Mechanics of Materials

Lecture 24 - Discontinuity Functions

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

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schedule

- 20 Apr - Beam Deflection (discontinuity functions)
- 20 Apr - Exam 3a Due by midnight
- 22 Apr - Beam Deflection (superposition)
- 24 Apr - Recitation, HW9 Due
- 27 Apr - Statically Indeterminate Beams

outline

- slope and displacement
- discontinuity functions

slope and displacement

curvature

- When talking about displacement in beams, we use the coordinates v and x, where v is the vertical displacement and x is the horizontal position
- In this notation, curvature is formally related to displacement according to

$$\frac{1}{\rho} = \frac{d^2v/dx^2}{[1+(dv/dx)^2]^{3/2}} = \frac{M}{EI}$$

curvature

- The previous equation is difficult to solve in general, but for cases of small displacement, $(dv/dx)^2$ will be negligible compared to 1, which then simplifies to

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

flexural rigidity

- In general, M, is a function of x, but EI (known as the flexural rigidity) is a constant along the length of the beam
- In this case, we can say

$$EI\frac{d^2v}{dx^2} = M(x)$$
$$EI\frac{d^3v}{dx^3} = V(x)$$
$$EI\frac{d^4v}{dx^4} = w(x)$$

boundary conditions

- If a support restricts displacement, but not rotation, we will have a boundary condition of $v = 0$ at that point
- Supports that restrict rotation give a boundary condition that $\theta = 0$

continuity conditions

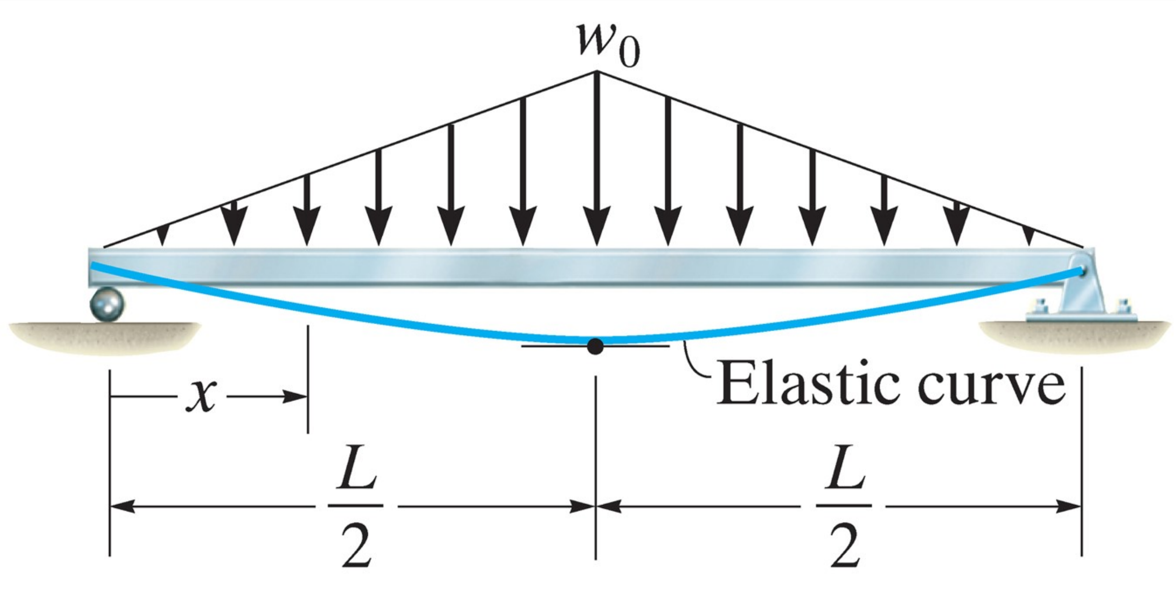
- If we have a piecewise function for $M(x)$, not all integration constants can be found from the boundary conditions
- Instead, we must also use continuity conditions to ensure that the slope and displacement are continuous at every point
- In other words, for two sets of functions, $\theta_1(x)$ and $v_1(x)$, $\theta_2(x)$, and $v_2(x)$, $\theta_1(a) = \theta_2(a)$ and $v_1(a) = v_2(a)$

slope

- For small displacements, we have

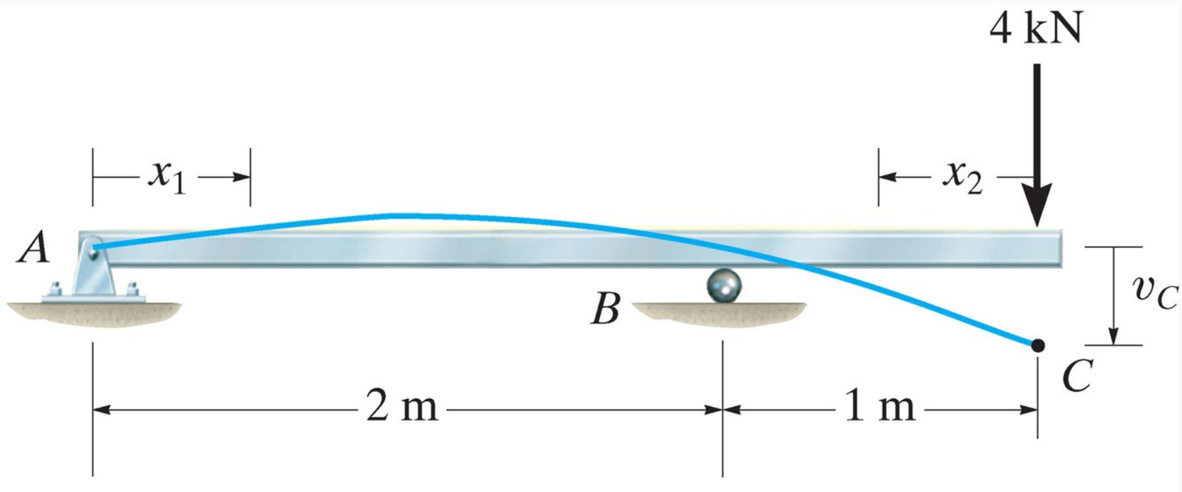
$$\theta \approx \tan(\theta) = dv/dx$$

example 12.1



Determine the maximum deflection if EI is constant.

example 12.4



Determine the displacement at C, EI is constant.

discontinuity functions

discontinuity functions

- Direct integration can be very cumbersome if multiple loads or boundary conditions are applied
- Instead of using a piecewise function, we can use discontinuity functions

Macaulay functions

- Macaulay functions can be used to describe various loading conditions, the general definition is

$$\langle x-a\rangle^n = \begin{cases} 0 & \text{for } x < a \\ (x-a)^n & \text{for } x \geq a \end{cases} n \geq 0$$

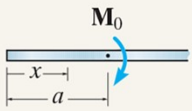
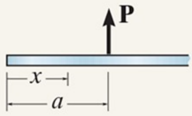
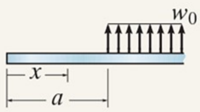
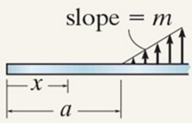
singularity functions

- Singularity functions are used for concentrated forces and can be written

$$w = P\langle x-a\rangle^{-1} = \begin{cases} 0 & \text{for } x \neq a \\ P & \text{for } x = a \end{cases}$$

discontinuity functions

TABLE 12–2

Loading	Loading Function $w = w(x)$	Shear $V = \int w(x)dx$	Moment $M = \int V dx$
	$w = M_0\langle x-a\rangle^{-2}$	$V = M_0\langle x-a\rangle^{-1}$	$M = M_0\langle x-a\rangle^{0}$
	$w = P\langle x-a\rangle^{-1}$	$V = P\langle x-a\rangle^{0}$	$M = P\langle x-a\rangle^{1}$
	$w = w_0\langle x-a\rangle^{0}$	$V = w_0\langle x-a\rangle^{1}$	$M = \frac{w_0}{2}\langle x-a\rangle^{2}$
	$w = m\langle x-a\rangle^{1}$	$V = \frac{m}{2}\langle x-a\rangle^{2}$	$M = \frac{m}{6}\langle x-a\rangle^{3}$

discontinuity functions

1. We add these up for each loading case along our beam
2. We integrate as usual to find displacement from load, slope, or moment

integration

- discontinuity functions follow special rules for integration
- when $n \geq 0$, they integrate like a normal polynomial
- when $n < 0$, they instead follow

$$\int \langle x - a \rangle^n dx = \langle x - a \rangle^{n+1}$$

signs

- we need to be careful to match the sign convention
- loads are defined as positive when they act upward
- moments are defined as positive when they act clockwise

example 12.5