AE 737 - Mechanics of Damage Tolerance

Lecture 7

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homework review

- Don’t cover problem number with staple
- Clearly indicate solution

schedule

- 11 Feb - Fracture Toughness
- 16 Feb - Residual Strength, Homework 3 Due, Homework 4 Assigned
- 18 Feb - Residual Strength
- 23 Feb - Multiple Site Damage, Homework 4 Due, Homework 5 Assigned
- 25 Feb - Mixed-mode Fracture

outline

1. R-curve
2. superposition
3. compounding

R-curve

R-curve

- For materials with some plasticity, the K_R Curve, or R Curve, is very important
- Sometimes called a ”resistance curve” it is generally dependent on
  - Thickness
  - Temperature
  - Strain rate
- When done correctly, K_R curves are not dependent on initial crack size or the specimen type used
- ASTM E561

K_c

- While we can look up plane stress K_c for various materials, it is best if we have a K_R curve
- We may not know if the table uses K_c using the tangent intersection method, or maximum stress intensity
- Even if tangent intersection method is used, K_c will different somewhat based on initial crack length

R-curve

- There are two main methods for plotting the R-curve
- Crack size is measured directly (possibly with a drawn-on scale and camera)
- Effective crack size is calculated from the load-displacement data

physical crack

- When the physical crack size is measured, we need to calculate the effective crack length (and effective stress intensity factor) at each data point
- The effective crack length calculated from the load-displacement data already has the plastic zone effect built in

- Using the slope data from our load-displacement curve, we can calculate the effective crack length using

$$EB\left(\frac{\Delta v}{\Delta P}\right) = \frac{2Y}{W}\sqrt{\frac{\pi a/W}{\sin(\pi a/W)}} \left[\frac{2W}{\pi Y}\cosh^{-1}\left(\frac{\cosh(\pi Y/W)}{\cos(\pi a/W)}\right) - \frac{1+\nu}{\sqrt{1+\left(\frac{\sin(\pi a/W)}{\sinh(\pi Y/W)}\right)^2}} + \nu\right] \tag{7.1}$$

secant compliance M(T)

- This equation is difficult to solve directly for a (for M(T) specimens)
- Instead it is generally solved iteratively
- The following equations are used to give a good initial guess to use in iterations

$$X = 1 - \exp\left[\frac{-\sqrt{[EB(\Delta v/\Delta P)]^2 - (2Y/W)^2}}{2.141}\right] \tag{7.2}$$

$$\begin{aligned}\frac{2a}{W} &= 1.2235X - 0.699032X^2 + 3.25584X^3 - 6.65042X^4 + \\ &\quad 5.54X^5 - 1.66989X^6\end{aligned} \tag{7.3}$$

secant compliance M(T)

- In the above equations, the following are the definitions of parameters used

$$
\begin{aligned}
E &= \quad \text{Young's Modulus} \\
\Delta v/\Delta P &= \quad \text{specimen compliance} \\
B &= \quad \text{specimen thickness} \\
W &= \quad \text{specimen width} \\
Y &= \quad \text{half span of the displacement measurement points} \\
a &= \quad \text{effective crack length} \\
\nu &= \quad \text{Poisson's ratio}
\end{aligned}
$$

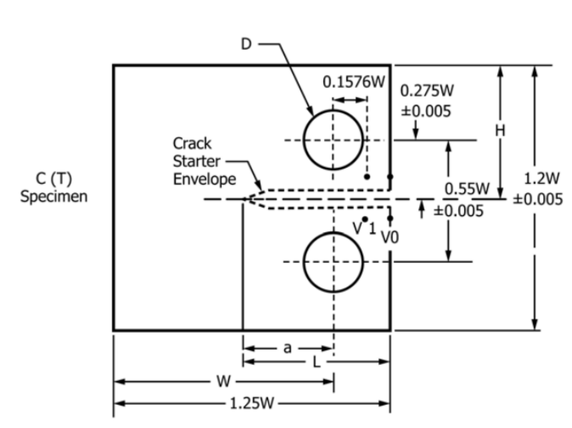
secant compliance C(T)

- For C(T) specimens, we use the following equations

$$EB\frac{\Delta v}{\Delta P} = A_0 + A_1\left(\frac{a}{W}\right) + A_2\left(\frac{a}{W}\right)^2 + A_3\left(\frac{a}{W}\right)^3 + A_4\left(\frac{a}{W}\right)^4 \qquad (7.4)$$

- The coefficients will differ based on where the displacement is measured from

secant compliance C(T)



secant compliance C(T)

location	A_0	A_1	A_2	A_3	A_4
V_0	120.7	-1065.3	4098.0	-6688.0	4450.5
V_1	103.8	-930.4	3610.0	-5930.5	3979.0

location	C_0	C_1	C_2	C_3	C_4	C_5
V_0	1.0010	-4.6695	18.460	-236.82	1214.90	-2143.6
V_1	1.0008	-4.4473	15.400	-180.55	870.92	-1411.3

- Where the initial guess for a is provided by

$$\frac{a}{W} = C_0 + C_1 U + C_2 U^2 + C_3 U^3 + C_4 U^4 + C_5 U^5 \tag{7.5}$$

- and U is given by

$$U = \frac{1}{1 + \sqrt{EB\frac{\Delta v}{\Delta P}}} \tag{7.6}$$

superposition

superposition vs. compounding

- In this course, we use ”superposition” to combine various loading conditions
- We use ”compounding” to combine various edge effects
- Both are very powerful tools and important concepts

superposition

- Sometimes we have to think out of the box to come up with a superposition
- Note: every super-posed solution must still satisfy equilibrium!
- On-board example: pressurized crack

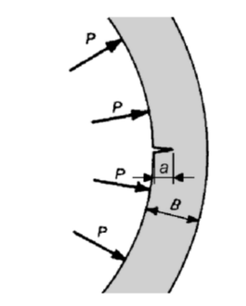


Figure 1: semi-elliptical surface flaw in a pressurized cylinder

group 2

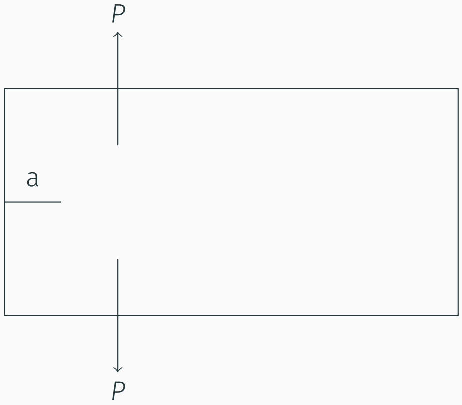


Figure 2: off-center point load on an edge-crack (like in a compact tension specimen)

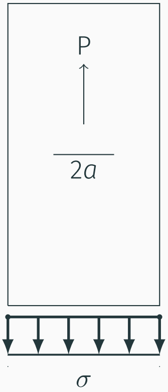


Figure 3: crack with applied force on one side and a remote stress on the other

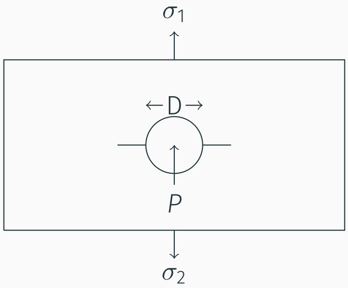


Figure 4: pin-loaded hole, find superposition such that remote stresses and local forces are separated

compounding

compounding

- Different types of boundaries create different correction factors to the usual stress intensity factor
- We often use β to indicate the total correction factor
- When multiple boundaries are present, we can combine them into one effective correction factor
- There are two general methods we use to create a compound correction factor

compounding method 1

- The first method uses linear superposition, and thus is restricted to cases where the effect of each boundary can be assumed to add linearly
- While in most cases this is not strictly true, it provides a reasonable approximation

$$K_r = \bar{K} + \sum_{i=1}^{N} (K_i - \bar{K}) \tag{7.7}$$

- Where N is the number of boundaries, $\bar{K}$ is the stress intensity factor with no boundaries present and K_i is the stress intensity factor associated with the i^{th} boundary.

- We can rewrite this equation as

$$K_r = \sigma\sqrt{\pi a}\beta_r = \sigma\sqrt{\pi a} + \sum_{i=1}^{N}(\sigma\sqrt{\pi a}\beta_i - \sigma\sqrt{\pi a}) \tag{7.8}$$

- Which leads to an expression for β_r as

$$\beta_r = 1 + \sum_{i=1}^{N}(\beta_i - 1) \tag{7.9}$$

compounding method 2

- An alternative empirical method approximates the boundary effect as

$$\beta_r = \beta_1 \beta_2 ... \beta_N \tag{7.10}$$

- If there is no interaction between the boundaries, method 1 and method 2 will give the same result

group 1

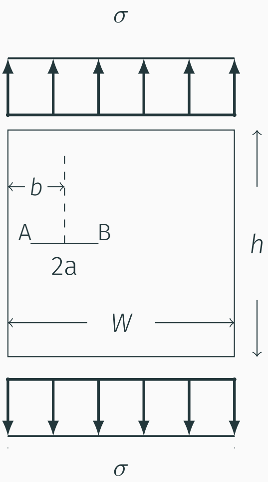


Figure 5: off-center crack, finite height

group 2

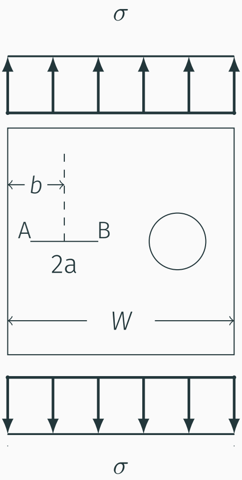


Figure 6: off-center crack, near a hole

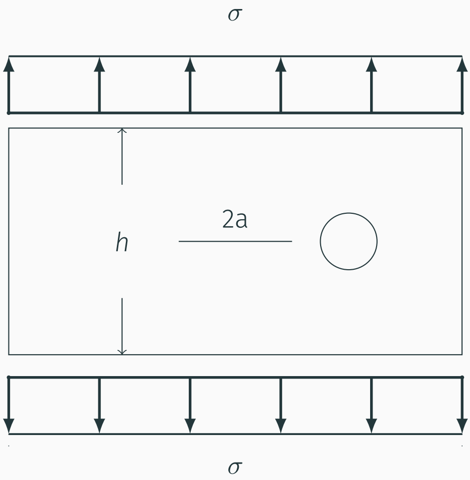


Figure 7: centered crack, near a hole, finite height

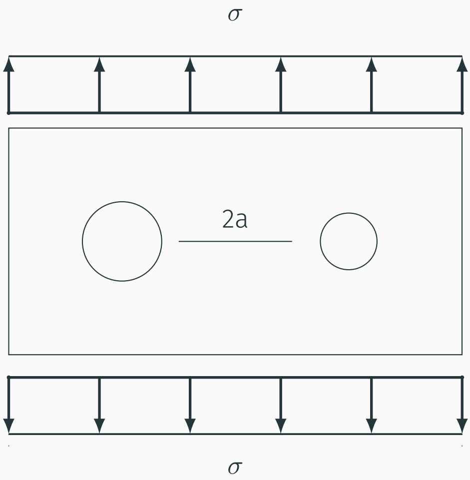


Figure 8: centered crack, near two holes