AE 737: Mechanics of Damage Tolerance

Lecture 17 - Crack Growth

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schedule

- 7 Apr - Crack growth
- 9 Apr - Boeing Method, HW7 Due
- 14 Apr - Cycle Counting
- 16 Apr - Crack retardation
- 21 Apr - Exam Review, HW8 Due
- 23 Apr - Exam 2

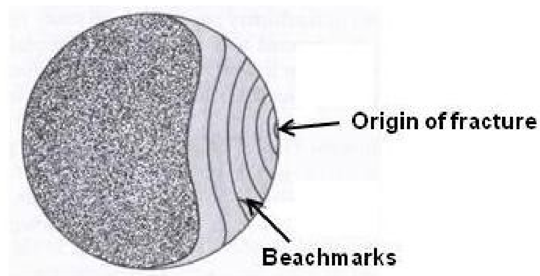
outline

- crack growth rate
- crack growth rate equations
- factors affecting crack propagation
- numerical algorithm

crack growth rate

fracture surface

fracture surface



Fatigue Fracture with Beachmarks

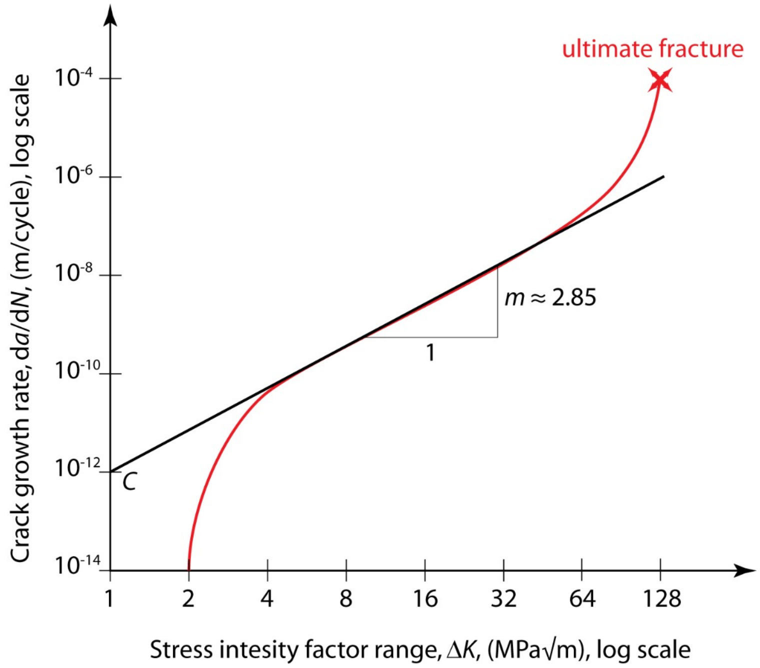
crack growth rate

- We can observe that fatigue damage occurs through crack propagation
- “cracks” and fracture mechanics have been omitted from all our fatigue discussion thus far
- It would be beneficial to predict at what rate a crack will extend

crack growth rate

- Crack growth rate can be measured experimentally
- Using a center-crack specimen, a fatigue load is applied
- The crack length is measured and plotted vs. the number of cycles
- The slope of this curve ($\frac{da}{dN}$) is then plotted vs. either K_{Imax} or ΔK_I on a log-log scale
- This chart is then commonly divided into three regions

da-dN vs K


ultimate fracture
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
C
$m \approx 2.85$
1
1
2
4
8
16
32
64
128
Crack growth rate, da/dN, (m/cycle), log scale
Stress intesity factor range, ΔK, (MPa√m), log scale

region I

- In Region I crack growth is very slow and/or difficult to measure
- In many cases, da/dN corresponds to the spacing between atoms!
- The point at which the da/dN curve intersects the x-axis (usually with a relatively vertical slope) is called the fatigue threshold
- Typically 3-15 $\text{ksi}\sqrt{\text{in}}$ for steel
- 3-6 $\text{ksi}\sqrt{\text{in}}$ for aluminum

region II

- Most important region for general engineering analysis
- Once a crack is present, most of the growth and life occurs in Region II
- Generally linear in the log-log scale

region III

- Unstable crack growth
- Usually neglected (we expect failure before Region III fully develops in actual parts)
- Can be significant for parts where we expect high stress and relatively short life

crack growth rate curve

- The crack growth rate curve is considered a material property
- The same considerations for thickness apply as with fracture toughness (K_c vs. K_{Ic})
- Is also a function of the load ratio, $R = \sigma_{min}/\sigma_{max}$

R effects

- While the x-axis can be either ΔK or K_{max}, the shape of the data is the same
- When we look at the effects of load ratio, R, the axis causes some differences on the plot
- With ΔK on the x-axis, increasing R will shift the curve up and to the left, shifting the fatigue threshold and fracture toughness on the graph as well

R effects

- With K_{max} on the x-axis, increasing R shifts the curve down and to the right, but fatigue threshold and fracture toughness keep same values
- In general, R dependence vanishes for $R > 0.8$ or $R < -0.3$. This effect is known as the band width

crack growth rate equations

crack growth rate equations

- There are many crack growth rate equations of varying complexity
- The “best” form to use will depend on design needs

growth equations

- The important features in curve-fit equations are
    1. Region II curve fit (linear on log-log scale)
    2. Region I curve fit (fatigue threshold)
    3. Region III curve fit (critical stress intensity)
    4. Stress ratio effects
    5. Band width of R-curves

paris law

- The original
- Fits the linear portion (Region II)
- Does not fit Region I, Region III, or have R-dependence

$$\frac{da}{dN} = C(\Delta K)^n$$

- Note: this assumes the x-axis is ΔK, but $\Delta K = (1 - R)K_{max}$, so we can easily convert

walker

- Region II is usually all that is needed for engineering, but R-dependence is often an important effect to capture
- Walker modified the Paris law to account for R-dependence

$$\frac{da}{dN} = C[(1-R)^m K_{max}]^n$$

- Gives a good fit for Region II with R-dependence and band width

forman

- The Forman equation was developed to capture the effects of Region II and Region III
- Also includes the effects of R, but does not control the band width of R effects

$$\frac{da}{dN} = \frac{C[(1-R)K_{max}]^n}{(1-R)K_c - (1-R)K_{max}}$$

modified forman

- The Forman equation can be modified to include the effect of band width

$$\frac{da}{dN} = \frac{C\left[(1-R)^m K_{max}\right]^n}{\left[(1-R)^m K_c - (1-R)^m K_{max}\right]^L}$$

collipriest

- The Collipriest equation fits Regions I, II and III, but has no R-dependence

$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1}\left[\frac{\log\left(\frac{K_{max}^2}{K_o K_c}\right)}{\log(K_c/K_o)}\right]$$

modified collipriest

- Following the same methods as before, we can modify the Collipriest equation for R-dependence and band width control

$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1}\left[\frac{\log\left(\frac{(1-R)^m K_{max}^2}{K_o K_c}\right)}{\log(K_c/K_o)}\right]$$

- For a cleaner graph, experimental data at different R-values is sometimes plotted vs. K_{eff}
- $K^*_{eff} = (1 - R)^m K_{max}$

nasgrow growth rate equation

- A very complicated curve fit is provided in the NASGROW growth rate equation

$$\frac{da}{dN} = C\left[\frac{1-f}{1-R}\Delta K\right]^n \frac{\left[1-\frac{\Delta K_{th}}{\Delta K}\right]}{\left[1-\frac{K_{max}}{K_{crit}}\right]}$$

- The curve fit parameters can be found in p. 307 of your text (or the NASGROLW/AFGROW documentation)

boeing-walker growth rate equation

- The Boeing-Walker growth equation is given as (for $R \geq 0$)

$$\frac{da}{dN} = 10^{-4}\left(\frac{1}{mT}\right)^{p}[K_{max}(1-R)^{q}]^{p}$$

conversion of constants

- Much of the data available to us is from Boeing, and given in terms of the Boeing-Walker equation
- We can re-write some other equations to more easily convert parameters between the various equations
- Walker-Boeing:

$$\frac{da}{dN} = 10^{-4}\left(\frac{1}{mT}\right)^{p}\left[\Delta K(1-R)^{q-1}\right]^{p}$$

- Walker-AFGROW:

$$\frac{da}{dN} = C_w\left[\Delta K(1-R)^{m-1}\right]^{n_w}$$

- Forman:

$$\frac{da}{dN} = \frac{C_F}{(1-R)K_c - \Delta K}(\Delta K)^{n_f}$$

conversion of constants

Walker-Boeing	Walker-AFGROW	Forman
$10^{-4}\left(\frac{1}{mT}\right)^p$	$C_w = 10^{-4}\left(\frac{1}{mT}\right)^p$	$C_F = (K_c - 1)10^{-4}\left(\frac{1}{mT}\right)^p$
q	$m = q$	
p	$n_w = p$	$n_f = p$

paris example

- A wide center-cracked panel with $C = 6.75 \times 10^{-10}$ and $n = 3.89$ (with units in ksi and inches)
- If the crack is initially 1 inch long, find the crack length after 5,000 cycles of 15 ksi loading
- What if the load cycles varied from 5 ksi to 15 ksi? ($m = 0.6$)

factors affecting crack propagation

factors affecting crack propagation

- thickness
- stress ratio
- temperature
- environment
- frequency
- crack orientation
- manufacturer
- heat treatment

thickness

- We already discussed the effects of thickness on fracture toughness
- The same effects are important in crack propagation
- In thin (plane stress) plates, cracks can be treated as through cracks
- In thick plates (plain strain), we generally need to consider the crack shape

thickness

- Cyclic life is primarily a function of K_i/K_c where K_i is the stress intensity factor in the first cycle
- Other experiments indicate a relationship between $\frac{d(a/Q)}{dN}$ and K_{max}
- Q is a shape parameter for elliptical flaws

temperature

- In general (for most aluminum alloys) cracks propagate more slowly with a decrease in temperature
- This trend is exactly opposite the trend for K_c
- The effect varies in different materials
- Most materials benefit from slightly lower temperatures, but as temperatures are further decreased the crack growth rate increases again

temperature

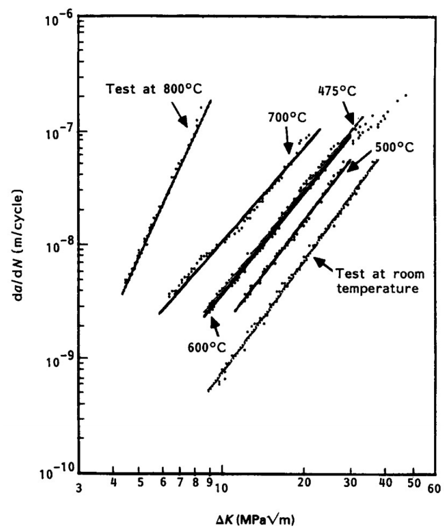


Fig. 2 Mid-range fatigue crack-growth rates with alternating stress intensity factor for 18%Cr–Nb ferritic stainless steel at room temperature, 475, 500, 600, 700 and 800°C; $R = 0.1$, $f = 15$ Hz

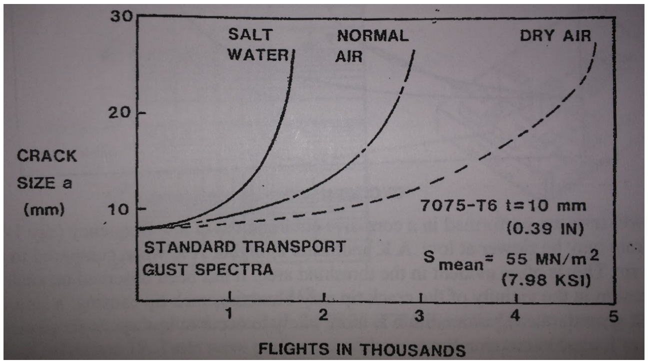
temperature

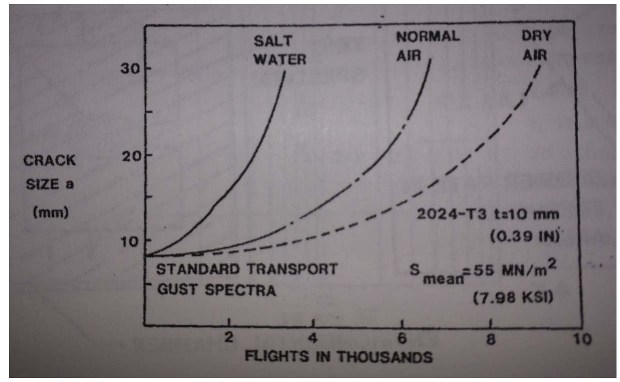
- In general, temperature effects can not be predicted well
- Instead, materials should be tested at a range of temperatures to establish a range of operating temperatures with corresponding crack growth data

environment

- There are many conditions in the environment that can affect crack growth
- Moisture greatly increases the crack growth rate
- Salt water increases crack growth rate even further
- These effects have varying strength depending on the material used

environment





environment

- Further, the shape of the applied load curve has a significant effect when combined with adverse environments
- Crack growth is faster when the load increases slowly and decreases rapidly
- Crack growth is slower when the load increases rapidly and decreases slowly

environment

- When the environment is corrosive, the test frequency is of particular importance
- At low frequencies, a corrosive environment increases the threshold, K^{th}
- However in Region II, crack growth is faster
- This effect can be explained by the corrosive environment blunting the crack tip

frequency

- There is conflicting information about the effect of frequency in the absence of a corrosive environment
- Some experiments have found a frequency dependence, while others have not
- Many claim that the frequency dependence is due to small amounts of water in air during frequency dependence experiment

crack orientation

- For rolled plates, a crack will generally propagate faster parallel to the rolling direction
- In many materials, however, the difference between orientations is not significant when compared to scatter, and it is often neglected
- Some materials behave very differently with different crack orientations (i.e. the slope of the paris law curve is different), so care should be taken based on the material used

manufacturer

- Different manufacturers of the same material can produce different crack growth rates
- Some reasons for this may be
    - Slight variation in composition
    - Site cleanliness (inclusions)
    - Heat treatment/cold rolling variations

heat and surface treatments

- Different heat and surface treatments are often applied
- They provide various benefits (corrosion resistance, residual stress, residual stress relief)
- But they will also affect the crack growth rate

numerical algorithm

numerical algorithm

- While the Paris Law can be integrated directly (for simple load cases), many of the other formulas cannot
- A simple numerical algorithm for determining incremental crack growth is

$$a_{i+1} = a_i + \left(\frac{da}{dN}\right)_i (\Delta N)_i$$

- This method is quite tedious by hand (need many a_i values for this to be accurate)
- But is simple to do in Excel, MATLAB, Python, or many other codes
- For most accurate results, use $\Delta N = 1$, but this is often unnecessary
- When trying to use large ΔN, check convergence by using larger and smaller Δ**N values

boeing-walker example

- Use the Boeing-Walker equation to find the crack length after 20000 cycles of 15 ksi load on a large, center-cracked sheet of bare 2024-T3 in dry air, with an initial crack of 0.5”
- Use the numerical algorithm with ΔN = 1000

convergence example

- compare the results from the previous example with $\Delta N = 10, 100, 10000$ and direct integration

variable load cases

- In practice variable loads are often seen
- The most basic way to handle these is to simply calculate the crack length after each block of loading
- We will discuss an alternate method, which is more convenient for flight “blocks” next class
- We will also discuss “retardation” models next class

variable load example

- For the same material as above (2024-T3, center-cracked, dry air), consider 20000 cycles with 15 ksi load followed by 10000 cycles of 5 - 20 ksi.