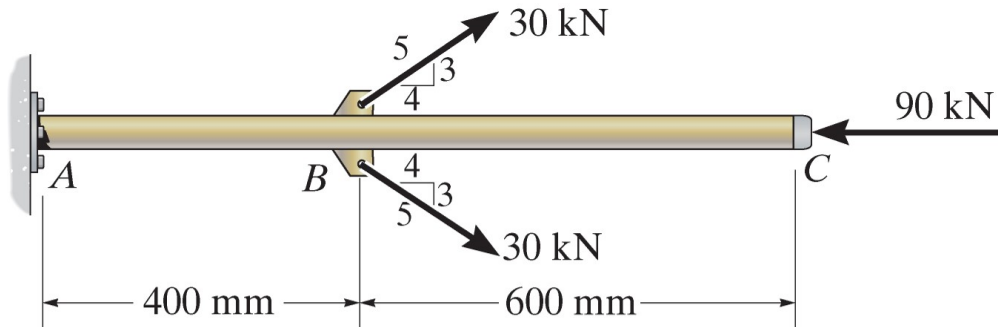Name:

Homework 3 Solutions

1. The 30 mm diameter steel rod is subjected to the loading shown. Find the displacement at C.
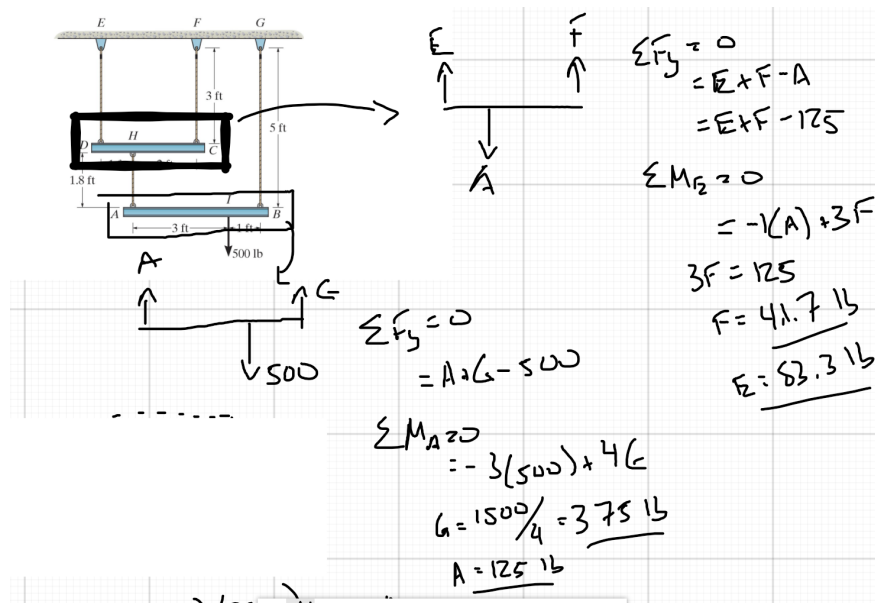

Solution: To find the displacement at C we need to find the total displacement of the rod. To do that we will need to add the effects of the loading at B with the loading at C.

- We can start by finding the displacement between AB, $\delta_{AB} = \frac{(2)(30\,\text{kN})(4/5)(400\,\text{mm})}{(\pi(15\,\text{mm})^2)(200\,\text{GPa})} = 0.136\,\text{mm}$

- And we add to that the displacement from the compressive force at C, $\delta_{AC} = \frac{(-90\,\text{kN})(1000\,\text{mm})}{(\pi(15\,\text{mm})^2)(200\,\text{GPa})} = -0.637\,\text{mm}$

- And the total displacement at C is $\delta_C = \delta_{AB} + \delta_{AC} = -0.501\,\text{mm}$

2. The load is supported by four stainless steel wires that are connected to rigid members AB and DC. Determine the vertical displacement of the 500 lb load if each wire has a cross-sectional area of $0.035\,\text{in}^2$.
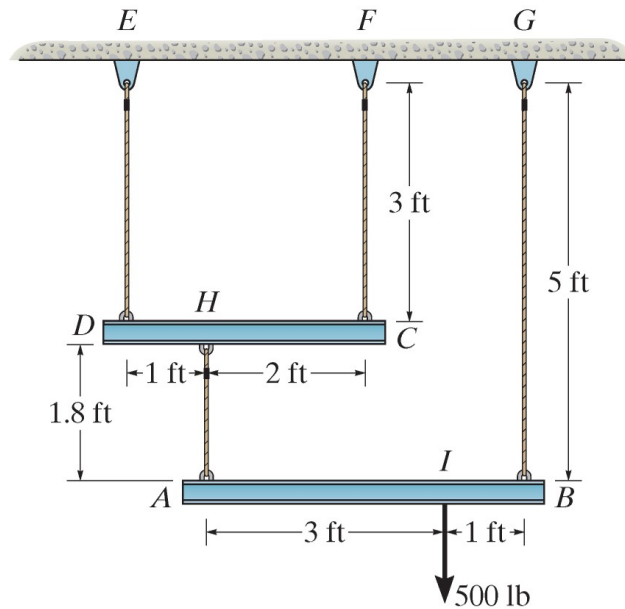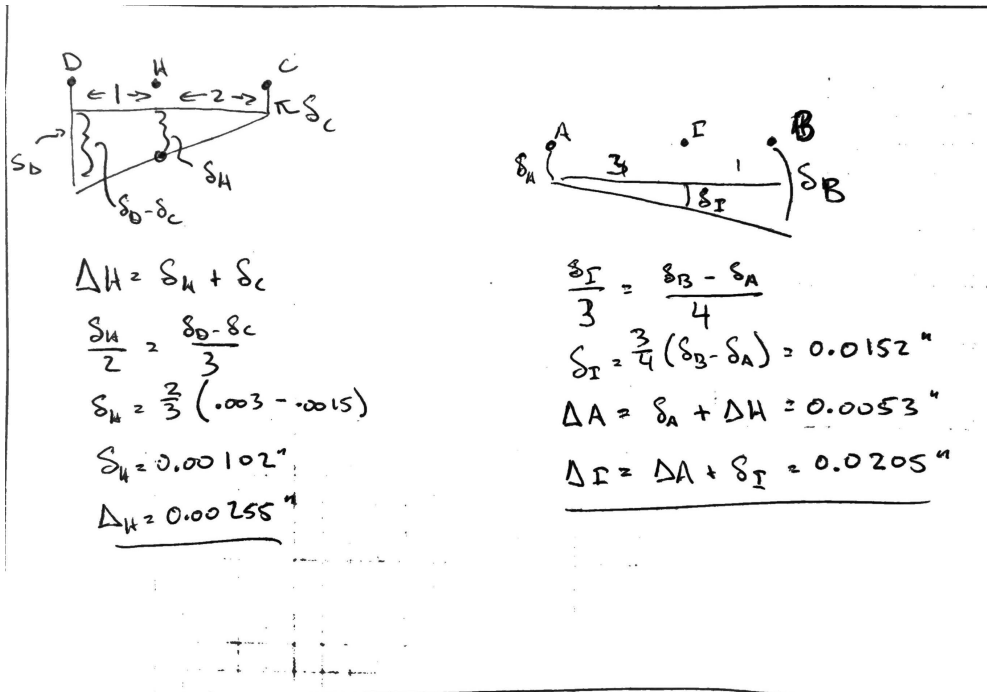
Solution:

- First we need to do some statics to find the force in each wire. We find (see image for FBD) that $A = 125\,\text{lb}$, $G = 375\,\text{lb}$, $E = 83.3\,\text{lb}$, and $F = 41.7\,\text{lb}$

- Next we need to find the amount each wire stretches using the deflection formula ($\delta = NL/AE$). We find $\delta_d = 0.003\,06\,\text{in}$, $\delta_C = 0.001\,53\,\text{in}$, $\delta_a = 0.002\,76\,\text{in}$, and $\delta_b = 0.0230\,\text{in}$

- We need to use similar triangles to find the displacement at H and add that to the deflection at A, then use similar triangles again to find the displacement of

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the force at I (see figure), note that I use δ to mean deflection due to wire stretch and a capital Δ to signify the actual displacement of a point (at A we need both wire deflection and movement of the point H to get the total displacement).

$$\Delta H = \delta_H + \delta_C$$

$$\frac{\delta_H}{2} = \frac{\delta_D - \delta_C}{3}$$

$$\delta_H = \frac{2}{3}(.003 - .0015)$$

$$\delta_H = 0.00102"$$

$$\Delta_H = 0.00255"$$

$$\frac{\delta_I}{3} = \frac{\delta_B - \delta_A}{4}$$

$$\delta_I = \frac{3}{4}(\delta_B - \delta_A) = 0.0152"$$

$$\Delta A = \delta_A + \Delta H = 0.0053"$$

$$\Delta I = \Delta A + \delta_I = 0.0205"$$

3. The column shown is constructed from concrete with A992 steel rebar. If the column is subjected to an axial force of 200 kip, find the required rod diameter such that 60% of the axial force is carried by the concrete.


4 in.

200 kip

3 ft
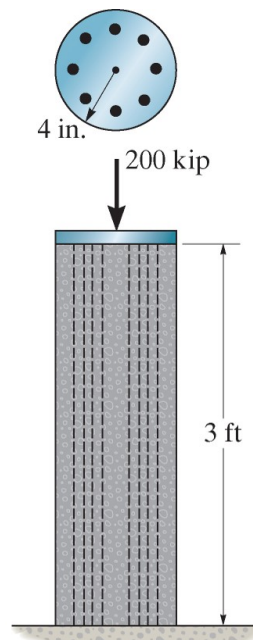
Solution:

- We know that the concrete and the steel rebar need to displace by the same amount, but we don't know how much that is

- We do know, however, how much load should be carried by each member, this means that $\frac{N_{st}L}{A_{st}E_{st}} = \frac{N_c L}{A_c E_c}$

- The length is the same, so that can be canceled, we just need to solve for the area, note that $A_c = \pi 4^2 - A_{st}$, so this leaves us with $\frac{N_{st}E_c}{N_c E_{st}} = \frac{A_{st}}{16\pi - A_{st}}$

- Substituting $N_{st} = 80\,\text{kip}$, $N_c = 120\,\text{kip}$, $E_c = 3.2\,\text{Msi}$ and $E_{st} = 29\,\text{Msi}$ we find $A_{st} = 3.44\,\text{in}^2$ which gives a steel rod diameter of $d = 2\sqrt{\left(\frac{3.44}{8\pi}\right)} = 0.740\,\text{in}$.

4. The post is made from 2024 aluminum and has a diameter of 50 mm. It is fixed at A and B and has a coiled spring attached to a rigid collar at C. If the spring is initially uncompressed, find the compression in the spring when a load of $P = 75\,\text{kN}$ is applied to the collar.
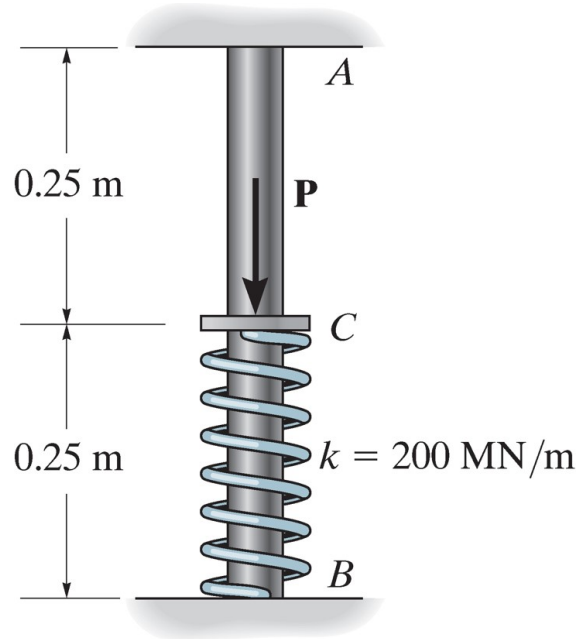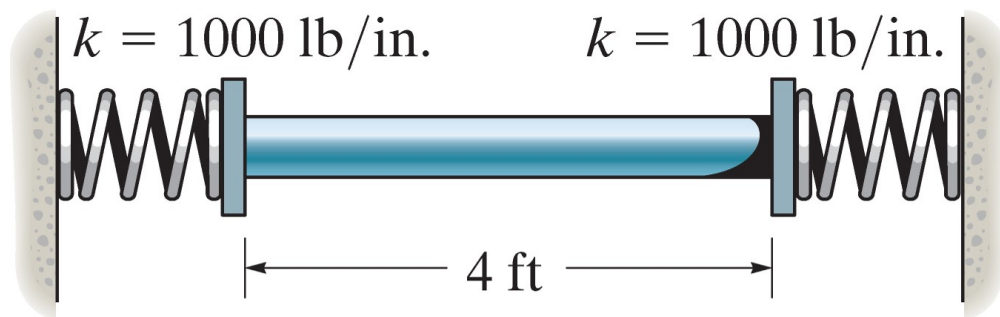

Solution:

- In this solution we will use the method of sections, we will have three unknown internal forces in this problem, the internal force between A and C (F_{AC}), the internal force between C and B (F_{BC}) and the force in the spring, F_s.

- All of these forces need to add to the applied load P, and all will influence deflection at C, which needs to be equal, thus we have $\delta_c(F_{AC}) = \delta_c(F_{BC}) = \delta_c(F_s)$

- Substituting deflection relationships for axial loading and springs gives $\frac{F_{AC}L_{AC}}{AE} = \frac{F_{BC}L_{BC}}{AE} = \frac{F_s}{k}$

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- Since $L_{AC} = L_{BC}$ and AE is constant, we can say that $F_A = F_C$, we can also relate F_s to F_A with $F_s = \frac{kF_{AC}L_{AC}}{AE}$ and finally we substitute into $F_{AC} + F_{BC} + F_s = P$ to find $F_{AC}\left(\frac{kL_{AC}}{AE}\right) + F_{BC} + F_{AC} = P = F_{AC}\left(\frac{kL_{AC}}{AE} + 2\right)$

- And we find that $F_{AC} = 31.7\,\text{kN}$ and $F_s = 11.5\,\text{kN}$ which gives a spring compression of $\delta_c = F_s/k = 0.058\,\text{mm}$

- As a quick sanity check, we can compare the effective spring constant of the aluminum rod (L/AE) to the spring constant for the spring, we find that $k_{al} = 550\,\text{MN/m}$, and it makes sense then that the force in the aluminum is so much higher than the spring for the same deflection

5. The assembly has shown fits snugly with no internal force at $70°F$. Find the average normal stress in each material when the temperature is raised to $200°F$.
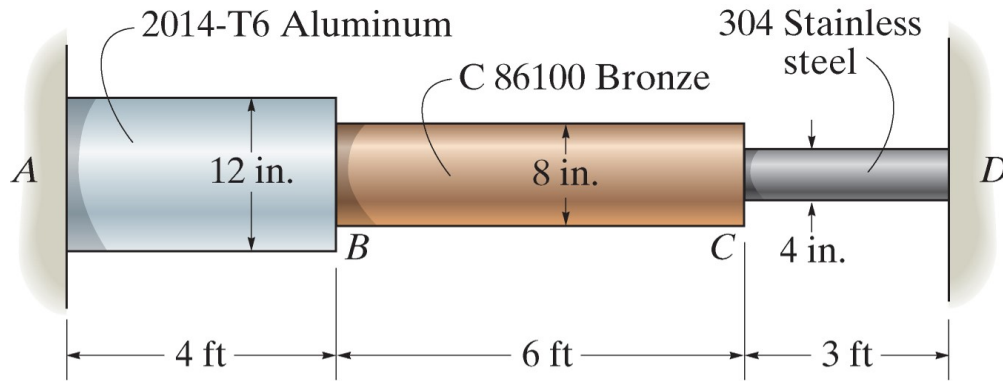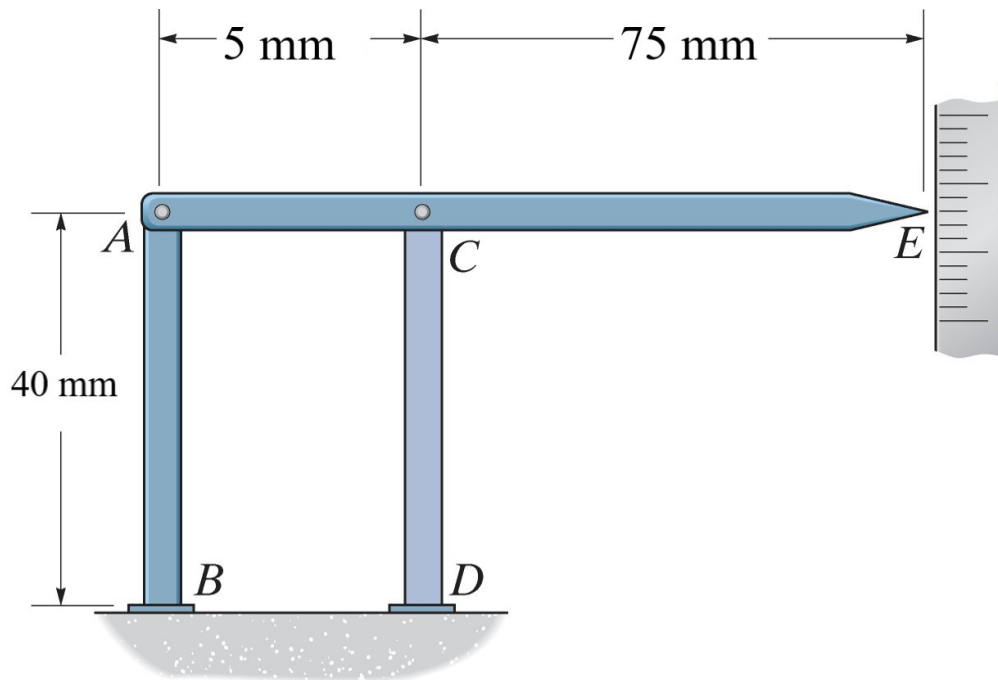

Solution:

- For this problem we will use the force method and begin by neglecting the fixed support at D.

- With no support at D each material can expand freely, with $\delta = \alpha\Delta TL$, we use $\alpha_{al} = 12.8 \times 10^{-6}/°F$, $\alpha_{br} = 12.2 \times 10^{-6}/°F$, and $\alpha_{st} = 9.6 \times 10^{-6}/°F$

- Substituting lengths (converted to inches), we find $\delta_{al} = 0.080\,\text{in}$, $\delta_{br} = 0.114\,\text{in}$, and $\delta_{st} = 0.045\,\text{in}$ for a total thermal deflection of $\delta_T = 0.239\,\text{in}$

- We need a reaction force to cancel this deflection exactly. From the method of sections, we can clearly see that the reaction force will also be exactly equal to the internal normal force in each material. Thus we need $\delta_M = \delta_T = \frac{NL_{al}}{E_{al}A_{al}} + \frac{NL_{br}}{E_{br}A_{br}} + \frac{NL_{st}}{E_{st}A_{st}}$

- Solving, we find $N = 1070\,\text{kip}$ and this gives stresses of $\sigma_{al} = 9.46\,\text{ksi}$, $\sigma_{br} = 21.3\,\text{ksi}$ and $\sigma_{st} = 85.1\,\text{ksi}$

6. The rod shown is made from aluminum and has a diameter of $0.25\,\text{in}$. If the rod is $4\,\text{ft}$ long when the springs are compressed $0.5\,\text{in}$ and the temperature of the rod is $40°F$, find the force in the rod when the temperature is $150°F$.

$k = 1000\,\text{lb/in.}$ $k = 1000\,\text{lb/in.}$

4 ft

Solution:

- We can simplify this problem slightly by considering the symmetry, we will consider the center to be the axis of symmetry, where there will be no displacement.

- For this problem, we will use the method of sections. We are given that before any thermal expansion, there is some compression in the springs, which will put some force into the aluminum, but since it is in equilibrium we can neglect it for now and only consider the additional deformation (and additional induced forces).

- Additional displacement will have a thermal and mechanical component for the aluminum, $\delta = \delta_T + \delta_M = x$ which will also equal the additional displacement x in the spring, since $F = kx$ we can say that $\delta_T + \delta_M = F_s/k$

- The normal force in the aluminum must be equal and opposite to the force in the spring, so we can further say that $\delta_T - \frac{F_s L}{AE} = \frac{F_s}{k}$

- We can further simplify to solve for F_s, $\alpha \Delta T L = \frac{F_s}{k} + \frac{F_s L}{AE}$ which gives $F_s = \alpha \Delta T L \left(\frac{1}{k} + \frac{L}{AE} \right)^{-1} = (12.8 \times 10^{-6}/°F)(110°F)(24\,\text{in}) \left(\frac{1}{1000\,\text{lb/in}} + \frac{24\,\text{in}}{0.0491\,\text{in}^2 10.6 \times 10^6\,\text{psi}} \right)^{-1}$

- Substituting values we find that $F_s = 32.3\,\text{lb}$, the total force in the rod however will also include $500\,\text{lb}$ from the initial spring compression, giving $F_s = N = 532\,\text{lb}$

7. The device shown is used to measure temperature. Bars AB and CD are made of tungsten ($\alpha = 4.5 \times 10^{-6}\ °C^{-1}$) and aluminum ($\alpha = 23 \times 10^{-6}\ °C^{-1}$), respectively. At room temperature ($20°C$) the rigid bar AE is horizontal. Find the vertical displacement of the pointer when the temperature is $200°C$.

Solution:

- We can use similar triangles to relate the displacement at the scale to the thermal displacement in each of the legs, see figure
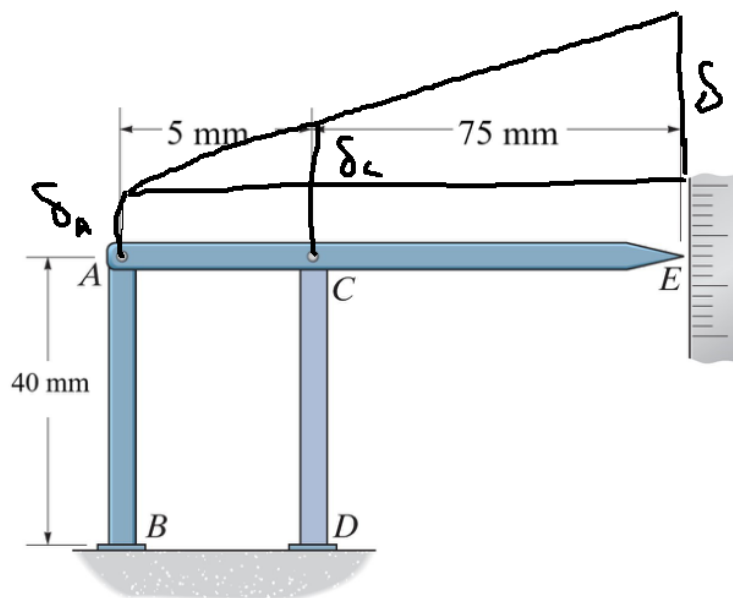
- We also need to add the total shift (below the triangle), so our final expression is
$\delta = \delta_a + \frac{80(\delta_c - \delta_a)}{5}$

- Substituting known values we find $\delta = 2.16\,\text{mm}$