# Homework 3

February 21, 2019

## 0.1 1

First we find $K_I$ without any consideration for plasticity. Since we have an edge-crack in a finite-width panel, we use (2.4a) and substitute the provided values.

In [12]:
```
import numpy as np
def beta(a,w):
    return 1.122 - 0.231*a/w + 10.55*(a/w)**2 - 21.71*(a/w)**3 + 30.82*(a/w)**4
def KI(a,w,s):
    return s*np.sqrt(np.pi*a)*beta(a,w)
a = 1.5
w = 6.
t = .25
s = 15. #ksi
sy = 65. #ksi
print KI(a,w,s)
```

```
48.99928079724123
```

We find $K_I = 49.0 \text{ ksi}\sqrt{\text{in}}$.
For plane stress, we use (6.6) with I=2, while for plane strain we set I=6.

In [13]:
```
#plane stress, I=2
I=2
rp = 1.0/(I*np.pi)*(KI(a,w,s)/sy)**2
print rp
```

```
0.09044255642790483
```

In [14]:
```
#calculate aeff, KI(aeff) until solution converges
KI_old = KI(a,w,s)
aeff = a + rp
KI_new = KI(aeff,w,s)
#Loop through until the percent error is less than 1%
while ((KI_old-KI_new)/(KI_old))**2 > 0.00001:
    rp = 1.0/(I*np.pi)*(KI(aeff,w,s)/sy)**2
    aeff = a + rp
    KI_old = KI_new
    KI_new = KI(aeff,w,s)
```

In [15]:
```
print rp
print KI_new
```

```
0.10321764158945292
52.396000450790474
```

So for plane stress we have: $K_I = 52.4$ ksi$\sqrt{\text{in}}$
In plane strain we follow the same procedure, with I=6

In [16]:
```
#plane strain, I=6
I=6
rp = 1.0/(I*np.pi)*(KI(a,w,s)/sy)**2
#calculate aeff, KI(aeff) until solution converges
KI_old = KI(a,w,s)
aeff = a + rp
KI_new = KI(aeff,w,s)
while ((KI_old-KI_new)/(KI_old))**2 > 0.00001:
    rp = 1.0/(I*np.pi)*(KI(aeff,w,s)/sy)**2
    aeff = a + rp
    KI_old = KI_new
    KI_new = KI(aeff,w,s)
print KI_old
print KI_new
```

```
49.97326249027704
50.01267113809021
```

And in plane strain we have $K_I = 50.0$ ksi$\sqrt{\text{in}}$
For $t = 0.25$, we can calculate $I$ directly using (6.7)

In [17]:
```
t=0.25
I = 6.7 - 1.5/t*(KI(a,w,s)/sy)**2
print I
```

```
3.290395949850567
```

We now proceed with the same solution method for $I = 3.29$

In [18]:
```
rp = 1.0/(I*np.pi)*(KI(a,w,s)/sy)**2
#calculate aeff, KI(aeff) until solution converges
KI_old = KI(a,w,s)
aeff = a + rp
KI_new = KI(aeff,w,s)
while ((KI_old-KI_new)/(KI_old))**2 > 0.00001:
    I = 6.7 - 1.5/t*(KI_new/sy)**2
    rp = 1.0/(I*np.pi)*(KI(aeff,w,s)/sy)**2
```

```
        aeff = a + rp
        KI_old = KI_new
        KI_new = KI(aeff,w,s)
    print KI_old
    print KI_new
```

```
51.083846403989384
51.139566264483655
```

```
In [19]: rp
```

```
Out[19]: 0.06566316998590092
```

As expected, we find $K_I$ somewhere between the plane strain and plane stress solutions, $K_I = 51.1\ \text{ksi}\sqrt{\text{in}}$

## 0.2 2

For an infinitely wide, center-cracked panel we use (2.1)

$$K_I = \sigma\sqrt{\pi a}$$

In plane strain, the plastic stress intensity factor, $K_{Ie}$ is given by

$$K_{Ie} = \sigma\sqrt{\pi(a + r_p)}$$

where (in plane strain)

$$r_p = \frac{1}{6\pi}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2$$

Substituting $r_p$ into $K_{Ie}$ gives

$$K_{Ie} = \sigma\sqrt{\pi\left(a + \frac{1}{6\pi}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2\right)}$$

We square both sides to find

$$K_{Ie}^2 = \sigma^2\pi\left(a + \frac{1}{6\pi}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2\right)$$

Multiplying out we get

$$K_{Ie}^2 = \sigma^2\pi a + \frac{\sigma^2}{6}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2$$

We can subtract the second term from both sides

$$K_{Ie}^2 - \frac{\sigma^2}{6}\left(\frac{K_{Ie}}{\sigma_{YS}}\right)^2 = \sigma^2\pi a$$

And simplify

$$K_{Ie}^2 \left(1 - \frac{\sigma^2}{6\sigma_{YS}^2}\right) = \sigma^2 \pi a$$

We can now divide both sides by $\left(1 - \frac{\sigma^2}{6\sigma_{YS}^2}\right)$ to find

$$K_{Ie}^2 = \frac{\sigma^2 \pi a}{1 - \frac{\sigma^2}{6\sigma_{YS}^2}}$$

We take the square root of both sides

$$K_{Ie} = \frac{\sigma\sqrt{\pi a}}{\sqrt{1 - \frac{\sigma^2}{6\sigma_{YS}^2}}}$$

We can now replace $\sigma\sqrt{\pi a}$ with $K_I$

$$K_{Ie} = \frac{K_I}{\sqrt{1 - \frac{\sigma^2}{6\sigma_{YS}^2}}}$$

And divide both sides by $K_I$

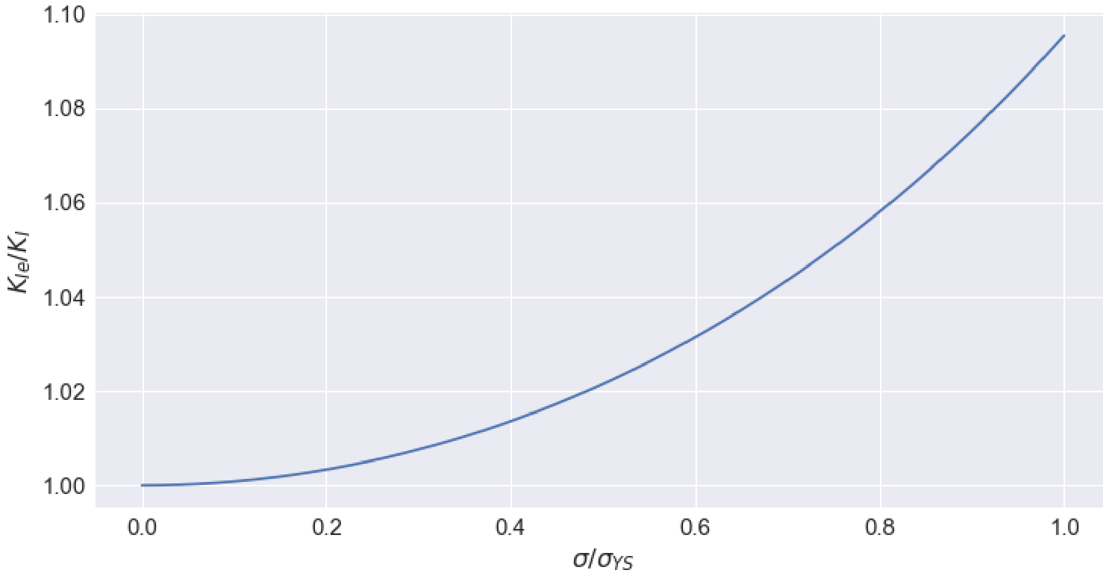
$$\frac{K_{Ie}}{K_I} = \frac{1}{\sqrt{1 - \frac{\sigma^2}{6\sigma_{YS}^2}}}$$

Now we are ready to generate our plot. Fracture mechanics is only valid when $\sigma < \sigma_{YS}$, so we consider $0 < \sigma < \sigma_{YS}$ for our plot.

```
In [20]: s_sys = np.linspace(0,1)
         KIe_KI = 1./(1.-s_sys**2/6.)**.5

         import matplotlib.pyplot as plt
         import seaborn as sb
         sb.set(font_scale=1.5)
         %matplotlib inline
         plt.figure(figsize=(12,6))
         plt.plot(s_sys,KIe_KI)
         plt.xlabel(r'$\sigma / \sigma_{YS}$')
         plt.ylabel(r'$K_{Ie} / K_I$')

Out[20]: Text(0,0.5,'$K_{Ie} / K_I$')
```




### 0.3 3

In this problem we are asked to find the ratio, $K_{Ie}/K_I$ for some specific conditions on a finite-width, center-cracked panel.

In this case we use (2.2a) for $K_I$ and we use (6.6) to find $r_p$, with $I = 2$ for plane stress and $I = 6$ for plane strain.

In [36]:
```
#4.12
def K_I(a,w,s):
    return s*np.sqrt(np.pi*a)*np.sqrt(1/np.cos(np.pi*a/w))

a = 1.
w = 7.
s = 45.
sy = 75.

#plane strain
KIa = K_I(a,w,s)
I = 6.
rp = 1.0/(I*np.pi)*(K_I(a,w,s)/sy)**2
#calculate aeff, KI(aeff) until solution converges
KI_old = K_I(a,w,s)
aeff = a + rp
KI_new = K_I(aeff,w,s)
while ((KI_old-KI_new)/(KI_old))**2 > 0.00001:
    rp = 1.0/(I*np.pi)*(K_I(aeff,w,s)/sy)**2
    aeff = a + rp
    KI_old = KI_new
```

```
            KI_new = K_I(aeff,w,s)
        print KI_old
        print KI_new
```

```
87.71532300053244
87.7391424415274
```

```
In [37]: print KI_new/KIa
```

```
1.0441450344891823
```

For plane strain we have $K_{Ie}/K_I = 1.04$

```
In [38]: #plane stress
        w=7.
        KIb = K_I(a,w,s)
        I = 2.
        rp = 1.0/(I*np.pi)*(K_I(a,w,s)/sy)**2
        #calculate aeff, KI(aeff) until solution converges
        KI_old = K_I(a,w,s)
        aeff = a + rp
        KI_new = K_I(aeff,w,s)
        while ((KI_old-KI_new)/(KI_old))**2 > 0.00001:
            rp = 1.0/(I*np.pi)*(K_I(aeff,w,s)/sy)**2
            aeff = a + rp
            KI_old = KI_new
            KI_new = K_I(aeff,w,s)
        print KI_old
        print KI_new
```

```
97.76029395762107
97.9830521282994
```

```
In [39]: print KI_new/KIb
```

```
1.1660533086705365
```

For plane stress with $W = 7$ we have $K_{Ie}/K_I = 1.17$

If the thickness of the panel was undecided, we can also plot the plasticity effect for varying thickness

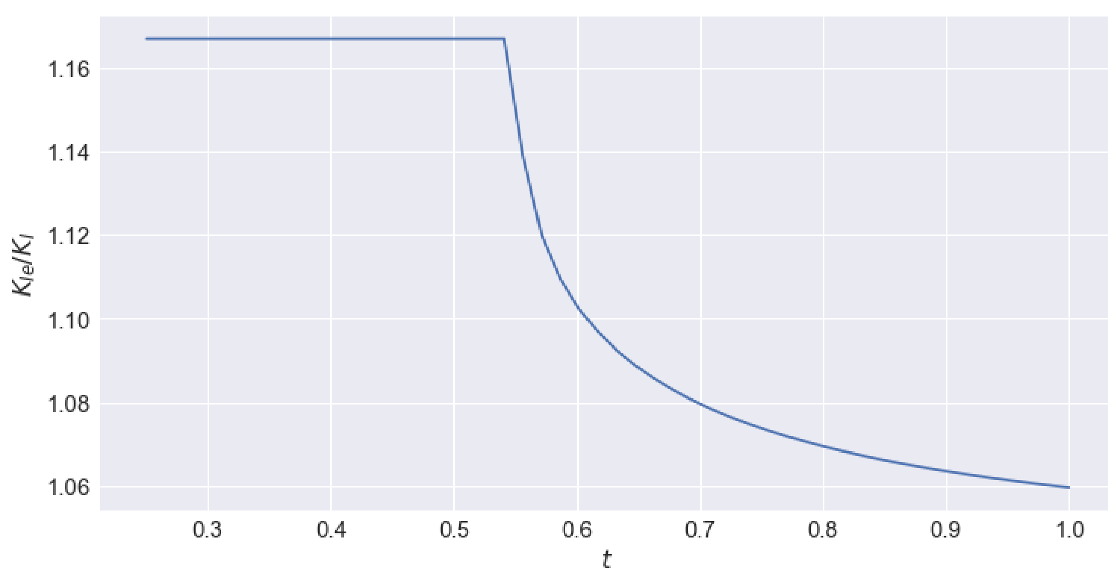
```
In [40]: t = np.linspace(1./4,1.)
        a = 1.
        W = 7.
        def calcI(t,K,sy):
```

```
        I = 6.7 - 1.5/t*(K/sy)**2
        for i in range(len(I)):
            if I[i] < 2.:
                I[i] = 2.
            elif I[i] > 6.:
                I[i] = 6.
        return I
    I = calcI(t,KI(a,W,s),sy)
    rp = 1.0/(I*np.pi)*(KI(a,w,s)/sy)**2
    #calculate aeff, KI(aeff) until solution converges
    KI_old = [K_I(a,w,s),0]
    aeff = a + rp
    KI_new = K_I(aeff,w,s)
    while ((max(KI_old)-max(KI_new))/(max(KI_old)))**2 > 0.00000000001:
        I = calcI(t,KI_new,sy)
        rp = 1.0/(I*np.pi)*(K_I(aeff,w,s)/sy)**2
        aeff = a + rp
        KI_old = KI_new
        KI_new = K_I(aeff,w,s)
    print max(KI_old)
    print max(KI_new)
```

```
98.0752187044578
98.07496495686982
```

```
In [41]: plt.figure(figsize=(12,6))
         plt.plot(t,KI_new/K_I(a,w,s))
         plt.xlabel(r'$t$')
         plt.ylabel(r'$K_{Ie} / K_I$')
```

```
Out[41]: Text(0,0.5,'$K_{Ie} / K_I$')
```



Here we see that the thicker the panel is, the lower the effect of plasticity. Panels less than 0.55" thick in this configuration are essentially in a state of plane stress.

### 0.4 4

First we calculate $K_I$ for the given plate using (2.4a)

```
In [27]: def beta(a,w):
             return 1.122 - 0.231*a/w + 10.55*(a/w)**2 - 21.71*(a/w)**3 + 30.82*(a/w)**4
         def KI(a,w,s):
             return s*np.sqrt(np.pi*a)*beta(a,w)
         a = 2.
         w = 6.
         s = 10. #ksi
         sy = 75. #ksi
         v = 0.3
         print KI(a,w,s)
```

```
44.95993689873641
```

For Von Mises yield theory in plane stress we have

$$r_p = \frac{K_I^2}{2\pi\sigma_{YS}^2}\cos^2\frac{\theta}{2}\left(1 + 3\sin^2\frac{\theta}{2}\right)$$

```
In [28]: th = np.linspace(0,2*np.pi,200)
         rp_vm_stress = KI(a,w,s)**2/(2*np.pi*sy**2)*np.cos(th/2)**2*(1+3*np.sin(th/2)**2)
```

For Von Mises yield theory in plane strain we have

$$r_p = \frac{K_I^2}{2\pi\sigma_{YS}^2}\cos^2\frac{\theta}{2}\left(1 - 4\nu + 4\nu^2 + 3\sin^2\frac{\theta}{2}\right)$$

```
In [29]: rp_vm_strain = KI(a,w,s)**2/(2*np.pi*sy**2)*np.cos(th/2)**2*(1-4*v+4*v**2+3*np.sin(th/2
```

For Tresca yield in plane stress we have

```
In [30]: rp_tr_stress = KI(a,w,s)**2/(2*np.pi*sy**2)*np.cos(th/2)**2*(1+np.sin(th/2))**2
```

For Tresca yield in plane strain we must first find $\theta_t$

```
In [31]: th1 = 2*np.arcsin(1-2*v)
```

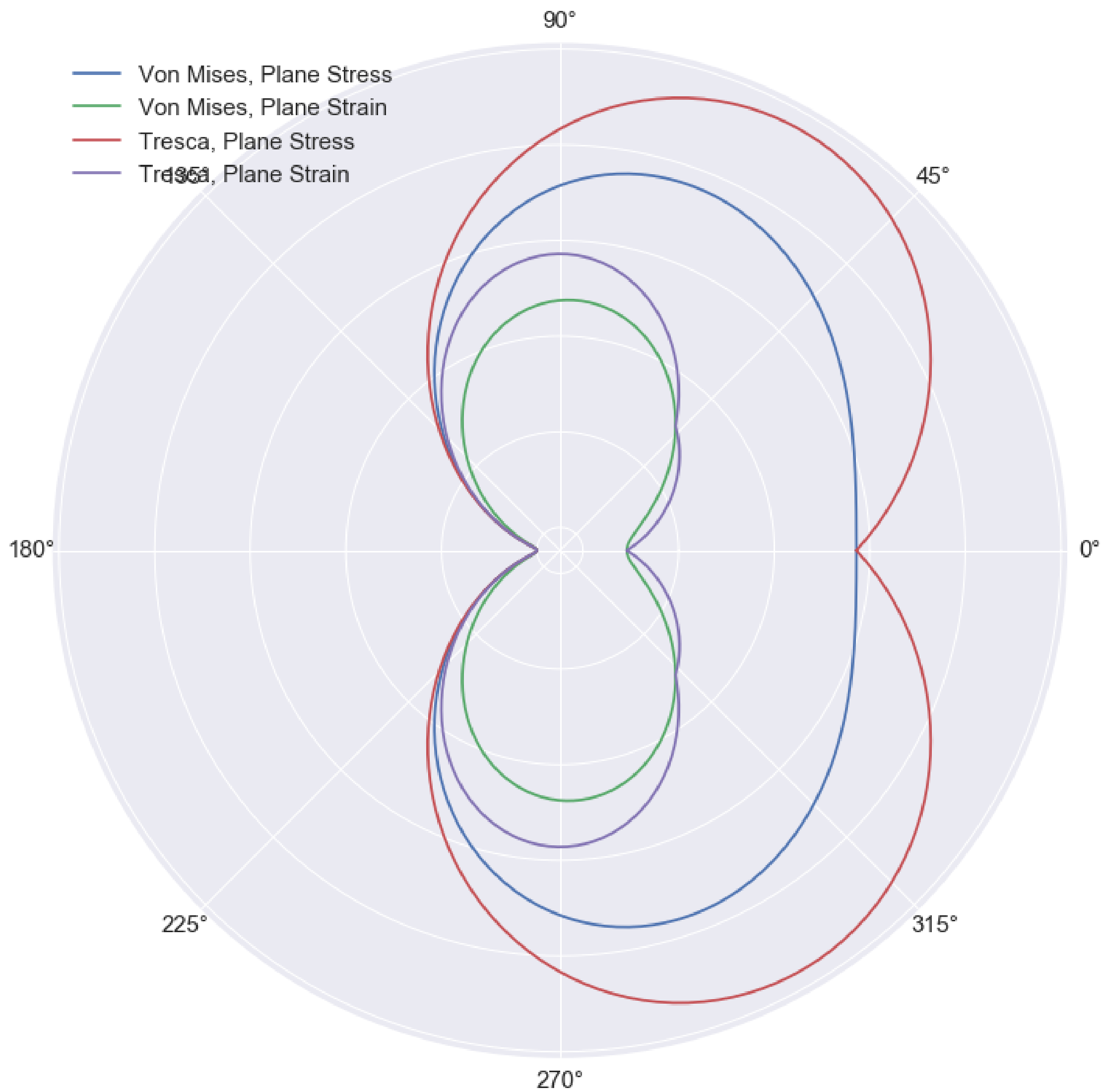
We then use the appropriate formulas, depending on whether $\theta_t < \theta < 2\pi - \theta_t$

```
In [32]: rp_tr_strain = np.zeros(len(th)) #initiate array of zeros
         for i in range(len(th)):
             if th[i] > th1 and th[i] < 2*np.pi - th1:
                 rp_tr_strain[i] = 2*KI(a,w,s)**2/(np.pi*sy**2)*np.cos(th[i]/2)**2*np.sin(th[i]/
             else:
                 rp_tr_strain[i] = KI(a,w,s)**2/(2*np.pi*sy**2)*np.cos(th[i]/2)**2*(1-2*v+np.sin
```

Now we make a polar plot to compare this plastic zone shapes

```
In [33]: fig = plt.figure(figsize=(12,12))
         ax = plt.subplot(111,projection='polar')
         ax.set_yticklabels([])
         ax.plot(th,rp_vm_stress,label='Von Mises, Plane Stress')
         ax.plot(th,rp_vm_strain,label='Von Mises, Plane Strain')
         ax.plot(th,rp_tr_stress,label='Tresca, Plane Stress')
         ax.plot(th,rp_tr_strain,label='Tresca, Plane Strain')
         ax.legend(loc='best')

Out[33]: <matplotlib.legend.Legend at 0xb47d4e0>
```


90°
45°
0°
315°
270°
225°
180°
135°
Von Mises, Plane Stress
Von Mises, Plane Strain
Tresca, Plane Stress
Tresca, Plane Strain