# AE333
# Mechanics of Materials

Lecture 10 - Torsion

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# schedule

- 21 Feb - Torsion, HW3 Due
- 24 Feb - Torsion
- 26 Feb - Bending
- 28 Feb - Bending, HW4 Due

# outline

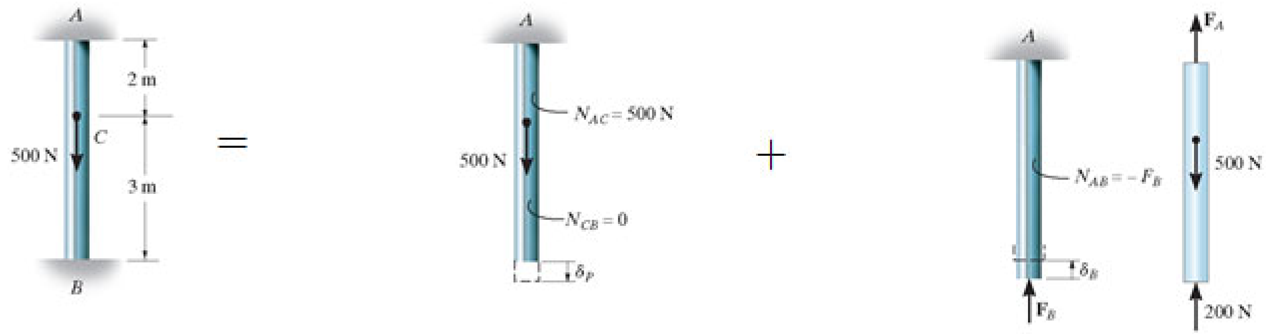
- force method
- thermal stress
- torsion
- power transmission

# force method

# force method

- One way to solve statically indeterminate problems is using the principle of superposition
- We choose one redundant support and remove it
- We then add it back as a force separately (without the other forces in the problem)

# force method



(d)

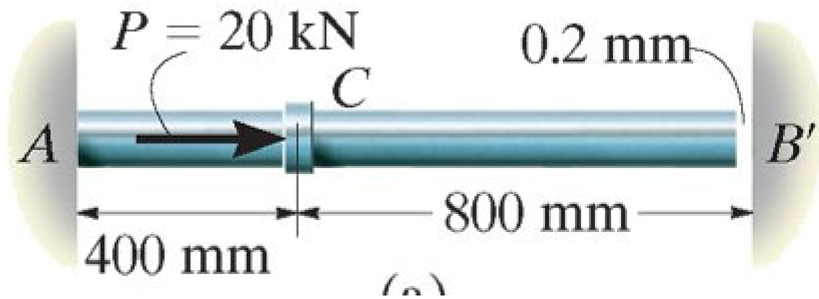
# force method

- We connect the two problems by requiring that the displacement in both frames adds to 0 to meet the support requirements
- This is referred to as the equation of compatibility

# procedure

- Choose one support as redundant, write the equation of compatibility
- Express the external load and redundant displacements in terms of load-displacement relationship
- Draw free body diagrams and use the equations of equilibrium to solve

# example 4.9




The steel rod shown has a diameter of 10 mm. Determine the reactions at A and B'.

# thermal stress

# thermal stress

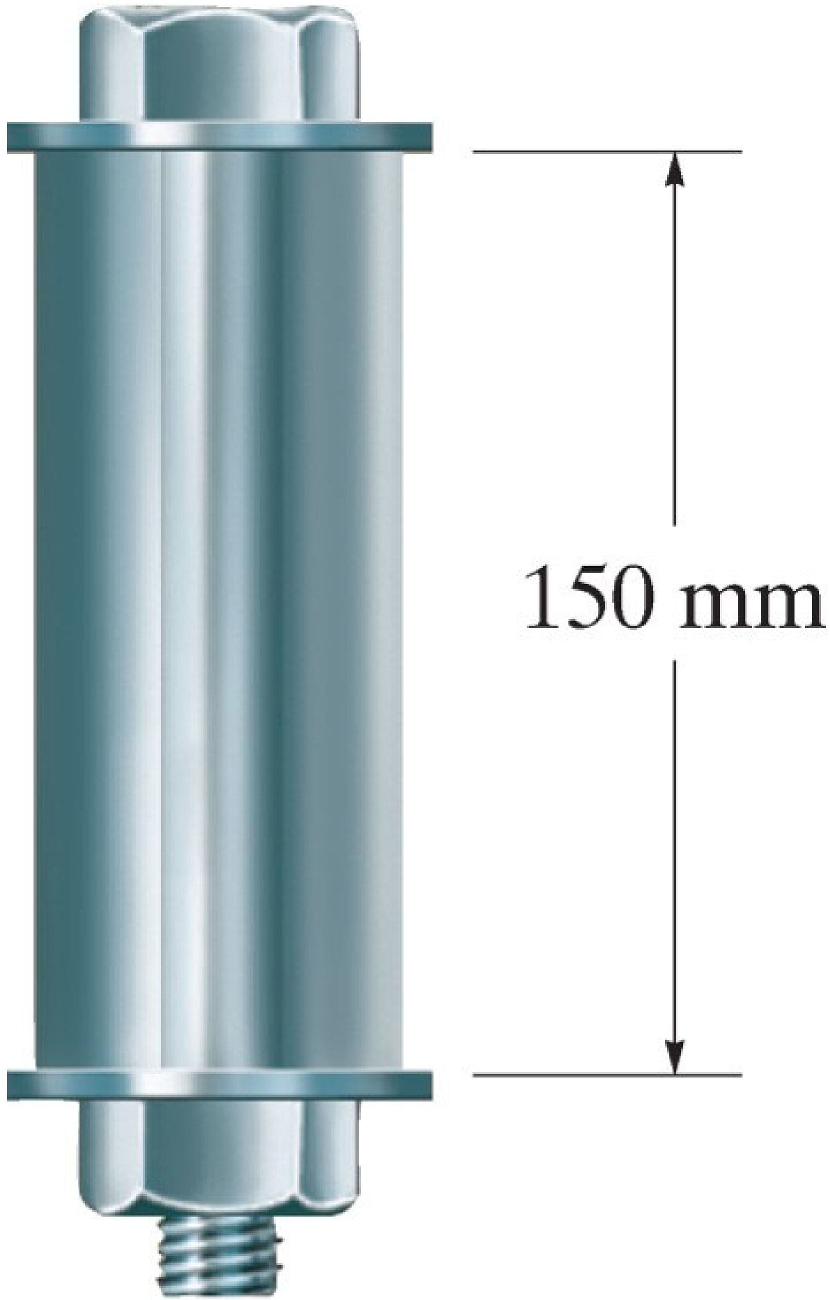
- A change in temperature causes a material to either expand or contract
- For most materials this is linear and can be described using the coefficient of linear expansion

$$\delta_T = \alpha \Delta T L$$

# thermal stress

- When a body is free to expand, the deformation can be readily calculated using
- If it is not free to expand, however, thermal stresses develop
- We can use the force method described previously to find the thermal stresses developed

# example 4.12



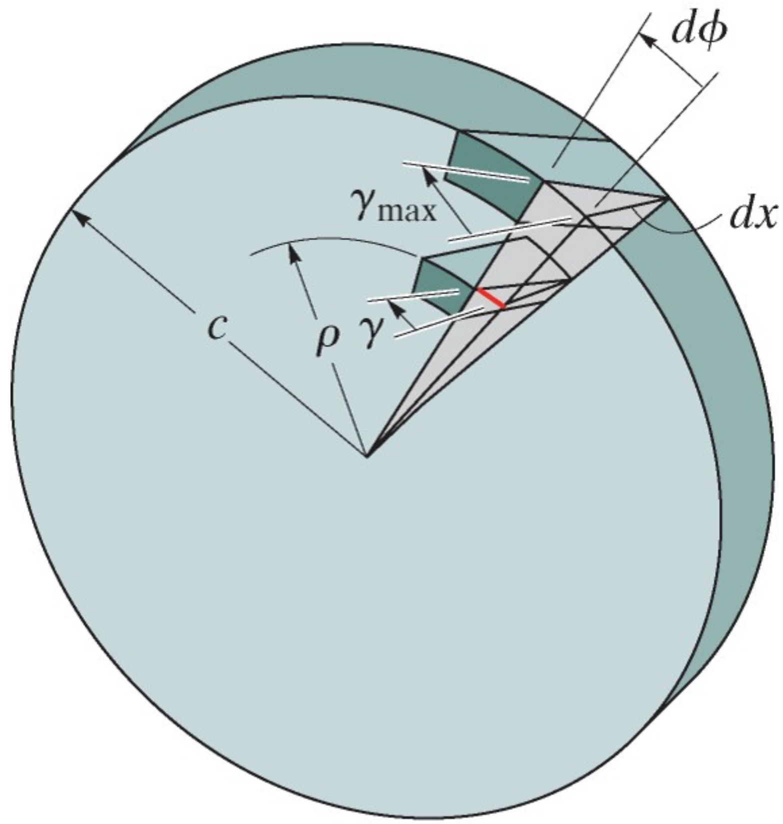

An aluminum tube with cross-section of 600 $mm^2$ is used as a sleeve for a steel bolt with cross-sectional area of 400 $mm^2$. When T=15 degrees Celsius there is negligible force and a snug fit, find the force in the bolt and sleeve when T=80 degrees Celsius.

**torsion**

# torsion

- Torque is a moment that tends to twist a member about its axis
- For small deformation problems, we assume that the length and radius do not change significantly under torsion
- The primary deformation we are concerned with in torsion is the angle of twist, denoted with $\phi$

# shear




The shear strain at points on the cross section increases linearly with $\rho$, i.e., $\gamma = (\rho/c)\gamma_{max}$.

# torsion formula

- For a linearly elastic material, Hooke's Law in shear will hold ($\tau = G\gamma$)
- This means that, like shear strain, shear stress will vary linearly along the radius

# torsion formula

- We can find the total force on an element, $dA$ by multiplying the shear stress by the area

$$dF = \tau dA$$

- The torque ($dT = \rho dF$) produced by this force is then

$$dT = \rho(\tau dA)$$

# torsion formula

- Integrating over the whole cross-section gives

$$T = \int_A \rho(\tau dA) = \frac{\tau_{max}}{c} \int_A \rho^2 dA$$

- The integral $\int_A \rho^2 dA$ is also called the Polar Moment of Inertia, $J$, so we can write

$$\tau_{max} = \frac{Tc}{J}$$

# polar moment of inertia

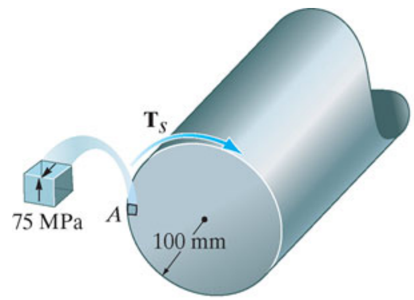
- We know that $J = \int_A \rho^2 dA$, so we can compute this for some common shapes
- For a solid circular cross-section, we have

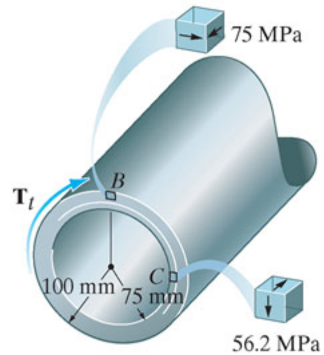
$$J = \int_0^c \rho^2 (2\pi\rho d\rho) = \frac{\pi}{2} c^4$$

- For a circular tube we have

$$J = \int_{c_1}^{c_2} \rho^2 (2\pi\rho d\rho) = \frac{\pi}{2} (c_2^4 - c_1^4)$$

# example 5.1




The allowable shear stress is 75 MPa. Determine the maximum torque that can be applied to each of the cross-sections shown and find the stress acting on a small element at A, B and C.

# power transmission

# power transmission

- Shafts and tubes are often connected to belts and drives, and the torque, speed, and power are all related
- Power is the rate of work done, for rotation problems, $P = T\omega$
- We are often given the frequency $f$ instead of the angular velocity, $\omega$, in this case $P = 2\pi fT$

# power units

- In SI Units, power is expressed in Watts 1 W = 1 N m / sec
- In Freedom Units, power is expressed in Horsepower 1 hp = 555 ft lb / sec

# shaft design

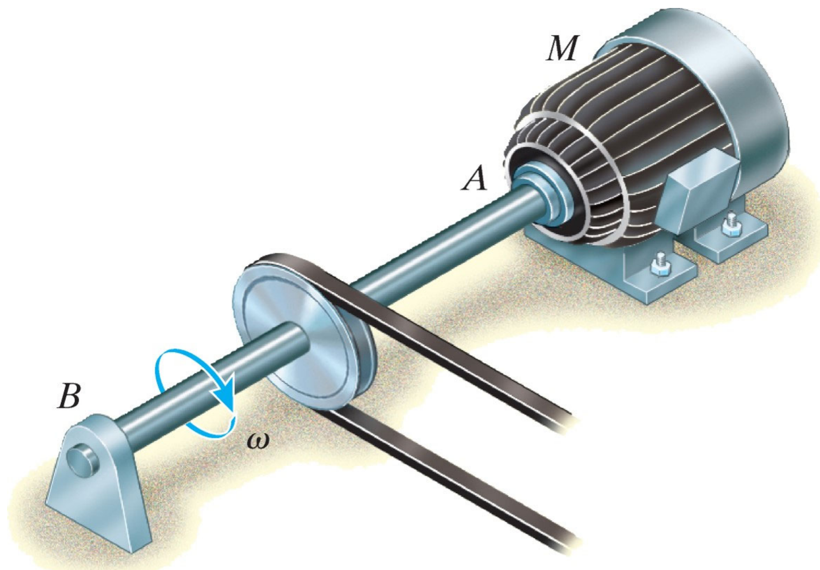
- We often know the power and frequency of a drive, and need to design a shaft such that the shear stress is acceptable
- We can easily find the torque as $T = P/2\pi f$, we can use this combined with the torsion equation

$$\tau_{max} = \frac{Tc}{J}$$

to find the appropriate shaft diameter.

- For solid shafts we can solve for $c$ uniquely, but not for hollow shafts

# example 5.4




The steel shaft shown is connected to a 5 hp motor that rotates at $\omega = 175$ rpm. If $\tau_{allow} = 14.5$ ksi, determine the required shaft diameter.