Homework 2

Problem 1

For Panel 1, we use equations (2.6) and (2.7) from the notes, or p. 52 from the text

$$K_I = \frac{P}{t\sqrt{\pi a}}\beta \tag{2.6}$$

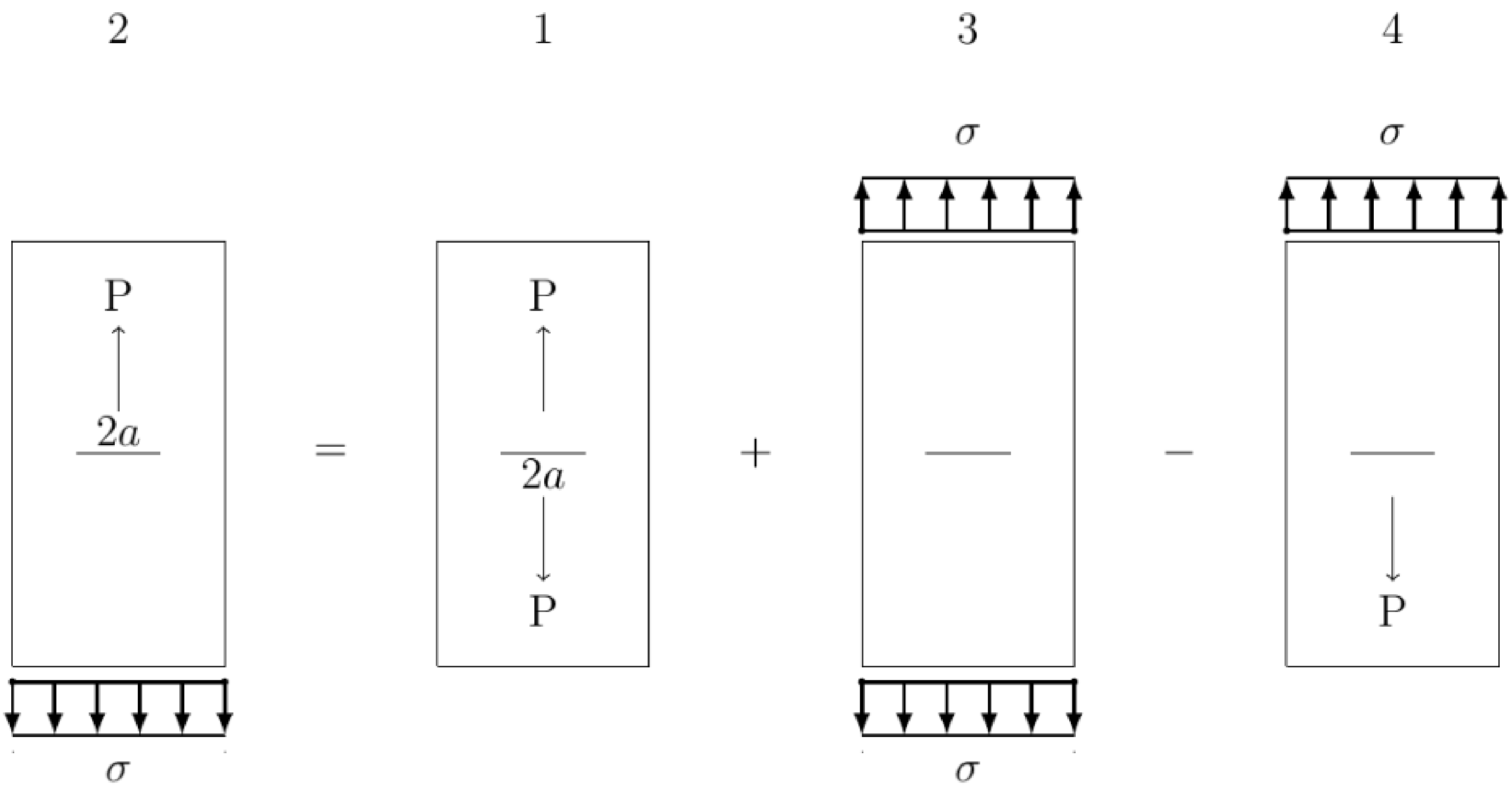
$$\beta = \frac{1 - 0.5\left(\frac{a}{W}\right) + 0.975\left(\frac{a}{W}\right)^2 - 0.16\left(\frac{a}{W}\right)^3}{\sqrt{1 - \left(\frac{a}{W}\right)}} \tag{2.7}$$

In Panel 2 we construct the following superposition

In [3]:

```
from IPython.display import Image
Image('2-1a.png')
```

Out[3]:



Since Panels 2 and 4 will give equivalent stress intensity factors, we can add Panel 4 to both sides to get

$$K_{I2} + K_{I4} = K_{I1} + K_{I3}$$
$$2K_{I2} = K_{I1} + K_{I3}$$
$$K_{I2} = \frac{1}{2}(K_{I1} + K_{I3})$$

We can now substutite the known equations for K_{I1} and K_{I3} to find

$$K_{I2} = \frac{1}{2}\left(\frac{P}{t\sqrt{\pi a}}\beta_1 + \sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)}\right)$$
$$\beta_1 = \frac{1 - 0.5\left(\frac{a}{W}\right) + 0.975\left(\frac{a}{W}\right)^2 - 0.16\left(\frac{a}{W}\right)^3}{\sqrt{1 - \left(\frac{a}{W}\right)}}$$

Problem 2

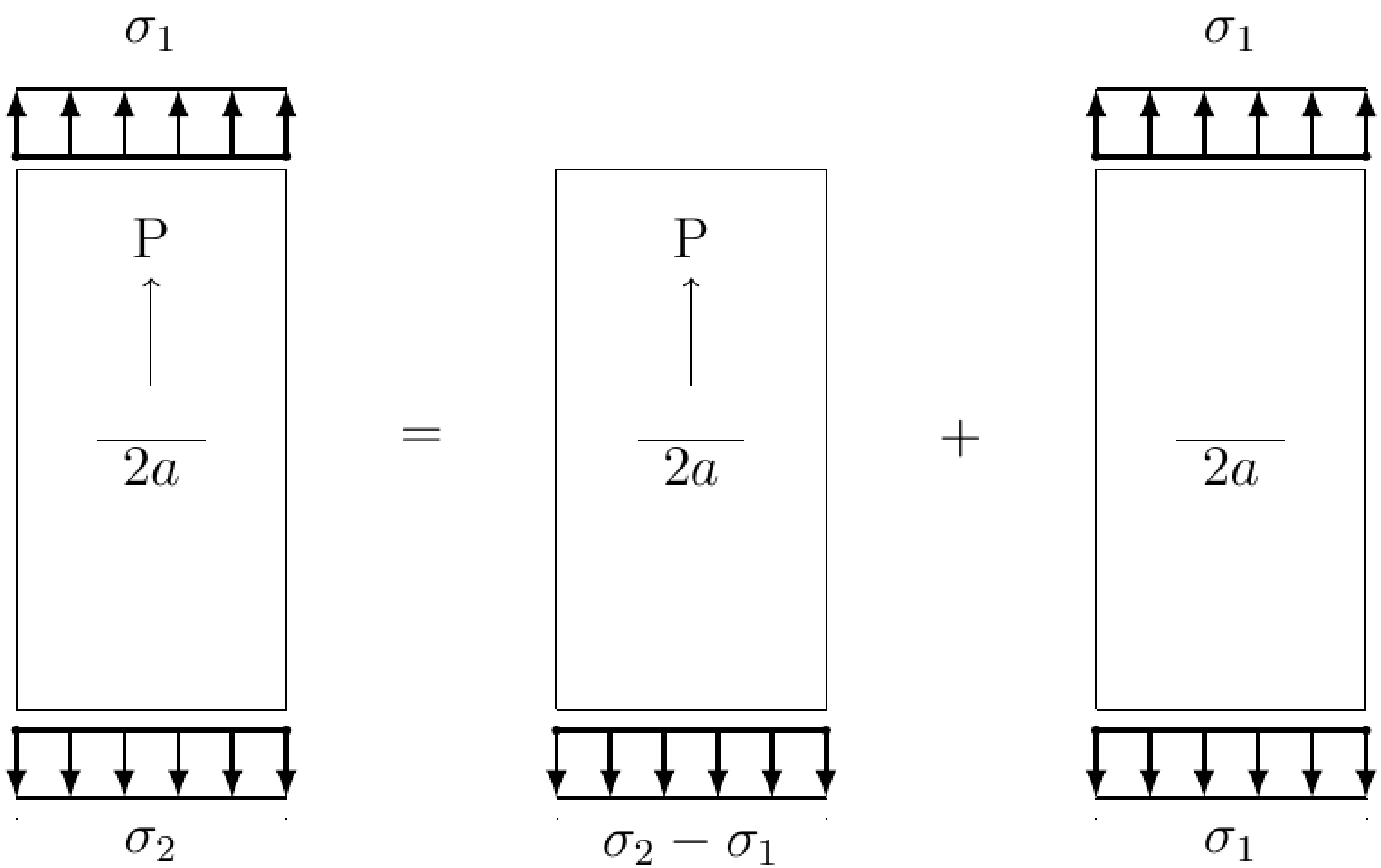
For a corner crack as indicated in Problem 2, one possible method is to superpose the lug corner-crack and bending corner-crack solutions (p. 63 and p. 64).

Problem 3

This problem is very similar to Panel 2 from Problem 1. If we superpose the solution from Problem 1 with a center crack under remote stress we will have the same conditions as here.

In [4]:
```
Image('2-3.png')
```

Out[4]:



Problem 4

In this case we will have two crack tips, one at A and one at B, with different stress intensity factors. We need to consider the effects of both edges, and the hole, but since no height dimension is given, we assume a negligible finite height effect.

From the left edge, we have $a = 1.25$ and $b = 2.45$, looking at the chart gives $\beta_A = 1.10$ and $\beta_B = 1.06$. From the right edge, we have $a = 1.25$ and $b = 6 - 2.45 = 3.55$, notice that we switch the A and B lines from the chart to match our labeling, and we find $\beta_A = 1.03$ and $\beta_B = 1.05$.

For the circular hole we have $R = 0.5$, $c = 2.55$, and $b = 2.05$. This gives $\beta_A = 1.02$ and $\beta_B = 1.07$. In summary we have

β_A	β_B
1.10	1.06
1.03	1.05
1.02	1.07

In [5]:

```
#method one
beta_a_1 = 1+ (1.10-1+1.03-1+1.02-1)
beta_b_1 = 1+ (1.06-1+1.05-1+1.07-1)

#method two
beta_a_2 = 1.10*1.03*1.02
beta_b_2 = 1.06*1.05*1.07

print beta_a_1
print beta_a_2

print beta_b_1
print beta_b_2
```

```
1.15
1.15566
1.18
1.19091
```

We have about 1% difference between methods 1 and 2 for both crack tip A and B, which means we have very little interaction of edge effects and we would expect both values to be fairly accurate.

We use $K_I = \sigma\sqrt{\pi a}\beta$ to find the stress intensity factor

In [6]:
```
import numpy as np
s = 15 #ksi
a = 1.25 #in

#method one
k_a_1 = s*np.sqrt(np.pi*a)*beta_a_1
k_b_1 = s*np.sqrt(np.pi*a)*beta_b_1

#method 2
k_a_2 = s*np.sqrt(np.pi*a)*beta_a_2
k_b_2 = s*np.sqrt(np.pi*a)*beta_b_2

print k_a_1
print k_a_2
print k_b_1
print k_b_2
```

```
34.18369794185185
34.351941185635226
35.0754465838132
35.39974583993982
```

Problem 5

Since we are dealing with cracks along a curved boundary, both "short" and "long," we will use the combined method to interpolate between the two solutions.

For a short crack, we start by finding the stress concentration factor from p. 84. For that chart, $r/d = \frac{0.25}{4-(0.1+0.25)}$, we also have $D/d = \frac{4}{4-(0.1+0.25)}$

In [7]:
```
r = 0.25
d = 4-(.1+.25)
D = 4
print r/d
print D/d
```

```
0.0684931506849
1.09589041096
```

We find on the chart that $K_{tn} = 2.5$. Next we relate the given panel to one with a hole in the center and a very short crack on one side of the hole. For this case we find $K_{tg} = 3.05$.

Our next task is to convert the net stress equation into global stress. We do this by making a free-body cut through the crack plane. (See Example 6 on p. 86 of text). Since the force on the cut face acts slightly off-center, we need to also include a bending moment for the body to be in equilibrium.

In [8]:
```
s = 6 #ksi, applied stress
t = 0.375
P = s*D*t #equivalent force from applied stress
M = P*(2-d/2) #moment equal to force times eccentricity of load at mid-plane
sn = P/(d*t) + M*(d/2)/(t*3.65**3/12) #net stress is force/area + My/I
```

$K_{tg}\sigma = K_{tn}\sigma_n$ (equate net-stress concentration factor to global stress concentration factor)

$$K_{tg} = K_{tn}\sigma_n/\sigma$$

In [9]:
```
K_tn = 2.5
K_tg = K_tn*sn/s
print K_tg
```

```
3.52786639144
```

We can now find what σ_A needs to be by equation $\sigma_{max,B}$ with $\sigma_{max,A}$

$$\sigma_{max,B} = \sigma_{max,A}$$
$$K_{tgB}\sigma = K_{tgA}\sigma_A$$
$$\frac{K_{tgB}}{K_{tgA}}\sigma = \sigma_A$$

In [10]:
```
sa = K_tg/3.05*s
print sa
```

```
6.94006503235
```

Since $K_{IA} = K_{IB}$ for short cracks, we can now find K_I using a crack on one side of a hole (eq 2.12)

In [11]:
```
import numpy as np
rc = 1 #for short cracks c = 0
B_3 = .7071+.7548+.3415+.6420+.9196
Fw = 1/np.cos(np.pi*.25/4)
Fww = 1 #because c=0
B_s = B_3*Fw*Fww*K_tg/3.05
B_s
```

Out[11]:
```
3.9684728996351324
```

So the β_S for short cracks is 3.97. For long cracks we use the formula for edge cracks in a finite panel, where the extra geometry, e is 0.35

In [12]:
```
e = 0.35
W = 4.0
c = np.linspace(0,1,100)
aw = (c+e)/W
B = 1.122 - 0.231*aw + 10.55*aw**2 - 21.71*aw**3 + 30.82*aw**4
B_l = np.sqrt((c+e)/c)*B
```

```
D:\Anaconda\lib\site-packages\ipykernel_launcher.py:6: RuntimeWarning: divide by zero encountered in divide
```

We now find the tangent curve to generate a cohesive plot

In [13]:
```
from scipy import interpolate
#interpolate our discrete points
spl = interpolate.splrep(c[1:],B_l[1:])
x1 = 0.0743 #guess, adjust until they match
fa = interpolate.splev(x1,spl,der=0)
fprime = interpolate.splev(x1,spl,der=1)
print fa-fprime*x1
print B_s #(to find x1)
```

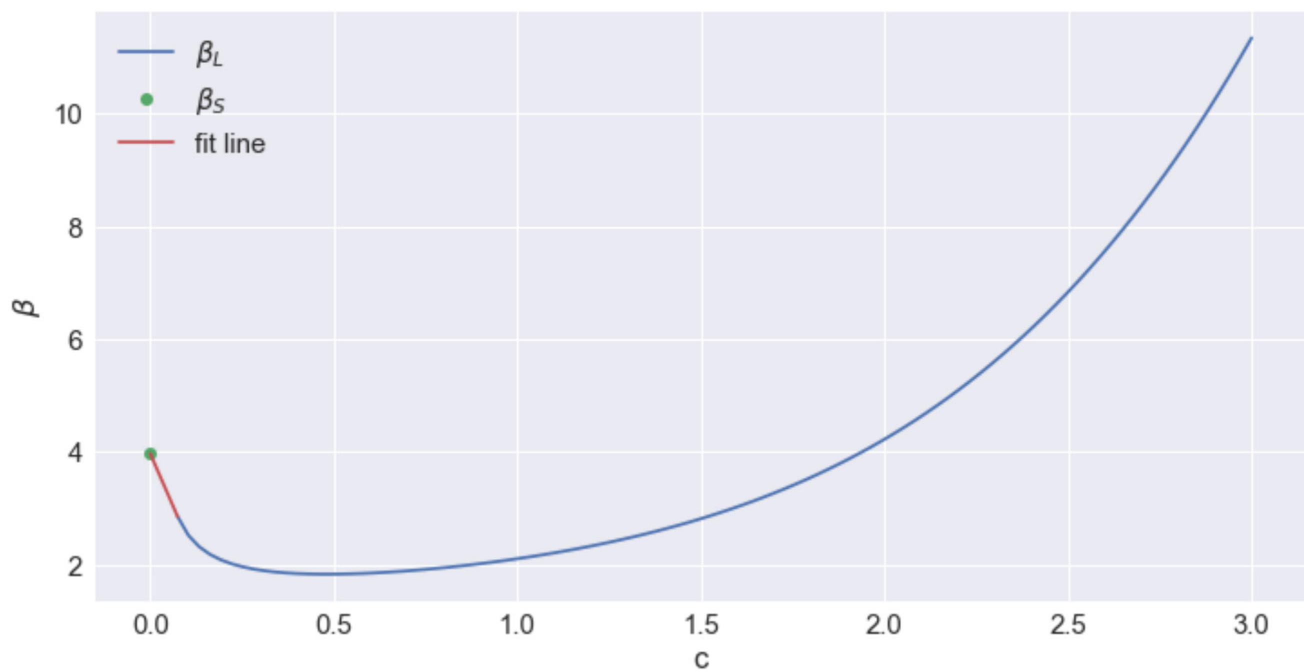
```
3.96788123325723
3.9684728996351324
```

And we plot the full result

In [14]:
```
import matplotlib.pyplot as plt
import seaborn as sb
sb.set(font_scale=1.5)
%matplotlib inline
plt.figure(figsize=(12,6))
c = np.linspace(x1,3,100)#only plot B_l from end of tangent
shortc = np.linspace(0,x1)
aw = (c+e)/W
B = 1.122 - 0.231*aw + 10.55*aw**2 - 21.71*aw**3 + 30.82*aw**4
B_l = np.sqrt((c+e)/c)*B
plt.figure(figsize=(12,6))
plt.plot(c,B_l,label=r'$\beta_L$')
plt.plot(0,B_s,'o',label=r'$\beta_S$')
plt.plot(shortc,fa+fprime*(shortc-x1),label='fit line')
plt.xlabel('c')
plt.ylabel(r'$\beta$')
plt.legend(loc='best')
```

Out[14]:
```
<matplotlib.legend.Legend at 0xb48b898>
```

```
<Figure size 864x432 with 0 Axes>
```



Visually, we can see that for part a we are using only the long crack formula, calculating K_I:

In [15]:
```
c = 0.4
a = c + e
aw = a/W
B = 1.122 - 0.231*aw + 10.55*aw**2 - 21.71*aw**3 + 30.82*aw**4
K_I = s*np.sqrt(np.pi*a)*B
print K_I
```

```
12.383412965569004
```

So $K_I = 12.38\text{ksi}\sqrt{\text{in.}}$

For part b we need to use our linear interpolation, we find that

In [16]:
```
c = 0.05
B = fa+fprime*(c-x1)
print B
```

```
3.218144130597965
```

Substituting this β in gives

In [17]:
```
K_I = s*np.sqrt(np.pi*c)*B
print K_I
```

```
7.652735088257687
```

So $K_I = 7.65\text{ksi}\sqrt{\text{in.}}$

If we used only the short-crack assumption, we would find $K_I = 3.97(6000)\sqrt{.05\pi}$

In [18]:
```
3.97*6*np.sqrt(np.pi*.05)
```

Out[18]:
```
9.440645622897518
```

Which gives a stress intensity factor of $K_I = 9.44$ ksi.

Even though we might assume a 0.05" crack is very short, using the short crack assumptions leads to an error of about 20%.

Problem 6

We formulate this problem much like Problem 4. For the left crack tip, we have $a/b = 0.33$, which gives $\beta_A = 1.04$ and $\beta_B = 1.03$. For the right crack tip (with A and B switched from the chart) we have $a/b = .05$, which gives $\beta_A \approx \beta_B \approx 1$. For the circular hole we have $R = 0.3$, $c = 0.6$, and $b = 0.3$. This gives $\beta_A = 1.19$ and $\beta_B = 1.31$. In summary we have

β_A	β_B
1.04	1.03
1.00	1.00
1.19	1.31

In [19]:

```
#method one
beta_a_1 = 1+ (1.04-1+1.00-1+1.19-1)
beta_b_1 = 1+ (1.03-1+1.00-1+1.31-1)

#method two
beta_a_2 = 1.04*1.*1.19
beta_b_2 = 1.03*1.0*1.31

print beta_a_1
print beta_a_2

print beta_b_1
print beta_b_2
```

```
1.23
1.2376
1.34
1.3493
```

Once again Methods 1 and 2 agree well, indicating little interaction. However we see in this problem that β is dominated by the hole, the edge effects are minimal. The stress intensities are:

In [20]:
```
a = 0.1 #in

#method one
k_a_1 = np.sqrt(np.pi*a)*beta_a_1
k_b_1 = np.sqrt(np.pi*a)*beta_b_1

#method 2
k_a_2 = np.sqrt(np.pi*a)*beta_a_2
k_b_2 = np.sqrt(np.pi*a)*beta_b_2

print k_a_1
print k_a_2
print k_b_1
print k_b_2
```

```
0.6894139196169452
0.6936737129414077
0.7510688229973225
0.7562814648285726
```

(multiplied by the applied stress)

In [ ]:
```
```