AE837

Advanced Mechanics of Damage Tolerance

Dr. Nicholas Smith
Wichita State University, Department of Aerospace Engineering September 12, 2019

upcoming schedule

- Sep 12 - Finite Size Effects, K-Dominance, HW 2 Due
- Sep 17 - Fracture Criterion
- Sep 19 - Exam Review, HW 3 Due
- Sep 24 - Exam 1

outline

- finite specimen effects
- williams crack tip fields
- K-Dominance

finite specimen effects

finite effects

- Up to this point, we have considered infinitely large panels
- In general, the effect of finite size is modeled using numerical techniques, such as finite elements
- These corrections will generally take the form of some dimensionless factor that is multiplied with the infinite stress intensity factor

$$K_I = \sigma\sqrt{\pi a}F(a/b, a/H)$$

finite specimen effects

- Finite specimen correction factors are often found using finite element analysis
- For finite width effects, Irwin proposed the following correction

$$F(a/b) = 1 + 0.128\left(\frac{a}{b}\right) - 0.288\left(\frac{a}{b}\right)^2 + 1.525\left(\frac{a}{b}\right)^3$$

williams crack tip fields

williams crack tip fields

- All the solutions we have found for the stress field near the crack tip have a singularity
- This singularity even has the same order (inverse square root)
- Williams approached the solution in a slightly different way to expand the terms

modes i and ii

- We start by considering the Airy stress function in polar coordinates expanded as a general series

$$\phi = \sum r^{\lambda_n + 1} F_n(\theta)$$

- Where λ_n are the Eigenvalues and F_n are the corresponding Eigenfunctions

modes i and ii

- To satisfy compatibility ($\nabla^2\nabla^2\phi = 0$) the Eigenfunctions must take the form

  $$F_n(\theta) = A_n \sin(\lambda_n + 1)\theta + B_n \cos(\lambda_n + 1)\theta + C_n \sin(\lambda_n - 1)\theta + D_n \cos(\lambda_n - 1)\theta$$

- Next the forms for the series are determined by satisfying the boundary conditions

mode i

- In mode one the solutions for the first two terms are

$$\sigma_{rr} = D_1 r^{-1/2}\left[-\frac{1}{4}\cos\frac{3\theta}{2} + \frac{5}{4}\cos\theta 2\right] + 2D_2\cos 2\theta + 2D_2 + O(r^{1/2})$$

$$\sigma_{\theta\theta} = D_1 r^{-1/2}\left[\frac{1}{4}\cos\frac{3\theta}{2} + \frac{3}{4}\cos\theta 2\right] - 2D_2\cos 2\theta + 2D_2 + O(r^{1/2})$$

$$\sigma_{r\theta} = D_1 r^{-1/2}\left[\frac{1}{4}\sin\frac{3\theta}{2} + \frac{1}{4}\sin\theta 2\right] - 2D_2\cos 2\theta + O(r^{1/2})$$

mode i

- We can see that the constant D_1 corresponds to the stress intensity factor
- The second term, D_2, corresponds to something known as the $T-$ stress
- The $T-$ stress influences the size and shape of the plastic zone

mode ii

- The Mode II problem can be solved in a similar fashion, but the Airy stress functions will be odd functions of θ (instead of even as in the mode I solution)

K-Dominance

K-Dominance

- We have now established that there are two parts to the stress field near a crack tip, a singular part and a non-singular part
- If we are sufficiently close to the crack tip, the singular part dominates fracture behavior
- We define the "K-Dominance Zone" to determine how close to the crack tip we need to be to ignore non-singular behavior

$$\Lambda = \frac{K_I/\sqrt{2\pi x}}{K_I/\sqrt{2\pi x} + |\text{non-singular part of } \sigma_{yy}|} = \frac{K_I/\sqrt{2\pi x}}{\sigma_{yy}}$$

K-Dominance

- The closer Λ is to 1, the more dominant the singularity
- We will consider an example case with a known solution for the stress field to see how the K-Dominance zone can be used

example

- For a Mode I crack under biaxial tension, we have

$$\sigma_{yy} = \frac{(x+a)\sigma_0}{\sqrt{x(x+2a)}}$$

- We recall that the stress intensity factor is $K_I = \sigma_0\sqrt{\pi a}$

example

$$\frac{K_I}{\sqrt{2\pi x}} = \frac{\sigma_0\sqrt{\pi a}}{\sqrt{2\pi x}} = \frac{\sigma_0}{\sqrt{2}\sqrt{x/a}}$$

$$\Lambda = \frac{K_I/\sqrt{2\pi x}}{\sigma_{yy}} = \frac{\sigma_0}{\sqrt{2}\sqrt{x/a}} \frac{a\sqrt{x/a}\sqrt{x/a+2}}{a(x/a+1)\sigma_0}$$

$$\Lambda = \frac{\sqrt{x/a+2}}{\sqrt{2}(x/a+1)}$$

example

- We can now solve for x/a for some Λ
- Find the size of the K-Dominance zone of $\Lambda = 0.95$
- We find $x \approx 0.07a$