Mechanics of Materials

Lecture 11 - Bending

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schedule

- 29 Sep - Bending
- 1 Oct - Homework 4 Due, Homework 3 Self-grade due
- 4 Oct - Bending
- 6 Oct - Transverse Shear
- 8 Oct - Homework 5 Due, Homework 4 Self-grade due
- (11 Oct) - Fall Break

outline

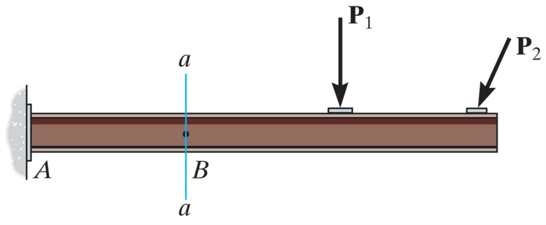
- shear and moment diagrams
- graphical method
- bending deformation

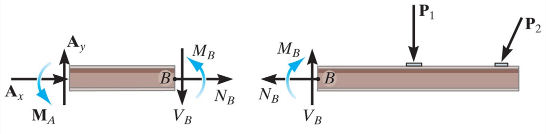
shear and moment diagrams

shear and moment diagrams

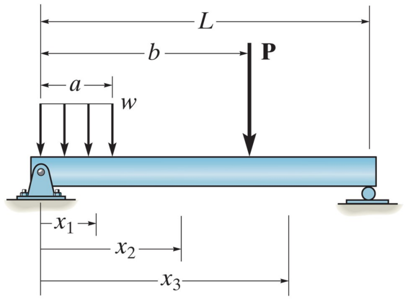
- The general approach to shear and moment diagrams is to first find the support reactions
- Next we section the beam and instead of finding the internal force and moment at a single point, we find it as a function of x
- Many beams will require piecewise sectioning
- We then draw this as a shear and moment diagram

sign convention

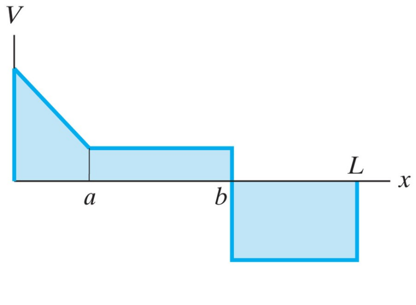




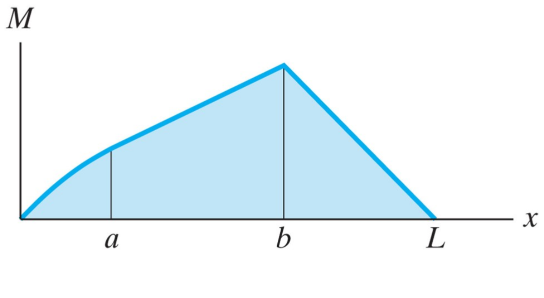
example beam



example beam



example beam

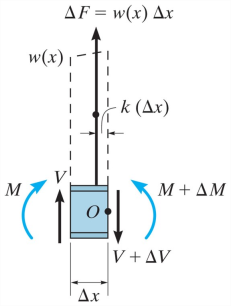


graphical method

relation between load, shear, moment

- When there are several forces, supports, or loading conditions applied to a beam, the piecewise method can be cumbersome
- In this section we will examine the differential relationships between distributed load, shear, and bending moments

distributed load



distributed load

- Consider a beam subjected to only distributed loading
- If we section this beam in the middle (to remove both supports) we can relate V to the loading function $w(x)$
- Considering the sum of forces in y:

$$
\begin{aligned}
V + w(x)\Delta x - (V + \Delta V) &= 0 \\
\Delta V &= w(x)\Delta x
\end{aligned}
$$

distributed load

- If we divide by Δx and let $\Delta x \to 0$ we find

$$\frac{dV}{dx} = w(x)$$

- Thus the slope of the shear diagram is equal to the distributed load function

moment diagram

- If we consider the sum of moments about O on the same section we find

$$(M + \Delta M) - (w(x)\Delta x)k\Delta x - V\Delta x - M = 0$$
$$\Delta M = V\Delta x + kw(x)\Delta x^2$$

- Dividing by Δx and letting $\Delta x \to 0$ gives

$$\frac{dM}{dx} = V$$

concentrated forces

- If we consider a concentrated force (instead of a distributed load) we find

$$\Delta V = F$$

- This means that concentrated loads will cause the shear diagram to “jump” by the amount of the concentrated force (causing a discontinuity on our graph)

couple moments

- If our section includes a couple moment, we find (from the moment equation) that

$$\Delta M = M_0$$

- Thus the moment diagram will have a jump discontinuity

example 7.9

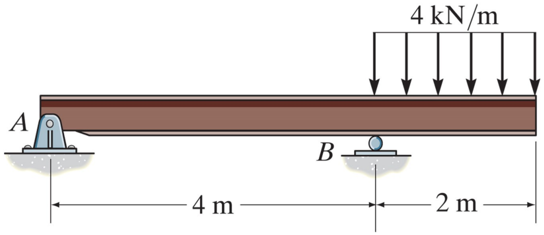
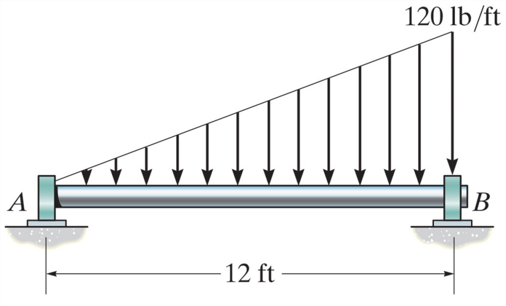


Figure 1: A beam is 6 meters long with pin supports at the left end, A, and at B, 4 meters to the right of A. From B to the right end of the beam is a uniform distributed load of 4 kN/m.

example 7.10

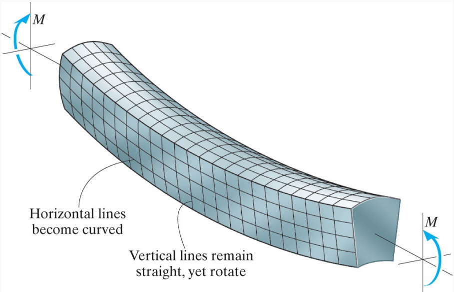


bending deformation

bending deformation

- If we drew a grid on a specimen in bending, we would find that vertical lines tend to stay straight (but rotate)
- Horizontal lines will become curved
- If bending lifts the ends up (like a smile), then the top face will be in compression (and expand), while the bottom face will be in tension (and contract)

bending deformation



After deformation

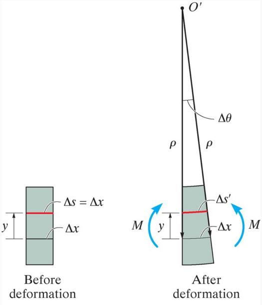
neutral axis

- Since the bottom is in tension and the top is in compression, there must be somewhere in between that is under no stress
- We call that the neutral axis, and assume it does not change in length
- We also assume that planar sections remain planar (no warping)
- Finally, Poisson's effects are neglected (cross-sections keep the same size and shape)

strain

- We will now consider an infinitesimal beam element before and after deformation
- Δx is located on the neutral axis and thus does not change in length after deformation
- Some other line segment, Δs is located y away from the neutral axis and changes its length to $\Delta s'$ after deformation

strain



strain

- We can now define strain at the line segment Δs as

$$\epsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

strain

- If we define ρ as the radius of curvature after deformation, thus $\Delta x = \Delta s = \rho \Delta \theta$
- The radius of curvature at Δs is $\rho - y$, thus we can write

$$\epsilon = \lim_{\Delta \theta \to 0} \frac{(\rho - y)\Delta \theta - \rho \Delta \theta}{\rho \Delta \theta}$$

- Which gives

$$\epsilon = -\frac{y}{\rho}$$