AE837

Advanced Mechanics of Damage Tolerance

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upcoming schedule

- Sep 5 - Mode II and III Westergaard
- Sep 10 - Stress Intensity Solutions
- Sep 12 - Finite Size Effects, K-Dominance, HW 2 Due
- Sep 17 - Fracture Criterion

outline

- complex stress intensity
- westergaard mode ii
- westergaard mode iii

complex stress intensity

complex notation

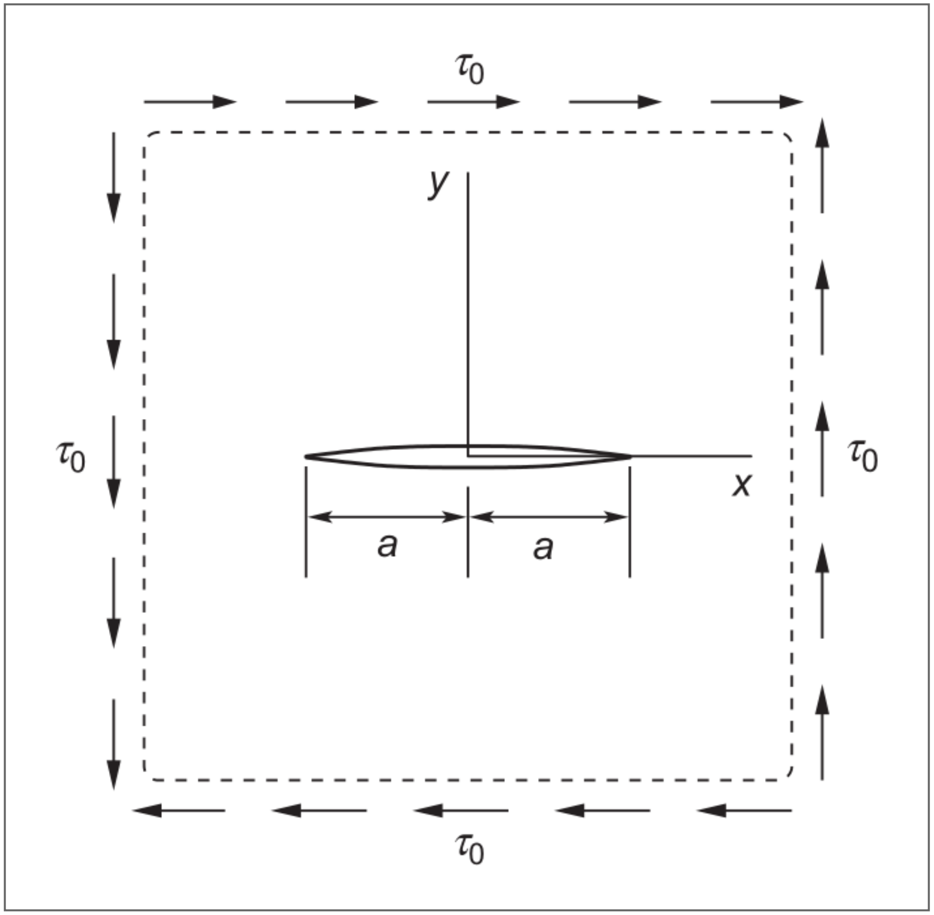
- it is often convenient to use complex notation to combine Modes I and II
  $K = K_I + iK_{II}$
- in terms of the Westergaard functions, we find
  $K\sqrt{2\pi}\lim_{z\to a}(\sqrt{z-a}(Z_I - iZ_{II}))$

mode iii

- for mode iii we find, in terms of the westergaard function
  $K_{III} = \sqrt{2\pi} \lim_{z \to a}(\sqrt{z-a} Z'_{III}(z))$
- a full derivation is found in Ch 3, pp 48-50

westergaard mode ii

mode ii



mode ii

- Boundary conditions at the crack tip ($y = 0, |x| < a$) are:
  $\sigma_{xy} = \sigma_{yy} = 0$
- Boundary conditions far away from the crack tip ($x^2 + y^2 \to \infty$) are:
  $\sigma_{xx} = \sigma_{yy} = 0$
  $\sigma_{xy} = \tau_0$

mode ii

- Consider the Westergaard function
  $$Z_{II}(z) = \frac{\tau_0 z}{\sqrt{z^2 - a^2}}$$

mode ii stress

- We calculate stress terms from Z_{II} as:
  $\sigma_{xx} = 2\mathrm{Im}Z_{II} + y\mathrm{Re}Z'_{II}$
  $\sigma_{yy} = -y\mathrm{Re}Z'_{II}$
  $\sigma_{xy} = \mathrm{Re}Z_{II} - y\mathrm{Im}Z'_{II}$

mode ii displacement

- And displacement as

$$2\mu u_x = \frac{1}{2}(\kappa+1)\mathrm{Im}\hat{Z}_{II} + y\mathrm{Re}Z_{II} + \frac{\kappa+1}{2}By$$
$$2\mu u_y = -\frac{1}{2}(\kappa-1)\mathrm{Re}\hat{Z}_{II} - y\mathrm{Im}Z_{II} - \frac{\kappa+1}{2}Bx$$

mode ii solution

- For the Westergaard function we find a stress field of

$$\sigma_{xx} = \frac{\tau_0 r}{\sqrt{r_1 r_2}}\left[2\sin\left(\theta - \frac{1}{2}\theta_1 - \frac{1}{2}\theta_2\right) - \frac{a^2}{r_1} r_2 \sin\theta \cos\frac{3}{2}(\theta_1 + \theta_2)\right]$$

$$\sigma_{yy} = \frac{\tau_0 a^2 r}{(r_1 r_2)^{3/2}} \sin\theta\cos\frac{3}{2}(\theta_1 + \theta_2)$$

$$\sigma_{xy} = \frac{\tau_0 r}{\sqrt{r_1 r_2}}\left[\cos\left(\theta - \frac{1}{2}\theta_1 - \frac{1}{2}\theta_2\right) - \frac{a^2}{r_1} r_2 \sin\theta \sin\frac{3}{2}(\theta_1 + \theta_2)\right]$$

westergaard mode iii

antiplane

- For antiplane problems, we can formulate in terms of displacement
  $u_x = u_y = 0 \qquad u_z = w(x, y)$
- This yields the strains
  $e_{xz} = \dfrac{1}{2}\dfrac{\partial w}{\partial x} \qquad e_{yz} = \dfrac{1}{2}\dfrac{\partial w}{\partial y}$

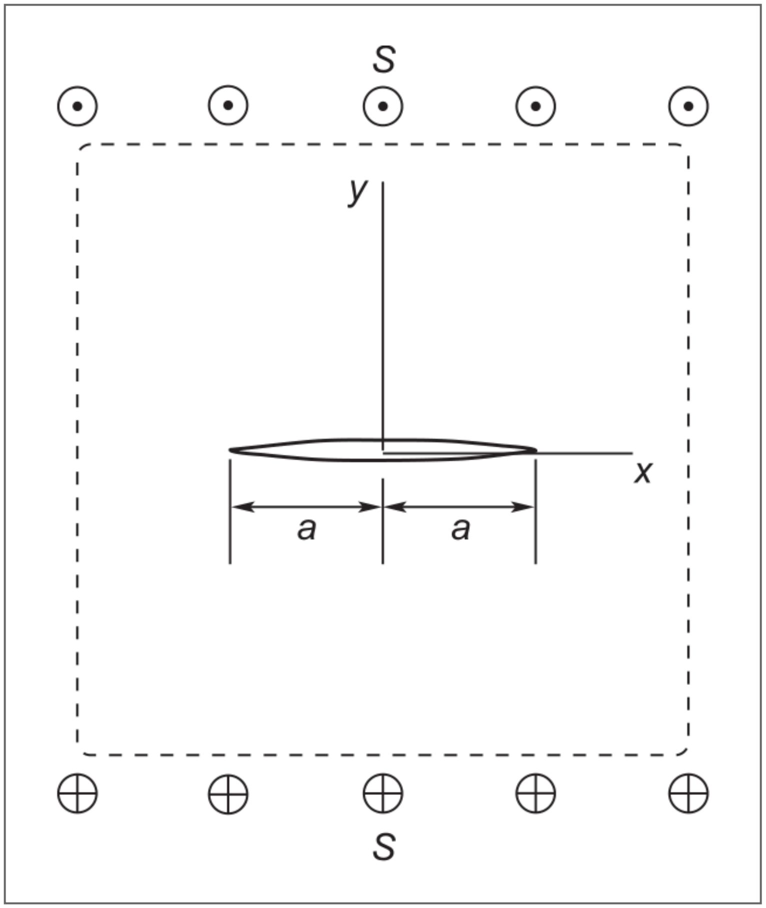
antiplane

- The stresses can be found with Hooke's Law
  $\sigma_{xz} = 2\mu e_{xz} \qquad \sigma_{yz} = 2\mu e_{yz}$
- The equlibrium equations then reduce to
  $$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{y} = 0$$
- Which, in terms of displacement, reduces to $\nabla^2 w = 0$

antiplane

- Therefore w must be a harmonic function, let
  $w = \frac{1}{\mu}\mathrm{Im}Z_{III}(z)$
- This gives the stresses as
  $\sigma_{xz} - i\sigma_{yz} = -iZ'_{III}(z)$

mode iii



mode iii

- The boundary conditions for a Mode III crack are
  $\sigma_{yz} = 0 \qquad \text{at} \qquad |x| < a| \text{ and } y = 0$
  $\sigma_{yz} = S \qquad \text{at} \qquad |y| \to \infty$

mode iii

- We choose
  $Z_{III} = S\sqrt{z^2 - a^2}$
- and find
  $$\sigma_{yz} = \frac{Sr}{\sqrt{r_1 r_2}}\cos\left(\theta - \frac{1}{2}\theta_1 - \frac{1}{2}\theta_2\right)$$
  $$\sigma_{xz} = \frac{Sr}{\sqrt{r_1 r_2}}\sin\left(\theta - \frac{1}{2}\theta_1 - \frac{1}{2}\theta_2\right)$$