# AE837

## Advanced Mechanics of Damage Tolerance

Dr. Nicholas Smith
Wichita State University, Department of Aerospace Engineering September 26, 2019

# upcoming schedule

- Sep 26 - Exam return, Direct Method
- Oct 1 - Virtual Crack Closure, J-integral
- Oct 3 - J-Integral, Cohesive Zone
- Oct 8 - eXtended Finite Element Method (XFEM)
- Oct 10 - XFEM, Homework 4 Due

# outline

- exam
- crack closure
- direct method in finite elements

# exam

# results

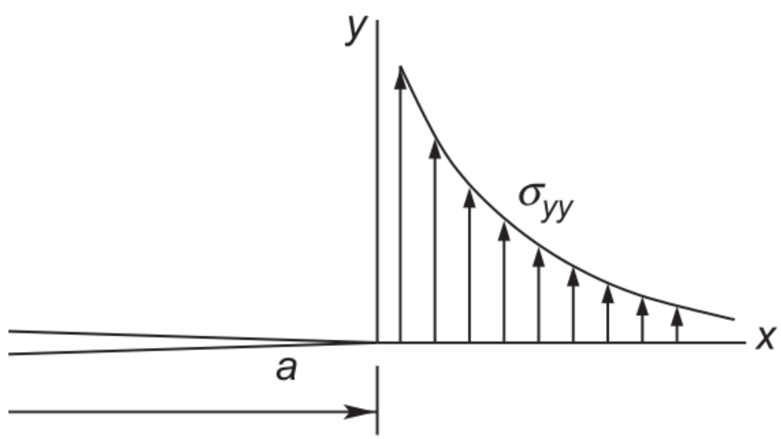
- High score was 93, curved scores by adding 7 (check my math)
- Traction boundary conditions
- Physical meaning of integrals

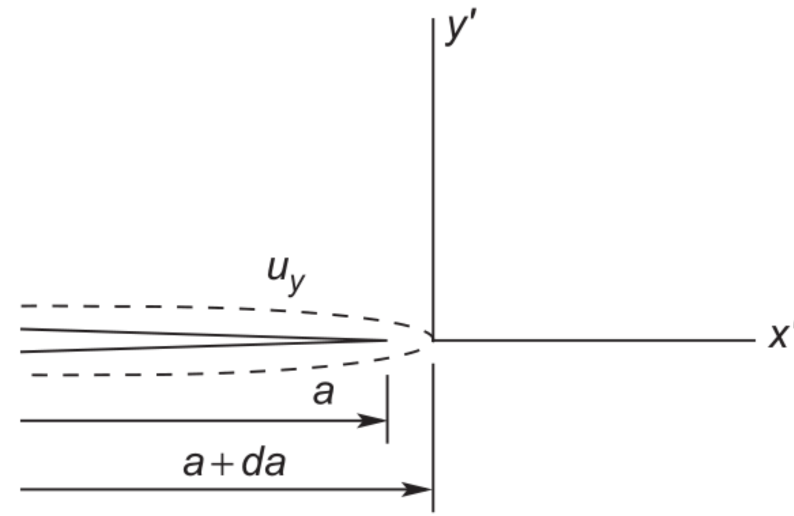
# crack closure

# crack closure

- We have discussed that $G_I$ and $K_I$ are related for perfectly elastic materials
- We will use the crack closure method to find this relationship

# crack closure



# crack closure

- The stress field ahead of the crack tip is given by

$$\sigma_{yy} = \frac{K_I(a)}{\sqrt{2\pi x}}$$

- And the displacement field at the former crack tip after some extension *da* is

$$u_y = \frac{\kappa + 1}{4\mu\pi} K_I \sqrt{2\pi(da - x)}$$

# crack closure

- If we consider a very small $da$, then we can consider that $K_I$ in both cases is equivalent
- The strain energy associated with crack extension can be thought of as the work done by $\sigma_{yy}$ to move $u_y$
- This must also be equal to the strain energy released, $G_I da$

$$G_I da = \int_0^{da} \sigma_{yy} u_y dx$$

# crack closure

- After substituting what we have already shown for $\sigma_{yy}$ and $u_y$ we find

$$G_I da = \frac{\kappa + 1}{4\mu\pi} K_I^2 \int_0^d a \sqrt{\frac{1 - x/a}{x/da}}$$

- After some integration tricks, we find

$$G_I = \frac{\kappa + 1}{8\mu} K_I^2$$

# kappa

- The parameter $\kappa$ helps convert between plane strain and plane stress
- In plane strain, $\kappa = 3 - 4v$
- In plane stress, $\kappa = (3 - v)/(1 + v)$

# direct method in finite elements

# direct method

- As we discussed previously, since we know both $\sigma_{yy}$ and $u_y$ as functions of $K_I$, we should be able to use those in finite element analysis to find $K_I$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi x}}$$

$$u_y = \frac{K_I(\kappa + 1)}{4\mu\pi}\sqrt{2\pi x}$$

# direct method

- This results in

$$K_I = \sigma_{yy}\sqrt{2\pi x}$$

$$K_I = \frac{2\mu u_y}{\kappa + 1}\frac{2\pi x}{x}$$

# accuracy

- This method will only be accurate in the K-dominance zone (it ignores non-singular stress)
- Needs a very fine mesh or $1/\sqrt{r}$ singularity elements