

AE 737 - MECHANICS OF DAMAGE TOLERANCE

LECTURE 19

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Last Updated: April 6, 2016 at 9:28am

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SCHEDULE

- 5 Apr - Crack Growth, Homework 7 due, Homework 8 assigned
- 7 Apr - Crack Growth, Stress Spectrum
- 12 Apr - Retardation, Boeing Commercial Method
- 14 Apr - Exam Review, Homework 8 Due
- 19 Apr - Damage Tolerance
- 21 Apr - Exam 2

FINAL PROJECT CLARIFICATION

- Some of the wording I used in the final project description is ambiguous, and based on what I learned in my fatigue class (not your text)
- By "crack growth" I intended "crack nucleation," which is a phrase to describe stress and strain based fatigue analysis
- "Crack propagation" is what I intended by what your text calls "crack growth", and refers to fracture mechanics-based fatigue analysis

- I have a meeting this Friday afternoon (4/8)
- Office hours will be Monday 4/11 from 3:00 - 5:00
- As always you can e-mail me to schedule another time, or ask your questions via e-mail

1. mean stress effects
2. multiaxial loading
3. crack growth rate
4. crack growth rate equations

MEAN STRESS EFFECTS

- In regions where plastic strain is significant, some applied mean stress is likely to be relaxed through cyclic plastic strain
- When the plastic strain is not significant, mean stress will exist
- Mean strain does not generally affect fatigue life

- Recall the Morrow approach for mean stress effects from the stress-based method

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma'_f} = 1 \quad (19.1)$$

- We can rearrange (and substitute) the equation such that

$$\sigma_a = \sigma'_f \left[\left(1 - \frac{\sigma_m}{\sigma'_f} \right)^{\frac{1}{b}} (2N_f) \right]^b \quad (19.2)$$

- If we compare to the stress-life equation ($\sigma_a = \sigma'_f(2N_f)^b$), we see that we can replace N_f with

$$N^* = N_f \left(1 - \frac{\sigma_m}{\sigma'_f} \right)^{\frac{1}{b}} \quad (19.3)$$

- We can now substitute N^* for N_f in the strain-life equation to find

$$\epsilon_a = \frac{\sigma'_f}{E} \left(1 - \frac{\sigma_m}{\sigma'_f} \right) (2N_f)^b + \epsilon'_f \left(1 - \frac{\sigma_m}{\sigma'_f} \right)^{\frac{c}{b}} (2N_f)^c \quad (19.4)$$

- Graphically, we can use the Morrow approach very easily using only the zero-mean stress graph
- From the zero-mean stress graph, find the point corresponding to your applied strain
- For a non zero mean stress, this point represents (ϵ_a, N^*) , we can now solve for N_f using 19.3

- While the Morrow equation agrees very well with many data, some are better fit with a modification
- In the modified version, it is assumed that the mean stress has no effect on the plastic term

$$\epsilon_a = \frac{\sigma'_f}{E} \left(1 - \frac{\sigma_m}{\sigma'_f} \right) (2N_f)^b + \epsilon'_f (2N_f)^c \quad (19.5)$$

- There is no convenient solution method for this form, and it generally must be solved numerically, or plotted with many families of σ_m

- The Smith, Watson, and Topper approach assumes that the life for any given state is dependent on the product $\sigma_{max}\epsilon_a$
- After some manipulation, this gives

$$\sigma_{max}\epsilon_a = \frac{(\sigma'_f)^2}{E} (2N_f)^{2b} + \sigma'_f \epsilon'_f (2N_f)^{b+c} \quad (19.6)$$

- This method can also be solved graphically if a plot of $\sigma_{max}\epsilon_a$ is made using zero-mean data. All we need to do is find the new $\sigma_{max}\epsilon_a$ point to find a new N_f

- All three methods discussed are in general use
- The Morrow method is very good for steel
- The modified Morrow method gives improved results in many materials
- The SWT approach is very good for general use, but is non-conservative with a compressive mean stress

MULTIAXIAL LOADING

- Multi-axial loading in strain-based fatigue analysis is still an active field of research
- We are currently only capable of handling proportional loads that are in-phase (i.e. have the same frequency)
- If we consider the principal directions where $\sigma_{2a} = \lambda\sigma_{1a}$, we find an expression for the strain-life as

$$\epsilon_{1a} = \frac{\frac{\sigma'_f}{E}(1 - \nu\lambda)(2N_f)^b + \epsilon'_f(1 - 0.5\lambda)(2N_f)^c}{\sqrt{1 - \lambda + \lambda^2}} \quad (19.7)$$

- Another approach is to consider the stress triaxiality factor

$$T = \frac{1 + \lambda}{\sqrt{1 - \lambda + \lambda^2}} \quad (19.8)$$

- Three notable cases of this are
 1. Pure planar shear: $\lambda = -1, T = 0$
 2. Uniaxial stress: $\lambda = 0, T = 1$
 3. Equal biaxial stress: $\lambda = 1, T = 2$
- Marloff suggests the following inclusion of stress triaxiality

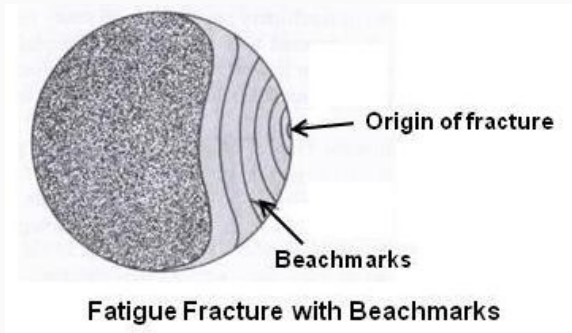
$$\bar{\epsilon}_a = \frac{\sigma'_f}{E} (2N_f)^b + 2^{1-T} \epsilon'_f (2N_f)^c \quad (19.9)$$

CRACK GROWTH RATE

FRACTURE SURFACE

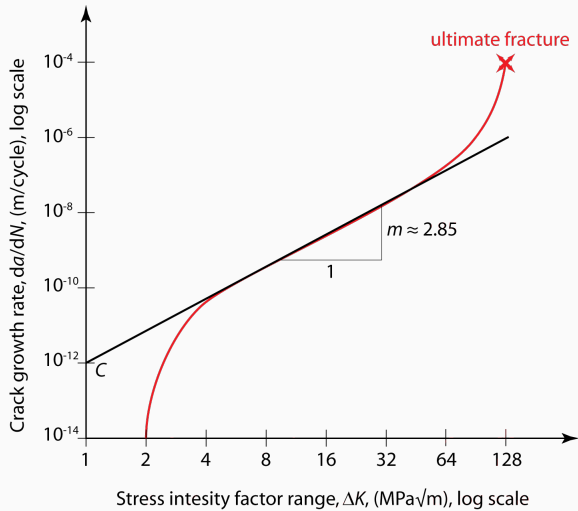


FRACTURE SURFACE



- We can observe that fatigue damage occurs through crack propagation
- "cracks" and fracture mechanics have been omitted from all our fatigue discussion thus far
- It would be beneficial to predict at what rate a crack will extend

- Crack growth rate can be measured experimentally
- Using a center-crack specimen, a fatigue load is applied
- The crack length is measured and plotted vs. the number of cycles
- The slope of this curve ($\frac{da}{dN}$) is then plotted vs. either $K_{I,max}$ or ΔK_I on a log-log scale
- This chart is then commonly divided into three regions



- In Region I crack growth is very slow and/or difficult to measure
- In many cases, da/dN corresponds to the spacing between atoms!
- The point at which the da/dN curve intersects the boundary between Region I and Region II is often called the fatigue threshold
- Typically $3-15 \text{ ksi}\sqrt{\text{in}}$ for steel
- $3-6 \text{ ksi}\sqrt{\text{in}}$ for aluminum

- Most important region for general engineering analysis
- Once a crack is present, most of the growth and life occurs in Region II
- Generally linear in the log-log scale

- Unstable crack growth
- Usually neglected (we expect failure before Region III fully develops in actual parts)
- Can be significant for parts where we expect high stress and relatively short life

- The crack growth rate curve is considered a material property
- The same considerations for thickness apply as with fracture toughness (K_c vs. K_{Ic})
- Is also a function of the load ratio, $R = \sigma_{min}/\sigma_{max}$

- While the x-axis can be either ΔK or K_{max} , the shape of the data is the same
- When we look at the effects of load ratio, R , the axis causes some differences on the plot
- With ΔK on the x-axis, increasing R will shift the curve up and to the left, shifting the fatigue threshold and fracture toughness on the graph as well
- With K_{max} on the x-axis, increasing R shifts the curve down and to the right, but fatigue threshold and fracture toughness keep same values
- In general, R dependence vanishes for $R > 0.8$ or $R < -0.3$. This effect is known as the band width

CRACK GROWTH RATE EQUATIONS

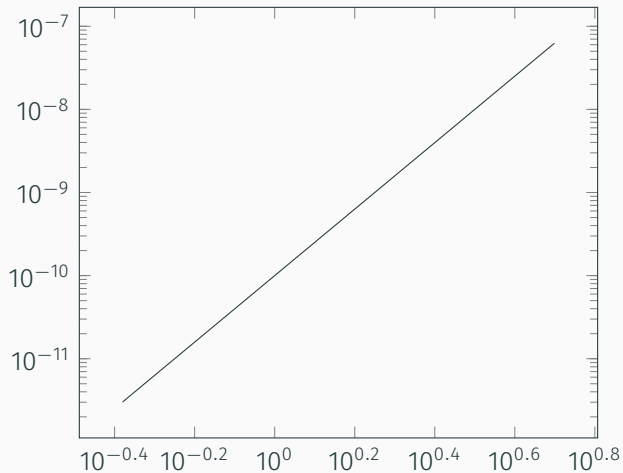
- There are many crack growth rate equations of varying complexity
- The "best" form to use will depend on design needs
- The important features in curve-fit equations are
 1. Region II curve fit (linear on log-log scale)
 2. Region I curve fit (fatigue threshold)
 3. Region III curve fit (critical stress intensity)
 4. Stress ratio effects
 5. Band width of R-curves

- The original
- Fits the linear portion (Region II)
- Does not fit Region I, Region III, or have R-dependence

$$\frac{da}{dN} = C(\Delta K)^n \quad (19.10)$$

- Note: this assumes the x-axis is ΔK , but $\Delta K = (1 - R)K_{max}$, so we can easily convert

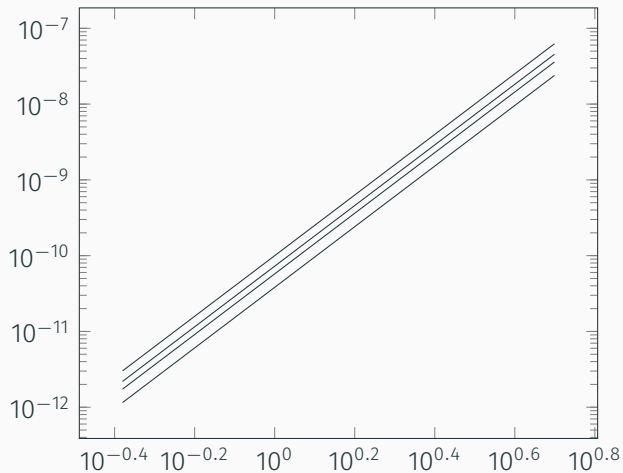
PARIS LAW



- Region II is usually all that is needed for engineering, but R-dependence is often an important effect to capture
- Walker modified the Paris law to account for R-dependence

$$\frac{da}{dN} = C [(1 - R)^m K_{max}]^n \quad (19.11)$$

- Gives a good fit for Region II with R-dependence and band width



- The Forman equation was developed to capture the effects of Region II and Region III
- Also includes the effects of R , but does not control the bandwidth of R effects

$$\frac{da}{dN} = \frac{C [(1 - R)K_{max}]^n}{(1 - R)K_c - (1 - R)K_{max}} \quad (19.12)$$

- The Forman equation can be modified to include the effect of band width

$$\frac{da}{dN} = \frac{C [(1 - R)^m K_{max}]^n}{[(1 - R)^m K_c - (1 - R)^m K_{max}]^L} \quad (19.13)$$

- The Collipriest equation fits Regions I, II and III, but has no R-dependence

$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1} \left[\frac{\log \left(\frac{K_{max}^2}{K_o K_c} \right)}{\log(K_c/K_o)} \right] \quad (19.14)$$

- Following the same methods as before, we can modify the Collipriest equation for R-dependence and band width control

$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1} \left[\frac{\log \left(\frac{(1-R)^m K_{max}^2}{K_o K_c} \right)}{\log(K_c/K_o)} \right] \quad (19.15)$$

- For a cleaner graph, experimental data at different R-values is sometimes plotted vs. K_{eff}

$$K_{eff} = (1 - R)^m K_{max} \quad (19.16)$$

NASGROW GROWTH RATE EQUATION

- A very complicated curve fit is provided in the NASGROW growth rate equation

$$\frac{da}{dN} = C \left[\frac{1-f}{1-R} \Delta K \right]^n \frac{\left[1 - \frac{\Delta K_{th}}{\Delta K} \right]}{\left[1 - \frac{K_{max}}{K_{crit}} \right]} \quad (19.17)$$

- The curve fit parameters can be found in p. 307 of your text (or the NASGROW/AFGROW documentation)

- The Boeing-Walker growth equation is given as (for $R \geq 0$)

$$\frac{da}{dN} = 10^{-4} \left(\frac{1}{mT} \right)^p [K_{max}(1 - R)^q]^p \quad (19.18)$$

CONVERSION OF CONSTANTS

- Much of the data available to us is from Boeing, and given in terms of the Boeing-Walker equation
- We can re-write some other equations to more easily convert parameters between the various equations
- Walker-Boeing:

$$\frac{da}{dN} = 10^{-4} \left(\frac{1}{mT} \right)^p [\Delta K (1 - R)^{q-1}]^p \quad (19.19)$$

- Walker-AFGROW:

$$\frac{da}{dN} = C_w [\Delta K (1 - R)^{m-1}]^{n_w} \quad (19.20)$$

- Forman:

$$\frac{da}{dN} = \frac{C_F}{(1 - R)K_c - \Delta K} (\Delta K)^{n_f} \quad (19.21)$$

CONVERSION OF CONSTANTS

Walker-Boeing	Walker-AFGROW	Forman
$10^{-4} \left(\frac{1}{mT}\right)^p$	$C_w = 10^{-4} \left(\frac{1}{mT}\right)^p$	$C_F = (K_c - 1)10^{-4} \left(\frac{1}{mT}\right)^p$
q	$m = q$	
p	$n_w = p$	$n_f = p$