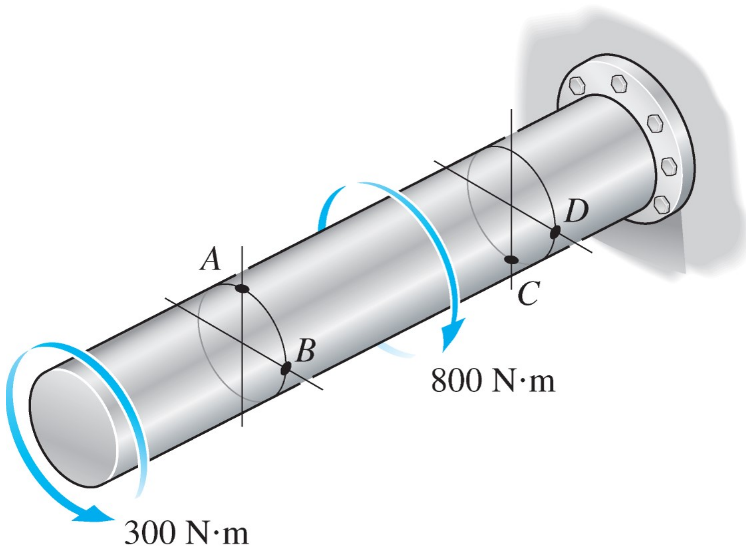
Name:

Homework 4 Solutions

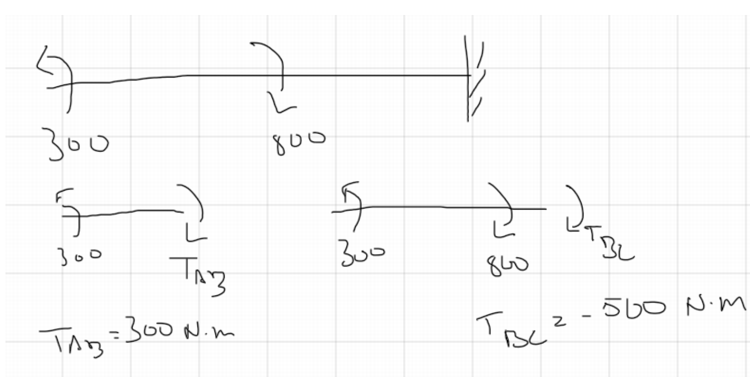
Due 29 Sep 2020

1. Determine the internal torque at each section and sketch the shear stress on a volume element at A, B, C, and D.

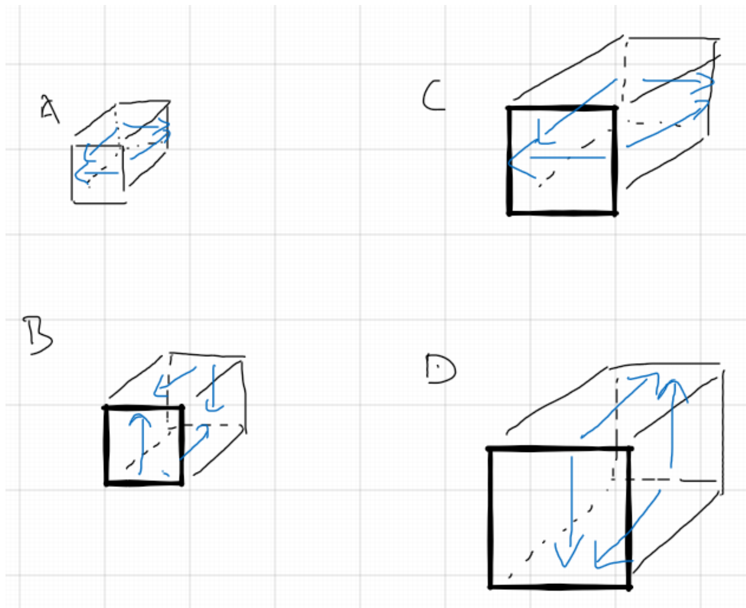


Solution:

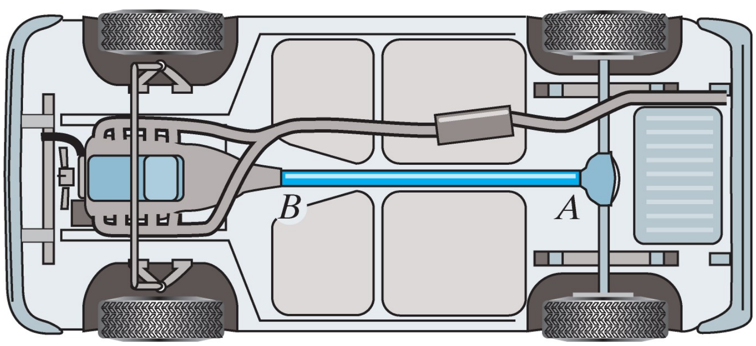
- Looking at the sections shown, we find $T_{AB} = 300\,\mathrm{N}\cdot\mathrm{m}$ and $T_{BC} = -500\,\mathrm{N}\cdot\mathrm{m}$



- and we can visualize the shear stress on the volume elements as shown



2. The drive shaft AB is to be designed as a thin-walled tube. The engine delivers 150 hp while the shaft turns 1000 rpm. Find the minimum thickness of the shaft's wall for an outer diameter of 2.0 in if the allowable shear stress is $\tau_{allow} = 6.0\,\text{ksi}$.



Solutions:

- We can relate power to torque using $P = 2\pi f T$, solving for T we find

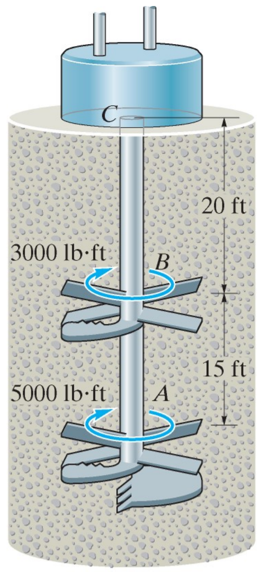
$$T = \frac{P}{2\pi f} = \frac{150 \text{ hp}}{2\pi 1000 \text{ rpm}} \frac{550 \text{ ft lb / s}}{\text{hp}} \frac{\text{rot}}{\text{min}} \frac{\text{min}}{60 \text{ sec}} = 788\,\text{ft}\cdot\text{lb} = 9554\,\text{in}\cdot\text{lb}$$

- Now we can relate the stress and torque to find the tube thickness, $\tau = \frac{Tc}{J}$, substituting for J we can solve

$$\begin{aligned} \tau &= \frac{Tc}{\pi/2(c^4 - c_i^4)} \\ c^4 - c_i^4 &= \frac{2Tc}{\tau} \\ c_i &= \left(c^4 - \frac{2Tc}{\tau}\right)^{1/4} \end{aligned}$$

- and we find $c_i = 1.76\,\text{in}$ which gives a wall thickness of $\frac{1}{8}$ in

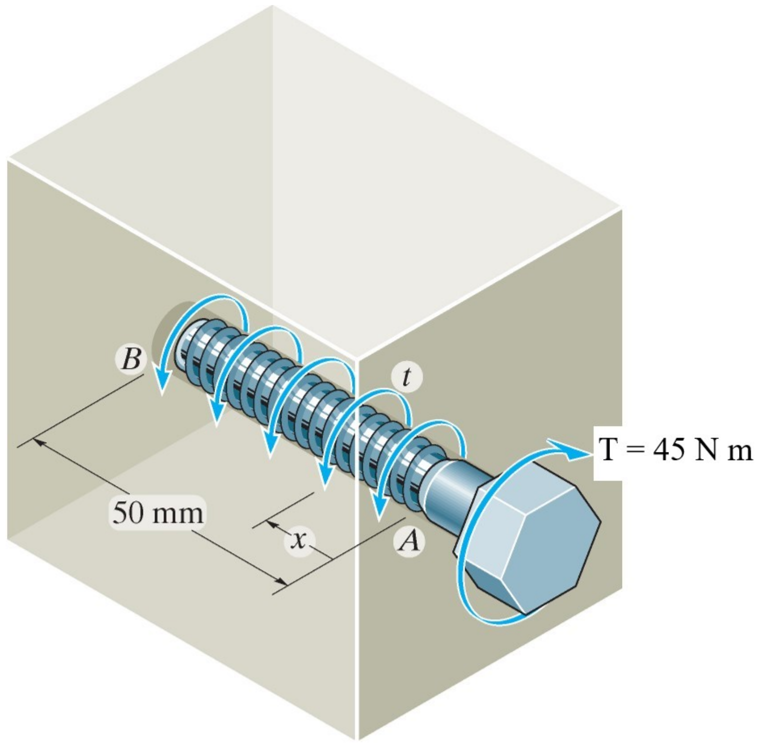
3. The soil mixer shown is connected to an A-36 steel tubular shaft with an inside diameter of 1.5 in and an outside diameter of 3.0 in. Determine the angle of twist at A relative to B and the absolute maximum shear stress for the torque shown.



Solutions:

- We start with a simple free body diagram and find that $T_{AB} = 5000\,\text{ft} \cdot \text{lb}$ and $T_{BC} = 8000\,\text{ft} \cdot \text{lb}$
- The angle of twist between A and B can be found with $\phi = \frac{TL}{JG}$ which for A-36 Steel is $\phi = \frac{60\,000\,\text{in} \cdot \text{lb}180\,\text{in}}{7.455\,\text{in}^4 11.0\,\text{Msi}} = 0.132\ \text{rad}\ = 7.55°$
- The shear stress can be found with $\tau = \frac{Tc}{J}$, to find the maximum we use T_{BC} and we find $\tau = \frac{96\,000\,\text{in} \cdot \text{lb}1.5\,\text{in}}{\pi/2(1.5^4 - .75^4)} = 19.3\,\text{ksi}$

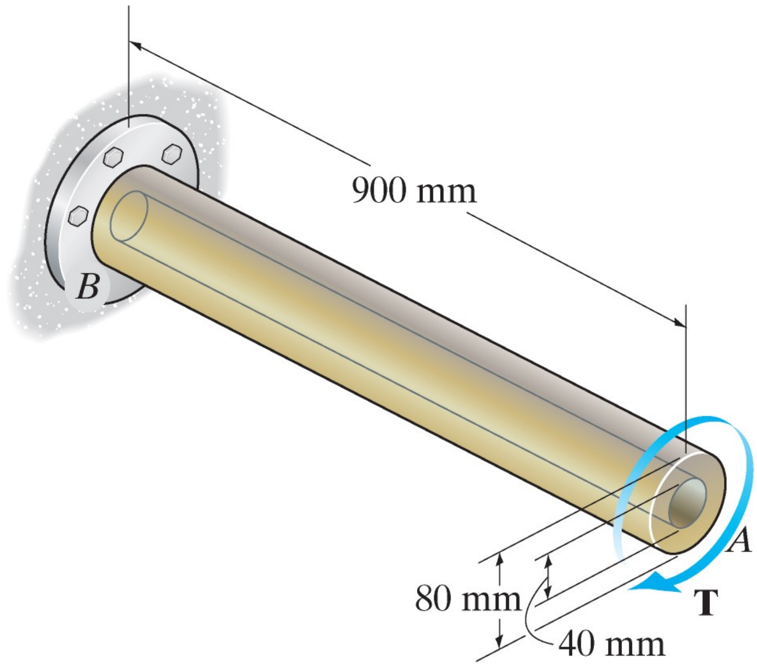
4. The A-36 Steel bolt shown is tightened such that there is a reactive torque on the shank. The reactive torque is expressed as $t = kx^2\,\text{N} \cdot \text{m/m}$ for x in meters. If a torque of $T = 45\,\text{N} \cdot \text{m}$ is applied to the bold head find the value of the constant k and the amount of twist in the shank. Assume the shank has a radius of 5 mm



Solution:

- We can see that the reactive torque in the shank must be equal to the applied torque, as there are no other reactions or applied torques
- The total torque exerted by this resistance will be the integral of t over the length of the shaft $T_t = \int_0^{50} kx^2 dx = \frac{1}{3}k(50)^3$, which gives $k = 0.00108$

5. The aluminum alloy tube (2014-T6, outside) is bonded to an A-36 steel rod (inside) as shown. If a torque of $7\,\mathrm{kN}\cdot\mathrm{m}$ is applied find the maximum shear stress in each material and plot the shear stress as a function of radial position.



Solution:

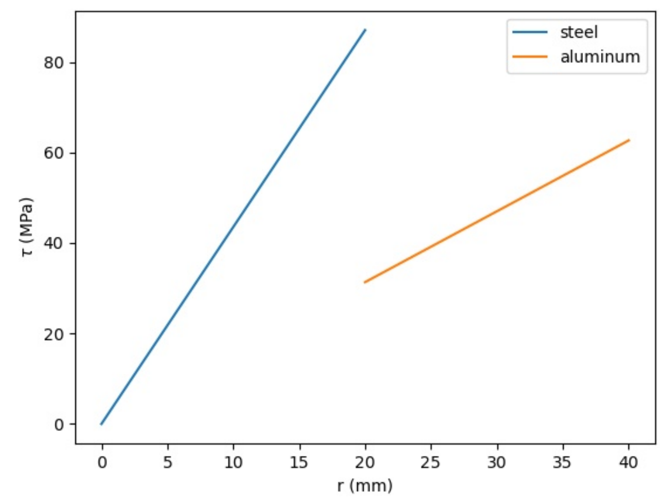
- This is a statically indeterminate problem, similar to an example we did in lecture. While we do not know how much of the applied torque is born by the rod and the tube, we do know that the total torque must be equal to the applied torque and that the angle of twist needs to be equal in both.

$$\phi_{al} = \phi_{st} = \left(\frac{TL}{JG}\right)_{al} = \left(\frac{TL}{JG}\right)_{st}$$

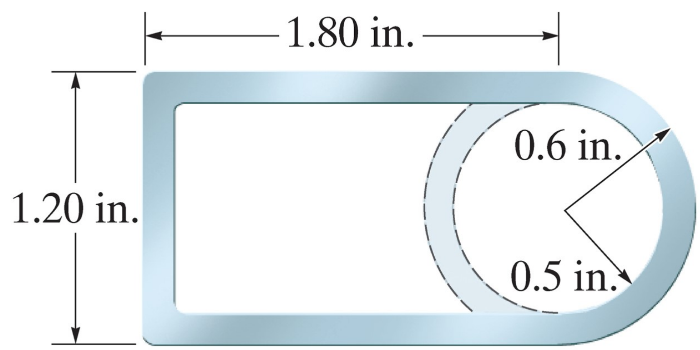
- The length is identical in both so that can be canceled, substituting for J and G in both and solving for T_{st} we find

$$T_{st} = \left(\frac{J_{st}G_{st}}{J_{al}G_{al}}\right) T_{al} = 0.185T_{al}$$

- We know that $T_{st} + T_{al} = 7\,\mathrm{kN \cdot m}$, substituting we can solve for $T_{al} = 5.91\,\mathrm{kN \cdot m}$ and $T_{st} = 1.09\,\mathrm{kN \cdot m}$
- We can now substitute known torques into $\tau = \frac{Tc}{J}$ to find the maximum shear stress $\tau_{st} = 87.0\,\mathrm{MPa}$ and $\tau_{al} = 62.7\,\mathrm{MPa}$



6. For an arbitrary maximum shear stress, compare the torque carrying capacity between two cross-sectional shapes: the circular tube shown (inside) and the rounded rectangular tube (outside) shown. For both cases the wall thickness is 0.1 in



Solution:

- To more directly compare, we'll start by rearranging terms, $\tau = \frac{Tc}{J}$ becomes $T = \left(\frac{J}{c}\right)\tau$
- For the non axisymmetric shape, we use shear flow, $T = 2qA_m = (2tA_m)\tau$
- The mean area is calculated as the area inside the midline of the tube, which means $A_m = 1.1(1.75) + \pi/2(.55^2)$
- Substituting known values we find $T_{round} = 0.176\tau$ and $T_{rect} = 0.480\tau$, this means the oddly shaped tube can hold a torque 2.7 times higher than the round tube while keeping the same shear stress