Mechanics of Materials

Lecture 9 - Torsion

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schedule

- 3 Mar - Torsion
- 5 Mar - Homework 3 Due
- 8 Mar - Torsion
- 10 Mar - Bending
- 12 Mar - Homework 4 Due, Homework 3 Self-grade due

outline

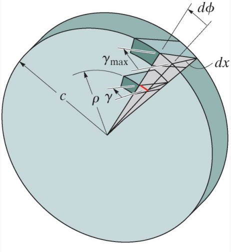
- torsion
- power transmission
- group problems

torsion

torsion

- Torque is a moment that tends to twist a member about its axis
- For small deformation problems, we assume that the length and radius do not change significantly under torsion
- The primary deformation we are concerned with in torsion is the angle of twist, denoted with ϕ

shear



The shear strain at points on the cross section increases linearly with ρ, i.e., $\gamma = (\rho/c)\gamma_{max}$.

torsion formula

- For a linearly elastic material, Hooke's Law in shear will hold ($\tau = G\gamma$)
- This means that, like shear strain, shear stress will vary linearly along the radius

torsion formula

- We can find the total force on an element, dA by multiplying the shear stress by the area

$$dF = \tau dA$$

- The torque ($dT = \rho dF$) produced by this force is then

$$dT = \rho(\tau dA)$$

torsion formula

- Integrating over the whole cross-section gives

$$T = \int_A \rho(\tau dA) = \frac{\tau_{max}}{c} \int_A \rho^2 dA$$

- The integral $\int_A \rho^2 dA$ is also called the Polar Moment of Inertia, J, so we can write

$$\tau_{max} = \frac{Tc}{J}$$

polar moment of inertia

- We know that $J = \int_A \rho^2 dA$, so we can compute this for some common shapes
- For a solid circular cross-section, we have

$$J = \int_0^c \rho^2 (2\pi \rho d\rho) = \frac{\pi}{2} c^4$$

- For a circular tube we have

$$J = \int_{c_1}^{c_2} \rho^2 (2\pi \rho d\rho) = \frac{\pi}{2}(c_2^4 - c_1^4)$$

example 5.1

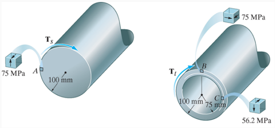


Figure 1: On left is a solid 100 mm radius tube, while on the right is a hollow tube with outer radius of 100 mm and inner radius of 75 mm. Element A is on the surface of the solid tube on the left, element B is on the outer surface of the hollow tube on the right and Element C is on the inner surface of the hollow tube

The allowable shear stress is 75 MPa. Determine the maximum torque that can be applied to each of the cross-sections shown and find the stress acting on a small element at A, B and C.

power transmission

power transmission

- Shafts and tubes are often connected to belts and drives, and the torque, speed, and power are all related
- Power is the rate of work done, for rotation problems, $P = T\omega$
- We are often given the frequency f instead of the angular velocity, ω, in this case $P = 2\pi f T$

power units

- In SI Units, power is expressed in Watts 1 W = 1 N m / sec
- In Freedom Units, power is expressed in Horsepower 1 hp = 550 ft lb / sec

shaft design

- We often know the power and frequency of a drive, and need to design a shaft such that the shear stress is acceptable
- We can easily find the torque as $T = P/2\pi f$, we can use this combined with the torsion equation

$$\tau_{max} = \frac{Tc}{J}$$

to find the appropriate shaft diameter. - For solid shafts we can solve for c uniquely, but not for hollow shafts

example 5.4

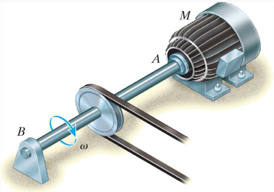


Figure 2: A rotating shaft connected to a motor

The steel shaft shown is connected to a 5 hp motor that rotates at $\omega = 175$ rpm. If $\tau_{allow} = 14.5$ ksi, determine the required shaft diameter.

group problems

group one

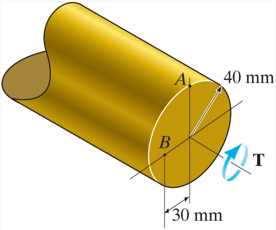


Figure 3: A 40 mm radius solid shaft. Point A is on the outer surface, Point B is 30 mm away from the center.

The solid circular shaft is subjected to an internal torque of 5 kN.m. Determine the shear stress at A and B and represent each state of stress on a volume element.

group two

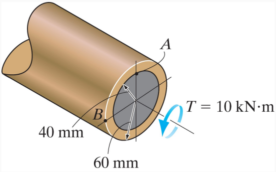


Figure 4: A hollow circular shaft with outer radius of 60 mm and inner radius of 40 mm. Point A is on the inner surface, Point B is on the outer surface.

The hollow circular shaft is subjected to an internal torque of 10 kN.m. Determine the shear stress at A and B and represent each state of stress on a volume element.

group three

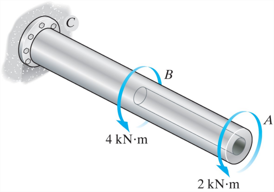


Figure 5: There is a fixed support at C, an applied torque of 4 kN.m at B (in the middle) and an applied torque of 2 kN.m at A (at the free end).

The circular shaft is hollow from A to B and solid from B to C. Determine the shear stress at A and B. The outer diameter is 80 mm and the wall thickness is 10 mm.

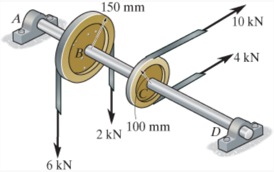


Figure 6: A shaft supports to pulleys, one with a 150 mm radius and tension of 6 kN at one end and 2 kN at the other other. The other pulley has a 100 mm radius and tensions of 10 kN and 4 kN.

Determine the maximum shear stress in the 40 mm diameter shaft.