AE837

Advanced Mechanics of Damage Tolerance

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upcoming schedule

- Oct 3 - J-Integral
- Oct 8 - Cohesive Zone, eXtended Finite Element Method (XFEM)
- Oct 10 - XFEM, Homework 4 Due
- Oct 15 - Fall Break (no class)

outline

- j-integral

j-integral

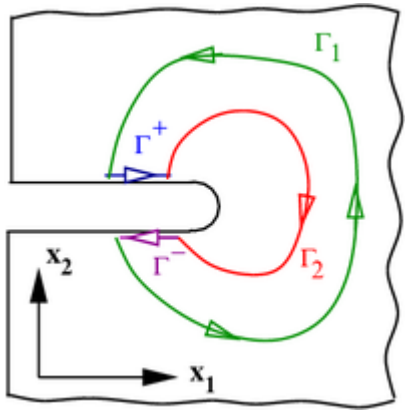
j-integral

- The J-Integral is defined as

$$\int_{\Gamma} \left(W dy - T_i \frac{\partial u_i}{\partial x} d\Gamma \right) = \int_{\Gamma} \left(W n_1 - \sigma_{ij} \frac{\partial u_i}{\partial x} n_j \right) d\Gamma$$

- Γ is an arbitrary contour beginning at the lower crack surface and end on the upper crack surface

j-integral



j-integral

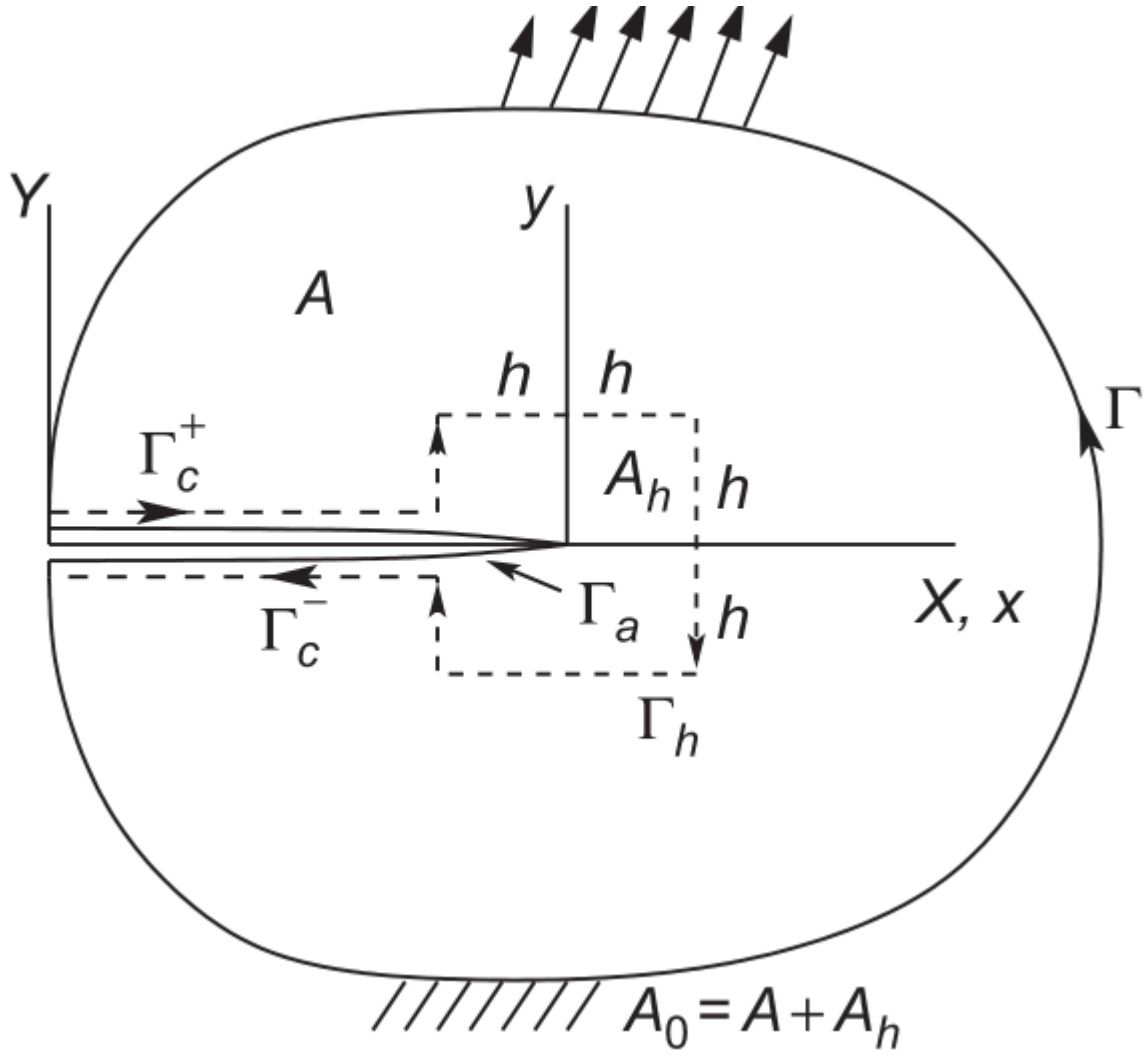
- The J-integral is path-idependent and represents the strain energy release rate
- We can prove this using the following principles from elasticity

$$\sigma_{ij,j} = 0 \qquad \text{(equilibrium)}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad \text{(strain-displacement)}$$

$$\sigma_{ij} = \frac{\partial W}{\partial e_{ij}} \qquad \text{(stress-strain)}$$

j-integral



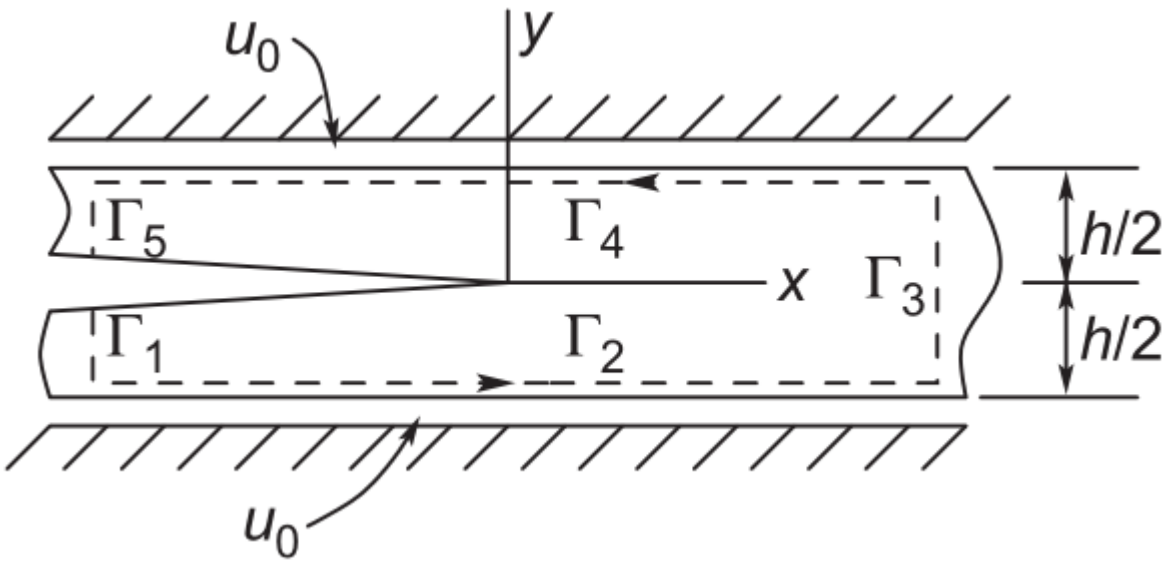
j-integral

- A_0 represents the area enclosed by the contour
- The potential energy can then be expressed as

$$\iint_{A_0} W dX dY - \int_{\Gamma_t} T_i u_i d\Gamma$$

- (worked on board)

examples



finite element

- many solvers have a built-in method for calculating the J-integral
- ABAQUS
- In COMSOL, we have some nice documentation for how to do it, but we need to set up the integral manually

fea implementation

- if you are using some other software, check the documentation, it may not be set up to calculate the J-integral for a symmetric model
- In COMSOL, we will only be calculating half of the J-integral (then we will double it)
- First we model the contours, then we will define local variables (the normal vector along various sections), local integrations (around the contour) and finally calculate J and K

screencast I

0:00 / 4:14

screencast II

0:00 / 6:41