# Mechanics of Materials

Lecture 5 - Axial Load

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# schedule

- 30 Aug - Axial Load (not on exam 1)
- 1 Sep - Exam Review
- 3 Sep - Homework 2 Due, Homework 1 Self-Grade Due
- 8 Sep - Exam 1
- 10 Sep - Project 1 Due
- 13 Sep - Axial Load

## outline

- saint venant's principle
- elastic axial deformation
- superposition
- statically indeterminate



# saint venant's principle

## saint venant's principle

- We use Saint Venant's principle to generalize various loading applications
- If we apply a concentrated force, near where we apply it (for example, along a pin), the stress will not be very uniform
- Far away from that point, however, the stress will be uniform, whether we apply the force with 1 pin, 2 pins, or via a uniform grip



## saint venant's principle

- We use *saint venant's principle* to replace difficult to model loads with easier ones
- There are two conditions
    1. The load must be statically equivalent
    2. Our region of interest must be far enough away from the point where the load was applied

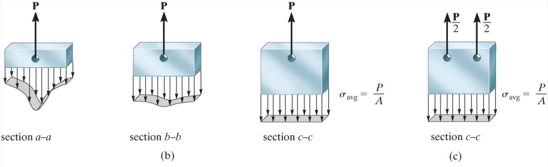



**Figure 1:** An image showing what the stress field looks like in a body both near to an applied load and far away.



# elastic axial deformation

# axially loaded member

- We can use Hooke's Law to find the deformation of a general body under axial loading (below the elastic limit)

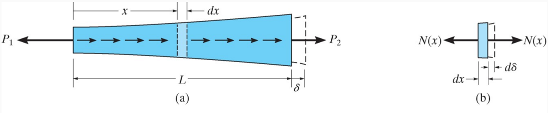



**Figure 2:** If we consider some homogeneous body with an applied load, we can look at a small section of this body with an applied load of N(x) which is initially dx wide, but under load stretches an additional d delta.



# axially loaded member

- For some differential element, we can consider the internal forces and stresses

$$\sigma = \frac{N(x)}{A(x)} = E(x)\epsilon(x) = E(x)\left(\frac{d\delta}{dx}\right)$$

- We can solve this for $d\delta$ to find

$$d\delta = \frac{N(x)dx}{A(x)E(x)}$$

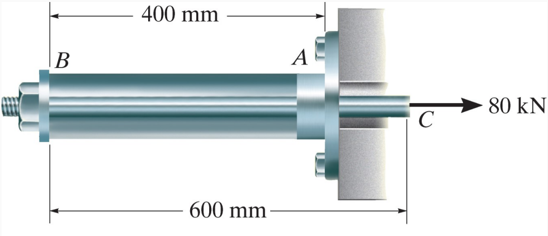
- We integrate this over the length of the bar to find the total displacement

## sign convention

- In general, we consider a force or stress to be positive if it causes tension and elongation
- It is negative if it causes compression and contraction

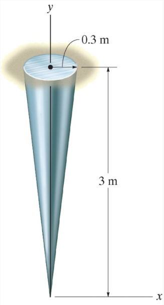


## example 4.2




A steel rod with a 10mm diameter is attached to a rigid collar passing through an aluminum tube with cross-sectional area of 400 mm2. Find the displacement at C if $E_{st} = 200$ GPa and $E_{al} = 70$ GPa.

## example 4.4




The cone shown has a specific weight of $\gamma = 6$ kN/m3 and E=9 GPa. Determine how far the end is displaced due to gravity.



# superposition

- Some problems are too complicated to solve all at once
- Instead, we break them up into two simpler problems
- Each "sub-problem" must still satisfy equilibrium
- Problem must be linear and the deformation should be small enough that it does not cause moment-equilibrium issues



# statically indeterminate

## statically indeterminate

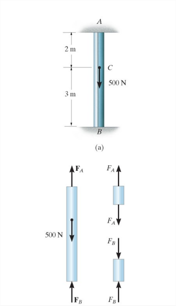
- There are many problems that are at least slightly over-constrained
- While this is common engineering practice, it creates too many variables for statics analysis
- These problems are called “statically indeterminate”



## statically indeterminate

- One extra equation we can use is called “compatibility” or the “kinematic condition”
- We know that at the displacement must be equal on both sides of any arbitrary section we make in a member
- We can separate a member into two parts, then use compatibility to relate the two unknown forces

# statically indeterminate






# example 4.7

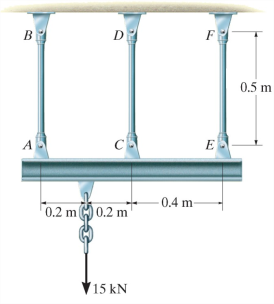



**Figure 3:** A 0.8 m long rigid horizontal bar is supported by hanging from 3 vertical rods. Rod

Assuming the bottom bar is rigid, find the force developed in each bar. AB and EF have cross-sectional areas of 50 mm2 while CD has a cross-sectional area of 30 mm2.