AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 21

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Last Updated: April 12, 2016 at 2:38pm

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- 12 Apr - Retardation, Boeing Commercial Method
- 14 Apr - Exam Review, Homework 8 Due
- 19 Apr - Damage Tolerance
- 21 Apr - Exam 2
- 26 Apr - Exam Solutions, Damage Tolerance
- 28 Apr - SPTE, AFGROW, Finite Elements

1. review
2. boeing method
3. cycle counting
4. crack growth retardation

REVIEW

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- When trying to use large ΔN, check convergence by using larger and smaller ΔN values

CONVERGENCE EXAMPLE

VARIABLE LOAD CASES

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- We will also discuss ”retardation” models next class

BOEING METHOD

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- In this form, the term $10^4\left(\frac{m_t}{z\sigma_{max}}\right)^p$ is strictly from the applied load and material, while $\int_{L_0}^{L_f}\frac{dL}{\left(n\sqrt{\pi L/n\beta}\right)^p}$ is from geometry

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- Using this method, G is typically looked up from a chart (such as on p. 369)

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- Which simplifies to

$$\sum_i\left(z\sigma_{max}\right)_i^p N_i = (S)^p \tag{21.11}$$

BOEING METHOD EXAMPLE

cycle counting

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- ASTM E1049-85 ”Standard Practices for Cycle Counting in Fatigue Analysis”

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5. When end of data is reached, count each range as 1/2-cycle

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4. Remaining cycles are counted backwards from end of history

CYCLE COUNTING EXAMPLE

CRACK GROWTH RETARDATION

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- We will discuss three retardation models, but no model has been shown to be perfect in all cases
- The Wheeler method reduces da/dN, the Willenborg model reduces ΔK, and the Closure model increases R (increases σ_{min})

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WHEELER EXAMPLE

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- The effective stress intensity factor is given by

$$K_{max,eff} = K_{max,i} - \left[K_{max,OL} \sqrt{1 - \frac{\Delta a_i}{r_{pol}}} - K_{max,i} \right] \tag{21.14}$$

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- And the correction factor, ϕ_i is given by

$$\phi_i \frac{1 - K_{TH}/K_{max,i}}{s_{ol} - 1} \tag{21.16}$$

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- Where C_{f0} is the value of the Closure Factor at $R = 0$

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- If the model we are using is in terms of ΔK we will also need to use $\Delta K = (1 - C_f)K_{max}$

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    4. Structures with large compressive loads are not generally subject to crack propagation problems