Exam 1 Equation Sheet

Stress transformation

$$\begin{bmatrix} \sigma_{rr} & \sigma_{r\theta} \\ \sigma_{r\theta} & \sigma_{\theta\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

Strain energy in simple geometries

General

$$U = \frac{1}{2} V \sigma_{ij} \epsilon_{ij} \tag{2}$$

Rod

$$U = \int_0^L \frac{P^2}{EA} dx \tag{3}$$

Beam

$$U = \int_0^L \frac{M^2}{2EI} dx \tag{4}$$

Westergaard Stress

Mode I

$$\sigma_{xx} = \mathrm{Re}\{Z_I\} - y\mathrm{Im}\{Z_I'\} + 2A \tag{5}$$
$$\sigma_{yy} = \mathrm{Re}\{Z_I\} + y\mathrm{Im}\{Z_I'\} \tag{6}$$
$$\sigma_{xy} = -y\mathrm{Re}\{Z_I'\} \tag{7}$$

Mode II

$$\sigma_{xx} = 2\mathrm{Im}\{Z_{II}\} + y\mathrm{Re}\{Z_{II}'\} \tag{8}$$
$$\sigma_{yy} = -y\mathrm{Re}\{Z_{II}'\} \tag{9}$$
$$\sigma_{xy} = \mathrm{Re}\{Z_{II}\} - y\mathrm{Im}\{Z_{II}'\} \tag{10}$$

Mode III

$$\sigma_{yz} = \mathrm{Re}\{Z_{III}'\} \tag{11}$$
$$\sigma_{xz} = \mathrm{Im}\{Z_{III}'\} \tag{12}$$

Stress Intensity Factor

$$K = \sqrt{2\pi} \lim_{z \to a}\{\sqrt{z-a}(Z_I - iZ_{II})\} \tag{13}$$
$$K_{III} = \sqrt{2\pi} \lim_{z \to a}\{\sqrt{z-a}(Z_{III}')\} \tag{14}$$

K-Dominance

$$\Lambda = \frac{K_I/\sqrt{2\pi x}}{K_I/\sqrt{2\pi x} + |\text{non-singlar part of } \sigma_{yy}|} \tag{15}$$