

## Lecture 18 - Deflection of Beams

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## schedule

- 21 April - Beam Deflection
- 26 April - Beam Deflection (superposition), HW 7  
Self-grade Due, HW 8 Due
- 28 April - Exam 3 (will only cover through strain transformation)
- 3 May - Stress concentration, buckling
- 5 May - Final exam review
- 6 May - Project 3 Due
- Homeworks 9-11 (posted to blackboard) are optional and provide some practice for exam

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- deflection of beams and shafts
- slope and displacement

## deflection of beams and shafts

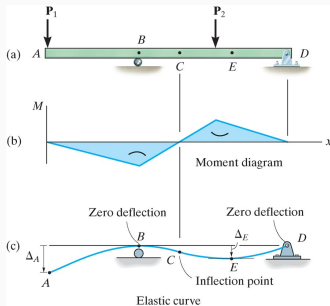
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## elastic curve

- Before finding the exact displacement, it is useful to sketch the approximate deformed shape of a beam
- For difficult beams, it is useful to draw the moment curve first
- Positive internal moment tends to bend the beam concave up, while negative moment tends to bend the beam concave down

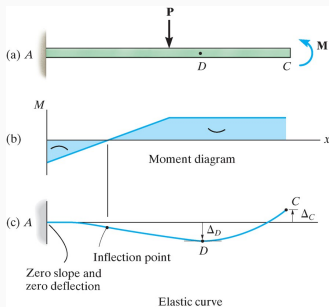
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## elastic curve



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## elastic curve



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## moment-curvature

- In Chapter 6 we compared the strain in a segment of a beam to the radius of curvature and found

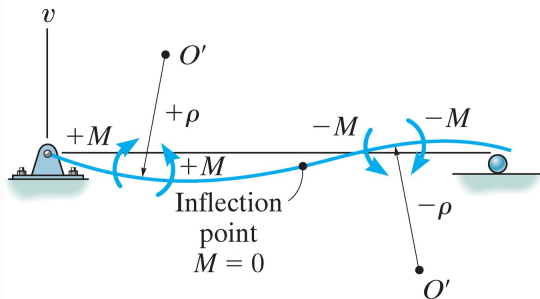
$$\frac{1}{\rho} = -\frac{\epsilon}{y}$$

- Since Hooke's Law applies,  $\epsilon = \sigma/E = -My/EI$ , substituting gives

$$\frac{1}{\rho} = \frac{M}{EI}$$

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## sign convention



$\rho$  is positive when the center of the arc is above the beam, negative when it is below.

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## slope and displacement

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- When talking about displacement in beams, we use the coordinates  $v$  and  $x$ , where  $v$  is the vertical displacement and  $x$  is the horizontal position
- In this notation, curvature is formally related to displacement according to

$$\frac{1}{\rho} = \frac{d^2v/dx^2}{[1 + (dv/dx)^2]^{3/2}} = \frac{M}{EI}$$

- The previous equation is difficult to solve in general, but for cases of small displacement,  $\left(\frac{dv}{dx}\right)^2$  will be negligible compared to 1, which then simplifies to

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

## flexural rigidity

- In general,  $M$ , is a function of  $x$ , but  $EI$  (known as the flexural rigidity) is a constant along the length of the beam
- In this case, we can say

$$EI \frac{d^2 v}{dx^2} = M(x)$$

$$EI \frac{d^3 v}{dx^3} = V(x)$$

$$EI \frac{d^4 v}{dx^4} = w(x)$$

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## boundary conditions

- If a support restricts displacement, but not rotation, we will have a boundary condition of  $v = 0$  at that point
- Supports that restrict rotation give a boundary condition that  $\theta = 0$

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- If we have a piecewise function for  $M(x)$ , not all integration constants can be found from the boundary conditions
- Instead, we must also use continuity conditions to ensure that the slope and displacement are continuous at every point
- In other words, for two sets of functions,  $\theta_1(x)$  and  $v_1(x)$ ,  $\theta_2(x)$  and  $v_2(x)$ ,  $\theta_1(a) = \theta_2(a)$  and  $v_1(a) = v_2(a)$

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## slope

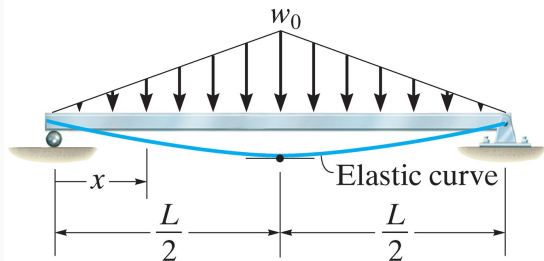
- For small displacements, we have

$$\theta \approx \tan(\theta) = dv/dx$$

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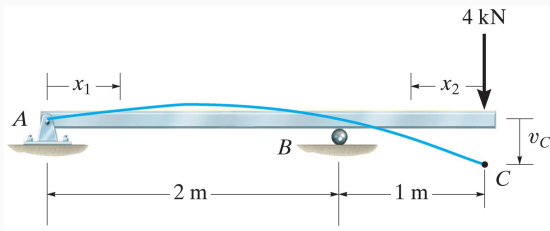
### example 12.1



Determine the maximum deflection if  $EI$  is constant.

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### example 12.4



Determine the displacement at C,  $EI$  is constant.

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