AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 12

Dr. Nicholas Smith

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Wichita State University, Department of Aerospace Engineering

SCHEDULE

- 1 Mar - Section 1 Review, Homework 5 Due
- 3 Mar - Section 1 Review, Homework 5 return
- 8 Mar - Exam 1
- 10 Mar - Exam return, Final Project discussion

OUTLINE

1. mixed mode fracture
2. exam
3. stress intensity
4. plastic zone

MIXED MODE FRACTURE

- Most cracks are primarily Mode I, but sometimes Mode II can also have an effect
- We can look at the combined stress field for Mode I and Mode II
- Recall the stress field near the crack tip

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{12.1a}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{12.1b}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{12.1c}$$

- For Mode II we have

$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \tag{12.2a}$$

$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{12.2b}$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{12.2c}$$

- In mixed-mode fracture problems, the crack will generally propagate in a different direction from the initial crack plane
- It is more convenient to handle this scenario in Polar Coordinates
- We can convert stress from Cartesian coordinates to Polar Coordinates using the stress transformation equations

$$\sigma_r = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \tag{12.3a}$$

$$\sigma_\theta = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta \tag{12.3b}$$

$$\tau_{r\theta} = -\sigma_x \sin\theta\cos\theta + \sigma_y \sin\theta\cos\theta + \tau_{xy}(\cos^\theta - \sin^2\theta) \tag{12.3c}$$

- When we convert the stress fields from Mode I and Mode II into polar coordinates and combine them, we find

$$\sigma_r = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right) \tag{12.4a}$$

$$\sigma_\theta = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right) \tag{12.4b}$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right) \tag{12.4c}$$

MAXIMUM CIRCUMFERENTIAL STRESS CRITERION

- The Maximum Circumferential Stress Criterion assumes that a crack will propagate in the principal direction
- In this direction, the shear stress is 0
- The fracture toughness is determined by the Mode I fracture toughness of the material
- **Note:** In this discussion, we will use K_{IC} to differentiate Mode I fracture toughness from Mode II fracture toughness. This does NOT necessarily mean we are referring to plane strain fracture toughness
- Thus fracture begins when

$$\sigma_\theta(\theta_P) = \sigma_\theta(\theta = 0, K_{II} = 0, K_I = K_{Ic}) = \frac{K_{IC}}{\sqrt{2\pi r}} \tag{12.5}$$

- Following the above assumptions, we can solve Equations 12.4c and 12.4b to find θ_P
- Note: This assumes that we know both K_I and K_{II}, in this class we have not discussed any Mode II stress intensity factors, so they will be given.
- Equation 12.4c in this case simplifies to

$$K_I \sin\theta_p + K_{II}(3\cos\theta_P - 1) = 0 \tag{12.6}$$

- and Equation 12.4b simplifies to

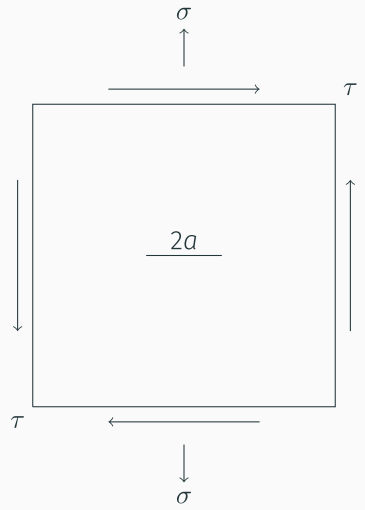
$$4K_{IC} = K_I\left(3\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\right) - 3K_{II}\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right) \tag{12.7}$$

- The general form for a Mode II stress intensity factor is

$$K_{II} = \tau\sqrt{\pi a}\beta' \tag{12.8}$$

Assuming $|\sigma| = 4|\tau|$, $K_{IC} = 60$ ksi$\sqrt{\text{in}}$, and $2a = 1.5$ in.

Note: Assume $\beta = \beta' = 1$



- In the maximum circumferential stress criterion, we found the principal direction in polar coordinates
- We can also find the principal direction in Cartesian coordinates
- If we make a free body cut along some angle θ we find, from equilibrium

$$0 = \sigma_\theta dA - \sigma_x dA \sin^2\theta - \sigma_y dA \cos^2\theta + 2\tau_{xy} dA \cos\theta \sin\theta \tag{12.9a}$$

$$\sigma_\theta = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta \tag{12.9b}$$

$$\frac{\partial \sigma_\theta}{\partial \theta} = (\sigma_x - \sigma_y) \sin 2\theta_p - 2\tau_{xy} \cos 2\theta_P \tag{12.9c}$$

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{12.9d}$$

- As before, we consider crack propagation purely due to Mode I
- In the principal stress criterion, we find the maximum Mode I stress as a function of the remote applied stress

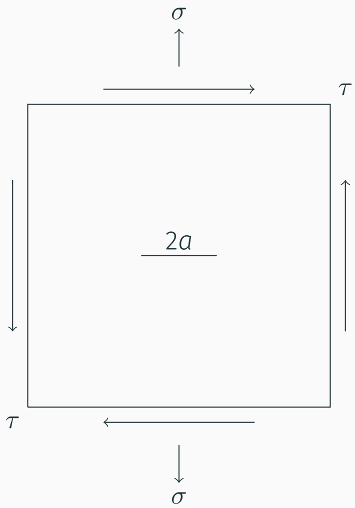
$$\sigma_{P1} = C\sigma \tag{12.10}$$

- We then find the remote failure stress by

$$\sigma_c = \frac{K_{IC}}{C\sqrt{\pi a}\beta} \tag{12.11}$$

Assuming $|\sigma| = 4|\tau|$, $K_{IC} = 60$ ksi$\sqrt{\text{in}}$, and $2a = 1.5$ in.

Note: Assume $\beta = \beta' = 1$



EXAM

- 4-5 questions
- Closed book, notes
- Equation sheet provided
- No T/F section, but the T/F questions in text can still be helpful
- Like homework, but simpler calculations
- In-class group review problems are also good practice for exam

- Stress Intensity
    - Geometry-specific formulas
    - Compounding
    - Superposition
    - Cracks on curved boundary

- Plastic Zone
    - Irwin’s first approximation
    - Irwin’s correction
    - Effective crack length
    - Plane strain vs. plane stress
    - Plastic zone shape

- Fracture Toughness
    - Plane strain fracture toughness - material property
    - Plane stress fracture toughness - not
    - Crack resistance curve
    - General test/analysis methodology (ASTM)
    - Thickness effects

- Residual Strength
    - Net section yield
    - Fedderson

- Stiffeners
    - Panel, stiffener, rivet residual strength
    - Positive slope in residual strength curve
    - Analyze residual strength curves

- Multiple site damage
    - Link-up equation
    - Modified link-up equation
    - Ductile/tough vs. stiff/brittle materials

- Mixed-mode fracture
    - Maximum Circumferential Stress
    - Principal Stress
    - Why is principal stress method bad?

STRESS INTENSITY

- Different types of boundaries create different correction factors to the usual stress intensity factor
- We often use β to indicate the total correction factor
- When multiple boundaries are present, we can combine them into one effective correction factor
- There are two general methods we use to create a compound correction factor

- The first method uses linear superposition, and thus is restricted to cases where the effect of each boundary can be assumed to add linearly
- While in most cases this is not strictly true, it provides a reasonable approximation

$$K_r = \bar{K} + \sum_{i=1}^{N} (K_i - \bar{K}) \tag{12.12}$$

- Where N is the number of boundaries, $\bar{K}$ is the stress intensity factor with no boundaries present and K_i is the stress intensity factor associated with the i^{th} boundary.

- We can rewrite this equation as

$$K_r = \sigma\sqrt{\pi a}\beta_r = \sigma\sqrt{\pi a} + \sum_{i=1}^{N}(\sigma\sqrt{\pi a}\beta_i - \sigma\sqrt{\pi a}) \tag{12.13}$$

- Which leads to an expression for β_r as

$$\beta_r = 1 + \sum_{i=1}^{N}(\beta_i - 1) \tag{12.14}$$

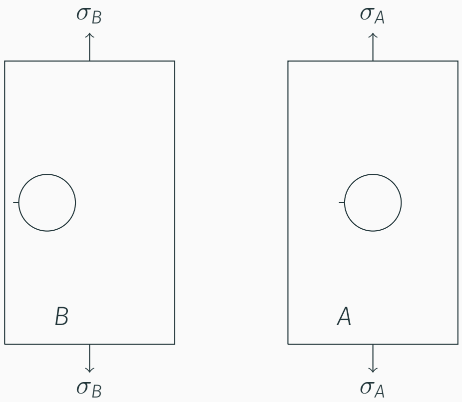
- An alternative empirical method approximates the boundary effect as

$$\beta_r = \beta_1 \beta_2 ... \beta_N \tag{12.15}$$

- If there is no interaction between the boundaries, method 1 and method 2 will give the same result

- For short cracks, we can use the *stress concentraction factor* on a curved boundary to determine the stress intensity factor
- The stress intensity factor only gives the maximum stress at the curved boundary, thus the longer the crack is, the farther away from the curved boundary (and maximum stress) it is.

- Suppose we want to determine the stress intensity on a panel, panel B
- We find a similar panel with a known stress intensity factor, panel A
- We adjust the applied load on panel A such that $K_{I,A} = K_{I,B}$
- The magnitude of this load adjustment is determined using the *stress concentration factors* in panels B and A
- Note the notation: K_t for stress concentration factor, K_I for stress intensity factor


σ_B
σ_A
B
A
σ_B
σ_A

- Since A is a fictional panel, we set the applied stress, σ_A such that

$$\sigma_{max,B} = \sigma_{max,A}$$

- Substituting stress concentration factors

$$K_{t,B}\sigma_B = K_{t,A}\sigma_A$$

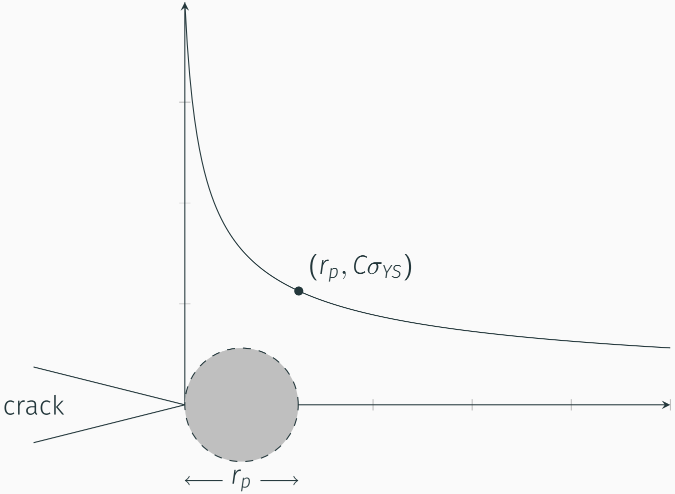
- Solving for σ_A

$$\sigma_A = \frac{K_{tB}}{K_tA}\sigma_B$$

- Since the crack is short and $\sigma_{max,A} = \sigma_{max,B}$ we can say

$$\begin{aligned} K_{I,B} &= K_{I,A} \\ &= \sigma_A \sqrt{\pi c} \beta_A \\ &= \frac{K_{t,B}}{K_{t,A}} \sigma_B \sqrt{\pi c} \beta_A \end{aligned}$$

PLASTIC ZONE



$(r_p, C\sigma_{YS})$
crack
r_p

- We use C "Plastic Constraint Factor" to convert between Plane Strain and Plane Stress solutions
- The plastic zone size can now be approximated by solving the equation $\sigma_{yy}(r = r_p) = C\sigma_{YS}$

$$\sigma_{yy}(r = r_p) = C\sigma_{YS} \tag{12.16a}$$

$$\frac{K_I}{\sqrt{2\pi r_p}} = C\sigma_{YS} \tag{12.16b}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{C\sigma_{YS}}\right)^2 \tag{12.16c}$$

- For plane stress (thin panels) we let $C = 1$ and find r_p as

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{12.17}$$

- And for plane strain (thick panels) we let $C = \sqrt{3}$ and find

$$r_p = \frac{1}{6\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{12.18}$$

- For panels which lie between plane strain and plane stress states, we use the following expression to estimate the plastic zone size

$$r_p = \frac{1}{I\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{12.19}$$

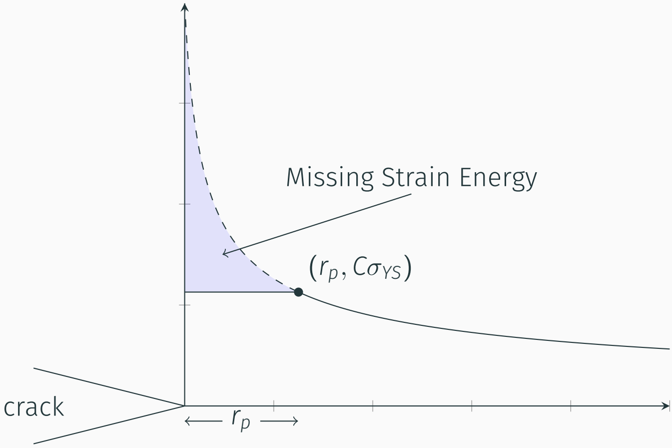
- Where I is defined as

$$I = 6.7 - \frac{1.5}{t}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \tag{12.20}$$

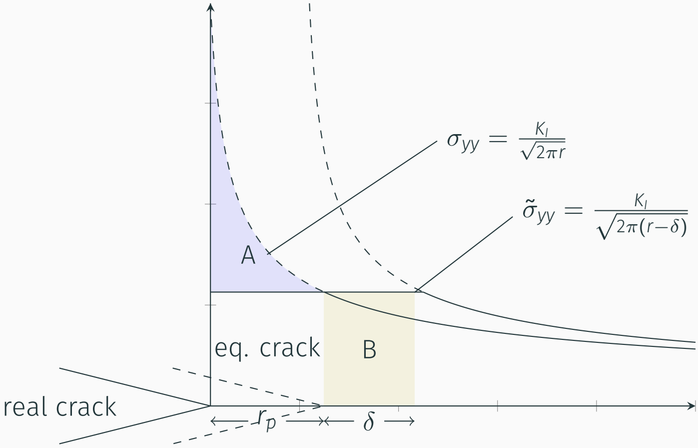
- And $2 \leq I \leq 6$

IRWIN'S SECOND APPROXIMATION

- If our material is perfectly elastic-plastic, no stresses above $C\sigma_{YS}$ will exist in the material
- This ignores the strain energy (represented by the area under the curve) in the plastic zone


Missing Strain Energy
$(r_p, C\sigma_{YS})$
crack
r_p

- To account for the additional strain energy, Irwin considered a plastic zone size increased by some δ
- He also needed to adjust the stress function, and considered an equivalent crack tip in these calculations



$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$
$\tilde{\sigma}_{yy} = \frac{K_I}{\sqrt{2\pi(r-\delta)}}$
A
B
eq. crack
real crack
r_p
δ

- We need $A = B$, so we set them equivalent and solve for δ.

$$A = \int_0^{r_p} \sigma_{yy} dr - r_p \sigma_{YS} \tag{12.21a}$$

$$= \int_0^{r_p} \frac{K_I}{\sqrt{2\pi r}} dr - r_p \sigma_{YS} \tag{12.21b}$$

$$= \frac{K_I}{\sqrt{2\pi}} \int_0^{r_p} r^{-1/2} dr - r_p \sigma_{YS} \tag{12.21c}$$

$$= \frac{2K_I \sqrt{r_p}}{\sqrt{2\pi}} - r_p \sigma_{YS} \tag{12.21d}$$

- We have already found r_p as

$$r_p = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_{YS}} \right)^2 \tag{12.21e}$$

- If we solve this for K_I we find

$$K_I = \sqrt{2\pi r_p} \sigma_{YS} \tag{12.21f}$$

- We can now substitute back into the strain energy of A

$$A = \frac{2\sqrt{2\pi r_p}\sigma_{YS}\sqrt{r_p}}{\sqrt{2\pi}} - r_p\sigma_{YS} \tag{12.21g}$$

$$= 2\sigma_{YS} r_p - r_p\sigma_{YS} \tag{12.21h}$$

$$= r_p\sigma_{YS} \tag{12.21i}$$

- B is given simply as $B = \delta\sigma_{YS}$, so we equate A and B to find δ

$$A = B \tag{12.21j}$$

$$r_p\sigma_{YS} = \delta\sigma_{YS} \tag{12.21k}$$

$$r_p = \delta \tag{12.21l}$$

- This means the plastic zone size is simply $2r_p$
- However, it also means that the effective crack length is $a + r_p$
- Since r_p depends on K_I, we must iterate a bit to find the "real" r_p and K_I