

AE 737 - MECHANICS OF DAMAGE TOLERANCE

LECTURE 16

Dr. Nicholas Smith

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Wichita State University, Department of Aerospace Engineering

- 24 Mar - Stress based fatigue
- 29 Mar - Influence of notches on fatigue, Homework 7 assigned, Homework 6 due
- 31 Mar - Strain based fatigue, project abstract due
- 5 Apr - Strain based fatigue, Homework 7 due

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- For now I do not have a due date for this project, but I may eventually allow students who have already completed a chart to complete a second one

OUTLINE

1. fatigue review
2. modeling real loads
3. mean stress effects
4. scatter
5. general stress

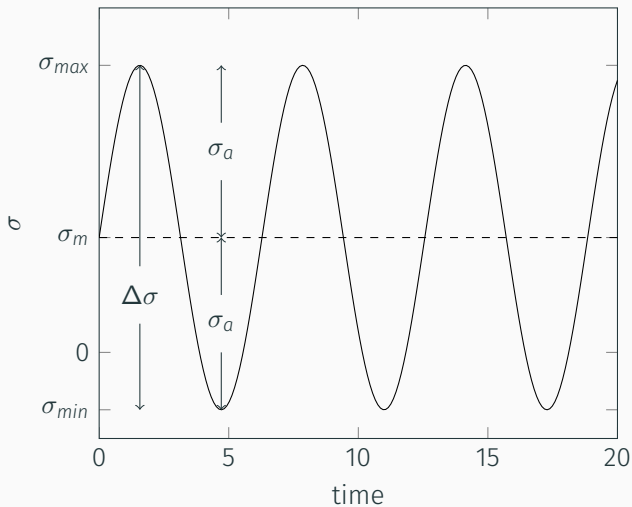
FATIGUE REVIEW

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- This is referred to as constant amplitude stressing

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- σ_m is the mean stress, and can sometimes be zero, but this is not always the case
- σ_a is the stress amplitude, and is the variation about the mean
- We can express all of these in terms of the maximum and minimum stress

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \quad (16.1)$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \quad (16.2)$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \quad (16.3)$$

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DEFINITION AND NOTATION

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- We must also be careful to note σ_y , in some cases $S < \sigma_y$ but at some locations $\sigma > \sigma_y$

SIMPLE TENSION

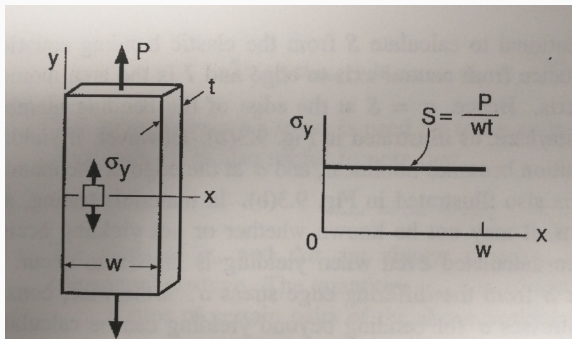


Figure 1: In this case $S = \sigma$

BENDING

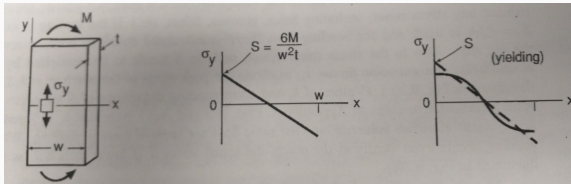


Figure 2: As long as $\sigma < \sigma_y$, σ varies linearly. If $\sigma > \sigma_y$ at any location, however, the relationship is non-linear

NOTCHES

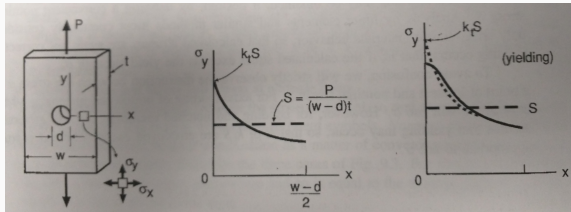


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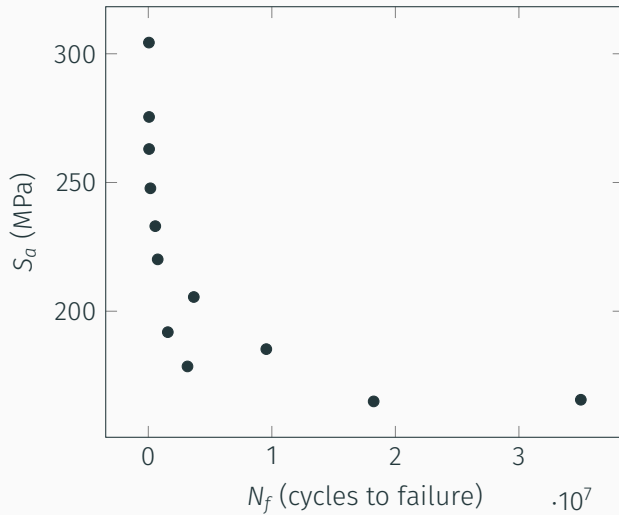
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- Servohydraulic machines generally have a speed of 10 - 100 Hz.
- At a speed of 100 Hz, it would take 28 hours for 10^7 cycles, 12 days for 10^8 cycles, and nearly 4 months for 10^9 cycles
- While some machines can test at very high speeds, the inertia of the sample can interfere with results

STRESS LIFE CURVES

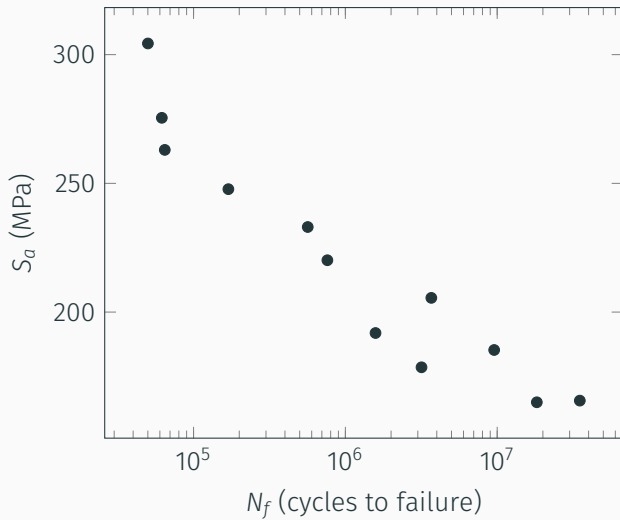


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- It is also very difficult to differentiate between low-cycle fatigue failure stresses
- Instead S-N curves are often plotted on a semi-log or log-log scale, so pay attention to the axes

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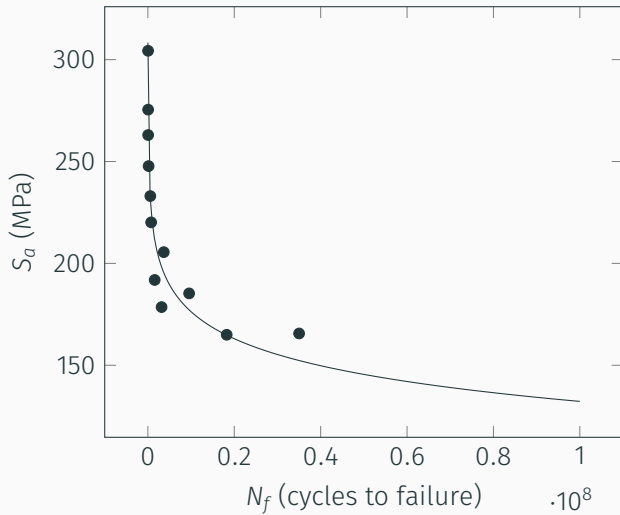
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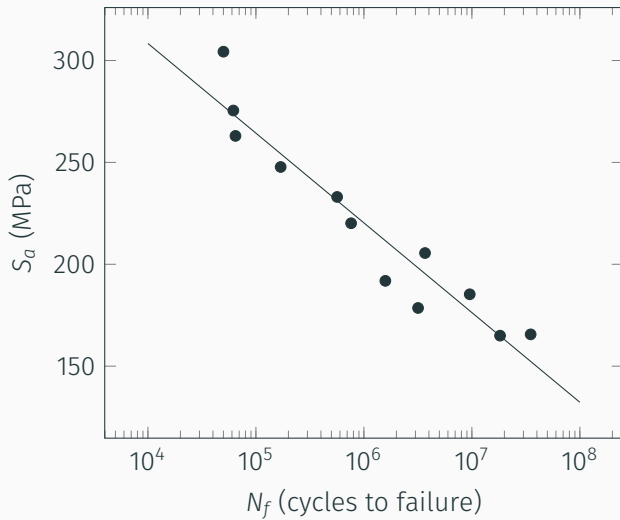
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- σ'_f and b are often considered material properties and can often be looked up on a table (p. 235)

CURVE FIT



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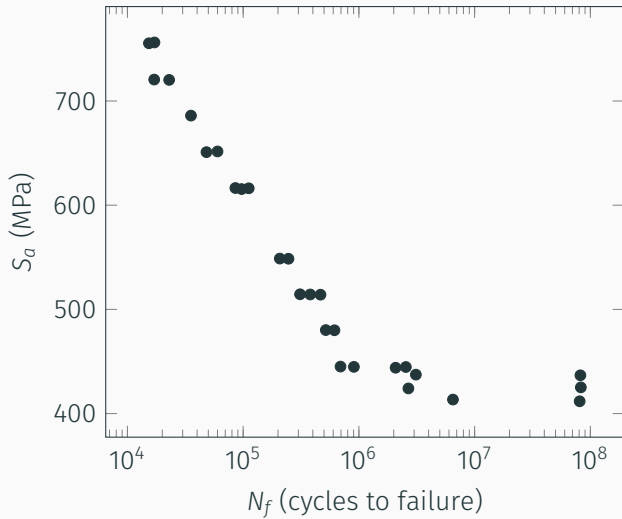
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- This phenomenon is not typical of aluminum or copper alloys, but is sometimes arbitrarily assigned using whatever the failure stress is at some large number of cycles (10^7 or 10^8)

FATIGUE LIMIT



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- "High cycle fatigue" generally is considered anything above 10^3 cycles, but varies somewhat by material
- High cycle fatigue occurs when the stress is sufficiently low that yielding effects do not dominate behavior
- When yielding effects do dominate behavior, the strain-based approach is more appropriate

MODELING REAL LOADS

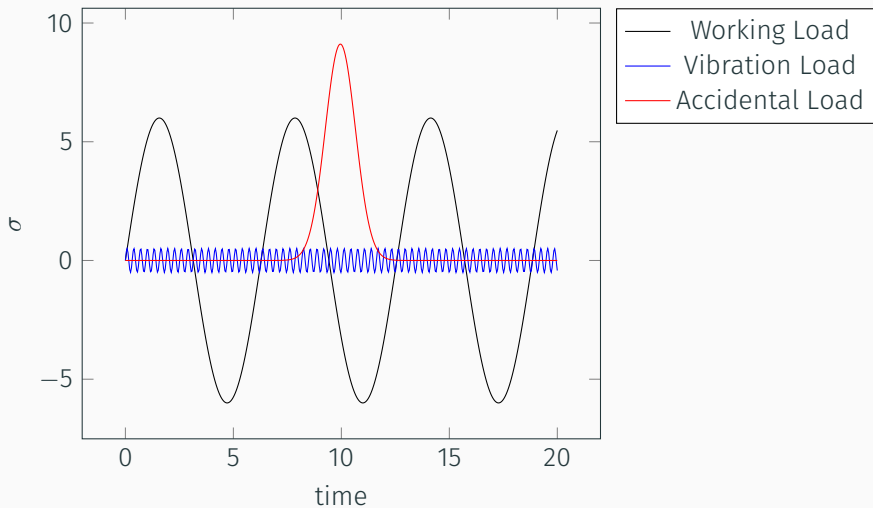
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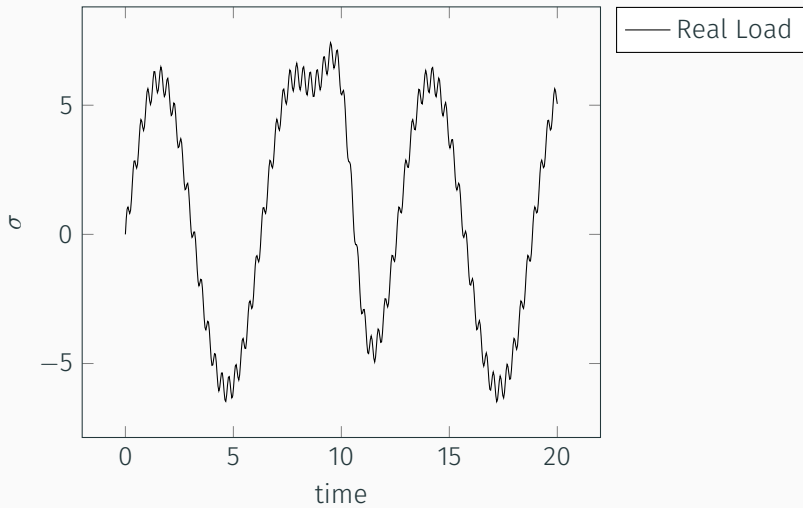
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- Accidental loads can occur at a much lower frequency than working loads

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sketch book p 239

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$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \dots = \sum \frac{N_i}{N_{fi}} = 1 \quad (16.6)$$

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$$B \left[\sum \frac{N_i}{N_{if}} \right]_{rep} = 1 \quad (16.7)$$

VARIABLE AMPLITUDE LOAD EXAMPLE

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MEAN STRESS EFFECTS

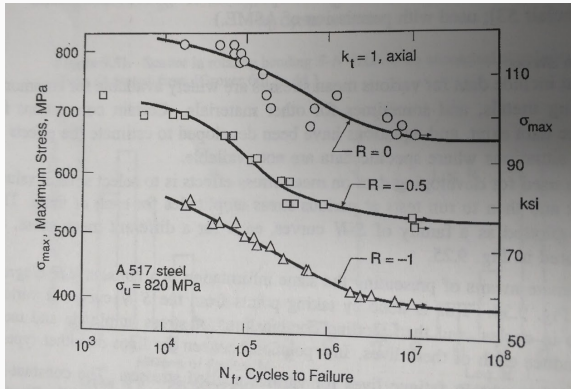
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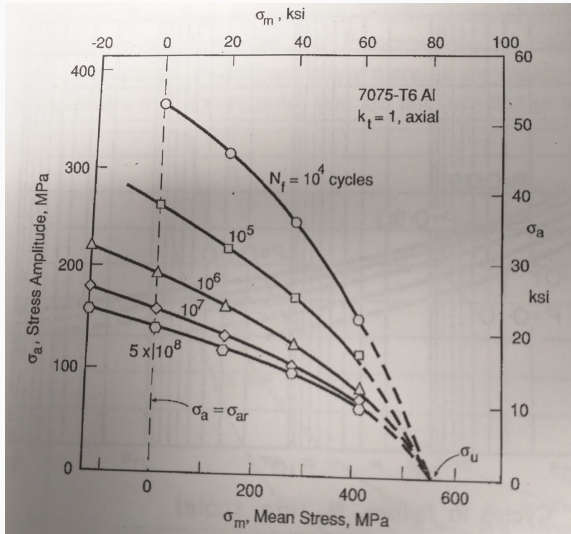
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- One useful way of representing these data, instead of many S-N curves, is a constant-life diagram
- It is created by taking points from the S-N curves and plotting a line through constant N_f values

S-N CURVES AT VARIABLE σ_m



CONSTANT LIFE DIAGRAM

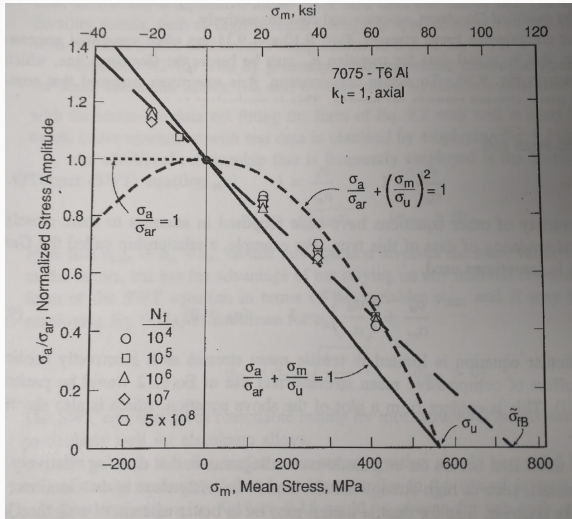


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- Plotting σ_a/σ_{ar} vs. σ_m provides a good way to group all the data together on one plot with the potential to fit a curve

NORMALIZED AMPLITUDE-MEAN DIAGRAM



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- In general, the Goodman line is a good fit for brittle materials (steels) while the Gerber parabola is a better fit for more ductile materials

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- For steels, $\sigma'_f \approx \tilde{\sigma}_{fB}$, but for aluminums these values can be significantly different, and better agreement is found using $\tilde{\sigma}_{fB}$.

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- In general, it is best to use a form that matches your data
- If data is lacking, the SWT (16.12) and Morrow (16.11) equations generally provide the best fit

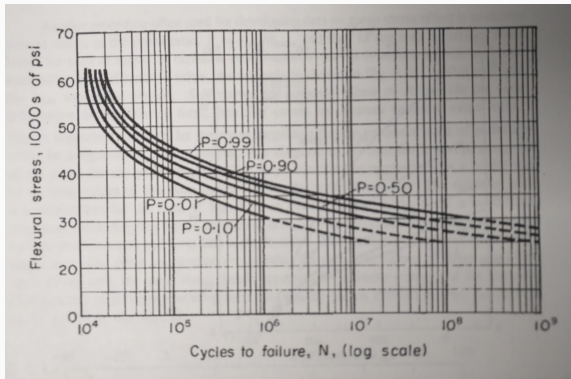
SCATTER

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- Quantifying this scatter requires many repetitions, which makes for time consuming tests
- In general, the scatter follows a lognormal distribution (or a normal distribution in $\log(N_f)$)

S-N-P CURVE



GENERAL STRESS

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- The effective mean stress is given by

$$\bar{\sigma}_m = \bar{\sigma}_{xm} + \bar{\sigma}_{ym} + \bar{\sigma}_{zm} \quad (16.14)$$

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- This is surprising, but agrees well with experiments