ae 737 - mechanics of damage tolerance

Lecture 15

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SCHEDULE

- 22 Mar - Stress based fatigue, Homework 6 assigned
- 24 Mar - Stress based fatigue
- 29 Mar - Influence of notches on fatigue, Homework 7 assigned, Homework 6 due
- 31 Mar - Strain based fatigue, project abstract due

1. fatigue
2. nominal and local stress
3. fatigue tests
4. fatigue life analysis
5. fatigue limit

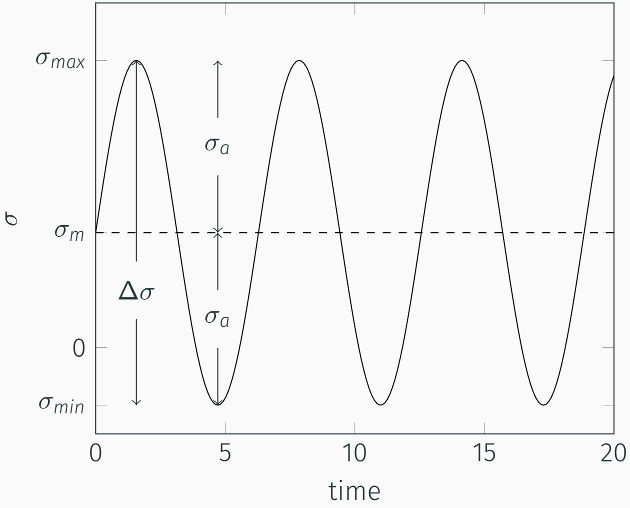
FATIGUE

FATIGUE

- We refer to damage from repeated, or cyclic loads as fatigue damage
- Some of the earliest work on fatigue began in the 1800’s
- Chains, railway axles, etc.
- An estimated 80% of failure expenses are due to fatigue

- There are three main approaches to fatigue analysis
- Stress based fatigue analysis
- Strain based fatigue analysis
- Fracture mechanics fatigue analysis

- One of the simplest assumptions we can make is that a load cycles between a constant maximum and minimum stress value
- This is a good approximation for many cases (axles, for example) and can also be easily replicated experimentally
- This is referred to as constant amplitude stressing



σ_{max}
σ_a
σ
σ_m
$\Delta\sigma$
σ_a
0
σ_{min}
0
5
10
15
20
time

- $\Delta\sigma$ is known as the stress range, and is the difference between max and min stress
- σ_m is the mean stress, and can sometimes be zero, but this is not always the case
- σ_a is the stress amplitude, and is the variation about the mean
- We can express all of these in terms of the maximum and minimum stress

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \tag{15.1}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \tag{15.2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{15.3}$$

- It is also common to describe some ratios
- The stress ratio, R is defined as

$$R = \frac{\sigma_{min}}{\sigma_{max}} \tag{15.4}$$

- And the amplitude ratio, A is defined as

$$A = \frac{\sigma_a}{\sigma_m} \tag{15.5}$$

- There are some useful relationships between the above equations

$$\Delta\sigma = 2\sigma_a = \sigma_{max}(1 - R) \tag{15.6a}$$

$$\sigma_m = \frac{\sigma_{max}}{2}(1 + R) \tag{15.6b}$$

$$R = \frac{1 - A}{1 + A} \tag{15.6c}$$

$$A = \frac{1 - R}{1 + R} \tag{15.6d}$$

NOMINAL AND LOCAL STRESS

- It is important to distinguish between the nominal (global) stress and the local stress at some point of interest
- We use σ for the stress at a point (local stress)
- We use S as the nominal (global) stress
- In simple tension, $\sigma = S$
- For many cases (bending, notches), $\sigma \neq S$ in general
- We must also be careful to note σ_y, in some cases $S < \sigma_y$ but at some locations $\sigma > \sigma_y$

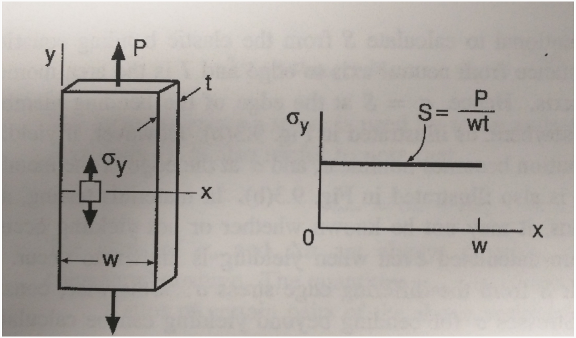


Figure 1: In this case $S = \sigma$

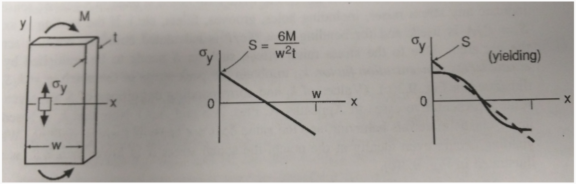


Figure 2: As long as $\sigma < \sigma_y$, σ varies linearly. If $\sigma > \sigma_y$ at any location, however, the relationship is non-linear

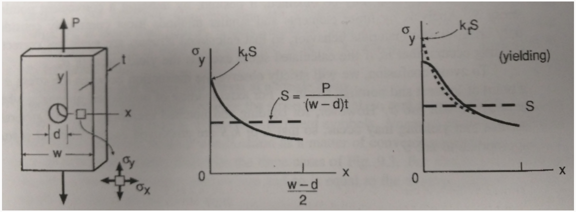
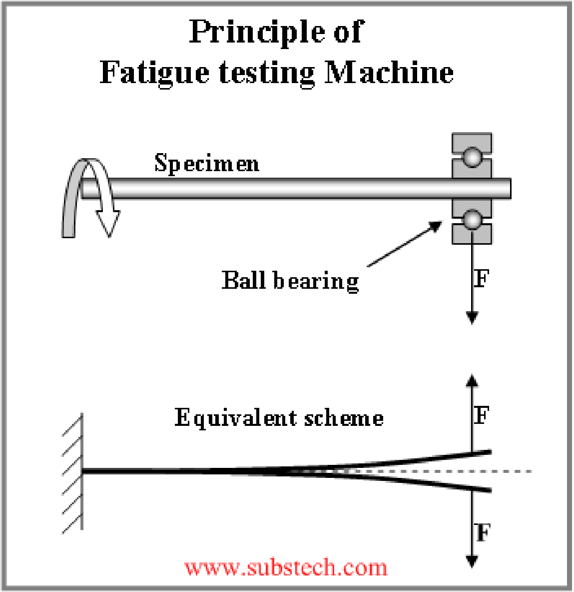


Figure 3: As long as $\sigma < \sigma_y$, σ varies linearly. If $\sigma > \sigma_y$ at any location, however, the relationship is non-linear

FATIGUE TESTS


Principle of
Fatigue testing Machine
Specimen
Ball bearing
F
Equivalent scheme
F
F
www.substech.com

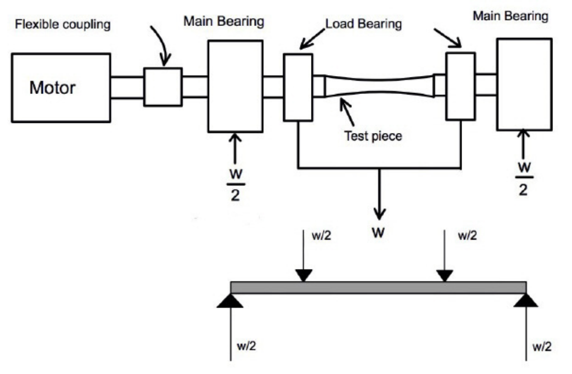


Figure 5: Four-point bend gives uniform stress (along top and bottom surfaces)

- The above rotating methods are very common, but in their current configurations can only be used for zero mean stress
- a reciprocating bend test can be used for non-zero mean stress

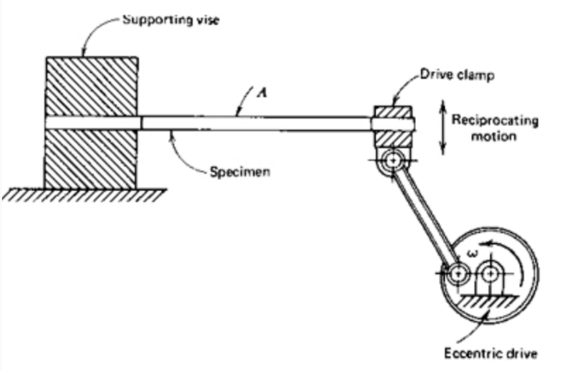


Figure 6: A reciprocating cantilever test allows for non-zero mean stress

AXIAL FATIGUE TEST

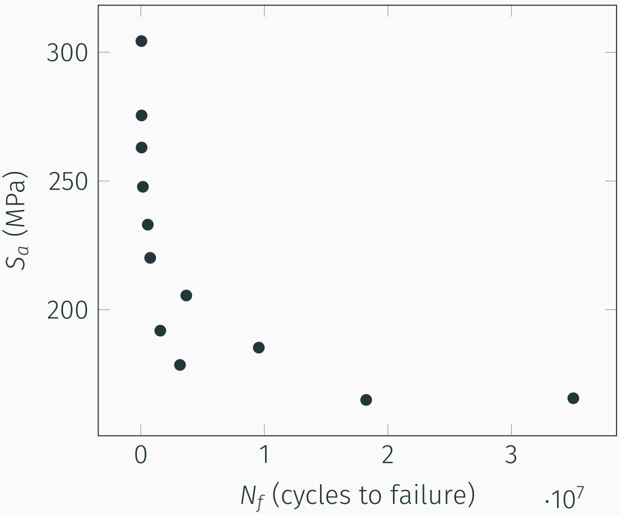
- The length of a fatigue test is determined by two factors
    1. How many cycles it takes for the specified load to cause failure
    2. The speed of the motor controlling the test
- Servohydraulic machines generally have a speed of 10 - 100 Hz.
- At a speed of 100 Hz, it would take 28 hours for 10^7 cycles, 12 days for 10^8 cycles, and nearly 4 months for 10^9 cycles
- While some machines can test at very high speeds, the inertia of the sample can interfere with results

FATIGUE LIFE ANALYSIS

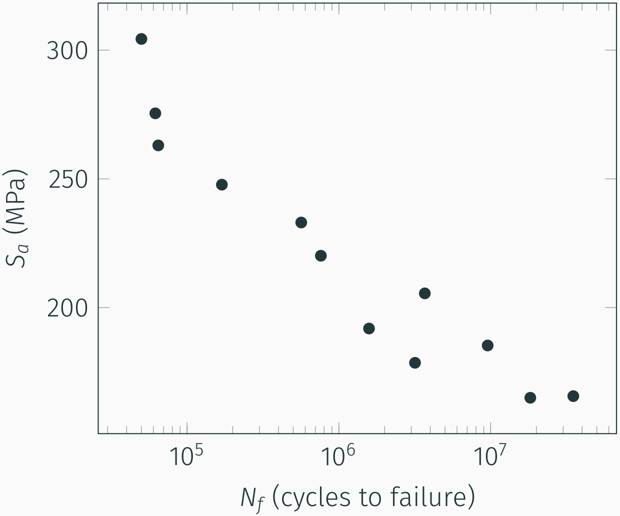
STRESS LIFE CURVES

- Stress-life curves, or S-N curves, are generated from test data to predict the number of cycles to failure
- In general, one set (or family) of S-N curves is generated using the same σ_m
- Usually S_a (the nominal stress equivalent of σ_a) is plotted versus N (the number of cycles)

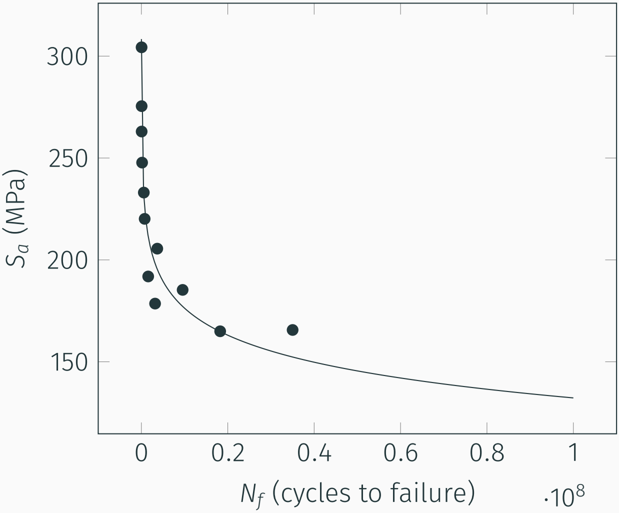
- Each individual point on an S-N curve represents one fatigue experiment
- To find enough data to form statistical significance, as well as to fit a curve across all levels of fatigue is very time-consuming
- In the following plot, if only one test was performed for each point, the total number of cycles tested would be about $7.3x10^7$
- For a 100 Hz machine, this represents over 200 hours of consecutive testing
- Each repetition would further increase the test time required

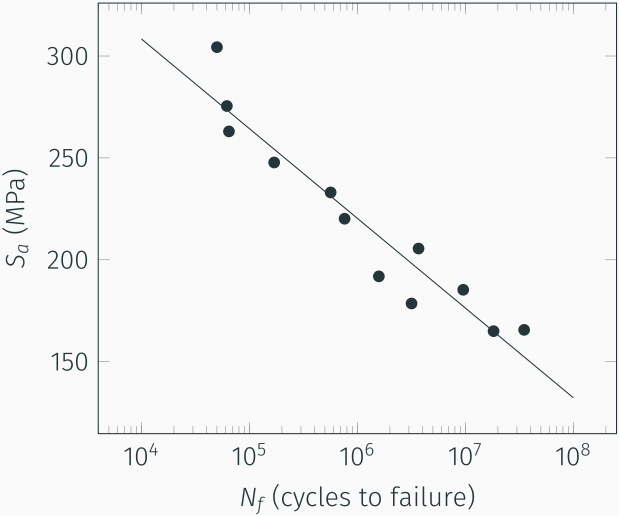

300
250
200
S_a (MPa)
0
1
2
3
N_f (cycles to failure)
$\cdot 10^7$

- On a linear scale, the data appear not to agree well with any standard fit
- It is also very difficult to differentiate between low-cycle fatigue failure stresses
- Instead S-N curves are often plotted on a semi-log or log-log scale, so pay attention to the axes


300
250
200
S_a (MPa)
10^5
10^6
10^7
N_f (cycles to failure)

- If the curve is nearly linear on a log-linear plot, we use the following form to fit the data
- $$\sigma_a = C + D \log N_f \tag{15.7}$$
- When the data are instead linear on a log-log scale, the following form is generally used
- $$\sigma_a = \sigma_f' \left(2N_f\right)^b \tag{15.8}$$
- σ_f' and b are often considered material properties and can often be looked up on a table (p. 235)

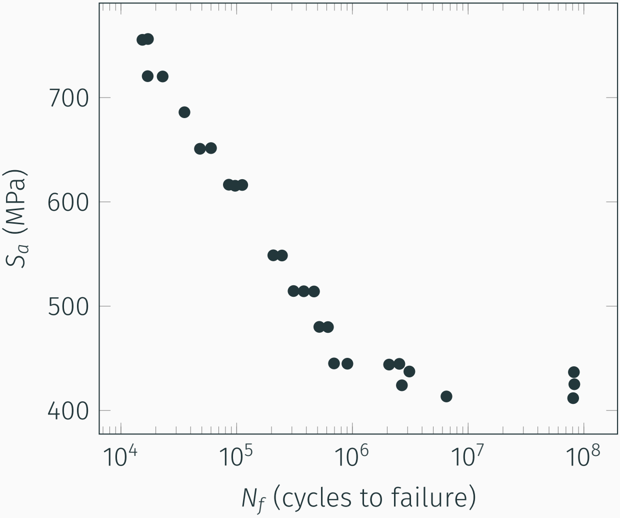

300
250
200
150
S_a (MPa)
0
0.2
0.4
0.6
0.8
1
N_f (cycles to failure)
$\cdot 10^8$



300
250
200
150
S_a (MPa)
10^4
10^5
10^6
10^7
10^8
N_f (cycles to failure)

FATIGUE LIMIT

- The fatigue limit, or endurance limit, is a feature of some materials where below a certain stress, no fatigue failure is observed
- Below the fatigue limit, this material is considered to have infinite life
- This most notably occurs in plain-carbon and low-alloy steels
- In these materials, σ_e is considered to be a material property
- This phenomenon is not typical of aluminum or copper alloys, but is sometimes arbitrarily assigned using whatever the failure stress is at some large number of cycles (10^7 or 10^8)


700
600
500
400
S_a (MPa)
10^4
10^5
10^6
10^7
10^8
N_f (cycles to failure)

HIGH AND LOW CYCLE FATIGUE

- Some other important terms are high cycle fatigue and low cycle fatigue
- ”High cycle fatigue” generally is considered anything above 10^3 cycles, but varies somewhat by material
- High cycle fatigue occurs when the stress is sufficiently low that yielding effects do not dominate behavior
- When yielding effects do dominate behavior, the strain-based approach is more appropriate