AE333

Mechanics of Materials

Lecture 13 - Bending

Dr. Nicholas Smith
Wichita State University, Department of Aerospace Engineering

February 28, 2020

schedule

- 28 Feb - Bending, HW4 Due
- 2 Mar - Bending
- 4 Mar - Bending
- 6 Mar - Transverse Shear, HW 5 Due

outline

- thin-walled tubes
- shear and moment diagrams
- graphical method

shear-moment diagrams

- Drawing shear-moment diagrams is a very important skill
- Learning MasteringEngineering’s drawing system is not as important (in my opinion)
- If you are more comfortable doing your shear-moment diagrams by hand, you may turn them into me in class on Monday and I will grade them by hand

thin-walled tubes

shear flow

- Thin-walled tubes of non-circular cross-sections are commonly found in aerospace and other applications
- We can analyze these using a technique called shear flow
- Because the walls of the tube are thin, we assume that the shear stress is uniformly distributed through the wall thickness

shear flow

- If we consider an arbitrary segment of a tube with varying thickness, we find from equilibrium that the product of the average shear stress and the thickness must be the same at every location on the cross-section

$$q = \tau_{avg} t$$

- q is called the shear flow

average shear stress

- We can relate the average shear stress to the torque by considering the torque produced about some point within the tubes boundary

$$T = \oint h\tau_{avg}t ds$$

- Where h is the distance from the reference point, ds is the differential arc length and t is the tube thickness.

average shear stress

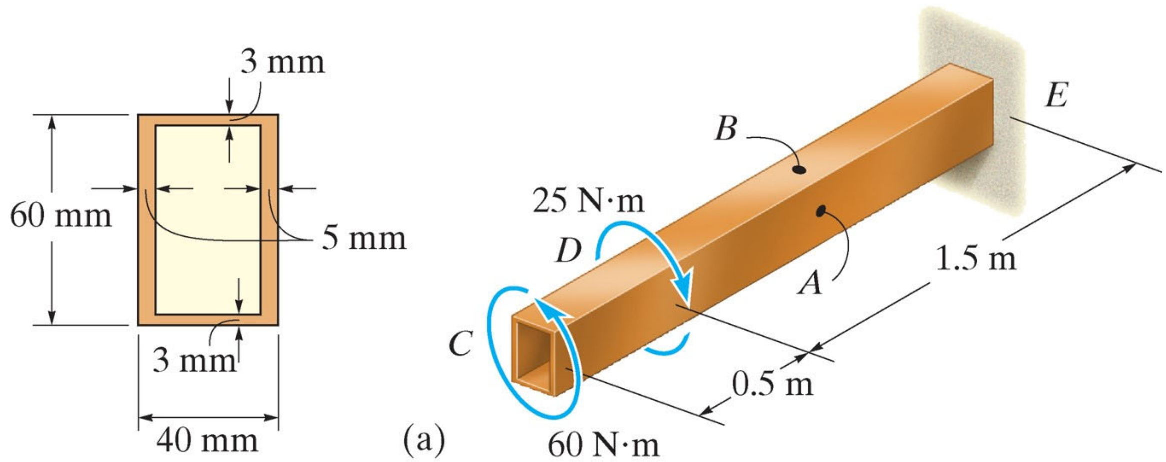
- We notice that $\tau_{avg}t$ is equal to the shear flow, q, which must be constant
- We can also simplify the integral by relating a similar area integral to the arc length integral

$$dA_m = 1/2hds$$

- Thus

$$T = \oint h\tau_{avg}tds = 2q\int dA_m = 2qA_m$$

example 5.13



(a)

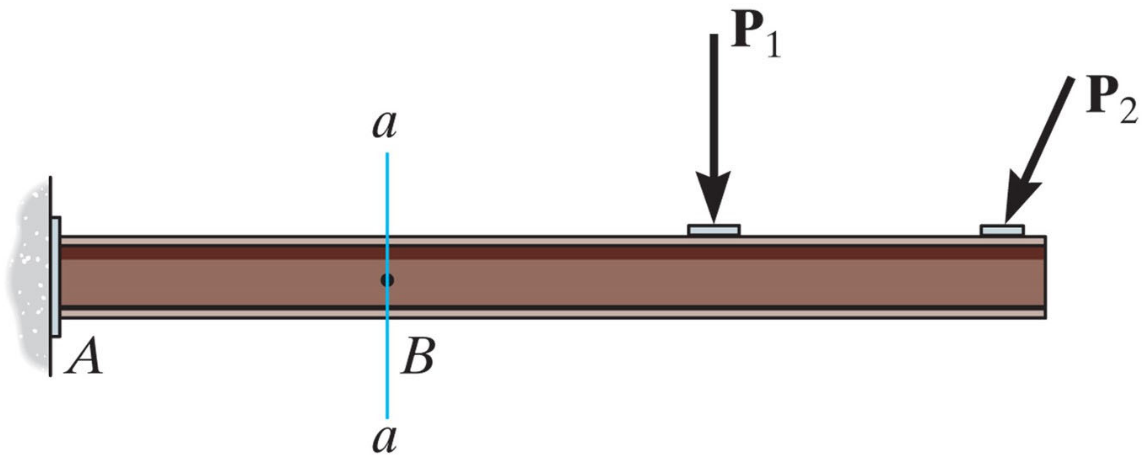
Determine the average shear stress at A and B.

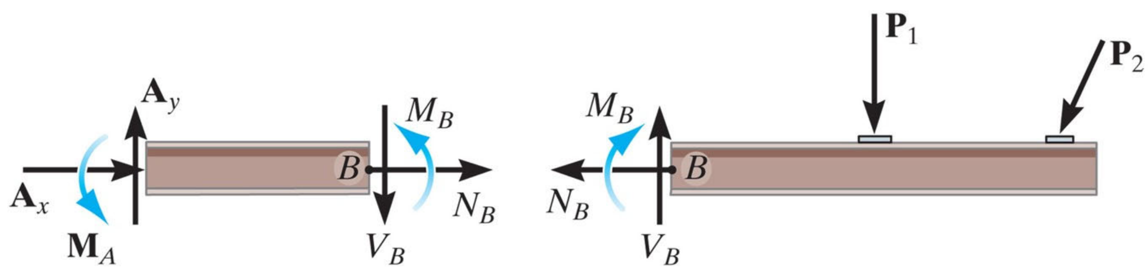
shear and moment diagrams

shear and moment diagrams

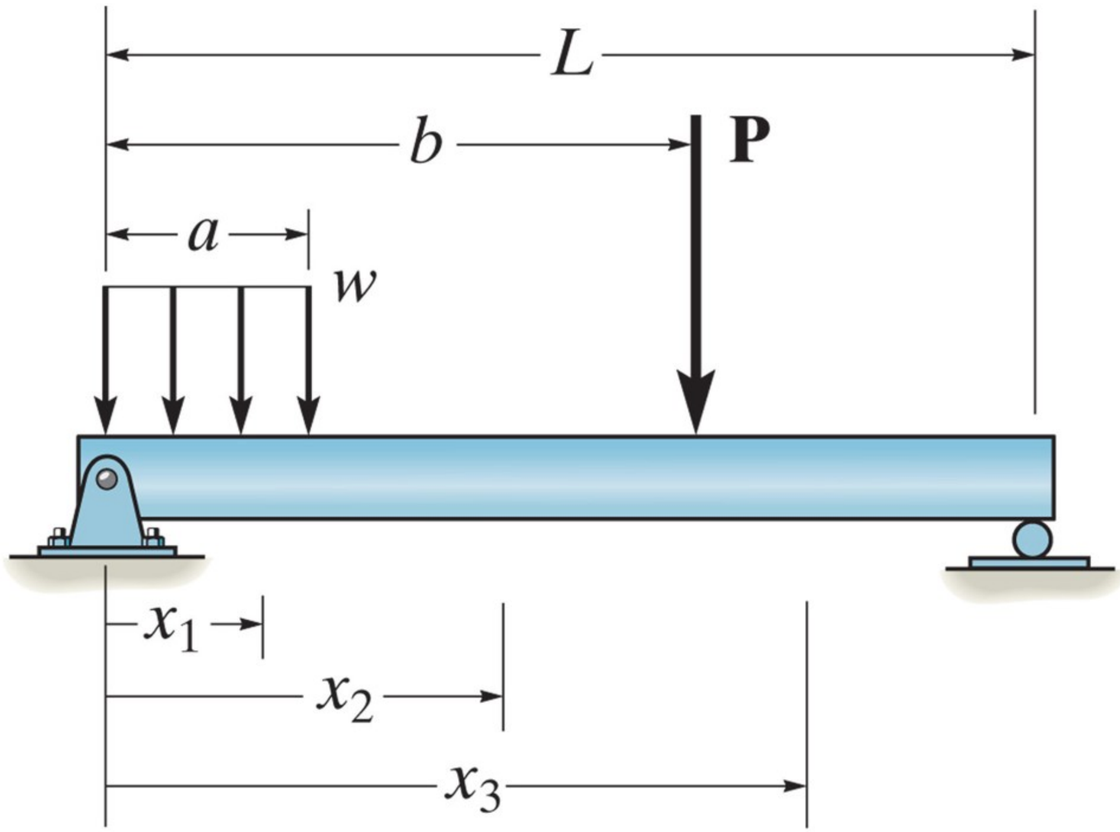
- The general approach to shear and moment diagrams is to first find the support reactions
- Next we section the beam and instead of finding the internal force and moment at a single point, we find it as a function of x
- Many beams will require piecewise sectioning
- We then draw this as a shear and moment diagram

sign convention

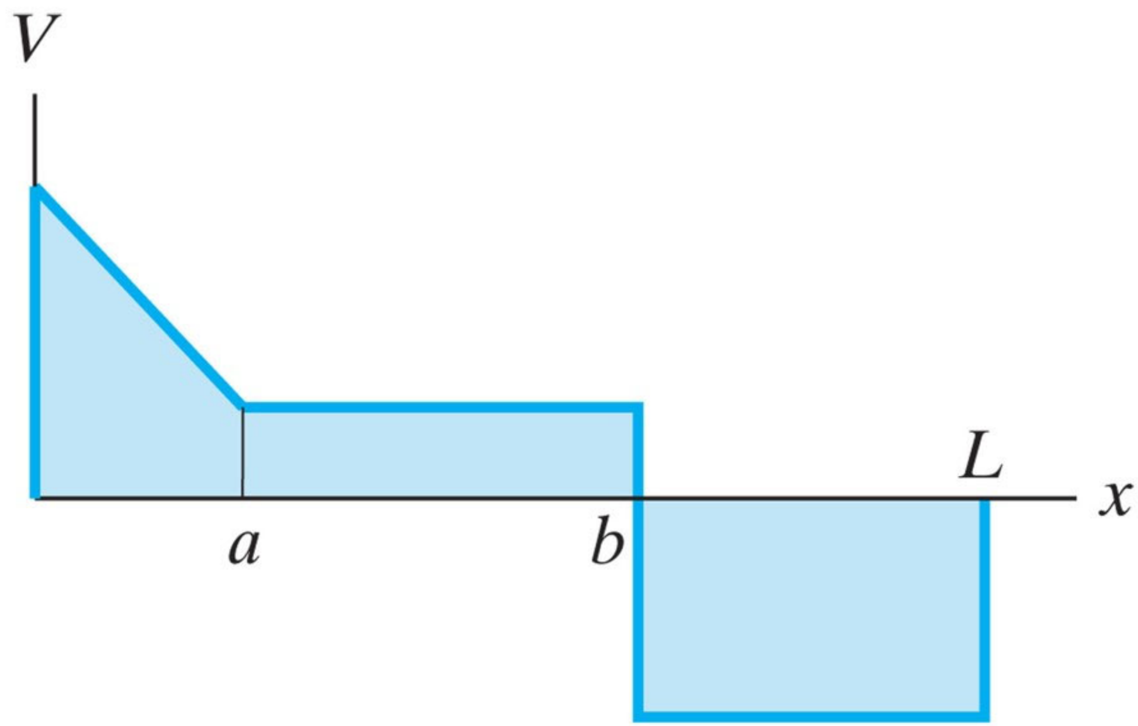




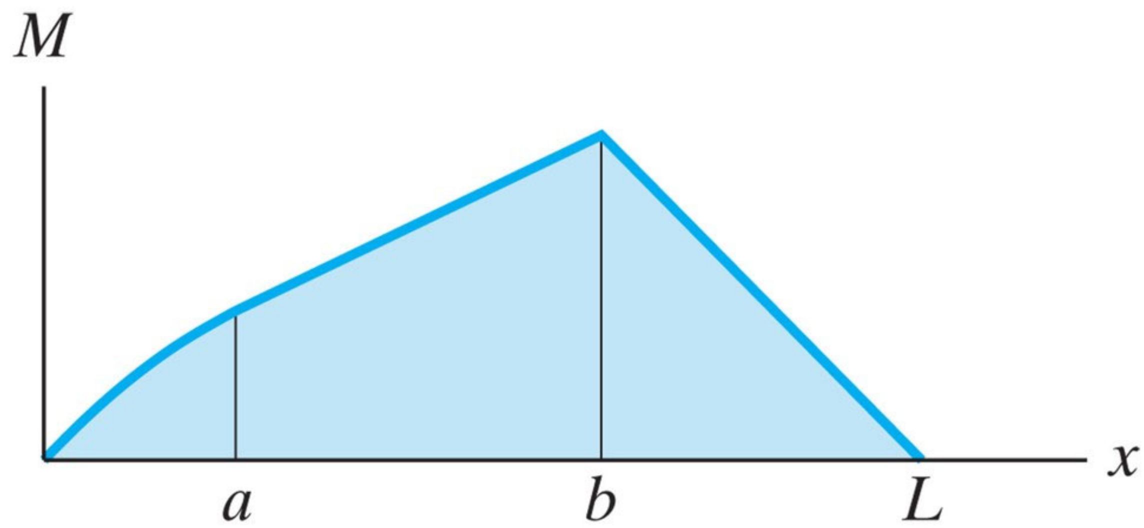
example beam



example beam



example beam

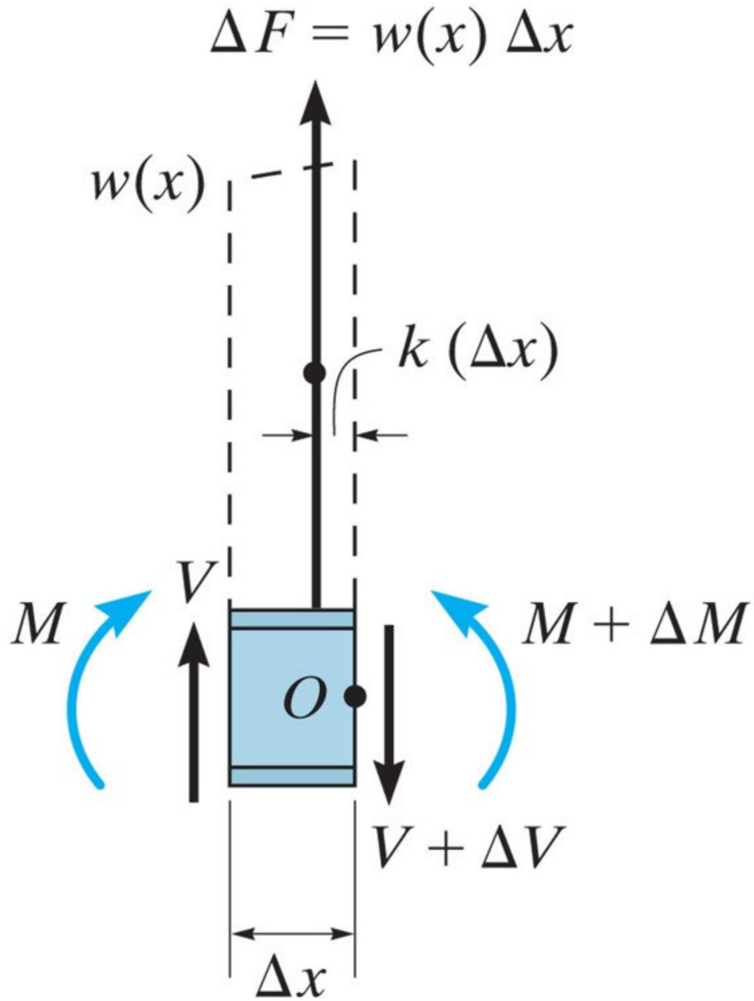


graphical method

relation between load, shear, moment

- When there are several forces, supports, or loading conditions applied to a beam, the piecewise method can be cumbersome
- In this section we will examine the differential relationships between distributed load, shear, and bending moments

distributed load



distributed load

- Consider a beam subjected to only distributed loading
- If we section this beam in the middle (to remove both supports) we can relate V to the loading function $w(x)$
- Considering the sum of forces in y:

$$
\begin{aligned}
V + w(x)\Delta x - (V + \Delta V) &= 0 \\
\Delta V &= w(x)\Delta x
\end{aligned}
$$

distributed load

- If we divide by Δx and let $\Delta x \to 0$ we find

$$\frac{dV}{dx} = w(x)$$

- Thus the slope of the shear diagram is equal to the distributed load function

moment diagram

- If we consider the sum of moments about O on the same section we find

$$
\begin{aligned}
(M + \Delta M) - (w(x)\Delta x)k\Delta x - V\Delta x - M = 0 \\
\Delta M = V\Delta x + kw(x)\Delta x^2
\end{aligned}
$$

- Dividing by Δx and letting $\Delta x \to 0$ gives

$$\frac{dM}{dx} = V$$

concentrated forces

- If we consider a concentrated force (instead of a distributed load) we find

$$\Delta V = F$$

- This means that concentrated loads will cause the shear diagram to “jump” by the amount of the concentrated force (causing a discontinuity on our graph)

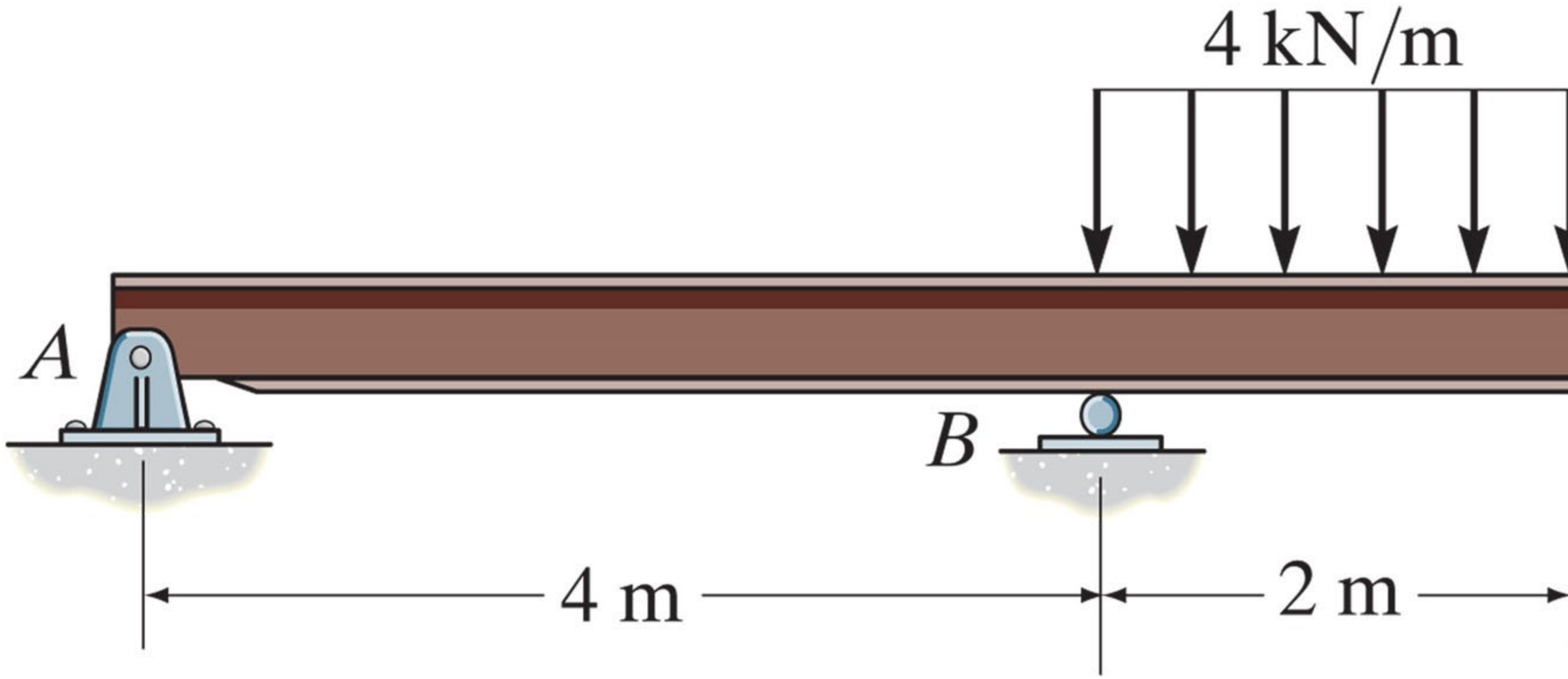
couple moments

- If our section includes a couple moment, we find (from the moment equation) that

$$\Delta M = M_0$$

- Thus the moment diagram will have a jump discontinuity

example 7.9



example 7.10