AE837

Advanced Mechanics of Damage Tolerance

Dr. Nicholas Smith
Wichita State University, Department of Aerospace Engineering September 10, 2019

upcoming schedule

- Sep 10 - Stress Intensity Solutions
- Sep 12 - Finite Size Effects, K-Dominance, HW 2 Due
- Sep 17 - Fracture Criterion
- Sep 19 - Exam Review, HW 3 Due

outline

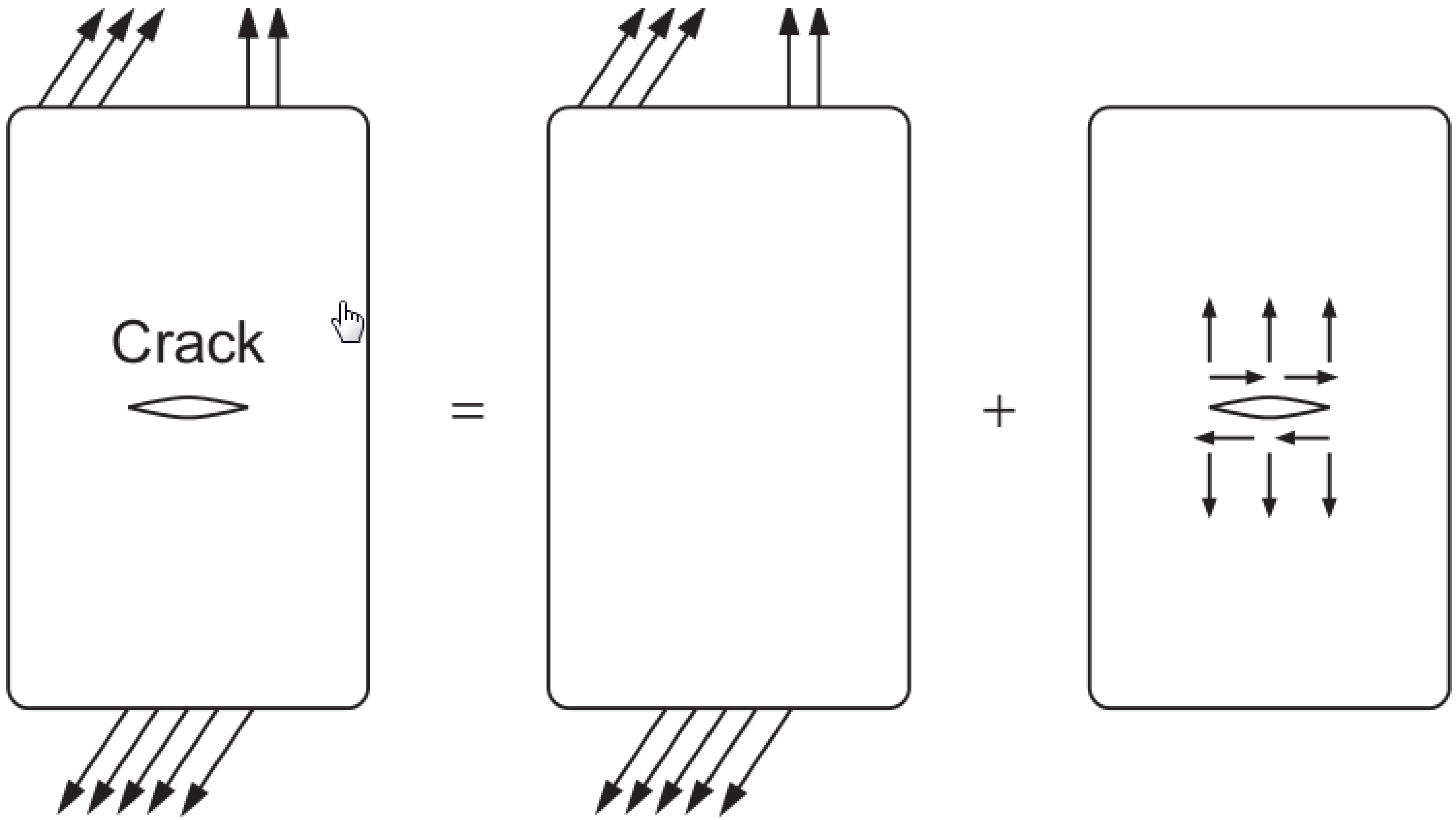
- crack with discrete load
- arbitrary face loads
- edge crack
- finite specimen effects

crack with discrete load

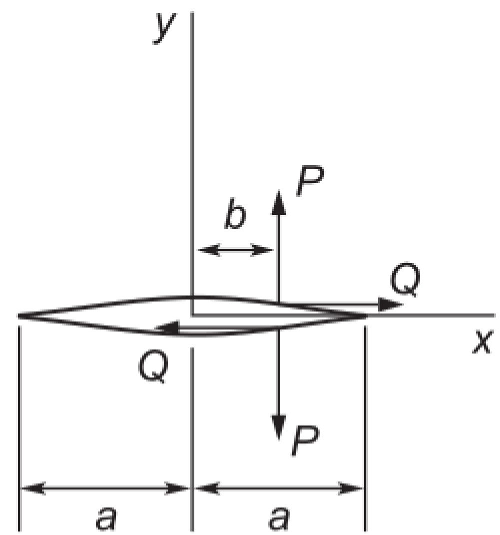
fundamental stress intensity solutions

- In general, we will not have infinite plates with uniform loading
- We can combine previous results with superposition and general elasticity solution techniques to find more useful cases

fundamental stress intensity solutions



concentrated splitting forces



splitting forces

- Consider the case shown on the previous slide, with concentrated splitting forces of magnitude P acting some distance $x=b$ from the center of a centered crack in an infinite plate

boundary conditions

- The boundary conditions for this problem are

$$
\begin{aligned}
\sigma_{yy} &= 0 \qquad \text{at} \qquad |x| \le a, x \ne b, \text{ and } y = 0 \\
\int_{-a}^{a} \sigma_{yy} dx &= -P \qquad \text{at} \qquad y = 0^+ \text{ and } y = 0^- \\
\sigma_{xy} &= 0 \qquad \text{at} \qquad |x| \le a, \text{ and } y = 0 \\
\sigma_{xx} = \sigma_{yy} &= \sigma_{xy} \to 0 \qquad \text{at} \qquad x^2 + y^2 \to \infty
\end{aligned}
$$

splitting forces

- Consider the Westergaard function

$$Z_I = \frac{P}{\pi(z-b)}\sqrt{\frac{a^2-b^2}{z^2-a^2}}$$

splitting forces

- We find that the boundary conditions are satisfied and the stress intensity factor at the right tip is

$$K_I = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a+b}{a-b}}$$

- And the left tip:

$$K_I = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a-b}{a+b}}$$

concentrated shear

- We can formulate a similar problem for a concentrated shear force, Q, acting along the crack face
- This gives boundary conditions of:

$$
\begin{aligned}
\sigma_{xy} &= 0 \qquad \text{at} \qquad |x| \leq a, x \neq b, \text{ and } y = 0 \\
\int_{-a}^{a} \sigma_{xy} dx &= -Q \qquad \text{at} \qquad y = 0^{+} \text{ and } y = 0^{-} \\
\sigma_{yy} &= 0 \qquad \text{at} \qquad |x| \leq a, \text{ and } y = 0 \\
\sigma_{xx} = \sigma_{yy} = \sigma_{xy} &\to 0 \qquad \text{at} \qquad x^2 + y^2 \to \infty
\end{aligned}
$$

concentrated shear

- We can show that the following Westergaard function can satisfy the boundary conditions

$$Z_{II} = \frac{Q}{\pi(z-b)}\sqrt{\frac{a^2-b^2}{z^2-a^2}}$$

- Which gives the following stress intensity factors

$$K_{II} = \frac{Q}{\sqrt{\pi a}}\sqrt{\frac{a+b}{a-b}} \text{ right}$$

$$K_{II} = \frac{Q}{\sqrt{\pi a}}\sqrt{\frac{a-b}{a+b}} \text{ left}$$

arbitrary face loads

arbitrary face load

- Assume a crack is subjected to some arbitrary distributed pressure, $p(x)$ at the crack face
- To find the stress intensity factor, we consider an infinitesimal element $d\xi$ at $x=\xi$ on the crack face
- The force exerted on this element is $p(\xi)\, d\xi$

arbitrary face load

- We can now use the previous solution for a concentrated force and consider the incremental effect of many infinitesimally small forces

$$dK_I = \frac{p(\xi)d\xi}{\sqrt{\pi a}} \sqrt{\frac{a+\xi}{a-\xi}} \text{ right}$$

$$dK_I = \frac{p(\xi)d\xi}{\sqrt{\pi a}} \sqrt{\frac{a-\xi}{a+\xi}} \text{ left}$$

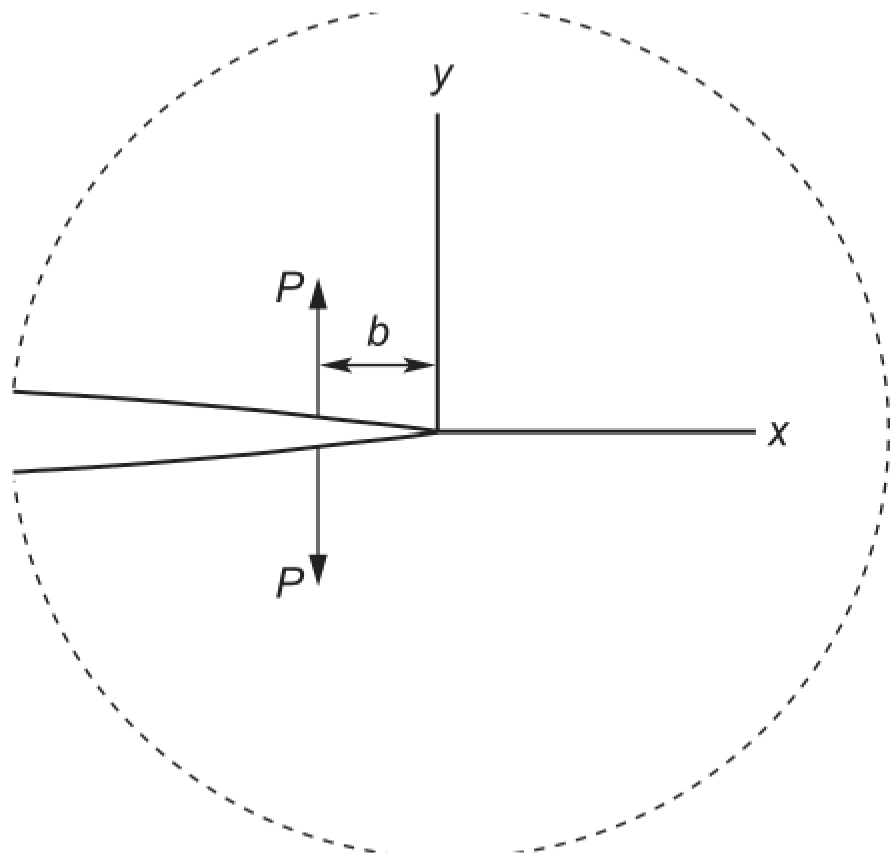
arbitrary face load

- We can find the total stress intensity factor by integrating from $\xi = -a \to a$
- The same procedure can be used to find the stress intensity for arbitrary shear loading along the crack face

edge crack

edge crack

- We can also consider the case of splitting forces on an edge crack



edge crack

- The boundary conditions for this problem are:

$$\sigma_{yy} = 0 \qquad \text{at} \qquad -\infty < x < 0, x \neq b, \text{ and } y = 0$$

$$\int_{-a}^{a} \sigma_{yy} dx = -P \qquad \text{at} \qquad y = 0^{+} \text{ and } y = 0^{-}$$

$$\sigma_{xy} = 0 \qquad \text{at} -\infty < x < 0 \text{ and } y = 0$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{xy} \rightarrow 0 \qquad \text{at} \qquad x^2 + y^2 \rightarrow \infty$$

edge crack

- The Westergaard function to solve this problem is given by:

$$Z_I = \frac{P}{\pi(z+b)}\sqrt{\frac{b}{z}}$$

- Which gives a stress intensity factor of

$$K_I = P\sqrt{\frac{2}{\pi b}}$$

finite specimen effects

finite specimen effects

- Up to this point, we have considered infinitely large panels
- In general, the effect of finite size is modeled using numerical techniques, such as finite elements
- These corrections will generally take the form of some dimensionless factor that is multiplied with the infinite stress intensity factor

$$K_I = \sigma\sqrt{\pi a}F(a/b, a/H)$$