AE333

Mechanics of Materials

Lecture 15 - Bending

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4 March, 2020

schedule

- 4 Mar - Bending
- 6 Mar - Exam 2 Review, HW 5 Due
- 9 Mar - Exam 2
- 11 Mar - Transverse Shear

outline

- bending deformation
- flexure formula
- moment of inertia
- group problems

shear-moment diagrams

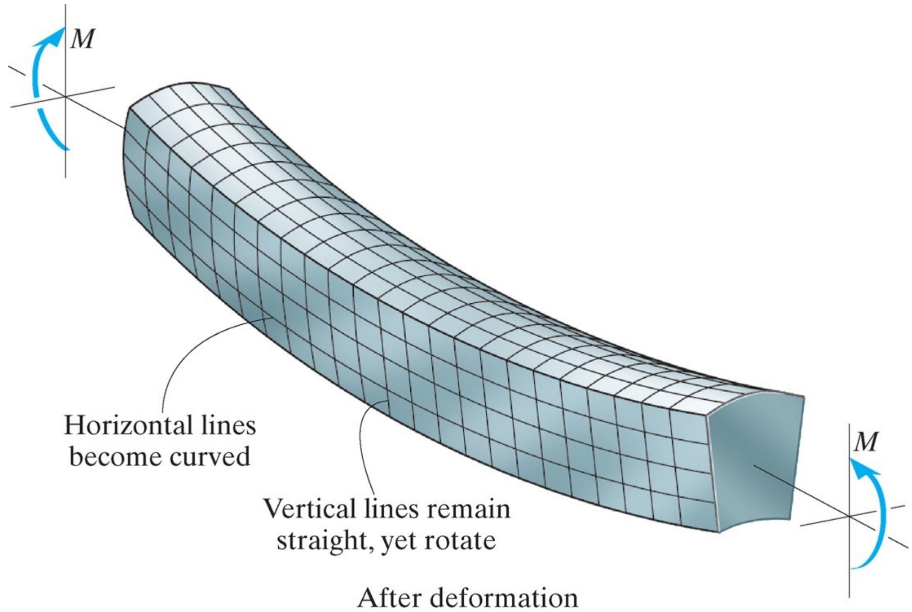
- Drawing shear-moment diagrams is a very important skill
- Learning MasteringEngineering’s drawing system is not as important (in my opinion)
- If you are more comfortable doing your shear-moment diagrams by hand, you may turn them into me in class on Monday and I will grade them by hand

bending deformation

bending deformation

- If we drew a grid on a specimen in bending, we would find that vertical lines tend to stay straight (but rotate)
- Horizontal lines will become curved
- If bending lifts the ends up (like a smile), then the top face will be in compression (and expand), while the bottom face will be in tension (and contract)

bending deformation



After deformation

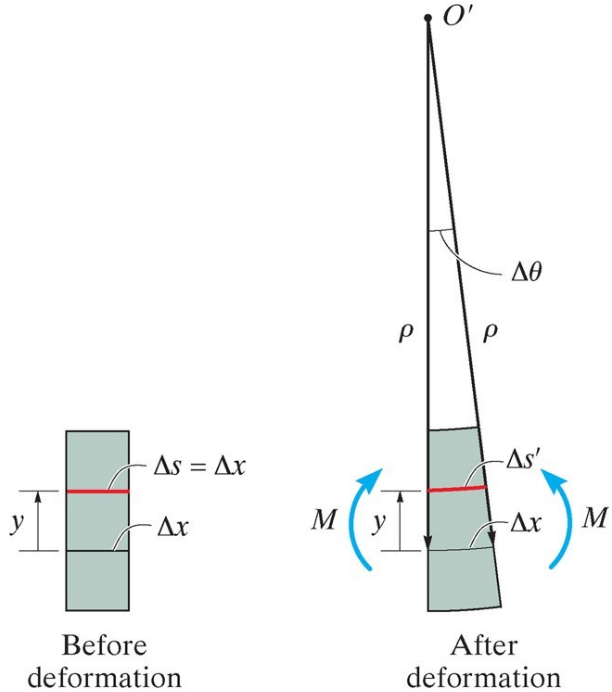
neutral axis

- Since the bottom is in tension and the top is in compression, there must be somewhere in between that is under no stress
- We call that the neutral axis, and assume it does not change in length
- We also assume that planar sections remain planar (no warping)
- Finally, Poisson's effects are neglected (cross-sections keep the same size and shape)

strain

- We will now consider an infinitesimal beam element before and after deformation
- Δx is located on the neutral axis and thus does not change in length after deformation
- Some other line segment, Δs is located y away from the neutral axis and changes its length to $\Delta s'$ after deformation

strain



O'
$\Delta\theta$
ρ
ρ
$\Delta s = \Delta x$
y
Δx
Before deformation
$\Delta s'$
M
y
Δx
M
After deformation

strain

- We can now define strain at the line segment Δs as

$$\epsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

strain

- If we define ρ as the radius of curvature after deformation, thus $\Delta x = \Delta s = \rho \Delta \theta$
- The radius of curvature at Δs is $\rho - y$, thus we can write

$$\epsilon = \lim_{\Delta\theta \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta}$$

- Which gives

$$\epsilon = -\frac{y}{\rho}$$

flexure formula

hooke’s law

- If the beam is a linear material that follows Hooke’s Law, the stress must be linearly proportional to the strain
- Thus the stress will vary linearly through the beam, just like the strain does

neutral axis

- We have already hypothesized that a neutral axis must exist, we will now find its location
- To be in equilibrium, the resultant force(s) produced by the stress must sum to zero, which means

$$\begin{aligned}\sum F_x = 0 = \int_A dF = \int_A \sigma dA \\ = \int_A -\left(\frac{y}{c}\right)\sigma_{max} dA \\ = -\frac{\sigma_{max}}{c}\int_A y dA\end{aligned}$$

neutral axis

- Since σ_{max} and c are both non-zero constants, we know that

$$\int_A y dA = 0$$

- Which can only be satisfied at the horizontal centroidal axis, so the neutral axis is the centroidal axis

bending moment

- The internal bending moment must be equal to the total moment produced by the stress distribution

$$
\begin{aligned}
M &= \int_A y dF = \int_A y(\sigma dA) \\
&= \int_A y \left(\frac{y}{c}\sigma_{max}\right) dA \\
&= \frac{\sigma_{max}}{c} \int_A y^2 dA
\end{aligned}
$$

bending moment

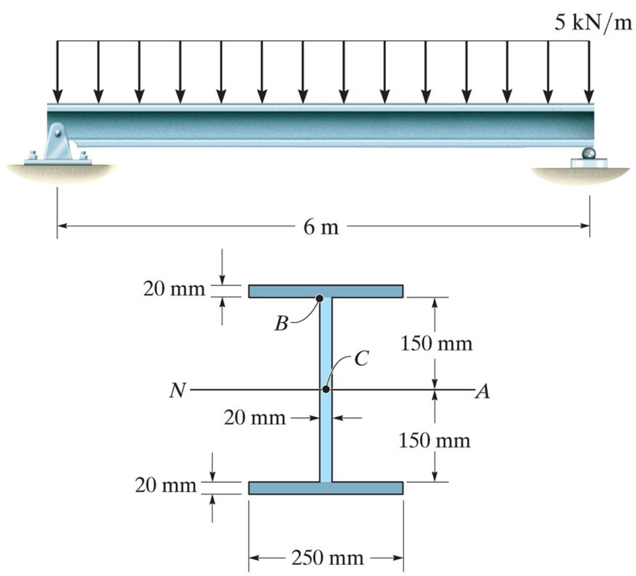
- We recognize that $\int_A y^2 dA = I$, and we re-arrange to write

$$\sigma_{max} = \frac{Mc}{I}$$

procedure

- Find the internal moment at the point of interest, or draw a moment diagram to find the maximum point (if needed)
- Determine the moment of inertia for the beam section
- Find the neutral axis and/or the distance from the neutral axis to the point of interest
- Use the flexure formula to find the stress

example 6.12



Find the maximum bending stress and draw the stress distribution through the thickness at that point.

moment of inertia

moment of inertia

- We know that $I = \int_A y^2 dA$
- For common shapes, this integral has been pre-calculated (about the centroid of the shape)
- For compound shapes, we use the parallel axis theorem to combine inertias from multiple areas

parallel axis theorem

- The parallel axis theorem is used to find the moment about any axis parallel to an axis passing through the centroid
- If we consider an axis parallel to the x-axis, separated by some dy we have

$$I_X = \int_A (y + dy)^2 dA$$

- Which gives

$$I_x = \int_A y^2 dA + 2dy \int_A y dA + dy^2 \int_A dA$$

parallel axis theorem

- The first integral is the moment of inertia about the original *x*-axis, which we will call $\bar{I}_x$
- The second integral is zero since the *x*-axis passes through the centroid
- This gives the parallel axis theorem

$$I_x = \bar{I}_x + Ady^2$$

parallel axis theorem

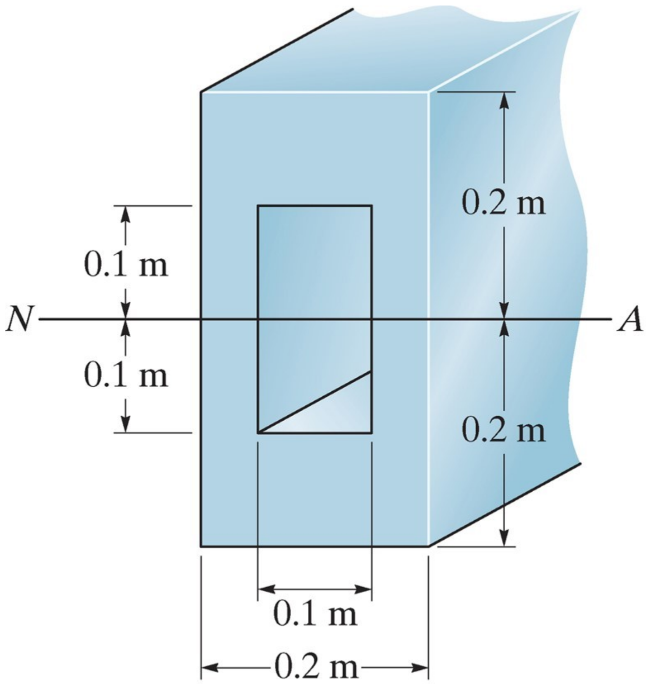
- Similarly for the y-axis and polar moment of inertia we find

$$I_y = \bar{I}_y + Adx^2$$

$$J_O = \bar{J}_C + Ad^2$$

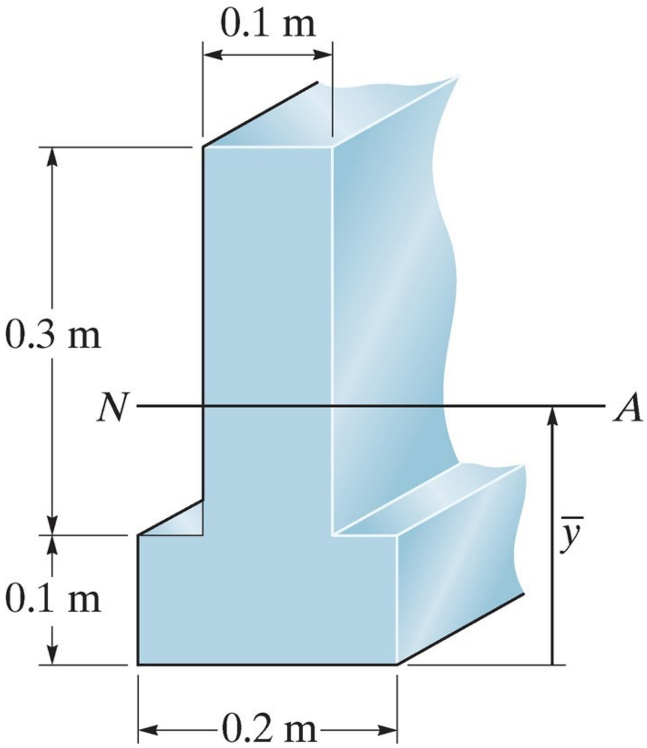
group problems

group one



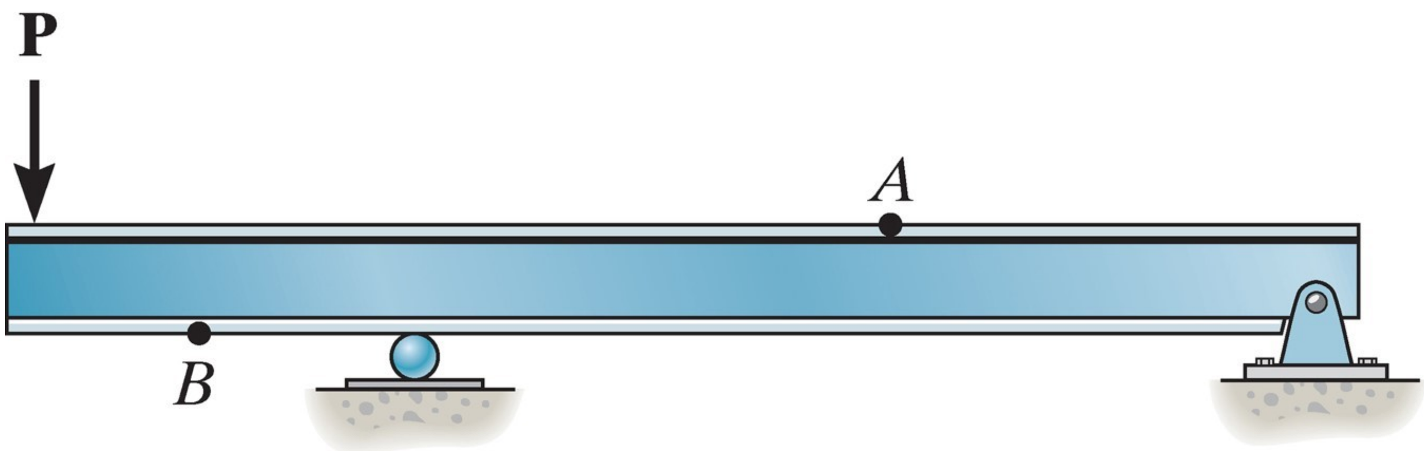
Determine the moment of inertia about the neutral axis

group two



Find the neutral axis and determine the moment of inertia about the neutral axis

group three



Show how the bending stress acts on a differential volume at point A and B.