AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 18

Dr. Nicholas Smith

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Wichita State University, Department of Aerospace Engineering

SCHEDULE

- 31 Mar - Strain based fatigue, project abstract due
- 5 Apr - Crack Growth, Homework 7 due, Homework 8 assigned
- 7 Apr - Crack Growth, Stress Spectrum
- 12 Apr - Retardation, Boeing Commercial Method
- 14 Apr - Exam Review, Homework 8 Due
- 19 Apr - Exam 2
- 21 Apr - Exam Solutions, Damage Tolerance

OUTLINE

1. strain based fatigue
2. general trends
3. other factors affecting fatigue
4. mean stress effects
5. multiaxial loading

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- The strain-based method gives greater detail, and validity at lower cycles
- It is still valid for high cycle fatigue
- Does not include crack growth analysis or fracture mechanics

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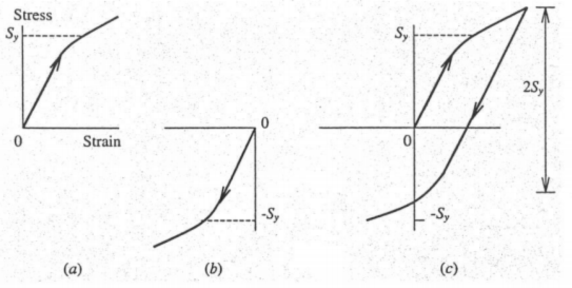
- The test is run in strain control (not load control)
- Generally plotted on log-log scale

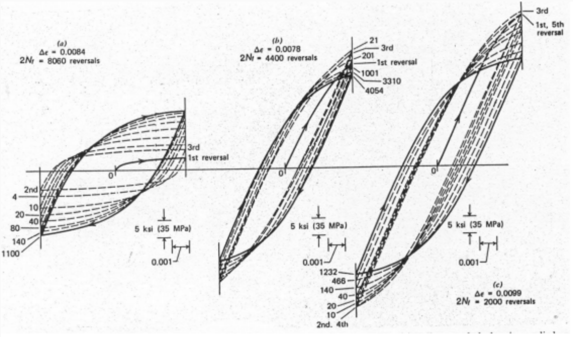
- We can separate the total strain into elastic and plastic components

PLASTIC AND ELASTIC STRAIN

- We can separate the total strain into elastic and plastic components

$$\epsilon_a = \epsilon_{ea} + \epsilon_{pa} \tag{18.1}$$


Stress
S_y
0
Strain
0
$-S_y$
S_y
0
$2S_y$
$-S_y$
(a)
(b)
(c)


(a)
Δε = 0.0084
2Nf = 8060 reversals
3rd
1st reversal
0
2nd
4
10
20
40
80
140
1100
5 ksi (35 MPa)
0.001
(b)
Δε = 0.0078
2Nf = 4400 reversals
21
3rd
201
1st reversal
1001
3310
4054
5 ksi (35 MPa)
0.001
1232
466
140
40
20
10
2nd. 4th
3rd
1st, 5th reversal
5 ksi (35 MPa)
0.001
(c)
Δε = 0.0099
2Nf = 2000 reversals

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$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} \tag{18.2}$$

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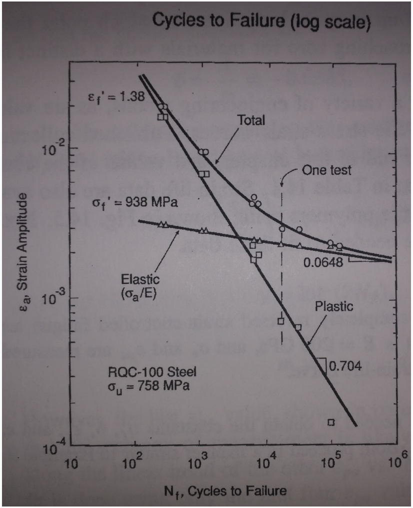
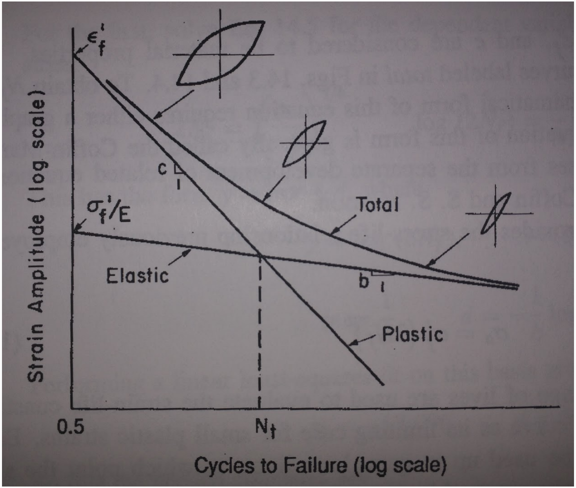
- Once for total strain, once for plastic strain, and once for elastic strain
- Since plastic strain and elastic strain vary by the number of cycles, a hysteresis loop from half the fatigue life is generally used
- This is considered representative of stable behavior


Cycles to Failure (log scale)
$\varepsilon_f' = 1.38$
Total
One test
$\sigma_f' = 938$ MPa
0.0648
Elastic
(σ_a/E)
Plastic
0.704
RQC-100 Steel
$\sigma_u = 758$ MPa
ε_a, Strain Amplitude
10^{-2}
10^{-3}
10^{-4}
10
10^2
10^3
10^4
10^5
10^6
N_f, Cycles to Failure



ϵ_f'
c
1
Total
σ_f'/E
Elastic
b
1
Plastic
Strain Amplitude (log scale)
0.5
N_t
Cycles to Failure (log scale)

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$$\epsilon_{ea} = \frac{\sigma_f'}{E}(2N_f)^b \tag{18.4}$$

LINES

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$$\epsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \epsilon_f'(2N_f)^c \tag{18.6}$$

Data from p. 270

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$$N_t = \frac{1}{2}\left(\frac{\sigma_f'}{\epsilon_f'}\right)^{\frac{1}{c-b}} \tag{18.7}$$

- If we consider the equation for the cyclic stress train curve

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- We can consider the plastic portion and solve for σ_a

$$\sigma_a = H' \epsilon_{pa}^{n'} \tag{18.9}$$

- We can eliminate $2N_f$ from the plastic strain equation

$$\epsilon_{pa} = \epsilon_f'(2N_f)^c \tag{18.10}$$

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  and substituting that into the plastic strain
- We then compare with 18.9 and find

$$H' = \frac{\sigma_f'}{(\epsilon_f')^{b/c}} \tag{18.12a}$$

$$n' = \frac{b}{c} \tag{18.12b}$$

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- One cause for this is data that do not lie on a straight line in the log-log domain
- For ductile materials at short lives, the true stresses and strains may differ significantly from engineering stress and strain

GENERAL TRENDS

- We can consider a tensile test as a fatigue test with $N_f = 0.5$

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- And similarly for strain $\tilde{\epsilon}_f \approx \epsilon'_f$

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- In turn this intersects with the elastic strain line much later, resulting a longer transition life for ductile materials

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- It is also noteworthy that strain-life for many metals pass through the point $\epsilon_a = 0.01$ and $N_f = 1000$ cycles
- Steels also follow a trend with Brinell Hardness, the higher they are on the HB scale, the lower their transition life

TYPICAL PROPERTY RANGES

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- A ”steep” elastic slope is around $b = -0.12$, common in soft metals
- While ”shallow” slopes are around $b = -0.05$, common for hardened metals

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- At a sufficiently elevated temperature for any material

SURFACE FINISH

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- In high-cycle fatigue, crack initiation is important (poor surface finish allows cracks to form earlier)
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$$b_s = \frac{\log\left(m_s(2N_e)^b\right)}{\log(2N_e)} \tag{18.13}$$

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    - Hot rolling can also cause decarburization

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    - Shot peeing introduces many small divots on surface, which can be detrimental in corrosion, but it does cause a residual compressive stress on the surface

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$$m_d = \left(\frac{d}{25.4\text{mm}}\right)^{-0.093} \tag{18.14}$$

- Which is then used to re-calculate material constants

$$\sigma'_{fd} = m_d \sigma'_f, \qquad \epsilon'_{fd} = m_d \epsilon'_f \tag{18.15}$$

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- If the temperature is significantly different between two sides of a part thermal stresses can also be introduced
- Low temperatures generally cause a material to behave in a more brittle fashion, which alters the fatigue life
- High temperatures cause problems with creep-relaxation and can also affect the crystalline structure

MEAN STRESS EFFECTS

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- When the plastic strain is not significant, mean stress will exist
- Mean strain does not generally affect fatigue life

- Recall the Morrow approach for mean stress effects from the stress-based method

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$$\sigma_a = \sigma'_f \left[\left(1 - \frac{\sigma_m}{\sigma'_f} \right)^{\frac{1}{b}} (2N_f) \right]^b \tag{18.17}$$

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- We can now substitute N^* for N_f in the strain-life equation to find

$$\epsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)(2N_f)^b + \epsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{c}{b}}(2N_f)^c \tag{18.19}$$

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- From the zero-mean stress graph, find the point corresponding to your applied strain
- For a non zero mean stress, this point represents (ϵ_a, N^*), we can now solve for N_f using 18.18

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$$\epsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_f}{\sigma_f'}\right)(2N_f)^b + \epsilon_f'(2N_f)^c \tag{18.20}$$

- There is no convenient solution method for this form, and it generally must be solved numerically, or plotted with many families of σ_m

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$$\sigma_{max}\epsilon_a = \frac{\left(\sigma_f'\right)^2}{E}(2N_f)^{2b} + \sigma_f'\epsilon_f'(2N_f)^{b+c} \tag{18.21}$$

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$$\sigma_{max}\epsilon_a = \frac{\left(\sigma_f'\right)^2}{E}(2N_f)^{2b} + \sigma_f'\epsilon_f'(2N_f)^{b+c} \tag{18.21}$$

- This method can also be solved graphically if a plot of $\sigma_{max}\epsilon_a$ is made using zero-mean data. All we need to do is find the new $\sigma_{max}\epsilon_a$ point to find a new N_f

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- The SWT approach is very good for general use, but is non-conservative with a compressive mean stress

EXAMPLE

MULTIAXIAL LOADING

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$$\epsilon_{1a} = \frac{\frac{\sigma_f'}{E}(1-\nu\lambda)(2N_f)^b + \epsilon_f'(1-0.5\lambda)(2N_f)^c}{\sqrt{1-\lambda+\lambda^2}} \tag{18.22}$$

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$$\bar{\epsilon_a} = \frac{\sigma_f'}{E}(2N_f)^b + 2^{1-T}\epsilon_f'(2N_f)^c \tag{18.24}$$