Mechanics of Materials

Lecture 3 - Strain

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schedule

- 8 Feb - Strain, Homework 1 Due
- 10 Feb - Mechanical Properties
- 15 Feb - Exam Review, Homework 2 Due, Homework 1 Self-grade Due
- 17 Feb - Exam 1
- 19 Feb - Project 1 Due

outline

- allowable stress design
- limit state design
- strain

allowable stress design

allowable stress

- Most of the time, we design structures so the stress is less than some limit
- By setting a conservative allowable stress, we account for some manufacturing tolerances, unintended loads, and variability in mechanical properties

factor of safety

- The factor of safety is the failure load divided by the allowable load

$$FS = \frac{F_{fail}}{F_{allow}}$$

- Since load and stress are linearly proportional, we could also define the factor of safety in terms of stress and it would be identical

factor of safety

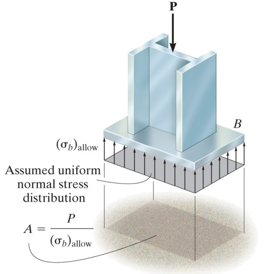
- Typical values for the factor of safety will vary based on application
- Aircraft and space vehicles might have a factor close to 1 to minimize weight
- Nuclear power plants might have a factor close to 3 since weight is not as important and failure would be catastrophic

simple connections

- We can rearrange the equations $\sigma = N/A$ and $\tau = V/A$ to size components based on some allowable stress

$$A = \frac{N}{\sigma_{allow}}$$
$$A = \frac{V}{\tau_{allow}}$$

bearing stress

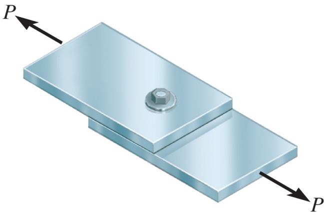


The area of the column base plate B is determined from the allowable bearing stress for the concrete.

embedded shear stress

The embedded length l of this rod in concrete can be determined using the allowable shear stress of the bonding glue.

lap joint shear



The area of the bolt for this lap joint is determined from the shear stress, which is largest between the plates.

limit state design

limit state design

- Allowable stress design accounts for uncertainty in the applied loading and the material properties in one factor of safety
- Limit state design separates these two into load and resistance factors

load factors

- The load factor combines uncertainty in various types of load
- For example, a building can have loading from a few different sources, such as its own weight, people in the building, and snow on top of the building
- Weight is considered a *dead load* and can usually be determined more precisely than moving things like people

load factors

- In this simple example, we consider a load factor, $\gamma_D = 1.2$ for the dead load, $\gamma_L = 1.6$ and $\gamma_S = 0.5$

$$R = 1.2D + 1.6L + 0.5S$$

- These load factors combine the concept of a safety factor with the probability that loads will occur

resistance factors

- Resistance factors, ϕ are used to express the probability a material will fail at its limit load
- If we are very confident in the failure stress of a material (i.e. steel has little variability), we might use $\phi = 0.9$
- If we are not as confident, (using a new material, or an organic material like wood with higher variability), we might use $\phi = 0.7$

design criteria

- If we call the nominal load P, then we can combine load and resistance factors using

$$\phi P \geq R$$

example 1-12

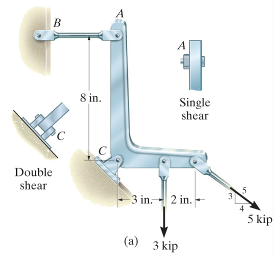


Figure 1: An l-shaped bracket has an 8 inch vertical leg and a 5 inch horizontal leg. A single shear pinned connector holds the point of the leg, A, in place while a

Determine to the nearest 1/4" the diameters of steel pins at A and C if the factor of safety in shear is 1.5 and the failure shear stress is 12 ksi.

example 1-15

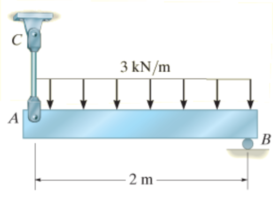


Figure 2: A 2 meter long beam is supported at the left end with a steel rod connecting vertically. It is subjected to a uniform load of 3 kilonewtons per meter, and a roller support at the right end

The 400 kg uniform bar, AB is supported by a steel rod AC and a roller at B. If it supports a live distributed loading, determine the required diameter of the rod. Use $\sigma_{fail} = 345$ MPa with $\phi = 0.9$, $\gamma_D = 1.2$, and $\gamma_L = 1.6$

strain

deformation

- When forces are applied to a body, it will change its shape and size
- We call these changes *deformation*
- Sometimes they are barely noticeable (steel), other times they are very significant (rubber)

strain

- Strain is a more precise measurement of the deformation of a body
- Normal strain is given as the change in length divided by the original length

$$\epsilon = \frac{L - L_0}{L_0}$$

- We can consider the average normal strain (over an entire body) or the local strain (take an infinitely small portion and calculate the strain there)

units

- Since we divide length by length, strain is unitless
- However it is customary to use *in/in* or for stiff specimens to use the phrase *microstrain* as a unit
- Strain can also sometimes be represented as a percent

shear strain

- Normal strain causes a line segment to expand or contract
- Deformation can also cause two lines to change their relative angle
- The change in angle between two originally perpendicular line segments is called shear strain

$$\gamma = \frac{\pi}{2} - \theta$$

shear strain

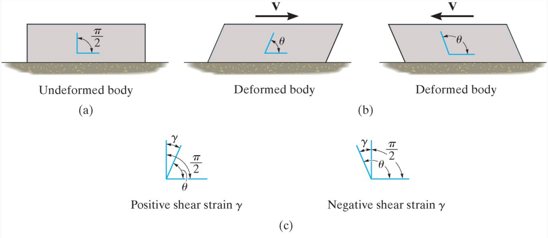


Figure 3: Three stages are shown, the first is a rectangular block at rest, with a fixed support on the ground. The second shows the block after it deforms with a horizontal force acting along the top to the right. The third shows the block after it deforms with a force acting along the top to the left. The first case causes a decrease in

cartesian components

- If we consider a very small cube/prism with sides of Δx, Δy, and Δz, normal strains will change the side lengths to

$$(1+\epsilon_x)\Delta x(1+\epsilon_y)\Delta y(1+\epsilon_z)\Delta z$$

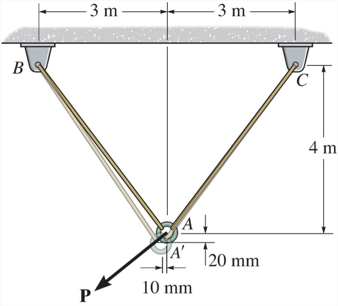
- And the shear strains will change the shape

$$\frac{\pi}{2}-\gamma_{xy} \qquad \frac{\pi}{2}-\gamma_{yz} \qquad \frac{\pi}{2}-\gamma_{xz}$$

small strain

- Most engineering analysis is based on the assumption of small strains
- This is valid for many materials (wood, metal), but not for rubbers and some other polymers
- When strains are small, we assume that the change in angle, $\Delta\theta$ is very small
- $\sin\Delta\theta \approx \Delta\theta$, $\cos\Delta\theta \approx 1$, $\tan\Delta\theta \approx \Delta\theta$

example 2.1



Find the normal strains in the two wires if A moves to A'

example 2.3

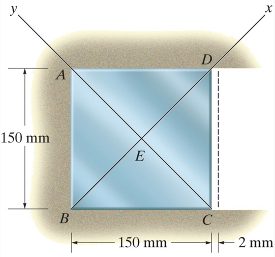


Figure 4: A 150 mm square block is constrained along the top, left, and bottom faces, but pushed 2 mm to the left on its right face. AC is the diagonal line going from the top left to the bottom right. E is the center point of the block (where the two diagonals intersect after deformation).

The plate is fixed along AB and held in horizontal guides along AD and BC. If the right side is displaced 2 mm find the average normal strain along AC and the shear strain at E relative to the x and y axes.