AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 8

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- $K_{Ie} > K_I$
- Sanity check on plots
- Significant figures
- polar plots in Excel - convert to x,y, set axis scales to be equivalent
- watch out for ”smoothing” in Excel (add more data points)

SCHEDULE

- 16 Feb - Residual Strength, Homework 3 Due, Homework 4 Assigned
- 18 Feb - Residual Strength
- 23 Feb - Multiple Site Damage, Homework 4 Due, Homework 5 Assigned
- 25 Feb - Mixed-mode Fracture
- 1 Mar - Section 1 Review, Homework 5 Due
- 3 Mar - Section 1 Review, Homework 5 return
- 8 Mar - Exam 1
- 10 Mar - Exam return, Final Project discussion

OUTLINE

1. compounding
2. thickness effects
3. residual strength
4. fedderson approach
5. proof testing

COMPOUNDING

- Different types of boundaries create different correction factors to the usual stress intensity factor
- We often use β to indicate the total correction factor
- When multiple boundaries are present, we can combine them into one effective correction factor
- There are two general methods we use to create a compound correction factor

COMPOUNDING METHOD 1

- The first method uses linear superposition, and thus is restricted to cases where the effect of each boundary can be assumed to add linearly
- While in most cases this is not strictly true, it provides a reasonable approximation

$$K_r = \bar{K} + \sum_{i=1}^{N} (K_i - \bar{K}) \tag{8.1}$$

- Where N is the number of boundaries, $\bar{K}$ is the stress intensity factor with no boundaries present and K_i is the stress intensity factor associated with the i^{th} boundary.

- We can rewrite this equation as

$$K_r = \sigma\sqrt{\pi a}\beta_r = \sigma\sqrt{\pi a} + \sum_{i=1}^{N}(\sigma\sqrt{\pi a}\beta_i - \sigma\sqrt{\pi a}) \tag{8.2}$$

- Which leads to an expression for β_r as

$$\beta_r = 1 + \sum_{i=1}^{N}(\beta_i - 1) \tag{8.3}$$

- An alternative empirical method approximates the boundary effect as

$$\beta_r = \beta_1 \beta_2 ... \beta_N \tag{8.4}$$

- If there is no interaction between the boundaries, method 1 and method 2 will give the same result

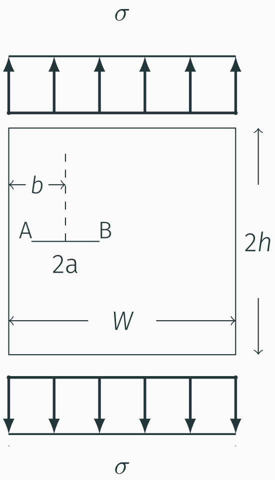

Figure 1: off-center crack, finite height, $2h = 1.6$, $b = .75$, $2a = 1.2$, $W = 4$

GROUP 2

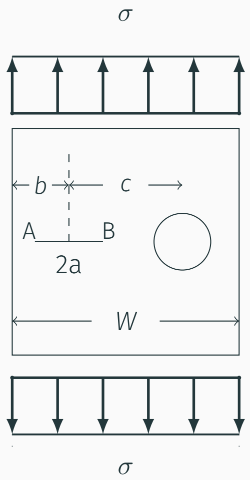

Figure 2: off-center crack, near a hole, $b = 0.5$, $2a = 0.6$, $R = 1.17$, $c = 1.67$, $W = 4$

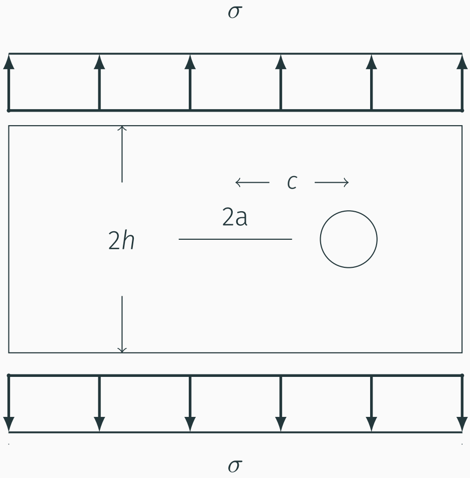

Figure 3: centered crack, near a hole, finite height, $2a = 2$, $W = 4$, $2h = 2$, $R = 0.25$, $c = 1.5$

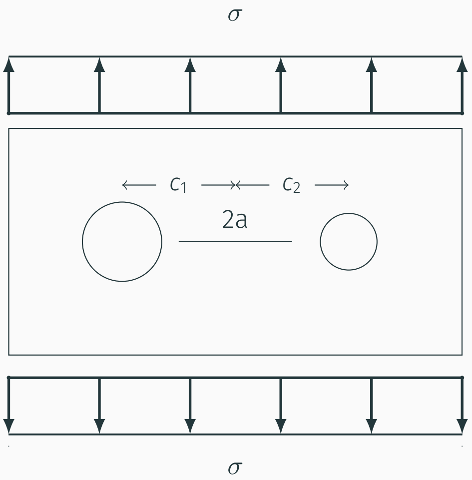

Figure 4: centered crack, near two holes, $2a = 2$, $R_1 = 0.5$, $c_1 = 1.75$, $R_2 = 0.375$, $c_2 = 1.875$

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- The phenomenon is not well-understood

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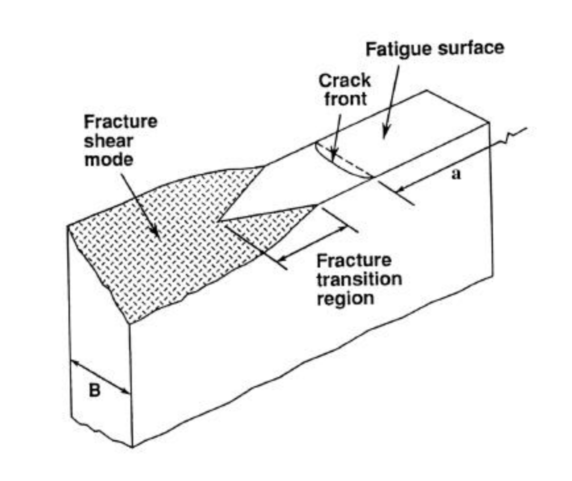
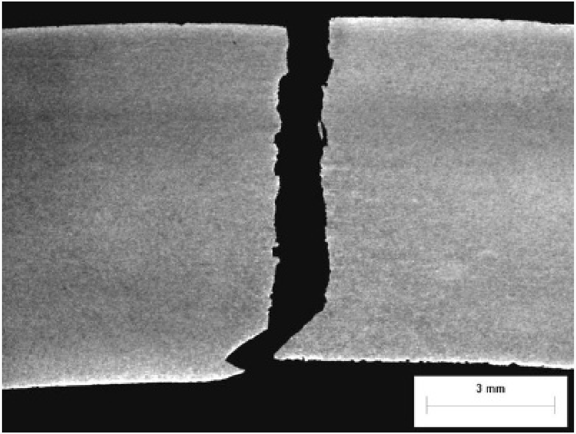
- This is more consistent with pure Mode I

SLANT FRACTURE



SHEAR LIP



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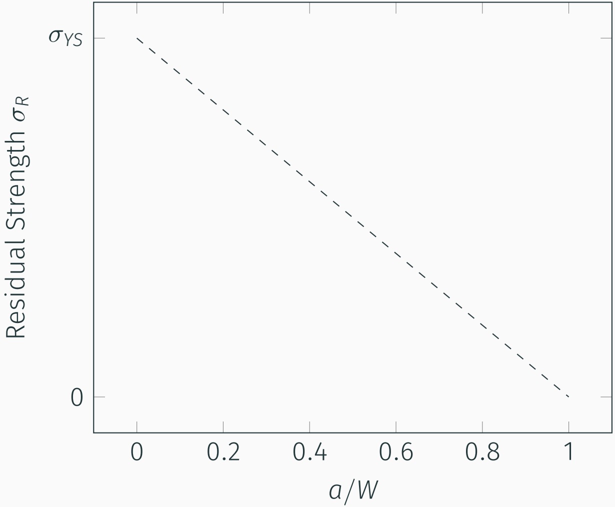
- The residual strength, σ_R is given in terms of the gross area, so as the crack grows the residual strength due to yield decreases
- We can relate the net-section stress to σ_R by

$$\sigma_R = \sigma_{YS} \frac{A_{net}}{A_{gross}} \tag{8.5}$$


σ_{YS}
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Residual Strength σ_R
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a/W

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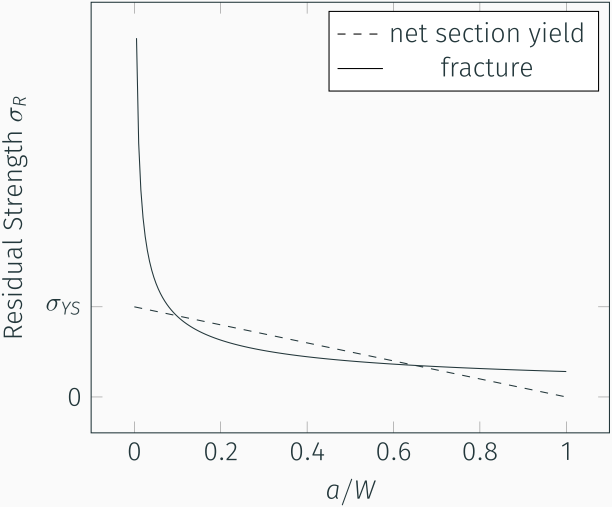
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    3. 2024-T3, $K_C = 144$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 42$ksi

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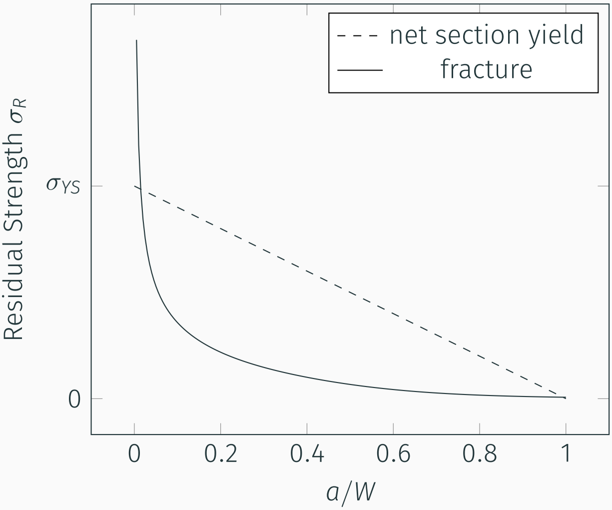
- $$\sigma_C = \sigma_{YS}\frac{W-a}{W} = \sigma_{YS}\frac{6-a}{6}$$
- And the fracture condition by

$$\sigma_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

With

$$\beta = 1.12 - 0.231\left(\frac{a}{W}\right) + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4$$

7178-T6



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fracture
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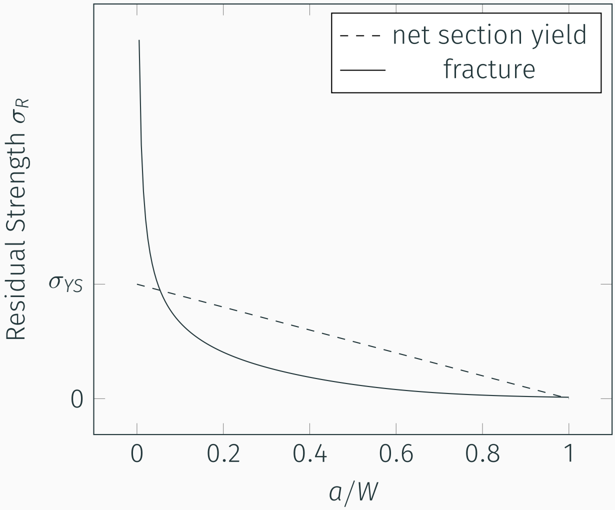

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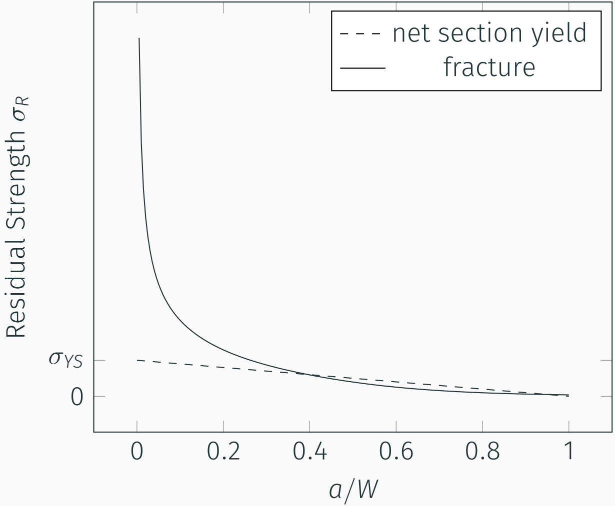

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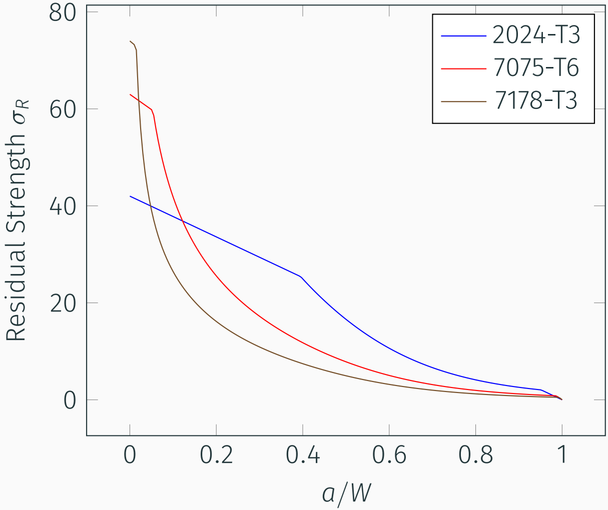

80
60
40
20
0
Residual Strength σ_R
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T-L	L-T

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- S-Basis - no statistical information available, standard value to be used

- F_{tu} - ultimate tensile strength
- F_{ty} - tensile yield strength
- F_{cy} - compressive yield strength
- F_{su} - ultimate shear strength
- F_{bru} - ultimate bearing strength
- F_{bry} - bearing yield strength
- E - tensile Young's Modulus
- E_c - compressive Young's Modulus
- G - shear modulus
- μ - Poisson's ratio

FEDDERSON APPROACH

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- Note: We could do something similar when the crack is very long, but we are generally less concerned with this region (failure will have already occurred)

PROOF TESTING

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- If the part does not fail in the proof test, we can assume that the largest flaw in the material is a_0

- Suppose we are concerned about edge cracks in a panel with $\sigma_{YS} = 65$ksi, $W = 5$"
- We have determined that the largest allowable crack is 0.4"
- The fracture toughness of this panel is $K_c = 140$ ksi$\sqrt{\text{in.}}$
- We can find the proof load

$$\begin{aligned}\sigma_c &= \frac{K_c}{\sqrt{\pi a_0}\beta} \\ &= \frac{140}{\sqrt{\pi 0.4}(1.161)} \\ &= 107.6\end{aligned}$$

- So the proof load would need to induce a gross section stress of 107.6 ksi.