Homework 1

February 1, 2018

0.1 1

Stress Intensity - measure of the magnitude of the local stress field

0.2 2

For a center-cracked panel with uniform remote stress, we must decide whether to use the Finite Width or Infinite Width formula. Since Width is given as a parameter, we will use the Finite Width equation.

$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi a/W)} \tag{2.2a}$$

For $W/2a = 4$ we can substitute to find

$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec(\pi/8)}$$

```
In [6]: import numpy as np
        K_I_center = np.sqrt(1/np.cos(np.pi/8))
        K_I_center
```

```
Out[6]: 1.0403807958110309
```

Thus

$$K_I = 1.040\sigma\sqrt{\pi a}$$

For an edge-cracked panel, since $a/w < 0.6$ we use the simpler finite-width formula:

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.71\left(\frac{a}{W}\right)^3 + 30.82\left(\frac{a}{W}\right)^4\right] \tag{2.4a}$$

Substituting $W/a = 4$ gives

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.231/4 + 10.55/16 - 21.71/4^3 + 30.82/4^4\right]$$

```
In [7]: K_I_edge = 1.12 - 0.231/4 + 10.55/16 - 21.71/(4**3) + 30.82/(4**4)
        K_I_edge
```

```
Out[7]: 1.5027968750000003
```

Thus

$$K_I = 1.50\sigma\sqrt{\pi a}$$

We can compare the two results, $K_{I,edge}/K_{I,center}$ to see the different effect the crack has for center and edge cracks

```
In [8]: K_I_edge/K_I_center
```

```
Out[8]: 1.4444681034586879
```

Thus a crack at the edge (for this given width) has a 44% stronger effect on the local stress than a center crack.

If we look at half of the center-crack specimen (so that it looks like an edge crack), there is an additional constraint (zero slope in y-deformation), this works to decrease the opening stress somewhat relative to the edge-cracked case.

0.3 3

For a through crack on one side of the hole, we use 2.12

$$K_I = \sigma\sqrt{\pi c}\beta$$

$$\beta = \beta_1 + \beta_2 \tag{2.12a}$$

$$\beta_1 = \beta_3 F_w F_{ww} \tag{2.12b}$$

$$\beta_2 = \frac{\sigma_{br}}{\sigma} F_4 F_w F_{ww} \tag{2.12c}$$

$$\beta_3 = 0.7071 + 0.7548\frac{R}{R+c} + 0.3415\left(\frac{R}{R+c}\right)^2 + 0.6420\left(\frac{R}{R+c}\right)^3 + 0.9196\left(\frac{R}{R+c}\right)^4 \tag{2.12d}$$

$$F_4 = 0.9580 + 0.2561\frac{c}{R} - 0.00193\left(\frac{c}{R}\right)^{2.5} - 0.9804\left(\frac{c}{R}\right)^{0.5} \tag{2.12e}$$

$$F_w = \sqrt{\sec\frac{\pi R}{W}\sec\frac{\pi(R+c/2)}{W-c}} \tag{2.12f}$$

$$F_{ww} = 1 - N^{-\frac{W}{D}}\left(\frac{2c}{W-D}\right)^{\frac{W}{D}+0.5} \tag{2.12g}$$

Where

$$\sigma_{br} = \frac{P}{Dt} = 0$$

We also check $2.65 - 0.24\left(2.75 - \frac{W}{D}\right)^2$ to find N

```
In [9]: c = .125
        D = .25
        W = 7.0
        R = D/2
        print 2.65 - 0.24*(2.75-W/D)**2
```

```
-150.365
```

Since this is less than 2.275, we use $N = 2.275$

```
In [10]: N = 2.275
         F_ww = 1-N**(-W/D)*(2*c/(W-D))**(W/D+0.5)
         F_w = np.sqrt(1/np.cos(np.pi*R/W)/np.cos(np.pi*(R+c/2)/(W-c)))
         F_4 = 0.958+0.2561*c/R-0.00193*(c/R)**2.5-0.9804*(c/R)**0.5
         beta_3 = 0.7071 + 0.7548*R/(R+c) + 0.3415*(R/(R+c))**2 + \
           0.6420*(R/(R+c))**3 + + 0.9196*(R/(R+c))**4
         beta_2 = 0
         beta_1 = beta_3*F_w*F_ww
         beta = beta_1 + beta_2
         s = 9000/(7*.157)
         K_I = s*np.sqrt(np.pi*c)*beta
         print K_I
```

```
6728.05561343
```

Which gives $K_I = 6.73$ ksi$\sqrt{\text{in.}}$
For symmetric through cracks around a hole we use the formula

$$K_I = \sigma\sqrt{\pi c}\beta$$

$$\beta = \beta_1 + \beta_2 \tag{2.11a}$$

$$\beta_1 = F_{c/R}F_wF_{ww} \tag{2.11b}$$

$$\beta_2 = \frac{\sigma_{br}}{\sigma}F_3F_wF_{ww} \tag{2.11c}$$

$$F_{c/R} = \frac{3.404 + 3.8172\frac{c}{R}}{1 + 3.9273\frac{c}{R} - 0.00695\left(\frac{c}{R}\right)^2} \tag{2.11d}$$

$$F_w = \sqrt{\sec\frac{\pi R}{W}\sec\frac{\pi(R+c)}{W}} \tag{2.11e}$$

$$F_{ww} = 1 - \left(\left(1.32\frac{W}{D} - 0.14\right)^{-\left(.98+\left(0.1\frac{W}{D}\right)^{0.1}\right)} - 0.02\right)\left(\frac{2c}{W-D}\right)^N \tag{2.11f}$$

$$F_3 = 0.098 + 0.3592e^{-3.5089\frac{c}{R}} + 0.3817e^{-0.5515\frac{c}{R}} \tag{2.11g}$$

Where

$$\sigma_{br} = \frac{P}{Dt} = 0$$

We also check $\frac{W}{D} + 2.5$

```
In [11]: c = .125
         D = .25
         W = 7.0
         R = D/2
         print W/D + 2.5
```

```
30.5
```

So we use $N = 4.5$ (2.11j)

```
In [12]: N = 4.5
         s_br = 0
         s = 9000/(7*.157)
         F_cr = (3.404+3.1872*c/R)/(1+3.9273*c/R - 0.00695*(c/R)**2)
         F_w = np.sqrt(1/np.cos(np.pi*R/W)/np.cos(np.pi*(R+c)/W))
         F_ww = 1-((1.32*W/D-0.14)**(-(0.98+(0.1*W/D)**0.1))-.02)*(2*c/(W-D))**N
         F_3 = 0.098 + 0.3592*np.exp(-3.5089*c/R)+0.3817*np.exp(-0.5515*c/R)
         beta_1 = F_cr*F_w*F_ww
         beta_2 = 0
         beta = beta_1+beta_2
         K_I = s*np.sqrt(np.pi*c)*beta
         K_I
```

```
Out[12]: 6901.6800916595457
```

Thus $K_I = 6.90$ ksi$\sqrt{\text{in.}}$

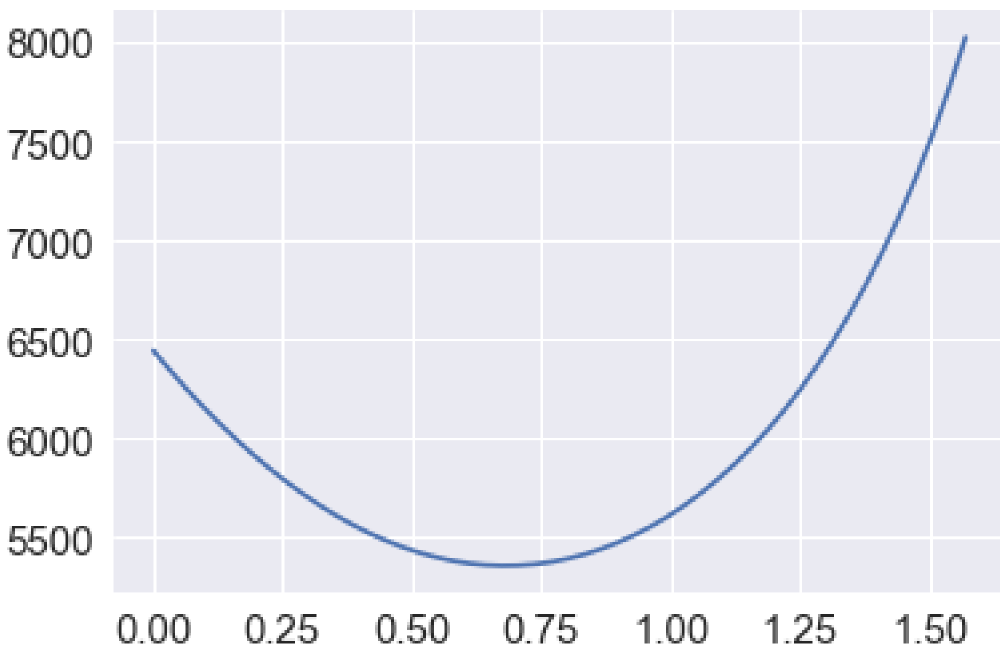
For a quarter circular crack we use (3.1) with $a = c$. With no minor axis, it is not obvious which direction in which direction the maximum stress intensity factor will be, so we vary ϕ and plot to find the maximum.

```
In [13]: import matplotlib.pyplot as plt
         import seaborn as sb
         sb.set(font_scale=1.5)
         %matplotlib inline

         a = c
         phi = np.linspace(0,np.pi/2)
         n = 1
         b = W/2
         r = R
         t=0.157
         fw = np.sqrt(1/np.cos(np.pi*r/(2*b))/np.cos((
                     np.pi*(2*r+n*c)/(4*(b-c)+2*n*c)*np.sqrt(a/t))))
         lam = 1/(1+c/r*np.cos(0.85*phi))
         g2 = (1+0.358*lam+1.425*lam**2-1.578*lam**3+2.156*lam**4)/(1+0.13*lam**2)
         M1 = 1.13 - 0.09*a/c
         M2 = -0.54 + 0.89/(0.2+a/c)
         M3 = 0.5 - 1/(0.65+a/c) + 14*(1-a/c)**24
         Q = 1+1.464*(a/c)**1.65
         g1 = 1+(0.1+0.35*(a/t)**2)*(1-np.sin(phi))**2
         g3 = (1+0.04*(a/c))*(1+0.1*(1-np.cos(phi))**2)*(0.85+0.15*(a/t)**.25)
         g4 = 1 - 0.7*(1-a/t)*(a/c-0.2)*(1-a/c)
         f_phi = ((a/c)**2*np.cos(phi)**2+np.sin(phi)**2)**.25
         Fch = (M1 +M2*(a/t)**2 + M3*(a/t)**4)*g1*g2*g3*g4*f_phi*fw
```

```
B = np.sqrt(1/Q)*Fch
KI_double = s*np.sqrt(np.pi*a)*B
KI_b = np.sqrt((4/np.pi + a*c/(2*t*r))/(4/np.pi+a*c/(t*r)))*KI_double
plt.figure()
plt.plot(phi,KI_b)
print max(KI_b)
```

```
8025.63965924
```



This gives a maximum stress intensity at $\phi = \pi/2$ (i.e. in the direction of the hole) of $K_I = 8.03\ \text{ksi}\sqrt{\text{in}}$.

0.4 4

For a semi-elliptical surface flaw, we use 2.15, with $b \to \infty$ and consider the maximum stress intensity factor when $\phi = 90°$

```
In [14]: a = 0.2
         c = 0.4 #2c = 0.8
         t = 0.56
         phi = np.pi/2
         fw = 1.0
         M1 = 1.13 - 0.09*a/c
         M2 = -0.52 + 0.89/(.2+a/c)
         M3 = 0.5-1.0/(.65+a/c)+14.0*(1.0-a/c)**4.0
         Q = 1 + 1.464*(a/c)**1.65
```

```
f_phi = ((a/c)**2*np.cos(phi)**2+np.sin(phi)**2)**.25
g = 1+ (0.1+.35*(a/c)**2)*(1-np.sin(phi))**2
Fs = (M1 + M2*(a/t)**2+M3*(a/t)**4)*g*f_phi*fw
B = np.sqrt(1/Q)*Fs
s = 20
KI = s*np.sqrt(np.pi*a)*B
print KI
```

```
15.5663712147
```

So we find $K_I = 15.6$ ksi$\sqrt{\text{in.}}$

0.5 5

While we have no direct formula for an I-Beam, we can estimate the stress intensity factor using the edge-crack bending formula (2.5a) and replacing σ with an effective stress using an I-Beam cross-section. i.e.

$$\sigma = \frac{6M}{tW^2} = \frac{My}{I}$$

We can calculate the inertia of the I-beam cross-section using the Parallel Axis Theorem

$$I = \sum \bar{I}_i + Ad_i^2$$

Since this I-beam is symmetric, we only need to find the inertia of the middle section, I_1 and the top section, I_2

```
In [15]: I_1 = 0.248*(8.14-2*.378)**3/12
         I_2 = 5.268*0.378**3/12
```

We can now apply the parallel axis theorem, we need to include the top and bottom sections, which are equal so we multiply the top section by 2

```
In [16]: I = I_1 + 2*(I_2+5.268*0.378*(8.14/2-.378/2)**2)
         print I
```

```
68.3545285908
```

So the inertia of this segmernt is 68.4 in.^{4}

Next we calculate the bending moment for this beam using a free-body diagram, we find the moment is a function of the distance from the applied load, $M = 3000x$ (ft-lb.), or in consistent units (in-lb.) $M = 3000(12)x = 36000x$ (in-lb.)

This is slightly different from the given formula (where M is constant), but we assume, as an estimate, that the bending at the crack should give a similar result, and take $x = 4$ feet.

We use this to calculate the effective stress as $\sigma = \frac{My}{I}$ with $y = W/2 = 4.07$

```
In [17]: M = 3000*4*12
         s = M*4.07/I
         s
```

Out[17]: 8574.121014107222

The crack will first need to propagate through the flange before it can affect the web, so we we consider $a/W = 0.3/5.268$, which we substitute into (2.5a)

```
In [18]: aw = 0.3/6.268
         beta = 1.122  - 1.4*aw + 7.33*aw**2 - 13.08*aw**3 +14.0*aw**4
         KI = s*np.sqrt(np.pi*0.3)*beta
         print KI
```

```
8910.06611044
```

Thus $K_I = 8.91\text{ksi}\sqrt{\text{in.}}$

0.6 6

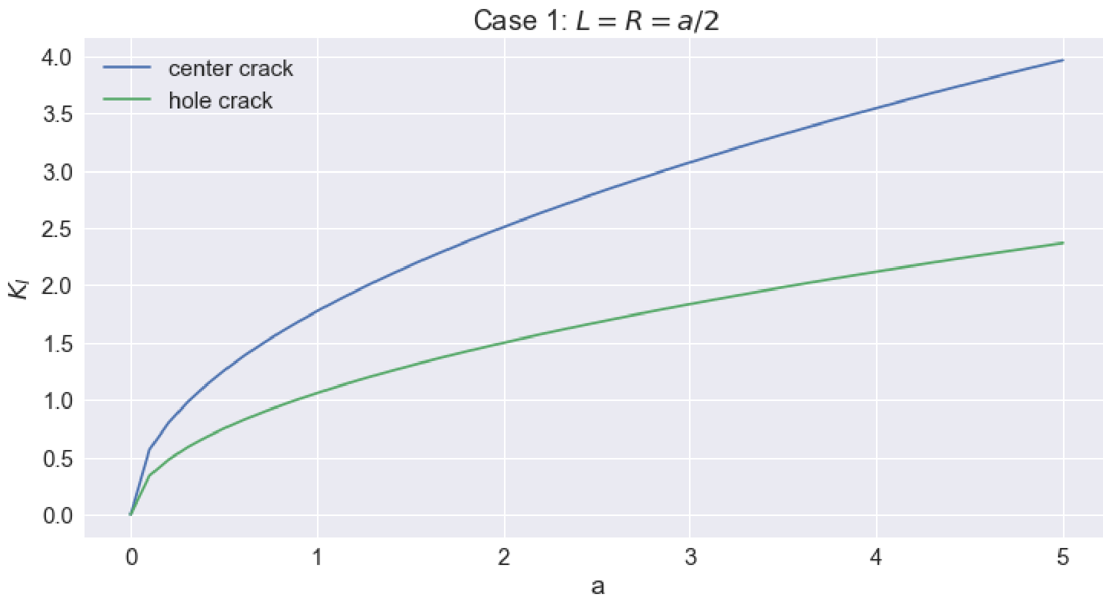
There are many different geometric configurations we could use to make this comparison. Here I will use two different configurations: one in which $R = L = a/2$ and one in which I hold R constant, making $L = a - R$. In both cases I assume the panel is very wide.

```
In [19]: #case 1, r = L = a/2
         a = np.linspace(0,5)
         r = a/2
         l = a/2
         K_I1 = np.sqrt(np.pi*a)

         #beta2 = 0, c/r = l/r = 1, w/d = infinity, r/w = 0
         Fcr = (.3404+3.8172)/(1+3.9273-0.00695)
         Fw = 1
         Fww = 1
         B = Fcr*Fw*Fww
         K_I2 = np.sqrt(np.pi*l)*B

         import matplotlib.pyplot as plt
         import seaborn as sb
         sb.set(font_scale=1.5)
         %matplotlib inline
         plt.figure(figsize=(12,6))
         plt.plot(a,K_I1,label='center crack')
         plt.plot(a,K_I2,label='hole crack')
         plt.legend(loc='best')
         plt.xlabel('a')
         plt.ylabel('$K_I$')
         plt.title('Case 1: $L = R = a/2$')
```

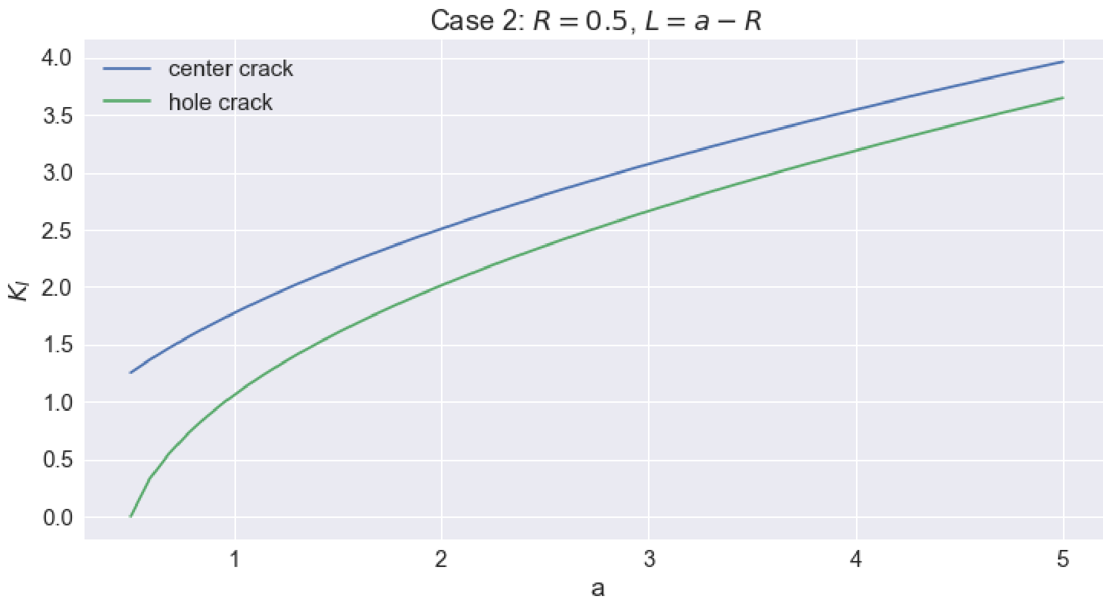
Out[19]: <matplotlib.text.Text at 0xb90a400>



In [20]:
```
#case 2, r = const, L = a - R
r = 0.5
a = np.linspace(r,5)
l = a - r
K_I1 = np.sqrt(np.pi*a)
#beta2 = 0, c/r = l/r, w/d = infinity, r/w = 0
cr = l/r
Fcr = (.3404+3.8172*cr)/(1+3.9273*cr-0.00695*cr**2)
Fw = 1
Fww = 1
B = Fcr*Fw*Fww
K_I2 = np.sqrt(np.pi*l)*B

plt.figure(figsize=(12,6))
plt.plot(a,K_I1,label='center crack')
plt.plot(a,K_I2,label='hole crack')
plt.legend(loc='best')
plt.xlabel('a')
plt.ylabel('$K_I$')
plt.title('Case 2: $R = 0.5$, $L = a - R$')
```

Out[20]:
```
<matplotlib.text.Text at 0xbb22da0>
```

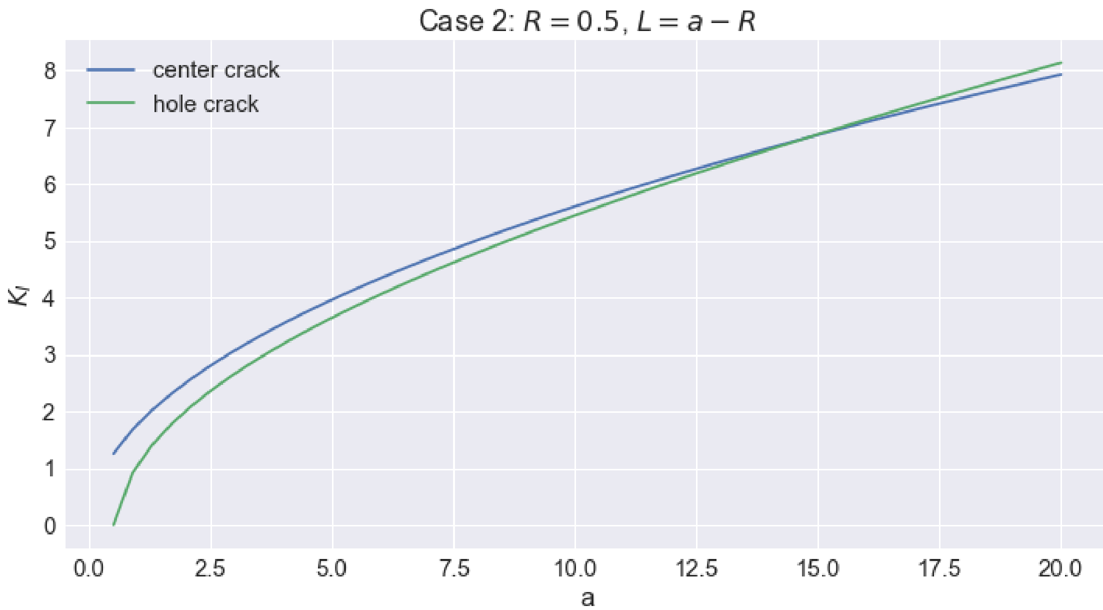


In both cases, the center crack has a larger stress intensity than the hole crack, although for a constant hole size, the longer the crack is the more the solutions appear to converge.

```
In [21]: #case 2, r = const, L = a - R
         r = 0.5
         a = np.linspace(r,20)
         l = a - r
         K_I1 = np.sqrt(np.pi*a)
         #beta2 = 0, c/r = l/r, w/d = infinity, r/w = 0
         cr = l/r
         Fcr = (.3404+3.8172*cr)/(1+3.9273*cr-0.00695*cr**2)
         Fw = 1
         Fww = 1
         B = Fcr*Fw*Fww
         K_I2 = np.sqrt(np.pi*l)*B

         plt.figure(figsize=(12,6))
         plt.plot(a,K_I1,label='center crack')
         plt.plot(a,K_I2,label='hole crack')
         plt.legend(loc='best')
         plt.xlabel('a')
         plt.ylabel('$K_I$')
         plt.title('Case 2: $R = 0.5$, $L = a - R$')

Out[21]: <matplotlib.text.Text at 0xb75a438>
```



Indeed, when we allow the crack to get longer, we find that the hole crack begins to have a larger stress intensity than the center crack.

0.7 7

In this problem we have the same configuration as in (2.11) (Case 11 on p 53 of text). We "plug and chug."

```
In [22]: c = 1.
         t = 0.25
         P = 9000.*t
         D = 0.375
         W = 6.
         r = D/2.
         cr = c/r
         Fcr = (.3404+3.8172*cr)/(1+3.9273*cr-0.00695*cr**2)
         Fw = np.sqrt(1/np.cos(np.pi*r/W)/np.cos(np.pi*(r+c)/W))
         N = W/D + 2.5
         if N > 4.5:
             N = 4.5
         Fww = 1-((1.32*W/D-0.14)**-(.98+(0.1*W/D)**.1) - 0.02)*(2*c/(W-D))**N
         F3 = 0.098 + 0.3592*np.exp(-3.5089*cr) + 0.3817*np.exp(-0.5515*cr)
         s = 6000.
         s_br = P/(D*t)
         B1 = Fcr*Fw*Fww
         B2 = s_br/s*F3*Fw*Fww
         B = B1 + B2
```

```
KI = s*np.sqrt(np.pi*c)*B
print KI
```

```
16844.8222112
```

So the stress intensity factor for this case is $K_I = 16.8\text{ksi}\sqrt{\text{in.}}$

0.8 8

For part a we have

```
In [23]: a = 6.0
         s = 10.0
         w = 15.0
         KIa = 1.12*s*np.sqrt(np.pi*a)
         print KIa
```

```
48.6260043063
```

Which gives $K_I = 48.6\text{ksi}\sqrt{\text{in.}}$
In part b we use (2.4a) (case 5 p 51)

```
In [24]: aw = a/w
         B = 1.122 - 0.231*aw + 10.55*aw**2 - 21.71*aw**3 + 30.82*aw**4
         KIb = s*np.sqrt(np.pi*a)*B
         print KIb
```

```
91.9184305974
```

Which gives $K_I = 91.9\text{ksi}\sqrt{\text{in.}}$
Here the difference is signficant. The assumptions that $\beta = 1.12$ is only appropriate when a/W is very small, as in the following where we consider $a = 0.5$

```
In [25]: a = 0.5
         s = 10.0
         w = 15.0
         KIa = 1.12*s*np.sqrt(np.pi*a)
         print KIa

         aw = a/w
         B = 1.122 - 0.231*aw + 10.55*aw**2 - 21.71*aw**3 + 30.82*aw**4
         KIb = s*np.sqrt(np.pi*a)*B
         print KIb
```

```
14.0371183379
14.1029950047
```

0.9 9

In this problem we compare (2.12) (case 12 p 53) to 3.1 (case 18 p. 56). For a through crack we have

```
In [26]: d = 0.375
         r = d/2
         c = .075
         w = 0.96
         t = 0.15
         s = 20
         rrc = r/(r+c)
         cr = c/r
         #no bearing stress, B2 = 0
         B3 = 0.7071 + 0.7548*rrc + 0.3415*rrc**2 + .6420*rrc**3 + 0.9196*rrc**4
         Fw = np.sqrt(1/np.cos(np.pi*r/w)/np.cos(np.pi*(r+c/2)/(w-c)))
         N = 2.65 - 0.24*(2.75-w/d)**2
         if N < 2.275:
             N = 2.275
         Fww = 1-N**(-w/d)*(2*c/(w-d))**(w/d+0.5)
         B = B3*Fw*Fww
         K_Ia = s*np.sqrt(np.pi*c)*B
         print K_Ia
```

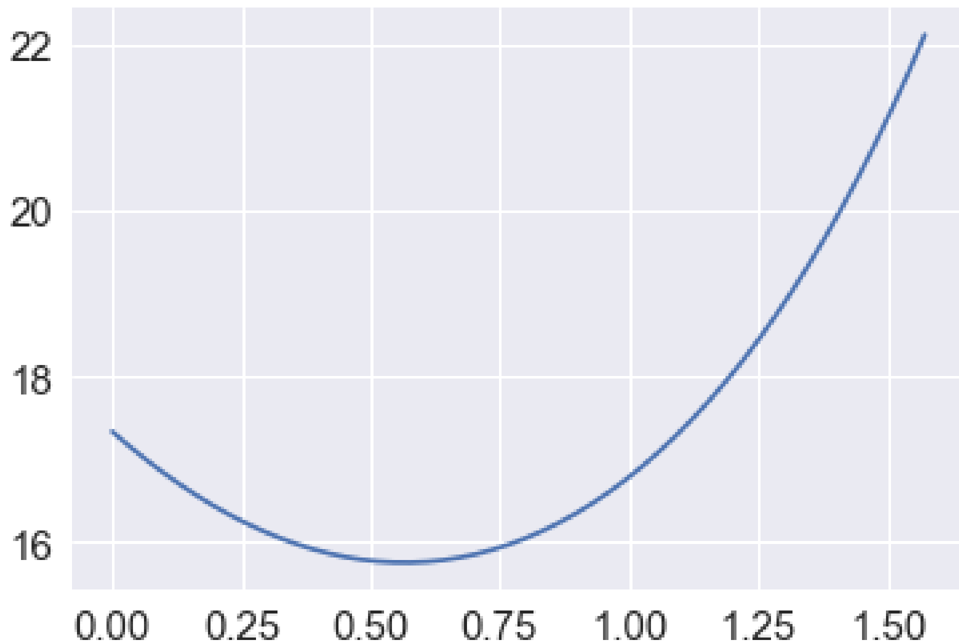
```
24.3127183417
```

Which gives a stress intensity of $K_I = 24.3\text{ksi}\sqrt{\text{in}}$.
For a quarter circular crack, we use 3.1 (with $a = c$).

```
In [27]: a = c
         phi = np.linspace(0,np.pi/2)
         n = 1
         b = w/2
         fw = np.sqrt(1/np.cos(np.pi*r/(2*b))/np.cos((
                     np.pi*(2*r+n*c)/(4*(b-c)+2*n*c)*np.sqrt(a/t))))
         lam = 1/(1+c/r*np.cos(0.85*phi))
         g2 = (1+0.358*lam+1.425*lam**2-1.578*lam**3+2.156*lam**4)/(1+0.13*lam**2)
         M1 = 1.13 - 0.09*a/c
         M2 = -0.54 + 0.89/(0.2+a/c)
         M3 = 0.5 - 1/(0.65+a/c) + 14*(1-a/c)**24
         Q = 1+1.464*(a/c)**1.65
         g1 = 1+(0.1+0.35*(a/t)**2)*(1-np.sin(phi))**2
         g3 = (1+0.04*(a/c))*(1+0.1*(1-np.cos(phi))**2)*(0.85+0.15*(a/t)**.25)
         g4 = 1 - 0.7*(1-a/t)*(a/c-0.2)*(1-a/c)
         f_phi = ((a/c)**2*np.cos(phi)**2+np.sin(phi)**2)**.25
         Fch = (M1 +M2*(a/t)**2 + M3*(a/t)**4)*g1*g2*g3*g4*f_phi*fw
         B = np.sqrt(1/Q)*Fch
         KI_double = s*np.sqrt(np.pi*a)*B
         KI_b = np.sqrt((4/np.pi + a*c/(2*t*r))/(4/np.pi+a*c/(t*r)))*KI_double
```

```
plt.figure()
plt.plot(phi,KI_b)
print max(KI_b)
```

```
22.1230461218
```



Thus the stress intensity factor for a quarter circular crack is only $K_I = 22.1\text{ksi}\sqrt{\text{in}}$.