

Lecture 9 - Torsion

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

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## schedule

- 15 Sep - Torsion
- 17 Sep - Homework 2 Self-Grade Due
- 20 Sep - Torsion
- 22 Sep - Bending
- 24 Sep - Homework 3 Due
- 27 Sep - Bending

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- torsion
- power transmission
- group problems

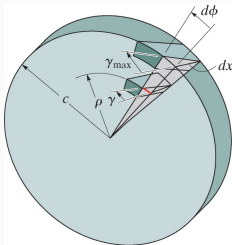
## torsion

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- Torque is a moment that tends to twist a member about its axis
- For small deformation problems, we assume that the length and radius do not change significantly under torsion
- The primary deformation we are concerned with in torsion is the angle of twist, denoted with  $\phi$

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## shear



The shear strain at points on the cross section increases linearly with  $\rho$ , i.e.,  $\gamma = (\rho/c)\gamma_{\max}$ .

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## torsion formula

- For a linearly elastic material, Hooke's Law in shear will hold ( $\tau = G\gamma$ )
- This means that, like shear strain, shear stress will vary linearly along the radius

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## torsion formula

- We can find the total force on an element,  $dA$  by multiplying the shear stress by the area

$$dF = \tau dA$$

- The torque ( $dT = \rho dF$ ) produced by this force is then

$$dT = \rho(\tau dA)$$

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- Integrating over the whole cross-section gives

$$T = \int_A \rho(\tau dA) = \frac{\tau_{max}}{c} \int_A \rho^2 dA$$

- The integral  $\int_A \rho^2 dA$  is also called the Polar Moment of Inertia,  $J$ , so we can write

$$\tau_{max} = \frac{Tc}{J}$$

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## polar moment of inertia

- We know that  $J = \int_A \rho^2 dA$ , so we can compute this for some common shapes
- For a solid circular cross-section, we have

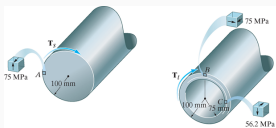
$$J = \int_0^c \rho^2 (2\pi\rho d\rho) = \frac{\pi}{2} c^4$$

- For a circular tube we have

$$J = \int_{c_1}^{c_2} \rho^2 (2\pi\rho d\rho) = \frac{\pi}{2} (c_2^4 - c_1^4)$$

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## example 5.1



**Figure 1:** On left is a solid 100 mm radius tube, while on the right is a hollow tube with outer radius of 100 mm and inner radius of 75 mm. Element A is on the surface of the solid tube on the left, element B is on the outer surface of the hollow tube on the right and Element C is on the inner surface of the hollow tube

The allowable shear stress is 75 MPa. Determine the maximum torque that can be applied to each of the cross-sections shown and find the stress acting on a small element at A, B and C.

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## power transmission

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## power transmission

- Shafts and tubes are often connected to belts and drives, and the torque, speed, and power are all related
- Power is the rate of work done, for rotation problems,  
 $P = T\omega$
- We are often given the frequency  $f$  instead of the angular velocity,  $\omega$ , in this case  $P = 2\pi fT$

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## power units

- In SI Units, power is expressed in Watts  $1 \text{ W} = 1 \text{ N m} / \text{sec}$
- In Freedom Units, power is expressed in Horsepower  $1 \text{ hp} = 550 \text{ ft lb} / \text{sec}$

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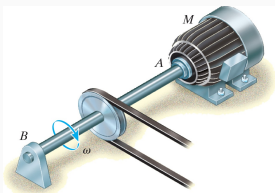
- We often know the power and frequency of a drive, and need to design a shaft such that the shear stress is acceptable
- We can easily find the torque as  $T = P/2\pi f$ , we can use this combined with the torsion equation

$$\tau_{max} = \frac{Tc}{J}$$

to find the appropriate shaft diameter. - For solid shafts we can solve for  $c$  uniquely, but not for hollow shafts

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### example 5.4



The steel shaft shown is connected to a 5 hp motor that rotates at  $\omega = 175$  rpm. If  $\tau_{allow} = 14.5$  ksi, determine the required shaft diameter.

**Figure 2:** A rotating shaft connected to a motor

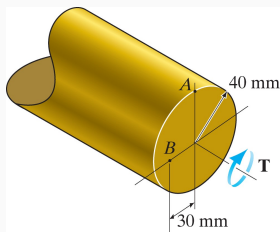
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## group problems

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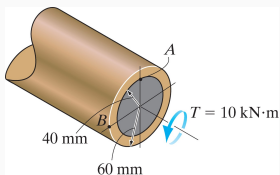
### group one



**Figure 3:** A 40 mm radius solid shaft. Point A is on the outer surface, Point B is 30 mm away from the center.

The solid circular shaft is subjected to an internal torque of 5 kN.m. Determine the shear stress at A and B and represent each state of stress on a volume element.

## group two

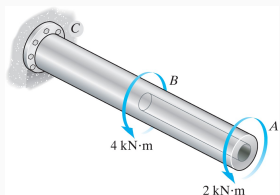


**Figure 4:** A hollow circular shaft with outer radius of 60 mm and inner radius of 40 mm. Point A is on the inner surface, Point B is on the outer surface.

The hollow circular shaft is subjected to an internal torque of 10 kN.m. Determine the shear stress at A and B and represent each state of stress on a volume element.

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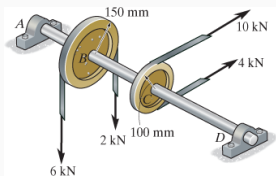
## group three



**Figure 5:** There is a fixed support at C, an applied torque of 4 kN.m at B (in the middle) and an applied torque of 2 kN.m at A (at the free end).

The circular shaft is hollow from A to B and solid from B to C. Determine the shear stress at A and B. The outer diameter is 80 mm and the wall thickness is 10 mm.

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Determine the maximum shear stress in the 40 mm diameter shaft.

**Figure 6:** A shaft supports to pulleys, one with a 150 mm radius and tension of 6 kN at one end and 2 kN at the other other. The other pulley has a 100 mm radius and tensions of 10 kN and 4 kN.