Mechanics of Materials

Lecture 15 - Stress Transformation

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schedule

- 15 Oct - Stress Transformation
- 20 Oct - Stress Transformation, HW 7 Due
- 22 Oct - Strain Transformation
- 27 Oct - Beam Deflection, HW 8 Due

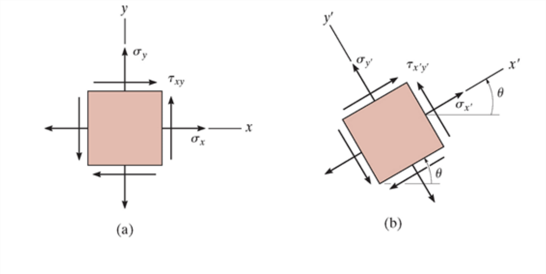
outline

- plane stress transformation
- general equations
- principal stresses
- mohr's circle

plane stress

- In general, the state of stress at a point is characterized by six stress components
- In practice, this is rare, as most stresses and forces act in the same plane
- This case is referred to as plane stress

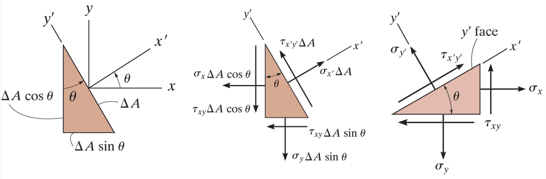
transformation



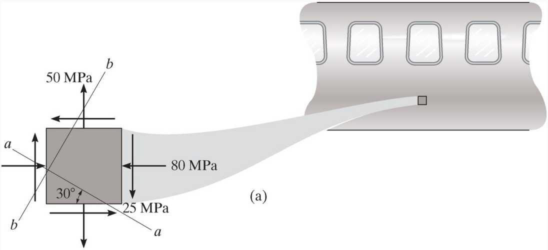
procedure

- If the state of stress ($\sigma_x, \sigma_y, \tau_{xy}$ is known for a known axis system x and y, we can find the stress relative to some rotated coordinate system
- We do this by considering a section of the element perpendicular to the x'
- Sum of forces in x and y will give σ_x' and τ_{xy}
- A second section is needed to find σ_y' perpendicular to the y' axis

procedure



example 9.1



Represent the state of stress shown on the fuselage section on an element rotated 30° clockwise from the position shown.

general equations

- We can follow the methodology from the previous section to develop equations for some arbitrary rotation and a completely general state of stress
- We use some trig identities to simplify the formulae

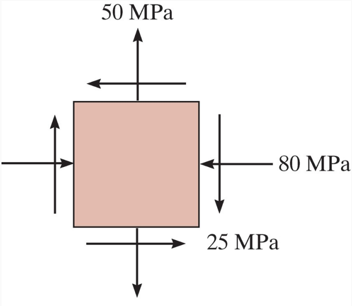
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

- To find σ_y' we can simply add 90° to θ

procedure

- The procedure in general is mostly “plug and chug”
- The only thing we need to be cautious about is sign convention: stresses are positive in tension, shear is positive with arrows pointing to the 1st and 3rd quadrants, θ is measured counter-clockwise from the x-axis

example 9.2



Determine the stress at this point on an element rotated 30° clockwise from the position shown.

principal stresses

- Since the local stresses only change with the rotation angle, we might like to find the angle with gives the maximum stress
- This is known as the principal direction, and the stresses are known as principal stresses
- We can find this direction by differentiating the equation for σ'_x

principal stress

- We find the angle as

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

- The principal stresses are then

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

maximum shear stress

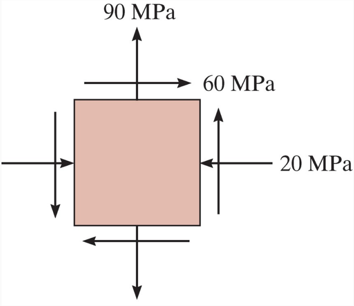
- Similarly, we might want to find the direction of maximum shear stress

$$\tan 2\theta_s = \frac{\sigma_y - \sigma_x}{2\tau_{xy}}$$

- And the maximum shear stress is

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

example 9.3



Find the principal stress for the stress state shown.

example 9.5

- When torsional loading T is applied to a circular bar it produces a state of pure shear stress.
- Find the maximum in-plane shear stress and the associated average normal stress
- Find the principal stresses

mohr's circle

- We can visualize plane stress transformation using a technique known as Mohr's circle
- If we re-write the stress transformation equations we find

$$\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right) = \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

mohr's circle

- If we square each equation and add them together, we find

$$\left[\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

mohr's circle

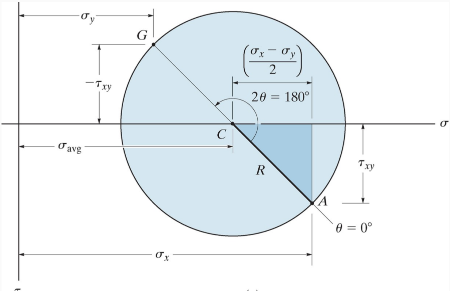
- Since σ_x, σ_y and τ_{xy} are known constants, we can write a more compact form by letting

$$(\sigma_{x'} - \sigma_{avg})^2 + \tau_{x'y'}^2 = R^2$$
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

mohr's circle

- Re-written in this way, we can see that the previous equation is the equation of a circle on the σ, τ axis
- The center of the circle is at $\tau = 0$ and $\sigma = \sigma_{avg}$
- The radius of the circle is $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
- Each point on the circle represents (σ_x', τ_{xy}')

mohr's circle



(c)

visual construction of Mohr's circle

- By convention, positive τ points down, use this convention to plot the center of the circle and a reference point at (σ'_x, τ'_{xy}) where the x' axis is coincident with the x axis
- Use these two points to sketch the circle

principal stress

- The principal stresses, $\sigma_1$and σ_2 are the coordinates where Mohr's circle intersects the σ axis
- The angles θ_{p1} and θ_{p2} can be found by calculating the angle between the reference line and the σ axis (note that this angle is equal to $2\theta_p$)
- Note that the direction from the reference point to the σ axis will be the same as the direction from the x axis to the principal axis

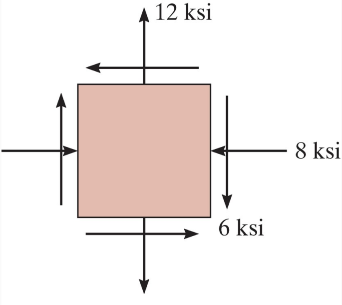
maximum shear stress

- The top and bottom of the circle represent the maximum shear stress
- The angles θ_{s1} and θ_{s2} can be found in a similar method to that described for the principal stress

stress on arbitrary plane

- To find the stress at some arbitrary plane some known angle θ away from our reference plane, we find the angle 2θ away from the reference line on Mohr's circle
- We can use trigonometry to find the value of the coordinates at that point
- We must draw our angle in the same direction as the desired rotation

example 9.9



Represent the state of stress shown on an element rotated 30° counterclockwise from the position shown.