AE333

Mechanics of Materials

Lecture 5 - Strain, Mechanical Properties

Dr. Nicholas Smith
Wichita State University, Department of Aerospace Engineering

February 3, 2020

schedule

- 3 Feb - Strain, Mechanical Properties
- 5 Feb - Mechanical Properties, Exam 1 Review
- 7 Feb - Axial Load, HW2 Due
- 10 Feb - Exam 1

outline

- strain
- stress-strain
- strain energy
- poisson’s ratio
- shear stress-strain

strain

deformation

- When forces are applied to a body, it will change its shape and size
- We call these changes *deformation*
- Sometimes they are barely noticeable (steel), other times they are very significant (rubber)

strain

- Strain is a more precise measurement of the deformation of a body
- Normal strain is given as the change in length divided by the original length

$$\epsilon = \frac{L - L_0}{L_0}$$

- We can consider the average normal strain (over an entire body) or the local strain (take an infinitely small portion and calculate the strain there)

units

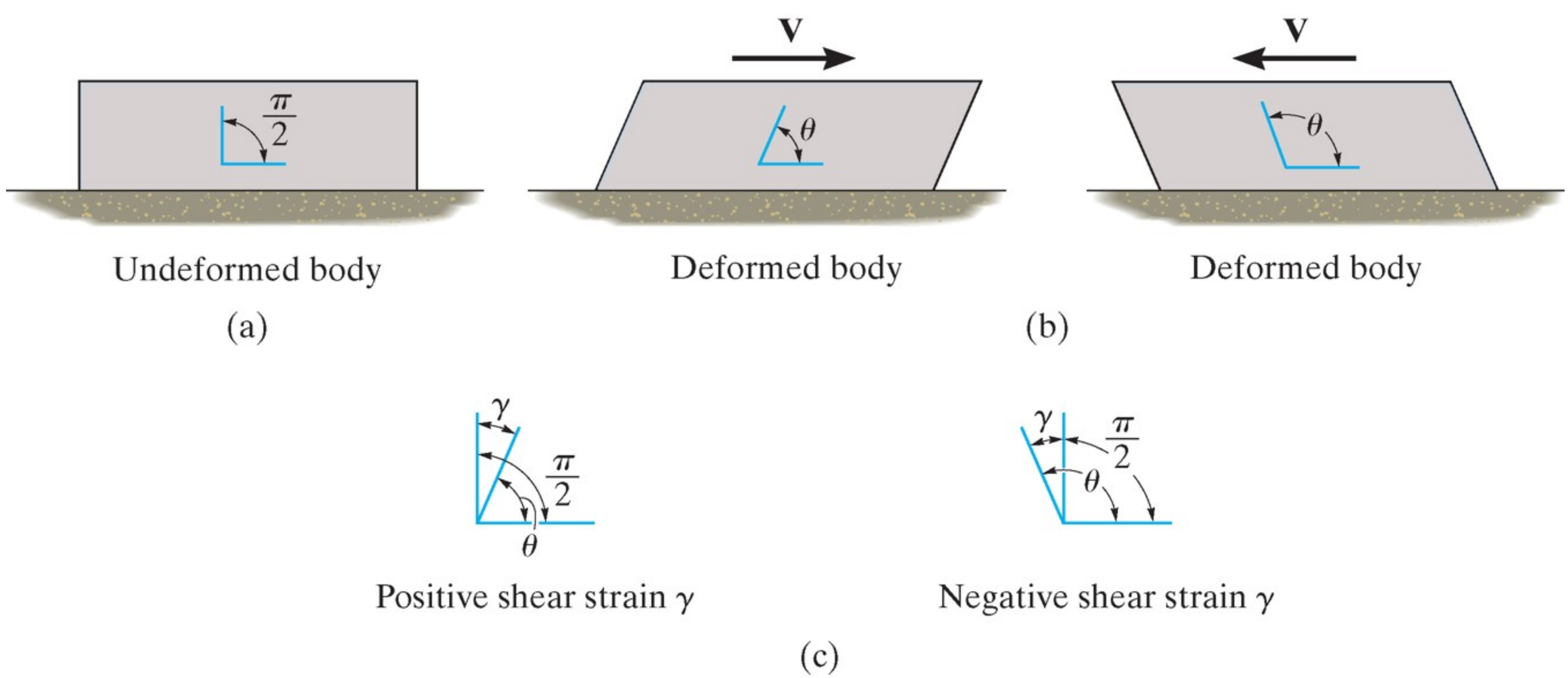
- Since we divide length by length, strain is unitless
- However it is customary to use *in/in* or for stiff specimens to use the phrase *microstrain* as a unit
- Strain can also sometimes be represented as a percent

shear strain

- Normal strain causes a line segment to expand or contract
- Deformation can also cause two lines to change their relative angle
- The change in angle between two originally perpendicular line segments is called shear strain

$$\gamma = \frac{\pi}{2} - \theta$$

shear strain



cartesian components

- If we consider a very small cube/prism with sides of Δx, Δy, and Δz, normal strains will change the side lengths to

$$(1+\epsilon_x)\Delta x(1+\epsilon_y)\Delta y(1+\epsilon_z)\Delta z$$

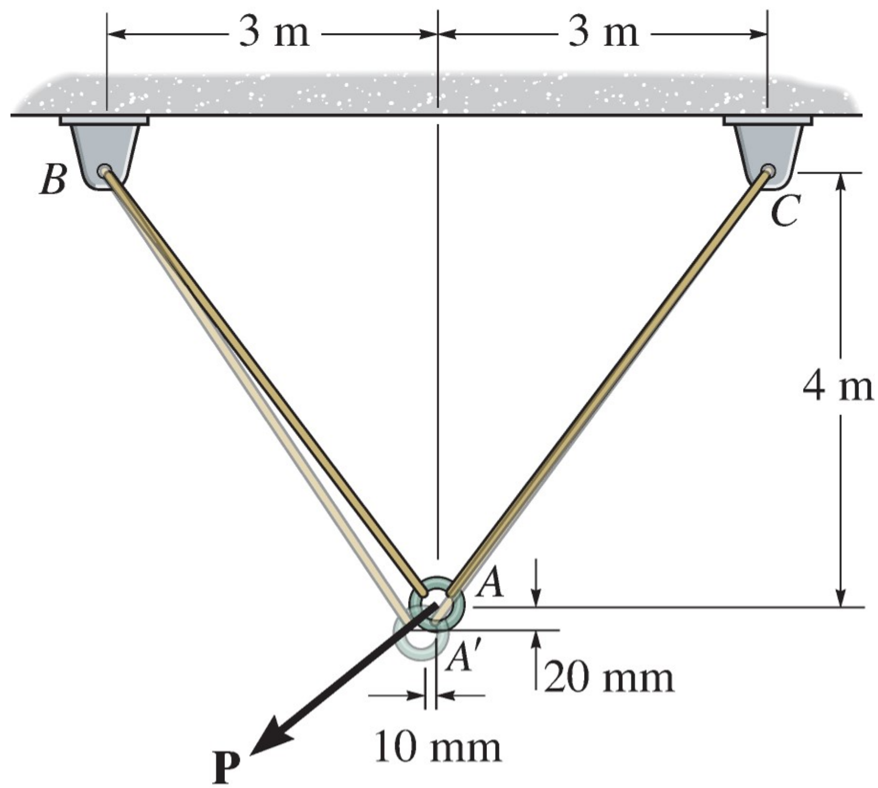
- And the shear strains will change the shape

$$\frac{\pi}{2}-\gamma_{xy} \qquad \frac{\pi}{2}-\gamma_{yz} \qquad \frac{\pi}{2}-\gamma_{xz}$$

small strain

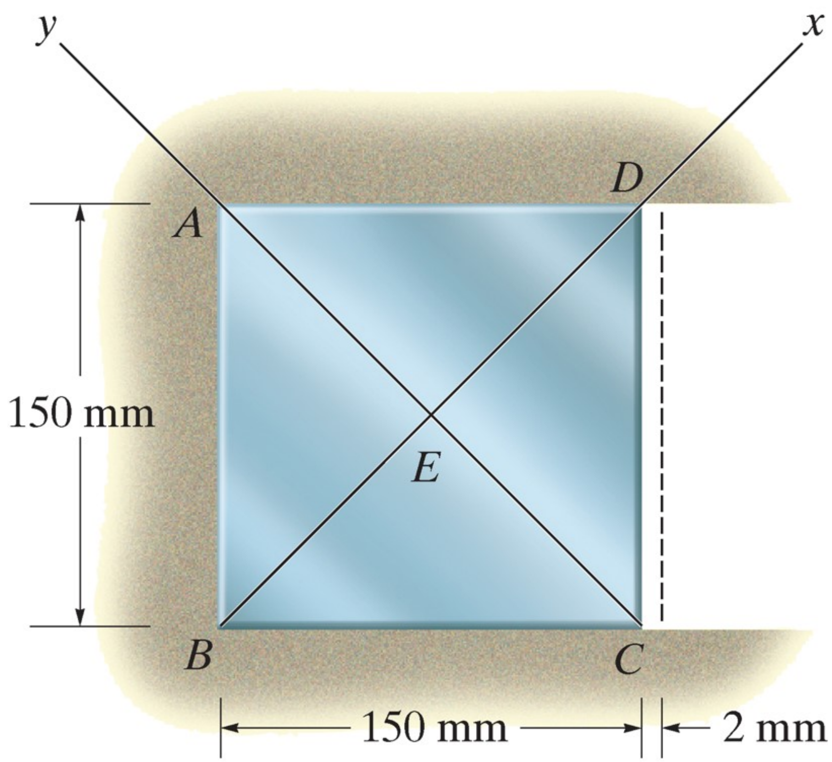
- Most engineering analysis is based on the assumption of small strains
- This is valid for many materials (wood, metal), but not for rubbers and some other polymers
- When strains are small, we assume that the change in angle, $\Delta\theta$ is very small
- $\sin\Delta\theta \approx \Delta\theta$, $\cos\Delta\theta \approx 1$, $\tan\Delta\theta \approx \Delta\theta$

example 2.1



Find the normal strains in the two wires if A moves to A'

example 2.3



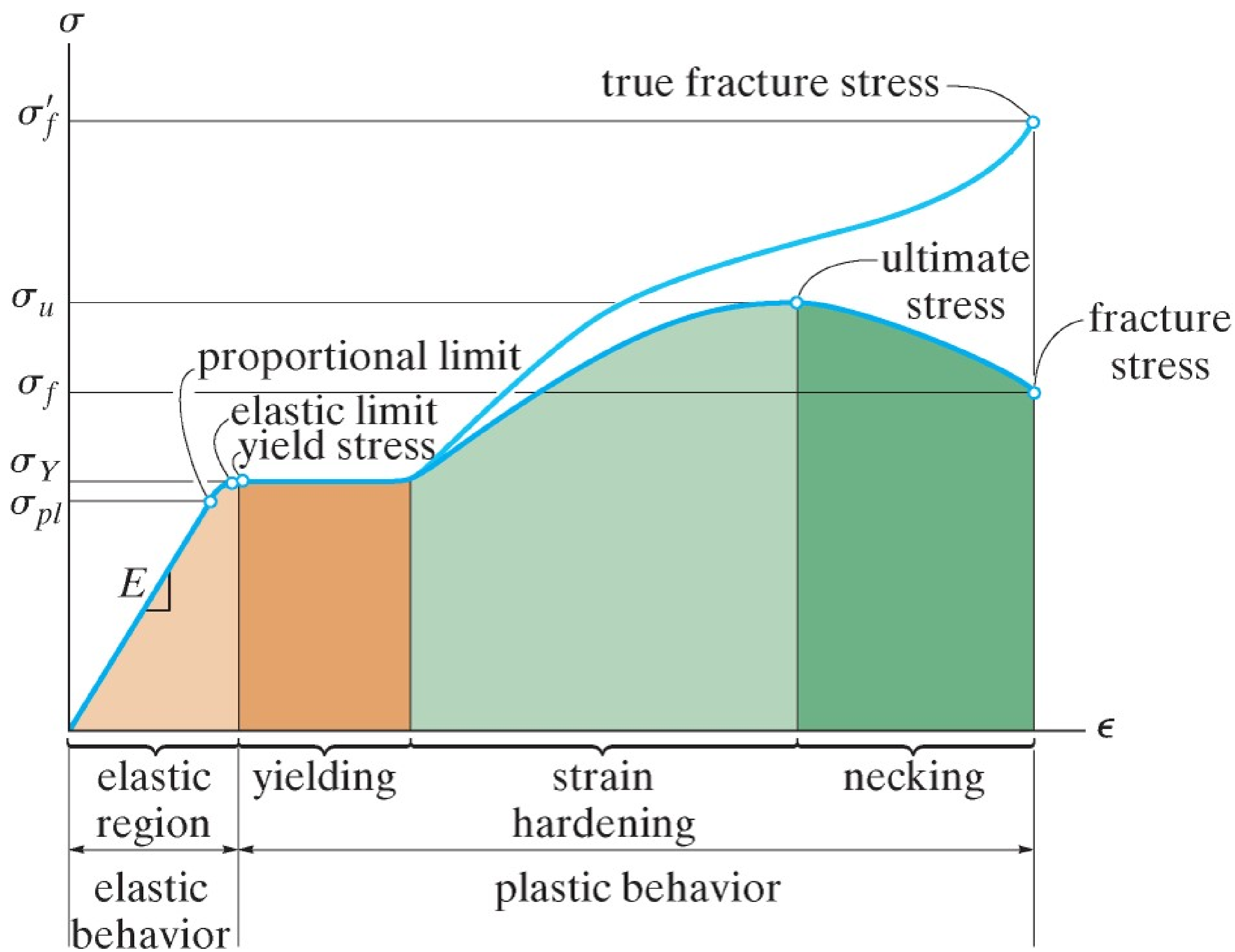
The plate is fixed along AB and held in horizontal guides along AD and BC. If the right side is displaced 2 mm find the average normal strain along AC and the shear strain at E relative to the x and y axes.

stress-strain

stress-strain

- Most engineering materials can be characterized by their stress-strain diagram
- Comes from a tensile or compressive test, where a load is applied (gives stress) and the strain is measured (via an extensometer or strain gauge)
- *Engineering stress* is plotted on the y-axis vs. *engineering strain* on the x-axis

stress-strain



Conventional and true stress–strain diagram for ductile material (steel) (not to scale)

elastic behavior

- Most of the time, the linear region is the one we are most interested in
- In this region, the material is elastic, meaning when the load is removed the material will return to its original state

elastic behavior

- Because the stress-strain curve is a straight line, we can relate stress and strain with a single constant
- This constant is known as the *modulus of elasticity* or *Young's modulus*

$$\sigma = E\epsilon$$

plastic behavior

- Yielding occurs when stress increases beyond the *yield stress* or *elastic limit*, this is when plastic deformation occurs, meaning the material will not go back to its original shape
- Strain hardening is common in many metals, and means as more stress is applied the material becomes more stiff

plastic behavior

- Necking occurs when the material begins to have a noticeable “neck” due to being stretched very thin and lower forces are required to deform the material

true stress-strain

- True stress and strain use the actual material cross-section (instead of the original cross-section) to calculate stress and strain
- In the elastic region the difference is negligible, so in many cases we just stick with engineering strain, even if we know it is *wrong*

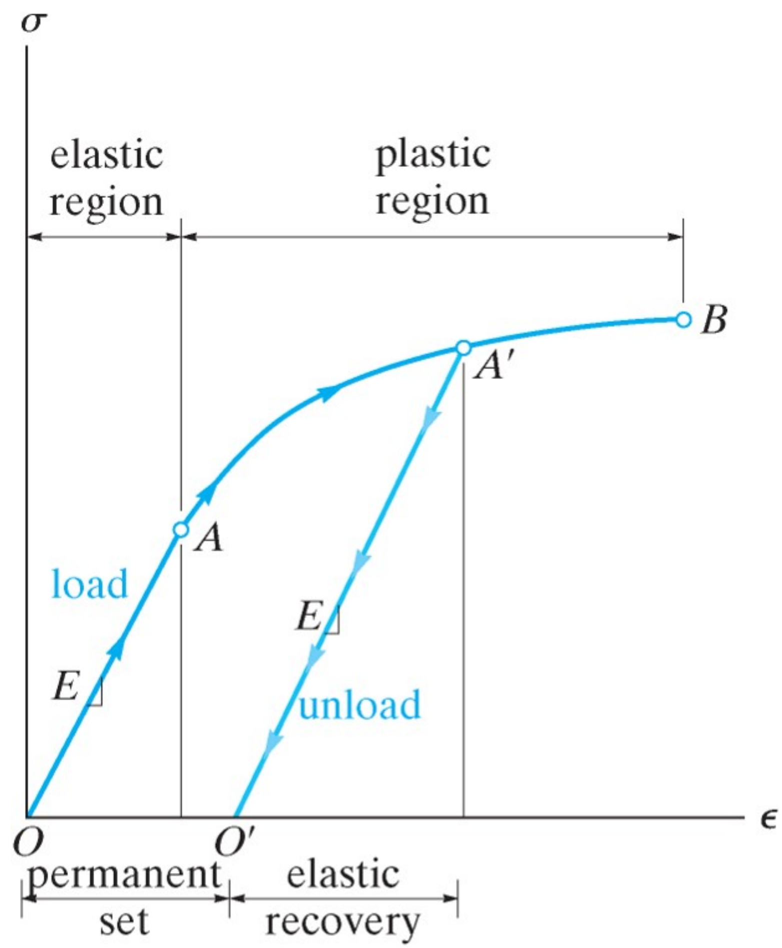
ductile materials

- Ductile materials can undergo large strains before failure
- One way to report how ductile a material is is known as percent elongation
- Steel, brass, molybdenum, and zinc exhibit similar ductile stress-strain characteristics
- Aluminum is often considered ductile, but its stress-strain behavior is a bit different

brittle materials

- Materials that exhibit little or no yielding before failure are called *brittle*
- Cast iron, concrete, and glass are common brittle materials
- Brittle materials fail easily in tension, but are very strong in compression

strain hardening



strain energy

strain energy

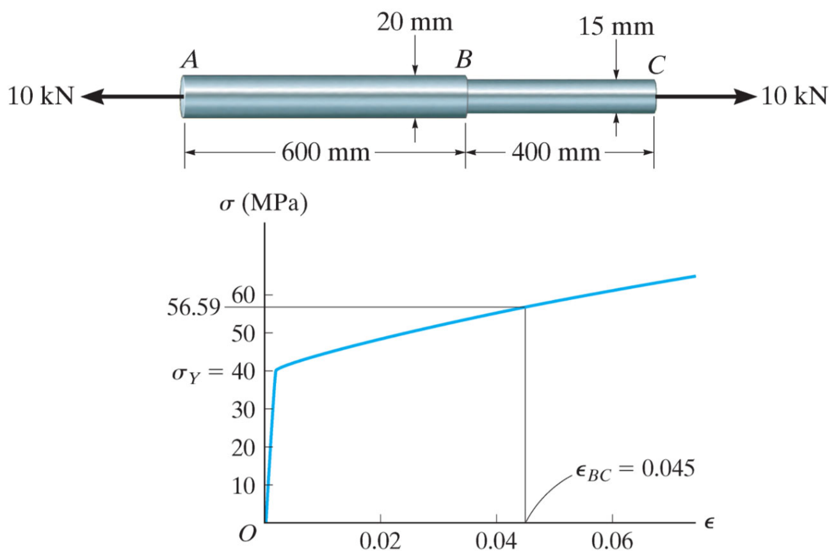
- Work in physics is defined as force times distance
- As a force is applied to a material, the energy from the work done by the load is stored in the material and called strain energy
- In engineering, we often use the strain energy density, or the amount of strain energy per unit volume of material

$$u = \frac{1}{2}\sigma\epsilon$$

toughness

- Graphically, the area under the stress strain curve represents the strain energy density
- We call the entire region (usually for a ductile material) the *toughness*
- Some hardened steels have a high failure strength, but are not very ductile, this gives them a lower toughness

example 3.3



The aluminum rod shown has a circular cross-section. Determine the elongation of the rod when load is applied using the given stress-strain diagram.

poisson’s ratio

poisson's ratio

- When a material is stretched in one direction, it tends to contract in the transverse direction
- The ratio of transverse to axial strain is called *Poisson's ratio*

$$\nu = -\frac{\epsilon_{transverse}}{\epsilon_{axial}}$$

shear stress-strain

shear stress-strain

- It can be experimentally difficult to obtain a state of pure shear, but a common method for many materials is to place a thin tube in torsion
- For most engineering materials, the shear stress-strain behavior is linear in the elastic region, but has a different constant relating stress to strain, known as the *Shear Modulus*

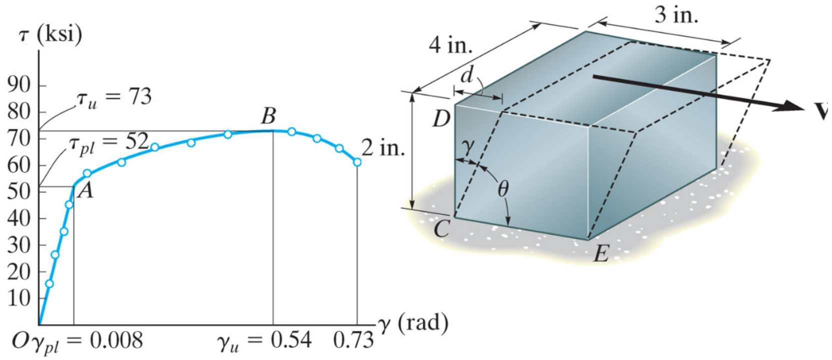
$$\tau = G\gamma$$

elastic constants

- For most materials, E, G and ν are related by the following expression

$$G = \frac{E}{2(1+\nu)}$$

example 3.5



Determine G for the specimen shown. Also find the maximum distance d, that the top could be displaced horizontally while remaining elastic. What force V is required to cause this displacement?