AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 17

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SCHEDULE

- 29 Mar - Influence of notches on fatigue, Homework 7 assigned, Homework 6 due
- 31 Mar - Strain based fatigue, project abstract due
- 5 Apr - Crack Growth, Homework 7 due, Homework 8 assigned
- 7 Apr - Crack Growth

OUTLINE

1. fatigue review
2. influence of notches
3. strain based fatigue

FATIGUE REVIEW

GROUP 1

- A part from AISI 4340 in a typical ”block” undergoes 100,000 cycles with $\sigma_{min} = 0$ ksi and $\sigma_{max} = 100$ ksi and an additional 10 cycles with $\sigma_{min} = 50$ ksi and $\sigma_{max} = 200$ ksi
- How many ”blocks” can this part support before failure?

GROUP 2

- Use the S-N-P chart on p. 245 for 7075-T6 Aluminum
- What is the probability of failure for 30 ksi at 10^6 cycles?
- To ensure that 99% of parts do not fail, after how many cycles should a fully reversed load of 35 ksi be inspected?
- How many cycles could the same part sustain if only 50% of parts are needed?

- The fatigue limit for AISI 4142 steel is 58 ksi for completely reversed fatigue loads.
- What is the fatigue limit for fatigue loads with $\sigma_m = 10, 20, 30$ ksi?

- A material made of 2024-T4 Aluminum undergoes the following load cycle
  - $\sigma_{x,min} = 10$, $\sigma_{x,max} = 50$
  - $\sigma_{y,min} = -20$, $\sigma_{y,max} = 20$
  - $\tau_{xy,min} = 0$, $\tau_{xy,max} = 30$
- How many cycles can it support before failure?

INFLUENCE OF NOTCHES

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- This relates local stress to the average, nominal stress
- The stress intensity factor can be used to characterize the ”strength” of a notch

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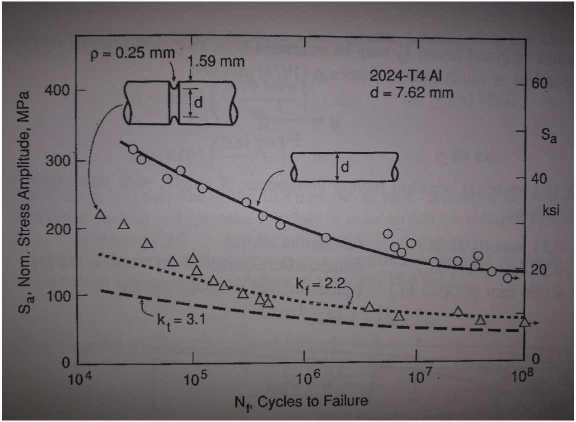
- Notches will have different effects, largely depending on their radius.
- The maximum possible fatigue notch factor is $k_f = k_t$


ρ = 0.25 mm
1.59 mm
d
2024-T4 Al
d = 7.62 mm
d
$k_f = 2.2$
$k_t = 3.1$
S_a, Nom. Stress Amplitude, MPa
400
300
200
100
0
60
S_a
40
ksi
20
0
10^4
10^5
10^6
10^7
10^8
N_f, Cycles to Failure

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NOTCH SENSITIVITY FACTOR

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- When $k_f = k_t$, $q = 1$, in which case the notch has its maximum effect

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Table 1: Table of α values for Peterson notch sensitivity equation

Material	α (mm)	α (in)
Aluminum alloys	0.51	0.02
Annealed or low-carbon steels	0.25	0.01
Quenched and tempered steels	0.064	0.0025

- For high-strength steels, a more specific α estimate can be found

$$\alpha = 0.025\left(\frac{2070}{\sigma_u}\right)^{1.8} \qquad \text{mm} \qquad \sigma_u \geq 550 \text{ MPa} \tag{17.4}$$

$$\alpha = 0.001\left(\frac{300}{\sigma_u}\right)^{1.8} \qquad \text{in} \qquad \sigma_u \geq 80 \text{ ksi} \tag{17.5}$$

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$$\alpha_{torsion} = 0.6\alpha \tag{17.6}$$

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ALTERNATIVE NOTCH SENSITIVITY FORMULATION

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- Where the material property β for steels is given by

$$\log \beta = -\frac{\sigma_u - 134}{586} \qquad \text{mm} \qquad \sigma_u \leq 1520 \text{ MPa} \tag{17.8}$$

$$\log \beta = -\frac{\sigma_u + 100}{85} \qquad \text{in} \qquad \sigma_u \leq 220 \text{ ksi} \tag{17.9}$$

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- For aluminum use the chart

S_u	150 (22)	300 (43)	600 (87)
β	2 (0.08)	0.6 (0.025)	0.5 (0.015)

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- Physical complexities are not fully modeled by these methods
- All of these have been developed for relatively ”mild” notches
- For sharp notches, best results are found by treating the notch as a crack

- Find the notch sensitivity factor for the following scenario

$$\rho = 0.25 \text{ in.}$$
$$\sigma_m = 0 \text{ ksi}$$
$$K_t = 3.0$$
$$\sigma_u = 84 \text{ ksi}$$

STRAIN BASED FATIGUE

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- The strain-based method gives greater detail, and validity at lower cycles
- It is still valid for high cycle fatigue
- Does not include crack growth analysis or fracture mechanics

- Similar to the S-N curves in stress-based fatigue analysis, we can plot the cyclic strain amplitude vs. number of cycles to failure

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- This is most commonly done using axial test machines (instead of rotating bending tests)
- The test is run in strain control (not load control)
- Generally plotted on log-log scale

- We can separate the total strain into elastic and plastic components

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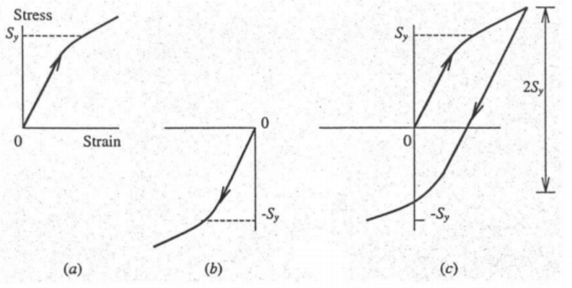
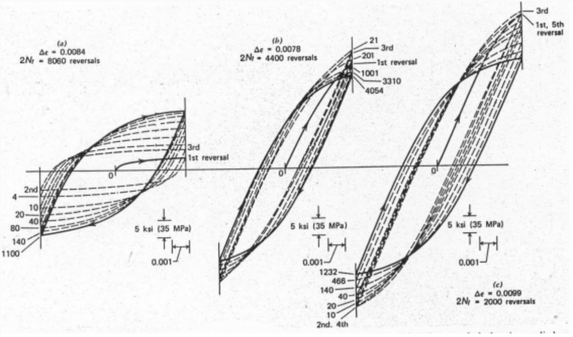
$$\epsilon_a = \epsilon_{ea} + \epsilon_{pa} \tag{17.10}$$

PLASTIC STRAIN




(a)
Δε = 0.0084
2Nf = 8060 reversals
3rd
1st reversal
0
2nd
4
10
20
40
80
140
1100
5 ksi (35 MPa)
0.001
(b)
Δε = 0.0078
2Nf = 4400 reversals
21
3rd
201
1st reversal
1001
3310
4054
0
5 ksi (35 MPa)
0.001
1232
466
140
40
20
10
2nd. 4th
3rd
1st, 5th reversal
0
5 ksi (35 MPa)
0.001
(c)
Δε = 0.0099
2Nf = 2000 reversals

- While strain-life data will generally just report ϵ_a and ϵ_{pa}, some will also tabulate a form for the cyclic stress-strain curve

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$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} \tag{17.11}$$

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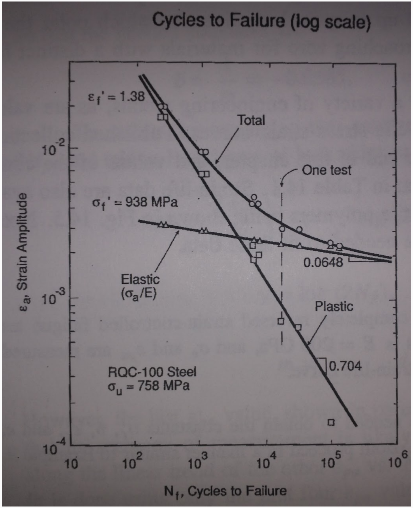
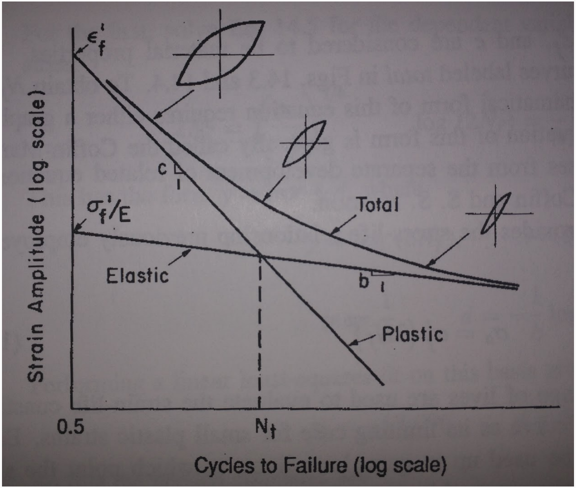
- On strain life curves, the strain is often plotted three times per each experiment
- Once for total strain, once for plastic strain, and once for elastic strain
- Since plastic strain and elastic strain vary by the number of cycles, a hysteresis loop from half the fatigue life is generally used
- This is considered representative of stable behavior


Cycles to Failure (log scale)
$\varepsilon_f' = 1.38$
Total
One test
$\sigma_f' = 938$ MPa
0.0648
Elastic
(σ_a/E)
Plastic
0.704
RQC-100 Steel
$\sigma_u = 758$ MPa
ε_a, Strain Amplitude
10^{-2}
10^{-3}
10^{-4}
10
10^2
10^3
10^4
10^5
10^6
N_f, Cycles to Failure


ϵ_f'
σ_f'/E
c
1
Total
Elastic
b
1
Plastic
Strain Amplitude (log scale)
0.5
N_t
Cycles to Failure (log scale)

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$$\epsilon_{ea} = \frac{\sigma_f'}{E}(2N_f)^b \tag{17.13}$$

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$$\epsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \epsilon_f'(2N_f)^c \tag{17.15}$$

EXAMPLE