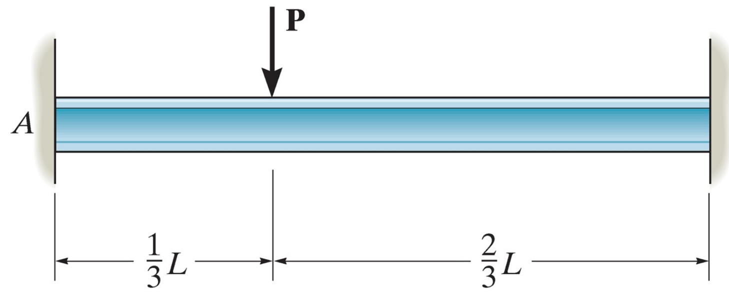
Name:

Homework 10 Solutions

Due 17 November 2020

1. Find the reactions at each support, assume EI is constant and neglect axial load.



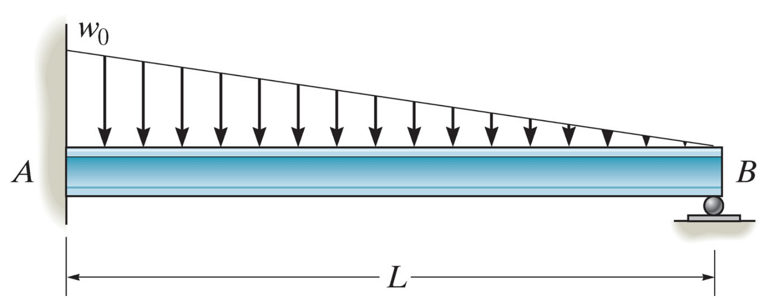
Solution:

- With a beam fixed at both ends we can find from statics that $A + B = P$ and from the sum of moments about A we find $M_B + BL = PL/3$.
- We have boundary conditions of $v(0) = v(L) = dv/dx(0) = dv/dx(L) = 0$.
- Because the internal moment equation for this problem is discontinuous (we would have one moment function before the load P and another after it) I will instead use discontinuity functions.
- $M(x) = Ax + M_A - P\langle x - L/3\rangle$
- Integrating twice gives

$$EI\frac{dv}{dx} = \frac{1}{2}Ax^2 + M_A x - \frac{P}{2}\langle x - L/3\rangle^2 + C_1$$
$$EIv = \frac{1}{6}Ax^3 + \frac{1}{2}M_A x^2 - \frac{P}{6}\langle x - L/3\rangle^3 + C_1 x + C_2$$

- Applying the boundary conditions at $x = 0$ gives $C_1 = C_2 = 0$.
- Applying the boundary conditions at $x = L$ gives $M_A = -\frac{4}{27}PL$ and $A = \frac{20}{27}P$
- Substituting into our original statics expressions gives $B = \frac{7}{27}P$ and $M_B = \frac{2PL}{27}$

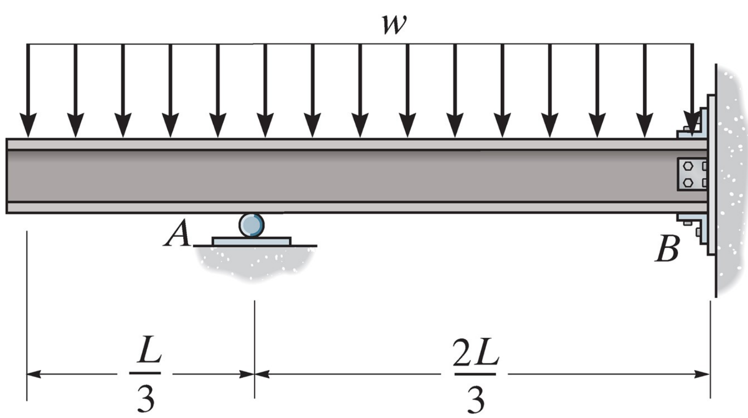
2. Find the maximum deflection in terms of some constant EI



Solution:

- For this problem it is convenient to use the superposition method
- We can add a cantilever beam with a distributed load as shown in the problem to a cantilever with an end load
- We need to choose an end-load such that the end deflections cancel, using the maximum deflections given in appendix C we find $\frac{w_0 L^4}{30EI} = \frac{PL^3}{3EI}$
- And we find $P = \frac{w_0 L}{10}$
- To find the maximum deflection we need to add the two expressions for deflection and find when their derivative is equal to zero. Since these are higher order polynomials, it is best to do this using either a graphing calculator or some other software
- There are 4 roots to the equation, however only one is within our bounds ($x \approx 0.553L$) which gives $v_max = -0.0239\frac{w_0 L^4}{EI}$

3. Find the maximum deflection in terms of some constant EI



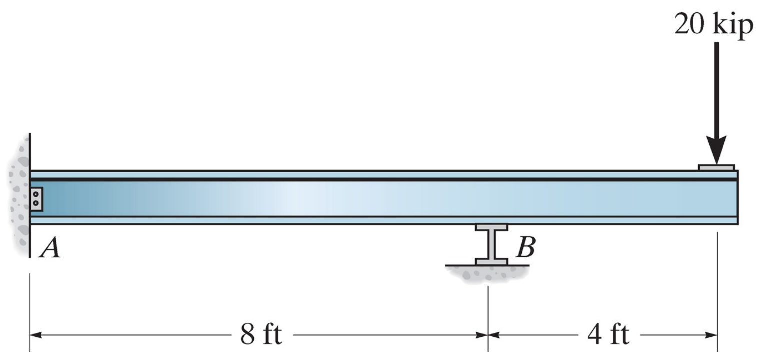
Solution:

- For this problem I will once again use direct integration with discontinuity functions.
- From statics we know that $A + B = wL$ and that $M_B = \frac{2AL}{3} - \frac{wL^2}{2}$
- Setting our origin at the left we have boundary conditions of $v(L/3) = v(L) = dv/dx(L) = 0$. NOTE: This problem would be much easier to work by hand setting the origin on the right hand side and moving left, but either case should give the same answer, if you did that just substitute $x_{new} = L - x_{old}$
- Our moment function is $M = -w/2x^2 + A\langle x - L/3\rangle$
- And integrating twice yields

$$EI\frac{dv}{dx} = -\frac{wx^3}{6} + \frac{A}{2}\langle x - L/3\rangle^2 + C_1$$
$$EIv = -\frac{wx^4}{24} + \frac{A}{6}\langle x - L/3\rangle^3 + C_1 x + C_2$$

- We find that $A = \frac{17wL}{24}$, $B = \frac{7wL}{24}$, $M_B = -\frac{wL^2}{36}$, $C_1 = \frac{wL^3}{108}$ and $C_2 = -\frac{5wL^4}{1944}$
- Using the same method as before, solving for $dv/dx = 0$ we find possible locations for the maximum deflection at $x = 0L, 0.405L, 0.720L$ so we compare the deflection at each location and find the maximum is at $x = 0$ and is $v(0) = \frac{-5wL^4}{1944EI}$

4. Find the reactions at each of the supports, assuming A is a fixed support and B is a roller.



Solution:

- Although this problem is simplest using discontinuity functions, I will demonstrate using direct integration without discontinuity functions for this solution
- We know from statics that $A + B = 20$ and that $M_A = 8B - 240$
- We find the following moment equations

$$M_1(x) = Ax + M_A$$
$$M_2(x) = Ax + M_A + B(x - 8)$$

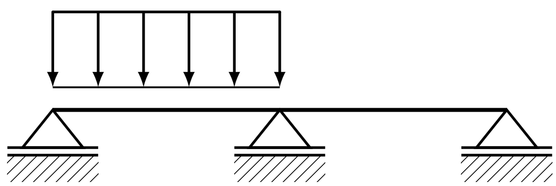
- Integrating twice gives

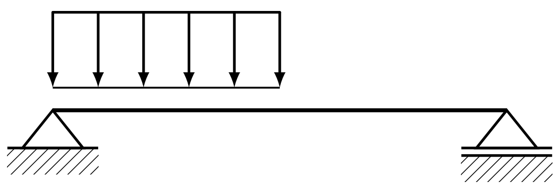
$$EIdv_1/dx = \frac{1}{2}Ax^2 + M_Ax + C_1$$
$$Eiv_1 = \frac{1}{6}Ax^3 + \frac{1}{2}M_Ax^2 + C_1x + C_2$$
$$EIdv_2/dx = \frac{1}{2}Ax^2 + M_Ax + \frac{B}{2}(x-8)^2 + C_3$$
$$EIv_2 = \frac{1}{6}Ax^3 + \frac{1}{2}M_Ax^2 + \frac{B}{6}(x-8)^3 + C_3x + C_4$$

- We have 7 unknowns with 2 statics equations, 3 boundary conditions ($v(0) = dv/dx(0) = v(8) = 0$) and 2 continuity equations ($v_1(8) = v_2(8)$, $dv_1/dx(8) = dv_2/dx(8)$), which in total makes 7 equations, so we should be able to solve for all the unknowns.
- It is quick to find $C_1 = C_2 = 0$ by applying boundary conditions at A.

- For the continuity conditions to be satisfied we can cancel the equivalent terms from each side to find $C_3 = 0$ and $C_4 = 0$
- Finally we apply $v_1(8) = 0$ to find $\frac{1}{6}A8^3 + \frac{1}{2}M_A 8^2 = 0$, or $8A + 3M_A = 0$.
- Combining with the statics equations, we solve to find $A = -15$, $B = 35$, and $M_A = 40$

5. Dr. Smith has decided to add an extra support at the center of his daughter's bed he is building. Compare the maximum deflection with and without the support assuming that the piece in question can be modeled as shown below. Note that the beam is 6 feet long with a support added exactly in the middle and his daughter weighs 30 pounds, neglect any horizontal reaction forces. Assume the beam is a solid rectangle made from pine 1.5 inches thick and 3 inches tall. **Solution:**





- With no redundant support, we have an exact match for this beam in Appendix C, which gives $v_{max} = 0.029\,\text{in}$
- We can use superposition again for the case with redundant reinforcement, but we treat the extra middle support as a point load such that the deflections at the center are equal, thus $\frac{5wL^4}{768EI} = \frac{PL^3}{48EI}$ which gives $P = \frac{5wL}{16} = 18.75\,\text{lb}$
- We will only use equations for $0 < x < L/2$ since that is where we expect the maximum deflection to occur and we find a maximum deflection at $x = 17.01\,\text{in}$ of $v_{max} = 0.0025\,\text{in}$, which reduces our maximum deflection by approximately a factor of 10.