# Mechanics of Materials

Lecture 13 - Transverse Shear

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# schedule

- 6 Oct - Transverse Shear
- 8 Oct - Homework 5 Due, Homework 4 Self-grade due
- (11 Oct) - Fall Break
- 13 Oct - Transverse Shear
- 15 Oct - Homework 6 Due, Homework 5 Self-grade due
- 18 Oct - Exam 2 Review
- 20 Oct - Exam 2

## outline

- shear in straight members
- the shear formula
- group problems



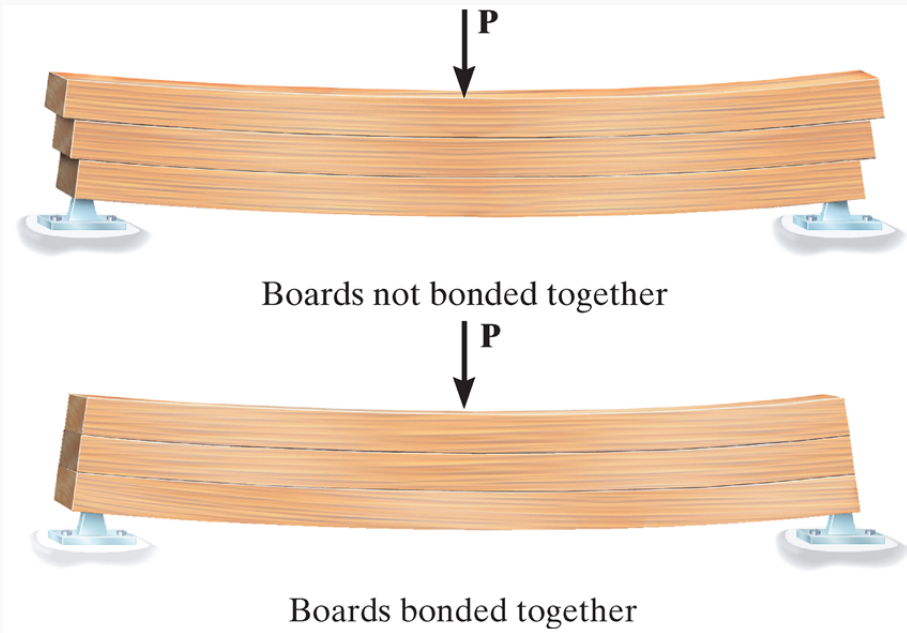
# shear in straight members

## shear

- We have discussed the internal stresses caused by the internal moment $M$
- There are also internal shear stresses caused by the internal shear force $V$
- We can illustrate the effect of internal shear stress by considering three boards, either resting on top of on another or bonded



## shear



# the shear formula

## shear formula

- Internal shear causes a more complicated deformation state, so we will use an indirect method to find the shear stress-strain distribution
- We will consider equilibrium along a section of a beam, then we will consider another section

# equilibrium

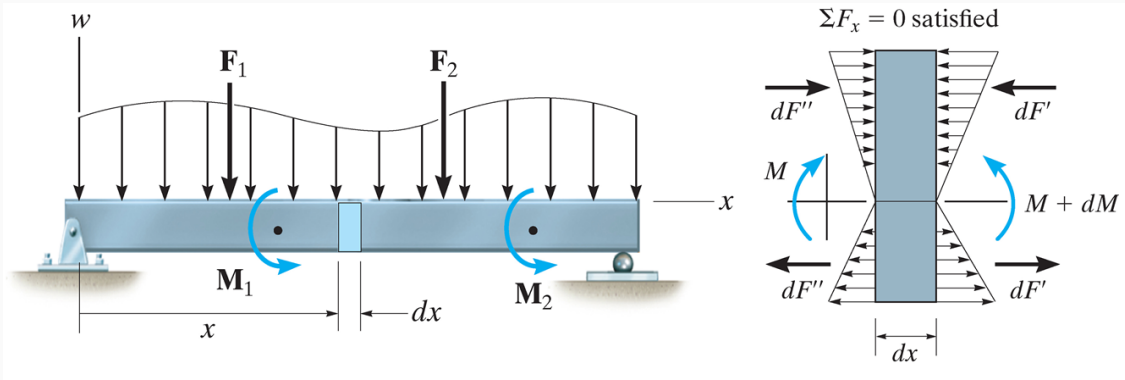
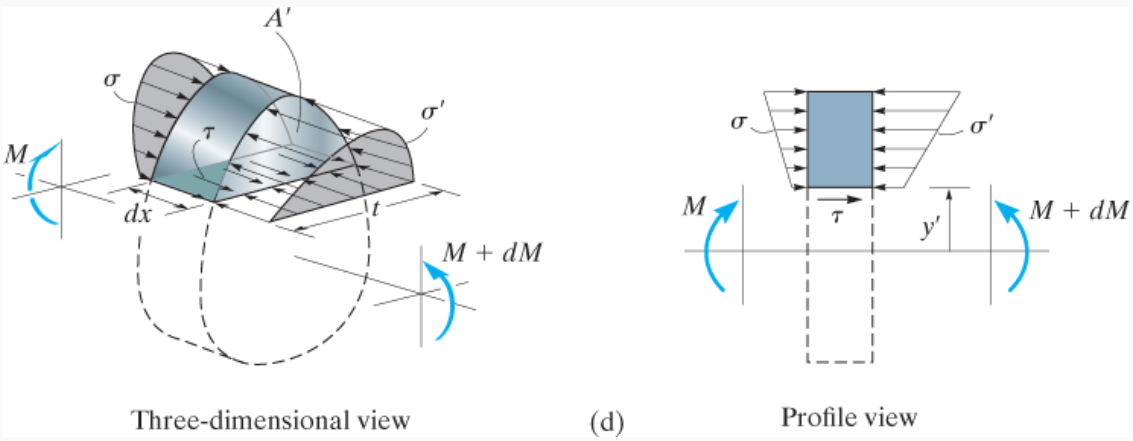



**Figure 1:** A free body diagram of an arbitrary beam.



# equilibrium



## equilibrium

- There must be a shear force along the bottom to equilibrate the different stresses on either side of the section
- If we assume that this shear is constant through the thickness, we obtain the following from equilibrium

$$\sum F_x = 0 = \int_{A'} \sigma' dA' - \int_{A'} \sigma dA' - \tau(tdx)$$



## equilibrium

$$0 = \int_{A'} \left(\frac{M + dM}{I}\right) ydA' - \int_{A'} \left(\frac{M}{I}\right) ydA' - \tau(tdx)$$

$$\left(\frac{dM}{I}\right) \int_{A'} ydA' = \tau(tdx)$$

$$\tau = \frac{1}{It} \left(\frac{dM}{dx}\right) \int_{A'} ydA'$$

## shear formula

- In this formula, recall that $V = \frac{dM}{dx}$
- We also call $Q$ the moment of the area $A'$ which is equal to $\int_{a'} ydA'$
- We can also write $Q$ in terms of the centroid

$$Q = \bar{y}'A'$$



## shear formula

- Simplified, the shear formula is

$$\tau = \frac{VQ}{It}$$

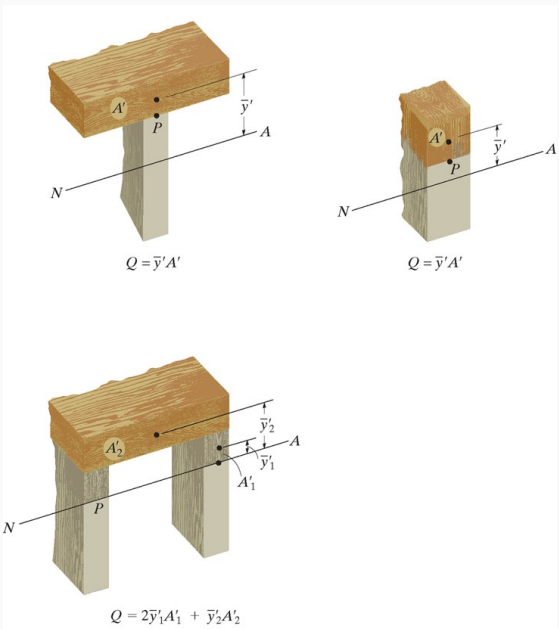
- $Q$ poses the greatest difficulty in calculations, so we will consider a few examples

# Q

- $Q$ represents the moment of the cross-sectional area above (or below) the point at which the shear stress is being calculated
- We apply the formula to that area



# Q



## shear formula limitations

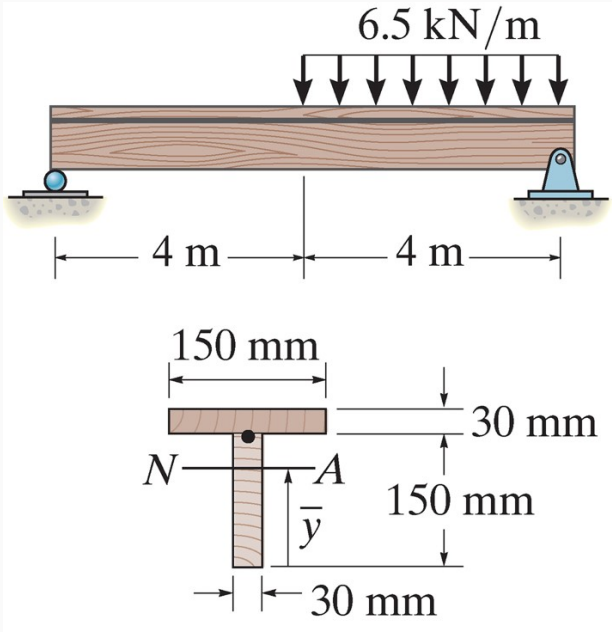
- A major assumption made is that the shear stress was constant through the thickness, essentially we have found the average shear
- This is more accurate the more slender a beam is (small $b$ and large $h$)
- The formula is also not accurate for cross sections that change or have boundaries that are not right angles



## procedure

- First we find the internal shear, $V$
- Find $I$, the moment of inertia (of the entire section) about the neutral axis
- Find $t$ from an imaginary cross-section at the point of interest
- Calculate $Q$ from either the area above or below the point of interest
- $\tau$ acts in the same direction as $V$ (and must be equilibrated on other faces)

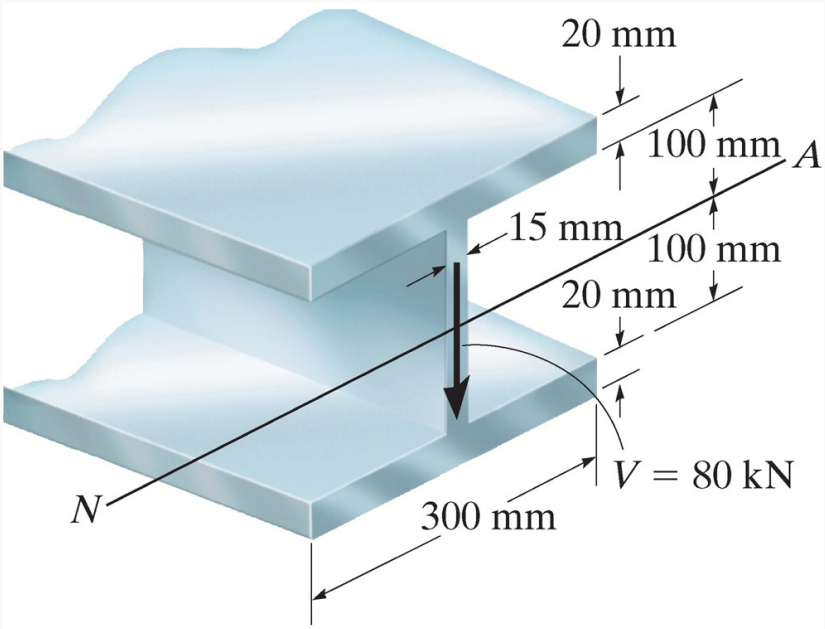
## example 7.1




Determine the maximum stress needed by a glue to hold the boards together.



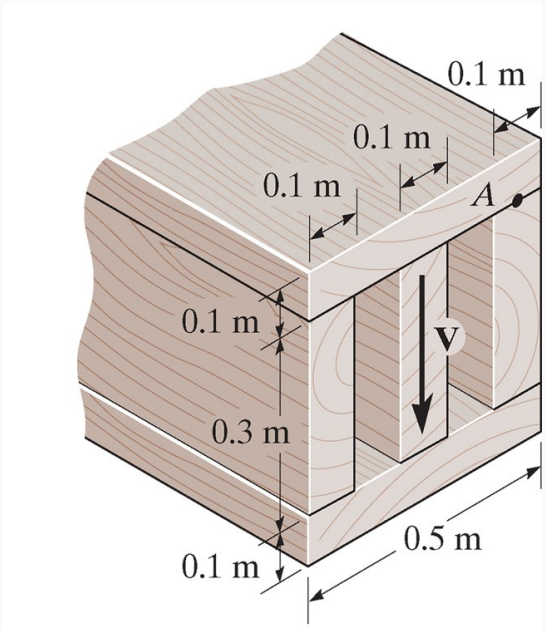
## example 7.3




Plot the shear stress distribution through the beam.

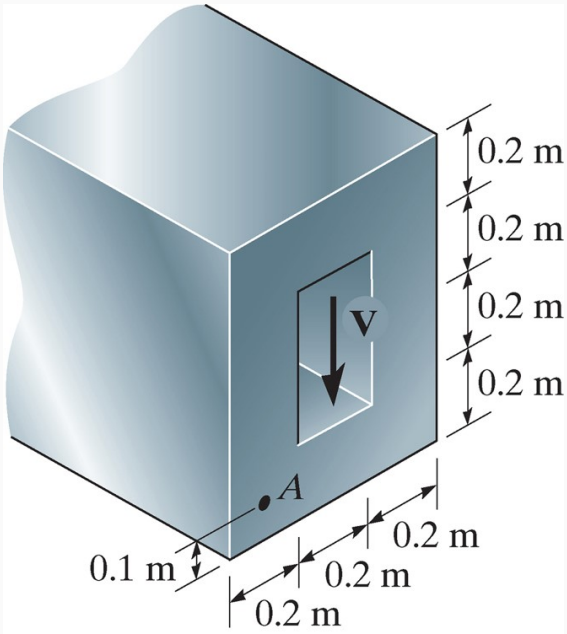
# group problems

## group one




Find Q and t that would be used to find the shear stress at A.

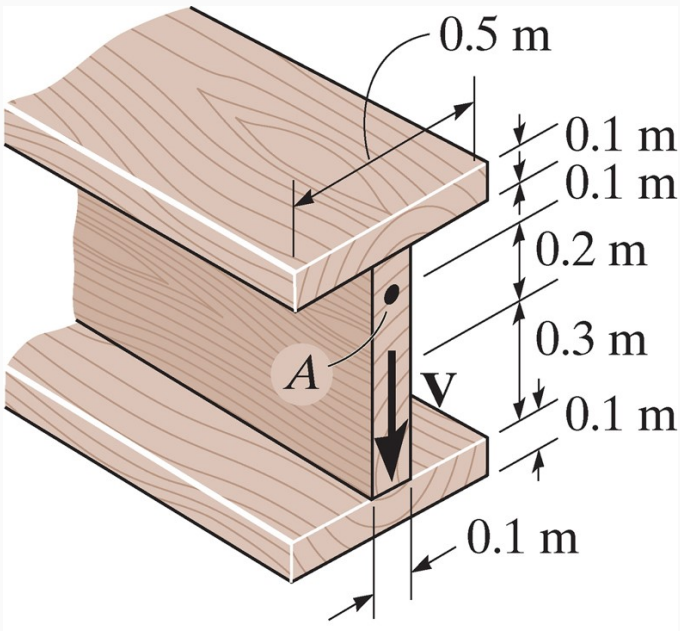
## group two




Find Q and t that would be used to find the shear stress at A.



## group three




Find Q and t that would be used to find the shear stress at A.