

## Lecture 17 - Crack Growth

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## schedule

- 29 Mar - Crack Growth
- 31 Mar - Boeing Method, HW 6 Due
- 5 Apr - Cycle counting
- 7 Apr - Crack Retardation, HW 7 Due, HW 6 Self-grade Due

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- crack growth rate
- crack growth rate equations
- factors affecting crack propagation
- numerical algorithm

## crack growth rate

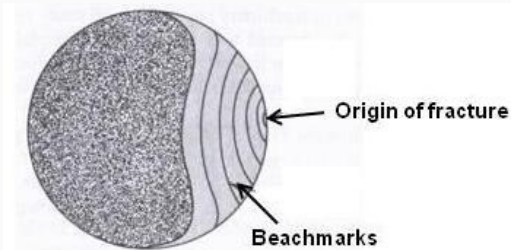
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## fracture surface



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## fracture surface



**Fatigue Fracture with Beachmarks**

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## crack growth rate

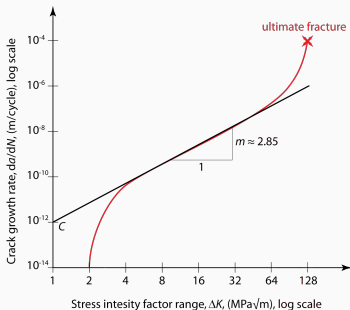
- We can observe that fatigue damage occurs through crack propagation
- “cracks” and fracture mechanics have been omitted from all our fatigue discussion thus far
- It would be beneficial to predict at what rate a crack will extend

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## crack growth rate

- Crack growth rate can be measured experimentally
- Using a center-crack specimen, a fatigue load is applied
- The crack length is measured and plotted vs. the number of cycles
- The slope of this curve ( $\frac{da}{dN}$ ) is then plotted vs. either  $K_{I\max}$  or  $\Delta K_I$  on a log-log scale
- This chart is then commonly divided into three regions

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## region I

- In Region I crack growth is very slow and/or difficult to measure
- In many cases,  $da/dN$  corresponds to the spacing between atoms!
- The point at which the  $da/dN$  curve intersects the x-axis (usually with a relatively vertical slope) is called the fatigue threshold
- Typically 3-15 ksi $\sqrt{in}$  for steel
- 3-6 ksi $\sqrt{in}$  for aluminum

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## region II

- Most important region for general engineering analysis
- Once a crack is present, most of the growth and life occurs in Region II
- Generally linear in the log-log scale

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## region III

- Unstable crack growth
- Usually neglected (we expect failure before Region III fully develops in actual parts)
- Can be significant for parts where we expect high stress and relatively short life

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- The crack growth rate curve is considered a material property
- The same considerations for thickness apply as with fracture toughness ( $K_c$  vs.  $K_{Ic}$ )
- Is also a function of the load ratio,  $R = \sigma_{min}/\sigma_{max}$

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## R effects

- While the x-axis can be either  $\Delta K$  or  $K_{max}$ , the shape of the data is the same
- When we look at the effects of load ratio,  $R$ , the axis causes some differences on the plot
- With  $\Delta K$  on the x-axis, increasing  $R$  will shift the curve up and to the left, shifting the fatigue threshold and fracture toughness on the graph as well

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- With  $K_{max}$  on the x-axis, increasing  $R$  shifts the curve down and to the right, but fatigue threshold and fracture toughness keep same values
- In general,  $R$  dependence vanishes for  $R > 0.8$  or  $R < -0.3$ . This effect is known as the band width

## crack growth rate equations

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## crack growth rate equations

- There are many crack growth rate equations of varying complexity
- The “best” form to use will depend on design needs

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## growth equations

- The important features in curve-fit equations are
  1. Region II curve fit (linear on log-log scale)
  2. Region I curve fit (fatigue threshold)
  3. Region III curve fit (critical stress intensity)
  4. Stress ratio effects
  5. Band width of R-curves

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- The original
- Fits the linear portion (Region II)
- Does not fit Region I, Region III, or have R-dependence

$$\frac{da}{dN} = C(\Delta K)^n$$

- Note: this assumes the x-axis is  $\Delta K$ , but  $\Delta K = (1 - R)K_{max}$ , so we can easily convert

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## walker

- Region II is usually all that is needed for engineering, but R-dependence is often an important effect to capture
- Walker modified the Paris law to account for R-dependence

$$\frac{da}{dN} = C [(1 - R)^m K_{max}]^n$$

- Gives a good fit for Region II with R-dependence and band width

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- The Forman equation was developed to capture the effects of Region II and Region III
- Also includes the effects of  $R$ , but does not control the band width of  $R$  effects

$$\frac{da}{dN} = \frac{C [(1 - R)K_{max}]^n}{(1 - R)K_c - (1 - R)K_{max}}$$

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## modified forman

- The Forman equation can be modified to include the effect of band width

$$\frac{da}{dN} = \frac{C [(1 - R)^m K_{max}]^n}{[(1 - R)^m K_c - (1 - R)^m K_{max}]^L}$$

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- The Collipriest equation fits Regions I, II and III, but has no R-dependence

$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1} \left[ \frac{\log \left( \frac{K_{max}^2}{K_o K_c} \right)}{\log(K_c/K_o)} \right]$$

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## modified collipriest

- Following the same methods as before, we can modify the Collipriest equation for R-dependence and band width control

$$\frac{da}{dN} = C_1 + C_2 \tanh^{-1} \left[ \frac{\log \left( \frac{(1-R)^m K_{max}^2}{K_o K_c} \right)}{\log(K_c/K_o)} \right]$$

- For a cleaner graph, experimental data at different R-values is sometimes plotted vs.  $K_{eff}$

$$K_{eff} = (1 - R)^m K_{max}$$

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## nasgrow growth rate equation

- A very complicated curve fit is provided in the NASGROW growth rate equation

$$\frac{da}{dN} = C \left[ \frac{1-f}{1-R} \Delta K \right]^n \frac{\left[ 1 - \frac{\Delta K_{th}}{\Delta K} \right]}{\left[ 1 - \frac{K_{max}}{K_{crit}} \right]}$$

- The curve fit parameters can be found in p. 307 of your text (or the NASGROLW/AFGROW documentation)

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## boeing-walker growth rate equation

- The Boeing-Walker growth equation is given as (for  $R \geq 0$ )

$$\frac{da}{dN} = 10^{-4} \left( \frac{1}{mT} \right)^P [K_{max}(1-R)^q]^P$$

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## conversion of constants

- Much of the data available to us is from Boeing, and given in terms of the Boeing-Walker equation
- We can re-write some other equations to more easily convert parameters between the various equations
- Walker-Boeing:

$$\frac{da}{dN} = 10^{-4} \left( \frac{1}{mT} \right)^p \left[ \Delta K (1 - R)^{q-1} \right]^p$$

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## conversion of constants

- Walker-AFGROW:

$$\frac{da}{dN} = C_w \left[ \Delta K (1 - R)^{m-1} \right]^{n_w}$$

- Forman:

$$\frac{da}{dN} = \frac{C_F}{(1 - R)K_c - \Delta K} (\Delta K)^{n_f}$$

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Walker-Boeing	Walker-AFGROW	Forman
$10^{-4} \left( \frac{1}{mT} \right)^P$	$C_w = 10^{-4} \left( \frac{1}{mT} \right)^P$	$C_F =$ $(K_c - 1)10^{-4} \left( \frac{1}{mT} \right)^P$
$q$	$m = q$	
$p$	$n_w = p$	$n_f = p$

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## paris example

- A wide center-cracked panel with  $C = 6.75 \times 10^{-10}$  and  $n = 3.89$  (with units in ksi and inches)
- If the crack is initially 1 inch long, find the crack length after 5,000 cycles of 15 ksi loading
- What if the load cycles varied from 5 ksi to 15 ksi? ( $m = 0.6$ )

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## factors affecting crack propagation

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### factors affecting crack propagation

- thickness
- stress ratio
- temperature
- environment
- frequency
- crack orientation
- manufacturer
- heat treatment



- We already discussed the effects of thickness on fracture toughness
- The same effects are important in crack propagation
- In thin (plane stress) plates, cracks can be treated as through cracks
- In thick plates (plain strain), we generally need to consider the crack shape

- Cyclic life is primarily a function of  $K_i/K_c$  where  $K_i$  is the stress intensity factor in the first cycle
- Other experiments indicate a relationship between  $\frac{d(a/Q)}{dN}$  and  $K_{max}$
- $Q$  is a shape parameter for elliptical flaws

- In general (for most aluminum alloys) cracks propagate more slowly with a decrease in temperature
- This trend is exactly opposite the trend for  $K_{Ic}$
- The effect varies in different materials
- Most materials benefit from slightly lower temperatures, but as temperatures are further decreased the crack growth rate increases again

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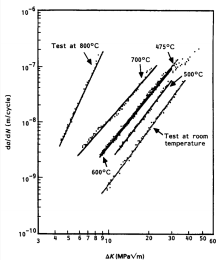


Fig. 2 Mid-range fatigue crack-growth rates with alternating stress intensity factor for 18%Cr-Nb ferritic stainless steel at room temperature, 475, 500, 600, 700 and 800°C;  $R = 0.1$ ,  $f = 16$  Hz

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## temperature

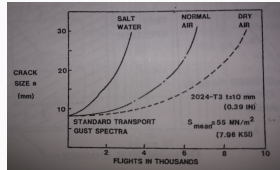
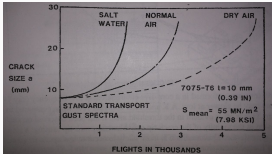
- In general, temperature effects can not be predicted well
- Instead, materials should be tested at a range of temperatures to establish a range of operating temperatures with corresponding crack growth data

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## environment

- There are many conditions in the environment that can affect crack growth
- Moisture greatly increases the crack growth rate
- Salt water increases crack growth rate even further
- These effects have varying strength depending on the material used

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- Further, the shape of the applied load curve has a significant effect when combined with adverse environments
- Crack growth is faster when the load increases slowly and decreases rapidly
- Crack growth is slower when the load increases rapidly and decreases slowly

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- When the environment is corrosive, the test frequency is of particular importance
- At low frequencies, a corrosive environment increases the threshold,  $K_{th}$
- However in Region II, crack growth is faster
- This effect can be explained by the corrosive environment blunting the crack tip

- There is conflicting information about the effect of frequency in the absence of a corrosive environment
- Some experiments have found a frequency dependence, while others have not
- Many claim that the frequency dependence is due to small amounts of water in air during frequency dependence experiment

## crack orientation

- For rolled plates, a crack will generally propagate faster parallel to the rolling direction
- In many materials, however, the difference between orientations is not significant when compared to scatter, and it is often neglected
- Some materials behave very differently with different crack orientations (i.e. the slope of the paris law curve is different), so care should be taken based on the material used

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## manufacturer

- Different manufacturers of the same material can produce different crack growth rates
- Some reasons for this may be
  - Slight variation in composition
  - Site cleanliness (inclusions)
  - Heat treatment/cold rolling variations

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- Different heat and surface treatments are often applied
- They provide various benefits (corrosion resistance, residual stress, residual stress relief)
- But they will also affect the crack growth rate

## numerical algorithm

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- While the Paris Law can be integrated directly (for simple load cases), many of the other formulas cannot
- A simple numerical algorithm for determining incremental crack growth is

$$a_{i+1} = a_i + \left( \frac{da}{dN} \right)_i (\Delta N)_i$$

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- This method is quite tedious by hand (need many  $a_i$  values for this to be accurate)
- But is simple to do in Excel, MATLAB, Python, or many other codes
- For most accurate results, use  $\Delta N = 1$ , but this is often unnecessary
- When trying to use large  $\Delta N$ , check convergence by using larger and smaller  $\Delta N$  values

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## boeing-walker example

- Use the Boeing-Walker equation to find the crack length after 20000 cycles of 15 ksi load on a large, center-cracked sheet of bare 2024-T3 in dry air, with an initial crack of 0.5"
- Use the numerical algorithm with  $\Delta N = 1000$

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## convergence example

- compare the results from the previous example with  $\Delta N = 10, 100, 1000$  and direct integration

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## variable load cases

- In practice variable loads are often seen
- The most basic way to handle these is to simply calculate the crack length after each block of loading
- We will discuss an alternate method, which is more convenient for flight “blocks” next class
- We will also discuss “retardation” models next class

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## variable load example

- For the same material as above (2024-T3, center-cracked, dry air), consider 20000 cycles with 15 ksi load followed by 10000 cycles of 5 - 20 ksi.

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