Mechanics of Materials

Lecture 13 - Transverse Shear

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schedule

- 22 Mar - Transverse Shear
- 22 Mar - Homework 5 Due, Homework 4 Self-grade due
- 24 Mar - Transverse Shear
- 29 Mar - Exam 2 Review
- 29 Mar - Homework 6 Due, Homework 5 Self-grade due
- 31 Mar - Exam 2

- shear in straight members
- the shear formula
- group problems

shear in straight members

- We have discussed the internal stresses caused by the internal moment M
- There are also internal shear stresses caused by the internal shear force V
- We can illustrate the effect of internal shear stress by considering three boards, either resting on top of on another or bonded

Boards not bonded together

Boards bonded together
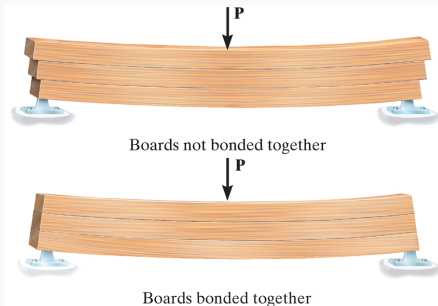
the shear formula

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shear formula

- Internal shear causes a more complicated deformation state, so we will use an indirect method to find the shear stress-strain distribution
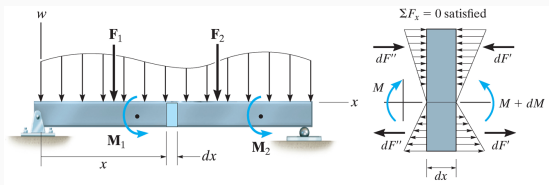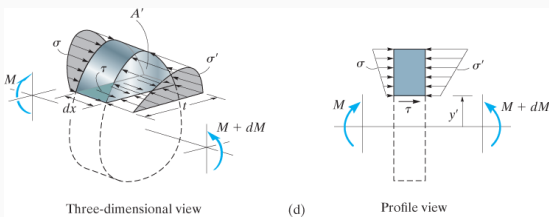- We will consider equilibrium along a section of a beam, then we will consider another section

Figure 1: A free body diagram of an arbitrary beam.

Three-dimensional view   (d)   Profile view

equilibrium

- There must be a shear force along the bottom to equilibrate the different stresses on either side of the section
- If we assume that this shear is constant through the thickness, we obtain the following from equilibrium

$$\sum F_x = 0 = \int_{A'} \sigma' \, dA' - \int_{A'} \sigma \, dA' - \tau(t dx)$$

equilibrium

$$0 = \int_{A'} \left(\frac{M + dM}{I} \right) y dA' - \int_{A'} \left(\frac{M}{I} \right) y dA' - \tau(t dx)$$

$$\left(\frac{dM}{I} \right) \int_{A'} y dA' = \tau(t dx)$$

$$\tau = \frac{1}{It} \left(\frac{dM}{dx} \right) \int_{A'} y dA'$$

shear formula

- In this formula, recall that $V = \frac{dM}{dx}$
- We also call Q the moment of the area A' which is equal to $\int_{a'} y \, dA'$
- We can also write Q in terms of the centroid

$$Q = \bar{y}' A'$$

shear formula

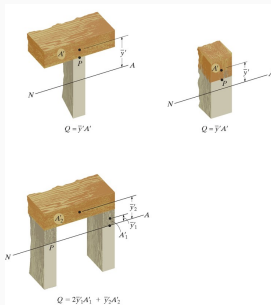- Simplified, the shear formula is

$$\tau = \frac{VQ}{It}$$

- Q poses the greatest difficulty in calculations, so we will consider a few examples

Q

- *Q* represents the moment of the cross-sectional area above (or below) the point at which the shear stress is being calculated
- We apply the formula to that area

Q

$Q = \bar{y}'A'$

$Q = \bar{y}'A'$

$Q = 2\bar{y}_1'A_1' + \bar{y}_2'A_2'$

shear formula limitations

- A major assumption made is that the shear stress was constant through the thickness, essentially we have found the average shear
- This is more accurate the more slender a beam is (small b and large h)
- The formula is also not accurate for cross sections that change or have boundaries that are not right angles
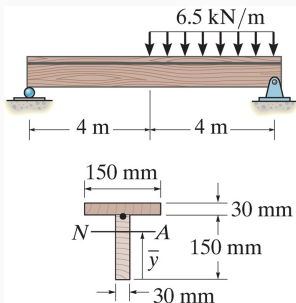
procedure

- First we find the internal shear, V
- Find I, the moment of inertia (of the entire section) about the neutral axis
- Find t from an imaginary cross-section at the point of interest
- Calculate Q from either the area above or below the point of interest
- τ acts in the same direction as V (and must be equilibrated on other faces)
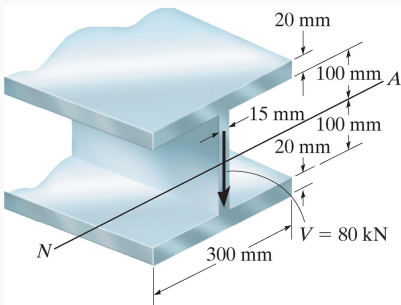
example 7.1

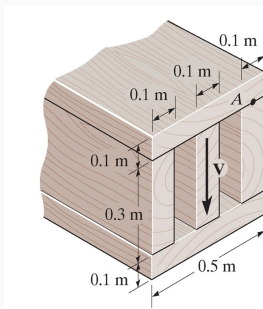Determine the maximum stress needed by a glue to hold the boards together.

example 7.3
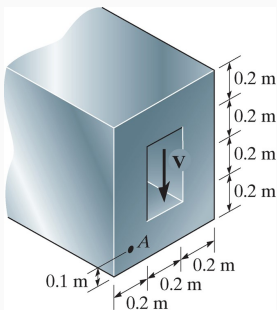

Plot the shear stress distribution through the beam.

group problems

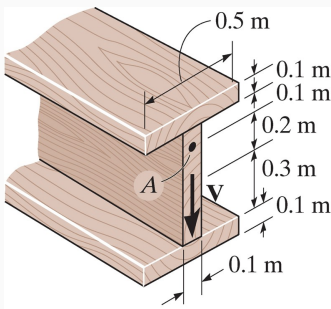Find Q and t that would be used to find the shear stress at A.

group two

Find Q and t that would be used to find the shear stress at A.

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group three

Find Q and t that would be used to find the shear stress at A.

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