AE 737: Mechanics of Damage Tolerance

Lecture 5 - Plastic Zone

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schedule

- 1 Feb - Plastic Zone
- 3 Feb - Plastic Zone, HW 2 Due, HW 1 Self-grade due
- 8 Feb - Fracture Toughness
- 10 Feb - Fracture Toughness, HW3 Due, HW 2 Self-grade due

outline

- plastic zone

plastic zone

plastic zone

- Previous developments assumed perfectly elastic materials
- Most common materials have some plasticity
- Any stress above the yield stress will undergo plastic deformation (no stress higher than σ_y will be present in the material)

plastic zone

- Plasticity helps retard crack propagation due to residual stresses
- After an overload, elastic regions will contract back to their undeformed shape
- The region which has undergone plastic deformation, however, holds its deformed shape
- This introduces a region of residual compressive stress near the crack tip
- Before the crack can propagate, a stress needs to overcome this residual stress

2D problems

- We often simplify the full 3D elasticity equations for planar problems
- For very thin panels, we assume that all out-of-plane stresses are 0
- This is called plane stress

plane stress

$$
\begin{aligned}
\sigma_z &= \tau_{xz} = \tau_{zy} = 0 \\
\epsilon_x &= \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} \\
\epsilon_y &= -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} \\
\epsilon_z &= -\nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} \\
\gamma_{xy} &= \frac{\tau_{xy}}{G} \\
\gamma_{xz} &= \gamma_{yz} = 0
\end{aligned}
$$

2D problems

- When instead a panel is very thick, we assume that any strains through the thickness are small relative to other strains
- $\epsilon_z = \gamma_{xz} = \gamma_{yz} = 0$
- This is known as plane strain

plane strain

$$\begin{aligned}
\epsilon_x &= \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \\
\epsilon_y &= -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \\
0 &= -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E} \\
\gamma_{xy} &= \frac{\tau_{xy}}{G} \\
\gamma_{xz} &= \gamma_{yz} = 0
\end{aligned}$$

Irwin's first approximation

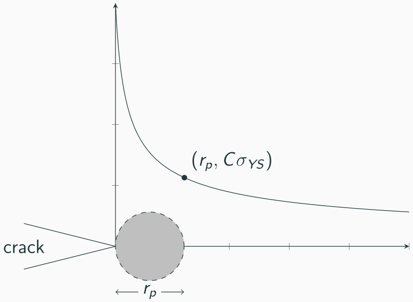
- If we recall the equation for opening stress (σ_y) near the crack tip

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{1.2}$$

- In the plane of the crack, when $\theta = 0$ we find

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}$$

Irwin's first approximation



Irwin's first approximation

- We use C, the *Plastic Constraint Factor* to convert between Plane Strain and Plane Stress solutions
- The plastic zone size can now be approximated

$$\sigma_{yy}(r = r_p) = C\sigma_{YS}$$

$$\frac{K_I}{\sqrt{2\pi r_p}} = C\sigma_{YS}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{C\sigma_{YS}}\right)^2$$

Irwin's first approximation

- For plane stress (thin panels) we let $C = 1$ and find r_p as

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

- And for plane strain (thick panels) we let $C = \sqrt{3}$ and find

$$r_p = \frac{1}{6\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

Intermediate panels

- For panels which lie between plane strain and plane stress states, we use the following expression to estimate the plastic zone size

$$r_p = \frac{1}{I\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

- Where I is defined as

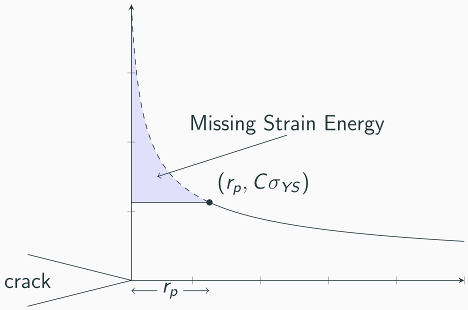
$$I = 6.7 - \frac{1.5}{t}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

- And $2 \leq I \leq 6$

Irwin's second approximation

- If our material is perfectly elastic-plastic, no stresses above $C\sigma_{ys}$ will exist in the material
- This ignores the strain energy (represented by the area under the curve) in the plastic zone

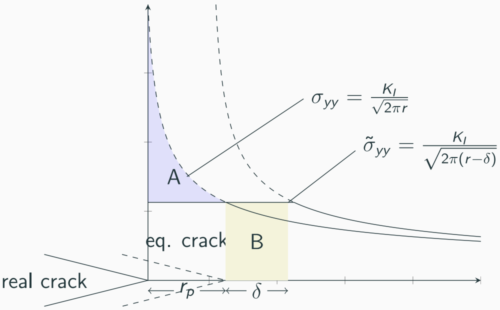
Irwin's second approximation



Irwin's second approximation

- To account for the additional strain energy, Irwin considered a plastic zone size increased by some δ
- He also needed to adjust the stress function, and considered an equivalent crack tip in these calculations

Irwin's second approximation



Irwin's second approximation

We need $A = B$, so we set them equivalent and solve for δ.

$$
\begin{aligned}
A &= \int_0^{r_p} \sigma_{yy} dr - r_p \sigma_{YS} \\
&= \int_0^{r_p} \frac{K_I}{\sqrt{2\pi r}} dr - r_p \sigma_{YS} \\
&= \frac{K_I}{\sqrt{2\pi}} \int_0^{r_p} r^{-1/2} dr - r_p \sigma_{YS} \\
&= \frac{2K_I\sqrt{r_p}}{\sqrt{2\pi}} - r_p \sigma_{YS}
\end{aligned}
$$

Irwin's second approximation

- We have already found r_p as

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

- If we solve this for K_I we find

$$K_I = \sqrt{2\pi r_p}\sigma_{YS}$$

Irwin's second approximation

- We can now substitute back into the strain energy of A

$$\begin{aligned} A &= \frac{2\sqrt{2\pi r_p}\sigma_{YS}\sqrt{r_p}}{\sqrt{2\pi}} - r_p\sigma_{YS} \\ &= 2\sigma_{YS} r_p - r_p\sigma_{YS} \\ &= r_p\sigma_{YS} \end{aligned}$$

Irwin's second approximation

- B is given simply as $B = \delta\sigma_{ys}$ so we equate A and B to find δ

$$A = B$$
$$r_p\sigma_{YS} = \delta\sigma_{YS}$$
$$r_p = \delta$$

Irwin's second approximation

- This means the plastic zone size is simply $2r_p$
- However, it also means that the effective crack length is $a + r_p$
- Since r_p depends on K_I, we must iterate a bit to find the "real" r_p and K_I

Example

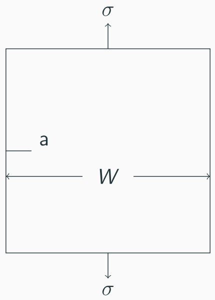


Figure 1: An edge crack of length a in a panel of width W is subjected to a remote load

equations

$$\beta = \left[1.122 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.71\left(\frac{a}{W}\right)^3 + 30.82\left(\frac{a}{W}\right)^4\right]$$

$$I = 6.7 - \frac{1.5}{t}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

$$r_p = \frac{1}{I\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$