Mechanics of Materials

Lecture 19 - Discontinuity Functions

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schedule

- 27 Oct - Beam Deflection (discontinuity functions), HW 8 Due, HW 7 Self-Grade Due
- 29 Oct - Beam Deflection (superposition)
- 3 Nov - Statically Indeterminate Beams, HW 9 Due, HW 8 Self-Grade Due
- 5 Nov - Statically Indeterminate Beams

outline

- discontinuity functions
- group problems

discontinuity functions

- Direct integration can be very cumbersome if multiple loads or boundary conditions are applied
- Instead of using a piecewise function, we can use discontinuity functions

Macaulay functions

- Macaulay functions can be used to describe various loading conditions, the general definition is

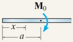
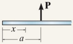
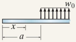
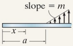
$$\langle x-a\rangle^n = \begin{cases} 0 & \text{for } x < a \\ (x-a)^n & \text{for } x \geq a \end{cases} n \geq 0$$

singularity functions

- Singularity functions are used for concentrated forces and can be written

$$w = P\langle x-a\rangle^{-1} = \begin{cases} 0 & \text{for } x \neq a \\ P & \text{for } x = a \end{cases}$$

discontinuity functions

TABLE 12–2			
Loading	Loading Function $w = w(x)$	Shear $V = \int w(x)dx$	Moment $M = \int V dx$
	$w = M_0\langle x-a\rangle^{-2}$	$V = M_0\langle x-a\rangle^{-1}$	$M = M_0\langle x-a\rangle^{0}$
	$w = P\langle x-a\rangle^{-1}$	$V = P\langle x-a\rangle^{0}$	$M = P\langle x-a\rangle^{1}$
	$w = w_0\langle x-a\rangle^{0}$	$V = w_0\langle x-a\rangle^{1}$	$M = \frac{w_0}{2}\langle x-a\rangle^{2}$
	$w = m\langle x-a\rangle^{1}$	$V = \frac{m}{2}\langle x-a\rangle^{2}$	$M = \frac{m}{6}\langle x-a\rangle^{3}$

discontinuity functions

1. We add these up for each loading case along our beam
2. We integrate as usual to find displacement from load, slope, or moment

integration

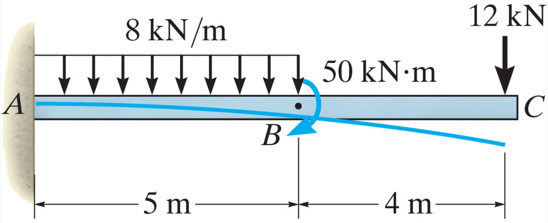
- discontinuity functions follow special rules for integration
- when $n \geq 0$, they integrate like a normal polynomial
- when $n < 0$, they instead follow

$$\int \langle x-a \rangle^n dx = \langle x-a \rangle^{n+1}$$

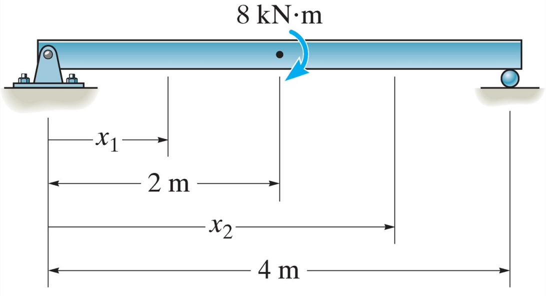
signs

- we need to be careful to match the sign convention
- loads are defined as positive when they act upward
- moments are defined as positive when they act clockwise

example 12.5

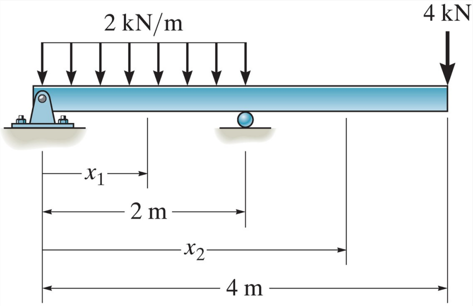


group one



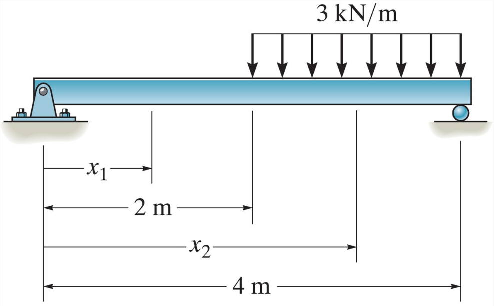
Find the maximum deflection using either direct integration or discontinuity functions.

group two



Find the maximum deflection using either direct integration or discontinuity functions.

group three



Find the maximum deflection using either direct integration or discontinuity functions.