Mechanics of Materials

Lecture 10 - Torsion

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schedule

- 17 Sep - Torsion
- 22 Sep - Bending, Homework 4 Due
- 24 Sep - Bending
- 29 Sep - Transverse Shear, Homework 5 Due

outline

- angle of twist
- statically indeterminate torsion
- solid non-circular shafts
- thin-walled tubes

angle of twist

- While in axial problems we examined the total deformation for the general case, in torsion we will examine the total angle of twist in general
- Using the method of sections, we can consider a differential disk which has some internal torque as a function of x, $T(x)$.
- On this section, the shear strain will be related to the angle of twist by the thickness of the section (dx) and the radial distance (ρ).

angle of twist

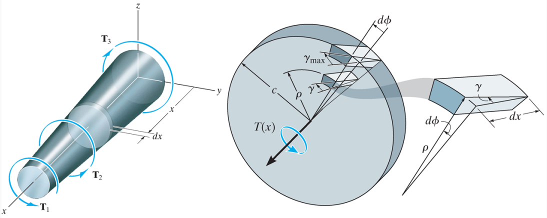


Figure 1: A section of an arbitrary cone is shown to depict how we can find the incremental change in angle from one end of the section to the next.

angle of twist

- γ and $d\phi$ are related by

$$d\phi = \gamma \frac{dx}{\rho}$$

- We also know that $\gamma = \tau / G$ and $\tau = T(x)\rho / J(x)$ substituting both this gives

$$d\phi = \frac{T(x)}{J(x)G(x)} dx$$

multiple torques

- If a shaft is subjected to multiple torques, or if there is a discontinuous change in shape or material we can sum the change in angle over various segments

$$\phi = \sum \frac{TL}{JG}$$

sign convention

- When we section a shaft and consider the internal torque, it is important to be consistent with our signs
- Both torque and angle of twist should follow the same convention
- The convention is to use the right hand rule with the thumb pointing normal to the cut, and the fingers curling in the positive direction

example 5.8

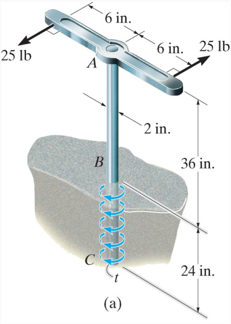


Figure 2: A wrench is attached to a post. 24 inches of the post are embedded in soil, while the other 36 inches continue

statically indeterminate torsion

- Just as in axial problems, we can now solve statically indeterminate torsion problems
- We will generally need one compatibility condition in addition to the equations of static equilibrium
- At any given section cut the angle of twist needs to be equal, or often the supports restrict the angle of twist and we can use that knowledge

example 5.10

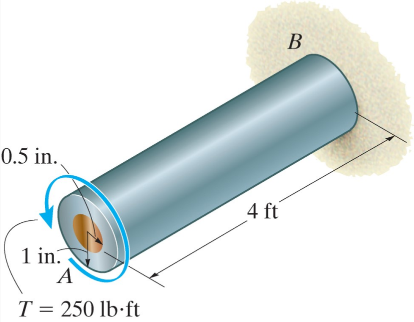
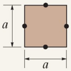
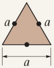
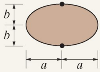


Figure 3: The brass core has a radius of 0.5 inches, while the steel tube has an outer radius of 1 inch. The shaft is 4 ft long and fixed at one end with a torque of 250 ft lb applied at the

non-circular shafts

- When a shaft is non-circular it is no longer axisymmetric, which causes cross-sections to bulge or warp
- Because of this deformation, in-depth analysis of non-circular shafts in torsion is beyond the scope of this class
- Using the theory of elasticity some standard shapes can be analyzed

theory of elasticity

Shape of cross section	τ_{max}	ϕ
Square 	$\frac{4.81\,T}{a^3}$	$\frac{7.10\,TL}{a^4G}$
Equilateral triangle 	$\frac{20\,T}{a^3}$	$\frac{46\,TL}{a^4G}$
Ellipse 	$\frac{2\,T}{\pi ab^2}$	$\frac{(a^2+b^2)TL}{\pi a^3b^3G}$

shear flow

- Thin-walled tubes of non-circular cross-sections are commonly found in aerospace and other applications
- We can analyze these using a technique called shear flow
- Because the walls of the tube are thin, we assume that the shear stress is uniformly distributed through the wall thickness

shear flow

- If we consider an arbitrary segment of a tube with varying thickness, we find from equilibrium that the product of the average shear stress and the thickness must be the same at every location on the cross-section

$$q = \tau_{avg} t$$

- q is called the shear flow

average shear stress

- We can relate the average shear stress to the torque by considering the torque produced about some point within the tubes boundary

$$T = \oint h \tau_{avg} t ds$$

- Where h is the distance from the reference point, ds is the differential arc length and t is the tube thickness.

average shear stress

- We notice that $\tau_{avg}t$ is equal to the shear flow, q, which must be constant
- We can also simplify the integral by relating a similar area integral to the arc length integral

$$dA_m = 1/2hds$$

- Thus

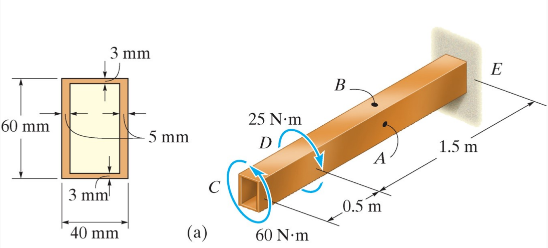
$$T = \oint h\tau_{avg}tds = 2q\int dA_m = 2qA_m$$

angle of twist

- The angle of twist for a thin-walled tube is given by

$$\phi = \frac{TL}{4A_m^2G}\oint\frac{ds}{t}$$

example 5.13



Determine the average shear stress at A and B.