AE333

Mechanics of Materials

Lecture 16 - Bending

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schedule

- 6 Mar - Exam 2 Review, HW 5 Due
- 9 Mar - Exam 2
- 11 Mar - Transverse Shear
- 13 Mar - Transverse Shear

outline

- compound centroids
- shear in straight members
- the shear formula

compound centroids

composite bodies

- Often we have to deal with bodies that are not described by a continuous function, but are made of different materials or different shapes
- We can use the same logic previously, but with a finite sum instead of an integral

$$
\begin{aligned}
\bar{x} \sum W &= \sum \tilde{x} W \\
\bar{y} \sum W &= \sum \tilde{y} W \\
\bar{z} \sum W &= \sum \tilde{z} W
\end{aligned}
$$

analysis procedure

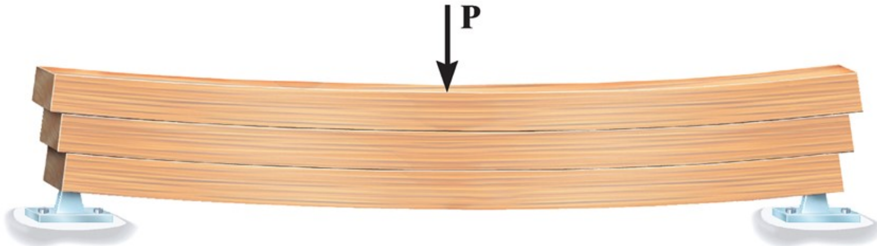
- Use a sketch to divide the body into sub-bodies
- If a body has a hole, it may be easier to treat that volume as whole and then subtract the hole
- Take note of any symmetry (an object symmetric about any axis will have a centroid along that axis)

shear in straight members

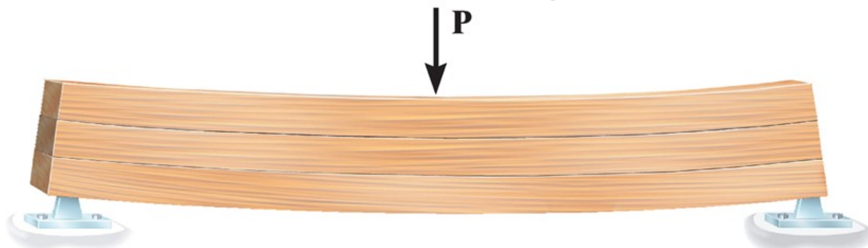
shear

- We have discussed the internal stresses caused by the internal moment M
- There are also internal shear stresses caused by the internal shear force V
- We can illustrate the effect of internal shear stress by considering three boards, either resting on top of on another or bonded

shear



Boards not bonded together



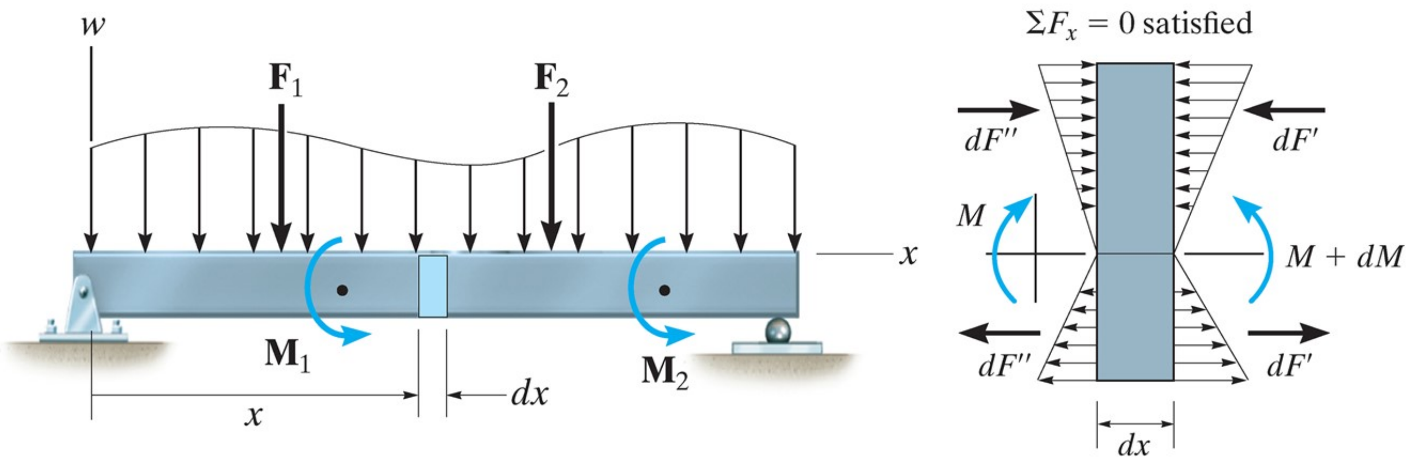
Boards bonded together

the shear formula

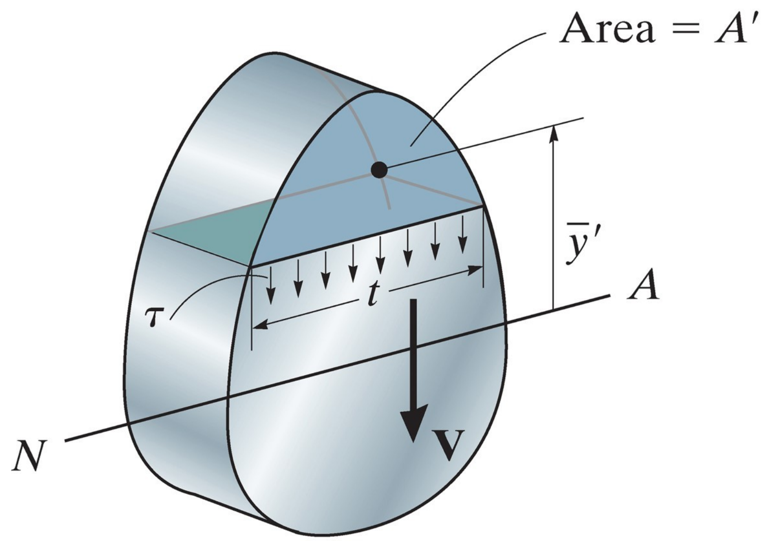
shear formula

- Internal shear causes a more complicated deformation state, so we will use an indirect method to find the shear stress-strain distribution
- We will consider equilibrium along a section of a beam, then we will consider another section

equilibrium



equilibrium



equilibrium

- There must be a shear force along the bottom to equilibrate the different stresses on either side of the section
- If we assume that this shear is constant through the thickness, we obtain the following from equilibrium

$$\sum F_x = 0 = \int_{A'} \sigma' dA' - \int_{A'} \sigma dA' - \tau(t dx)$$

equilibrium

$$0 = \int_{A'} \left(\frac{M + dM}{I} \right) y dA' - \int_{A'} \left(\frac{M}{I} \right) y dA' - \tau(tdx)$$

$$\left(\frac{M}{I} \right) \int_{A'} y dA' = \tau(tdx)$$

$$\tau = \frac{1}{It} \left(\frac{dM}{dx} \right) \int_{A'} y dA'$$

shear formula

- In this formula, recall that $V = \frac{dM}{dx}$
- We also call Q the moment of the area A' which is equal to $\int_{a'} y dA'$
- We can also write Q in terms of the centroid

$$Q = \bar{y}' A'$$

shear formula

- Simplified, the shear formula is

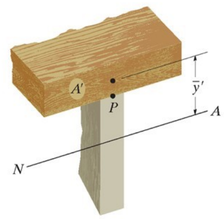
$$\tau = \frac{VQ}{It}$$

- Q poses the greatest difficulty in calculations, so we will consider a few examples

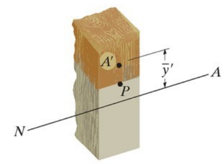
Q

- Q represents the moment of the cross-sectional area above (or below) the point at which the shear stress is being calculated
- We apply the formula to that area

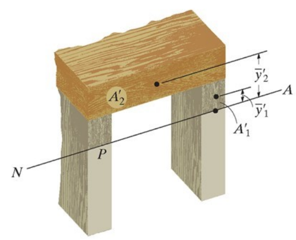
Q



$Q = \bar{y}'A'$



$Q = \bar{y}'A'$



$Q = 2\bar{y}'_1A'_1 + \bar{y}'_2A'_2$

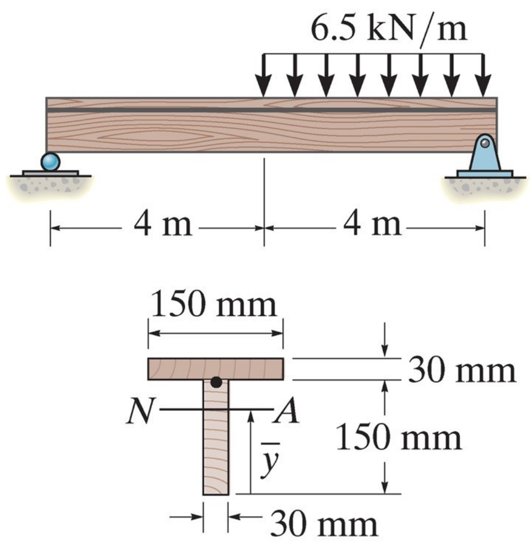
shear formula limitations

- A major assumption made is that the shear stress was constant through the thickness, essentially we have found the average shear
- This is more accurate the more slender a beam is (small b and large h)
- The formula is also not accurate for cross sections that change or have boundaries that are not right angles

procedure

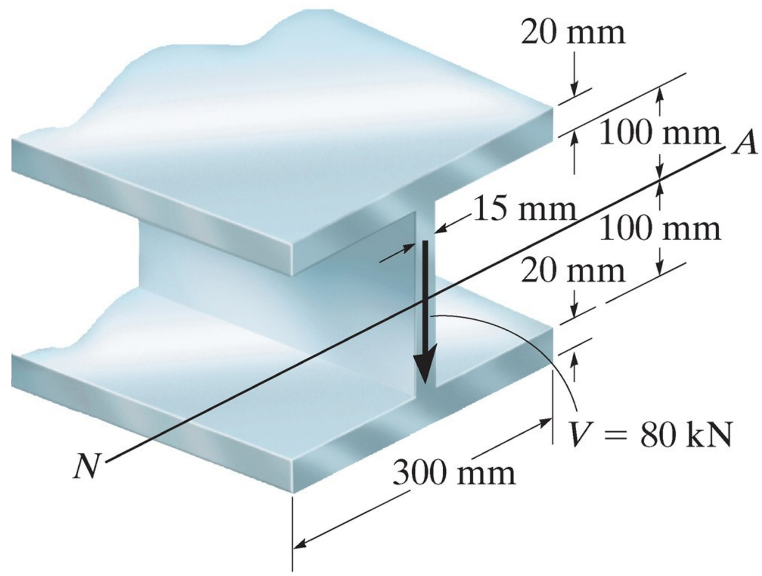
- First we find the internal shear, V
- Find I, the moment of inertia (of the entire section) about the neutral axis
- Find t from an imaginary cross-section at the point of interest
- Calculate Q from either the area above or below the point of interest
- τ acts in the same direction as V (and must be equilibrated on other faces)

example 7.1



Determine the maximum stress needed by a glue to hold the boards together.

example 7.3



Plot the shear stress distribution through the beam.