Mechanics of Materials

Lecture 18 - Deflection of Beams

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schedule

- 14 April - Beam Deflection
- 19 April - Beam Deflection (discontinuity functions), HW 7 Self-grade Due, HW 8 Due
- 21 April - Beam Deflection (superposition)
- 26 April - Statically indterminate beams

outline

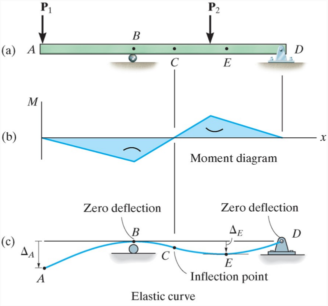
- deflection of beams and shafts
- slope and displacement

deflection of beams and shafts

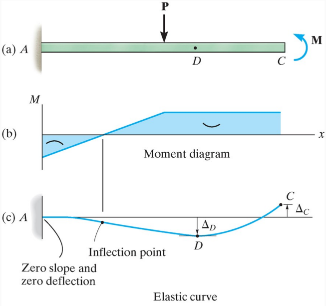
elastic curve

- Before finding the exact displacement, it is useful to sketch the approximate deformed shape of a beam
- For difficult beams, it is useful to draw the moment curve first
- Positive internal moment tends to bend the beam concave up, while negative moment tends to bend the beam concave down

elastic curve



elastic curve



moment-curvature

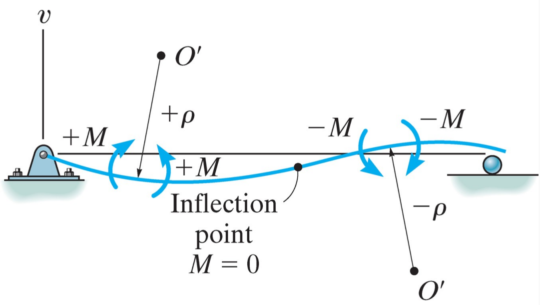
- In Chapter 6 we compared the strain in a segement of a beam to the radius of curvature and found

$$\frac{1}{\rho} = -\frac{\epsilon}{y}$$

- Since Hooke's Law applies, $\epsilon = \sigma/E = -My/EI$, substituting gives

$$\frac{1}{\rho} = \frac{M}{EI}$$

sign convention



ρ is positive when the center of the arc is above the beam, negative when it is below.

slope and displacement

curvature

- When talking about displacement in beams, we use the coordinates v and x, where v is the vertical displacement and x is the horizontal position
- In this notation, curvature is formally related to displacement according to

$$\frac{1}{\rho} = \frac{d^2v/dx^2}{[1 + (dv/dx)^2]^{3/2}} = \frac{M}{EI}$$

curvature

- The previous equation is difficult to solve in general, but for cases of small displacement, $\left(\frac{dv}{dx}\right)^2$ will be negligible compared to 1, which then simplifies to

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

flexural rigidity

- In general, M, is a function of x, but EI (known as the flexural rigidity) is a constant along the length of the beam
- In this case, we can say

$$EI\frac{d^2v}{dx^2} = M(x)$$
$$EI\frac{d^3v}{dx^3} = V(x)$$
$$EI\frac{d^4v}{dx^4} = w(x)$$

boundary conditions

- If a support restricts displacement, but not rotation, we will have a boundary condition of $v = 0$ at that point
- Supports that restrict rotation give a boundary condition that $\theta = 0$

continuity conditions

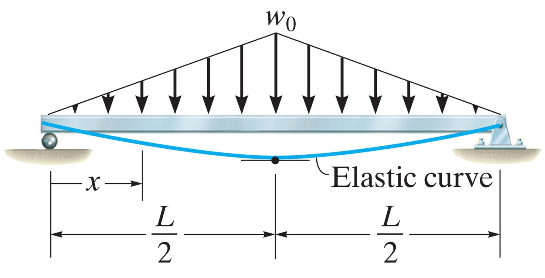
- If we have a piecewise function for $M(x)$, not all integration constants can be found from the boundary conditions
- Instead, we must also use continuity conditions to ensure that the slope and displacement are continuous at every point
- In other words, for two sets of functions, $\theta_1(x)$ and $v_1(x)$, $\theta_2(x)$ and $v_2(x)$, $\theta_1(a) = \theta_2(a)$ and $v_1(a) = v_2(a)$

slope

- For small displacements, we have

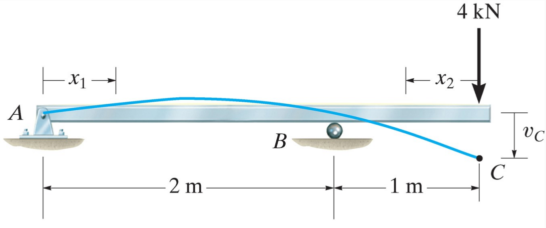
$$\theta \approx \tan(\theta) = dv/dx$$

example 12.1



Determine the maximum deflection if EI is constant.

example 12.4



Determine the displacement at C, EI is constant.