Mechanics of Materials

Lecture 11 - Bending

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

22 September, 2020

schedule

- 22 Sep - Bending, Homework 4 Due, Homework 3 Self-Grade Due
- 24 Sep - Bending
- 29 Sep - Transverse Shear, Homework 5 Due, Homework 4 Self-Grade Due
- 1 Oct - Transverse Shear

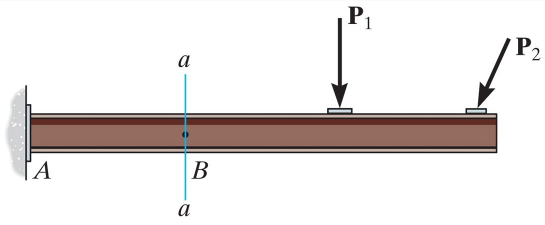
outline

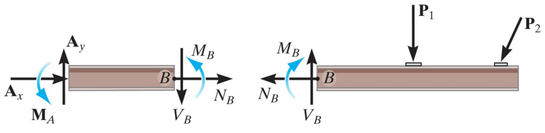
- shear and moment diagrams
- graphical method
- bending deformation
- flexure formula

shear and moment diagrams

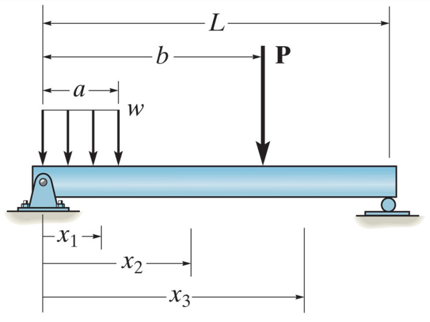
- The general approach to shear and moment diagrams is to first find the support reactions
- Next we section the beam and instead of finding the internal force and moment at a single point, we find it as a function of x
- Many beams will require piecewise sectioning
- We then draw this as a shear and moment diagram

sign convention

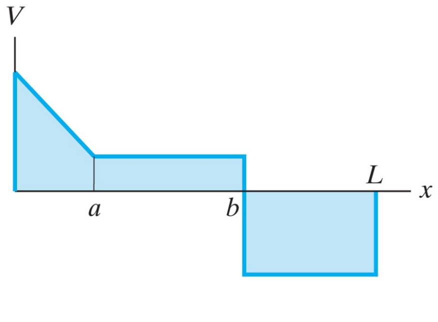




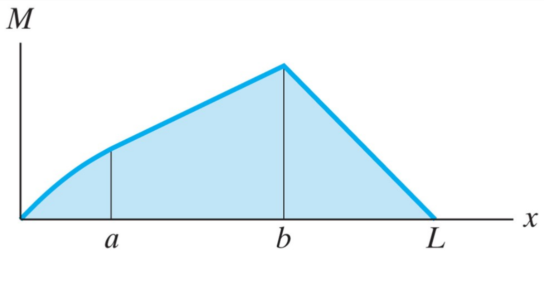
example beam



example beam



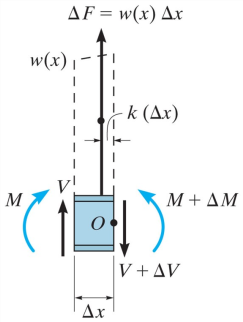
example beam



relation between load, shear, moment

- When there are several forces, supports, or loading conditions applied to a beam, the piecewise method can be cumbersome
- In this section we will examine the differential relationships between distributed load, shear, and bending moments

distributed load



distributed load

- Consider a beam subjected to only distributed loading
- If we section this beam in the middle (to remove both supports) we can relate V to the loading function $w(x)$
- Considering the sum of forces in y:

$$\begin{aligned} V + w(x)\Delta x - (V + \Delta V) = 0 \\ \Delta V = w(x)\Delta x \end{aligned}$$

distributed load

- If we divide by Δx and let $\Delta x \to 0$ we find

$$\frac{dV}{dx} = w(x)$$

- Thus the slope of the shear diagram is equal to the distributed load function

moment diagram

– If we consider the sum of moments about O on the same section we find

$$(M + \Delta M) - (w(x)\Delta x)k\Delta x - V\Delta x - M = 0$$
$$\Delta M = V\Delta x + kw(x)\Delta x^2$$

– Dividing by Δx and letting $\Delta x \to 0$ gives

$$\frac{dM}{dx} = V$$

concentrated forces

– If we consider a concentrated force (instead of a distributed load) we find

$$\Delta V = F$$

– This means that concentrated loads will cause the shear diagram to “jump” by the amount of the concentrated force (causing a discontinuity on our graph)

couple moments

- If our section includes a couple moment, we find (from the moment equation) that

$$\Delta M = M_0$$

- Thus the moment diagram will have a jump discontinuity

example 7.9

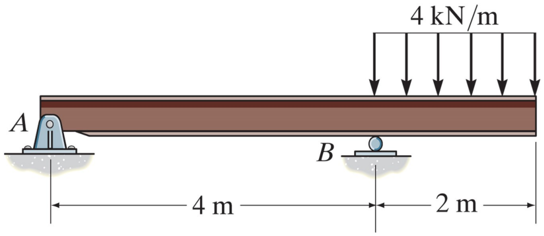
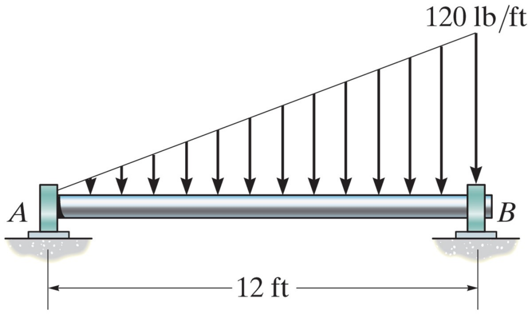


Figure 1: A beam is 6 meters long with pin supports at the left end, A, and at B, 4 meters to the right of A. From B to the right end of the beam is a uniform distributed load of 4 kN/m.

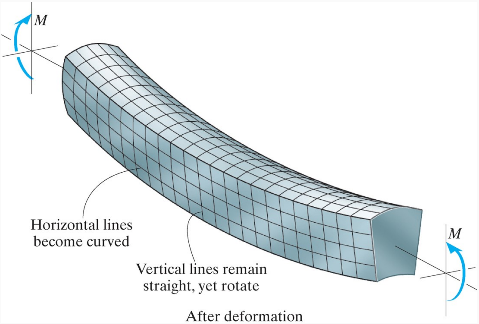
example 7.10



bending deformation

- If we drew a grid on a specimen in bending, we would find that vertical lines tend to stay straight (but rotate)
- Horizontal lines will become curved
- If bending lifts the ends up (like a smile), then the top face will be in compression (and expand), while the bottom face will be in tension (and contract)

bending deformation



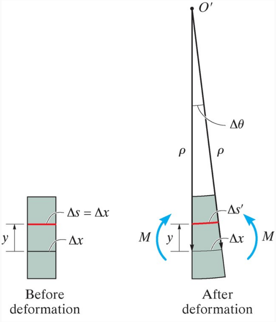
neutral axis

- Since the bottom is in tension and the top is in compression, there must be somewhere in between that is under no stress
- We call that the neutral axis, and assume it does not change in length
- We also assume that planar sections remain planar (no warping)
- Finally, Poisson's effects are neglected (cross-sections keep the same size and shape)

strain

- We will now consider an infinitesimal beam element before and after deformation
- Δx is located on the neutral axis and thus does not change in length after deformation
- Some other line segment, Δs is located y away from the neutral axis and changes its length to $\Delta s'$ after deformation

strain



strain

- We can now define strain at the line segment Δs as

$$\epsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

strain

- If we define ρ as the radius of curvature after deformation, thus $\Delta x = \Delta s = \rho \Delta\theta$
- The radius of curvature at Δs is $\rho - y$, thus we can write

$$\epsilon = \lim_{\Delta\theta \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta}$$

- Which gives

$$\epsilon = -\frac{y}{\rho}$$

hooke's law

- If the beam is a linear material that follows Hooke's Law, the stress must be linearly proportional to the strain
- Thus the stress will vary linearly through the beam, just like the strain does

neutral axis

- We have already hypothesized that a neutral axis must exist, we will now find its location
- To be in equilibrium, the resultant force(s) produced by the stress must sum to zero, which means

$$\begin{aligned}\sum F_x = 0 = \int_A dF = \int_A \sigma dA \\ = \int_A -\left(\frac{y}{c}\right)\sigma_{max} dA \\ = -\frac{\sigma_{max}}{c}\int_A y dA\end{aligned}$$

neutral axis

– Since σ_{max} and c are both non-zero constants, we know that

$$\int_A y dA = 0$$

– Which can only be satisfied at the horizontal centroidal axis, so the neutral axis is the centroidal axis

bending moment

– The internal bending moment must be equal to the total moment produced by the stress distribution

$$\begin{aligned} M &= \int_A y dF = \int_A y(\sigma dA) \\ &= \int_A y \left(\frac{y}{c}\sigma_{max}\right) dA \\ &= \frac{\sigma_{max}}{c} \int_A y^2 dA \end{aligned}$$

bending moment

– We recognize that $\int_A y^2 dA = I$, and we re-arrange to write

$$\sigma_{max} = \frac{Mc}{I}$$

procedure

– Find the internal moment at the point of interest, or draw a moment diagram to find the maximum point (if needed)
– Determine the moment of inertia for the beam section
– Find the neutral axis and/or the distance from the neutral axis to the point of interest
– Use the flexure formula to find the stress

example 6.12

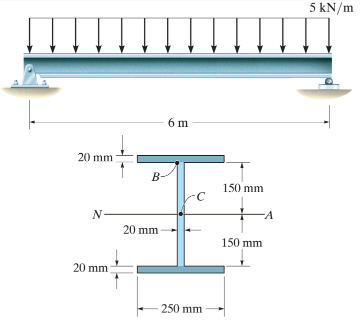


Figure 2: A 6 meter long beam is pinned at both ends and subjected to a uniformly distributed load of 5 kN/m.

Find the maximum bending stress and draw the stress distribution through the thickness at that point.