AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 2

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OUTLINE

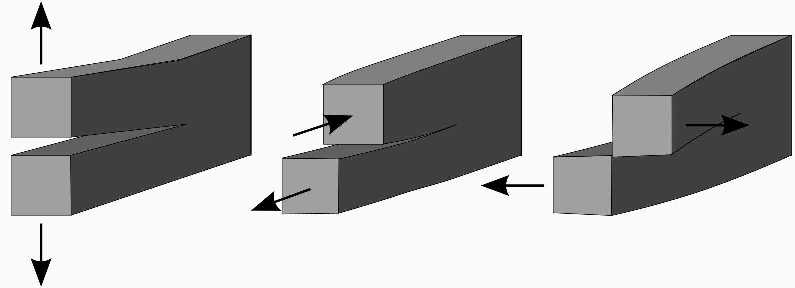
OFFICE HOURS

- I will e-mail everyone in the course a Doodle link we can use to find the optimal office hours
- Let me know if you do not receive the e-mail, you may need to update your information in Blackboard
- Take advantage of office hours, this is time that I have already set aside for you
- If the regular office hours do not work for your schedule, send me an e-mail and we can work out a time to meet
- While in person visits are often the most helpful, I will always try to answer questions as best as I can via e-mail

FRACTURE MECHANICS

- *Linear Elastic Fracture Mechanics* is the study of the propagation of cracks in materials
- There are some corrections we add to account for plasticity
- In this course we will not follow the full mathematical development of fracture mechanics (there is a separate course dedicated to that)
- Instead we will take some results and apply them

- In fracture mechanics we consider three different modes
- Mode I is known as the ”opening mode”
- Mode II is known as the ”sliding mode”
- Mode III is known as the ”tearing mode”



Mode I: Opening Mode II: Sliding Mode III: Tearing

STRESS INTENSITY

STRESS INTENSITY

- A key finding from Linear Elastic Fracture Mechanics (LEFM) is known as the *Stress Intensity Factor*
- The stress intensity factor is often written in this form

$$K = \sigma\sqrt{\pi a}\beta$$

- Where K is the stress intensity factor, σ is the applied stress, a is the crack length, and β is a dimensionless parameter depending on geometry
- Be careful that although the notation is similar, the *Stress Intensity Factor* is different from the *Stress Concentration Factor* from strength of materials
- We are usually most concerned with Mode I, but there will be a unique stress intensity factor for each mode, we label these K_I, K_{II}, and K_{III}
- If no subscript is given, assume Mode I

- For brittle materials (where ”linear” fracture mechanics assumptions hold true) we can find the full stress field near the crack in terms of the stress intensity factor

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

- Similarly for Mode II we find

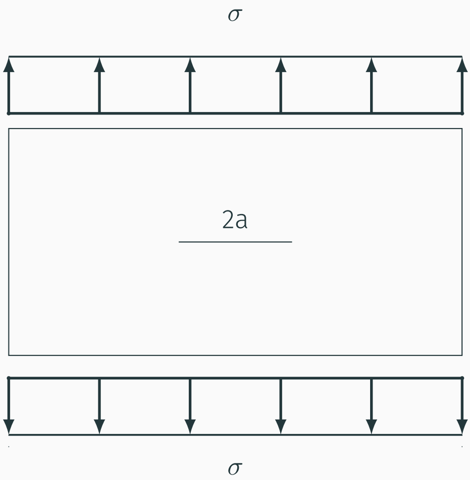
$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$
$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

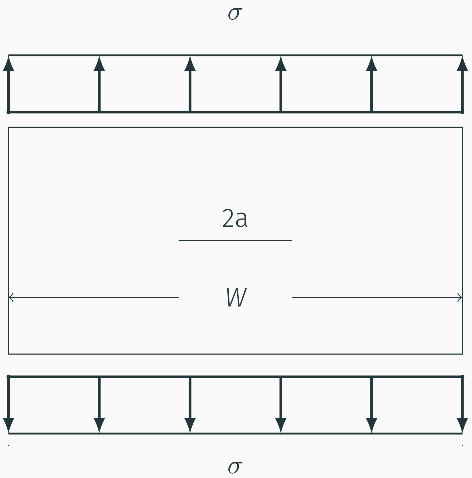
- And for Mode III

$$\tau_{xz} = \frac{-K_{III}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}$$
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}$$

COMMON STRESS INTENSITY FACTORS

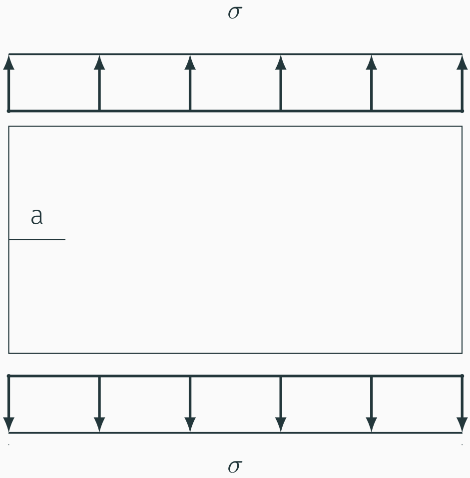
$$K_I = \sigma\sqrt{\pi a} \tag{2.1}$$



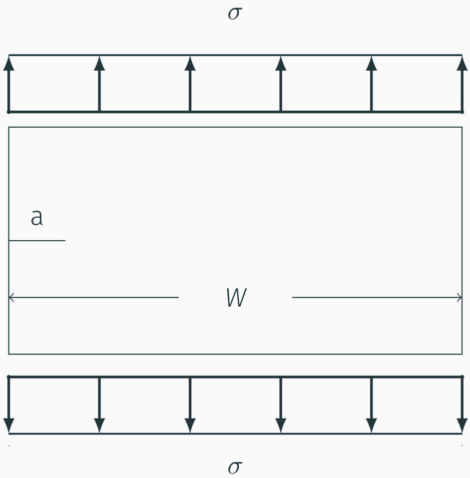

σ
2a
W
σ

- $$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)} \tag{2.2a}$$
- Accurate within 0.3% for $2a/W \leq 0.7$
- within 1.0% for $2a/W = -.8$
- $$K_I = \sigma\sqrt{\pi a}\left[1.0 - 0.025\left(\frac{2a}{W}\right)^2 + 0.06\left(\frac{2a}{W}\right)^4\right]\sqrt{\sec(\pi a/W)} \tag{2.2b}$$
- Accurate within 0.1% for all crack lengths.

$$K_I = 1.122\sigma\sqrt{\pi a} \tag{2.3}$$



EDGE CRACK, FINITE WIDTH



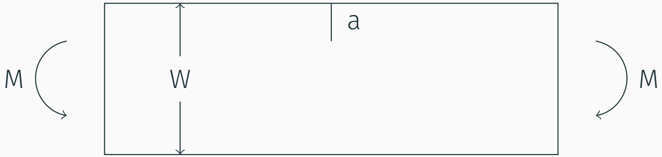
$$\beta = \left[1.12 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.71\left(\frac{a}{W}\right)^3 + 30.82\left(\frac{a}{W}\right)^4\right] \tag{2.4a}$$

- Within 0.5% accuracy for $\frac{a}{W} < 0.6$

$$\beta = \frac{0.752 + 2.02\frac{a}{W} + 0.37\left(1 - \sin\frac{\pi a}{2W}\right)^3}{\cos\frac{\pi a}{2W}}\sqrt{\frac{2W}{\pi a}\tan\frac{\pi a}{2W}} \tag{2.4b}$$

- Within 0.5% accuracy for $0 < \frac{a}{W} < 1.0$

EDGE CRACK, BENDING MOMENT



EDGE CRACK, BENDING MOMENT

- The usual form for stress intensity still applies

$$K_I = \sigma\sqrt{\pi a}\beta$$

- Where

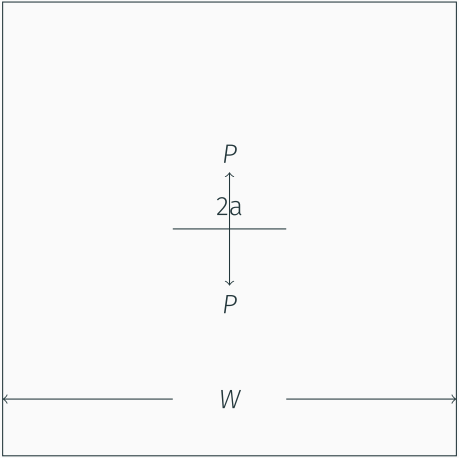
$$\sigma = \frac{6M}{tW^2}$$

$$\beta = 1.122 - 1.40\left(\frac{a}{W}\right) + 7.33\left(\frac{a}{W}\right)^2 - 13.08\left(\frac{a}{W}\right)^3 + 14.0\left(\frac{a}{W}\right)^4 \tag{2.5a}$$

- valid within 0.2% accuracy for $\frac{a}{W} \leq 0.6$

$$\beta = \frac{0.923 + 0.199\left(1 - \sin\frac{\pi a}{2W}\right)^4}{\cos\frac{\pi a}{2W}}\sqrt{\frac{2W}{\pi a}\tan\frac{\pi a}{2W}} \tag{2.5b}$$

- valid within 0.5% for any $\frac{a}{W}$

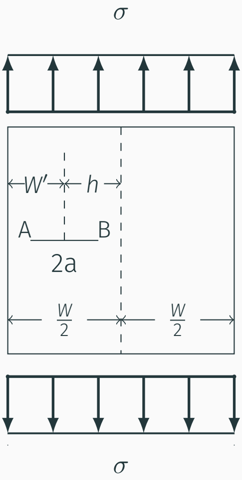

P
2a
P
W

- With an applied load we use a slightly modified form for the stress intensity factor

$$K_I = \frac{P}{t\sqrt{\pi a}}\beta \tag{2.6}$$

- With β in this case given as

$$\beta = \frac{1 - 0.5\left(\frac{a}{W}\right) + 0.975\left(\frac{a}{W}\right)^2 - 0.16\left(\frac{a}{W}\right)^3}{\sqrt{1 - \left(\frac{a}{W}\right)}} \tag{2.7}$$



$$K_{IA} = \sigma\sqrt{\pi a}\beta_c\beta_A \text{ and } K_{IB} = \sigma\sqrt{\pi a}\beta_c\beta_B \tag{2.8}$$

$$F_{A,B} = \frac{1}{(1-\lambda^{1.8})^{1.08}}\left[\frac{2(1-\cos\theta_1)(1+\cos\theta_2)}{\theta_1\sin\theta_1(cos\theta_1+\cos\theta_2)}\right]\frac{\frac{\pi}{2}\lambda}{\tan\frac{\pi}{2}\lambda}$$

$$\lambda = \frac{a}{W'} \leq 0.9$$

$$\epsilon = \frac{h}{W} < 1.0$$

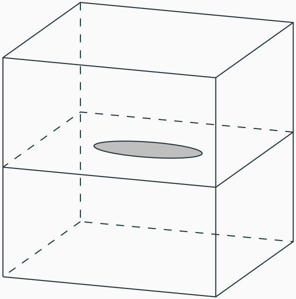
$$\theta_1 = \frac{\pi\lambda}{2}(1-\epsilon)$$

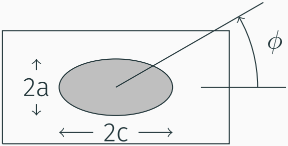
$$\theta_2 = \pi\left[\frac{\lambda}{2}(1-\epsilon)\pm\epsilon\right]$$

Use + in θ_2 for $K_{I,A}$ and - for $K_{I,B}$

3D crack shapes

- The previous stress intensity factors all assume a 2D problem
- Through the thickness, it is assumed that the crack length is the same
- In many cases this is not an accurate assumption
- We will now consider 3D crack shapes and their effect on the stress intensity factor




2a
2c
ϕ

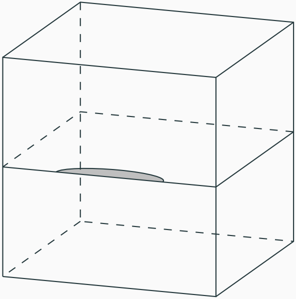
$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4} \tag{2.9}$$

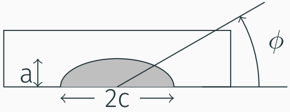
$$Q = \Phi^2 - 0.212\left(\frac{\sigma}{\sigma_y}\right)^2 \text{(2nd term usually ignored)}$$

$$\Phi = \int_0^{\pi/2}\left[1 - \left(\frac{c^2-a^2}{c^2}\sin^2\phi\right)\right]^2 d\phi$$

$$\approx \frac{\pi}{2}\left[1 - \frac{1}{4}\frac{c^2-a^2}{c^2} - \frac{3}{64}\left(\frac{c^2-a^2}{c^2}\right)^2 - \ldots\right]$$

$$\Phi \approx \frac{\pi}{2}\left[1 - \frac{1}{4}\frac{c^2-a^2}{c^2}\right] \text{(sufficient for most cases)}$$



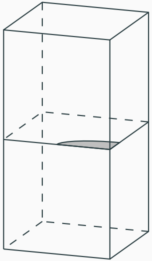


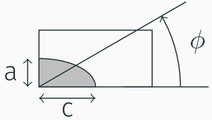
$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4} \quad (1.1)$$

(2.10)

Note: $1.1 \approx \sqrt{1.2}$, which is known as the front surface correction factor. A more accurate correction factor is given as

$$1 + .12\left(1 - \frac{a}{c}\right)$$





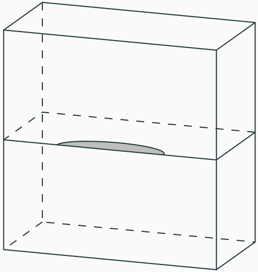
$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4} \quad (1.1)(1.1)$$

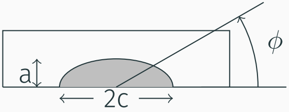
(2.11)

Note: $1.1 \approx \sqrt{1.2}$, which is known as the front surface correction factor. A more accurate correction factor is given as

$$1 + .12\left(1 - \frac{a}{c}\right)$$

SEMI-ELLIPTICAL SURFACE FLAW IN FINITE BODY

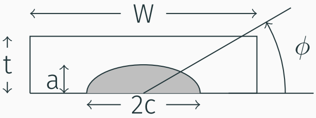




$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4} \quad (1.1) M_K \qquad (2.12)$$

Front surface correction same as for the infinite body.

M_K is the back surface correction, and can be looked up on a chart in your textbook.

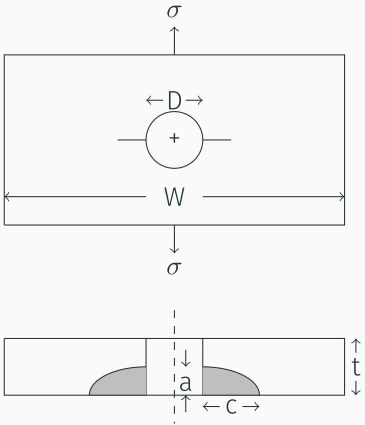


- An alternative method for calculating the finite thickness and finite width correction factors uses the following formula

$$K_I = \sigma\sqrt{\pi c}\sqrt{\frac{a}{cQ'}}\left[M_1 + \left(\sqrt{\frac{Q'c}{a}} - M_1\right)\left(\frac{a}{t}\right)^P\right]\sqrt{\sec\left(\frac{\pi c}{W}\sqrt{\frac{a}{t}}\right)} \tag{2.13}$$

FINITE THICKNESS CORRECTION

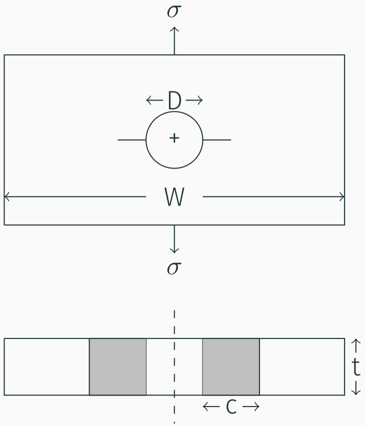
$$
\begin{aligned}
M_1 &= 1.2 - 0.1\frac{a}{c} && \text{for} \quad 0.02 \leq \frac{a}{c} \leq 1 \\
&= \sqrt{\frac{c}{a}}\left(1 + 0.1\frac{a}{c}\right) && \text{for} \quad \frac{a}{c} > 1 \\
Q' &= 1 + 1.47\left(\frac{a}{c}\right)^{1.64} && \text{for} \quad \frac{a}{c} \leq 1 \\
&= 1 + 1.47\left(\frac{c}{a}\right)^{1.64} && \text{for} \quad \frac{a}{c} > 1 \\
P &= 2 + 8\left(\frac{a}{c}\right)^{3}
\end{aligned}
$$


σ
D
W
σ
a
t
c

- For corner cracks under uniform applied stress, we have

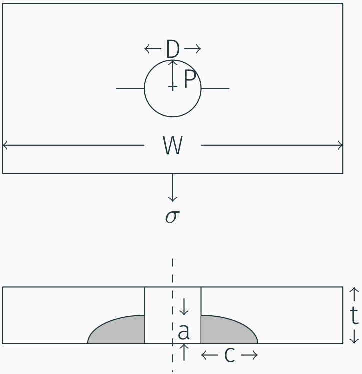
$$K_I = \sigma\sqrt{\pi c}\sqrt{\frac{a}{cQ'}}\left[M_1 + \left(\sqrt{\frac{Q'c}{a}} - M_1\right)\left(\frac{a}{t}\right)^P\right]$$
$$\sqrt{\sec\left(\frac{\pi}{2}\frac{D+bc}{W-2C+bc}\sqrt{\frac{a}{t}}\right)} f_b\sqrt{\sec\frac{\pi D}{2W}} \tag{2.14}$$

CRACKS AROUND A HOLE



- For through cracks under uniform applied stress, we have

$$K_I = \sigma\sqrt{\pi c}\sqrt{\sec\left(\frac{\pi}{2}\frac{D+bc}{W-2C+bc}\right)}f_b\sqrt{\sec\frac{\pi D}{2W}} \tag{2.15}$$


D
P
W
σ
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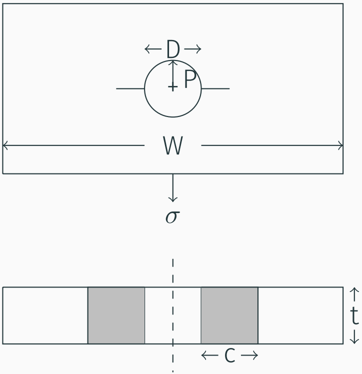
c

- For corner cracks under fastener load, we have

$$K_I = \frac{P}{Wt}\sqrt{\pi c}\sqrt{\frac{a}{cQ'}}\left[M_1 + \left(\sqrt{\frac{Q'c}{a}} - M_1\right)\left(\frac{a}{t}\right)^P\right] \sqrt{\sec\left(\frac{\pi}{2}\frac{D+bc}{W-2C+bc}\sqrt{\frac{a}{t}}\right)} f_b \sqrt{\sec\frac{\pi D}{2W}} G_b \tag{2.16}$$

CRACKS AROUND A HOLE



- For through cracks under fastener load, we have

$$K_I = \frac{P}{Wt}\sqrt{\pi c}\sqrt{\sec\left(\frac{\pi}{2}\frac{D+bc}{W-2C+bc}\right)}f_b\sqrt{\sec\frac{\pi D}{2W}}G_b \tag{2.17}$$

$$
\begin{aligned}
P &= 2 + 8\left(\frac{a}{c}\right)^3 & \lambda &= \frac{1}{1+2C/D} \\
M_1 &= 1.2 - 0.1\left(\frac{a}{c}\right) & \text{for} \quad .02 &\le \frac{a}{c} \le 1 \\
&= \sqrt{\frac{c}{a}}\left[1 + 0.1\left(\frac{a}{c}\right)\right] & \text{for} \quad \frac{a}{c} &> 1 \\
f_b = f_1 &= .707 - .18\lambda + 6.55\lambda^2 - 10.54\lambda^3 + 6.85\lambda^4 & \text{for} \quad b &= 1 \\
f_b = f_2 &= 1 - .15\lambda + 3.46\lambda^2 - 4.47\lambda^3 + 3.52\lambda^4 & \text{for} \quad b &= 2 \\
G_1 &= \frac{1}{2} + \frac{W}{\pi(D+c)}\sqrt{\frac{D}{D+2c}} & \text{for} \quad b &= 1 \\
G_2 &= \frac{1}{2} + \frac{W}{\pi(D+2c)} & \text{for} \quad b &= 2
\end{aligned}
$$

EXAMPLES

EXAMPLE 1

1. 1.1 Determine the value of K_I for a center-cracked panel with $W/2a = 3$ and a uniformly applied remote stress, σ.
   1.2 Determine the value of K_I for an edge-cracked panel with $W/a = 3$ and a uniformly applied remote stress, σ.
   1.3 Compare these two results. Note that in both cases the panel width to crack length ratio is the same.

EXAMPLE 1

- Based on the ratio of crack length to width, we choose (2) over (1)
$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)}$$
- This gives $K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi/6)}$
- If we normalize by the infinite width solution, we find $K_{I,f}/K_{I,i} \approx 1.075$

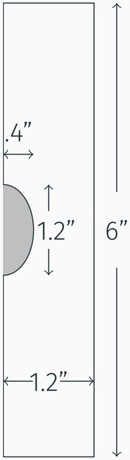
- Once again, based on the ratio of crack length to width, we choose (4) over (3)

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4\right]$$

- This gives $K_I \approx 1.786\sigma\sqrt{\pi a}$
- If we normalize by the infinite width solution, we find $K_{I,f}/K_{I,i} \approx 1.595$

EXAMPLE 1

- Comparing the two cases, we see that the finite width effects are much more significant for the edge-crack specimen
- The edge-crack specimen is also overall more effected by a crack of that relative length.
- Why are they not the same?



- Find maximum value of K_I for semi-elliptical surface flaw
- $\sigma = 20$kpsi (in opening direction)

- Here we will compare three different equations
- (8), which is for infinite width and thickness

$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4}(1.1) \tag{8}$$

- (10), which is for infinite width, but finite thickness

$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{1/4}(1.1)M_K \tag{10}$$

- And (11), which accounts for finite width and thickness

$$K_I = \sigma\sqrt{\pi c}\sqrt{\frac{a}{cQ'}}\left[M_1 + \left(\sqrt{\frac{Q'c}{a}} - M_1\right)\left(\frac{a}{t}\right)^P\right]\sqrt{\sec\left(\frac{\pi c}{W}\sqrt{\frac{a}{t}}\right)} \tag{11}$$