Homework 8

April 15, 2019

1 1

1.1 a

First we find K_t from p. 82 of the text. Since we would prefer to use the global stress instead of net stress, we use the upper curve. a/W for this chart is $0.25/7 = 0.0357$, which means $K_t = 3$.

1.2 b

We can estimate k_f using the Peterson method, with $\alpha = 0.02$ in. for aluminum alloys. This gives

$$q = \frac{1}{1 + \alpha/\rho}$$

with the radius, $\rho = 0.125$ in.

```
In [1]: import numpy as np
        D = 0.25
        rho = 0.25/2.0 #in.
        alpha = 0.02 #in.
        q = 1/(1+alpha/rho)
        print q
```

```
0.862068965517
```

With $q = 0.862$ we can now find k_f

```
In [2]: Kt = 3.0
        kf = 1+ q*(Kt-1)
        print kf
```

```
2.72413793103
```

$k_f = 2.72$

1.3 c

We can now find the number of cycles to failure using a nominal stress amplitude, $S_a = k_f S$. In this case we find the stress as $S = 30/(7 * 0.157) = 27.3$ ksi, which gives a nominal stress amplitude of $S_a = 74.4$ ksi

In [3]:
```
P = 30.0 #k-lbs
W = 7.0
t = 0.157
S = P/(W*t)
print S
Sa = kf*S
print Sa
```

```
27.2975432211
74.3622729127
```

This can now be substituted into the equation for the S-N curve using data given on p. 235.

$$S_a = \sigma_f'(2N_f)^b$$

Solved for N_f gives

$$N_f = \frac{1}{2}\left(\frac{S_a}{\sigma_f'}\right)^{1/b}$$

In [4]:
```
sfp = 131.0
b = -0.102
Nf = 0.5*(Sa/sfp)**(1/b)
print Nf
```

```
128.806287448
```

We predict 128.8 cycles to failure in this case.

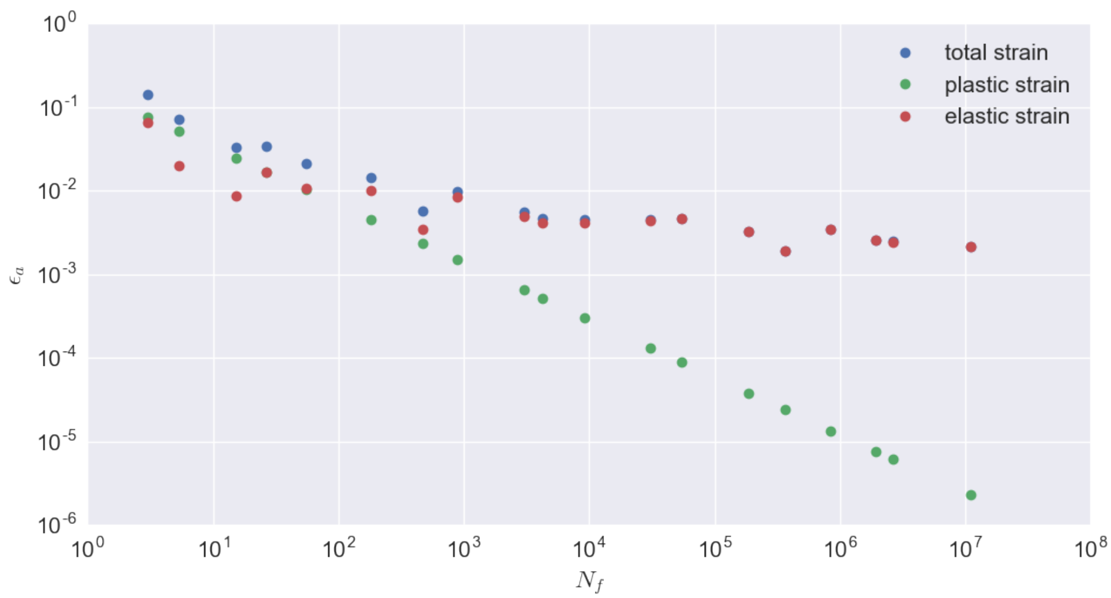
2 2

First we load the given data and plot it

In [7]:
```
#load plotting libraries
from matplotlib import pyplot as plt
import seaborn as sb
sb.set(font_scale=1.5)
%matplotlib inline

data = np.loadtxt('../hw8.txt')

ea = data[:,0] #total strain
```

```
epa = data[:,1] #plastic strain
eea = ea - epa #elastic strain
nf = data[:,2] #cycles to failure

plt.figure(figsize=(12,6))
plt.loglog(nf,ea,'o',label='total strain')
plt.loglog(nf,epa,'o',label='plastic strain')
plt.loglog(nf,eea,'o',label='elastic strain')
plt.legend(loc='best')
plt.xlabel('$N_f$')
plt.ylabel('$\epsilon_a$')
```

```
Out[7]: <matplotlib.text.Text at 0xa3e2668>
```



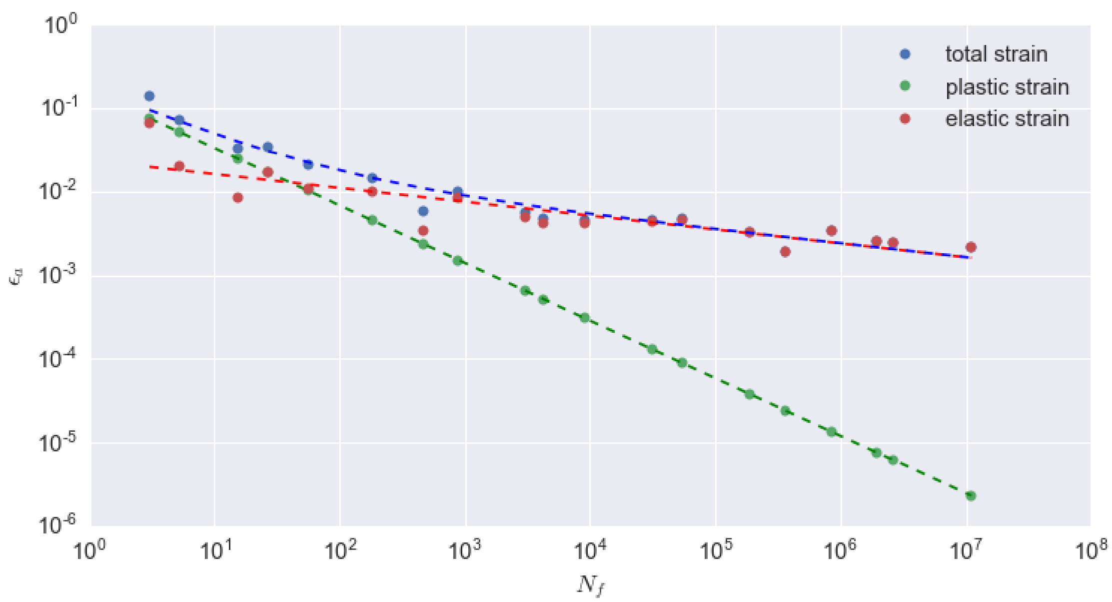
To find the constants, σ'_f, b, ϵ'_f, and c we fit linear curves (in the log-log space) to the plastic and elastic strain data.

```
In [8]: from scipy.optimize import curve_fit

        def func(x,lsfp,b):
            return lsfp + b*x #log(sfp) + b*log(2Nf)

        def func2(Nf,sfp,b):
            return sfp*(2*Nf)**b

        #linear fit in log-log domain
        popt,pcov = curve_fit(func,np.log10(2*nf),np.log10(epa))
        efp = 10**popt[0]
```

```
c = popt[1]
popt,pcov = curve_fit(func,np.log10(2*nf),np.log10(eea))
E = 10.3e3 #ksi
sfp = 10**popt[0]*E
b = popt[1]
```

In [66]:
```
plt.figure(figsize=(12,6))
plt.loglog(nf,ea,'o',label='total strain')
plt.loglog(nf,epa,'o',label='plastic strain')
plt.loglog(nf,func2(nf,efp,c),'g--')
plt.loglog(nf,eea,'o',label='elastic strain')
plt.loglog(nf,func2(nf,sfp/E,b),'r--')
plt.loglog(nf,func2(nf,sfp/E,b)+func2(nf,efp,c),'b--')
plt.legend(loc='best')
plt.xlabel('$N_f$')
plt.ylabel('$\epsilon_a$')
```

Out[66]: <matplotlib.text.Text at 0x1598e940>



We see a good fit, so we now examine the constants

In [67]:
```
print sfp, b, efp, c
```

```
273.692934459 -0.165634191231 0.262092470055 -0.688091438879
```

And we find $\sigma_f' = 273.7$ ksi, $b = -0.166$, $\epsilon_f' = 0.262$ and $c = -0.688$.

3 3

We can find the transition life, N_t by finding the point of intersection between the plastic strain and elastic strain curves. Visually, we would expect it to occur near 20 cycles, but we can solve for it directly.

```
In [68]: Nt = 0.5*(sfp/efp/E)**(1/(c-b))
         print Nt
```

```
39.9550761435
```

And we find $N_t = 40$

4 4

For the Morrow approach we use

$$\epsilon_a = \frac{\sigma'_f}{E}\left(1-\frac{\sigma_m}{\sigma'_f}\right)(2N_f)^b + \epsilon'_f\left(1-\frac{\sigma_m}{\sigma'_f}\right)^{c/b}(2N_f)^c$$

For the modified Morrow approach, we use the same equation but only adjust the elastic portion

$$\epsilon_a = \frac{\sigma'_f}{E}\left(1-\frac{\sigma_m}{\sigma'_f}\right)(2N_f)^b + \epsilon'_f(2N_f)^c$$

And finally, in the Smith, Watson and Topper approach we use

$$\epsilon_a = \frac{1}{\sigma_{max}}\left[\frac{\left(\sigma'_f\right)^2}{E}(2N_f)^{2b} + \sigma'_f\epsilon'_f(2N_f)^{b+c}\right]$$

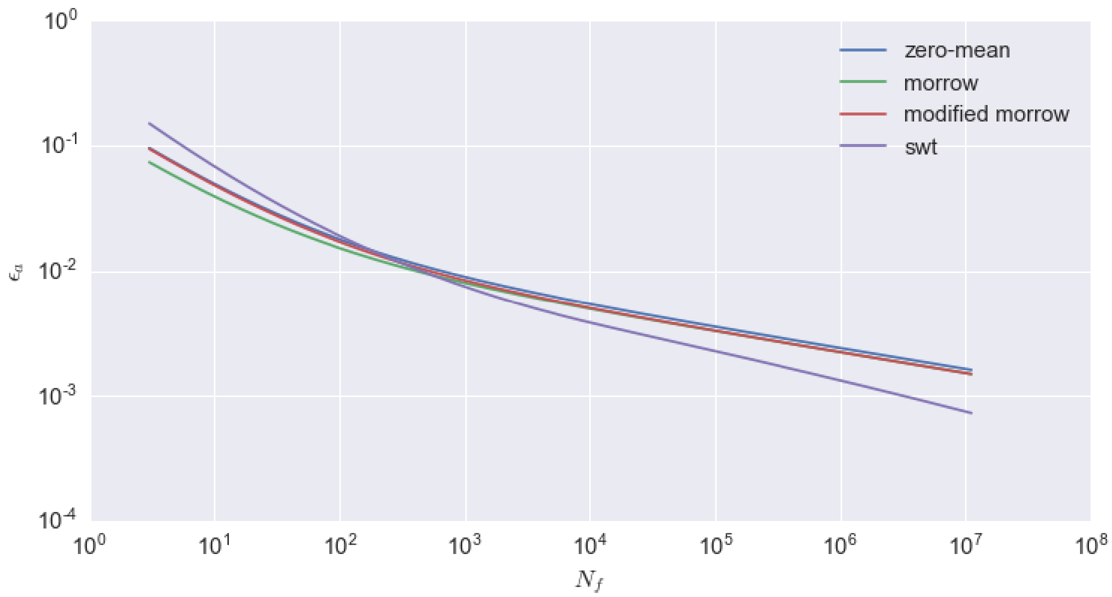
To find σ_{max}, we add σ_m to the stress amplitude, σ_a. σ_a can be found using the stress-strain curve.

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}$$

```
In [69]: sm = 20.0 #ksi, mean stress
         Nf = np.logspace(np.log10(min(nf)),np.log10(max(nf)),200)
         ea_morrow = sfp/E*(1-sm/sfp)*(2*Nf)**b + efp*(1-sm/sfp)**(c/b)*(2*Nf)**c
         ea_mod = sfp/E*(1-sm/sfp)*(2*Nf)**b + efp*(2*Nf)**c
         #numerical solver
         from scipy.optimize import fsolve
         Hp = 142.0
         n = 0.106
         ea_smooth = func2(Nf,sfp/E,b)+func2(Nf,efp,c)
         swt = lambda sa: -ea_smooth + sa/E + (sa/Hp)**(1/n)
         sa = fsolve(swt,func2(Nf,sfp,b))
         smax = sm + sa
```

```
ea_swt = (sfp**2/E*(2*Nf)**(2*b)+sfp*efp*(2*Nf)**(b+c))/smax
plt.figure(figsize=(12,6))
plt.loglog(Nf,ea_smooth,label='zero-mean')
plt.loglog(Nf,ea_morrow,label='morrow')
plt.loglog(Nf,ea_mod,label='modified morrow')
plt.loglog(Nf,ea_swt,label='swt')
plt.xlabel('$N_f$')
plt.ylabel('$\epsilon_a$')
plt.legend(loc='best')
```

```
Out[69]: <matplotlib.legend.Legend at 0x158c8828>
```



From these we see that the modified Morrow equation matches the zero-mean data nearly exactly at low-cycles, and converges to the morrow method for higher cycles, the Morrow method itself is almost uniformly lower than the zero-mean data, while the SWT shows a different trend, with an increase in life before the transition region, and a stronger decrease after. Without test data we cannot comment on which is the best fit, but they do provide different estimates.