AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 13

Dr. Nicholas Smith

Last Updated: March 3, 2016 at 1:43pm

Wichita State University, Department of Aerospace Engineering

SCHEDULE

- 3 Mar - Section 1 Review, Homework 5 return
- 8 Mar - Exam 1
- 10 Mar - Exam return, Final Project discussion
- 22 Mar - Stress based fatigue, Homework 6 assigned
- 24 Mar - Stress based fatigue

OUTLINE

1. exam notes
2. stress intensity
3. fracture toughness
4. residual strength
5. stiffeners
6. multiple site damage
7. mixed mode fracture
8. extra credit

EXAM NOTES

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- 4 questions

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- Equation sheet is posted on Blackboard

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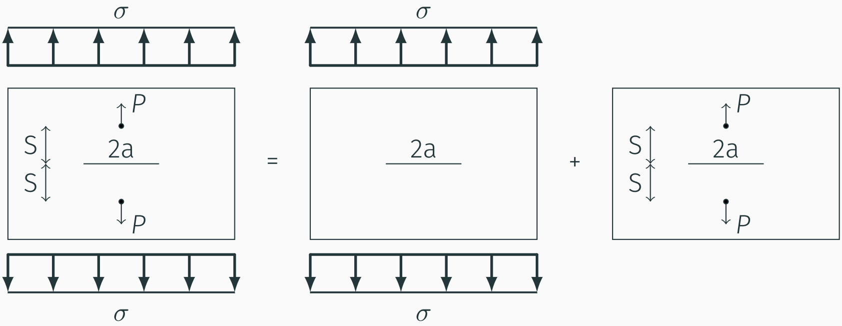
- Experimental

- Since the stress intensity factor is derived using Linear Elasticity, the principle of superposition applies
- Multiple applied loads can be superposed to find the effective stress intensity factor of the combined loading

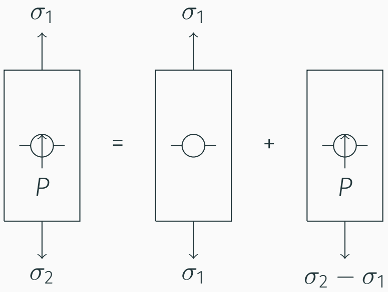

σ
P
S
2a
S
P
σ
=
σ
2a
σ
+
P
S
2a
S
P

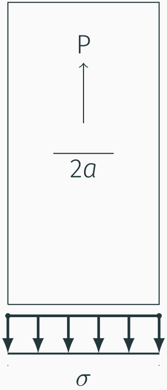
$$K_I = K_{I(\sigma)} + K_{I(P)}$$

$$K_I = \sigma\sqrt{\pi a} + \frac{P}{t\sqrt{\pi a}} \frac{1 - 0.5\left(\frac{a}{W}\right) + 0.975\left(\frac{a}{W}\right)^2 - 0.16\left(\frac{a}{W}\right)^3}{\sqrt{1 - \left(\frac{a}{W}\right)}}$$

- Sometimes, the superposition needed to solve a problem is not obvious
- It can be helpful to subtract a known solution from the problem

SUPERPOSITION




P
2a
σ

FRACTURE TOUGHNESS

- The critical load at which a cracked specimen fails produces a critical stress intensity factor
- The ”critical stress intensity factor” is known as K_c
- For Mode I, this is called K_{Ic}
- The critical stress intensity factor is also known as fracture toughness

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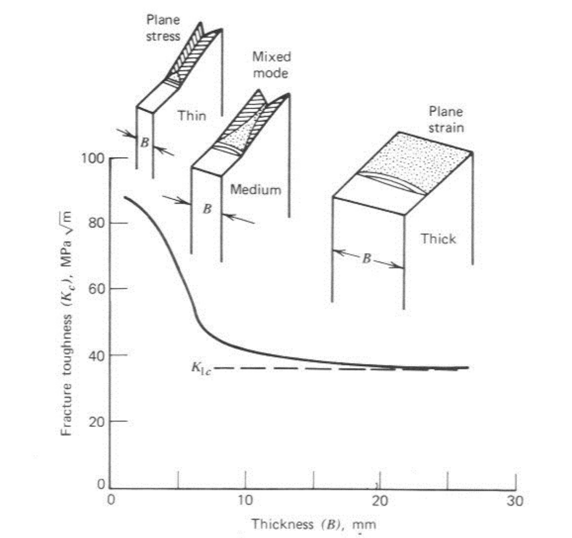
- NOTE: ”Fracture Toughness” can also refer to G_{Ic}, which is analogous to K_{Ic}, but not the same

- Fracture toughness is a material property, but it is only well-defined in certain conditions
- Brittle materials
- Plane strain (smaller plastic zone)
- In these cases ASTM E399-12 is used.

FRACTURE TOUGHNESS



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- For an elastic-plastic material, once the load is large enough to extend the crack, it will extend slightly
- The load must be continually increased until a critical value causes unstable crack growth

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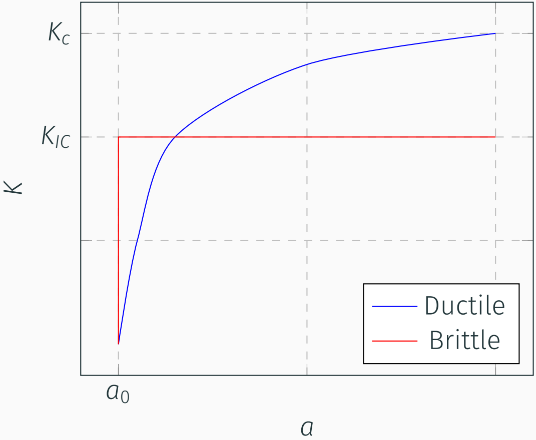
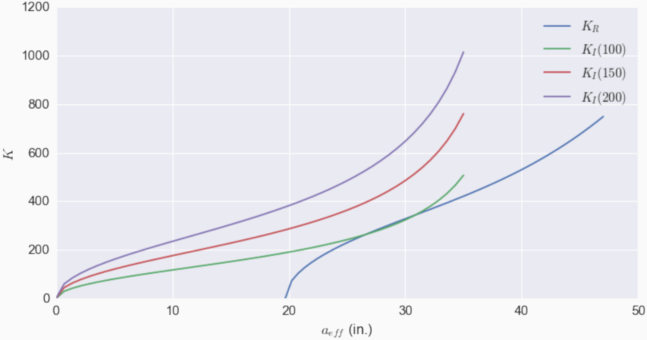
- These are then used to create a ”K-curve”, plotting K_I vs. a


K_c
K_{IC}
K
a_0
a
Ductile
Brittle

k_r CURVE



1200
1000
800
600
400
200
0
K
0
10
20
30
40
50
a_{eff} (in.)
K_R
$K_I(100)$
$K_I(150)$
$K_I(200)$

RESIDUAL STRENGTH

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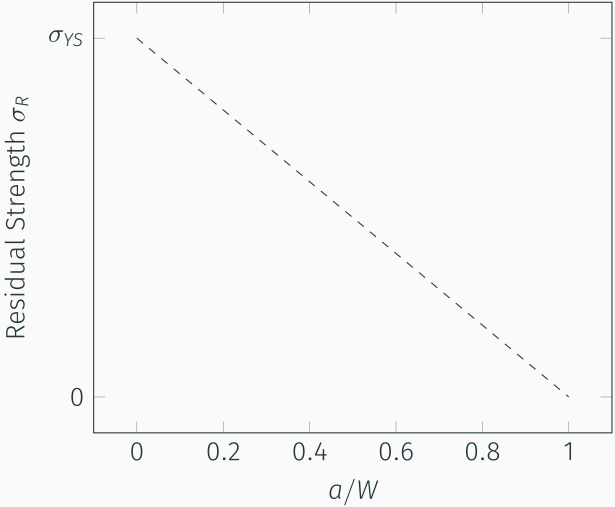
- The residual strength, σ_R is given in terms of the gross area, so as the crack grows the residual strength due to yield decreases
- We can relate the net-section stress to σ_R by

$$\sigma_R = \sigma_{YS} \frac{A_{net}}{A_{gross}} \tag{13.2}$$

RESIDUAL STRENGTH



- For brittle fracture to occur, we need to satisfy the condition

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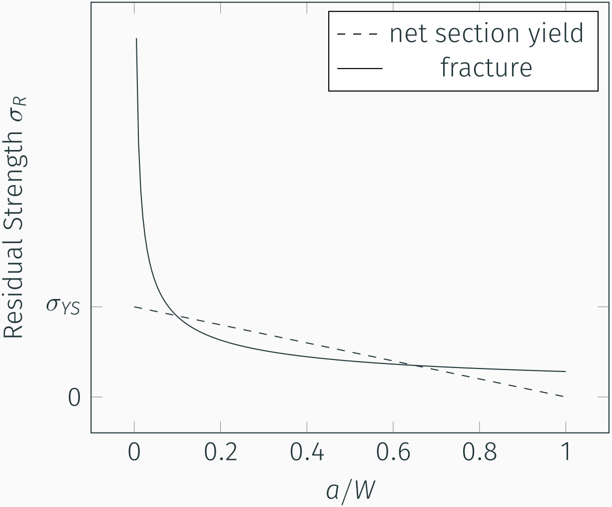
- $$\sigma_R = \sigma_C = \frac{K_C}{\sqrt{\pi a}\beta} \tag{13.3}$$


net section yield
fracture
Residual Strength σ_R
σ_{YS}
0
0
0.2
0.4
0.6
0.8
1
a/W

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    3. 2024-T3, $K_C = 144$ ksi$\sqrt{\text{in.}}$, $\sigma_{YS} = 42$ksi

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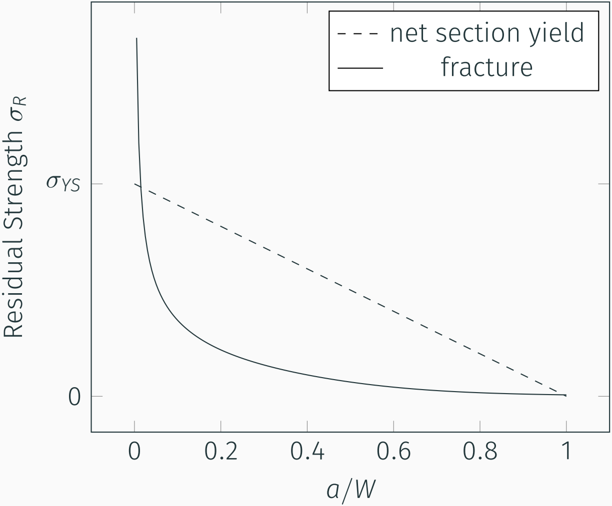
- $$\sigma_C = \sigma_{YS}\frac{W-a}{W} = \sigma_{YS}\frac{6-a}{6}$$
- And the fracture condition by

$$\sigma_C = \frac{K_C}{\sqrt{\pi a}\beta}$$

With

$$\beta = 1.12 - 0.231\left(\frac{a}{W}\right) + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4$$

7178-T6



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fracture
Residual Strength σ_R
σ_{YS}
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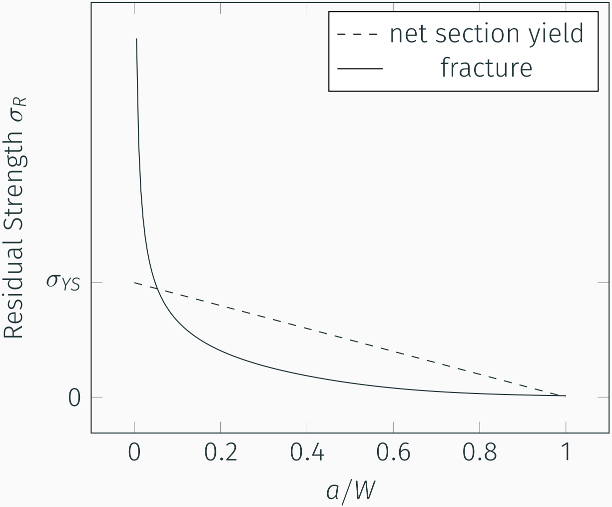
7075-T6



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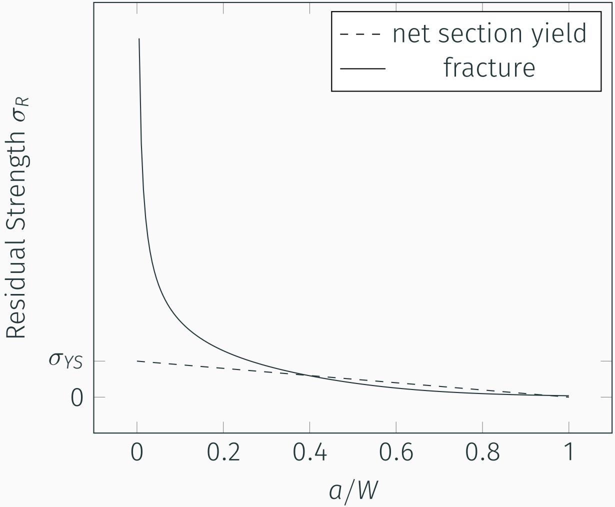
2024-T3


net section yield
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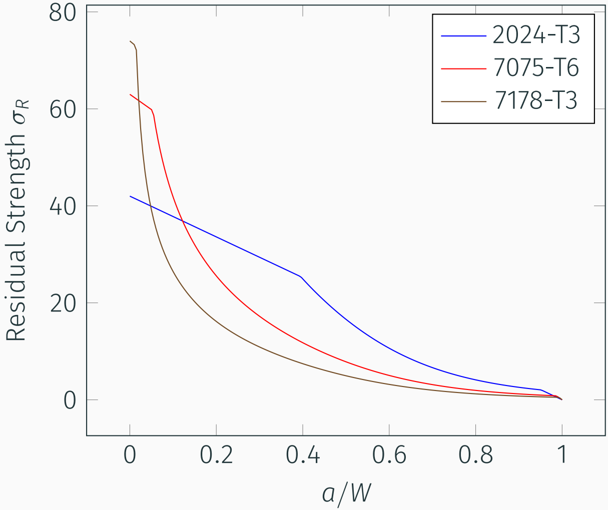
COMPARISON



80
60
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20
0
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0.2
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2024-T3
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STIFFENERS

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- When σ_R is exceeded, the panel fails due to unstable crack growth
- Stiffeners reverse this trend to some extent, but causing some sections of residual strength curve to have positive slope
- When the slope of the residual strength curve is positive, crack growth is stable
- Thus in some cases, we can predict some amount of crack growth

CRITICAL CRACK LENGTH

sketch

RESIDUAL STRENGTH

sketch

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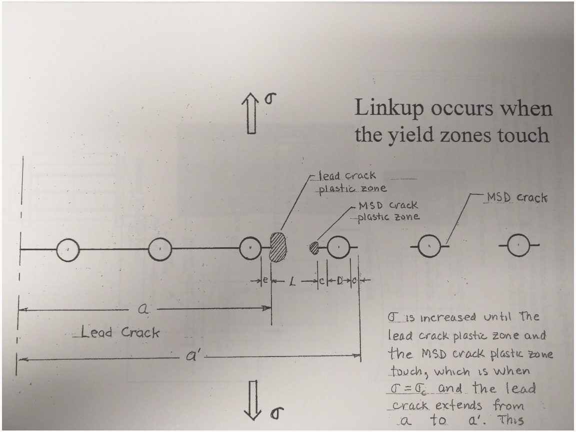
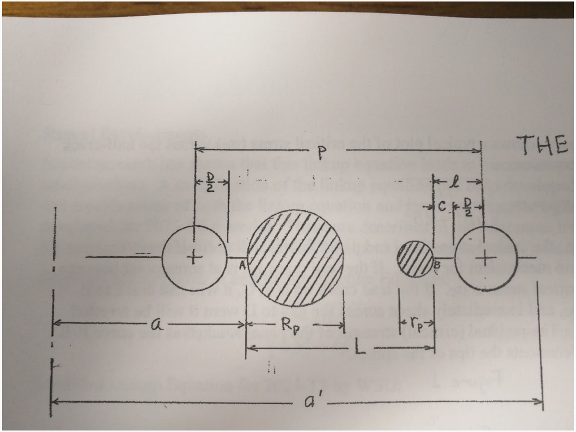
- ”link up” occurs when the plastic zones between two adjacent cracks touch


σ
Linkup occurs when the yield zones touch
lead crack plastic zone
MSD crack plastic zone
MSD crack
e
L
c
D
c
a
Lead Crack
a'
σ
σ is increased until the lead crack plastic zone and the MSD crack plastic zone touch, which is when σ = σc and the lead crack extends from a to a'. This


THE
P
D/2
ℓ
C
D/2
A
B
a
Rp
rp
L
a'

- We know that

$$R_p = \frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2 \tag{13.4}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2 \tag{13.5}$$

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- Where we define the stress intensity factors at a and L as

$$K_{Ia} = \sigma\sqrt{\pi a}\beta_a \tag{13.6}$$

$$K_{Il} = \sigma\sqrt{\pi l}\beta_l \tag{13.7}$$

LINKUP EQUATION

- Since fast cracking occurs when $R_p + r_p = L$, we solve for the condition where $R_p + r_p < L$

$$\frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2 + \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2 < L \tag{13.8a}$$

$$\frac{1}{2\pi\sigma_{YS}^2}\left[K_{Ia}^2 + K_{Il}^2\right] < L \tag{13.8b}$$

$$\frac{1}{2\pi\sigma_{YS}^2}\left[\sigma^2\pi a\beta_a^2 + \sigma^2\pi l\beta_l^2\right] < L \tag{13.8c}$$

$$\frac{\sigma^2}{2\sigma_{YS}^2}\left[a\beta_a^2 + l\beta_l^2\right] < L \tag{13.8d}$$

$$\frac{\sigma_c^2}{2\sigma_{YS}^2}\left[a\beta_a^2 + l\beta_l^2\right] = L \tag{13.8e}$$

$$\sigma_c = \sigma_{YS}\sqrt{\frac{2L}{a\beta_a^2 + l\beta_l^2}} \tag{13.8f}$$

CAVEATS

- The link-up equation is not a good predictor for materials with a small plastic zone size
- Even for ductile materials, some fine tuning of the equation is needed
- In practice, MSD predictions are based on experiments

MIXED MODE FRACTURE

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- Maximum circumferential stress finds the principal stress direction near the crack tip
- Assumes crack will propagate due to maximum opening stress
- Maximum principal stress theory finds the maximum principal stress, neglecting crack tip stress field
- Also assumes crack propagates in Mode I direction

EXTRA CREDIT

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- Start out with a limit of one figure per student
- Use Google Doc to write the page of a figure you are working on (so we don't repeat)

- Google Doc: `https://docs.google.com/spreadsheets/d/1ay4HfJQG2mF-nyr3fgtDr0lHDQoWP30TIMynKGxiwp0/edit?usp=sharing`
- Chart tracer: `http://arohatgi.info/WebPlotDigitizer/app/?`

REVIEW PROBLEMS

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- p. 415 problem 6
- p. 418 problem 9
- p. 419 problem 10-11
- p. 421 problem 13
- p. 423 problem 17
- p. 424 problem 3
- p. 425 problem 5
- p. 426 problem 1
- p. 427 problem 3
- p. 429 problem 6
- p. 432 problem 9
- p. 433 problem 14
- p. 434 problem 3
- p. 437 problem 8