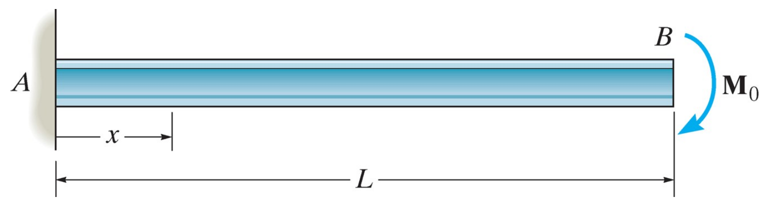
Name:

Homework 9 Solutions

Due 3 December 2021

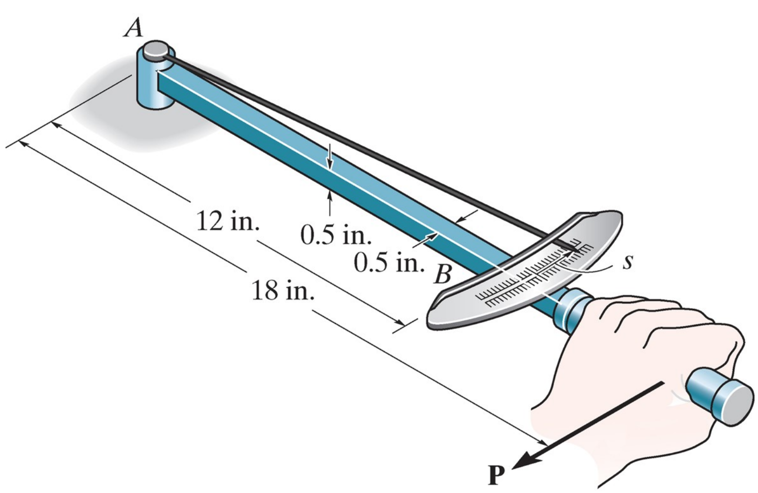
1. Find the deflection (as a function of x) for a cantilevered beam with an applied end moment. Assume EI is constant and express answer in terms of applied moment and EI.



Solution:

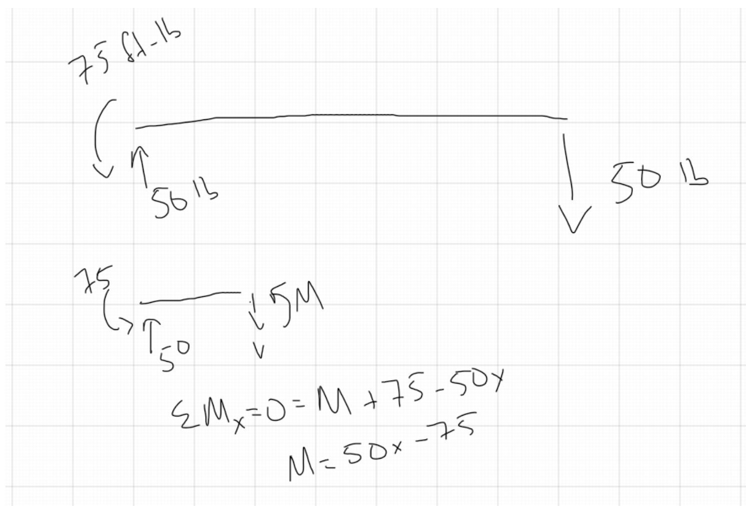
- Statics to find the reactions gives that there is only a reaction moment, equal and opposite to the applied moment. Taking an internal section, we find the applied moment is negative.
- This gives $M(x) = -M_0 = \frac{d^2v}{dx^2}EI$
- Integrating twice gives $-\frac{1}{2}M_0x^2 + C_1x + C_2 = vEI$
- We can now apply the boundary conditions at the fixed end, $v(0) = 0$ and $\frac{dv}{dx}(0) = 0$ to find that $C_1 = C_2 = 0$
- This gives the deflection as $v(x) = -\frac{M_0}{2EI}x^2$

2. A torque wrench is used to tighten the nut on a bolt. If the dial indicates that a torque of $75\,\mathrm{ft \cdot lb}$ when the bolt is fully tightened, find the force P on the handle and the distance that the needle moves along the scale. Assume that only the section AB bends and the cross section is a solid $0.5\,\mathrm{in}$ square with $E = 29\,\mathrm{Msi}$



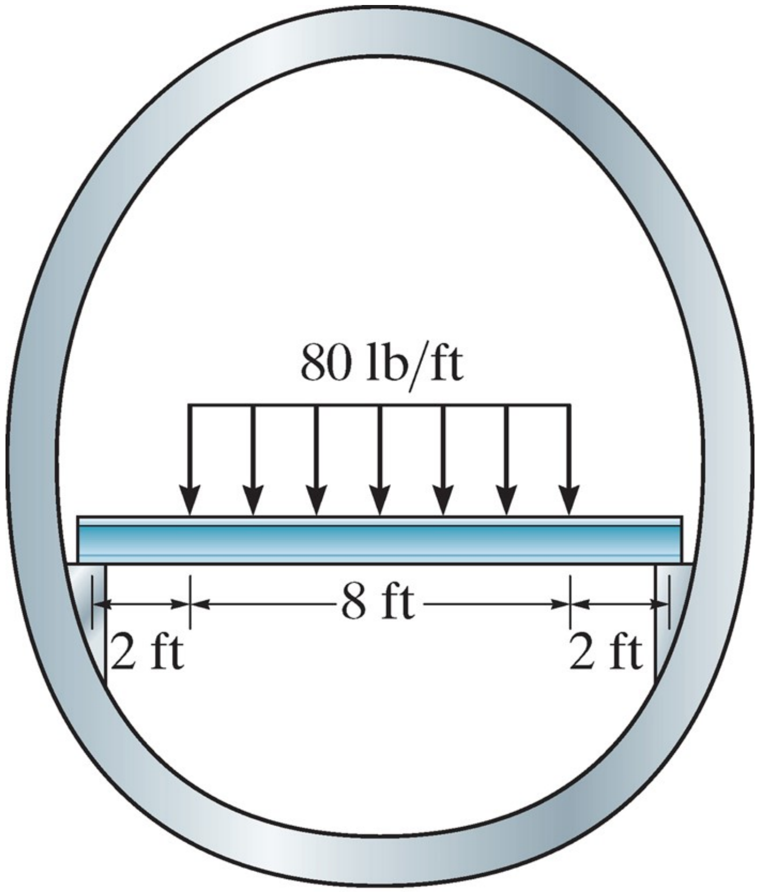
Solution:

- We can consider the socket end of the wrench to be a cantilevered support. Even though it does not perfectly stop rotation like most "fixed" supports, it does resist rotation and in this case we are given the reaction moment as the torque of the bolt.
- We can now use statics to find the force P that corresponds to the appropriate reaction moment and we find $P = 50\,\mathrm{lb}$
- We now need to section our torque wrench to find the moment as a function of x.



- We integrate twice to find that $vEI = \frac{50}{6}x^3 - \frac{75}{2}x^2 + C_1x + C_2$
- Notice that to keep consistent units, we need to use x in feet, not inches (or convert 75 ftlb into inch-pounds), with boundary conditions of $v(0) = 0$ and $\frac{dv}{dx}(0) = 0$ we find that $C_1 = C_2 = 0$.
- Now we need to substitute values at $x = 12\,\mathrm{in}$ (and calculate the moment of inertia). At this point it is easiest to convert to inch-pounds so that our length units are consistent. We find that $I = 0.0052\,\mathrm{in}^4$ and $75\,\mathrm{ft}\cdot\mathrm{lb} = 900\,\mathrm{in}\cdot\mathrm{lb}$ which gives $v(12) = -0.334\,\mathrm{in}$

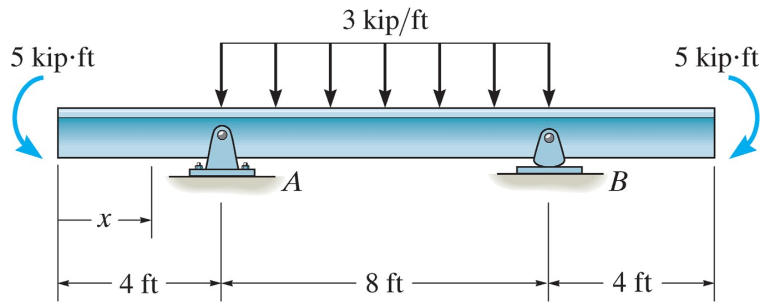
3. The floor beam of an airplane is subjected to the loading shown. Assuming the fuselage exerts only vertical reactions on the ends of the beam, determine the maximum deflection of the beam in terms of some constant flexural rigidity, EI.



Solution:

- For this problem I will use the discontinuity function approach in my solutions
- We have 80 lb/ft acting over 8 feet, which means 640 lbs, and each reaction will be 320 lbs.
- This gives our moment equation as $M = 320 - 40\langle x-2\rangle^2 + 40\langle x-10\rangle^2$
- Integrating twice gives $vEI = 160x^2 - 10/3\langle x-2\rangle^4 + 10/3\langle x-10\rangle^4 + C_1x + C_2$
- We can find the unknown constants by substituting boundary conditions of $v(0) = 0$ and $v(12) = 0$.
- We find $C_2 = 0$ and $C_1 = 853.3$
- The maximum deflection, since this is symmetric, will occur at the middle, so we can substitute $x = 6$ to find $v = \frac{1}{EI}(10026.7)$

4. Find the deflection as a function of x for the beam shown in terms of some constant EI.



Solution:

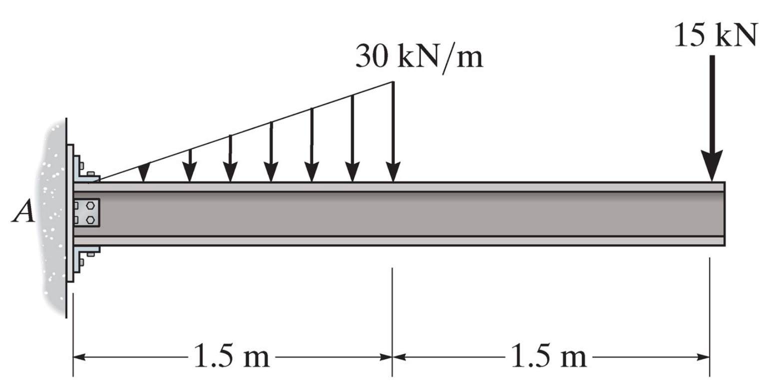
- From statics we find the reaction forces are both 12 kip
- Using the discontinuity function method, we find the moment as a function of x to be $M(x) = -5\,\text{kip}\cdot\text{ft} + 12\,\text{kip}\langle x-4\rangle - 3/2\,\text{kip/ft}\langle x-4\rangle^2 + 12\,\text{kip}\langle x-12\rangle + 3/2\,\text{kip}\langle x-12\rangle^2$
- Integrating twice gives $EIv(x) = -\frac{5}{2}x^2 + 2\langle x-4\rangle^3 - \frac{1}{8}\langle x-4\rangle^4 + 2\langle x-12\rangle^3 + \frac{1}{8}kip\langle x-12\rangle^4 + C_1x + C_2$
- In this problem the boundary conditions are $v(4) = 0$ and $v(12) = 0$ which gives

$$
\begin{aligned}
0 &= -\frac{5}{2}(4)^2 + C_1(4) + C_2 \\
0 &= -\frac{5}{2}(12)^2 + 2(8) - \frac{1}{8}(8)^4 + C_1(12) + C_2
\end{aligned}
$$

- We find $C_1 = 102$ and $C_2 = -368$ which gives the total deflection, in terms of EI as

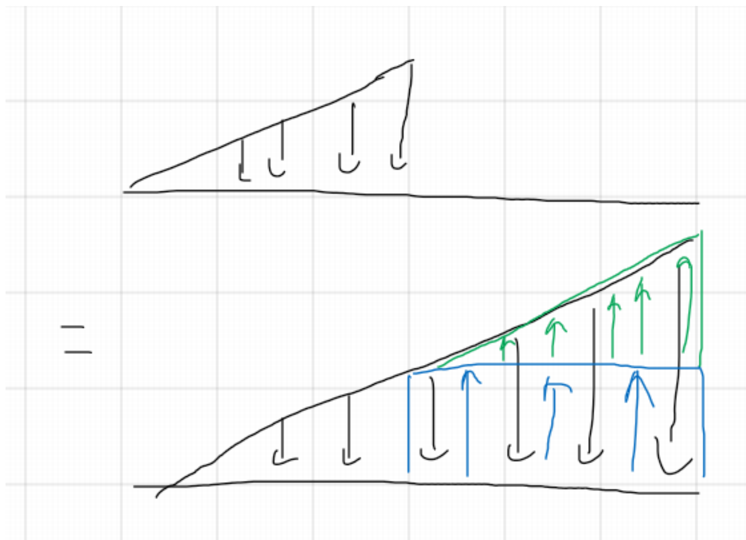
$$
v(x) = \frac{1}{EI}\left(-\frac{5}{2}x^2 + 2\langle x-4\rangle^3 - \frac{1}{8}\langle x-4\rangle^4 + 2\langle x-12\rangle^3 + \frac{1}{8}kip\langle x-12\rangle^4 + 102x - 368\right)
$$

5. Find the deflection as a function of x for the beam shown in terms of some constant EI.



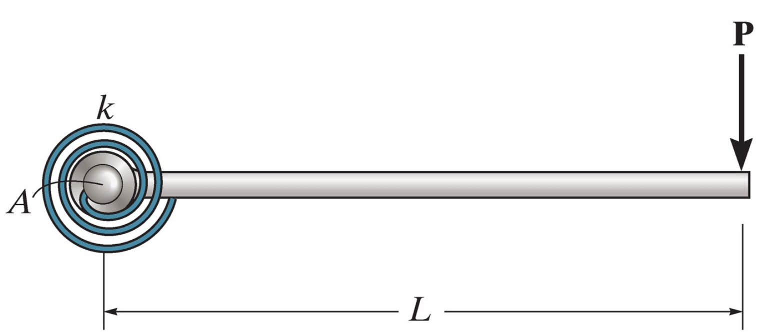
Solution:

- Once again we start with statics to find the reactions. We find the vertical force at A is $37.5\,\mathrm{kN}$ while the reaction moment is $67.5\,\mathrm{kN}\cdot\mathrm{m}$
- It is a bit tricky to apply the distributed load correctly, see the drawing to show the necessary superposition, starting at 0 the discontinuity function applies a linearly increasing distributed load on the entire beam, and we need both a uniform and linearly increasing distributed load to cancel it.



- All told this gives $M(x) = 60x - 90 - 5x^3 + 5\langle x - 1.5\rangle^3 + \frac{45}{2}\langle x - 1.5\rangle^2$
- Integrating twice gives $EIv(x) = 10x^3 - 45x^2 - \frac{1}{4}x^5 - \frac{1}{4}\langle x - 1.5\rangle^5 + \frac{15}{8}\langle x - 1.5\rangle^4 + C_1x + C_2$
- Applying boundary conditions of $v(0) = 0$ and $dv/dx(0) = 0$ gives $C_1 = C_2 = 2$ and our final deflection equation is $v(x) = \frac{1}{EI}\left(10x^3 - 45x^2 - \frac{1}{4}x^5 - \frac{1}{4}\langle x - 1.5\rangle^5 + \frac{15}{8}\langle x - 1.5\rangle^4\right)$

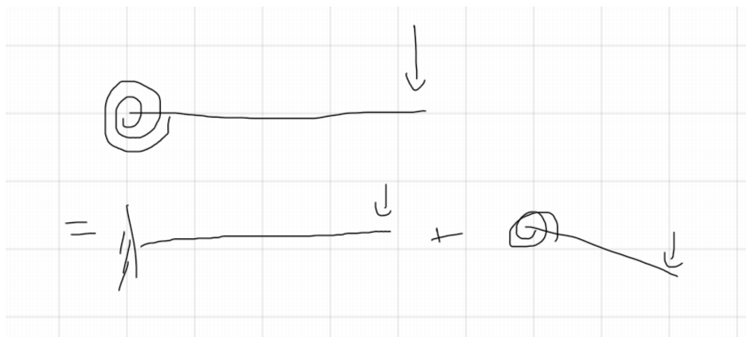
6. The rod is pinned at the end A and attached to a torsional spring with stiffness k (with k expressed in torque per radian of rotation). For a perpendicular force, P, as shown find the displacement of the force in terms of some constant EI.



Solution:

- We will use the superposition method to solve this problem.

- Some of the deflection will occur from the rod bending like a cantilever beam, while the rest will come from rigid rotation about the spring, we can add these two solutions together.



- The rigid spring rotation will occur with the spring torque $k\theta$ equal to the applied moment, PL, we can find the angle in terms of the other parameters as $\theta = \frac{PL}{k}$. We know that $\tan\theta = \frac{v_1}{L}$, so we can say that $v_1 = -L\tan\frac{PL}{k}$ (negative sign added to indicate that deflection goes down).
- The cantilever portion we can obtain directly from Appendix C, where we find $v_2 = -\frac{Px^2}{6EI}(3L - x)$
- We add the two solutions to find the total deflection $v = v_1 + v_2 = -L\tan\frac{PL}{k} - \frac{Px^2}{6EI}(3L - x)$