Homework 4

February 19, 2019

0.1 1

a. We know $\sigma_c = 30$ ksi, so we substitute that into (6.1) to find K_C. Note that we will need to iterate to find r_p for this material.

```
In [1]: import numpy as np

        sc = 30.0 #ksi
        a0 = 2.0 #in.
        sy = 100.0 #ksi
        t = 0.1 #in.

        KI = sc*np.sqrt(np.pi*a0)
        I = 6.7 - 1.5/t*(KI/sy)**2
        #I is less than 2, so we force 2
        if I < 2:
            I = 2
        #This is plane stress
        rp = 1.0/(I*np.pi)*(KI/sy)**2
        aeff = a0+rp
        KI_new = sc*np.sqrt(np.pi*aeff)
        while ((KI_new-KI)/(KI))**2 > 0.00001:
            rp = 1.0/(I*np.pi)*(KI_new/sy)**2
            aeff = a0 + rp
            KI = KI_new
            KI_new = sc*np.sqrt(np.pi*aeff)
        print KI
        print KI_new
```

```
76.87220421844556
76.94664961273321
```

So we find the fracture toughness, $K_C = 76.9$ ksi$\sqrt{\text{in.}}$.

b. Since we found this to be in a state of plane stress, K_{IC} is unknown, a thicker sample under plane strain conditions would need to be tested to find K_{IC}.

c. We can use this same K_C for a material with the same thickness, so we find

```
In [2]: a0 = 9.0/2.0 #in
        rp = 1.0/(I*np.pi)*(KI/sy)**2
        aeff = a0+rp
        sc = KI_new/np.sqrt(np.pi*aeff)
        print sc
```

```
20.254285132190763
```

We find that a remote stress of 20.3 ksi will fracture a specimen with a 9-in. crack.

d. This K_C is only valid for 0.1-in thick plates, so we cannot determine the fracture stress in a thicker panel.

e. For a 0.1 in. center-crack we find

```
In [3]: a0 = 0.1/2.0 #in
        rp = 1.0/(I*np.pi)*(KI_new/sy)**2
        aeff = a0+rp
        sc = KI_new/np.sqrt(np.pi*aeff)
        print sc
```

```
114.30983268362391
```

This stress is higher than the yield stress of the material, at which point LEFM are not valid, so we are unsure of the fracture stress (although we would predict general failure due to net yield at 100 ksi).

0.2 2

a. Here we look up the appropriate K_C or K_{IC} values from the tables given in the text. 1.5-in thick aluminum is considered thick enough to be in a state of plane strain, so we use $K_{IC} = 32$ ksi$\sqrt{\text{in.}}$ and use (2.4a) for a finite width edge-crack.

```
In [4]: a = 2.0 #in
        W = 8.0 #in
        KIC = 32.0 #ksi sqrt(in)
        sy = 50.0 #ksi
        t = 1.5 #in
        beta = 1.122 -0.231*a/W + 10.55*(a/W)**2 -21.71*(a/W)**3 + 30.82*(a/W)**4
        sc = KIC/(np.sqrt(np.pi*a)*beta)
        print sc
```

```
8.483638679038224
```

Although for this thickness we assumed a state of plane strain, which would have a negligible plastic zone size, we can check to see if this is a valid assumption.

```
In [5]: I = 6.7 - 1.5/t*(KIC/sy)**2
        if I > 6.0:
            I = 6
        #we find that we are in plane strain, check r_p
        rp = 1.0/(I*np.pi)*(KIC/sy)**2
        aeff = a+rp
        beta = 1.122 -0.231*aeff/W + 10.55*(aeff/W)**2 -21.71*(aeff/W)**3 + 30.82*(aeff/W)**4
        sc = KIC/(np.sqrt(np.pi*aeff)*beta)
        print sc
```

```
8.39375004618113
```

We see that the plastic zone size has a very minimal effect, changing σ_c from 8.48 ksi to 8.39 ksi. This is why it is often neglected in plane strain problems.

b. For an identical panel in the transverse direction, we have $K_{IC} = 29$ ksi$\sqrt{\text{in.}}$ we change this value and use all the same equations as in part a.

```
In [6]: a = 2.0 #in
        W = 8.0 #in
        KIC = 29.0 #ksi sqrt(in)
        sy = 50.0 #ksi
        t = 1.5 #in
        beta = 1.122 -0.231*a/W + 10.55*(a/W)**2 -21.71*(a/W)**3 + 30.82*(a/W)**4
        sc = KIC/(np.sqrt(np.pi*a)*beta)
        print sc
        I = 6.7 - 1.5/t*(KIC/sy)**2
        if I > 6.0:
            I = 6
        #we find that we are in plane strain, check r_p
        rp = 1.0/(I*np.pi)*(KIC/sy)**2
        aeff = a+rp
        beta = 1.122 -0.231*aeff/W + 10.55*(aeff/W)**2 -21.71*(aeff/W)**3 + 30.82*(aeff/W)**4
        sc = KIC/(np.sqrt(np.pi*aeff)*beta)
        print sc
```

```
7.68829755287839
7.621314531784794
```

For this panel, including the small plastic zone correction, we anticipate a failure stress of $\sigma_c = 7.62$ ksi.

c. This panel is thin enough to be within the plane stress/transition regime, so we look it up on the chart on page 113, where we find $K_C = 60$ ksi$\sqrt{\text{in.}}$. We use the same equations as before, with the new material properties and thickness for this panel

```
In [7]: a = 2.0 #in
        W = 8.0 #in
        KC = 60.0 #ksi sqrt(in)
        sy = 72.0 #ksi
        t = 0.5 #in
        beta = 1.122 -0.231*a/W + 10.55*(a/W)**2 -21.71*(a/W)**3 + 30.82*(a/W)**4
        sc = KC/(np.sqrt(np.pi*a)*beta)
        print sc
        I = 6.7 - 1.5/t*(KC/sy)**2

        #we find that we are in the transition between plane strain and plane stress, check r_p
        rp = 1.0/(I*np.pi)*(KC/sy)**2
        aeff = a+rp
        beta = 1.122 -0.231*aeff/W + 10.55*(aeff/W)**2 -21.71*(aeff/W)**3 + 30.82*(aeff/W)**4
        sc = KC/(np.sqrt(np.pi*aeff)*beta)
        print sc
```

```
15.906822523196668
15.538392032797738
```

After including the plastic zone size, we find $\sigma_c = 15.5$ ksi.

d. For the same panel at -65°F, we find $K_C = 46$ ksi$\sqrt{\text{in.}}$, all other parameters are the same

```
In [8]: a = 2.0 #in
        W = 8.0 #in
        KC = 46.0 #ksi sqrt(in)
        sy = 72.0 #ksi
        t = 0.5 #in
        beta = 1.122 -0.231*a/W + 10.55*(a/W)**2 -21.71*(a/W)**3 + 30.82*(a/W)**4
        sc = KC/(np.sqrt(np.pi*a)*beta)
        print sc
        I = 6.7 - 1.5/t*(KC/sy)**2
        #we find that we are in the transition between plane strain and plane stress, check r_p
        rp = 1.0/(I*np.pi)*(KC/sy)**2
        aeff = a+rp
        beta = 1.122 -0.231*aeff/W + 10.55*(aeff/W)**2 -21.71*(aeff/W)**3 + 30.82*(aeff/W)**4
        sc = KC/(np.sqrt(np.pi*aeff)*beta)
        print sc
```

```
12.195230601117446
12.054214554665117
```

We find that $\sigma_c = 12.1$ ksi

e. For this alloy of steel we find $K_C = 72$ ksi, substituting into the equations gives

In [9]:
```
a = 2.0 #in
W = 8.0 #in
KC = 107.0 #ksi sqrt(in)
sy = 140.0 #ksi
t = 1.0 #in
beta = 1.122 -0.231*a/W + 10.55*(a/W)**2 -21.71*(a/W)**3 + 30.82*(a/W)**4
sc = KC/(np.sqrt(np.pi*a)*beta)
print sc
I = 6.7 - 1.5/t*(KC/sy)**2
if I > 6:
    I = 6
#we find that we are in the plane strain region, check r_p
rp = 1.0/(I*np.pi)*(KC/sy)**2
aeff = a+rp
beta = 1.122 -0.231*aeff/W + 10.55*(aeff/W)**2 -21.71*(aeff/W)**3 + 30.82*(aeff/W)**4
sc = KC/(np.sqrt(np.pi*aeff)*beta)
print sc
```

```
28.36716683303406
27.92693191923586
```

Which gives $\sigma_c = 27.9$ ksi.

0.3 3

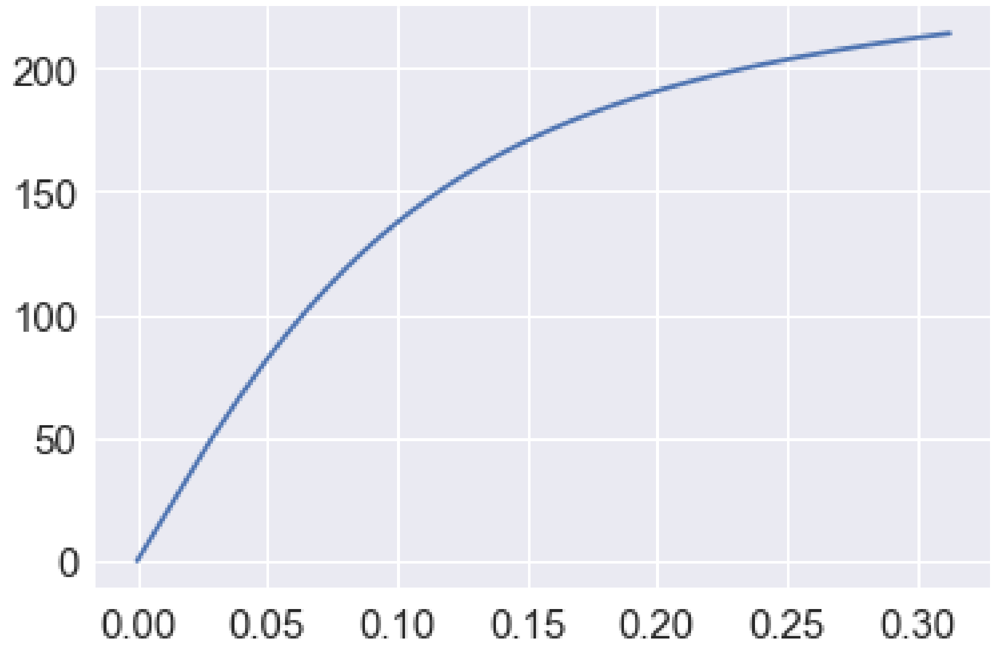
In this problem we are given load-displacement data and are tasked with using that data to create an R-Curve, and use the tangent curve method to find K_C for this test. We first load the data and define material properties

In [39]:
```
#load optimization and plotting libraries
from scipy.optimize import minimize
from scipy import interpolate
from matplotlib import pyplot as plt
import seaborn as sb
sb.set(font_scale=1.5)
%matplotlib inline

#load fictitious test data
data = np.loadtxt('../HW4-3.txt')
v = data[:,0]*1.1
P = data[:,1]*1.1
plt.plot(v,P)

#material properties (also fake)
s_ys = 48.0 #kpsi
B = 0.1 #in.
W = 50.0 #in.
E = 18000.0 #kpsi
```

```
Y = 0.75 #in.
nu = 0.33 #unitless
```



The load-displacement curve looks reasonable and our data has been entered, so now we can use equations (6.14)-(6.16) to calculate the effective crack length at each point.

In [40]:
```
#calculate dv/dp from the data
dvdp = (v[1:]-v[0])/(P[1:]-P[0])

#use 6.15 and 6.16 to get initial guess for a_eff
X = 1. - np.exp(-np.sqrt((E*B*dvdp)**2-(2*Y/W)**2)/2.141)
a_guess = W/2.*(1.2235*X-0.699032*X**2 + 3.25584*X**3 - 6.65042*X**4 + 5.54*X**5 - 1.66

#objective function for minimization
def myobj(x,args=(dvdp,)):
    return (E*B*args - 2*Y/W*np.sqrt(np.pi*x/W/np.sin(np.pi*x/W))*(2*W/np.pi/Y*np.arcco

#run minimization once for every dvdp value
a_opt = []
for i in range(len(a_guess)):
    x0 = a_guess[i] #initial value
    res = minimize(myobj,x0,method='nelder-mead',args=(dvdp[i],))
    a_opt.append(res.x[0])
    #print res.fun #check function value to make sure optimization was successful
```

In [41]:
```
X
```

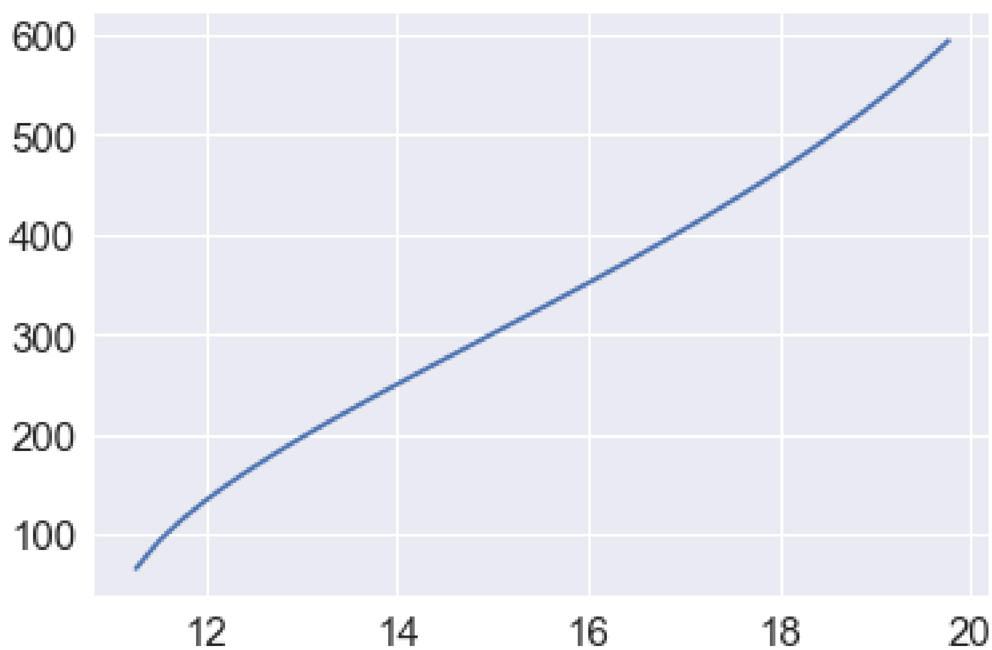
```
Out[41]: array([0.38478505, 0.39305654, 0.40125212, 0.40936772, 0.41740727,
                0.42537183, 0.43325604, 0.44106906, 0.44880742, 0.45647569,
                0.46406935, 0.4715959 , 0.47905025, 0.48643633, 0.49375637,
                0.50100641, 0.50819271, 0.51531344, 0.52236936, 0.52936282,
                0.53629275, 0.54315965, 0.54996657, 0.55671129, 0.56339562,
                0.57002069, 0.57658698, 0.58309591, 0.58954507, 0.5959378 ,
                0.60227395, 0.60855545, 0.61477855, 0.62094877, 0.62706288,
                0.63312259, 0.63913008, 0.64508359, 0.65098352, 0.65683017,
                0.6626252 , 0.66836863, 0.67406095, 0.67970223, 0.68529231,
                0.69083156, 0.69631999, 0.70175887, 0.70714829])
```

Now that we have determined the effective crack length at each point in the test, we can also calculate K_I for each test point, and plot the resulting K_R curve.

```
In [42]: #KI for center crack in finite panel
         def ki(a,W=W,B=B,P=100):
             return P/(B*W)*(np.pi*a)**.5*(1./np.cos(np.pi*a/W))**.5

         K_r = []
         for i in range(len(a_opt)):
             K_r.append(ki(a_opt[i],W=W,B=B,P=P[i+1]))
         plt.plot(a_opt,K_r)
```

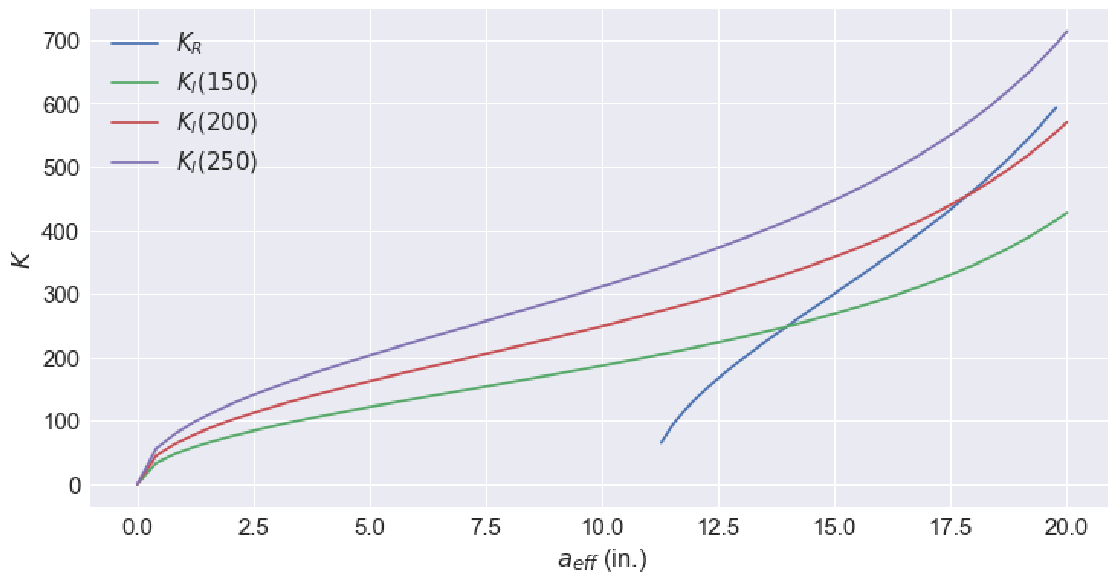
```
Out[42]: [<matplotlib.lines.Line2D at 0xd4901d0>]
```



Now we need to find K_c. We do this by finding the point of intersection for a constant-stress K_I curve which is tangent to the K_R curve at the point of intersection. First let's plot a few constant-stress K_I curves to see what a good initial guess for the optimizer should be.

In [43]:
```
plt.figure(figsize=(12,6))
#r-curve plot
plt.plot(a_opt,K_r,label='$K_R$')
#constant-stress KI-curves
a = np.linspace(0,20)
plt.plot(a,[ki(i,P=150) for i in a],label='$K_I(150)$')
plt.plot(a,[ki(i,P=200) for i in a],label='$K_I(200)$')
plt.plot(a,[ki(i,P=250) for i in a],label='$K_I(250)$')
plt.legend(loc='best')
plt.xlabel('$a_{eff}$ (in.)')
plt.ylabel('$K$')
```

Out[43]:
```
Text(0,0.5,'$K$')
```



We see that our initial load should probably be around 200 k-lbs, so we use that in the optimization

In [44]:
```
#interpolate the kr_curve
kr_smooth = interpolate.splrep(a_opt,K_r)

#objective function for minimization
#x is array of variables to be optimized
def myobj(x):
    a_crit = x[0] #critical crack length, point of intersection
    P_crit = x[1] #critical load
    ki_smooth = interpolate.splrep(a,[ki(i,P=P_crit) for i in a])
    #squared difference of function values
    d1 = (interpolate.splev(a_crit,kr_smooth,der=0)-interpolate.splev(a_crit,ki_smooth,
    #squared difference of slopes (check for tangent)
```

```
        d2 = (interpolate.splev(a_crit,kr_smooth,der=1)-interpolate.splev(a_crit,ki_smooth,
        return d1 + d2
    #initial guess
    x0 = np.array([18,200])
    res = minimize(myobj,x0,method='nelder-mead')
    print res.x[0], res.x[1]
    print interpolate.splev(res.x[0],kr_smooth)
```

```
19.064523060619198 209.9997363207441
536.5809938415185
```

We find a_{crit} is 19.1 in. with $K_C = 537$ ksi $\sqrt{\text{in}}$ (with a load of P=210 k-lb).