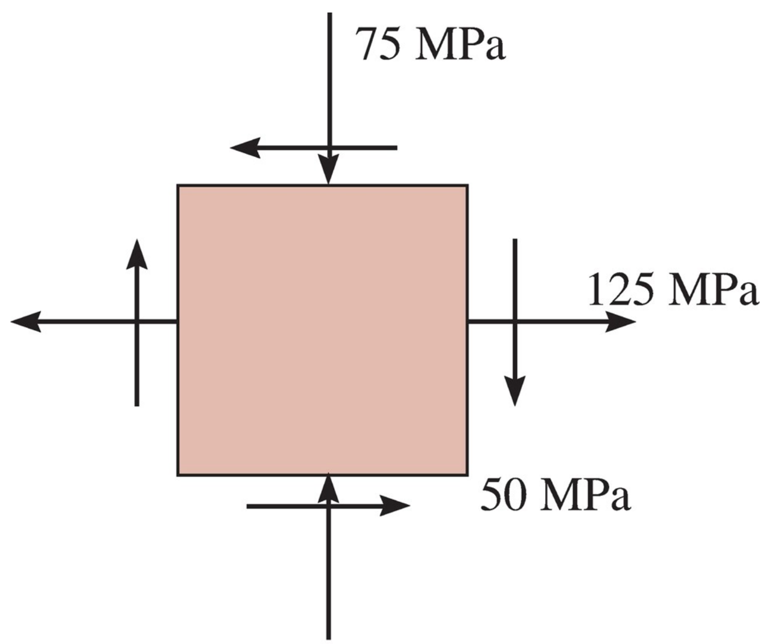
Name:

Homework 8 Solutions

Due 3 November 2020

1. Find the stress state on an axis rotated 30° counter-clockwise from the x and y axis.



Solution:

- There are three approaches that could be used to solve this problem, the first we discussed would be to section it and set up triangles, the second would be to use the general formulas and the third would be to use Mohr's circle. The most straight-forward to show in this solution is the formula, but you are free to use any method.
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$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
- This gives $\sigma_{x'} = 31.7\,\text{MPa}$, $\sigma_{y'} = 18.3\,\text{MPa}$ and $\tau_{x'y'} = -111.6\,\text{MPa}$

2. For the stress state in Problem 1, find the principal stresses and maximum (in-plane) shear stress **Solution:**

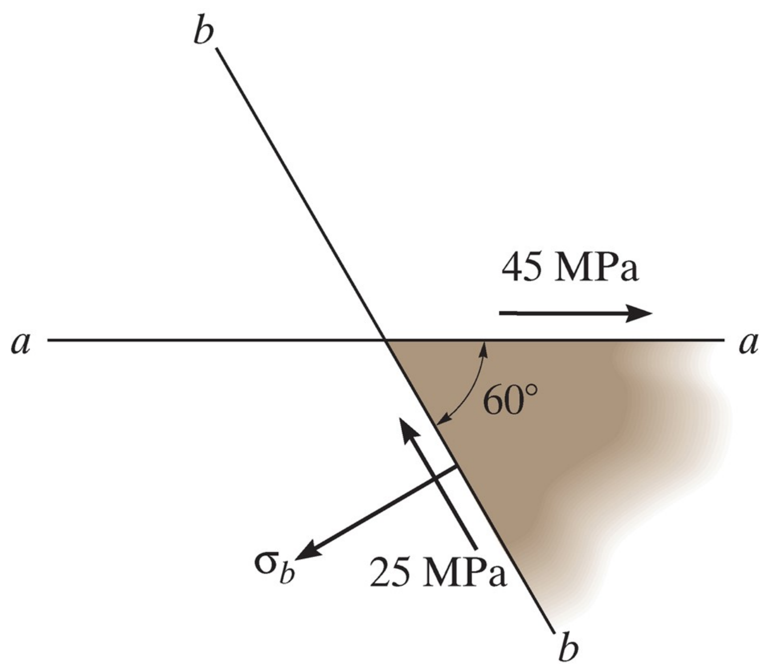
- Once again, we have a few different solutions, but we will continue using the formula

- $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
- This gives $\sigma_1 = 136.8\,\text{MPa}$ and $\sigma_2 = -86.8\,\text{MPa}$
- The maximum in plane shear stress is given by

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- Which gives $\tau_{max} = 111.8\,\text{MPa}$

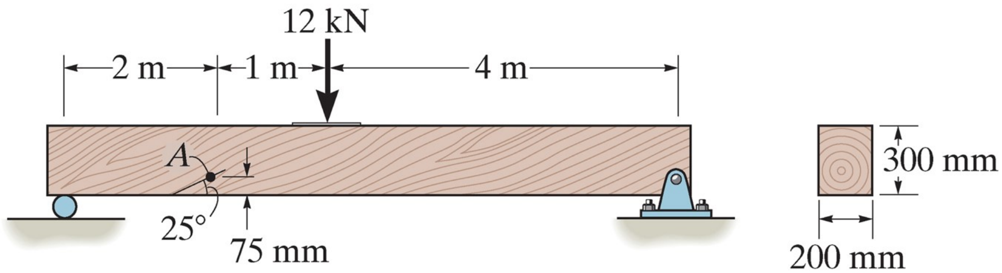
3. The stress state along two planes is known as shown. Find the normal stresses on plane $b-b$ and the principal stresses.



Solution:

- We start by interpreting the problem a little bit, although they are not perpendicular, we can notice that the shear stresses shown for the τ_{xy} and τ_{ab} are acting in different directions, so whatever our convention, one of them should be negative while the other is positive.
- Next we use the formula for $\tau_{x'y'}$ which we can use to find $-\sigma_x + \sigma_y = 5.77\,\text{MPa}$
- From the drawing we see that $\sigma_y = 0$, therefore we know $\sigma_x = -5.77\,\text{MPa}$, which we can now use to find $\sigma_b = -37.5\,\text{MPa}$ and $\sigma_a = 10.8\,\text{MPa}$
- We can also find the principal stresses as $\sigma_1 = 21.4\,\text{MPa}$ and $\sigma_2 = -48.1\,\text{MPa}$

4. The wood beam is subjected to a load of 12 kN. If grains of wood in the beam at point A make an angle of 25° with the horizontal as shown find the normal and shear stress that act perpendicular to the grains.

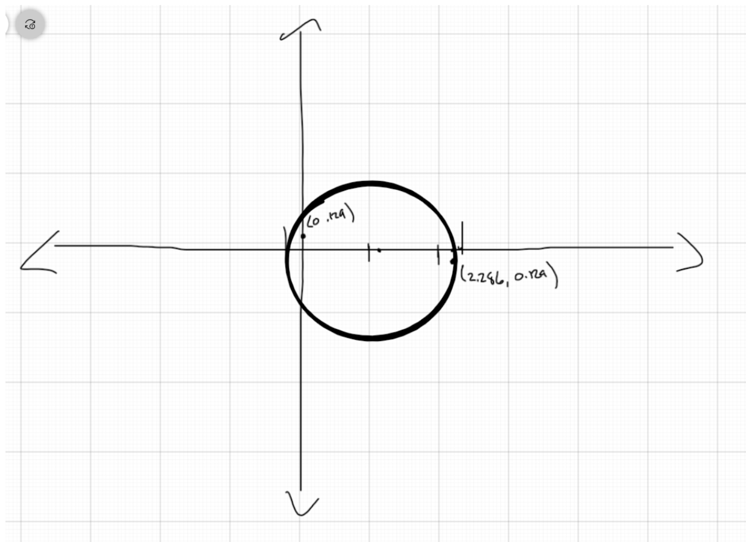


Solution:

- We first need to use the flexure and transverse shear formulas to find the stress state at point A
- Starting with statics, we find the left reaction force to be $F_1 = 6.86\,\mathrm{kN}$ and the right reaction force $F_2 = 5.14\,\mathrm{kN}$
- This gives $V = 6.86\,\mathrm{kN}$ and $M = 13.71\,\mathrm{kN} \cdot \mathrm{m}$ at A
- For the cross section we find $I = 450 \times 10^6\,\mathrm{mm}^4$
- At A we have $y = 75\,\mathrm{mm}$
- To calculate Q we find $\bar{A} = 15 \times 10^3\,\mathrm{mm}^2$ and $\bar{y} = 112.5\,\mathrm{mm}$ which gives $Q = 1.688\,\mathrm{mm}^3$
- This gives a stress state of $\sigma_x = 2.286\,\mathrm{MPa}$ and $\tau_{xy} = 128.6\,\mathrm{Pa}$
- To find the normal and shear stresses in the direction of the wood fibers, we use the stress transformation equations with $\theta = 25°$
- We find $\sigma_{x'} = 1.98\,\mathrm{MPa}$ (in the fiber direction) and $\sigma_{y'} = 0.31\,\mathrm{MPa}$ (perpendicular to the fiber direction) and $\tau_{x'y'} = -0.79\,\mathrm{MPa}$

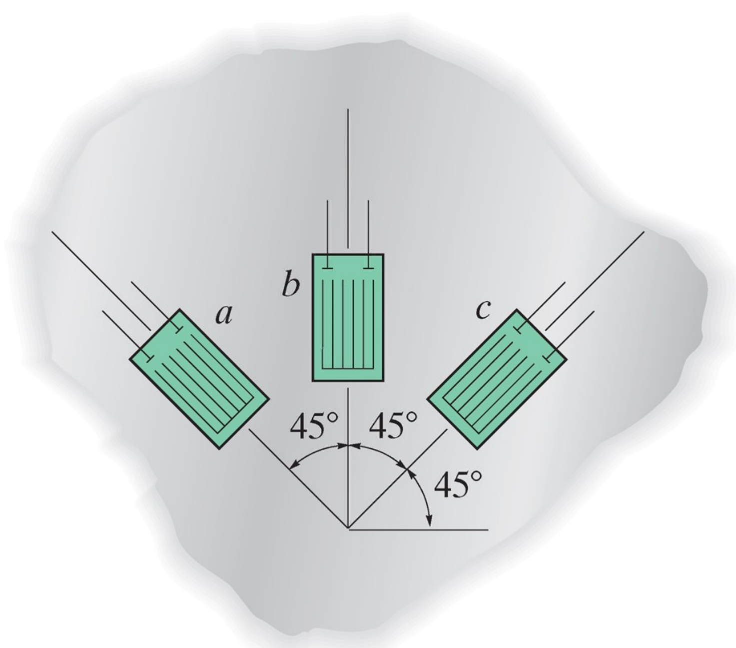
5. Draw Mohr's circle for the stress state in problem 4 and use it to estimate the principal stresses. **Solution:**

- Here we will ignore the work done previously and draw mohr's circle directly for the initial stress state, $\sigma_x = 2.286\,\mathrm{MPa}$, $\sigma_y = 0\,\mathrm{MPa}$ and $\tau_{xy} = 0.129\,\mathrm{MPa}$



- Although my digital drawing is not ideal, it is clear to see that the principal stresses will be fairly close to the current stress state, with perhaps $\sigma_1 = 2.4\,\text{MPa}$ and $\sigma_2 = -0.1\,\text{MPa}$

6. The 45° strain rosette is mounted on the surface of a shell with the following readings: $\epsilon_a = -200\mu\epsilon$, $\epsilon_b = 300\mu\epsilon$ and $\epsilon_c = 250\mu\epsilon$. Find the in-plane principal strains.



Solution:

- From the figure, we can see that $\epsilon_b = \epsilon_y$, the other strain measurements we will need to find using the strain transformation equations.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

- If we set up the equations for the two angles we have left, $\theta_a = 45°$ and $\theta_c = 135°$

$$\epsilon_a = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_a + \frac{\gamma_{xy}}{2}\sin 2\theta_a$$

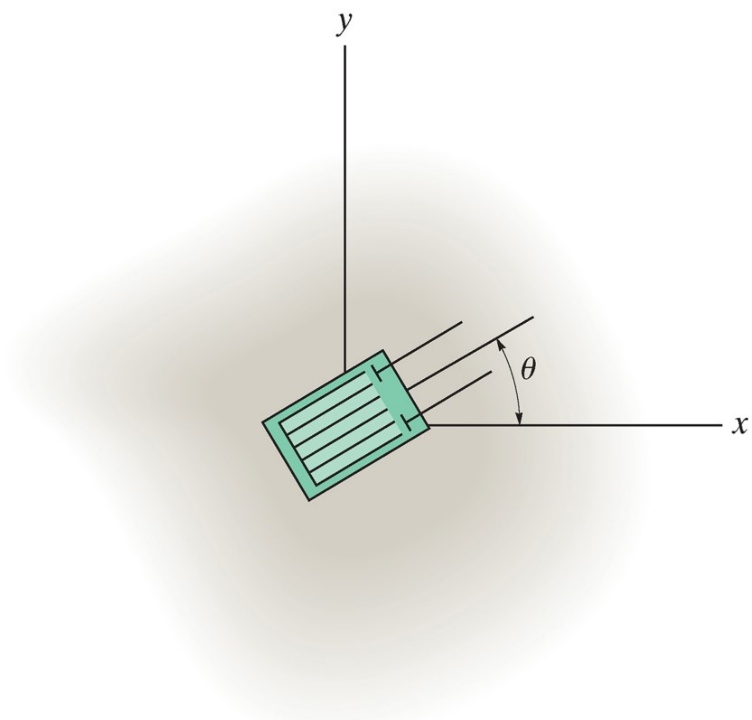
$$\epsilon_c = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_c + \frac{\gamma_{xy}}{2}\sin 2\theta_c$$

- Note that $\sin(2\theta_a) = 1$, $\sin(2\theta_c) = -1$, $\cos(2\theta_a) = 0$, and $\cos(2\theta_c) = 0$

$$-200\mu\epsilon = \frac{\epsilon_x + 300\mu\epsilon}{2} + \frac{\gamma_{xy}}{2}$$
$$250\mu\epsilon = \frac{\epsilon_x + 300\mu\epsilon}{2} - \frac{\gamma_{xy}}{2}$$

- From which we find that $\epsilon_x = -250\mu\epsilon$ and $\gamma_{xy} = -225\mu\epsilon$

7. A material is subjected to principal stresses σ_x and σ_y. Find the orientation, θ, of the strain gage so that its reading of normal strain corresponds only to σ_y, not σ_x. The relevant material constants for this problem are E and ν (express answer in terms of these)



Solution:

- Notice that the problem states we have applied principal stresses, this means that in this plane there are no shear stresses applied
- The reason we cannot simply align the strain gage in the y-direction is that this would include some Poisson's contraction from stress in the x-direction.

- We start by writing the equation for $\sigma_{x'}$ and $\sigma_{y'}$ in the direction of our strain gage. For no shear stress it is more convenient to substitute $\cos 2\theta = 2\cos^2\theta - 1$ which gives

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$$
$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta$$

- We can now write an expression for $\epsilon_{x'}$ using Hooke's Law (and neglecting out-of-plane terms)

$$\epsilon_{x'} = \frac{1}{E}(\sigma_{x'} - \nu\sigma_{y'})$$

- Substituting the previous equations for $\sigma_{x'}$ and $\sigma_{y'}$ gives

$$\epsilon_{x'} = \frac{1}{E}(\sigma_x \cos^2\theta + \sigma_y \sin^2\theta - \nu[\sigma_x \sin^2\theta + \sigma_y \cos^2\theta)]$$

- To eliminate σ_x from this equation we need $\sigma_x \cos^2\theta - \nu\sigma_x \sin^2\theta = 0$
- Factoring σ_x we have

$$\sigma_x(\cos^2\theta - \nu\sin^2\theta = 0)$$

- To solve for any σ_x we set the portion in parentheses equal to 0 and solve for θ, which gives $\theta = \tan^{-1}\left(\frac{1}{\sqrt{\nu}}\right)$