Name:

Homework 3
Due 19 September 2019

1. An edge crack with applied splitting forces can be solved by the Westergaard function

$$Z_I = \frac{P}{\pi(b+z)}\sqrt{\frac{b}{z}}$$

use this Westergaard function to find the stress intensity factor.

Solution: The standard formula to find the stress intensity factor from a given Westergaard function assumes that the axis is "zeroed" at the center of the crack.

$$K = \sqrt{2\pi}\lim_{z\to a}\{\sqrt{z-a}(Z_I - iZ_{II})\}$$

In this case, we can see that the singularity is at $z=0$, and thus the axis is "zeroed" at the crack tip, this is also a Mode I problem, so we can say

$$K_I = \sqrt{2\pi}\lim_{z\to 0}\{\sqrt{z}Z_I\} = \sqrt{2\pi}\lim_{z\to 0}\{\sqrt{z}\frac{P}{\pi(b+z)}\sqrt{\frac{b}{z}}\}$$

After simplifying the limit we find

$$K_I = P\sqrt{\frac{2}{\pi b}}$$

2. An infinite plate containing a crack of length $2a$ is subjected to a uniform pressure p as shown. Determine the stress intensity factor.

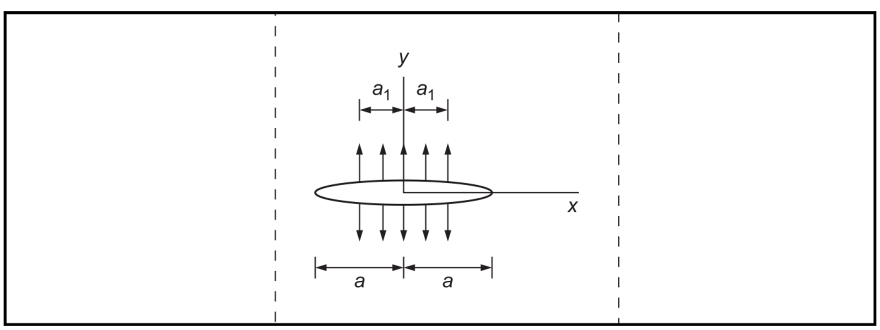

Figure 1: Illustration of Problem 2

Solution: We know that for uniform pressure we can integrate the concentrated force solution such that

$$K_I^R = \frac{1}{\sqrt{\pi a}}\int_{-a}^{a} p(\xi)\sqrt{\frac{a+\xi}{a-\xi}}d\xi$$

$$K_I^L = \frac{1}{\sqrt{\pi a}}\int_{-a}^{a} p(\xi)\sqrt{\frac{a-\xi}{a+\xi}}d\xi$$

In this case, $p(\xi)$ is a piecewise function with $p(\xi) = p$ when $-a_1 < \xi < a_1$ and 0 otherwise, thus the integrals become

$$K_I^R = \frac{p}{\sqrt{\pi a}} \int_{-a_1}^{a_1} \sqrt{\frac{a+\xi}{a-\xi}} d\xi$$
$$K_I^L = \frac{p}{\sqrt{\pi a}} \int_{-a_1}^{a_1} \sqrt{\frac{a-\xi}{a+\xi}} d\xi$$

Computing the integral is not trivial, so the solution could be left in this form, or computed using Maple or Wolfram Alpha

$$K_I^R = K_I^L = 2P\sqrt{\frac{a}{\pi}} \tan^{-1}\left(\frac{a_1}{\sqrt{a^2 - a_1^2}}\right)$$