AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 21

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Last Updated: April 12, 2016 at 2:38pm

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- 12 Apr - Retardation, Boeing Commercial Method
- 14 Apr - Exam Review, Homework 8 Due
- 19 Apr - Damage Tolerance
- 21 Apr - Exam 2
- 26 Apr - Exam Solutions, Damage Tolerance
- 28 Apr - SPTE, AFGROW, Finite Elements

OUTLINE

1. review
2. boeing method
3. cycle counting
4. crack growth retardation

REVIEW

- While the Paris Law can be integrated directly (for simple load cases), many of the other formulas cannot
- A simple numerical algorithm for determining incremental crack growth is

$$a_{i+1} = a_i + \left(\frac{da}{dN}\right)_i (\Delta N)_i \tag{21.1}$$

- This method is quite tedious by hand (need many a_i values for this to be accurate)
- But is simple to do in Excel, MATLAB, Python, or many other codes
- For most accurate results, use $\Delta N = 1$, but this is often unnecessary
- When trying to use large ΔN, check convergence by using larger and smaller ΔN values

CONVERGENCE EXAMPLE

- In practice variable loads are often seen
- The most basic way to handle these is to simply calculate the crack length after each block of loading
- We will discuss an alternate method, which is more convenient for flight ”blocks” next class
- We will also discuss ”retardation” models next class

BOEING METHOD

- Whether integrating numerically or analytically, it is time-consuming to consider multiple repeated loads
- It is particularly difficult to consider flight loads, which can vary by ”mission”
- For example, an aircraft may fly three different routes, in no particular order, but with a known percentage of time spent in each route
- Traditional methods would use a random mix of each load spectra
- The Boeing Method combines each repeatable load spectrum into one single equivalent cycle

- The Boeing method is derived by separating the geometry effects from load and material effects in the Boeing-Walker equation.

$$\frac{da}{dN} = \left[\frac{1}{n}\right]\frac{dL}{dN} = 10^{-4}\left[\frac{k_{max}Z}{m_T}\right]^p \tag{21.2}$$

$$\frac{dL}{dN} = n10^{-4}\left[\frac{k_{max}Z}{m_T}\right]^p \tag{21.3}$$

$$\frac{dN}{dL} = \frac{1}{n}10^{4}\left[\frac{m_T}{k_{max}Z}\right]^p \tag{21.4}$$

$$\int_0^N dN = \frac{10^4}{n}\int_{L_0}^{L_f}\left[\frac{m_T}{k_{max}Z}\right]^p dL \tag{21.5}$$

$$N = 10^4\left(\frac{m_t}{z\sigma_{max}}\right)^p\int_{L_0}^{L_f}\frac{dL}{\left(n\sqrt{\pi L/n}\beta\right)^p} \tag{21.6}$$

- In this form, the term $10^4\left(\frac{m_t}{z\sigma_{max}}\right)^p$ is strictly from the applied load and material, while $\int_{L_0}^{L_f}\frac{dL}{\left(n\sqrt{\pi L/n}\beta\right)^p}$ is from geometry
- If we now define G to account for crack geometry

$$G = \left[\int_{L_0}^{L_f}\frac{dL}{\left(n\sqrt{\pi L/n}\beta\right)^p}\right]^{-1/p} \tag{21.7}$$

- And define $z\sigma_{max} = S$ as the equivalent load spectrum, then we have

$$N = 10^4\left(\frac{m_t/G}{S}\right)^p \tag{21.8}$$

- Using this method, G is typically looked up from a chart (such as on p. 369)

- To replace a repeated load spectrum with an equivalent load, we need to invert the relationship
- Equation 21.8 gives cycles per crack growth, inverting gives crack growth per cycle

$$\text{crack growth per cycle} = 10^{-4}\left(\frac{m_t/G}{S}\right)^{-p} \tag{21.9}$$

- If we consider a general, repeatable ”block”, we have

$$10^{-4}\left(m_t/G\right)^{-p}\sum_i\left(\frac{1}{z\sigma_{max}}\right)_i^{-p}N_i = 10^{-4}\left(\frac{m_t/G}{S}\right)^{-p} \tag{21.10}$$

- Which simplifies to

$$\sum_i\left(z\sigma_{max}\right)_i^p N_i = (S)^p \tag{21.11}$$

BOEING METHOD EXAMPLE

BOEING METHOD EXAMPLE - CONT.

cycle counting

CYCLE COUNTING

- As illustrated in our previous example, cycle counting method can make a difference for variable amplitude loads
- Two common methods for cycle counting that give similar results are known as the ”rainflow” and ”range-pair” methods
- ASTM E1049-85 ”Standard Practices for Cycle Counting in Fatigue Analysis”

1. Read next peak or valley. S is the starting point, Y is the first range, X is the second range
2. If $X < Y$ advance points (S remains same, Y and X change)
3. If $X \geq Y$ and Y contains S, count Y as 1/2-cycle, discard S and go to 1
4. If $X \geq Y$ and Y does not contain S, count Y as 1 cycle, discard both points in Y and go to 1 (S remains same)
5. When end of data is reached, count each range as 1/2-cycle

1. Read next peak or valley. Y is the first range, X is the second range
2. If $X < Y$ advance points
3. If $X \geq Y$, count Y as 1 cycle and discard both points in Y, go to 1
4. Remaining cycles are counted backwards from end of history

CYCLE COUNTING EXAMPLE

CRACK GROWTH RETARDATION

- When an overload is applied, the plastic zone is larger
- This zone has residual compressive stresses, which slow crack growth until the crack grows beyond this over-sized plastic zone
- We will discuss three retardation models, but no model has been shown to be perfect in all cases
- The Wheeler method reduces da/dN, the Willenborg model reduces ΔK, and the Closure model increases R (increases σ_{min})

- As long as crack is within overload plastic zone, we scale da/dN by some ϕ

$$(a_i + r_{pi}) = (a_{ol} + r_{pol}) \tag{21.12}$$

- And ϕ is given by

$$\phi_i = \left[\frac{r_{pi}}{a_{ol} + r_{pol} - a_i}\right]^m \tag{21.13}$$

- and the constant, m is to be determined experimentally

- Once again, we consider that retardation occurs when $(a_i + r_{pi}) = (a_{ol} + r_{pol})$
- Willenborg assumes that the residual compressive stress in the plastic zone creates an effective, $K_{max,eff}$, where $K_{max,eff} = K_{max} - K_{comp}$
- The effective stress intensity factor is given by

$$K_{max,eff} = K_{max,i} - \left[K_{max,OL}\sqrt{1 - \frac{\Delta a_i}{r_{pol}}} - K_{max,i} \right] \qquad (21.14)$$

- Galagher and Hughes observed that the Willenborg model stops cracks when they still propagate
- They proposed a correction to the model

$$K_{max,eff} = K_{max,i} - \phi_i \left[K_{max,OL} \sqrt{1 - \frac{\Delta a_i}{r_{pol}}} - K_{max,i} \right] \tag{21.15}$$

- And the correction factor, ϕ_i is given by

$$\phi_i \frac{1 - K_{TH}/K_{max,i}}{s_{ol} - 1} \tag{21.16}$$

WILLENBORG EXAMPLE

- Once again, we consider that retardation occurs when $(a_i + r_{pi}) = (a_{ol} + r_{pol})$
- Within the overloaded plastic zone, the opening stress required can be expressed as

$$\sigma_{OP} = \sigma_{max}(1 - (1 - C_{f0})(1 + 0.6R)(1 - R)) \tag{21.17}$$

- Commonly this is expressed using the Closure Factor, C_f

$$C_f = \frac{\sigma_{OP}}{\sigma_{max}} = (1 - (1 - C_{f0})(1 + 0.6R)(1 - R)) \tag{21.18}$$

- Where C_{f0} is the value of the Closure Factor at $R = 0$

- When using the closure model, we replace R with C_f
- If the model we are using is in terms of ΔK we will also need to use $\Delta K = (1 - C_f)K_{max}$

CLOSURE EXAMPLE

- Just as a tensile ”overload” retards crack growth, we might expect a compressive ”underload” to accelerate crack growth
- This effect is not usually modeled for a few reasons
    1. Compressive underloads are uncommon in airframes
    2. The acceleration effect is minimal
    3. Analysis is generally adjusted with experimental data, so acceleration can be built in to current model
    4. Structures with large compressive loads are not generally subject to crack propagation problems