AE 737 - MECHANICS OF DAMAGE TOLERANCE

Lecture 11

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SCHEDULE

- 25 Feb - Multiple Site Damage, Mixed-mode Fracture, Homework 4 Due, Homework 5 Assigned
- 1 Mar - Section 1 Review, Homework 5 Due
- 3 Mar - Section 1 Review, Homework 5 return
- 8 Mar - Exam 1
- 10 Mar - Exam return, Final Project discussion

a vs. a_{eff}

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- Ductile/tough material vs. brittle/stiff material?
- Plane stress vs. plane strain?
- Charts/FE data

OUTLINE

1. stiffener review
2. multiple site damage
3. mixed mode fracture

STIFFENER REVIEW

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- As cracks get long, the relative difference between a and a_{eff} is minor
- An active field of research is to integrate failsafes and crack stoppers in one part
- Manufacturing methods for composites are very different than for metals and damage tolerant designs need to adjust

STIFFENERS

- Group 1 - Sketch and describe the effect of crack stoppers on panel residual strength
- Group 2 - Sketch a residual strength curve for a typical stiffened panel and describe how to find regions of stable and un-stable crack growth.
- Group 3 - Describe the effect of stiffener cross-sectional area using the figure on p. 186
- Group 4 - What does the text mean when it says unstable cracking will begin at shorter crack lengths?

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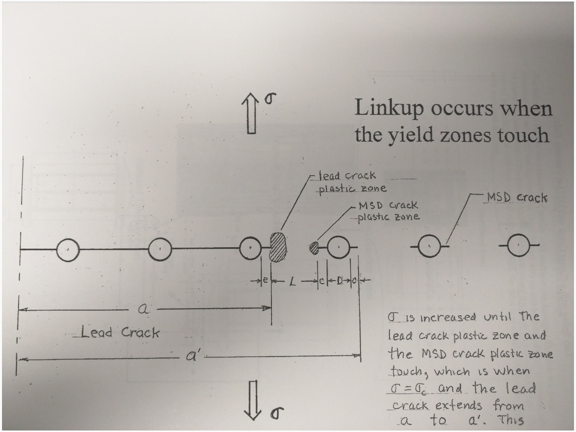
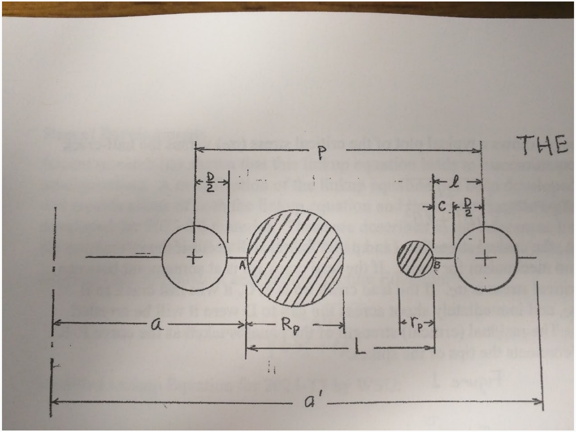
- ”link up” occurs when the plastic zones between two adjacent cracks touch


σ
Linkup occurs when
the yield zones touch
lead crack
plastic zone
MSD crack
plastic zone
MSD crack
e
L
c
D
c
a
Lead Crack
a'
σ
σ is increased until the
lead crack plastic zone and
the MSD crack plastic zone
touch, which is when
σ = σc and the lead
crack extends from
a to a'. This
σ


THE
P
D/2
ℓ
C
D/2
A
B
a
R_p
r_p
L
a'

- We know that

$$R_p = \frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2 \tag{11.1}$$

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- Where we define the stress intensity factors at a and L as

$$K_{Ia} = \sigma\sqrt{\pi a}\beta_a \tag{11.3}$$

$$K_{Il} = \sigma\sqrt{\pi l}\beta_l \tag{11.4}$$

- Since fast cracking occurs when $R_p + r_p = L$, we solve for the condition where $R_p + r_p < L$

$$\frac{1}{2\pi}\left(\frac{K_{Ia}}{\sigma_{YS}}\right)^2 + \frac{1}{2\pi}\left(\frac{K_{Il}}{\sigma_{YS}}\right)^2 < L \tag{11.5a}$$

$$\frac{1}{2\pi\sigma_{YS}^2}\left[K_{Ia}^2 + K_{Il}^2\right] < L \tag{11.5b}$$

$$\frac{1}{2\pi\sigma_{YS}^2}\left[\sigma^2\pi a\beta_a^2 + \sigma^2\pi l\beta_l^2\right] < L \tag{11.5c}$$

$$\frac{\sigma^2}{2\sigma_{YS}^2}\left[a\beta_a^2 + l\beta_l^2\right] < L \tag{11.5d}$$

$$\frac{\sigma_c^2}{2\sigma_{YS}^2}\left[a\beta_a^2 + l\beta_l^2\right] = L \tag{11.5e}$$

$$\sigma_c = \sigma_{YS}\sqrt{\frac{2L}{a\beta_a^2 + l\beta_l^2}} \tag{11.5f}$$

EXAMPLE

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MODFIED LINKUP EQUATIONS

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- This does not agree with test data
- Even the 2024 predictions don’t agree well with test data
- WSU used some empirical parameters to modify the linkup equations and better predict residual strength when multiple site damage is present

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- The same equation can also be used for 2524 with $A_1 = 0.1905$, $A_2 = 0.9683$ for A-basis yield and $A_1 = 0.2024$, $A_2 = 1.0719$ for B-basis yield

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$$\sigma_{c,mod} = \frac{K_c}{\sqrt{\pi a}(0.856 - 0.496 \ln(L))} \tag{11.8}$$

EXAMPLE

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- Recall the stress field near the crack tip

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{11.9a}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{11.9b}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{11.9c}$$

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$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \tag{11.10a}$$

$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{11.10b}$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{11.10c}$$

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$$\sigma_r = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \tag{11.11a}$$

$$\sigma_\theta = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta \tag{11.11b}$$

$$\tau_{r\theta} = -\sigma_x \sin\theta\cos\theta + \sigma_y \sin\theta\cos\theta + \tau_{xy}(\cos^\theta - \sin^2\theta) \tag{11.11c}$$

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$$\sigma_r = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right) \tag{11.12a}$$

$$\sigma_\theta = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right) \tag{11.12b}$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right) \tag{11.12c}$$

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- Thus fracture begins when

$$\sigma_\theta(\theta_P) = \sigma_\theta(\theta = 0, K_{II} = 0, K_I = K_{Ic}) = \frac{K_{IC}}{\sqrt{2\pi r}} \tag{11.13}$$

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- Equation 11.12c in this case simplifies to

$$K_I \sin\theta_p + K_{II}(3\cos\theta_P - 1) = 0 \tag{11.14}$$

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- and Equation 11.12b simplifies to

$$4K_{IC} = K_I\left(3\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\right) - 3K_{II}\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right) \tag{11.15}$$

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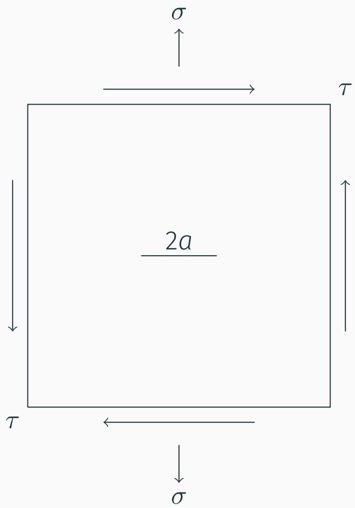
- The general form for a Mode II stress intensity factor is

$$K_{II} = \tau\sqrt{\pi a}\beta' \tag{11.16}$$

Assuming $|\sigma| = 4|\tau|$, $K_{IC} = 60$ ksi$\sqrt{\text{in}}$, and $2a = 1.5$ in.

Note: Assume $\beta = \beta' = 1$



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PRINCIPAL STRESS CRITERION

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$$0 = \sigma_\theta dA - \sigma_x dA \sin^2\theta - \sigma_y dA \cos^2\theta + 2\tau_{xy} dA \cos\theta \sin\theta \tag{11.17a}$$

$$\sigma_\theta = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta \tag{11.17b}$$

$$\frac{\partial \sigma_\theta}{\partial \theta} = (\sigma_x - \sigma_y) \sin 2\theta_p - 2\tau_{xy} \cos 2\theta_P \tag{11.17c}$$

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{11.17d}$$

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- We then find the remote failure stress by

$$\sigma_c = \frac{K_{IC}}{C\sqrt{\pi a}\beta} \tag{11.19}$$

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