Mechanics of Materials

Lecture 25 - Superposition

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

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schedule

- 22 Apr - Beam Deflection (superposition)
- 24 Apr - Recitation, HW9 Due
- 27 Apr - Statically Indeterminate Beams
- 29 Apr - Beam Review
- 1 May - Recitation, HW 10 Due

outline

- discontinuity functions
- group problems
- superposition

discontinuity functions

discontinuity functions

- Direct integration can be very cumbersome if multiple loads or boundary conditions are applied
- Instead of using a piecewise function, we can use discontinuity functions

Macaulay functions

- Macaulay functions can be used to describe various loading conditions, the general definition is

$$\langle x-a\rangle^n = \begin{cases} 0 & \text{for } x < a \\ (x-a)^n & \text{for } x \geq a \end{cases} n \geq 0$$

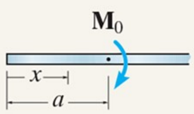
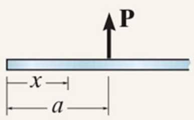
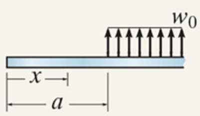
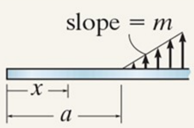
singularity functions

- Singularity functions are used for concentrated forces and can be written

$$w = P\langle x-a\rangle^{-1} = \begin{cases} 0 & \text{for } x \neq a \\ P & \text{for } x = a \end{cases}$$

discontinuity functions

TABLE 12–2

Loading	Loading Function $w = w(x)$	Shear $V = \int w(x)dx$	Moment $M = \int V dx$
	$w = M_0\langle x-a\rangle^{-2}$	$V = M_0\langle x-a\rangle^{-1}$	$M = M_0\langle x-a\rangle^{0}$
	$w = P\langle x-a\rangle^{-1}$	$V = P\langle x-a\rangle^{0}$	$M = P\langle x-a\rangle^{1}$
	$w = w_0\langle x-a\rangle^{0}$	$V = w_0\langle x-a\rangle^{1}$	$M = \frac{w_0}{2}\langle x-a\rangle^{2}$
	$w = m\langle x-a\rangle^{1}$	$V = \frac{m}{2}\langle x-a\rangle^{2}$	$M = \frac{m}{6}\langle x-a\rangle^{3}$

discontinuity functions

1. We add these up for each loading case along our beam
2. We integrate as usual to find displacement from load, slope, or moment

integration

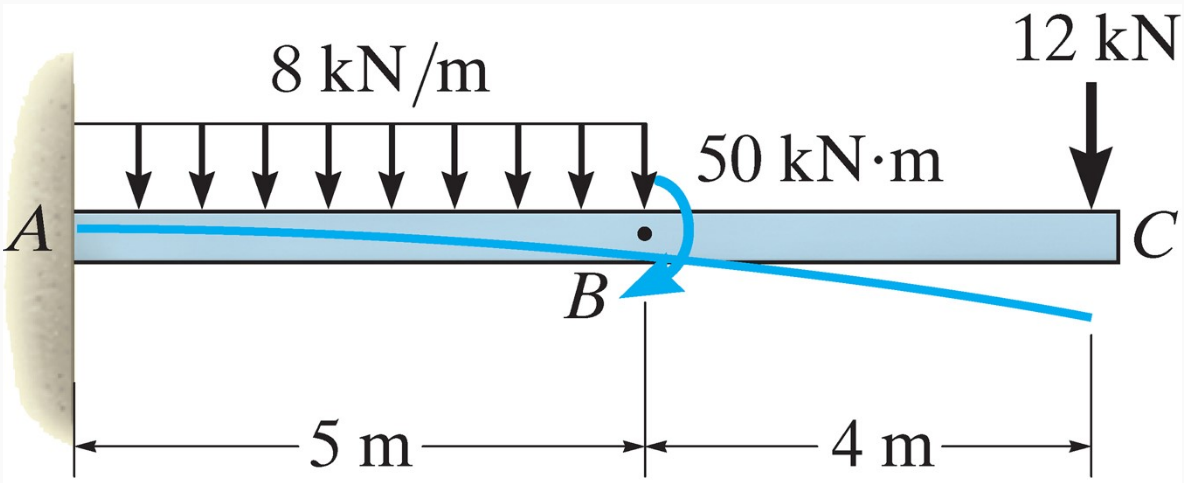
- discontinuity functions follow special rules for integration
- when $n \geq 0$, they integrate like a normal polynomial
- when $n < 0$, they instead follow

$$\int \langle x - a \rangle^n dx = \langle x - a \rangle^{n+1}$$

signs

- we need to be careful to match the sign convention
- loads are defined as positive when they act upward
- moments are defined as positive when they act clockwise

example 12.5



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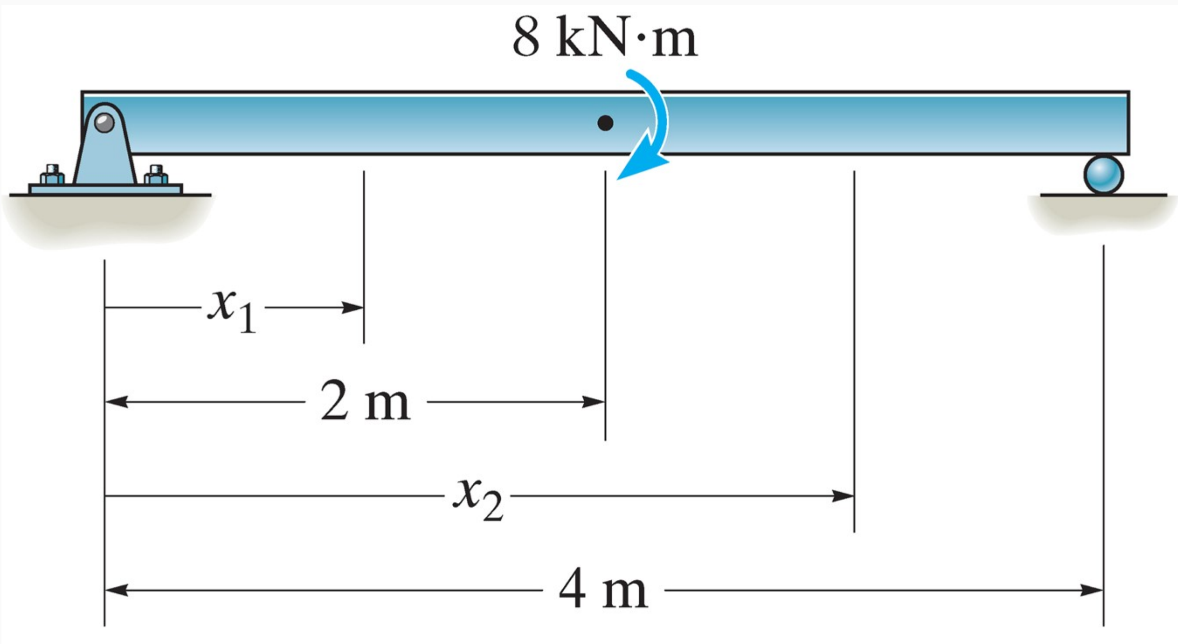
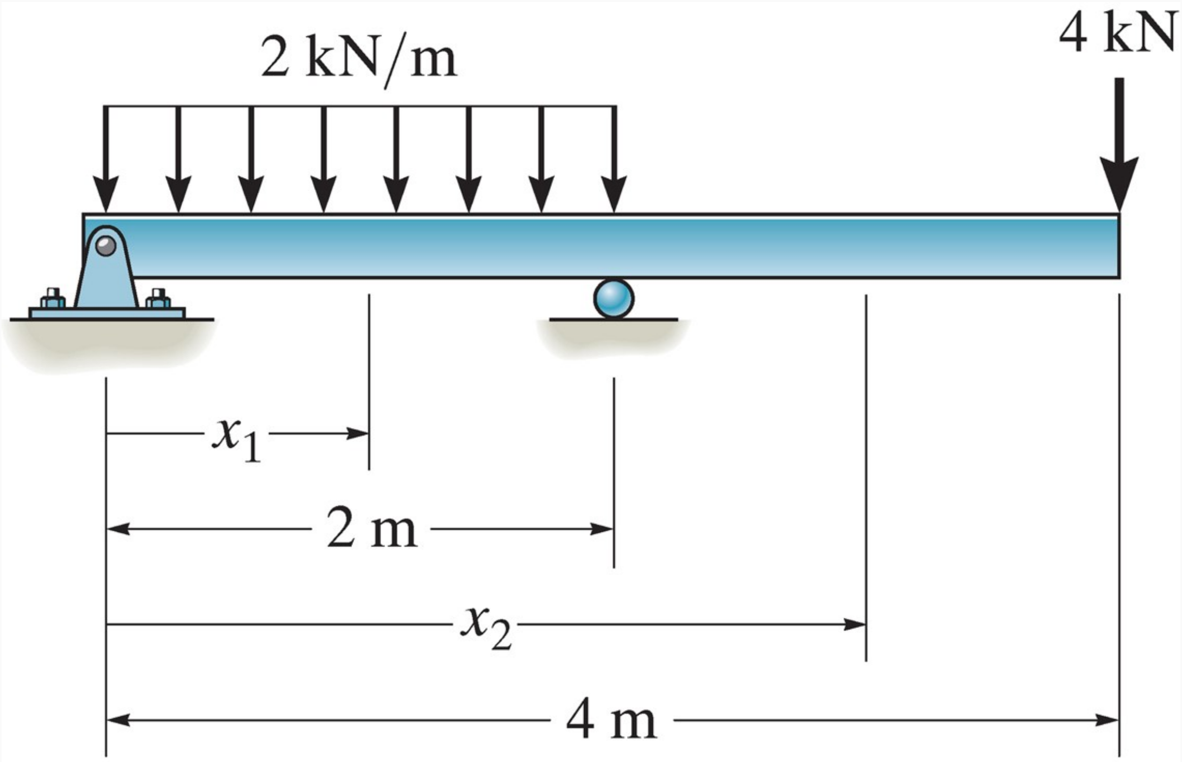
group problems

group one



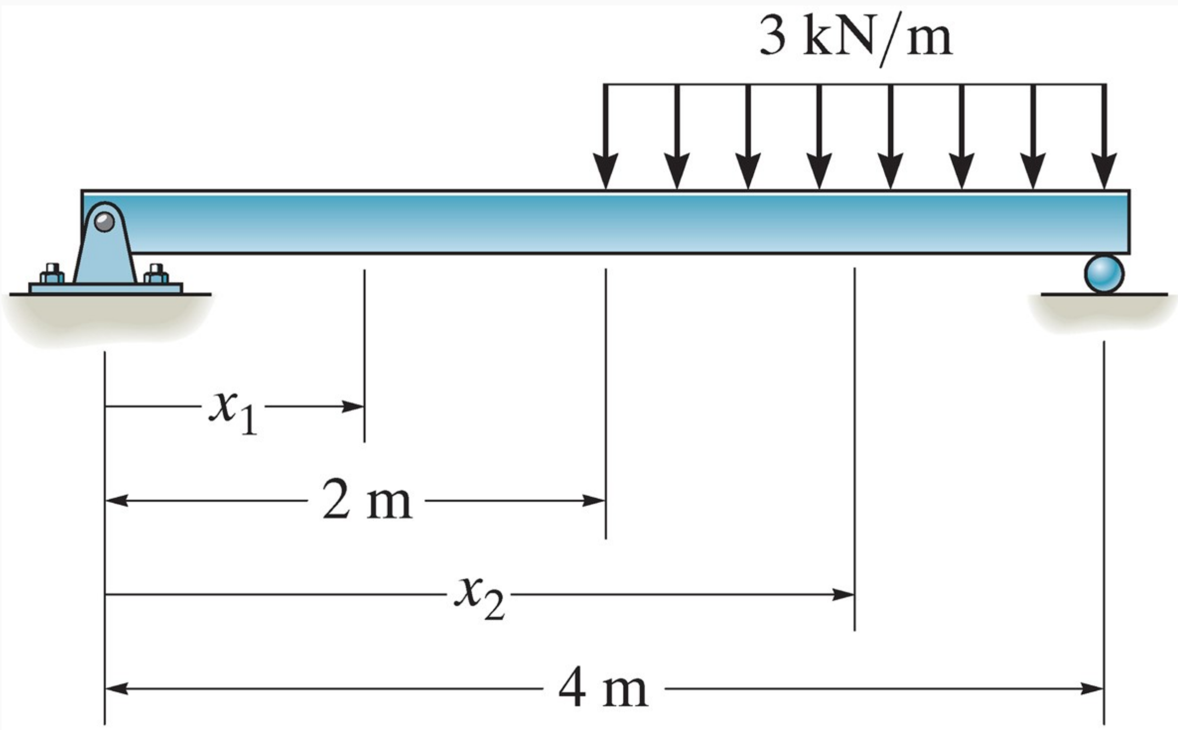
Find the maximum deflection using either direct integration or discontinuity functions.

group two



Find the maximum deflection using either direct integration or discontinuity functions.

group three



Find the maximum deflection using either direct integration or discontinuity functions.

superposition

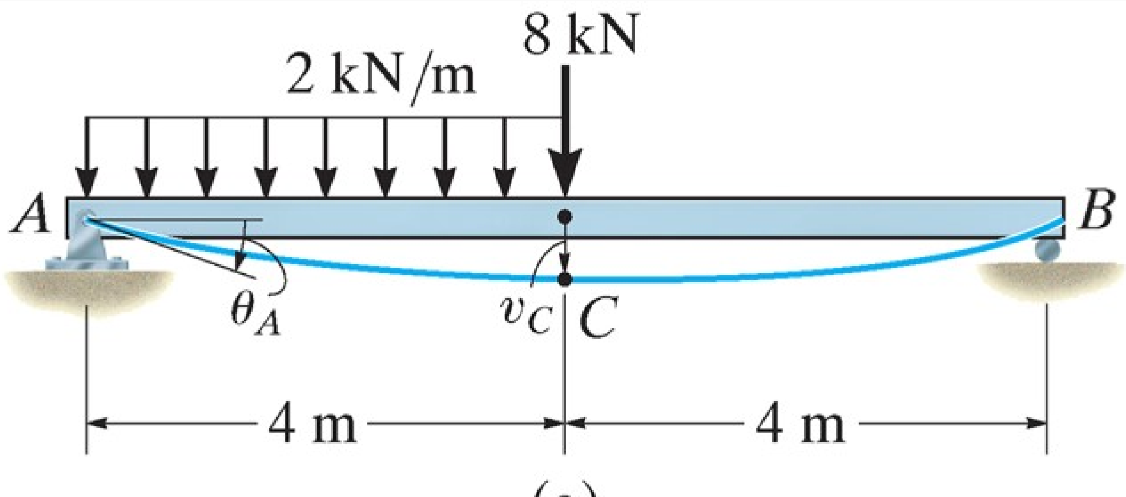
superposition

- The differential equation $EId4v/dx4 = w(x)$ satisfies the requirements for superposition
- $w(x)$ is linearly related to $v(x)$
- Load does not significantly change the shape of the beam

superposition

- This means we can superpose multiple deflection solutions from simpler cases
- Appendix C in the text has many solutions that can be superposed

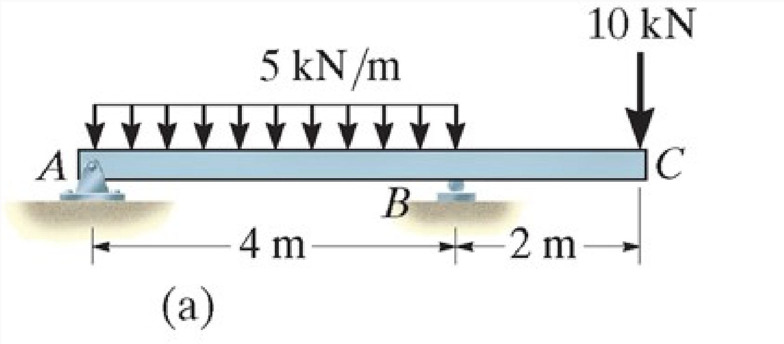
example 12.13



(a)

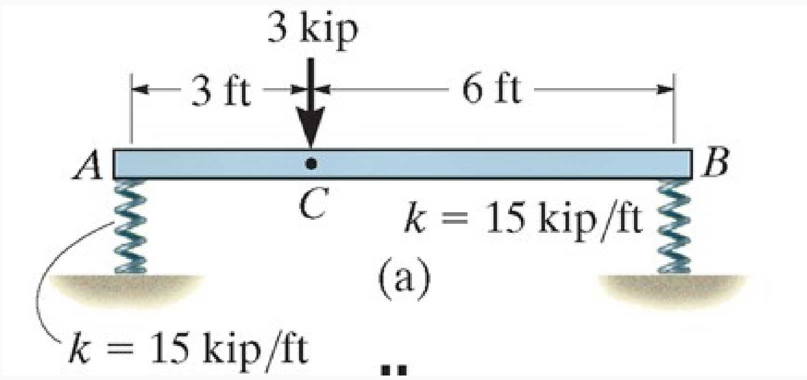
Use superposition to find the displacement at C and the slope at A

example 12.15



Use superposition to find the displacement at C

example 12.16



The steel bar is supported by springs with k=15 kip/ft originally unstretched. For the force shown, determine the displacement at C. Take $E_{st} = 29$ Msi and $I = 12$ in^4.