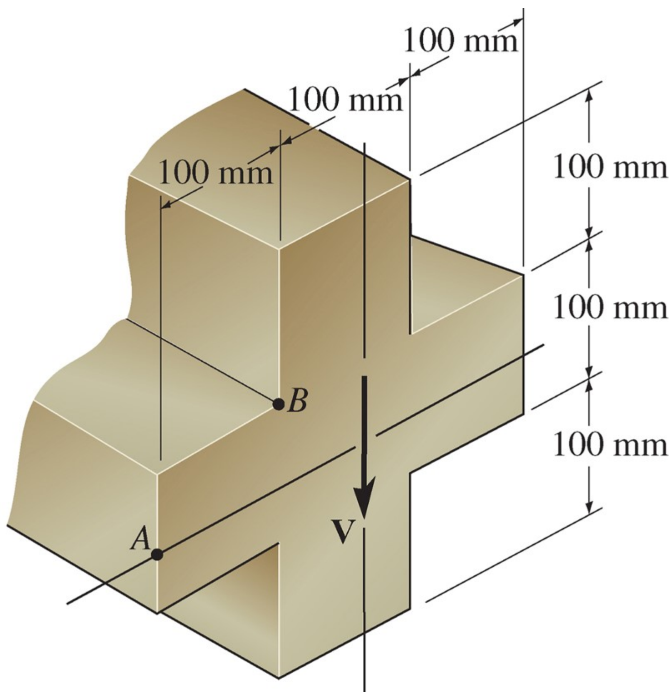
Name:

Homework 6 Solutions

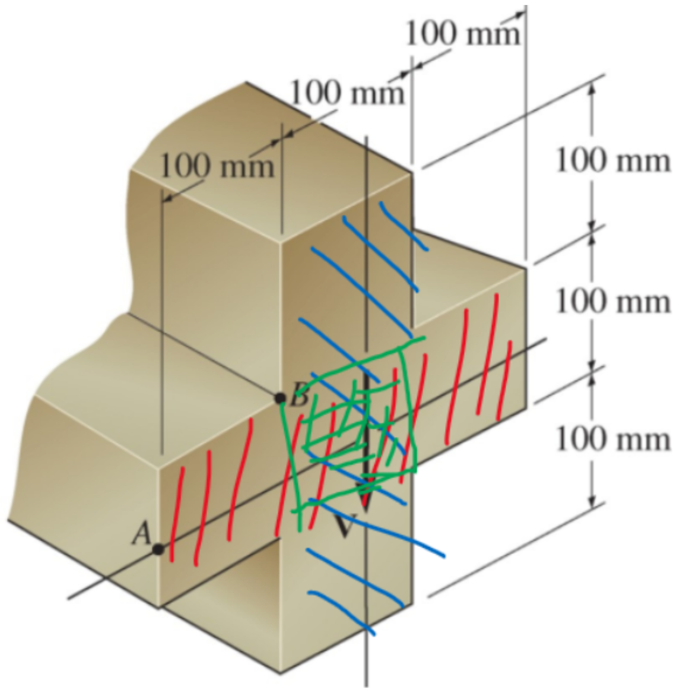
Due 13 October 2020

1. Find the shear stress at points A and B when $V = 450\,\text{kN}$. Draw the state of stress on a volume element at each point.

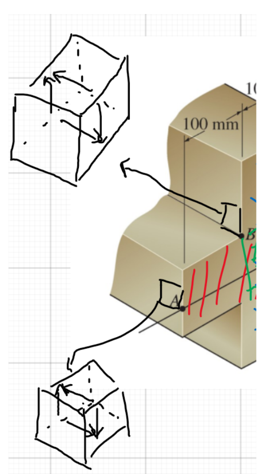


Solution:

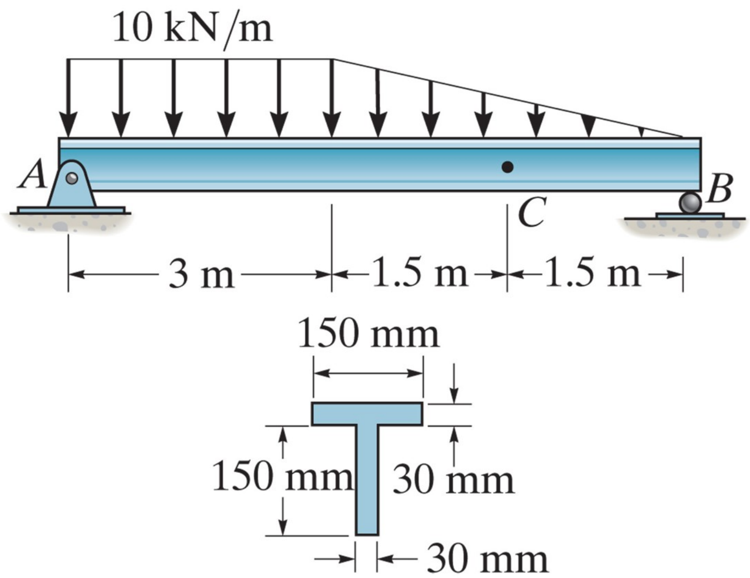
- Since we are given V, we just need to find Q, I, and t at each point
- There are several ways to go about calculating I, I will use a method that might seem counter-intuitive. I will consider the two rectangles shown in red and blue (which overlap), and then subtract the overlapping square (but only once). Another alternative would be to calculate one rectangle and then add the two squares left over (note that this way requires using the parallel axis theorem).



- This gives $I = 2.42 \times 10^8\,\mathrm{mm}^4$Note: since this section is symmetric, the neutral axis (centroid) is in the center and all our inertias reference that
- To find Q we find the area above points A and B respectively, we find $A'_B = 1 \times 10^4\,\mathrm{mm}^2$, $A'_A = 2.5 \times 10^4\,\mathrm{mm}^2$, $\bar{y}'_B = 100\,\mathrm{mm}$ and $\bar{y}'_A = 55\,\mathrm{mm}$ which gives $Q_B = 1 \times 10^6\,\mathrm{mm}^3$ and $Q_A = 1.375 \times 10^6\,\mathrm{mm}^3$
- t can be found from the geometry with $t_b = 100\,\mathrm{mm}$ and $t_a = 300\,\mathrm{mm}$
- This gives $\tau_B = 18.6\,\mathrm{MPa}$ and $\tau_A = 8.53\,\mathrm{MPa}$ and the stress state on representative volume elements looks like:

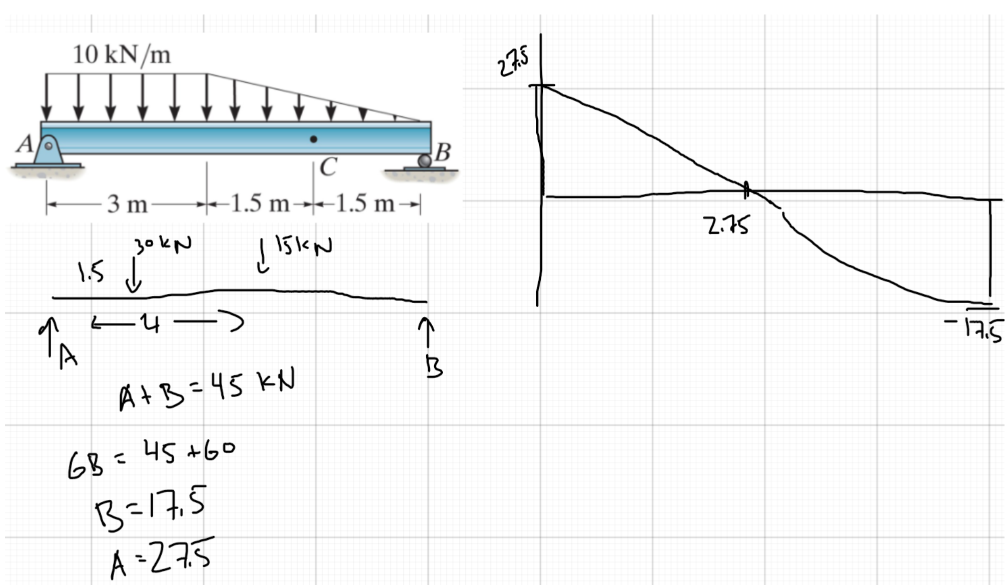


2. Find the maximum shear stress acting on the beam shown.



Solution:

- We start with a free body diagram and a shear diagram

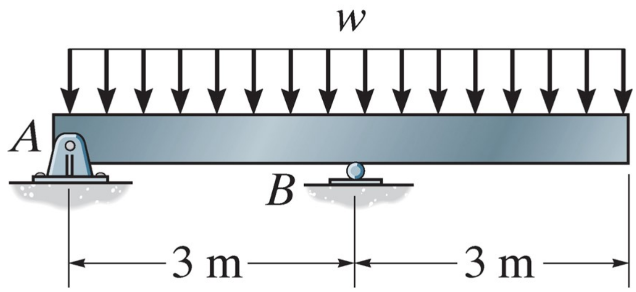


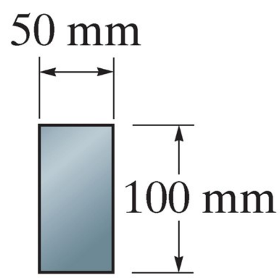
- We see that the maximum shear force is 27.5 kN, so now we find I, Q, and t
- This cross-section is not symmetric, so we begin by finding the neutral axis $\bar{y} = \frac{(75)(30)(150)+(165)(150)(30)}{2(150)(30)} = 120\,\mathrm{mm}$ (measured from the bottom)
- Next we find the inertia, $I = \frac{1}{12}(30)(150^3) + (30)(150)(75-120)^2 + \frac{1}{12}(150)(30^3) + (150)(30)(120-75)^2 = 2.7 \times 10^7\,\mathrm{mm}^4$
- Typically, the maximum shear stress occurs at the center (unless there is a thin section far away from the center, as occurred in problem 1), since the thinnest

region is at the center, we expect the maximum shear to occur at the center, and we calculate Q there. We can find Q by considering the area either above or below the point, and in this case the area below is much easier. $Q = (30)(120)(120 - 60) = 2.16 \times 10^5\,\text{mm}^3$

- Finally, we use $t = 30\,\text{mm}$ to calculate the shear stress as $7.33\,\text{MPa}$

3. The overhanging beam is subjected to a uniform load of $w = 75\,\text{kN/m}$. Find the maximum shear stress in the beam.

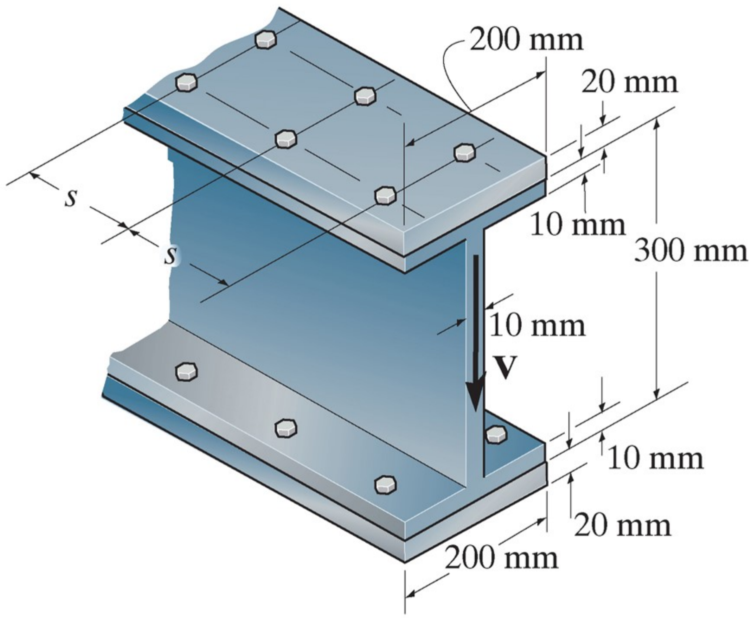




Solution:

- From statics analysis, we find the reaction force at A is 0 and the maximum shear force of $V = 225\,\text{kN}$ occurs at B
- Next we find $I = \frac{1}{12}bh^3 = \frac{1}{12}50(100^3) = 4.17 \times 10^6\,\text{mm}^4$
- Since this is a solid rectangular cross-section, the maximum shear will occur at the middle and we can find $Q = \bar{y}'A' = (25)(50)(50) = 6.25 \times 10^4\,\text{mm}^3$
- We can now find the maximum shear stress $\tau = \frac{VQ}{IT} = 67.5\,\text{MPa}$

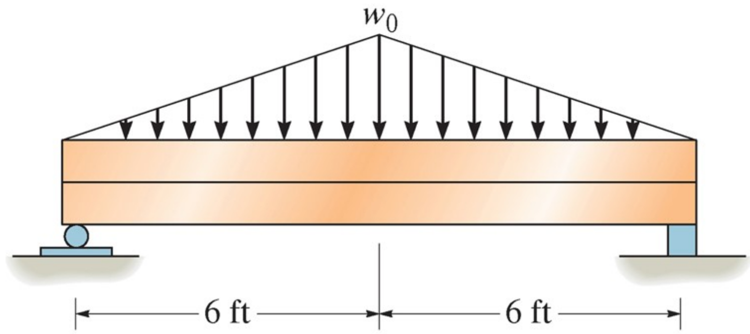
4. Two identical 20 mm plates are bolted to the top and bottom of a flange to form a built-up beam. For a shear force of $V = 400\,\text{kN}$ find the maximum bolt spacing, s, if each bolt has a shear strength of 45 kN

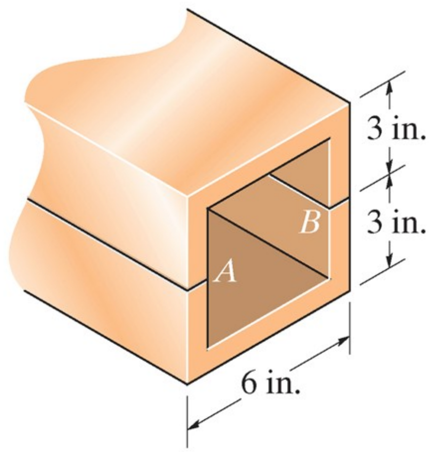


Solution:

- We start by calculating the inertia of the entire cross-section, since this is needed to find the shear flow, q
- $I = \frac{1}{12}(10)(280^3) + 2\left[\frac{1}{12}(200)(30^3) + (200)(30)(155^2)\right] = 3.075 \times 10^8\,\mathrm{mm}^4$
- Next we need to find the moment of area, Q, for the area to be bolted on $Q = \bar{y}'A' = (160)(20)(200) = 6.4 \times 10^5\,\mathrm{mm}^3$
- We can now solve for the shear flow, $q = \frac{VQ}{I} = 833\,\mathrm{N/mm}$
- Next we use $F = qs$ to solve for the spacing with $F = 45\,\mathrm{kN}$ and we find $s = 54.1\,\mathrm{mm}$

5. The beam shown is made by gluing two 1/2 in c-channel strips together as sown. If the glue has a maximum shear stress of $\tau = 600\,\mathrm{psi}$ find the maximum intensity, w_0, of the triangular distributed loading.





Solution:

- Since this is symmetric we can quickly see that the highest shear force will occur at each support and is equal to $V = w_0 6/4$
- The easiest way to find the inertia is to consider the tube to be solid and subtract the inside $I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(5)(5^3) = 55.9\,\text{in}^4$
- Q can be found in the same fashion, by considering the entire top half (as if it were solid) and subtracting the empty space, $Q = (6)(3)(1.5) - (5)(2.5)(1.25) = 11.4\,\text{in}^3$
- Finally we find $t = 2(0.5) = 1\,\text{in}$ and we can solve $\tau = \frac{VQ}{It}$ for w_0 (note that we need to convert length units to all be in inches) to find $w_0 = 164\,\text{lb/in} = 1.97\,\text{kip/ft}$