# Mechanics of Materials

Lecture 8 - Axial Load, Torsion

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# schedule

- 1 Mar - Axial Load
- 3 Mar - Torsion
- 5 Mar - Homework 3 Due
- 8 Mar - Torsion
- 10 Mar - Bending
- 12 Mar - Homework 4 Due, Homework 3 Self-grade due

# outline

- superposition
- statically indeterminate
- force method
- thermal stress
- torsion



# superposition

## superposition

- Some problems are too complicated to solve all at once
- Instead, we break them up into two simpler problems
- Each "sub-problem" must still satisfy equilibrium
- Problem must be linear and the deformation should be small enough that it does not cause moment-equilibrium issues



# statically indeterminate

## statically indeterminate

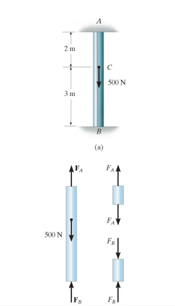
- There are many problems that are at least slightly over-constrained
- While this is common engineering practice, it creates too many variables for statics analysis
- These problems are called “statically indeterminate”



## statically indeterminate

- One extra equation we can use is called “compatibility” or the “kinematic condition”
- We know that at the displacement must be equal on both sides of any arbitrary section we make in a member
- We can separate a member into two parts, then use compatibility to relate the two unknown forces

## statically indeterminate






## example 4.7

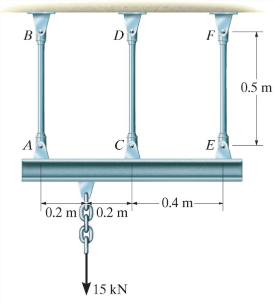



**Figure 1:** A 0.8 m long rigid horizontal bar is supported by hanging from 3 vertical rods. Rod

Assuming the bottom bar is rigid, find the force developed in each bar. AB and EF have cross-sectional areas of 50 mm2 while CD has a cross-sectional area of 30 mm2.

# force method

## force method

- One way to solve statically indeterminate problems is using the principle of superposition
- We choose one redundant support and remove it
- We then add it back as a force separately (without the other forces in the problem)

# force method

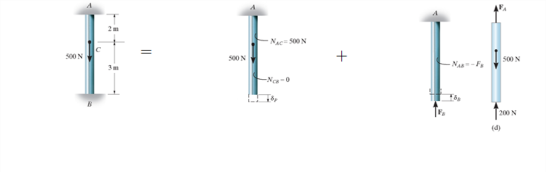



**Figure 2:** An illustration of the force method, we have the same statically indeterminate problem as before, a 5 m long, vertically-oriented bar is fixed at both ends, with a 500 N downward load applied 2 m from the top. We set this equivalent to a bar with the same load, but no support on the bottom end. We then add a force which will provide enough displacement to cancel out the displacement introduced by removing the load.



# force method

- We connect the two problems by requiring that the displacement in both frames adds to 0 to meet the support requirements
- This is referred to as the equation of compatibility

## procedure

- Choose one support as redundant, write the equation of compatibility
- Express the external load and redundant displacements in terms of load-displacement relationship
- Draw free body diagrams and use the equations of equilibrium to solve



## example 4.9

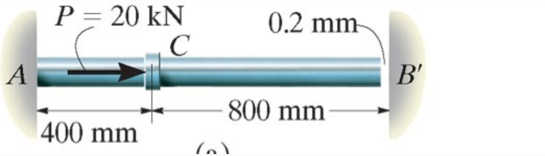



**Figure 3:** A 1200 mm long horizontal rod is fixed at its left end and has a fixed support 0.2 mm away from its right end. A 20 kN load is applied to the right 400 mm away from its left end.

The steel rod shown has a diameter of 10 mm. Determine the reactions at A and B'.

# thermal stress

## thermal stress

- A change in temperature cases a material to either expand or contract
- For most materials this is linear and can be described using the coefficient of linear expansion

$$\delta_T = \alpha \Delta T L$$

## thermal stress

- When a body is free to expand, the deformation can be readily calculated using
- If it is not free to expand, however, thermal stresses develop
- We can use the force method described previously to find the thermal stresses developed



## example 4.12

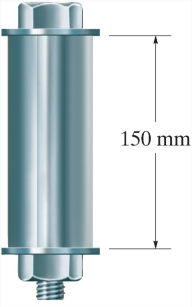



**Figure 4:** An aluminum tube used as a sleeve for a steel bolt. The tube is 150 mm long.

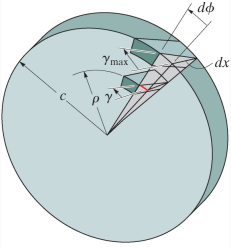
An aluminum tube with cross-section of 600 mm2 is used as a sleeve for a steel bolt with cross-sectional area of 400 mm2. When T=15 degrees Celsius there is negligible force and a snug fit, find the force in the bolt and sleeve when T=80 degrees Celsius.

# torsion

## torsion

- Torque is a moment that tends to twist a member about its axis
- For small deformation problems, we assume that the length and radius do not change significantly under torsion
- The primary deformation we are concerned with in torsion is the angle of twist, denoted with $\phi$

## shear




The shear strain at points on the cross section increases linearly with $\rho$, i.e., $\gamma = (\rho/c)\gamma_{max}$.



## torsion formula

- For a linearly elastic material, Hooke's Law in shear will hold ($\tau = G\gamma$)
- This means that, like shear strain, shear stress will vary linearly along the radius

## torsion formula

- We can find the total force on an element, $dA$ by multiplying the shear stress by the area

$$dF = \tau dA$$

- The torque ($dT = \rho dF$) produced by this force is then

$$dT = \rho(\tau dA)$$



## torsion formula

- Integrating over the whole cross-section gives

$$T = \int_A \rho(\tau dA) = \frac{\tau_{max}}{c} \int_A \rho^2 dA$$

- The integral $\int_A \rho^2 dA$ is also called the Polar Moment of Inertia, $J$, so we can write

$$\tau_{max} = \frac{Tc}{J}$$

## polar moment of inertia

- We know that $J = \int_A \rho^2 dA$, so we can compute this for some common shapes
- For a solid circular cross-section, we have

$$J = \int_0^c \rho^2 (2\pi\rho d\rho) = \frac{\pi}{2} c^4$$

- For a circular tube we have

$$J = \int_{c_1}^{c_2} \rho^2 (2\pi\rho d\rho) = \frac{\pi}{2}(c_2^4 - c_1^4)$$



## example 5.1

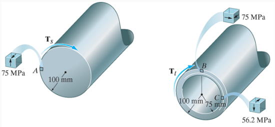



**Figure 5:** On left is a solid 100 mm radius tube, while on the right is a hollow tube with outer radius of 100 mm and inner radius of 75 mm. Element A is on the surface of the solid tube on the left, element B is on the outer surface of the hollow tube on the right and Element C is on the inner surface of the hollow tube

The allowable shear stress is 75 MPa. Determine the maximum torque that can be applied to each of the cross-sections shown and find the stress acting on a small element at A, B and C.