Mechanics of Materials

Lecture 13 - Transverse Shear

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

29 September, 2020

schedule

- 29 Sep - Transverse Shear, Homework 5 Due, Homework 4 Self-Grade Due
- 1 Oct - Transverse Shear
- 6 Oct - Exam review, Homework 6 Due, Homework 5 Self-Grade Due
- 8 Oct - Exam 2

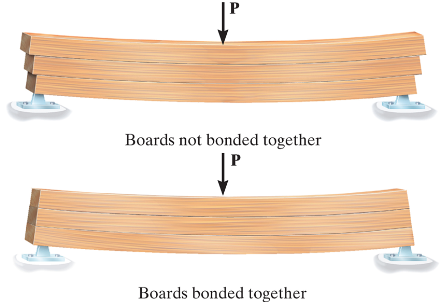
outline

- shear in straight members
- the shear formula
- group problems
- shear flow in built-up members

shear

- We have discussed the internal stresses caused by the internal moment M
- There are also internal shear stresses caused by the internal shear force V
- We can illustrate the effect of internal shear stress by considering three boards, either resting on top of on another or bonded

shear



shear formula

- Internal shear causes a more complicated deformation state, so we will use an indirect method to find the shear stress-strain distribution
- We will consider equilibrium along a section of a beam, then we will consider another section

equilibrium

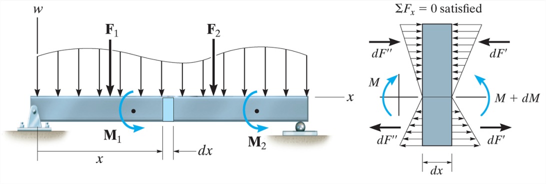
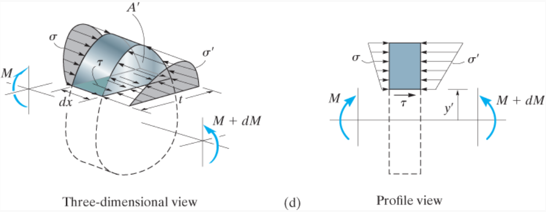


Figure 1: A free body diagram of an arbitrary beam.

equilibrium



equilibrium

- There must be a shear force along the bottom to equilibrate the different stresses on either side of the section
- If we assume that this shear is constant through the thickness, we obtain the following from equilibrium

$$\sum F_x = 0 = \int_{A'} \sigma' dA' - \int_{A'} \sigma dA' - \tau(tdx)$$

equilibrium

$$\begin{aligned} 0 &= \int_{A'} \left(\frac{M + dM}{I}\right) ydA' - \int_{A'} \left(\frac{M}{I}\right) ydA' - \tau(tdx) \\ \left(\frac{dM}{I}\right) \int_{A'} ydA' &= \tau(tdx) \\ \tau &= \frac{1}{It} \left(\frac{dM}{dx}\right) \int_{A'} ydA' \end{aligned}$$

shear formula

- In this formula, recall that $V = \frac{dM}{dx}$
- We also call Q the moment of the area A' which is equal to $\int_{a'} y dA'$
- We can also write Q in terms of the centroid

$$Q = \bar{y}' A'$$

shear formula

- Simplified, the shear formula is

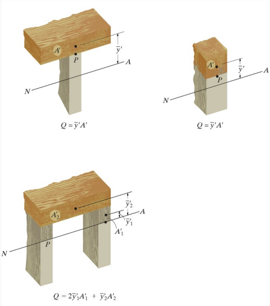
$$\tau = \frac{VQ}{It}$$

- Q poses the greatest difficulty in calculations, so we will consider a few examples

Q

- Q represents the moment of the cross-sectional area above (or below) the point at which the shear stress is being calculated
- We apply the formula to that area

Q



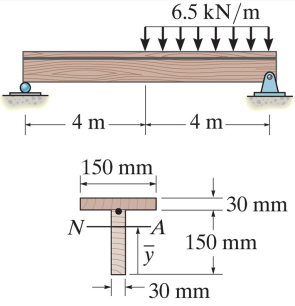
shear formula limitations

- A major assumption made is that the shear stress was constant through the thickness, essentially we have found the average shear
- This is more accurate the more slender a beam is (small b and large h)
- The formula is also not accurate for cross sections that change or have boundaries that are not right angles

procedure

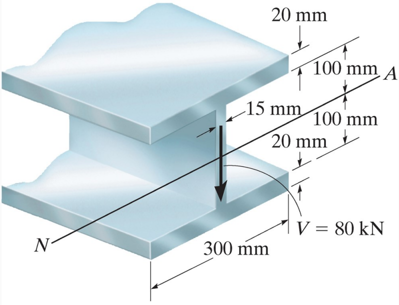
- First we find the internal shear, V
- Find I, the moment of inertia (of the entire section) about the neutral axis
- Find t from an imaginary cross-section at the point of interest
- Calculate Q from either the area above or below the point of interest
- τ acts in the same direction as V (and must be equilibrated on other faces)

example 7.1



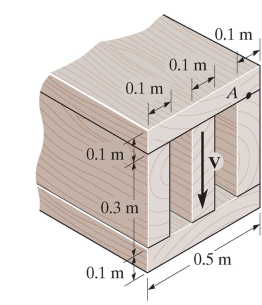
Determine the maximum stress needed by a glue to hold the boards together.

example 7.3



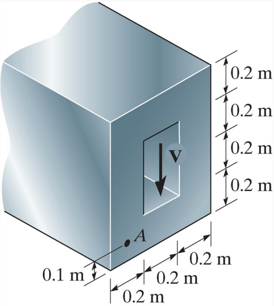
Plot the shear stress distribution through the beam.

group one



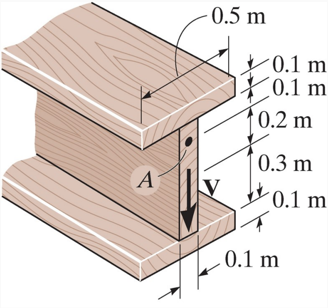
Find Q and t that would be used to find the shear stress at A.

group two



Find Q and t that would be used to find the shear stress at A.

group three



Find Q and t that would be used to find the shear stress at A.