11.3 Selected Stress Intensity Factor Cases

This section will present a catalog of stress-intensity factor solutions for some typical crack geometries. Many of these solutions are found in computer programs and handbooks. Tables 11.3.1 through 11.3.5 summarize the solutions that are presented. The solutions are categorized by the location of the crack, either embedded, in a plate (surface or edge), or at a hole, in Tables 11.3.1 through 11.3.3. Solutions for cracks in a cylinders and sphere are summarized in Table 11.3.4, and the ASTM standard specimens are listed in Table 11.3.5. Table 11.3.6 includes listings of the parameters used in the drawings and equations as well as their definitions.

Following these tables, the equations for the stress-intensity factor solutions are given. The solutions are presented in the same order as listed in the tables.

The remote loading solutions are presented in the form:

$$K_i = F_i \sigma \sqrt{\pi a} \tag{11.3.1}$$

where the coefficient F_i is expressed as a function of geometry, and i indicates the loading type. Some of the cases considered can be used to develop more complex solutions through the methods of superposition and compounding

Table 11.3.1. Embedded Cracks

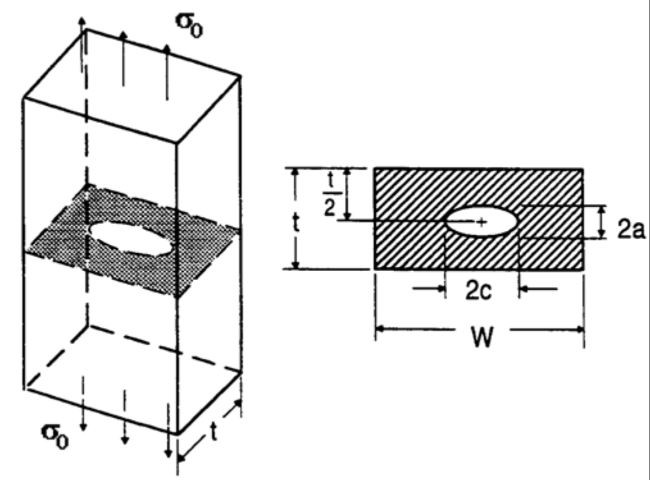
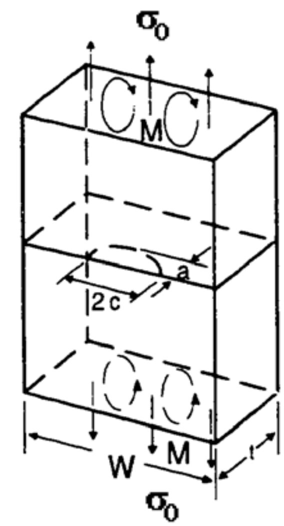
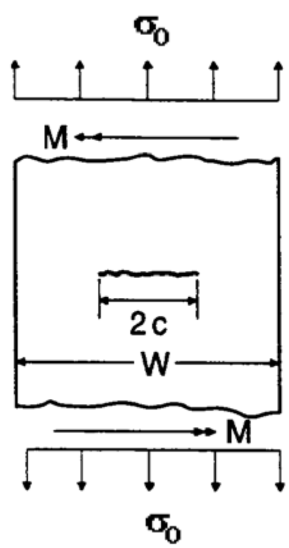
Description	Illustration	References
Embedded Crack in a Plate		Newman & Raju [1984] Forman, et al. [1998]

Table 11.3.2. Cracks in a Plate

Description	Illustration	References
Surface Crack in Plate		Newman & Raju [1984] Forman, et al. [1989]
Through Crack in the Center of a Plate		Fedderson [1966] Paris & Sih [1964] Roberts & Kibler [1971] Forman, et al. [1998]

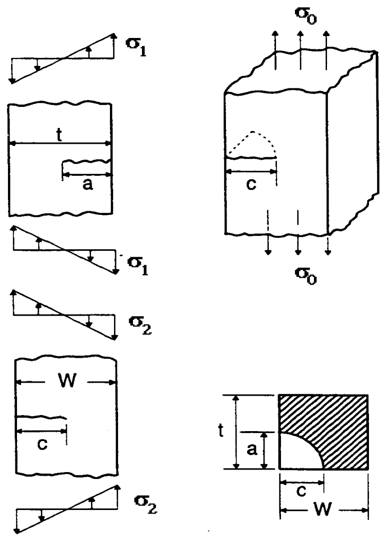
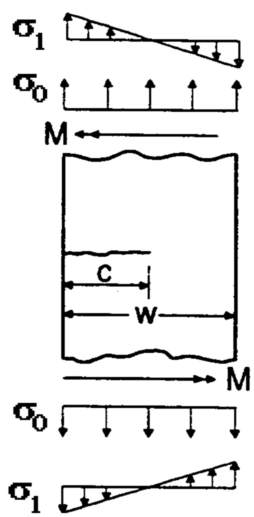
Corner Crack at the Edge of a Plate		Raju & Newman [1988] Forman, et al. [1998]
Through Crack at the Edge of a Plate		Tada, et al. [1973] Forman, et al. [1998]

Table 11.3.3. Cracks from Holes

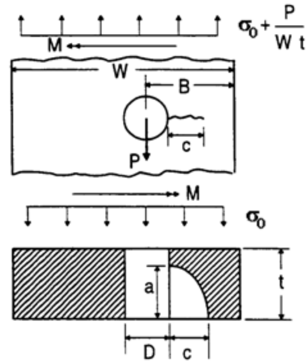
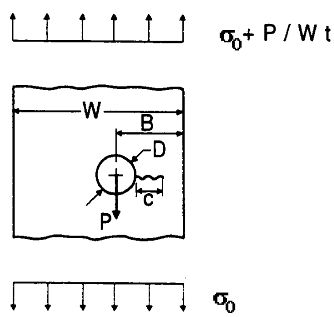
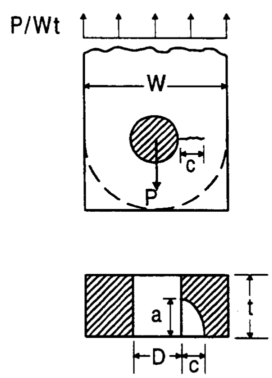
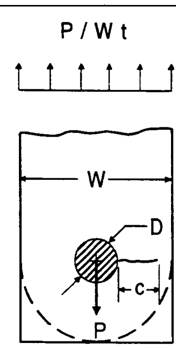
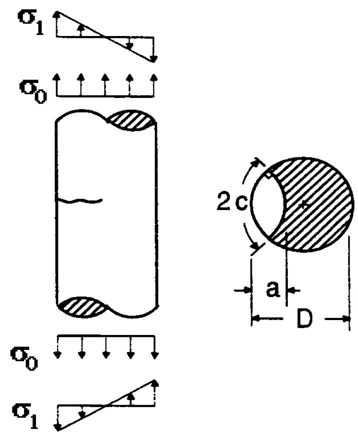
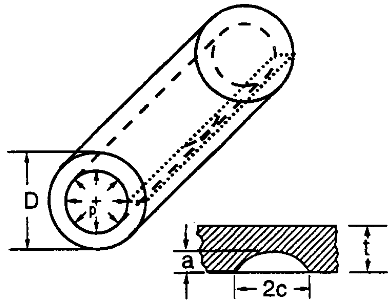
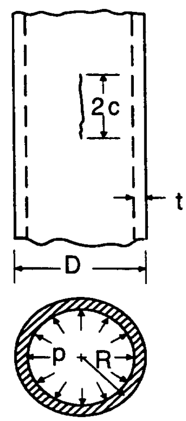
Description	Illustration	References
Radial Corner Crack from a Hole		Newman & Raju [1984] Forman, et al. [1989]
Radial Through Crack from a Hole		Shivakumar & Hsu [1977] Zatz, et al. [1981] Isida [1973] Forman, et al. [1989]
Corner Crack from a Hole in a Lug		Newman & Raju [1984] Forman & Mettu [1992] Forman, et al. [1998]
Through Crack from a Hole in a Lug		Shivakumar & Hsu [1977] Zatz, et al. [1981] Forman, et al. [1989]

Table 11.3.4. Cracks in Cylinders and Spheres

Description	Illustration	References
Surface Crack in a Solid Cylinder		Forman & Shivakumar [1986] Forman, et al. [1998]
Longitudinal Surface Crack in a Cylinder		Newman & Raju [1979] Forman, et al. [1989]
Longitudinal Through Crack in a Cylinder		Newman [1976] Forman, et al. [1998]

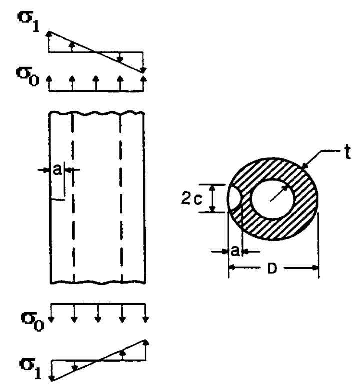
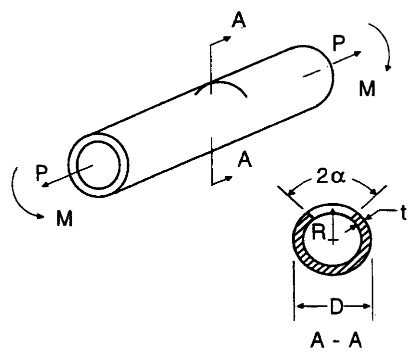
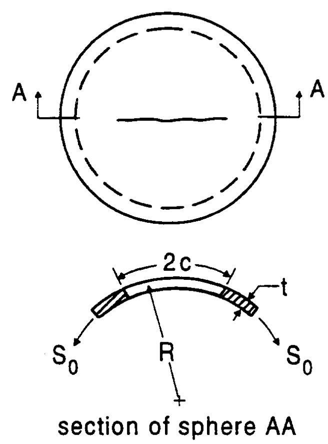
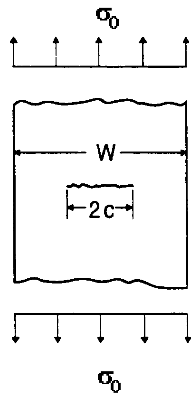
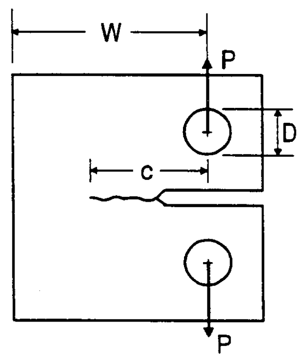
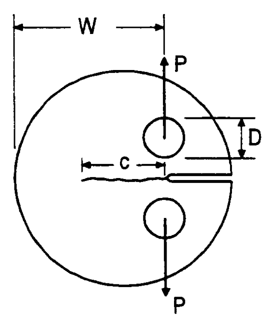
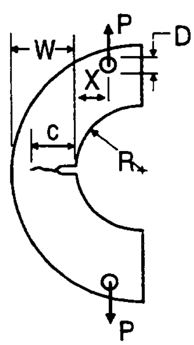
Thumbnail Crack on a Hollow Cylinder		Raju & Newman [1984] Forman, et al. [1989]
Circular Through Crack in a Cylinder		Forman, et al. [1985] Forman, et al. [1998]
Through Crack in a Sphere		Erdogan & Kibler [1969] Forman, et al. [1998]

Table 11.3.5. ASTM Standard Specimens

Description	Illustration	References
Standard Center-Cracked Tension Specimen		Fedderson [1966]
Standard Compact Specimen		ASTM E399 [2000]
Standard Round Compact Specimen		ASTM E399 [2000]
Standard Arc-Shaped Specimen		ASTM E399 [2000]

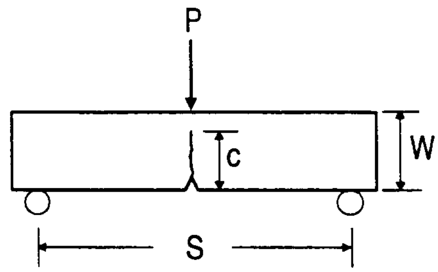
Standard Bend Specimen		ASTM E399 [2000]

Table 11.3.6. Description of Parameters Used for SIF Solutions

Parameter	Description
a	Crack Depth
c	Crack Length
t	Thickness
W	Width
D	Hole diameter; cylinder diameter
B	Distance from hole center to edge of plate Thickness (ASTM standard solutions)
R	Cylinder radius
σ_0	Remote tension stress
σ_1 *and* σ_2	Bending stresses
σ_3	Bearing stress

Table 11.3.7. Stress Intensity Solutions for Embedded Cracks

Embedded Crack in a Plate	$K_0 = F_0 \sigma_0 \sqrt{\pi a}$ $F_0 = M_0 g f_\phi f_w f_x$ $M_0 = M_1 + M_2 (a/t)^2 + M_3 (a/t)^4$ $M_2 = \dfrac{0.05}{\left(0.11 + (a/c)^{\frac{3}{2}}\right)}$ $M_3 = \dfrac{0.29}{\left(0.23 + (a/c)^{\frac{3}{2}}\right)}$ $g = 1 - \left[\dfrac{(a/t)^4 \left(2.6 - 2(a/t)\right)^{\frac{1}{2}}}{\left(1 + 4(a/c)\right)} \right] \lvert \cos\phi \rvert$ $f_w = \left\{ \sec\left[\left(\dfrac{\pi a}{W} \right) \sqrt{\dfrac{a}{t}} \right] \right\}^{\frac{1}{2}}$ $\phi = 0^\circ \quad for \quad \dfrac{dc}{dN}$ $\phi = 90^\circ \quad for \quad \dfrac{da}{dN}$ See Tables 11.3.11 for f_ϕ and f_x equations

Table 11.3.8. Stress Intensity Solutions for Cracks in a Plate

Surface Crack in a Plate	*Tension* $K_0 = F_0 \sigma_0 \sqrt{\pi a}$ *Bending* $K_1 = F_1 \sigma_1 \sqrt{\pi a}$	$F_0 = M_0 g_1 f_\phi f_w f_x$ $F_1 = H_c F_0$ $f_w = \sqrt{sec\left(\frac{\pi c}{W}\sqrt{\frac{a}{t}}\right)}$ $\phi = 10^\circ \quad for \ \frac{dc}{dN}$ $\phi = 90^\circ \quad for \ \frac{da}{dN}$ See Table 11.3.12 for M_0, g_1, f_ϕ , and f_x equations
Through Crack in the Center of a Plate	*Tension* $K_0 = F_0 \sigma_0 \sqrt{\pi a}$ *Bending* $K_1 = F_1 \sigma_1 \sqrt{\pi a}$	$F_0 = \left\{ sec\left(\pi \frac{a}{W}\right) \right\}^{\frac{1}{2}}$ $F_1 = \frac{F_0}{2}$
Corner Crack at the Edge of a Plate	*Tension* $K_0 = F_0 \sigma_0 \sqrt{\pi a}$ *Bending* $K_1 = F_1 \sigma_1 \sqrt{\pi a}$	$F_i = f_x f_\phi f_a f_i$ $f_i = \left(\frac{a}{c}, \frac{a}{t}, \frac{c}{W}\right) \quad for \ i = 0, 1, 2$ See Tables 11.3.12 for f_ϕ , f_a and f_x equations and Table 11.3.9 for f_i
Through Crack at the Edge of a Plate	*Tension* $K_0 = F_0 \sigma_0 \sqrt{\pi a}$ *Bending* $K_1 = F_1 \sigma_1 \sqrt{\pi a}$ $K_2 = F_2 \sigma_2 \sqrt{\pi a}$	$F_0 = sec\,\beta\left(\frac{tan\,\beta}{\beta}\right)^{\frac{1}{2}}\left[0.752 + 2.02\left(\frac{a}{W}\right) + 0.37(1 - sin\,\beta)^3\right]$ $F_1 = \frac{F_0}{2}$ $F_2 = sec\,\beta\left(\frac{tan\,\beta}{\beta}\right)^{\frac{1}{2}}\left[0.923 + 0.199(1 - sin\,\beta)^4\right]$ $\beta = \frac{\pi a}{2W}$

Table 11.3.9. Calculation of f_i for Corner Crack Solution

$f_i = \left(\frac{a}{c}, \frac{a}{t}, \frac{a}{W}\right)$ obtained from interpolating in f_0, f_1, f_2 tables as follows

1. Four data points, $f_{Ii}\{(a/c)_j,\ a/t,\ c/W\}|_{j=1,2,3,4}$, are calculated using cubic spline interpolation, where $(a/c)_j$ are listed tabular values of 0.2, 0.4, 0.5, 1.0, 2.0, 2.5, and 5.0, and, in general, $(a/c)_{j=1,2} < a/c$ and $(a/c)_{j=3,4} > a/c$.
2. $f_i(a/c)$ are then calculated from the above four data points using piecewise Hermite polynomial interpolation.

Table of F_0 Values

		c/W = 0.0		c/W = 0.1		c/W = 0.2		c/W = 0.5		c/W = 0.8		c/W = 1.0	
a/c	a/t	a-tip	c-tip	a-tip	c-tip	a-tip	c-tip	a-tip	c-tip	a-tip	c-tip	a-tip	c-tip
0.2	0.0	1.037	1.280	1.041	1.285	1.043	1.291	1.070	1.330	1.102	1.390	1.128	1.441
	0.1	1.078	1.311	1.083	1.318	1.087	1.322	1.116	1.355	1.145	1.408	1.169	1.452
	0.2	1.157	1.374	1.161	1.380	1.169	1.388	1.207	1.420	1.240	1.470	1.268	1.513
	0.5	1.515	1.752	1.536	1.787	1.571	1.833	1.732	1.993	1.944	2.243	2.124	2.448
	0.8	2.031	2.498	2.098	2.663	2.196	2.832	2.749	3.528	3.623	4.603	4.378	5.491
	1.0	2.475	3.286	2.578	3.585	2.749	3.931	3.790	5.340	5.523	7.514	7.026	9.311
0.4	0.0	1.073	1.173	1.077	1.177	1.082	1.183	1.130	1.244	1.201	1.314	1.254	1.365
	0.1	1.094	1.198	1.097	1.201	1.104	1.206	1.161	1.267	1.233	1.343	1.289	1.398
	0.2	1.131	1.241	1.135	1.246	1.147	1.257	1.227	1.337	1.306	1.417	1.375	1.488
	0.5	1.317	1.488	1.339	1.521	1.378	1.567	1.577	1.749	1.865	2.072	2.117	2.349
	0.8	1.636	1.985	1.691	2.069	1.780	2.198	2.318	2.781	3.239	3.816	4.066	4.723
	1.0	1.941	2.504	2.015	2.638	2.167	2.861	3.111	3.972	4.813	5.875	6.355	7.559
0.5	0.0	1.086	1.158	1.090	1.160	1.097	1.165	1.150	1.220	1.235	1.302	1.308	1.381
	0.1	1.102	1.179	1.106	1.180	1.113	1.185	1.178	1.245	1.271	1.339	1.350	1.424
	0.2	1.130	1.211	1.134	1.217	1.147	1.228	1.238	1.310	1.345	1.417	1.439	1.511
	0.5	1.272	1.414	1.294	1.446	1.335	1.492	1.550	1.684	1.879	2.045	2.161	2.355
	0.8	1.546	1.827	1.596	1.899	1.684	2.018	2.224	2.574	3.169	3.609	4.010	4.516
	1.0	1.801	2.260	1.871	2.368	2.021	2.558	2.931	3.568	4.595	5.380	6.163	7.059
1.0	0.0	1.138	1.138	1.142	1.141	1.145	1.144	1.236	1.192	1.416	1.343	1.601	1.523
	0.1	1.141	1.142	1.144	1.144	1.154	1.152	1.261	1.220	1.470	1.399	1.683	1.609
	0.2	1.144	1.145	1.152	1.154	1.172	1.172	1.309	1.267	1.565	1.486	1.801	1.685
	0.5	1.198	1.232	1.220	1.251	1.267	1.309	1.547	1.547	2.075	2.056	2.555	2.514
	0.8	1.364	1.413	1.399	1.470	1.486	1.565	2.056	2.075	3.171	3.171	4.196	4.162
	1.0	1.481	1.615	1.545	1.686	1.685	1.801	2.514	2.555	4.162	4.190	5.977	5.977

Table of F_1 Values

		c/W = 0.0		c/W = 0.1		c/W = 0.2		c/W = 0.5		c/W = 0.8		c/W = 1.0	
a/c	a/t	a-tip	c-tip	a-tip	c-tip	a-tip	c-tip	a-tip	c-tip	a-tip	c-tip	a-tip	c-tip
0.2	0.0	1.037	1.280	1.041	1.285	1.043	1.291	1.070	1.330	1.102	1.390	1.128	1.441
	0.1	0.939	1.287	0.940	1.289	0.945	1.294	0.975	1.336	1.029	1.400	1.077	1.458
	0.2	0.855	1.295	0.862	1.296	0.870	1.302	0.910	1.360	0.972	1.435	1.025	1.510
	0.5	0.683	1.475	0.689	1.486	0.706	1.520	0.820	1.632	0.956	1.829	1.070	1.990
	0.8	0.392	1.762	0.428	1.811	0.469	1.898	0.730	2.231	1.135	2.811	1.494	3.204
	1.0	0.056	2.050	0.093	2.129	0.165	2.266	0.572	2.793	1.264	3.745	1.883	4.577
0.4	0.0	1.073	1.173	1.077	1.177	1.082	1.183	1.130	1.244	1.201	1.314	1.254	1.365
	0.1	0.941	1.152	0.943	1.160	0.956	1.170	1.015	1.214	1.087	1.307	1.188	1.396
	0.2	0.820	1.148	0.828	1.157	0.842	1.168	0.911	1.212	0.997	1.333	1.124	1.455
	0.5	0.515	1.195	0.538	1.210	0.562	1.236	0.694	1.378	0.877	1.603	1.027	1.807
	0.8	0.194	1.340	0.217	1.360	0.247	1.400	0.488	1.705	0.903	2.243	1.255	2.739
	1.0	-0.026	1.490	-0.018	1.503	0.035	1.573	0.357	2.044	1.028	2.857	1.698	3.599
0.5	0.0	1.086	1.158	1.090	1.160	1.097	1.165	1.150	1.220	1.235	1.302	1.308	1.381
	0.1	0.946	1.130	0.952	1.139	0.965	1.148	1.027	1.192	1.117	1.297	1.233	1.417
	0.2	0.808	1.114	0.820	1.126	0.840	1.140	0.915	1.183	1.019	1.320	1.167	1.482
	0.5	0.475	1.124	0.490	1.140	0.526	1.164	0.660	1.313	0.873	1.573	1.055	1.831

	0.8	0.129	1.223	0.150	1.243	0.184	1.281	0.422	1.570	0.838	2.099	1.197	2.654
	1.0	-0.094	1.334	-0.079	1.343	-0.032	1.407	0.274	1.854	0.934	2.594	1.600	3.375
1.0	0.0	1.138	1.138	1.142	1.141	1.145	1.144	1.236	1.192	1.416	1.343	1.601	1.523
	0.1	0.965	1.087	0.977	1.097	0.993	1.111	1.094	1.176	1.288	1.348	1.488	1.573
	0.2	0.785	1.047	0.810	1.060	0.838	1.080	0.960	1.167	1.180	1.368	1.408	1.650
	0.5	0.345	0.982	0.375	1.000	0.419	1.033	0.590	1.194	0.942	1.574	1.270	2.012
	0.8	-0.070	0.961	-0.043	0.983	-0.006	1.031	0.228	1.280	0.698	1.831	1.189	2.551
	1.0	-0.352	0.964	-0.323	0.990	-0.279	1.043	-.005	1.407	0.637	2.028	1.154	2.992

Table of F_0 Values

		c/W = 0.0		**c/W = 0.1**		**c/W = 0.2**		**c/W = 0.5**		**c/W = 0.8**		**c/W = 1.0**	
a/c	**a/t**	**a-tip**	**c-tip**	**a-tip**	**c-tip**	**a-tip**	**c-tip**	**a-tip**	**c-tip**	**a-tip**	**c-tip**	**a-tip**	**c-tip**
0.2	0.0	1.037	1.280	1.006	1.054	0.976	0.822	0.900	0.138	0.800	-0.566	0.740	-1.033
	0.1	1.078	1.311	1.050	1.080	1.020	0.848	0.955	0.150	0.866	-0.550	0.805	-1.018
	0.2	1.157	1.374	1.119	1.123	1.090	0.896	1.039	0.190	0.952	-0.522	0.885	-0.996
	0.5	1.515	1.752	1.469	1.492	1.440	1.259	1.400	0.530	1.313	-0.276	1.250	-0.814
	0.8	2.031	2.498	1.997	2.282	2.009	2.081	2.124	1.447	2.200	0.614	2.300	0.058
	1.0	2.475	3.286	2.470	3.085	2.558	2.967	2.873	2.536	3.320	1.821	3.700	1.347
0.4	0.0	1.070	1.175	1.050	1.000	1.010	0.796	0.940	0.215	0.845	-0.335	0.769	-0.714
	0.1	1.095	1.198	1.070	1.015	1.037	0.812	0.970	0.242	0.875	-0.324	0.806	-0.700
	0.2	1.131	1.241	1.100	1.039	1.074	0.852	1.010	0.276	0.922	-0.284	0.859	-0.658
	0.5	1.317	1.488	1.281	1.288	1.271	1.112	1.250	0.563	1.196	-0.045	1.150	-0.419
	0.8	1.630	1.985	1.629	1.798	1.652	1.635	1.772	1.199	1.912	0.649	1.998	0.282
	1.0	1.941	2.504	1.970	2.318	2.044	2.167	2.376	1.861	2.778	1.548	3.177	1.194
0.5	0.0	1.086	1.158	1.055	0.989	1.020	0.789	0.942	0.244	0.854	-0.269	0.792	-0.625
	0.1	1.102	1.179	1.074	1.000	1.040	0.809	0.968	0.272	0.884	-0.255	0.825	-0.603
	0.2	1.130	1.211	1.100	1.025	1.070	0.846	1.004	0.310	0.930	-0.212	0.878	-0.561
	0.5	1.272	1.414	1.241	1.230	1.234	1.067	1.216	0.566	1.187	0.025	1.157	-0.311
	0.8	1.546	1.827	1.538	1.649	1.560	1.502	1.701	1.123	1.851	0.652	1.938	0.362
	1.0	1.801	2.260	1.851	2.075	1.926	1.939	2.271	1.685	2.680	1.435	3.068	1.132
1.0	0.0	1.138	1.138	1.087	0.965	1.047	0.785	0.982	0.345	0.961	-0.070	0.964	-0.352
	0.1	1.141	1.142	1.097	0.977	1.060	0.810	1.000	0.375	0.983	-0.043	0.990	-0.323
	0.2	1.144	1.145	1.111	0.993	1.080	0.838	1.033	0.419	1.031	-0.006	1.043	-0.279
	0.5	1.192	1.236	1.176	1.094	1.167	0.960	1.194	0.590	1.280	0.228	1.407	-0.005
	0.8	1.343	1.416	1.348	1.288	1.368	1.280	1.574	0.942	1.831	0.698	2.028	0.637
	1.0	1.523	1.601	1.573	1.488	1.650	1.408	2.012	1.270	2.551	1.189	2.992	1.154

Table 11.3.10. Stress Intensity Solutions for Cracks from Holes

Radial Corner Crack from a Hole	*Tension* $K_0 = F_0 \sigma_0 \sqrt{\pi a}$ *Bending* $K_1 = F_1 \sigma_1 \sqrt{\pi a}$ *Bearing* $K_3 = F_3 \sigma_3 \sqrt{\pi a}$	$F_0 = G_0 G_w$ $F_1 = G_1 G_w H_c$ $F_3 = \left(\frac{G_0 D}{2W} + G_1 \right) G_w$ See Tables 11.3.11 and 11.3.12 for additional equations
Radial Through Crack from a Hole	*Tension* $K_0 = F_0 \sigma_0 \sqrt{\pi a}$ *Bearing* $K_3 = F_3 \sigma_3 \sqrt{\pi a}$	$F_0 = G_0 G_w$ $F_3 = \left(\frac{G_0 D}{2W} + G_1 \right) G_w$ $G_w = \left[\frac{sec\ \lambda\ (sin\ \beta)}{\beta} \right]^{\frac{1}{2}}$ See Tables 11.3.11 and 11.3.12 for additional equations
Corner Crack from a Hole in a Lug	*Bearing* $K_3 = F_3 \sigma_3 \sqrt{\pi a}$	$F_3 = \left(\frac{G_0 D}{2W} + G_1 \right) G_w$ See Tables 11.3.11 and 11.3.12 for additional equations

Through Crack from a Hole in a Lug	*Bearing* $K_3 = F_3 \sigma_3 \sqrt{\pi a}$	$F_3 = \left(\frac{G_0 D}{2W} + G_1 \right) G_w G_L G_2$ $G_L = \left[sec \left(\frac{\pi D}{2W} \right) \right]^{\frac{1}{2}}$ $G_w = \left(sec\ \lambda \right)^{\frac{1}{2}}$ $G_2 = C_1 + C_2 \left(\frac{c}{b} \right) + C_3 \left(\frac{c}{b} \right)^2 + C_4 \left(\frac{c}{b} \right)^3$ $b = \frac{W - D}{2}$ $C_1 = 0.688 + 0.772 \left(\frac{D}{W} \right) + 0.613 \left(\frac{D}{W} \right)^2$ $C_2 = 4.948 - 17.318 \left(\frac{D}{W} \right) + 16.785 \left(\frac{D}{W} \right)^2$ $C_3 = -14.297 + 62.994 \left(\frac{D}{W} \right) - 69.818 \left(\frac{D}{W} \right)^2$ $C_4 = 12.35 - 58.664 \left(\frac{D}{W} \right) + 66.387 \left(\frac{D}{W} \right)^2$ See Tables 11.3.11 and 11.3.12 for additional equations

Table 11.3.11. Additional Equations Used for Calculating SIF at Holes

	Thru Cracks	**Part-thru Cracks**
λ	$\left(\frac{\pi}{2}\right)\left(\frac{D+a}{2B-a}\right)$	$\left(\frac{\pi}{2}\right)\left(\sqrt{\frac{a}{t}}\right)\left(\frac{D+a}{2B-a}\right)$
G_0	$f_0\left(z_0\right)$	$\frac{f_0\left(z_0\right)}{d_0}$
G_1	$f_1\left(z_0\right)$	$f_1\left(z_0\right)\left(\frac{g_p}{d_0}\right)$
G_2		$\frac{f_0\left(z_2\right)}{d_2}$
G_w		$M_0\ g_1\ g_3\ g_4\ f_w\ f_\phi\ f_x$
z	$\left(1+\frac{2a}{D}\right)^{-1}$	$\left[1+2\left(\frac{a}{D}\right)cos\left(\mu\phi\right)\right]^{-1}$
ϕ		$\phi=10^\circ\ for\ \frac{dc}{dN}$ $\phi=80^\circ\ for\ \frac{da}{dN}$

Table 11.3.12. Additional Equations Used for Calculating SIF for Cracks In a Plate and Cracks at Holes

Parameter	**Equation**	
$f_0(z)$	$0.7071 + 0.7548z + 0.3415z^2 + 0.642z^3 + 0.9196z^4$	
$f_1(z)$	$0.078z + 0.7588z^2 - 0.4293z^3 + 0.0644z^4 + 0.651z^5$	
M_0	$m_1 + m_2\left(\frac{a}{t}\right)^2 + m_3\left(\frac{a}{t}\right)^4$	
H_c	$H_1 + (H_2 - H_1)\, sin^p\, \phi^3$	
H_1	$1 + G_{11}\left(\frac{a}{t}\right) + G_{12}\left(\frac{a}{t}\right)^2 + G_{13}\left(\frac{a}{t}\right)^3$	
H_2	$1 + G_{21}\left(\frac{a}{t}\right) + G_{22}\left(\frac{a}{t}\right)^2 + G_{23}\left(\frac{a}{t}\right)^3$	
$z_{0,1}$	$\left[1 + 2\left(\frac{a}{D}\right) cos\,(\mu_{0,1}\,\phi)\right]^{-1}$	
$d_{0,1}$	$1 + 0.13z_{0,1}^2$	
μ_0	0.85	
μ_1	$0.85 - 0.25v^{0.25}$	
g_p	$\left(\frac{W + D}{W - D}\right)^{0.5}$	
β	$\frac{D}{B} - \frac{2D}{W}$	
	$\frac{a}{c} \le 1$	$\frac{a}{c} > 1$
f_x	$\left[1 + 1.464\left(\frac{a}{c}\right)^{1.65}\right]^{-\frac{1}{2}}$	$\left[1 + 1.464\left(\frac{c}{a}\right)^{1.65}\right]^{-\frac{1}{2}}$
f_ϕ	$\left[\left(\frac{a}{c} cos\,\phi\right)^2 + sin^2\,\phi\right]^{\frac{1}{4}}$	$\left[cos^2\,\phi + \left(\frac{c}{a} sin\,\phi\right)^2\right]^{\frac{1}{4}}$
f_a, M_1	1	$\sqrt{\frac{c}{a}}$
m_1	$1.13 - 0.09\left(\frac{a}{c}\right)$	$\left(\frac{a}{c}\right)^{-\frac{1}{2}} + 0.04\left(\frac{a}{c}\right)^{-\frac{3}{2}}$

m_2	$-0.54+\frac{0.89}{(0.2+(a/c))}$	$0.2(a/c)^{-4}$
m_3	$0.5-\frac{1}{(0.65+(a/c))}+14(1-(a/c))^{24}$	$-0.11\left(\frac{a}{c}\right)^{-4}$
g_1	$1+\left(0.1+0.35\left(\frac{a}{t}\right)^2\right)(1-\sin\phi)^2$	$1+\left(0.1+\frac{0.35}{(a/c)}(a/t)^2\right)(1-\sin\phi)^2$
g_3	$\left(1+0.04\left(\frac{a}{c}\right)\right)\left[1+0.1(1-\cos\phi)^2\right]$ $\left(0.85+0.15\left(\frac{a}{t}\right)^{\frac{1}{4}}\right)$	$\left(1.13-\frac{0.09}{\left(\frac{a}{c}\right)}\right)\left[1+0.1(1-\cos\phi)^2\right]$ $\left(0.85+0.15\left(\frac{a}{t}\right)^{\frac{1}{4}}\right)$
g_4	$1-0.7\left(1-\left(\frac{a}{t}\right)\right)\left(\left(\frac{a}{c}\right)-0.2\right)\left(1-\left(\frac{a}{c}\right)\right)$	1
p	$0.1+1.3\left(\frac{a}{t}\right)+1.1\left(\frac{a}{c}\right)-0.7\left(\frac{a}{c}\right)\left(\frac{a}{t}\right)$	$0.2+\frac{1}{(a/c)}+0.6\left(\frac{a}{t}\right)$
G_{11}	$-0.43-0.74\left(\frac{a}{c}\right)-0.84\left(\frac{a}{c}\right)^2$	$-2.07+\frac{0.06}{(a/c)}$
G_{12}	$1.25-1.19\left(\frac{a}{c}\right)+4.39\left(\frac{a}{c}\right)^2$	$4.35+\frac{0.16}{(a/c)}$
G_{13}	$-1.94+4.22\left(\frac{a}{c}\right)-5.51\left(\frac{a}{c}\right)^2$	$-2.93-\frac{0.3}{(a/c)}$
G_{21}	$-1.5-0.04\left(\frac{a}{c}\right)-1.73\left(\frac{a}{c}\right)^2$	$-3.64+\frac{0.37}{(a/c)}$
G_{22}	$1.71-3.17\left(\frac{a}{c}\right)+6.84\left(\frac{a}{c}\right)^2$	$5.87-\frac{0.49}{(a/c)}$
G_{23}	$-1.28+2.71\left(\frac{a}{c}\right)-5.22\left(\frac{a}{c}\right)^2$	$-4.32+\frac{0.53}{(a/c)}$

Table 11.3.13. Cracks in Cylinders and Spheres

Surface Crack in a Solid Cylinder	*Tension* $K_0 = F_0 \sigma_0 \sqrt{\pi a}$ *Bending* $K_1 = F_1 \sigma_1 \sqrt{\pi a}$	$F_0 = G\left[0.752 + 1.286\beta + 0.37Y^3\right]$ $F_1 = G\left[0.923 + 0.199Y^4\right]$ $G = 0.92\left(\frac{2}{\pi}\right) sec\ \beta \left[\frac{(tan\ \beta)}{\beta}\right]^{\frac{1}{2}}$ $Y = 1 - sin\ \beta$ $\beta = \left(\frac{\pi}{2}\right)\left(\frac{a}{D}\right)$
Longitudinal Surface Crack in a Cylinder	$K_0 = F_0 \sigma_0 \sqrt{\pi a}$	$F_0 = 0.97\ M_0 g_1 f_\phi f_c f_i f_x$ $f_c = \left[\frac{(1+k^2)}{(1-k^2)} + 1 - 0.5\sqrt{v}\right]\left[\frac{2t}{D-2t}\right]$ $k = 1 - \frac{2t}{D}$ $f_i = 1.0$ *for internal crack* $f_i = 1.1$ *for external crack* $\phi = 10^\circ$ *for* $\frac{dc}{dN}$ $\phi = 90^\circ$ *for* $\frac{da}{dN}$ See Table 11.3.12 for M_0, g_1, f_ϕ, and f_x equations
Longitudinal Through Crack in a Cylinder	$K_0 = F_0 \sigma_0 \sqrt{\pi a}$	$F_0 = (1 + 0.52\lambda + 1.29\lambda^2 - 0.074\lambda^3)^{\frac{1}{2}}$ $\lambda = \frac{a}{\sqrt{Rt}}$

Thumbnail Crack on a Hollow Cylinder	*Tension* $K_0 = F_0 \sigma_0 \sqrt{\pi a}$ *Bending* $K_1 = F_1 \sigma_1 \sqrt{\pi a}$	$F_0 = G\left[\left(A_0 + B_0\left(\frac{a}{t}\right)\right) sin^2 \phi + \left(C_0 + D_0\left(\frac{a}{t}\right)^2\right) cos^2 \phi\right]$ $F_1 = G\left[\left(A_1 + B_1\left(\frac{a}{t}\right)\right) sin^2 \phi + \left(C_1 + D_1\left(\frac{a}{t}\right)^2\right) cos^2 \phi\right]$ $A_0 = 1.135 - 0.135\left(\frac{a}{c}\right)$ $B_0 = 0.5 - 0.663\left(\frac{a}{c}\right) + 0.266\left(\frac{a}{c}\right)^2 + \left(0.713 - 1.286\left(\frac{a}{c}\right) + 0.651\left(\frac{a}{c}\right)^2\right)\left(\frac{2t}{D}\right)$ $C_0 = 0.56 + 0.555\left(\frac{a}{c}\right)$ $D_0 = 0.876 - 0.465\left(\frac{a}{c}\right) - \left(0.86 - 0.217\left(\frac{a}{c}\right)\right)\left(\frac{2t}{D}\right)$ $A_1 = 1.093 - 0.1\left(\frac{a}{c}\right)$ $B_1 = 0.936 - 1.758\left(\frac{a}{c}\right) + 0.903\left(\frac{a}{c}\right)^2 - \left(0.598 + 0.417\left(\frac{a}{c}\right)\right)\left(\frac{2t}{D}\right)$ $C_1 = 0.556 + 0.548\left(\frac{a}{c}\right)$ $D_1 = 0.943 - 0.518\left(\frac{a}{c}\right) - \left(2.382 - 2.226\left(\frac{a}{c}\right) + 0.9\left(\frac{a}{c}\right)^2\right)\left(\frac{2t}{D}\right)$ $Y = 1 - 0.385\left(\frac{2t}{D}\right)\left(\frac{(a/t)}{(a/c)}\right)\left(2.14\left(\frac{a}{c}\right) - 1.557\left(\frac{a}{c}\right)^2 + 0.417\left(\frac{a}{c}\right)^3\right)$ $f_x = \left[1 + 4.464\left(\frac{a}{c}\right)^{1.65}\right]^{-\frac{1}{2}} \qquad G = \frac{f_x}{Y}$ $\phi = 10^\circ \quad for \ \frac{dc}{dN}$ $\phi = 90^\circ \quad for \ \frac{da}{dN}$

<table>
<tr>
<td>Circular Through Crack in a Cylinder</td>
<td><u>Tension</u>
$K_0 = F_0 \sigma_0 \sqrt{\pi a}$

<u>Bending</u>
$K_1 = F_1 \sigma_1 \sqrt{\pi a}$</td>
<td>$F_0 = \left(\dfrac{I_0}{2\pi\alpha} \right)^{\frac{1}{2}}$
$F_1 = \left(\dfrac{I_1}{2\pi\alpha} \right)^{\frac{1}{2}}$
$I_0 = \left[\sqrt{8}\left(f^2 - 1\right) + \dfrac{\pi\beta^2}{b} \right] \dfrac{\alpha^2}{k}$
$I_1 = \left[\sqrt{8}\left(g^2 - 1\right) + \dfrac{\pi\beta^2}{b} \right] \dfrac{\alpha^2}{k}$
$f = 1 + \dfrac{h\left(1 - \alpha \cot \alpha\right)}{2\alpha}$
$g = \left[1 + \dfrac{h\left(\alpha + \alpha \cot^2 \alpha - \cot \alpha\right)}{4} \right] \dfrac{\left(\sin \alpha\right)}{\alpha}$
$h = \dfrac{\sqrt{2}}{\left\{ \cot\left[\dfrac{\left(\pi - \alpha\right)}{\sqrt{2}} \right] + \sqrt{2} \cot \alpha \right\}}$
$b = \dfrac{\alpha}{2k}$
$k = \sqrt{\dfrac{t}{R}} \left[12\left(1 - v^2\right)\right]^{-\frac{1}{4}}$
$\beta = 1 + \left(\dfrac{\pi}{16}\right) b^2 - 0.0293\, b^3 \quad \text{for} \quad b \le 1$
$\beta = \left(\dfrac{\sqrt{8b}}{\pi}\right)^{0.5} + \left(\dfrac{0.179}{b}\right)^{0.885} \quad \text{for} \quad b > 1$
$\alpha = c / R$</td>
</tr>
<tr>
<td>Through Crack in a Sphere</td>
<td>$K_0 = F_0 \sigma_0 \sqrt{\pi a}$</td>
<td>$F_0 = \sqrt{1 + 3\lambda^2}$
$\lambda = \dfrac{a}{\sqrt{Rt}}$</td>
</tr>
</table>

Table 11.3.14. Stress Intensity Solutions for ASTM Standard Specimens

Standard Center-cracked Tension Specimen	$K_0 = F_0 \sigma_0 \sqrt{\pi a}$ $F_0 = \left[sec\left(\frac{\pi a}{W} \right) \right]^{1/2}$
Standard Compact Specimen	$K_0 = F_0 \sigma_0 \sqrt{\pi a}$ $F_0 = D(\pi a W)^{-1/2} \left[\left(2 + \frac{a}{W} \right) \left(1 - \frac{a}{W} \right)^{-3/2} \right] G$ $G = 0.886 + 4.64\left(\frac{a}{W} \right) - 13.32\left(\frac{a}{W} \right)^2 + 14.72\left(\frac{a}{W} \right)^3 - 5.6\left(\frac{a}{W} \right)^4$
Standard Round Compact Specimen	$K_0 = F_0 \sigma_0 \sqrt{\pi a}$ $F_0 = D\left(\pi\, aW \right)^{-\frac{1}{2}} \left[\left(2 + \frac{a}{W} \right) \left(1 - \frac{a}{W} \right)^{-\frac{3}{2}} \right] G$ $G = 0.76 + 4.8\left(\frac{a}{W} \right) - 11.58\left(\frac{a}{W} \right)^2 + 11.43\left(\frac{a}{W} \right)^3 - 4.08\left(\frac{a}{W} \right)^4$
Standard Arc-Shaped Specimen	$K_0 = F_0 \sigma_0 \sqrt{\pi a}$ $F_0 = D\left(\pi\, a W \right)^{-\frac{1}{2}} \left[\frac{3X}{W} + 1.9 + 1.1\left(\frac{a}{W} \right) \right] GY$ $G = 1 + 0.25\left(1 - \frac{a}{W} \right)^2 \left[1 - \frac{R}{R+W} \right]$ $Y = \sqrt{\frac{a}{W}} \left(1 - \frac{a}{W} \right)^{-\frac{3}{2}} \left[3.74 - 6.3\left(\frac{a}{W} \right) + 6.32\left(\frac{a}{W} \right)^2 - 2.43\left(\frac{a}{W} \right)^3 \right]$
Standard Bend Specimen	$K_0 = F_0 \sigma_0 \sqrt{\pi a}$ $F_0 = \dfrac{\left[1.99 - \left(\frac{a}{W} \right) \left(1 - \frac{a}{W} \right) \left(2.15 - 3.93\left(\frac{a}{W} \right) + 2.7\left(\frac{a}{W} \right)^2 \right) \right]}{\sqrt{\pi} \left(1 + \frac{2a}{W} \right) \left(1 - \frac{a}{W} \right)^{\frac{3}{2}}}$