Name:

Homework 5

Due 17 October 2019

1. Calculate the strain energy release rate of the DCB loaded as shown using both the compliance method ($G = \partial U/\partial a$) and the J-Integral method (assume the thickness is 1).

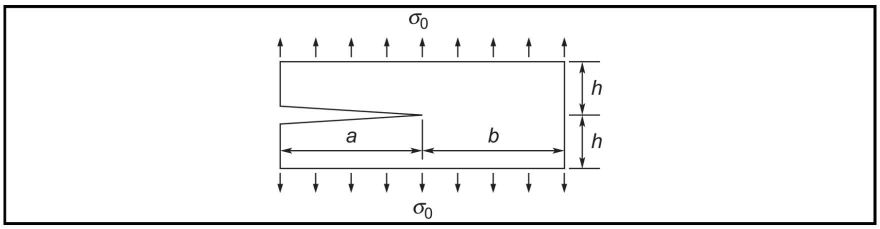

Figure 1: Figure for Problem 1

Solution: For the compliance method, we can divide the beam into 3 sections as usual, where the strain energy from the two "DCB" segments is identical, and for a cantilever beam under uniform load we have $M = \sigma_0 x^2/2$.

$$U_1 = U_2 = \int_0^a \frac{M^2}{2EI} dx \tag{1}$$

$$= \frac{\sigma_0^2}{8EI} \int_0^a x^4 dx \tag{2}$$

$$U_1 = U_2 = \frac{\sigma_0^2 a^5}{40EI} \tag{3}$$

Since we also have strain energy in the "fixed" segment we will consider that as well. If all stress is acting in the y-direction, then the strain energy reduces to

$$U_3 = \frac{1}{2} V \sigma_{ij} \epsilon_{ej} \tag{4}$$

$$= \frac{1}{2}(L-a)(2h)(b)\sigma_0 \left(\frac{\sigma_0}{E}\right) U_3 \qquad = bh(L-a)\frac{\sigma_0^2}{E} \tag{5}$$

We now use $U_T = \frac{1}{b}(U_1 + U_2 + U_3)$ and $G = \frac{\partial U_T}{\partial a}$ to find

$$G = \frac{\sigma_0^2 a^4}{4EIb} + \frac{\sigma_0^2 h}{E} \tag{6}$$

For J-integral method, we trace a path around the outside of the DCB as usual, noting that

$$\Gamma_1 \& \Gamma_5 : W = 0 \qquad T_i = 0 \tag{7}$$

$$\Gamma_2 \& \Gamma_4 : dy = 0 \tag{8}$$

$$\Gamma 3 : T_i = 0 \tag{9}$$

If we calculate each integral separately, we have $J = J_1 + J_2 + J_3 + J_4 + J_5$, where $J_1 = J_5 = 0$ and $J_2 = J_4$, thus we can write

$$J = 2J_2 + J_3 \tag{10}$$

On J_2, we notice that $\frac{\partial u}{\partial x}$ is 0 everywhere, but $\frac{\partial v}{\partial x}$ is non-zero in the cantilever beam portion. Assuming it does behave as a cantilever beam under uniform load, we have

$$v = \frac{\sigma_0}{48EI}(x^4 - 4L^3x - 3x^4) \tag{11}$$

Defining our coordinate system with origin at crack tip with x positive to the left, we can write

$$J_2 = \int_0^a -\sigma_0 \frac{\partial v}{\partial x} dx \qquad = \frac{\sigma_0^2 a^4}{8EI} \tag{12}$$

On J_3 we need only concern ourselves with $W dy$, since $T_i = 0$. This gives

$$J_3 = \int_{-h}^{h} \frac{1}{2}\sigma_0(\sigma_0/E)dy \tag{13}$$

$$J_3 = \frac{\sigma_0^2 h}{E} \tag{14}$$

Substituting in to find the total J-integral we find

$$J = 2J_2 + J_3 = \frac{\sigma_0^2 a^4}{4EI} + \frac{\sigma_0^2 h}{E} \tag{15}$$

which only differs from the compliance method by a thickness term.