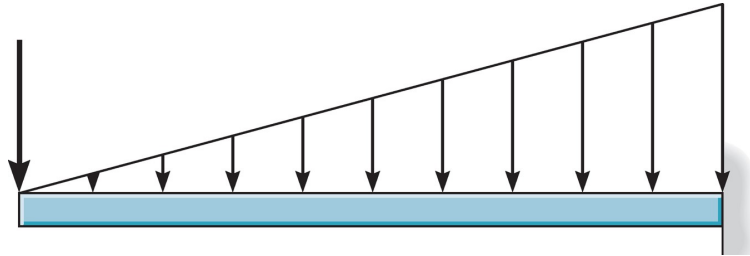Name:

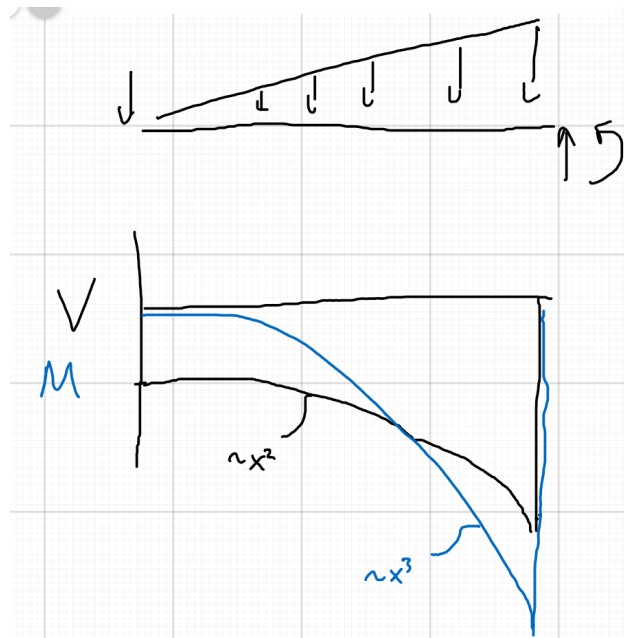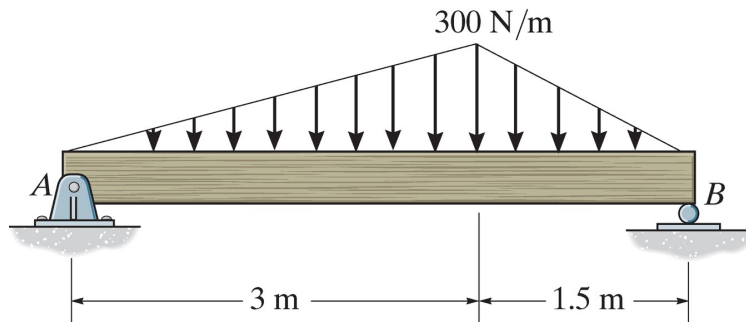# Homework 5 Solutions
Due 6 Oct 2020

1. Draw a free-body diagram of the beam and sketch the general shape of the shear-moment diagram

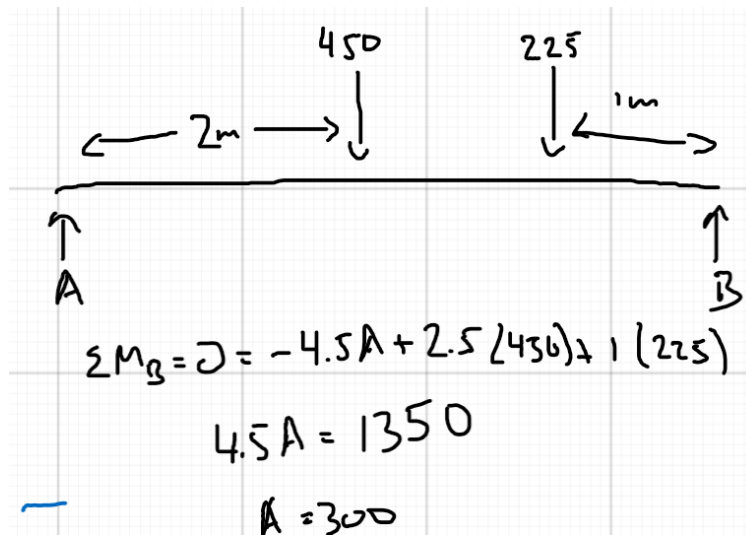   **Solution:**

2. Draw the shear-moment diagram for the simply supported beam
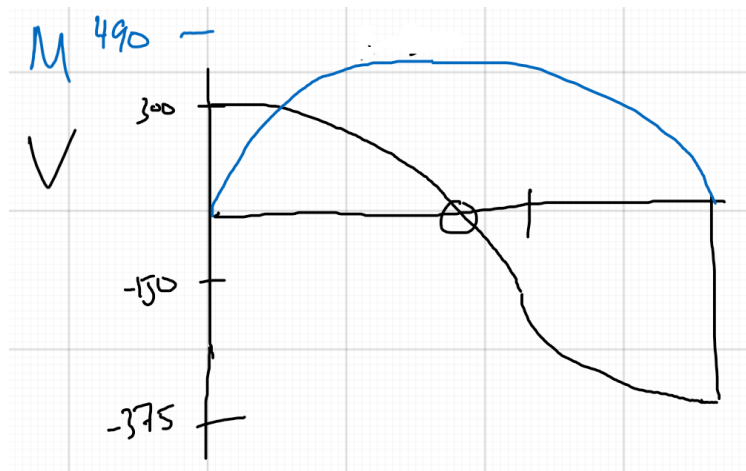
300 N/m

A

B

3 m — 1.5 m

Solution:

- We start by setting up a free body diagram to find the unknown support, A.
- NOTE: If we trust our process, we don't actually need to solve for B (although once A is found it is not difficult to do)

450   225

$\leftarrow\ 2m\ \rightarrow$ $\leftarrow$ 1m $\rightarrow$

A   B

$\Sigma M_B = 0 = -4.5A + 2.5(450) + 1(225)$

$4.5A = 1350$

$A = 300$

- Now we have enough information to draw the general shape of the graph
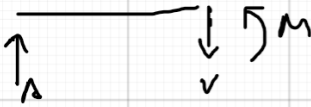

M   490   —

V   300

-150

-375

- However, it is not obvious where the line crosses the x-axis, or the value of the maximum moment. To find both we section the beam and solve for $V(x)$ and $M(x)$ directly, but we need only consider the region near where the function crosses the axis. (This means we do not need to deal with a piecewise function).

$$\frac{300}{3} = \frac{\omega}{x}$$

$$\omega = 100 x$$

$$(100 x) \frac{x}{2}$$

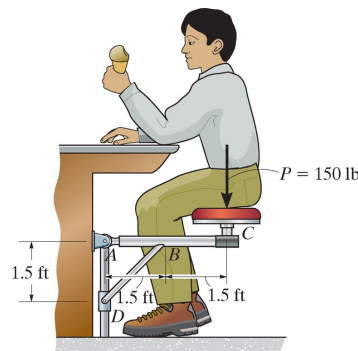$$\downarrow \frac{x}{3}$$

$$\downarrow 5 M$$

$$V = A - 50 x^2 \implies 0 = A - 50 x_o^2$$
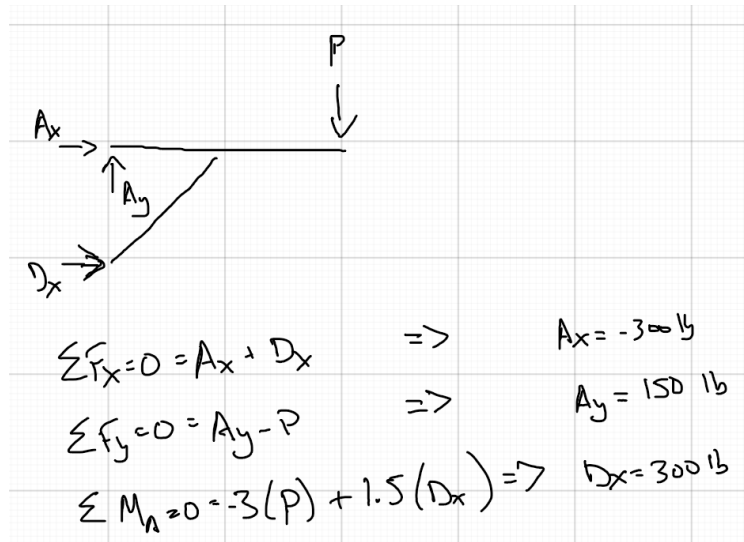$$x_o = 2.45$$

$$M = A x - \frac{50}{3} x^3$$

$$M(2.45) \approx 490$$

3. Members ABC and BD are rigidly connected at B while the smooth collar at D is allowed to move along the vertical post. Draw the shear-moment diagram for ABC.
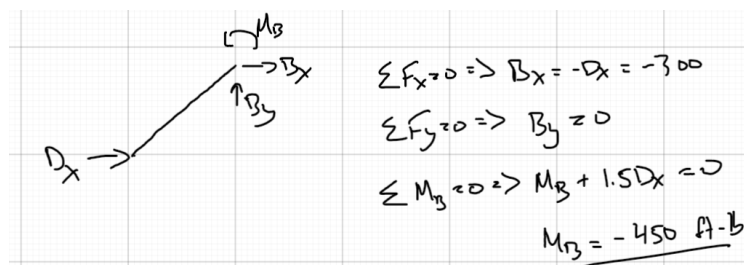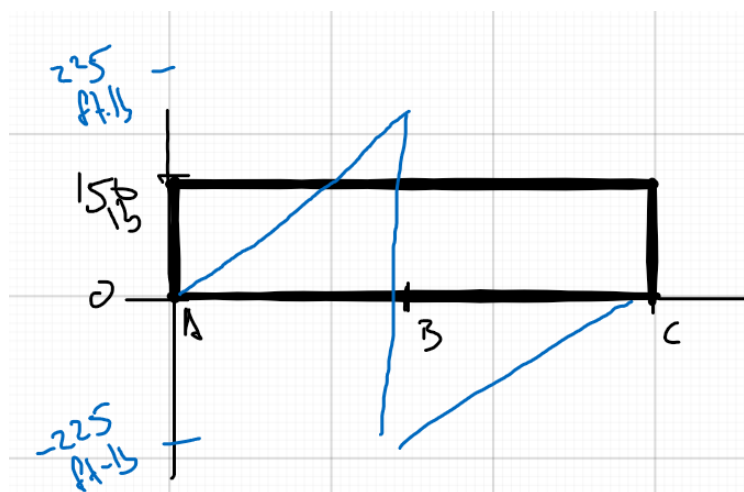

Solution:

- There are possibly a few different ways to consider the free body diagram for this problem. Initially, I decided to use the rigidly connected portion. Notice that since the support at D slides vertically (and is connected via a pin to allow rotation), it only exerts a reaction forces in the x direction.

3

P

$A_x \rightarrow$

$\uparrow A_y$

$D_x \rightarrow$

$\Sigma F_x = 0 = A_x + D_x \quad \Rightarrow \quad A_x = -300 \text{ lb}$

$\Sigma F_y = 0 = A_y - P \quad \Rightarrow \quad A_y = 150 \text{ lb}$

$\Sigma M_A = 0 = -3(P) + 1.5(D_x) \Rightarrow D_x = 300 \text{ lb}$

- Since the problem asks for the shear-moment diagram of ABC, we need to remove the connection BD, to determine the internal forces at that section, I now considered a FBD of BD

M_B

$\rightarrow B_x$

$\uparrow B_y$

$D_x \rightarrow$

$\Sigma F_x = 0 \Rightarrow B_x = -D_x = -300$

$\Sigma F_y = 0 \Rightarrow B_y = 0$

$\Sigma M_B = 0 \Rightarrow M_B + 1.5 D_x = 0$

$M_B = -450 \text{ ft-lb}$

- This gives the shear-moment diagram as

225
ft-lb

158
lb

0

-225
ft-lb

A

B

C

4. If the beam is subjected to a bending moment of $M = 10\,\text{kN}\cdot\text{m}$ find the bending stress in the beam at points A and B

4
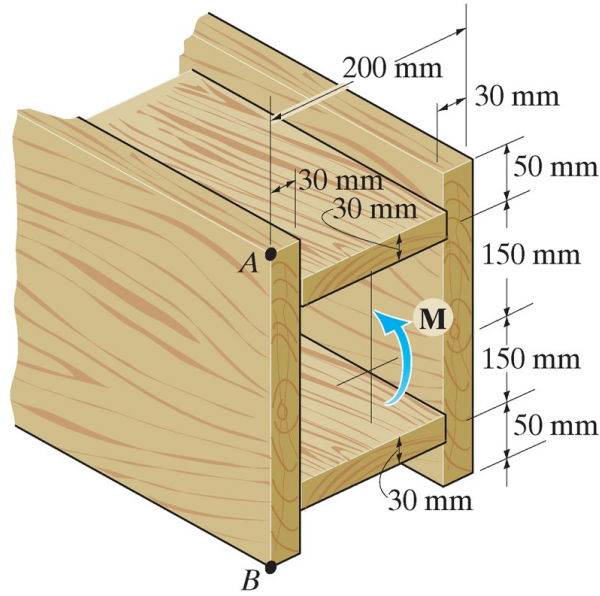
Solution:

- We will be using the flexure formula $\sigma = \frac{My}{I}$, since the moment is given we just need to find the neutral axis and the inertia

- The section is symmetric, so the neutral axis is simply the middle of the section

- Due to the symmetry we can find the total inertia as $I = 2I_1 + 2I_2$
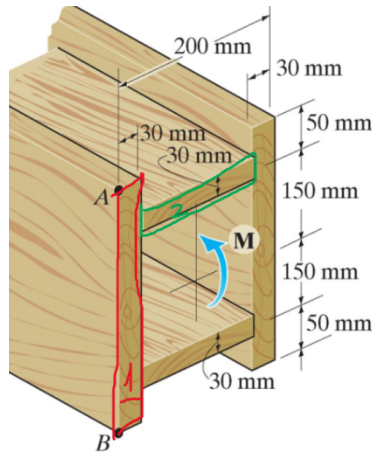
- This gives $I = 2\left(\frac{(30)(400^3)}{12}\right) + 2\left(\frac{(140)(30^3)}{12} + (140)(30)(150^2)\right) = 509.6 \times 10^6 \,\mathrm{mm}^4$

- With $y = 200\,\mathrm{mm}$ this gives $\sigma_A = -3.92\,\mathrm{MPa}$ and $\sigma_B = 3.92\,\mathrm{MPa}$

5. Find the absolute maximum bending stress in the beam assuming that the support at B exerts a uniformly distributed reaction on the beam. The cross section is rectangular with a base of $3\,\mathrm{in}$ and a height of $5\,\mathrm{in}$

5

14 kip

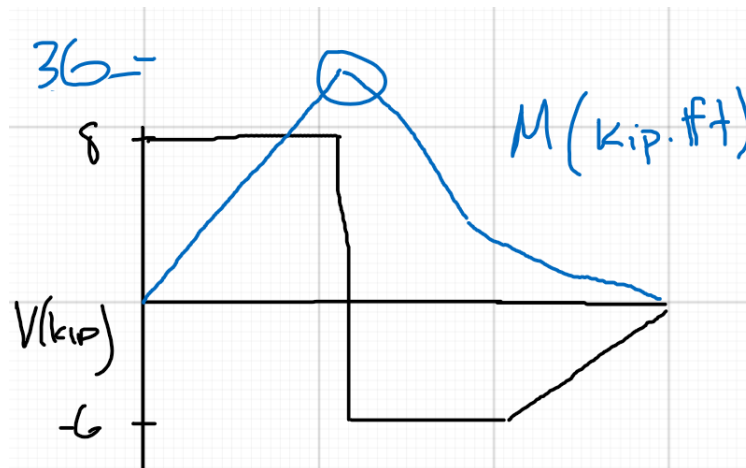A |←——— 4.5 ft ———→|←——— 4.5 ft ———→|←— 3 ft —→| B
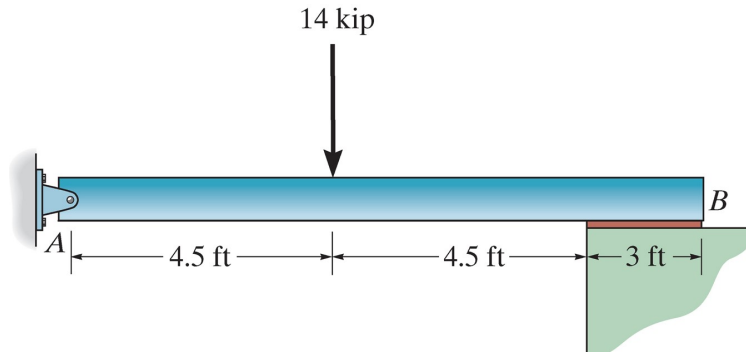
Solution:

- To solve for the distributed load, we first treat it as a concentrated force located in the middle, giving the free body diagram shown.
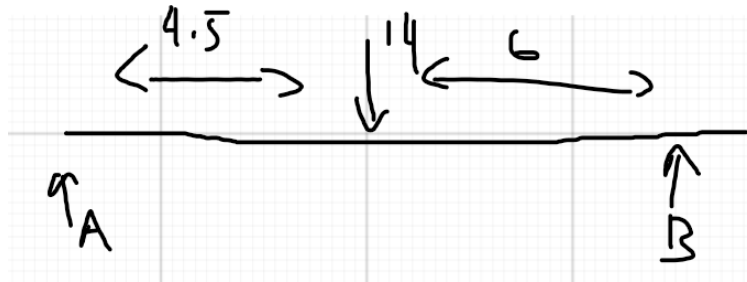

- We find $A = 8\,\text{kip}$ and $B = 6\,\text{kip}$
- The distributed load at B is $6\,\text{kip}/3\,\text{ft} = 2\,\text{kip/ft}$
- We can draw a shear-moment diagram to find the maximum moment, which we need to find the maximum bending stress
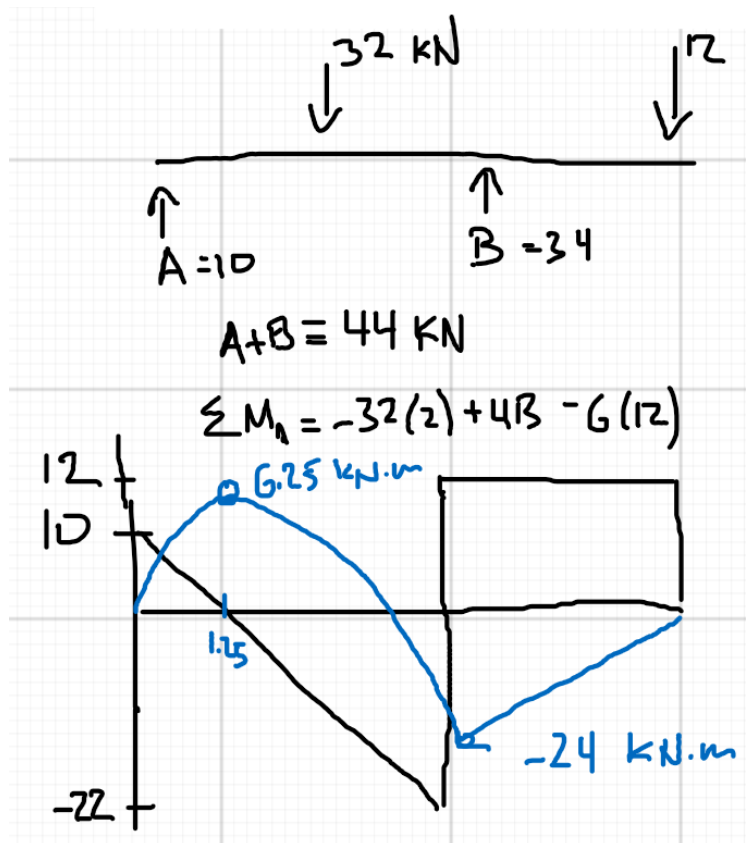

- The inertia is just $I = \frac{(3)(5^3)}{12} = 31.25\,\text{in}^4$
- We find the stress $\sigma = 36\,\text{kip} \cdot \text{ft} 2.5\,\text{in}/31.25\,\text{in}^4 = 34.6\,\text{ksi}$

6

6. If the allowable bending stress is $\sigma_{allow} = 6\,\text{MPa}$, find the maximum dimension, d of the beams cross sectional area.
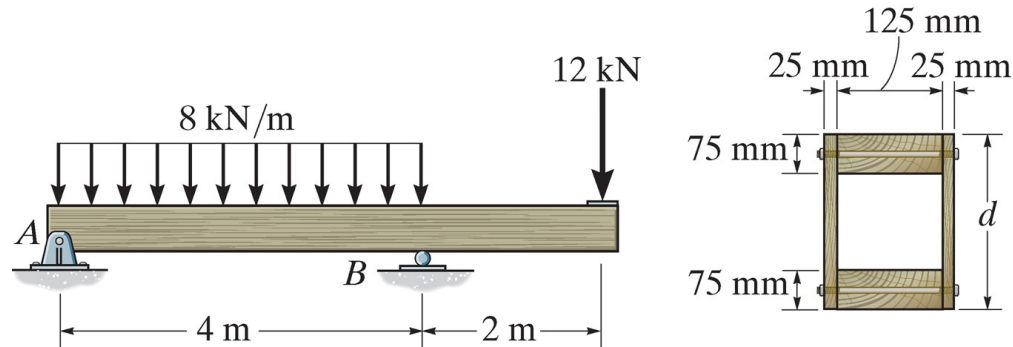
Solution:

- We start by finding the maximum bending moment with the shear-bending diagram

$$32\text{ kN} \qquad 12$$

$$A = 10 \qquad B = 34$$

$$A + B = 44\text{ KN}$$

$$\Sigma M_A = -32(2) + 4B - 6(12)$$

$$6.25\text{ kN.m}$$

$$1.25$$

$$-24\text{ kN.m}$$

$$12, 10, -22$$

- We now need to find an expression for the inertia, $I = 2\left(\frac{25d^3}{12}\right) + 2\left(\frac{125(75^3)}{12} + 125(75)\left(\frac{d}{2} - \frac{75}{2}\right)^2\right)$

- And finally we can set up an expression for d, $\sigma = 6\,\text{MPa} = \frac{24d/2}{I} = \frac{12d}{25d^3/6 + 125(75^3)/6 + 250(75)(d/2 - 75/2)}$

- Before solving we need to make a note about unit consistency, since σ is in MPa and our length units are in mm we need to convert the moment from kN m to N mm, we find $24\,\text{kNm} = 24 \times 10^6\,\text{Nmm}$

- After simplifying, we find a cubic equation with three solutions for d, one of them is non-physical (negative), and we choose the larger of the remaining solutions, $d = 409\,\text{mm}$

7. These garden shears were manufactured using an inferior material. Using a load of 50 lb and appropriate dimensions find the maximum bending stress and show why failure occurred at this location

Solution:

- It is actually unnecessary (and a bit difficult) to model the entire set of shears (and would depend on what is being cut), instead we can directly consider a handle section as shown
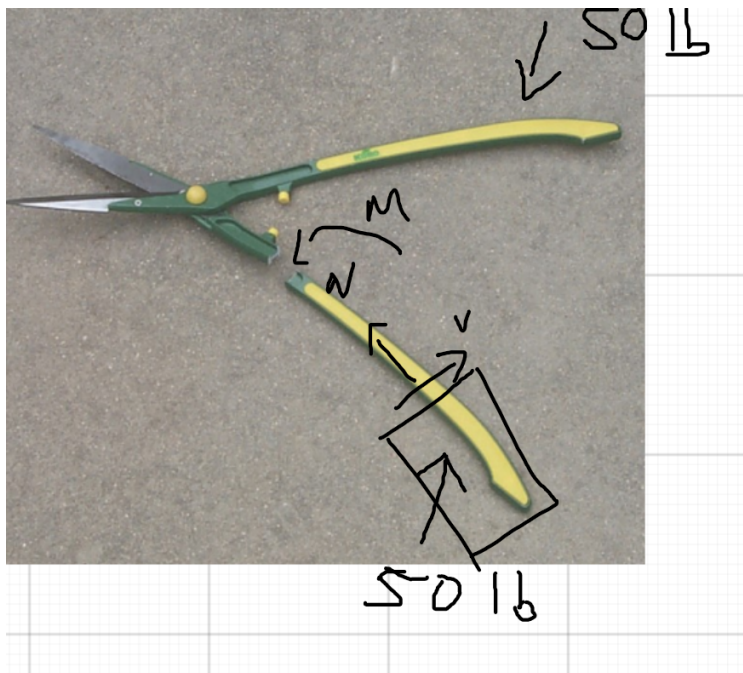
- We can see that the moment we find increases linearly until we reach the next support. Right at the point where the bending moment would be increasing to its maximum point, the cross-section also changes to decrease the inertia, I, which will also increase the stress.

- Assuming 3 inches of handle length between where we apply the load and the next support and an I-beam that is 1/2" tall, 1/16" thick and 1/4" wide in the I-beam flanges we find $I = 1.78 \times 10^{-3}\,\mathrm{in}^4$ and $\sigma = (50\,\mathrm{lb})(3\,\mathrm{in})(1/4\,\mathrm{in})/1.78 \times 10^{-3}\,\mathrm{in}^4 = 84.3\,\mathrm{ksi}$

- This stress would be enough to cause failure in most steel alloys (and other metals) listed in the textbook, so clearly a heavier casting (with greater inertia) is needed to prevent this failure