AE837

Advanced Mechanics of Damage Tolerance

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upcoming schedule

- Oct 1 - Virtual Crack Closure, J-integral
- Oct 3 - J-Integral, Cohesive Zone
- Oct 8 - eXtended Finite Element Method (XFEM)
- Oct 10 - XFEM, Homework 4 Due
- Oct 15 - Fall Break (no class)

outline

- comsol setup
- finite element demo
- virtual crack closure
- j-integral

comsol setup

comsol install

- I need your computer's "hostname" to whitelist you
- You can download COMSOL here
- To run COMSOL, you either need to be on campus or you need to use a VPN (to have an on-campus IP address)
- When installing, choose "License Format -> port number @ hostname"
    - port number 1718
    - aecomsol.wichita.edu as hostname

comsol tutorials

- If you have not used COMSOL before, they have a pretty good library of tutorials
- For example, to see how they perform the J-integral you can follow this tutorial
- The application gallery has many more documented examples you can follow along with

finite element demo

objectives

- We will use the direct method to find the stress intensity factor, K_I, of an edge crack
- The analytic solution is

$$K_I = (1.122 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.71\left(\frac{a}{W}\right)^3)\sigma\sqrt{\pi a}$$

- And K_I in terms of stress and displacement is

$$K_I = \sigma_{yy}\sqrt{2\pi x}$$
$$K_I = \frac{2\mu u_y}{\kappa + 1}\sqrt{\frac{2\pi}{x}}$$

boundary conditions

- The first thing to consider is symmetry
- It is easiest to cut our model in half vertically and treat it as symmetric
- If we don't do this, we need a way of cutting the nodes where the crack is (or joining them where it isn't)
- For an edge crack, we will have a symmetry condition on the symmetric portion and a boundary load on top, otherwise everything else is traction free

screencast

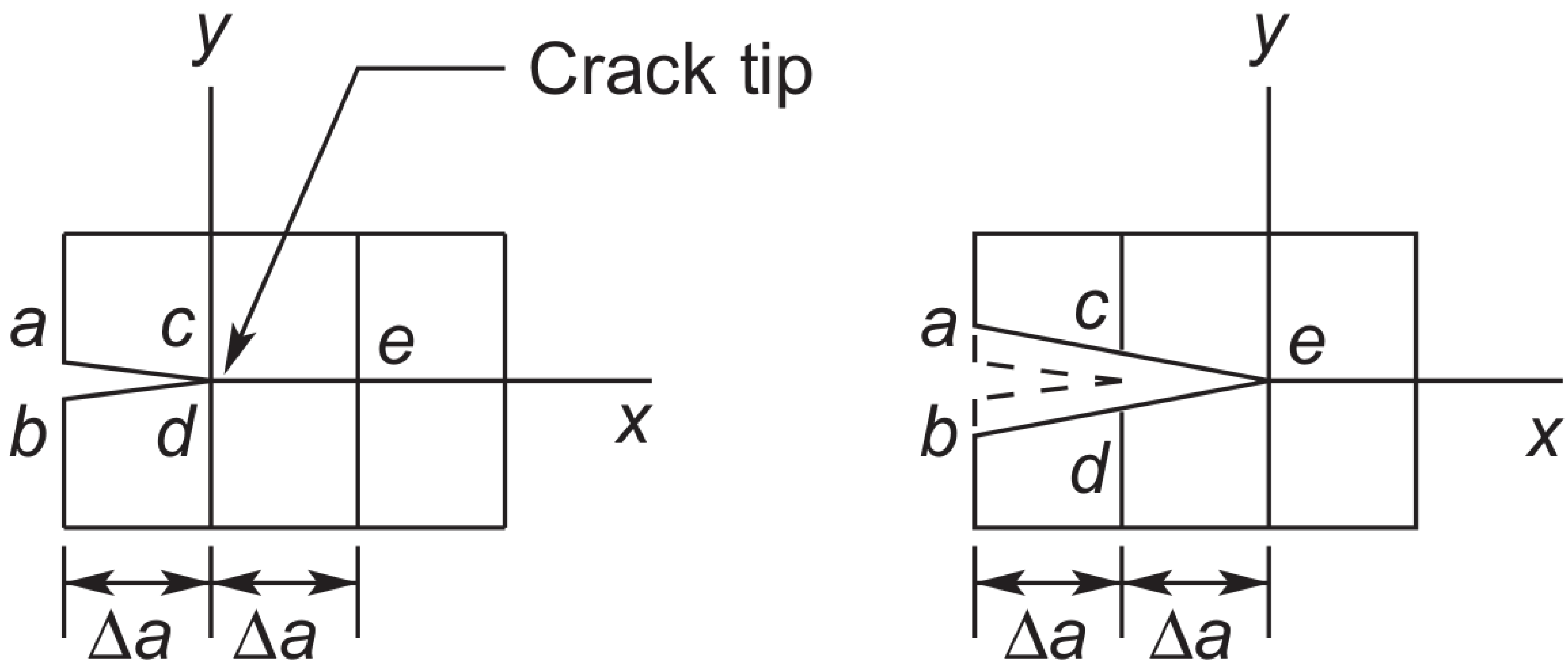
0:00 / 6:57

virtual crack closure

vcct

- As can be seen from the results previously found, the direct method leaves something to be desired, and is very mesh-dependent
- An alternative approach is to use the same energy method (virtual crack closure) that we used to relate G_I to K_I in finite elements
- We consider two cases, one before and one after some crack extension, da

vcct illustration



vcct

- Since this is an energy approach, we will be directly finding G_I, but for elastic materials we can easily convert this to K_I

$$G_I = \frac{1}{2da} F_y^{(c)} \left(u_y^{(c)} - u_y^{(d)} \right)$$

- Where the force at c is taken before extension (or after closure) and the displacement is taken after extension (or before closure)

modified version

- It is a little bit cumbersome to work with two finite element solutions for some da
- It has been shown that with no loss of accuracy, for small da ($da/a \leq 0.05$), we can use the nodes right in front of and behind the crack tip, eliminating the need for a two-step model
- Note: for this method to work, the mesh near the crack tip must be uniform

demo

- We will add to the previous model, however in this case we need to partition the line of symmetry a second time to enforce a uniform mesh both directions around the crack tip
- To compare G_I and K_I we will want to convert one or the other, since we had found K_I previously, we will convert G_I to K_I

$$K_I = \sqrt{\frac{EG_I}{1-\nu^2}} \qquad \text{plane strain}$$

$$K_I = \sqrt{EG_I} \qquad \text{plane stress}$$

comsol notes

- I have had some trouble with the reaction forces calculated in COMSOL
- In your homework, don't worry if you don't get a very good value for K_I
- In my screencast I show how to switch to linear elements, which performed better for me
- I also got a better result when I used the reaction force at the crack tip, instead of da away
- This did not match my experience with ABAQUS

screencast

0:00 / 7:14

j-integral

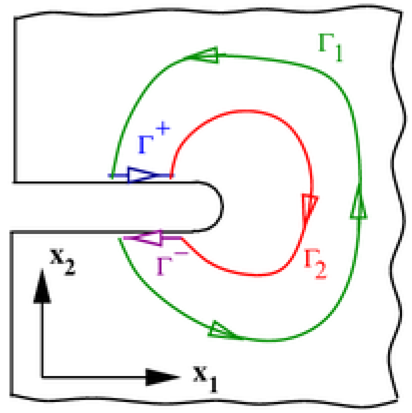
j-integral

- The J-Integral is defined as

$$\int_{\Lambda}\left(W dy - T_i \frac{\partial u_i}{\partial x} d\Lambda\right) = \int_{\Lambda}\left(W n_1 - \sigma_{ij}\frac{\partial u_i}{\partial x} n_j\right) d\Lambda$$

- \Lambda is an arbitrary contour beginning at the lower crack surface and end on the upper crack surface

j-integral



j-integral

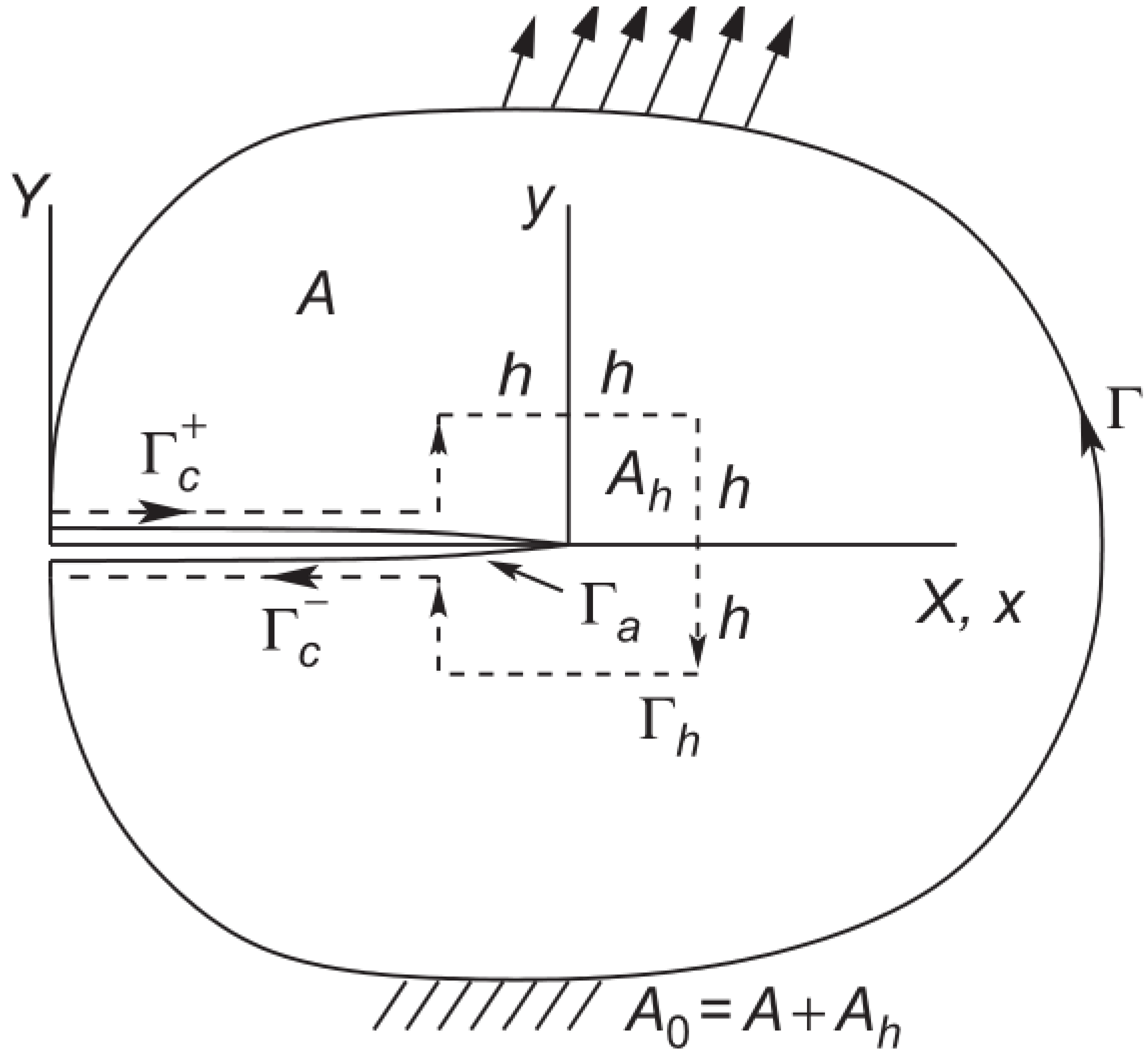
- The J-integral is path-idependent and represents the strain energy release rate
- We can prove this using the following principles from elasticity

$$\sigma_{ij,j} = 0 \qquad \text{(equilibrium)}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad \text{(strain-displacement)}$$

$$\sigma_{ij} = \frac{\partial W}{\partial e_{ij}} \qquad \text{(stress-strain)}$$

j-integral



j-integral

- A_0 represents the area enclosed by the contour
- The potential energy can then be expressed as

$$\iint_{A_0} W dX dY - \int_{\Lambda_t} T_i u_i d\Lambda$$

- (worked on board)

examples