

# **AE837**

## **Advanced Mechanics of Damage Tolerance**

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering September 10, 2019

# upcoming schedule

- Sep 10 - Stress Intensity Solutions
- Sep 12 - Finite Size Effects, K-Dominance, HW 2 Due
- Sep 17 - Fracture Criterion
- Sep 19 - Exam Review, HW 3 Due

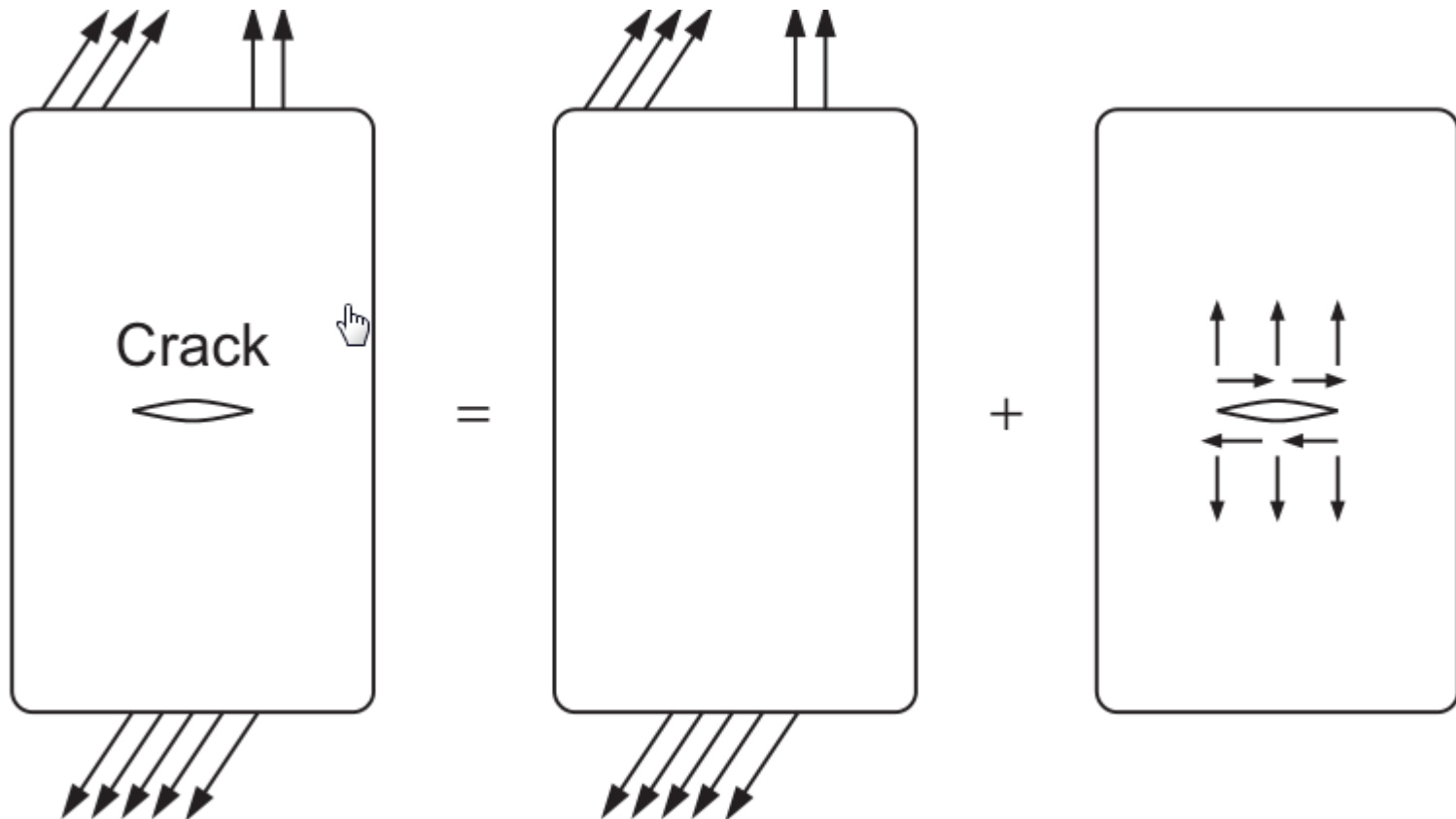
# outline

- crack with discrete load
- arbitrary face loads
- edge crack
- finite specimen effects

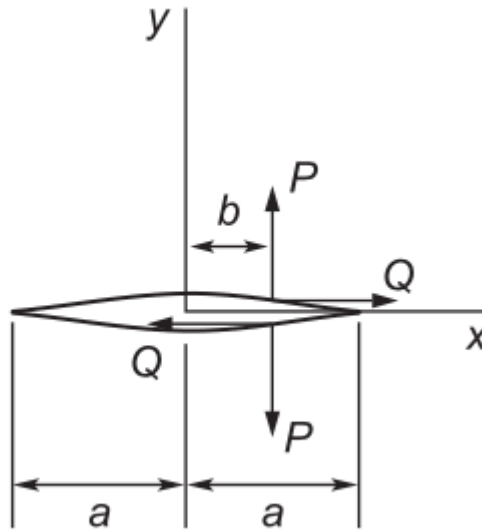
**crack with discrete  
load**

# fundamental stress intensity solutions

- In general, we will not have infinite plates with uniform loading
- We can combine previous results with superposition and general elasticity solution techniques to find more useful cases



# concentrated splitting forces



# splitting forces

- Consider the case shown on the previous slide, with concentrated splitting forces of magnitude  $P$  acting some distance  $x=b$  from the center of a centered crack in an infinite plate

# boundary conditions

- The boundary conditions for this problem are

$$\begin{aligned}\sigma_{yy} &= 0 & \text{at} & \quad |x| \leq a, x \neq b, \text{ and } y = 0 \\ \int_{-a}^a \sigma_{yy} dx &= -P & \text{at} & \quad y = 0^+ \text{ and } y = 0^- \\ \sigma_{xy} &= 0 & \text{at} & \quad |x| \leq a, \text{ and } y = 0 \\ \sigma_{xx} = \sigma_{yy} = \sigma_{xy} &\rightarrow 0 & \text{at} & \quad x^2 + y^2 \rightarrow \infty\end{aligned}$$



# splitting forces

- Consider the Westergaard function

$$Z_I = \frac{P}{\pi(z-b)} \sqrt{\frac{a^2 - b^2}{z^2 - a^2}}$$

# splitting forces

- We find that the boundary conditions are satisfied and the stress intensity factor at the right tip is

$$K_I = \frac{P}{\sqrt{\pi a}} \sqrt{\frac{a+b}{a-b}}$$

- And the left tip:

$$K_I = \frac{P}{\sqrt{\pi a}} \sqrt{\frac{a-b}{a+b}}$$

# concentrated shear

- We can formulate a similar problem for a concentrated shear force,  $Q$ , acting along the crack face
- This gives boundary conditions of:

$$\begin{aligned}\sigma_{xy} &= 0 & \text{at} & \quad |x| \leq a, x \neq b, \text{ and } y = 0 \\ \int_{-a}^a \sigma_{xy} dx &= -Q & \text{at} & \quad y = 0^+ \text{ and } y = 0^- \\ \sigma_{yy} &= 0 & \text{at} & \quad |x| \leq a, \text{ and } y = 0 \\ \sigma_{xx} = \sigma_{yy} = \sigma_{xy} &\rightarrow 0 & \text{at} & \quad x^2 + y^2 \rightarrow \infty\end{aligned}$$

# concentrated shear

- We can show that the following Westergaard function can satisfy the boundary conditions

$$Z_{II} = \frac{Q}{\pi(z-b)} \sqrt{\frac{a^2 - b^2}{z^2 - a^2}}$$

- Which gives the following stress intensity factors

$$K_{II} = \frac{Q}{\sqrt{\pi a}} \sqrt{\frac{a+b}{a-b}} \text{ right}$$

$$K_{II} = \frac{Q}{\sqrt{\pi a}} \sqrt{\frac{a-b}{a+b}} \text{ left}$$

# **arbitrary face loads**

# arbitrary face load

- Assume a crack is subjected to some arbitrary distributed pressure,  $p(x)$  at the crack face
- To find the stress intensity factor, we consider an infinitesimal element  $d\xi$  at  $x=\xi$  on the crack face
- The force exerted on this element is  $p(\xi) d\xi$

# arbitrary face load

- We can now use the previous solution for a concentrated force and consider the incremental effect of many infinitesimally small forces

$$dK_I = \frac{p(\xi)d\xi}{\sqrt{\pi a}} \sqrt{\frac{a+\xi}{a-\xi}} \text{ right}$$

$$dK_I = \frac{p(\xi)d\xi}{\sqrt{\pi a}} \sqrt{\frac{a-\xi}{a+\xi}} \text{ left}$$

# arbitrary face load

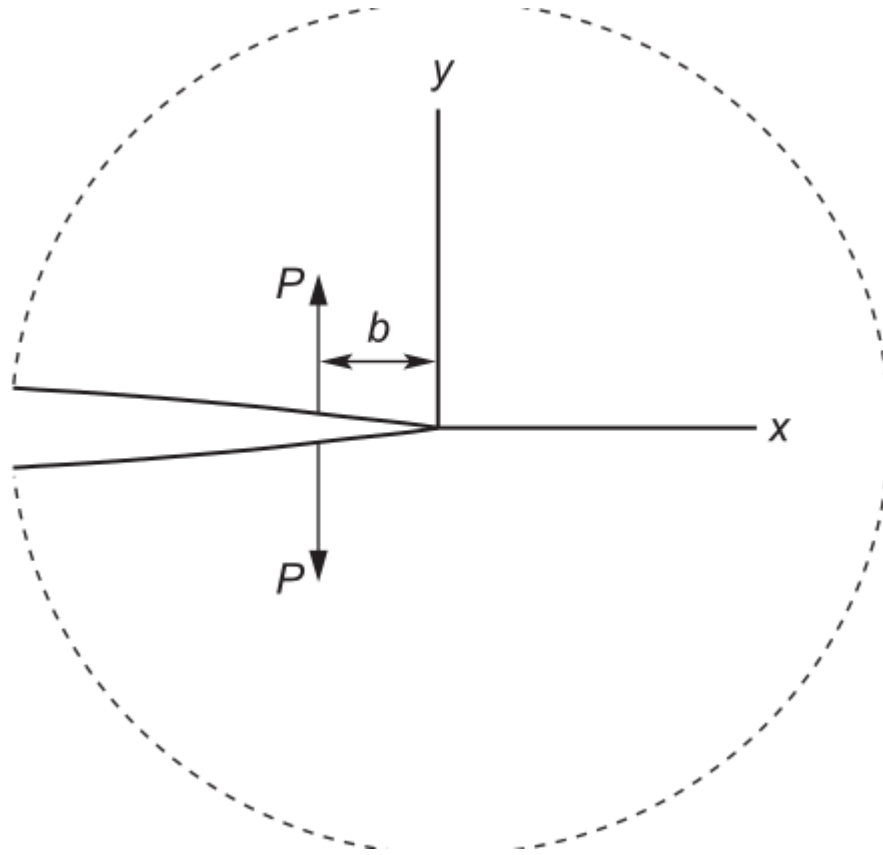
- We can find the total stress intensity factor by integrating from  $\xi = -a \rightarrow a$
- The same procedure can be used to find the stress intensity for arbitrary shear loading along the crack face



# edge crack

# edge crack

- We can also consider the case of splitting forces on an edge crack



# edge crack

- The boundary conditions for this problem are:

$$\begin{aligned}\sigma_{yy} &= 0 & \text{at} & & -\infty < x < 0, x \neq b, \text{ and } y = 0 \\ \int_{-a}^a \sigma_{yy} dx &= -P & \text{at} & & y = 0^+ \text{ and } y = 0^- \\ \sigma_{xy} &= 0 & \text{at} & & -\infty < x < 0 \text{ and } y = 0 \\ \sigma_{xx} = \sigma_{yy} = \sigma_{xy} &\rightarrow 0 & \text{at} & & x^2 + y^2 \rightarrow \infty\end{aligned}$$

# edge crack

- The Westergaard function to solve this problem is given by:

$$Z_I = \frac{P}{\pi(z+b)} \sqrt{\frac{b}{z}}$$

- Which gives a stress intensity factor of

$$K_I = P \sqrt{\frac{2}{\pi b}}$$

# **finite specimen effects**

# finite specimen effects

- Up to this point, we have considered infinitely large panels
- In general, the effect of finite size is modeled using numerical techniques, such as finite elements
- These corrections will generally take the form of some dimensionless factor that is multiplied with the infinite stress intensity factor

$$K_I = \sigma \sqrt{\pi a} F(a/b, a/H)$$