

Lecture 16 - Stress Transformation

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schedule

- 27 Oct - Exam 2 Return, Stress Transformation
- 29 Oct - Project 2 Due, HW 6 Self-Grade Due
- 1 Nov - Mohr's Circle
- 3 Nov - Strain Transformation
- 5 Nov - HW 7 Due
- 8 Nov - Beam Deflection

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- plane stress transformation
- general equations
- principal stresses
- mohr's circle

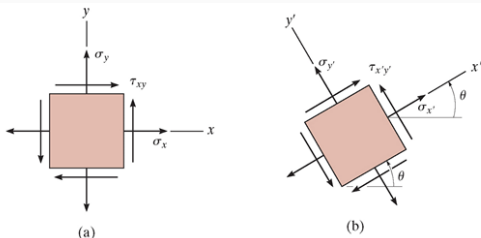
plane stress transformation

plane stress

- In general, the state of stress at a point is characterized by six stress components
- In practice, this is rare, as most stresses and forces act in the same plane
- This case is referred to as plane stress

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transformation

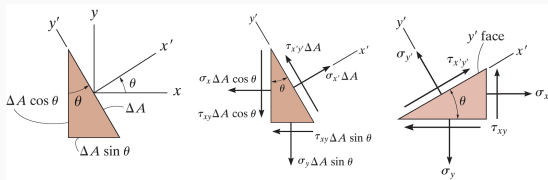


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- If the state of stress ($\sigma_x, \sigma_y, \tau_{xy}$) is known for a known axis system x and y , we can find the stress relative to some rotated coordinate system
- We do this by considering a section of the element perpendicular to the x'
- Sum of forces in x and y will give $\sigma'_{x'}$ and $\tau_{x'y'}$
- A second section is needed to find $\sigma'_{y'}$ perpendicular to the y' axis

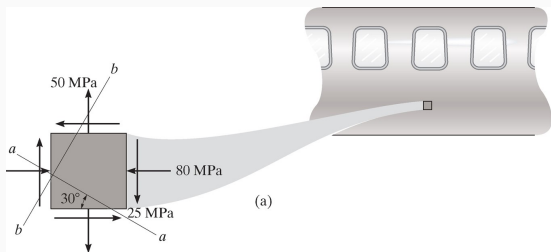
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procedure



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example 9.1



Represent the state of stress shown on the fuselage section on an element rotated 30° clockwise from the position shown.

general equations

- We can follow the methodology from the previous section to develop equations for some arbitrary rotation and a completely general state of stress
- We use some trig identities to simplify the formulae

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

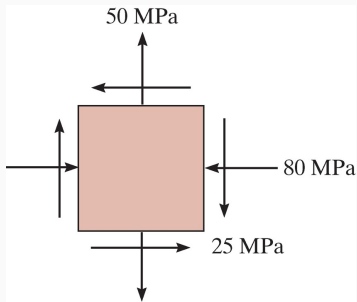
- To find σ'_y we can simply add 90° to θ

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procedure

- The procedure in general is mostly “plug and chug”
- The only thing we need to be cautious about is sign convention: stresses are positive in tension, shear is positive with arrows pointing to the 1st and 3rd quadrants, θ is measured counter-clockwise from the x-axis

example 9.2



Determine the stress at this point on an element rotated 30° clockwise from the position shown.

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principal stresses

principal stresses

- Since the local stresses only change with the rotation angle, we might like to find the angle with gives the maximum stress
- This is known as the principal direction, and the stresses are known as principal stresses
- We can find this direction by differentiating the equation for σ'_x

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principal stress

- We find the angle as

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

- The principal stresses are then

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

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maximum shear stress

- Similarly, we might want to find the direction of maximum shear stress

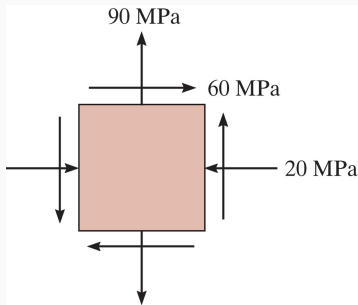
$$\tan 2\theta_s = \frac{\sigma_y - \sigma_x}{2\tau_{xy}}$$

- And the maximum shear stress is

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

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example 9.3



Find the principal stress for the stress state shown.

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- When torsional loading T is applied to a circular bar it produces a state of pure shear stress.
- Find the maximum in-plane shear stress and the associated average normal stress
- Find the principal stresses

mohr's circle

mohr's circle

- We can visualize plane stress transformation using a technique known as Mohr's circle
- If we re-write the stress transformation equations we find

$$\begin{aligned}\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right) &= \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta + \tau_{xy} \sin 2\theta \\ \tau_{x'y'} &= -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta\end{aligned}$$

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mohr's circle

- If we square each equation and add them together, we find

$$\left[\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

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- Since σ_x , σ_y and τ_{xy} are known constants, we can write a more compact form by letting

$$\begin{aligned}(\sigma_{x'} - \sigma_{avg})^2 + \tau_{x'y'}^2 &= R^2 \\ \sigma_{avg} &= \frac{\sigma_x + \sigma_y}{2} \\ R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\end{aligned}$$

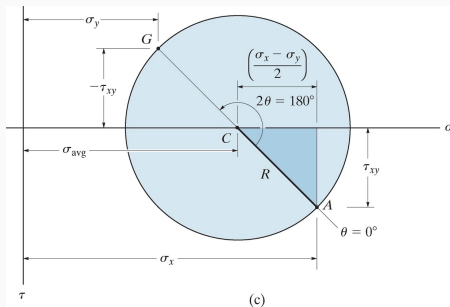
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mohr's circle

- Re-written in this way, we can see that the previous equation is the equation of a circle on the σ, τ axis
- The center of the circle is at $\tau = 0$ and $\sigma = \sigma_{avg}$
- The radius of the circle is $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
- Each point on the circle represents (σ'_x, τ'_{xy})

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mohr's circle



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visual construction of Mohr's circle

- By convention, positive τ points down, use this convention to plot the center of the circle and a reference point at (σ'_x, τ'_{xy}) where the x' axis is coincident with the x axis
- Use these two points to sketch the circle

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principal stress

- The principal stresses, σ_1 and σ_2 are the coordinates where Mohr's circle intersects the σ axis
- The angles θ_{p1} and θ_{p2} can be found by calculating the angle between the reference line and the σ axis (note that this angle is equal to $2\theta_p$)
- Note that the direction from the reference point to the σ axis will be the same as the direction from the x axis to the principal axis

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maximum shear stress

- The top and bottom of the circle represent the maximum shear stress
- The angles θ_{s1} and θ_{s2} can be found in a similar method to that described for the principal stress

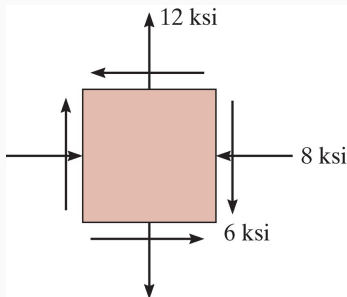
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stress on arbitrary plane

- To find the stress at some arbitrary plane some known angle θ away from our reference plane, we find the angle 2θ away from the reference line on Mohr's circle
- We can use trigonometry to find the value of the coordinates at that point
- We must draw our angle in the same direction as the desired rotation

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example 9.9



Represent the state of stress shown on an element rotated 30° counterclockwise from the position shown.

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