AE837

Advanced Mechanics of Damage Tolerance

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upcoming schedule

- Aug 29 - Griffith Fracture
- Sep 3 - Elastic Stress Field, Homework 1 Due
- Sep 5 - Elastic Stress Field
- Sep 10 - Elastic Stress Field

outline

- relation among energies

relation among energies

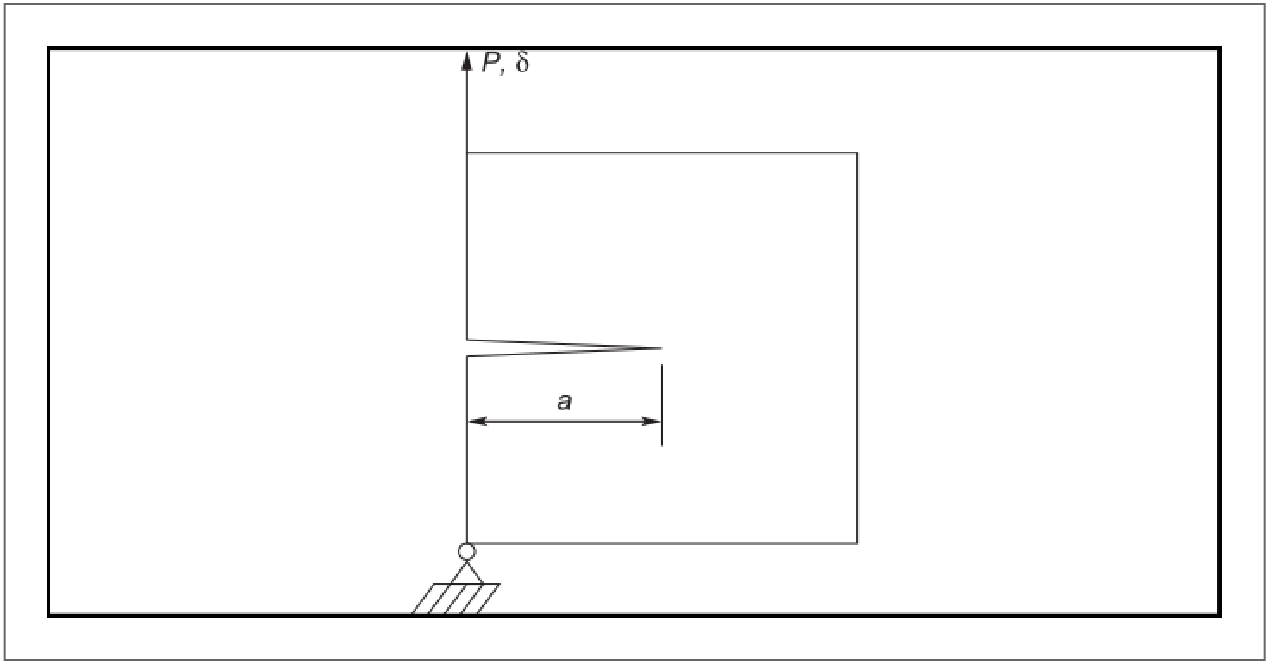
energy balance

- Griffith's theory is founded on the principle of energy balance
- During crack extension, the external work done, dW_e must equal the increment of surface energy, dW_s, and the increment of elastic strain energy, dU
  $dW_s + dU = dW_e$

energy balance

- For a conservative field we can write this as
  $$\frac{\partial}{\partial a}(W_s + U + V) = 0$$
- Where a negative potential, V, implies positive external work done dW_e

example



example

- The displacement at the top edge will be proportional to the applied load by some elastic compliance of the specimen
  $\delta = SP$
- Note that this compliance, S, will be a function of the crack length, a
- The strain energy can be expressed as
  $$U = \int_{\delta=0}^{\delta=SP} P d\delta = \int_{\delta=0}^{\delta=SP} \frac{\delta}{S} d\delta$$

example

- After integrating
  $$U = \frac{1}{2S}(\delta^2)|_0^{SP} = \frac{1}{2}SP^2$$

example

- To find the incremental strain energy increase (where both P and S should be treated as variable), we find
  $$dU = \frac{1}{2}P^2dS + SPdP$$
- We will now consider two loading cases, one with fixed displacement and the other with fixed loading force

fixed displacement

- Under constant displacement we have
  $\delta = SP = \text{constant}$
- This means that the derivative of displacement will be zero, hence
  $d\delta = SdP + PdS = 0$
- and
  $SdP = -PdS$

fixed displacement

- Substituting into the previous equation gives
  $$dU = -\frac{1}{2}P^2 dS$$
- Since $d\delta = 0$, the external work, $dW_e = 0$ and we find
  $$dW_s = -dU = \frac{1}{2}P^2 dS$$

fixed load

- If instead of fixing displacement we fix the applied load we have $dP = 0$
- Which gives
  $dU = \frac{1}{2}P^2dS$
- The strain energy increases (while under fixed displacement it decreased)

fixed load

- Further, we can find the external work done as $dW_e = Pd\delta = P^2dS$
- And thus, from energy balance, we find the surface energy $dW_s = \frac{1}{2}P^2dS$
- In this case the external work is equally split between strain energy and surface energy

comparison

- In both cases, the energy released is $dW = dW_e - dU$

$$dW = 0 - (-\frac{1}{2}P^2 dS) \qquad \text{fixed displacement}$$

$$dW = P^2 dS - \frac{1}{2}P^2 dS \qquad \text{fixed load}$$

- And we see that the energy released is independent of the load type

example - stable crack

- During experimental characterization, it is often desirable to measure crack growth
- To do this accurately, the crack growth must be stable
- For crack growth to be stable, the strain energy should decrease as crack length increases ($\partial G/\partial a \leq 0$)
- If we recall G for the Double-Cantilever Beam (DCB) specimen

$$G = \frac{P^2 a^2}{bEI}$$

example - stable crack

- Under fixed-load conditions, we find
$$\frac{dG}{da} = \frac{2P^2a}{bEI}$$
- This is always positive, and thus results in unstable crack growth
- Under fixed-displacement conditions, we substitute for P in terms of displacement using $P = \delta/S$

example - stable crack

- From beam theory we can express S more precisely as
  $$S = \frac{2a^3}{3EI}$$
  $$\frac{dG}{da} = -\frac{9\delta^2 EI}{ba^5}$$
- Which is always stable, so for DCB tests, displacement control is generally used

example - peel of thin film