CHAPTER 1

Introduction

1.1 FAILURE OF SOLIDS

Failure of solids and structures can take various forms. A structure may fail without breaking the material, such as in elastic buckling. However, failure of the material in a structure surely will lead to failure of the structure. Two general forms of failure in solids are excessive permanent (plastic) deformation and breakage. Plasticity can be viewed as an extension of elasticity for decribing the mechanical behavior of solids beyond yielding. The theory of plasticity has been studied for more than a century and has long been employed for structural designs. On the other hand, the latter form of failure is usually regarded as the strength of a solid, implying the total loss of load-bearing capability of the solid. For brittle solids, this form of failure often causes the body under load to break into two or more separated parts.

Unlike plasticity, the prediction of the strength of solid materials was all based on phenomenological approaches before the inception of fracture mechanics. Many phenomenological failure criteria in terms of stress or strain have been proposed and calibrated against experimental results. In the commonly used failure criteria, such as the maximum principal stress or strain criterion, a failure envelope in the stress or strain space is constructed based on limited experimental strength data. Failure is assumed to occur when the maximum normal stress at a point in the material exceeds the strength envelope, that is,

$$\sigma_1 \geq \sigma_f$$

where σ_1 (> 0) is a principal stress and σ_f is the tensile strength of the solid. The failure envelope has also been modified to distinguish the difference between tensile and compressive strengths and to account for the effects of stress interactions.

In general, the classical phenomenological failure theories predict failure of engineering materials and structures with reasonable accuracy in applications where the stress field is relatively uniform. These theories are often unreliable in the presence of high-stress gradients resulting from cutouts. Moreover, there were many premature structural failures at stresses that were well below the critical values specified in the classical failure theories.

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The most frequently cited example is the failure of Liberty cargo ships built during World War II. Among roughly 2700 all-welded hull ships, more than 100 were seriously fractured and about 10 were fractured in half [1-1]. It was demonstrated [1-2] that cracks were first initiated at the stress concentration locations and then propagated in the hull, resulting in the catastrophic failure. Other significant examples include fuselage failure in Comet passenger jet airplanes from 1953 to 1955 [1-3] and failure of heavy rotors in steam turbines from 1955 to 1956 [1-4].

The aforementioned historical events led researchers to recognize that defects are the original cause of failure and in strength predictions, materials cannot be always assumed free of defects. Cracks and other forms of defects may be introduced during materials manufacturing and processing, as well as during service. For instance, rapid quenching of cast irons results in microcracks in the material. Cyclic stresses induce cracks in the connections of the structural components. The stresses at the crack tip are much higher than the material strength, which is measured under a state of uniform stress in laboratory condition. The high stresses near the crack tip drive the crack to extend, leading to the eventual catastrophic failure of the material. Failure caused by crack propagation is usually called *fracture failure*. The classical failure criteria assume that materials are free of defects, and hence are not capable of predicting fracture failure, or failure of materials containing crack-like flaws.

1.2 FRACTURE MECHANICS CONCEPTS

Fracture mechanics is a subject of engineering science that deals with failure of solids caused by crack initiation and propagation. There are two basic approaches to establish fracture criteria, or crack propagation criteria: crack tip stress field (local) and energy balance (global) approaches. In the crack tip field approach, the crack tip stress and displacement states are first analyzed and parameters governing the near-tip stress and displacement fields are identified. Linear elastic analysis of a cracked body shows that stresses around the crack tip vary according to $r^{-1/2}$, where r is the distance from the tip. It is clear that stresses become unbounded as r approaches the crack tip. Such a singular stress field makes the classical strength of materials failure criteria inapplicable.

A fundamental concept of fracture mechanics is to accept the theoretical stress singularity at the crack tip but not use the stress directly to determine failure/crack extension. This is based on the fact that the tip stress is limited by the yield stress or the cohesive stress between atoms and singular stresses are the results of linear elasticity. It is also recognized that the singular stress field is a convenient representation of the actual finite stress field if the discrepancy between the two lies in a small region near the crack tip. This notion is referred to as small-scale yielding.

The stresses near the tip of a crack in linearly elastic solids have the following universal form independent of applied loads and the geometry of the cracked body

(Chapter 3):

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{1}{2}\theta\left(1 - \sin\frac{1}{2}\theta\sin\frac{3}{2}\theta\right)$$
$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{1}{2}\theta\left(1 + \sin\frac{1}{2}\theta\sin\frac{3}{2}\theta\right) \tag{1.1}$$
$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{1}{2}\theta\cos\frac{1}{2}\theta\cos\frac{3}{2}\theta$$

where K_I is the so-called stress intensity factor, which depends on the applied load and crack geometry and (r, θ) are the polar coordinates centered at the crack tip. Here it is assumed that the loads and the geometry are symmetric about the crack line. Equation (1.1) shows that K_I is a measure of the stress intensity near the crack tip.

Based on this obervation, Irwin [1-5] proposed a fracture criterion which states that crack growth occurs when the stress intensity factor reaches a critical value, that is,

$$K_I = K_{Ic} \tag{1.2}$$

where K_{Ic} is called fracture toughness, a material constant determined by experiment. The preceding fracture criterion for cracked solids is fundamentally different from the classical failure criteria based on stresses. It does not directly use stresses or strains, but a proportionality factor in the stress field around the crack tip. K_I is proportional to the applied load but has a dimension of $MPa - \sqrt{m}$ in the SI unit system and $ksi - \sqrt{in}$ in the US customary unit system. K_{Ic} is a new material parameter introduced in fracture mechanics that characterizes the resistance of a material to crack extension.

The criterion in Eq. (1.2) is based on linear elasticity with which the inverse square root singular stress field exists and the stress intensity factor is well defined. The actual fracture process at the crack tip cannot be described using the linear elasticity theory. The rationality of the criterion lies in the condition that the fracture process zone is sufficiently small so that it is well contained inside the singular stress field Eq. (1.1) characterized by the stress intensity factor K_I.

The second approach for establishing a fracture criterion is based on the consideration of global energy balance during crack extension. The potential energy of a cracked solid under a given load is first determined and its variation with a virtual crack extension is then examined. Consider a two-dimensional elastic body with a crack of length a. The total potential energy per unit thickness of the system is denoted by $\Pi = \Pi(a)$. Note that the potential energy is a function of the crack length. For a small crack extension da, the decrease in the potential energy is $-d\Pi$. Griffith [1-6] proposed that this energy decrease in the cracked body would be absorbed into the surface energy of the newly created crack surface. Denote the surface energy per unit area by γ, which can be calculated from solid state physics. The total surface energy of the new crack surface equals $2da\gamma$. The Griffith energy balance equation becomes

$$-d\Pi = 2da\gamma \qquad \text{or} \qquad -\frac{d\Pi}{da} = 2\gamma$$

The energy release rate G proposed by Irwin [1-7] is defined as the decrease in potential energy per unit crack extension under constant load, that is,

$$G = -\frac{d\Pi}{da}$$

The crack growth or failure criterion using the energy balance approach is established as

$$G = G_c = 2\gamma \tag{1.3}$$

The fracture criterion given before is also fundamentally different from the classical failure criteria. It involves the total energy of the cracked body as well as the surface energy of the solid, which exists only in atomistic scale considerations. Like K_{Ic}, G_{Ic} is also a new material constant introduced in fracture mechanics to measure the resistance to fracture. G_{Ic} has a dimension of J/m^2, or kJ/m^2. The fracture criteria Eqs. (1.2) and (1.3) are actually equivalent (Chapter 4). However, the experimentally measured critical energy release rate for engineering materials, especially metals, is significantly larger than 2γ. This is because plastic deformations in the crack tip region also contribute significantly to the crack growth resistance. For perfectly brittle solids, it has been shown by MD simulations that $G_{Ic} = 2\gamma$ is valid in NaCl single crystal if the crack length is equal to or greater than 10 times the lattice constant [1-8].

Fracture mechanics introduces two novel concepts: stress intensity factor and energy release rate. These two quantities distinguish fracture mechanics from the classical failure criteria. In using the stress intensity factor-based fracture criterion to predict failure of a material or structure, one first needs to calculate the stress intensity factor for the given load and geometry. The second step is to measure the fracture toughness. Once the stress intensity factor and the fracture toughness are known, Eq. (1.2) can be used to determine the maximum allowable load that will not cause crack growth for a given crack length, or the maximum allowable crack length that will not propagate under the design load. The advantage of the stress intensity factor approach is its ease in the calculation of stress intensity factors and the easy measurement of fracture toughness. In contrast to the stress intensity factor approach, the energy release rate-based fracture criterion Eq. (1.3) is more naturally extended to cases where nonlinear effects need to be accounted for because the energy concept is universal.

Stress intensity factor and energy release rate lay the foundation of linear elastic fracture mechanics (LEFM). In LEFM, the cracked solid is treated as a linearly elastic medium and nonlinear effects are assumed to be minimal and can be ignored. While a modified stress intensity factor approach may be used to predict fracture of a cracked solid when plastic deformations are small and confined in the near-tip region, the approach, along with the energy release rate, would become futile when the cracked solid undergoes large-scale plastic deformations. Several fracture parameters have been proposed to predict fracture of solids under nonlinear deformation conditions, for example, the J-integral, the crack tip opening displacement (CTOD), and the crack tip opening angle (CTOA). Failure criteria based on these parameters, however, have not been as successful as the stress intensity factor and the energy release rate critetia in LEFM.

1.3 HISTORY OF FRACTURE MECHANICS

This section briefly describes the historical development of fracture mechanics from Griffith's pioneering work on brittle fracture of glass in 1920s, to Irwin's stress intensity factor concept and fracture criterion in 1950s, and to elastic-plastic fracture mechanics research in 1960s and early 1970s. A brief introduction of recent development of fracture mechanics research since 1990s is also included.

1.3.1 Griffith Theory of Fracture

The advent of fracture mechanics is usually credited to the poineering work of A. A. Griffith on brittle fracture of glass [1-6]. This paper was basically his PhD thesis work at Cambrige University under the guidance of G. I. Taylor. It had been known before Griffith's work that the theoretical fracture strength of glass determined based on the breaking of atomic bonds exceeds the strength of laboratory specimens by one to two orders of magnitude. Griffith believed that this huge discrepancy could be due to microcracks in the glass and that these cracks could propagate under a load level that is much smaller than the theoretical strength.

Griffith adopted an energy balance approach to determine the strength of cracked solids, that is, the work done during a crack extension must be equal to the surface energy stored in the newly created surfaces. To calculate the strain energy in a cracked body, he derived the stress field in an infinite plate with a through-thickness central crack under biaxial loading from Inglis's solution [1-9] for an elliptical hole in an elastic plate by reducing the minor axis to zero. Using this solution, Griffith was able to calculate the total potential energies before and after crack extension. The difference of the potential enegies of the two states were set equal to the corresponding gain in surface energy.

It follows from the Griffith theory that the fracture strength (the remote applied stress) of a solid with a crack is proportional to the square root of the surface energy and is inversely proportional to the square root of the crack size, that is,

$$\sigma_f \propto \sqrt{\frac{\gamma_c E}{a}}$$

where σ_f is the applied failure stress, γ_c is the specific surface energy, a is half the crack length, and E is Young's modulus.

The preceding relationship points out a specific functional form between the failure stress and the crack size. The Griffith theory represents a breakthrough in the strength theory of solids. It successfully explains why there is an order of magnitude difference between the theoretical strength and experimentally measured failure load for a solid. In particular, it provides a well-defined physical mechanism that controls the failure process, which is lacking in the classical phenomenological failure theories. The original work of A. A. Griffith dealt with fracture of brittle glass. In metals, plastic deformations develop around the crack tip and the measured fracture strength is much greater than that predicted by the Griffith theory. Orowan [1-10] and Irwin [1-11] suggested to add to 2γ the plastic work γ_p associated with the creation of new crack surfaces. For metals, γ_p is much larger than the surface energy 2γ, and hence the modified Griffith theory by Orowan and Irwin explained the high fracture strength of metals.

1.3.2 Fracture Mechanics as an Engineering Science

Although the basic energy concept of fracture mechanics was presented by A. A. Griffith in 1920, it was only after the 1950s that fracture mechanics was accepted as an engineering science with successful practical applications mainly as a result of Irwin's work ([1-5] and [1-7]). Irwin first introduced the energy release rate to establish a fracture criterion as in Eq. (1.3). He then defined the stress intensity factor K and derived the relationship between the energy release rate G and the stress intensity factor K based on Westergaard's solutions for the stress and displacement fields in a cracked plate [1-12]. Because of the $G-K$ relationship, Irwin proposed to use the stress intensity factor as a fracture parameter, which is a more direct approach for fracture mechanics applications as described by Eq. (1.2).

At the same time, Williams [1-13] derived the asymptotic stress field near a crack tip with the leading term exhibiting an inverse square root singularity under general planar loading conditions. The Williams solution, with both symmetric and asymmetric terms, gives a universal expression for the crack tip stress field independent of external loads and crack geometries. The load and crack geometry influence the crack tip singular stresses through the stress intensity factors K_I and K_{II}, which govern the intensity of the singular stress field. Williams' solution provides a justification for adopting the stress intensity factors to establish fracture criteria.

The stress intensity factor fracture criterion assumes that materials behavior is linearly elastic, which is a good assumption for brittle materials such as glass and ceramics. For ductile metals at room and elevated temperatures, however, plastic yielding occurs around the crack tip due to the stress singularity predicted in the elastic solution. For linear elastic fracture mechanics to be applicable to metals, the plastic deformation zone around the crack tip must be smaller than the dominance zone of the stress intensity factor. Irwin [1-14] estimated the size of plastic deformation zone near the crack tip and found that the plastic zone size is proportional to the square of the stress intesnity factor to the yield strength ratio if the plastic zone is small.

With the fracture criterion Eq. (1.2) in hand and the knowledge of the crack tip plastic zone size, the American Society for Testing and Materials (ASTM) formed a Special Technical Committee (ASMT STC, subsequently ASTM Committee E-24) to develop the standard for measuring K_{Ic}, the plane strain fracture toughness (or simply fracture toughness) for metallic materials. In the meantime, great efforts were made in 1960s and 1970s to develop analytical and numerical methods to compute stress intensity factors. Most of the approaches and techniques are included in a multi-volume fracture mechanics monograph, *Mechanics of Fracture*, edited by G. C. Sih and his coworkers [1-15, 1-17]. Stress intensity factors for various crack geometries under a variety of loading conditions are compiled in the handbook by Tada et al. [1-18].

The LEFM based on stress intensity factor K and energy release rate G has been very successful in predicting fracture of metals when the crack tip plastic zone is smaller than the K-dominance zone—also termed small-scale yielding (SSY). Under large-scale yielding conditions, however, the LEFM generally becomes inadequate and fracture criteria based on plasticity of the cracked solids have to be used. Irwin [1-14] introduced an effective stress intensity factor concept to take the crack tip plasticity effect into account. The effective stress intensity factor is obtained by replacing the crack length with an effective crack length that is equal to the original length plus half the plastic zone size.

Dugdale [1-19] presented a strip yielding zone model to determine the plastic zone size in thin cracked sheets. Wells [1-20] and [1-21] proposed to use the crack opening dispalcement (COD) as a fracture parameter. The COD criterion is equivalent to the effective stress intensity factor criterion under modest yielding conditions but can be extended to large-scale yielding when it is combined with the COD equation from the Dugdale model. Rice [1-22] generalized the energy release rate concept to nonlinear elastic materials or elastic-plastic materials described by the deformation plasticity and found that the energy release rate can be represented by a line integral, the so-called path-independent J-integral.

Begley and Landes [1-23] later proposed to use the J-integral for predicting elastic-plastic crack initiation and experimentally measured the critical value of J at crack initiation. In 1968, Rice and Rosengren [1-24] and Hutchinson [1-25] published their work on the crack tip plastic stress field (HRR field) in the framework of deformation plasticity. The HRR field shows that the J-integral characterizes the intensity of the singular stress field in a similar way to the role of stress intensity factor in LEFM. Becasue the HRR field is based on the deformation plasticity, the J-integral in general may be used for crack initiation only. In other words, the HRR field disappears as the crack extends and unloading (a behavior that the deformation plasticity theory cannot model) takes place.

1.3.3 Recent Developments in Fracture Mechanics Research

In recent years, LEFM has found many new applications mostly dealing with new materials such as nonhomogeneous and anisotropic fiber-reinforced composites.

The main issues that arise in these new applications include, for example, coupled thermal-mechanical loads in microelectronic packaging and multiscale issues in treating composite materials as homogeneous solids. Because of the increasing interest in nanotechnology, fracture of nanostructured materials has recently attracted the attention of many researchers. Molecular dynamics (MD) simulations are employed to model crack extension in atomistic systems. Researchers have attempted to answer the question regarding the applicability of continuum theory-based LEFM in solids at nano scale ([1-8] and [1-26]). For instance, are the stress intensity factor and energy release rate introduced in LEFM still valid, and how can one evaluate their values? Other issues involve the definition of cracks that are equivalent to cracks adopted in continuum LEFM.

A new form of fracture model called cohesive zone model (CZM) has evolved from LEFM but has taken a different treatment of the crack tip stress and strain fields. The main motivation in CZM was to avoid the seemingly unrealistic stress singularity at the crack tip. The idea of the CZM is credited to Barenblatt [1-27], who assumed that failure would occur by decohesion of the upper and lower surfaces of a volumeless cohesive zone ahead of the crack tip. In the cohesive zone the separation displacement of the two surfaces that bound the cohesive zone follows a cohesive traction law. Crack growth occurs when the opening displacement at the tail of the cohesive zone (physical crack tip) reaches a critical value at which the cohesive traction vanishes. Clearly, the cohesive modeling approach does not involve stress singularities and material failure is controlled by quantities such as displacements and stresses, which are consistent with the usual strength of materials theory.

Since Needleman [1-28] introduced the cohesive element technique in the finite element framework for fracture studies, CZM has emerged as a popular tool for simulating fracture processes in materials and structures due to the computational convenience. Although many researchers have reported successful results using the CZM approach, many issues remain to be resolved including the physics of the cohesive zone, a rational way to develop the cohesive traction law, and the uniqueness of the cohesive traction with respect to variation of loads and specimen geometry.

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