AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 10

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- 23 Feb - Residual Strength, Multiple Site Damage
- 25 Feb - Multiple Site Damage, Mixed-mode Fracture, Homework 4 Due, Homework 5 Assigned
- 1 Mar - Section 1 Review, Homework 5 Due
- 3 Mar - Section 1 Review, Homework 5 return
- 8 Mar - Exam 1
- 10 Mar - Exam return, Final Project discussion

OUTLINE

1. residual strength review
2. stiffeners
3. severed stiffeners
4. crack stoppers
5. multiple site damage

RESIDUAL STRENGTH REVIEW

RESIDUAL STRENGTH REVIEW

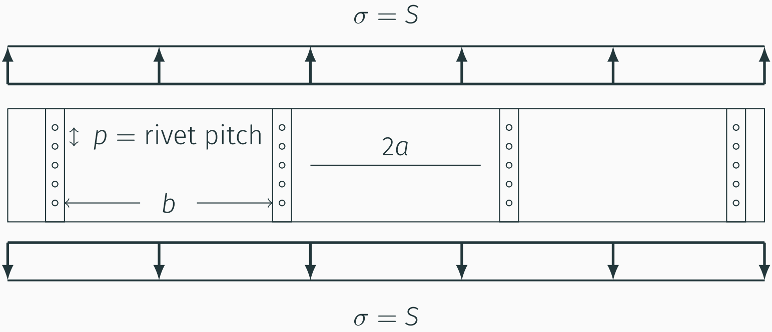
- Group 1 - Sketch a residual strength curve for a single material (include fracture and net-section yield)
- Group 2 - Sketch and describe the difference in residual strength between stiff/brittle materials and ductile/tough materials
- Group 3 - Find the proof load needed to ensure no center-cracks less than 0.01" are present in a material with $K_C = 120$ ksi$\sqrt{\text{in.}}$
- Group 4 - Sketch the Fedderson approach to residual strength. How is this different from the traditional approach? Why is it beneficial?

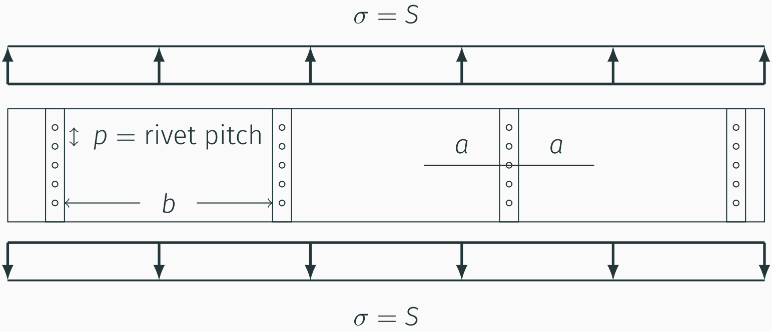
STIFFENERS

STIFFENED PANELS

- In aircraft the skin/stringer system provides many benefits (resistance to buckling)
- Stringers also act as stiffeners to resist crack propagation in the skin
- Panels in these configurations are generally very wide relative to expected crack dimensions
- Cracks are generally modeled either as centered between stiffeners or centered under a stiffener
- We need to consider the residual strength of the panel, the stiffener, and the rivets

CENTERED BETWEEN STIFFENERS




$\sigma = S$
$p =$ rivet pitch
a
a
b
$\sigma = S$

- For equilibrium to be satisfied, we know that

$$\left(\frac{PL}{AE}\right)_{Skin} = \left(\frac{PL}{AE}\right)_{Stiffener}$$

- Since L is the same, we find

$$\frac{S}{E} = \frac{S_S}{E_S}$$

- Where the subscript $_S$ indicates stiffener values, we can express the remote stress in the stiffener as

$$S_S = S\frac{E_S}{E} \tag{10.1}$$

- The critical stress in the skin is determined the same way as it was in the residual strength chapter
- The only exception is that the stiffener contributes to β

$$S_C = \frac{K_C}{\sqrt{\pi a}\beta} \tag{10.2}$$

- The maximum stress in a stiffener will be increased near a crack
- We represent the ratio of maximum force in stiffener to remote force with the Stiffener Load Factor, L

$$L = \frac{\text{max force in stiffener}}{\text{remote force applied to stiffener}} \tag{10.3a}$$

$$= \frac{S_{S,max}A_S}{S_S A_S} \tag{10.3b}$$

$$= \frac{S_{S,max}}{S\frac{E_S}{E}} \tag{10.3c}$$

$$LS\frac{E_S}{E} = S_{S,max} \tag{10.3d}$$

$$LS\frac{E_S}{E} = \sigma_{YS} \tag{10.3e}$$

$$S_C = \frac{\sigma_{YS}E}{LE_S} \tag{10.3f}$$

- We can define a similar rivet load factor to relate maximum stress in the rivet to remote stress in the skin

$$L_R = \frac{\tau_{max} A_R}{Sbt} \tag{10.4a}$$

$$L_R = \frac{\tau_{YS} A_R}{Sbt} \tag{10.4b}$$

$$S_c = \frac{\tau_{YS} A_R}{L_R bt} \tag{10.4c}$$

- CC Poe found that panels could be related by a parameter he defines as μ

$$\mu = \frac{A_S E_S}{A_S E_S + AE} \tag{10.5}$$

- Where A_S is the cross-sectional area of a stiffener, E_S is stiffener modulus
- A is the skin cross-sectional area (per stiffener) $A = bt$ and E is the modulus of the skin
- pp 167 - 178 give β, L and L_R for various skin/stiffener configurations
- These values were determined using a finite element model

EXAMPLES

- quantitative example (p. 179-180)
- qualitative notes on behavior (p. 181-182)

SEVERED STIFFENERS

- Sometimes the stiffeners fail before the panel
- T. Swift conducted some parametric studies on panels with a severed stiffener
- When the crack is short (and near the severed stiffener) the residual strength is lower due to the broken stiffener
- As the crack nears the next stiffener, residual strength is very similar to a panel with all stiffeners intact

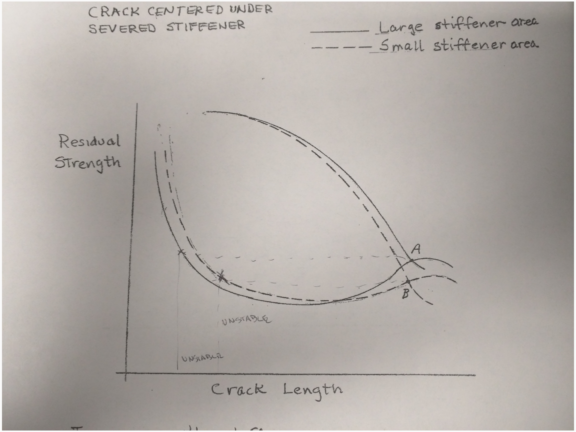
- Swift considers the difference in stress at different points in the stiffener
- Instead of one general load factor (L), he uses $SCFO$ and $SCFI$
- We can find the critical value of remote stress at the outer flange as

$$\sigma_C = \frac{\sigma_U}{SCFO} \tag{10.6}$$

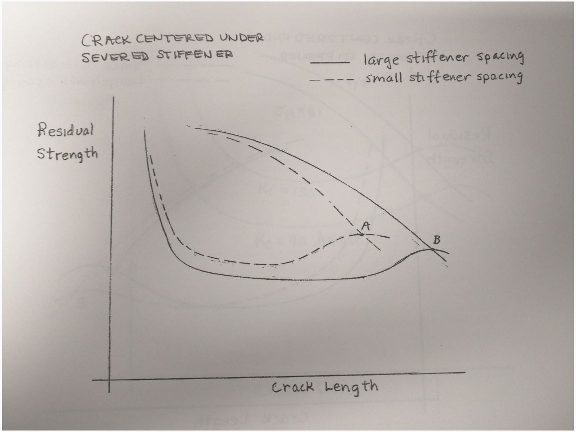
- And similarly at the inner flange

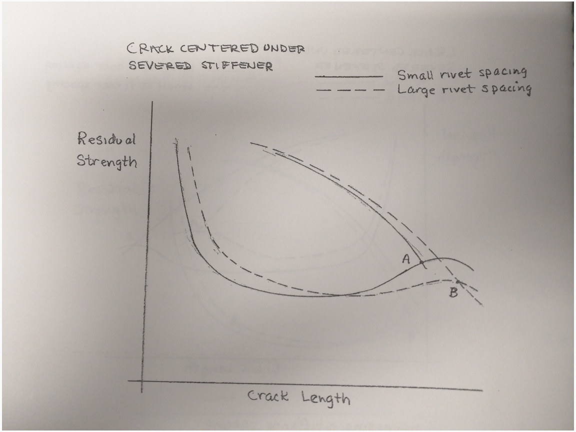
$$\sigma_C = \frac{\sigma_U}{SCFI} \tag{10.7}$$

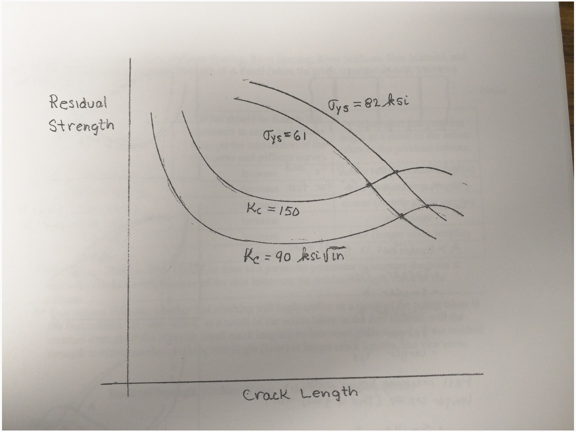
- Swift's parametric study did not consider rivet failure


CRACK CENTERED UNDER SEVERED STIFFENER
Large stiffener area
Small stiffener area
Residual Strength
Crack Length
A
B
UNSTABLE
UNSTABLE

STIFFENER SPACING




CRACK CENTERED UNDER
SEVERED STIFFENER
Small rivet spacing
Large rivet spacing
Residual
Strength
A
B
Crack Length


Residual Strength
$\sigma_{ys} = 82$ ksi
$\sigma_{ys} = 61$
$K_c = 150$
$K_c = 90$ ksi$\sqrt{in}$
Crack Length

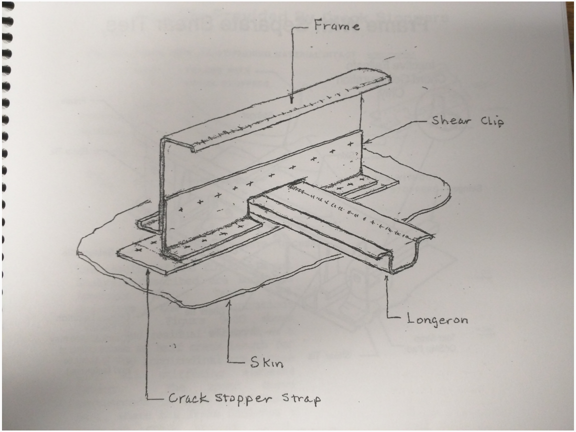
- If we consider the case from Swift's data most similar to our previous example:

$$P = 1.0 \text{ in}$$
$$A_{st} = 0.2538 \text{ in}^2$$
$$b = 10.0 \text{ in}$$

- So we use the tables for Case 10

CRACK STOPPERS

CRACK STOPPER



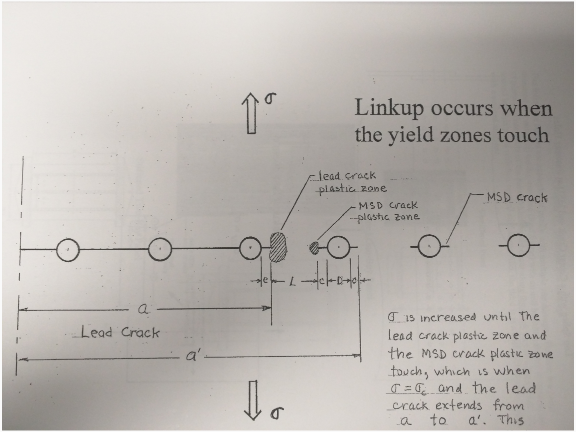
OPTIMAL CRACK STOPPER

- Swift found that the ideal crack stopper has a cross-sectional area approximately equal to 1/4 the stiffener area
- The ideal material was titanium (as opposed to steel or aluminum).
- Aluminum did not transfer enough load to the stiffeners, steel transferred too much

- Compare cases 1, 3, and 5

MULTIPLE SITE DAMAGE

- Often damage can accumulate among multiple sources
- This is very common when there are a series of holes, each can develop cracks with a potential to link up
- ”link up” occurs when the plastic zones between two adjacent cracks touch


Linkup occurs when the yield zones touch
σ
lead crack plastic zone
MSD crack plastic zone
MSD crack
e
L
c
D
c
a
Lead Crack
a′
σ
σ is increased until the lead crack plastic zone and the MSD crack plastic zone touch, which is when σ = σc and the lead crack extends from a to a′. This

- We know that

$$R_p = \frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2 \tag{10.8}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2 \tag{10.9}$$

- Where we define the stress intensity factors at a and L as

$$K_{Ia} = \sigma\sqrt{\pi a}\beta_a \tag{10.10}$$

$$K_{Il} = \sigma\sqrt{\pi l}\beta_l \tag{10.11}$$

- Since fast cracking occurs when $R_p + r_p = L$, we solve for the condition where $R_p + r_p < L$

$$\frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2 + \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2 < L \tag{10.12a}$$

$$\frac{1}{2\pi\sigma_{YS}^2}\left[K_{Ia}^2 + K_{Il}^2\right] < L \tag{10.12b}$$

$$\frac{1}{2\pi\sigma_{YS}^2}\left[\sigma^2\pi a\beta_a^2 + \sigma^2\pi l\beta_l^2\right] < L \tag{10.12c}$$

$$\frac{\sigma^2}{2\sigma_{YS}^2}\left[a\beta_a^2 + l\beta_l^2\right] < L \tag{10.12d}$$

$$\frac{\sigma_c^2}{2\sigma_{YS}^2}\left[a\beta_a^2 + l\beta_l^2\right] = L \tag{10.12e}$$

$$\sigma_c = \sigma_{YS}\sqrt{\frac{2L}{a\beta_a^2 + l\beta_l^2}} \tag{10.12f}$$