

## Lecture 3 - Strain

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## schedule

- 23 Aug - Strain, Homework 1 Due
- 25 Aug - Mechanical Properties
- 30 Aug - Exam Review, Homework 2 Due, Homework 1 Self-grade Due
- 1 Sep - Exam 1
- 4 Sep - Project 1 Due

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- allowable stress design
- limit state design
- strain

## allowable stress design

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## allowable stress

- Most of the time, we design structures so the stress is less than some limit
- By setting a conservative allowable stress, we account for some manufacturing tolerances, unintended loads, and variability in mechanical properties

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## factor of safety

- The factor of safety is the failure load divided by the allowable load

$$FS = \frac{F_{fail}}{F_{allow}}$$

- Since load and stress are linearly proportional, we could also define the factor of safety in terms of stress and it would be identical

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## factor of safety

- Typical values for the factor of safety will vary based on application
- Aircraft and space vehicles might have a factor close to 1 to minimize weight
- Nuclear power plants might have a factor close to 3 since weight is not as important and failure would be catastrophic

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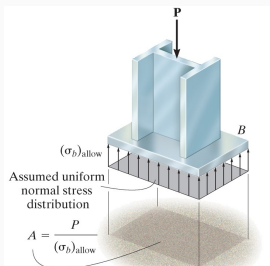
## simple connections

- We can rearrange the equations  $\sigma = N/A$  and  $\tau = V/A$  to size components based on some allowable stress

$$A = \frac{N}{\sigma_{allow}}$$
$$A = \frac{V}{\tau_{allow}}$$

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## bearing stress



The area of the column base plate  $B$  is determined from the allowable bearing stress for the concrete.

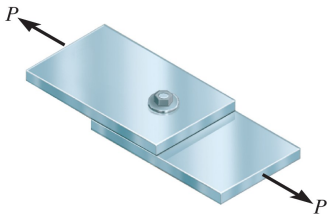
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## embedded shear stress



The embedded length  $l$  of this rod in concrete can be determined using the allowable shear stress of the bonding glue.

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The area of the bolt for this lap joint is determined from the shear stress, which is largest between the plates.

## limit state design

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## limit state design

- Allowable stress design accounts for uncertainty in the applied loading and the material properties in one factor of safety
- Limit state design separates these two into load and resistance factors

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## load factors

- The load factor combines uncertainty in various types of load
- For example, a building can have loading from a few different sources, such as its own weight, people in the building, and snow on top of the building
- Weight is considered a *dead load* and can usually be determined more precisely than moving things like people

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## load factors

- In this simple example, we consider a load factor,  $\gamma_D = 1.2$  for the dead load,  $\gamma_L = 1.6$  and  $\gamma_S = 0.5$

$$R = 1.2D + 1.6L + 0.5S$$

- These load factors combine the concept of a safety factor with the probability that loads will occur

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## resistance factors

- Resistance factors,  $\phi$  are used to express the probability a material will fail at its limit load
- If we are very confident in the failure stress of a material (i.e. steel has little variability), we might use  $\phi = 0.9$
- If we are not as confident, (using a new material, or an organic material like wood with higher variability), we might use  $\phi = 0.7$

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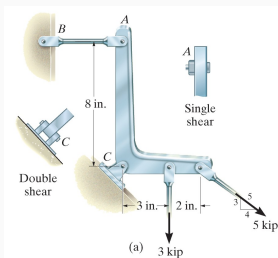


- If we call the nominal load  $P$ , then we can combine load and resistance factors using

$$\phi P > R$$

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## example 1-12

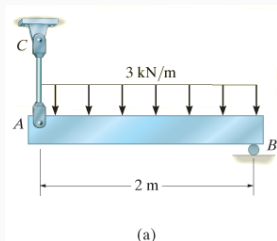


Determine to the nearest  $1/4$ " the diameters of steel pins at A and C if the factor of safety in shear is 1.5 and the failure shear stress is 12 ksi.

**Figure 1:** An I-shaped bracket has an 8 inch vertical leg and a 5 inch horizontal leg. A single shear pinned connector holds the point of the leg, A, in place while a

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## example 1-15



The 400 kg uniform bar,  $AB$  is supported by a steel rod  $AC$  and a roller at  $B$ . If it supports a live distributed loading, determine the required diameter of the rod. Use

$$\sigma_{fail} = 345 \text{ MPa with } \phi = 0.9, \\ \gamma_D = 1.2, \text{ and } \gamma_L = 1.6$$

**Figure 2:** A 2 meter long beam is supported at the left end with a steel rod connecting vertically. It is subjected to a uniform load of 3 kilonewtons per meter, and a roller support at the right end.

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## strain

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## deformation

- When forces are applied to a body, it will change its shape and size
- We call these changes *deformation*
- Sometimes they are barely noticeable (steel), other times they are very significant (rubber)

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## strain

- Strain is a more precise measurement of the deformation of a body
- Normal strain is given as the change in length divided by the original length

$$\epsilon = \frac{L - L_0}{L_0}$$

- We can consider the average normal strain (over an entire body) or the local strain (take an infinitely small portion and calculate the strain there)

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- Since we divide length by length, strain is unitless
- However it is customary to use *in/in* or for stiff specimens to use the phrase *microstrain* as a unit
- Strain can also sometimes be represented as a percent

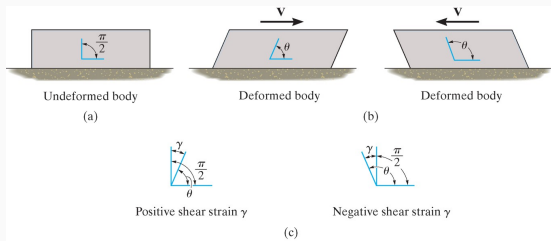
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## shear strain

- Normal strain causes a line segment to expand or contract
- Deformation can also cause two lines to change their relative angle
- The change in angle between two originally perpendicular line segments is called shear strain

$$\gamma = \frac{\pi}{2} - \theta$$

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**Figure 3:** Three stages are shown, the first is a rectangular block at rest, with a fixed support on the ground. The second shows the block after it deforms with a horizontal force acting along the top to the right. The third shows the block after it deforms with a force acting along the top to the left. The first case causes a decrease in

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## cartesian components

- If we consider a very small cube/prism with sides of  $\Delta x$ ,  $\Delta y$ , and  $\Delta z$ , normal strains will change the side lengths to

$$(1 + \epsilon_x)\Delta x(1 + \epsilon_y)\Delta y(1 + \epsilon_z)\Delta z$$

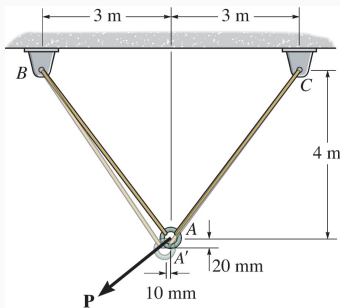
- And the shear strains will change the shape

$$\frac{\pi}{2} - \gamma_{xy} \quad \frac{\pi}{2} - \gamma_{yz} \quad \frac{\pi}{2} - \gamma_{xz}$$

- Most engineering analysis is based on the assumption of small strains
- This is valid for many materials (wood, metal), but not for rubbers and some other polymers
- When strains are small, we assume that the change in angle,  $\Delta\theta$  is very small
- $\sin \Delta\theta \approx \Delta\theta$ ,  $\cos \Delta\theta \approx 1$ ,  $\tan \Delta\theta \approx \Delta\theta$

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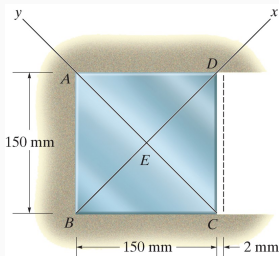
### example 2.1



Find the normal strains in the two wires if A moves to A'

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## example 2.3



The plate is fixed along AB and held in horizontal guides along AD and BC. If the right side is displaced 2 mm find the average normal strain along AC and the shear strain at E relative to the x and y axes.

**Figure 4:** A 150 mm square block is constrained along the top, left, and bottom faces, but pushed 2 mm to the left on its right face. AC is the diagonal line going from the top left to the bottom right. E is the center point of the block (where the two diagonals intersect after deformation).