AE333
Mechanics of Materials

Lecture 22 - Mohr's Circle

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schedule

- 10 Apr - Mohr's Circle
- 13 Apr - Stress Transformation, HW7 Due
- 15 Apr - Strain Transformation
- 17 Apr - Beam Deflection

outline

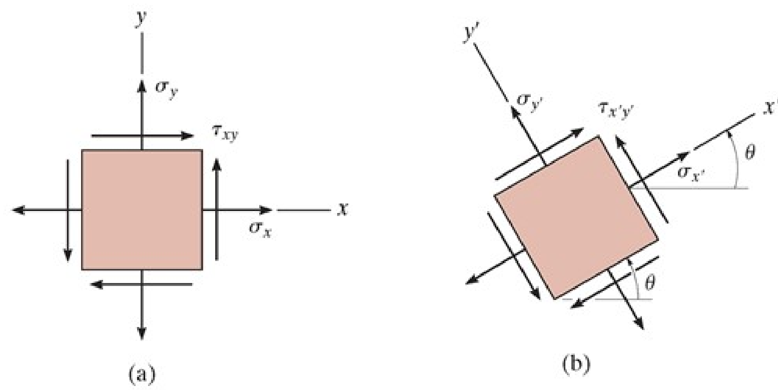
- plane stress transformation
- general equations
- principal stresses

plane stress transformation

plane stress

- In general, the state of stress at a point is characterized by six stress components
- In practice, this is rare, as most stresses and forces act in the same plane
- This case is referred to as plane stress

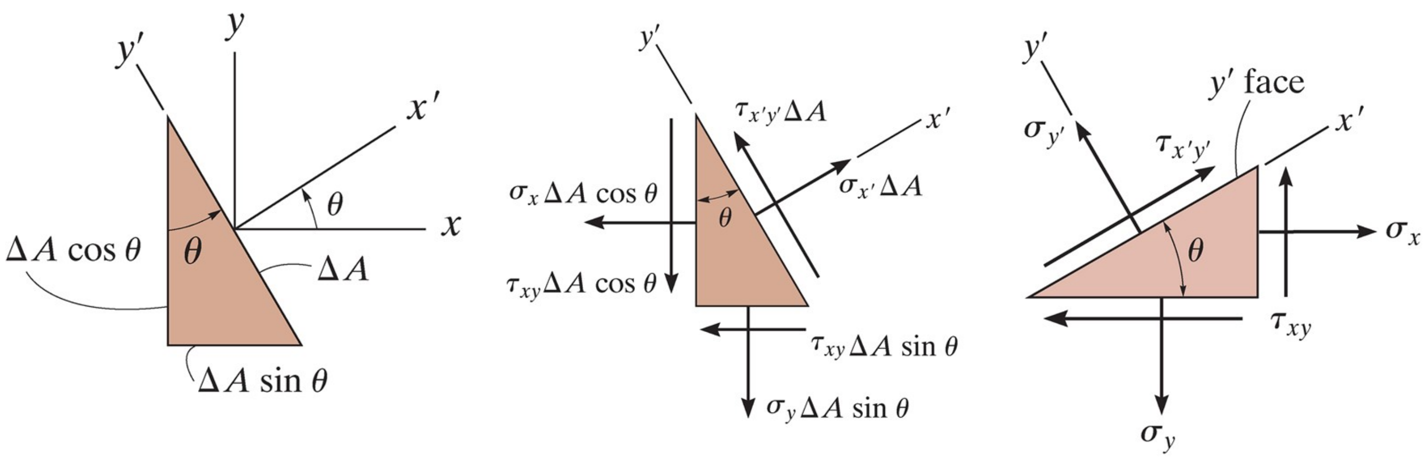
transformation



procedure

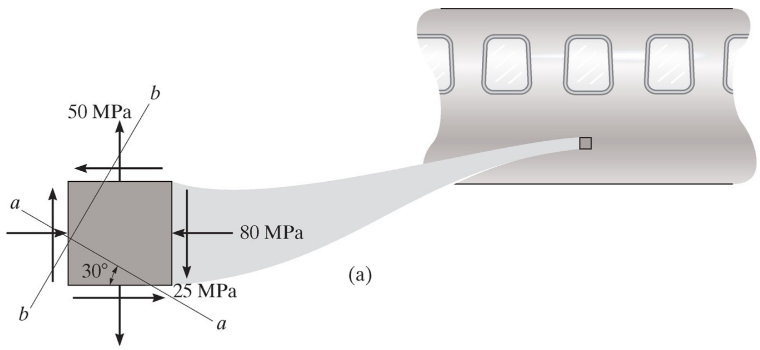
- If the state of stress ($\sigma_x, \sigma_y, \tau_{xy}$) is known for a known axis system x and y, we can find the stress relative to some rotated coordinate system
- We do this by considering a section of the element perpendicular to the $x^{'}$ axis
- Sum of forces in x and y will give $\sigma_{x'}$ and $\tau_{x'y'}$
- A second section is needed to find $\sigma_{y'}$, perpendicular to the $y^{'}$ axis

procedure



y
y′
x′
x
θ
ΔA cos θ
θ
ΔA
ΔA sin θ
y′
$\tau_{x'y'}\Delta A$
x′
$\sigma_x \Delta A \cos\theta$
θ
$\sigma_{x'}\Delta A$
$\tau_{xy}\Delta A \cos\theta$
$\tau_{xy}\Delta A \sin\theta$
$\sigma_y \Delta A \sin\theta$
y′
y′ face
x′
$\sigma_{y'}$
$\tau_{x'y'}$
θ
σ_x
τ_{xy}
σ_y

example 9.1



Represent the state of stress shown on the fuselage section on an element rotated 30° clockwise from the position shown.

general equations

general equations

- We can follow the methodology from the previous section to develop equations for some arbitrary rotation and a completely general state of stress
- We use some trig identities to simplify the formulae

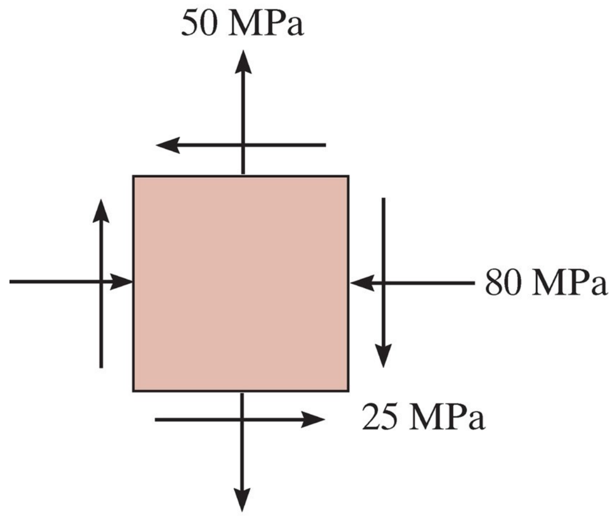
$$\begin{aligned}
\sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\
\tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta
\end{aligned}$$

- To find $\sigma_{y'}$ we can simply add 90° to θ

procedure

- The procedure in general is mostly “plug and chug”
- The only thing we need to be cautious about is sign convention: stresses are positive in tension, shear is positive with arrows pointing to the 1st and 3rd quadrants, θ is measured counter-clockwise from the x-axis

example 9.2



Determine the stress at this point on an element rotated 30° clockwise from the position shown.

principal stresses

principal stresses

- Since the local stresses only change with the rotation angle, we might like to find the angle with gives the maximum stress
- This is known as the principal direction, and the stresses are known as principal stresses
- We can find this direction by differentiating the equation for $\sigma_{x'}$

principal stress

- We find the angle as

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

- The principal stresses are then

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

maximum shear stress

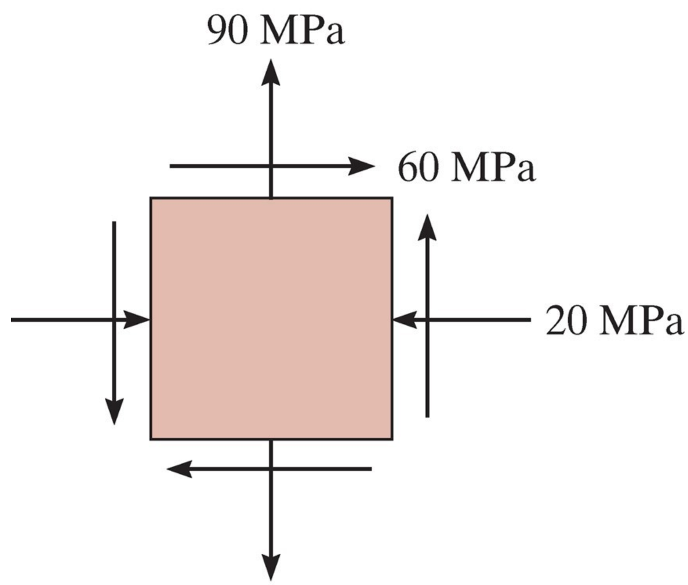
- Similarly, we might want to find the direction of maximum shear stress

$$\tan 2\theta_s = \frac{\sigma_y - \sigma_x}{2\tau_{xy}}$$

- And the maximum shear stress is

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

example 9.3



Find the principal stress for the stress state shown.

example 9.5

- When torsional loading T is applied to a circular bar it produces a state of pure shear stress.
- Find the maximum in-plane shear stress and the associated average normal stress
- Find the principal stresses