results that are generally less accurate than the Morrow equation, we will only compare the Morrow and SWT equations.

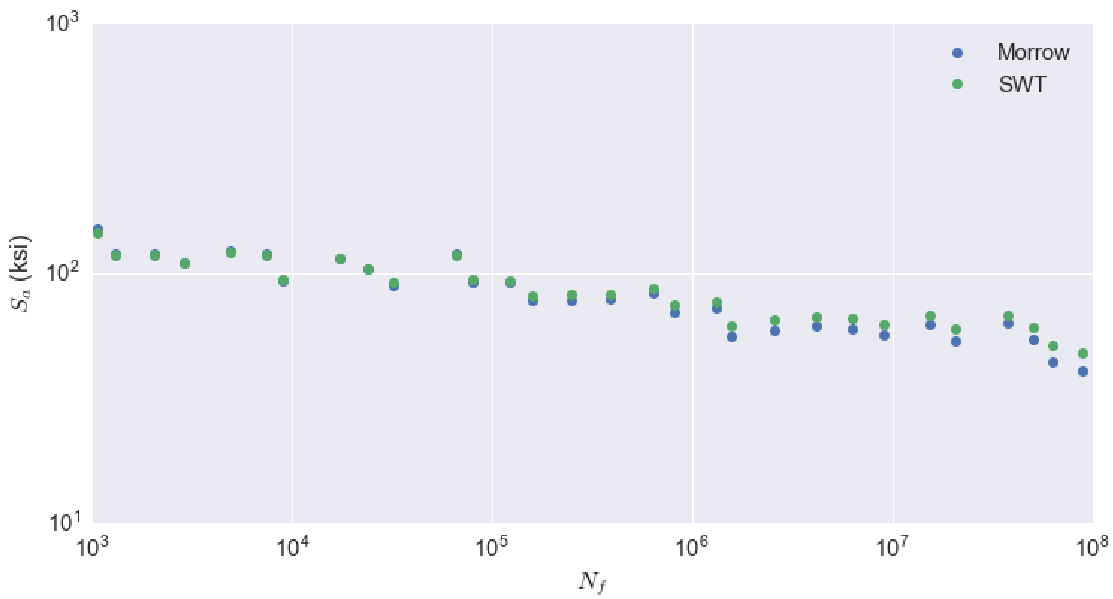
We can find $\sigma_{max}$ by adding $\sigma_a$ to $\sigma_m$

```
In [8]: s_m = 30.0 #ksi, mean stress
        s_ar_morrow = y/(1-s_m/s_f)
        s_ar_swt = np.sqrt((y+s_m)*y)
```

We can compare the effects of these two methods of shifting the S-N curve graphically

```
In [9]: plt.figure(figsize=(12,6))
        plt.loglog(x,s_ar_morrow,'o',label='Morrow')
        plt.loglog(x,s_ar_swt,'o',label='SWT')
        plt.legend(loc='best')
        plt.xlabel('$N_f$')
        plt.ylabel('$S_a$ (ksi)')

Out[9]: <matplotlib.text.Text at 0xc3a02e8>
```

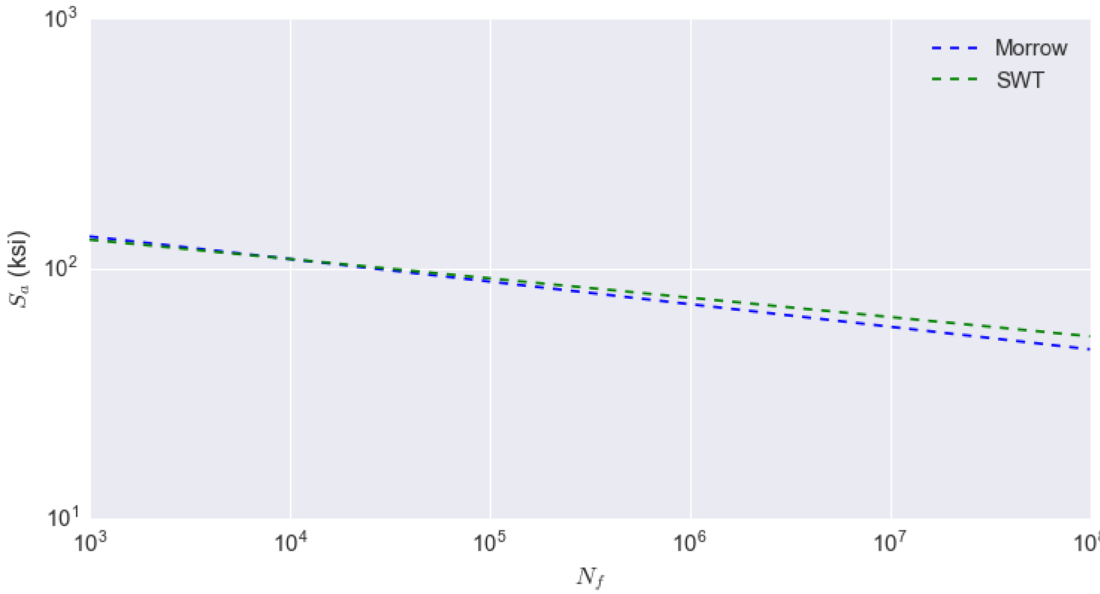



The data points are very similar, with a slight divergence at very high cycles. It is a little easier to compare the best-fit lines

```
In [10]: popt_morrow, pcov_morrow = curve_fit(fitfunc,x, s_ar_morrow)
         popt_swt, pcov_swt = curve_fit(fitfunc,x, s_ar_swt)

         x_fit = np.logspace(3,8)
         plt.figure(figsize=(12,6))
         plt.loglog(x_fit,fitfunc(x_fit, popt_morrow[0],popt_morrow[1]),'b--',label='Morrow')
         plt.loglog(x_fit,fitfunc(x_fit, popt_swt[0],popt_swt[1]),'g--',label='SWT')
         plt.legend(loc='best')
         plt.xlabel('$N_f$')
         plt.ylabel('$S_a$ (ksi)')
```

Out[10]: <matplotlib.text.Text at 0xcff25f8>




## 0.3 3

For variable amplitude loading, we will use Miner's rule.

$$\sum \frac{n_i}{N_{if}} = 1$$

In this problem we have two load "blocks", one with zero mean stress and stress amplitude of 50 ksi and one with a mean stress of 30 ksi and a stress amplitude of 60 ksi. If we define the number of cycles, $n$, the number of times this combination of loads can repeat then Miner's rule will give

$$\sum n \left( \frac{20}{N_{1f}} + \frac{5}{N_{2f}} \right)$$

We use the data from problem 1 to find the zero mean stress amplitude life and the data from problem 2 to find the 30 ksi mean stress amplitude life.

For zero mean stress, we use

$$\sigma_a = \sigma_f'(2N_f)^b$$

substituting known values and solving for $N_f$ gives

```
In [11]: sa = 50. #ksi
         nf1 = 0.5*(sa/s_f)**(1./b)
         print nf1
```

```
12165376.2157
```

Or $N_{1f} = 12.2$ million cycles. We can use either of the models in problem 2 to find $N_{2f}$. In either case, we will find an effective stress amplitude, which we will plug into the same formula as for the zero-amplitude stress to find the number of cycles to failure at that effective load.

```
In [12]: sa = 60. #ksi
         sm = 30. #ksi
         sa_swt = np.sqrt((sa+sm)*sa)
         sa_morrow = sa/(1-sm/s_f)
         print sa_swt
         print sa_morrow
```

```
73.4846922835
68.9562081004
```

We find effective loads of 73.5 and 69.0 ksi for the SWT and Morrow methods, respectively. We substitute to find the cycles to failure:

```
In [13]: nf2_swt = 0.5*(sa_swt/s_f)**(1./b)
         nf2_morrow = 0.5*(sa_morrow/s_f)**(1./b)
         print nf2_swt
         print nf2_morrow
```

```
168507.738348
341679.272371
```

Which gives 169,000 and 342,000 cycles, respectively. Substituting into Miner's Rule we find.

```
In [14]: n_swt = 1./(20./nf1+5./nf2_swt)
         n_morrow = 1./(20./nf1+5./nf2_morrow)
         print n_swt
         print n_morrow
```

```
31932.3159592
61434.0560585
```

Overall the assumption of a mean-stress model has a large effect on the cycles we predict in this case, either 32,000 cycles for the SWT method or 61,000 cycles with the Morrow method.

### 0.4 4

For mixed-mode loading, we can use the effective stress amplitudes

```
In [21]: sx = 27.0/2
         sy = 13.0/2
         sz = 0.0/2
         tauxy = 8.0/2
         tauxz = 0.0
```

```
tauyz = 0.0

sa_eff = 1.0/np.sqrt(2)*np.sqrt((sx-sy)**2+(sy-sz)**2+(sz-sx)**2+6*(tauxy**2+tauyz**2+t
```

Now we need to account for the mean stress. There are multiple ways to do this, but for this problem we will use the Modified Morrow formula

```
In [24]: sxm = 27.0/2
         sym = 13.0/2

         sm = sxm + sym

         sfp = 131 #ksi, found on Table 9.1 (p. 235)
         b = -0.102 #found on Table 9.1 (p. 235)

         sa = sa_eff/(1-sm/sfp)

         Nf = ((sa/sfp)**(1.0/b))/2.0

         print Nf
```

```
436966353.019
```

This results in 437 million cycles to failure.

### 0.5 4 (Assuming zero-mean stress mixed-mode loading)

For mixed-mode loading, we can use the effective stress amplitudes

```
In [16]: sx = 27.0
         sy = 13.0
         sz = 0.0
         tauxy = 8.0
         tauxz = 0.0
         tauyz = 0.0

         sa_eff = 1.0/np.sqrt(2)*np.sqrt((sx-sy)**2+(sy-sz)**2+(sz-sx)**2+6*(tauxy**2+tauyz**2+t
```

We can now use the data on Table 9.1 (p. 235) to find the properties for a zero-mean stress S-N curve for 2024-T4 aluminum, we find $\sigma_f' = 131$ ksi and $b = -0.102$

If we substitute $\sigma_a = \bar{\sigma}_a$, we can substitute that result into (15.8) and solve for $N_f$

$$N_f = \frac{\left(\frac{\bar{\sigma}_a}{\sigma_f'}\right)^{1/b}}{2}$$

```
In [17]: sf = 131.0
         b=-0.102

         Nf = ((sa_eff/sf)**(1.0/b))/2.0
```

```
In [18]: Nf
```

```
Out[18]: 2480556.8531319159
```

This means that at this constant amplitude stress level, we can expect failure after 2.48 million cycles.

```
In [19]: sa_eff
```

```
Out[19]: 27.18455443813637
```