AE 737: Mechanics of Damage Tolerance

Lecture 15 - Stress based fatigue

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schedule

- 24 Mar - Stress-based Fatigue
- 26 Mar - Project Abstract Due
- 29 Mar - Strain-based Fatigue
- 31 Mar - Crack Growth
- 2 Apr - Homework 6 Due

outline

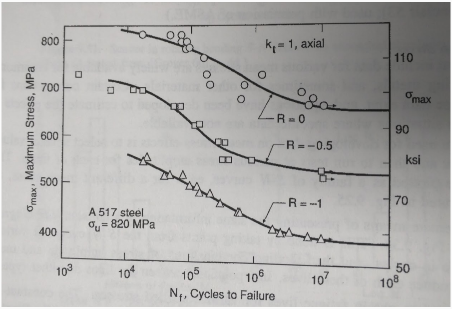
- mean stress effects
- scatter
- general stress
- influence of notches
- fatigue review
- strain based fatigue
- variable amplitude strains
- general trends

mean stress effects

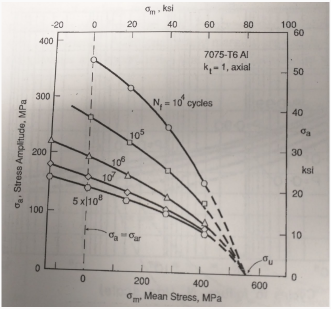
mean stress

- Since mean stress has an effect on fatigue life, sometimes a family of S-N curves at varying mean stress values is created
- S-N curves for these are reported in different ways, but commonly σ_{max} replaces σ_a on the y-axis
- One useful way of representing these data, instead of many S-N curves, is a constant-life diagram
- It is created by taking points from the S-N curves and plotting a line through constant *Nf* values

S-N curves at variable σ_m



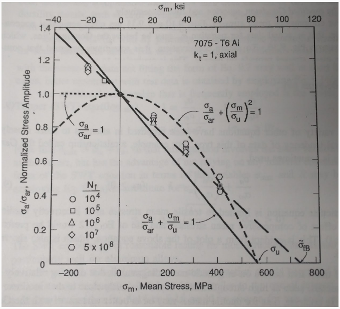
constant life diagram



normalizing

- One very useful way to plot this data is to normalize the amplitude by the zero-mean amplitude
- We call the zero-mean amplitude σ_{ar}
- Plotting σ_a/σ_{ar} vs. σ_m provides a good way to group all the data together on one plot with the potential to fit a curve

normalized amplitude-mean diagram



Goodman line

- The first work on this problem was done by Goodman, who proposed the line

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$$

- This equation can also be used for fatigue limits, since they are just a point on the S-N curves

$$\frac{\sigma_e}{\sigma_{er}} + \frac{\sigma_m}{\sigma_u} = 1$$

modifications

- While the Goodman line gives a good approximation to convert non-zero mean stress S-N curves, it is somewhat overly conservative at high mean stresses
- It is also non-conservative for negative mean stresses
- An alternative fit is known as the Gerber Parabola

$$\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1$$

- In general, the Goodman line is a good fit for brittle materials (steels) while the Gerber parabola is a better fit for more ductile materials

modifications

- The Goodman line can also be improved by replacing σ_u with the corrected true fracture strength $\tilde{\sigma}_{fB}$ or the constant σ_f' from the S-N curve fit

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f'} = 1$$

- This is known as the Morrow Equation
- For steels, $\sigma_f' \approx \tilde{\sigma}_{fB}$, but for aluminums these values can be significantly different, and better agreement is found using $\tilde{\sigma}_{fB}$.

modifications

- One more relationship that has shown particularly good results with aluminum alloys is the Smith, Watson, and Topper equations (SWT)

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a}$$

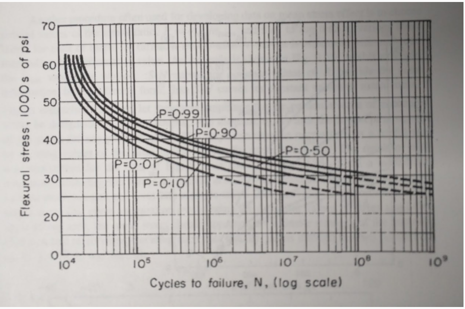
- In general, it is best to use a form that matches your data
- If data is lacking, the SWT and Morrow equations generally provide the best fit

scatter

fatigue scatter

- One of the challenges with fatigue is that there is generally considerable scatter in the data
- Quantifying this scatter requires many repetitions, which makes for time consuming tests
- In general, the scatter follows a lognormal distribution (or a normal distribution in log(Nf))

S-N-P Curve



general stress

general stress

- Often combined loads from different sources introduce stresses which are not uni-axial
- For ductile materials, good agreement has been found using an effective stress amplitude, similar to the octahedral shear yield criterion

$$\bar{\sigma}_a = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{xa}-\sigma_{ya})^2+(\sigma_{ya}-\sigma_{za})^2+(\sigma_{za}-\sigma_{xa})^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)}$$

- The effective mean stress is given by

$$\bar{\sigma}_m = \bar{\sigma}_{xm} + \bar{\sigma}_{ym} + \bar{\sigma}_{zm}$$

- This effective stress can be used in all other relationships, including mean stress relationships
- Note that mean shear stress has no effect on the effective mean stress
- This is surprising, but agrees well with experiments
- When yielding effects do dominate behavior, the strain-based approach is more appropriate

influence of notches

notch effects

- In this discussion, we use "notch" to refer to any geometric feature that increases the local stress (such as holes, fillets, grooves, etc.)
- We discussed notches and stress concentration factors in terms of stress concentration factors
- In our fatigue notation, $\sigma_{max} = K_t S$
- This relates local stress to the average, nominal stress
- The stress intensity factor can be used to characterize the "strength" of a notch

notch effects

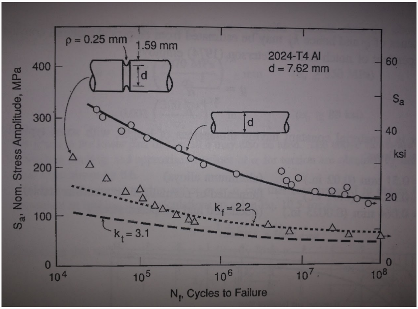
- We might expect the fatigue life of a notched specimen to be similar to a pristine specimen with $S_{a,pristine} = \sigma_{max,notched}$
- If we look at actual test data, however, this estimate would be overly conservative
- Even when the stress is adjusted for some fatigue notch factor, k_f it is only valid at longer cycles ($N_f > 10^6$)

notch effects

$$k_f = \frac{\sigma_{ar}}{S_{ar}}$$

- Notches will have different effects, largely depending on their radius.
- The maximum possible fatigue notch factor is $k_f = k_t$

notch effects



notch sensitivity factor

- To avoid generating fatigue data for every possible notch configuration, some empirical relationships have been developed
- A useful concept in these methods is the notch sensitivity factor

$$q = \frac{k_f - 1}{k_t - 1}$$

- When $k_f = 1$, $q = 0$, in which case the notch has no effect
- When $k_f = k_t$, $q = 1$, in which case the notch has its maximum effect

peterson notch sensitivity

- Peterson developed the following relationship

$$q = \frac{1}{1 + \frac{\alpha}{\rho}}$$

- Where ρ is the radius of the notch
- α is a material property

peterson notch sensitivity

Material	α (mm)	α (in)
Aluminum alloys	0.51	0.02
Annealed or low-carbon steels	0.25	0.01
Quenched and tempered steels	0.064	0.0025

peterson notch sensitivity

- For high-strength steels, a more specific α estimate can be found

$$\alpha = 0.025 \left(\frac{2070}{\sigma_u}\right)^{1.8} \quad \text{mm} \qquad \sigma_u \geq 550 \text{ MPa}$$

$$\alpha = 0.001 \left(\frac{300}{\sigma_u}\right)^{1.8} \quad \text{in} \qquad \sigma_u \geq 80 \text{ ksi}$$

peterson notch sensitivity

- α predictions are valid for bending and axial fatigue
- For torsion fatigue, a good estimate can be found
- $\alpha_{torsion} = 0.6\alpha$

alternative

- An alternative formulation for q was developed by Neuber

$$q = \frac{1}{1 + \sqrt{\frac{\beta}{\rho}}}$$

- Where the material property β for steels is given by

$$\log \beta = -\frac{\sigma_u - 134}{586} \quad \text{mm} \qquad \sigma_u \leq 1520 \text{ MPa}$$
$$\log \beta = -\frac{\sigma_u + 100}{85} \quad \text{in} \qquad \sigma_u \leq 220 \text{ ksi}$$

alternative

- For aluminum use the chart MPa (ksi) and mm (in.)

S_u	150 (22)	300 (43)	600 (87)
β	2 (0.08)	0.6 (0.025)	0.5 (0.015)

notch sensitivity factors

- While the above methods are useful, they should be regarded as estimates only
- Physical complexities are not fully modeled by these methods
- All of these have been developed for relatively "mild" notches
- For sharp notches, best results are found by treating the notch as a crack

- Find the notch sensitivity factor for the following scenario

$$\begin{aligned} \rho &= 0.25 \text{ in.} \\ \sigma_m &= 0 \text{ ksi} \\ K_t &= 3.0 \\ \sigma_u &= 84 \text{ ksi} \end{aligned}$$

fatigue review

group 1

- A part from AISI 4340 in a typical "block" undergoes 100,000 cycles with $\sigma_{min} = 0$ ksi and $\sigma_{max} = 100$ ksi and an additional 10 cycles with $\sigma_{min} = 50$ ksi and $\sigma_{max} = 200$ ksi
- How many "blocks" can this part support before failure?

group 2

- Use the S-N-P chart on p. 245 for 7075-T6 Aluminum
- What is the probability of failure for 30 ksi at 106 cycles?
- To ensure that 99% of parts do not fail, after how many cycles should a fully reversed load of 35 ksi be inspected?
- How many cycles could the same part sustain if only 50% of parts are needed?

group 3

- The fatigue limit for AISI 4142 steel is 58 ksi for completely reversed fatigue loads.
- What is the fatigue limit for fatigue loads with $\sigma_m = 10, 20, 30$ ksi?

group 4

- A material made of 2024-T4 Aluminum undergoes the following load cycle
  - $\sigma_{x,min} = 10$, $\sigma_{x,max} = 50$
  - $\sigma_{y,min} = -20$, $\sigma_{x,max} = 20$
  - $\tau_{xy,min} = 0$, $\tau_{xy,max} = 30$
- How many cycles can it support before failure?

strain based fatigue

strain based fatigue

- The strain based fatigue method uses local stresses and strains (instead of global, nominal values)
- The strain-based method gives greater detail, and validity at lower cycles
- It is still valid for high cycle fatigue (but gives same result as stress-based fatigue)
- Does not include crack growth analysis or fracture mechanics

strain life curve

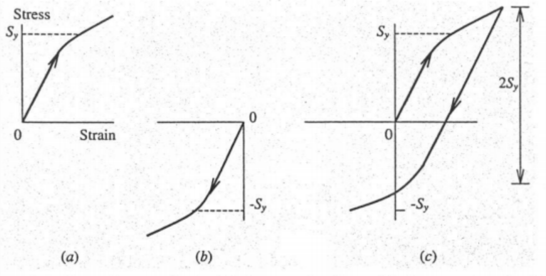
- Similar to the S-N curves in stress-based fatigue analysis, we can plot the cyclic strain amplitude vs. number of cycles to failure
- This is most commonly done using axial test machines (instead of rotating bending tests)
- The test is run in strain control (not load control)
- Generally plotted on log-log scale

plastic and elastic strain

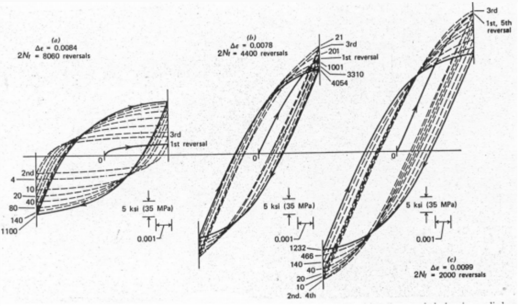
- We can separate the total strain into elastic and plastic components

$$\epsilon_a = \epsilon_{ea} + \epsilon_{pa}$$

plastic strain



hysteresis loops



cyclic stress strain curve

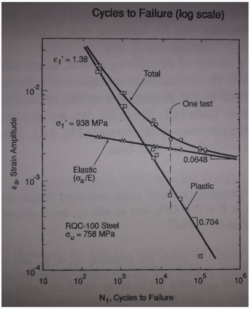
- While strain-life data will generally just report ϵ_a and ϵ_{pa} some will also tabulate a form for the cyclic stress-strain curve

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

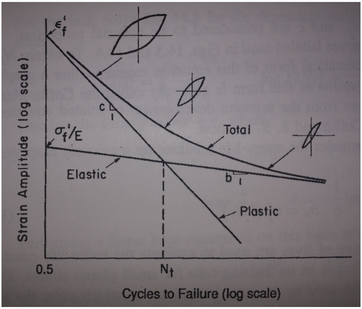
plastic and elastic strain

- On strain life curves, the strain is often plotted three times per each experiment
- Once for total strain, once for plastic strain, and once for elastic strain
- Since plastic strain and elastic strain vary by the number of cycles, a hysteresis loop from half the fatigue life is generally used
- This is considered representative of stable behavior

experimental data



trends



lines

- We notice that the data for elastic and plastic strains are represented by straight lines, in the log-log scale
- If we recall the form used for a straight line in log-log plots for S-N curves:
- $\sigma_a = \sigma'_f (2N_f)^b$
- We can convert this to find the elastic component of strain

$$\epsilon_{ea} = \frac{\sigma'_f}{E}(2N_f)^b$$

lines

- We can use the same form with new constants for the plastic component of strain
- $\epsilon_{pa} = \epsilon'_f (2N_F)^c$
- We can combine the elastic and plastic portions to find the total strain-life curve

$$\epsilon_a = \frac{\sigma'_f}{E}(2N_f)^b + \epsilon'_f (2N_f)^c$$

example

ϵ_a	σ_a (MPa)	ϵ_{pa}	N_f
0.0202	631	0.01695	227
0.0100	574	0.00705	1030
0.0045	505	0.00193	6450
0.0030	472	0.00064	22250
0.0023	455	(0.00010)	110000

transition life

- With the strain-based fatigue method we are better equipped to discuss the difference between high and low-cycle fatigue
- Low-cycle fatigue is dominated by plastic effects, while high-cycle fatigue has little plasticity
- We can find the intersection of the plastic strain and elastic strain lines
- This point is N_t, the transition fatigue life

$$N_t = \frac{1}{2}\left(\frac{\sigma_f'}{\epsilon_f'}\right)^{\frac{1}{c-b}}$$

inconsistencies in constants

- If we consider the equation for the cyclic stress train curve

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

- We can consider the plastic portion and solve for σ_a
- $\sigma_a = H'\epsilon_{pa}^{n'}$

inconsistencies in constants

- We can eliminate $2N_f$ from the plastic strain equation
- $\epsilon_{pa} = \epsilon_f'(2N_f)^c$
- By solving the stress-life relationship for $2N_f$
- $\sigma_a = \sigma_f'(2NF)^b$
- and substituting that into the plastic strain

inconsistencies in constants

- We then compare with stress-life equations and find

$$H' = \frac{\sigma_f'}{(\epsilon_f')^{b/c}}$$
$$n' = \frac{b}{c}$$

inconsistencies in constants

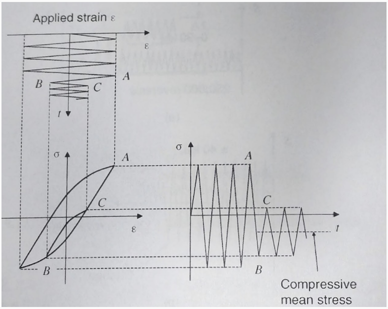
- However, in practice these constants are fit from different curves
- In some cases there can be large inconsistencies in these values
- One cause for this is data that do not lie on a straight line in the log-log domain
- For ductile materials at short lives, the true stresses and strains may differ significantly from engineering stress and strain

variable amplitude strains

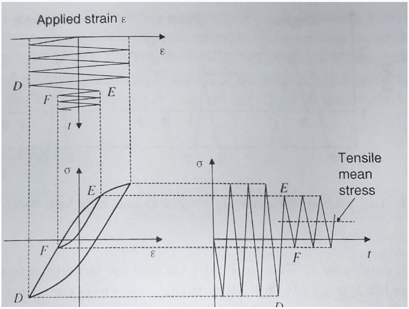
variable amplitude strains

- As with stresses, we can apply variable amplitude strains
- However, when the change is made will affect whether there is a tensile or compressive mean stress

compressive mean



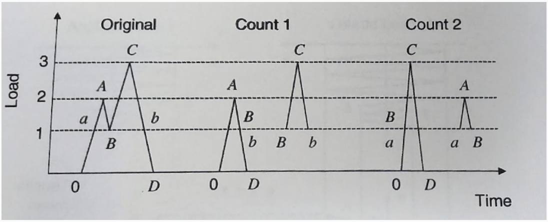
tensile mean



cycle counting

- In all fatigue methods (stress, strain, and crack propagation) the way we count load cycles can have an effect on our results
- To avoid being non-conservative, we need to always count the largest amplitudes first
- We will discuss some specific cycle-counting algorithms during crack propagation

cycle counting



true fracture strength

- We can consider a tensile test as a fatigue test with $N_f = 0.5$
- We would then expect the true fracture strength $\tilde{\sigma}_f \approx \sigma'_f$
- And similarly for strain $\tilde{\epsilon}_f \approx \epsilon'_f$

ductile materials

- Since ductile materials experience large strains before failure, we expect relatively large ϵ_f' and relatively small σ_f'
- This will cause a less steep slope in the plastic strain line
- In turn this intersects with the elastic strain line much later, resulting a longer transition life for ductile materials

brittle materials

- Brittle materials exhibit the opposite effect, with relatively low ϵ_f' and relatively high `\sigma_f^\prime`
- This results in a steeper plastic strain line
- And shorter transition life

tough materials

- Tough materials have intermediate values for both ϵ'_f and σ'_f
- This gives a transition life somewhere between brittle and ductile materials
- It is also noteworthy that strain-life for many metals pass through the point $\epsilon_a = 0.01$ and $N_f = 1000$ cycles
- Steels also follow a trend with Brinell Hardness, the higher they are on the HB scale, the lower their transition life

typical property ranges

- Most common engineering materials have $-0.8 < c < -0.5$, most values being very close to $c =$ -0.6
- The elastic strain slope generally has $b =$ -0.085
- A “steep” elastic slope is around $b =$ -0.12, common in soft metals
- While “shallow” slopes are around $b =$ -0.05, common for hardened metals