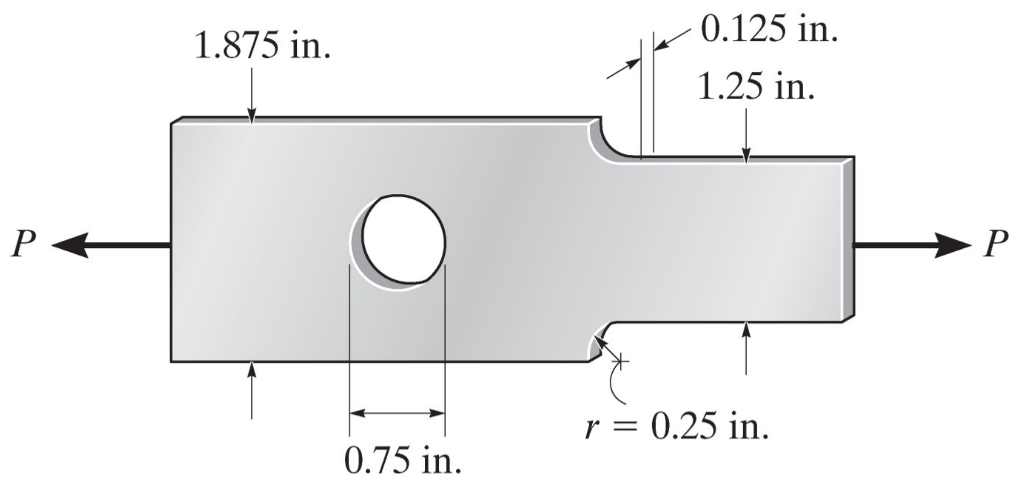
Name:

Homework 10

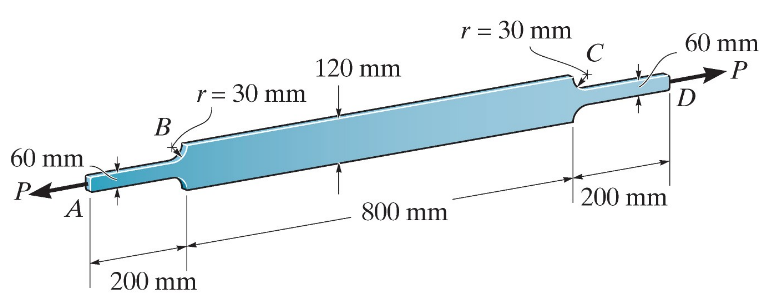
1. Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 4\,\text{kip}$



Solution:

- To find the maximum normal stress we need to find the maximum stress from the hole and the fillet and determine which is greater.
- We start by finding the stress concentration factor for the hole. We have $2r/w = 0.4$ which gives $K = 2.2$. Using $\sigma_{avg} = \frac{N}{(w-2r)t}$ we find $\sigma_{max} = 62.6\,\text{ksi}$
- For the fillet we find $w/h = 1.5$ and $r/h = 0.2$ which gives $K = 1.7$. Using $\sigma_{avg} = \frac{N}{ht}$ we find $\sigma_{max} = 43.5\,\text{ksi}$
- We find that the maximum normal stress is caused by the hole and has a value of $\sigma_{max} = 62.6\,\text{ksi}$

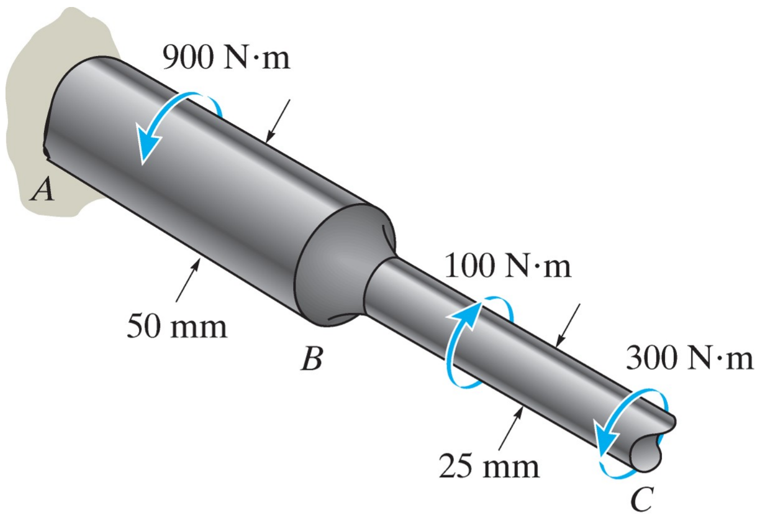
2. The A-36 steel plate has a thickness of $12\,\text{mm}$. If $\sigma_{allow} = 135\,\text{MPa}$, determine the maximum axial load, P that it can support.



Solution:

- In this case we use $\sigma_{allow} = K\sigma_{avg}$ to find the maximum P that we can support
- For the fillet geometry shown we have $w/h = 2$ and $r/h = 0.5$ which gives $K = 1.4$
- We can now solve $135\,\text{MPa} = \frac{1.4P}{60 \cdot 12}$ for P to find $P = 69.4\,\text{kN}$

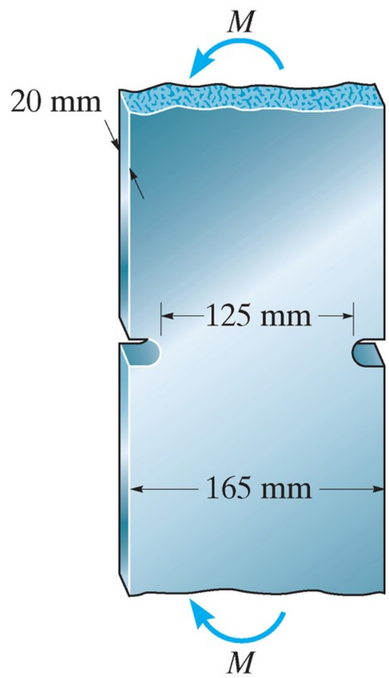
3. The shaft is fixed to the wall at A and is subjected to the torques shown. Find the maximum shear stress in the shaft. A fillet weld having a radius of 2.75 mm is used to connect the shafts at B



Solution:

- From statics we find internal torques of $1100\,\text{N}\cdot\text{m}$ between the wall and the first torque, $200\,\text{N}\cdot\text{m}$ between the first and second torques and $300\,\text{N}\cdot\text{m}$ between the second and third torques. There is a fillet between the first and second torques to increase the maximum stress.
- We calculate shear stress from torsional load in the usual way with $\tau = \frac{Tc}{J}$, which gives $\tau_1 = 44.8\,\text{MPa}$, $\tau_2 = 65.2\,\text{MPa}$ and $\tau_3 = 97.8\,\text{MPa}$
- Applying the stress concentration factor to the region with the fillet (τ_2) we find $D/d = 2.0$ and $r/d = 0.22$ we find $K \approx 1.21$ which gives $\tau_2 = 78.9\,\text{MPa}$
- This is still less than τ_3, which means maximum shear stress in the shaft is $\tau_3 = 97.8\,\text{MPa}$

4. If the radius of each notch on the plate is 10 mm find the largest moment M that can be applied. The maximum allowable bending stress is $\sigma_{allow} = 190\,\text{MPa}$



Solution:

- We can use the bending stress formula $190\,\text{MPa} = \frac{KMy}{I}$ to solve for M.
- We can find K from the chart with $r/h = .08$ and $b/r = 2.0$, which gives $K = 2.1$.
- With $y = 62.5\,\text{mm}$ and $I = 3.26 \times 10^6\,\text{mm}^4$ we find $M = 4.71\,\text{kN} \cdot \text{m}$

5. The W8 x 67 flange 2014-T6 aluminum column can be assumed to be fixed at its base and pinned at its top. Find the largest axial force, P, that can be applied without causing buckling.

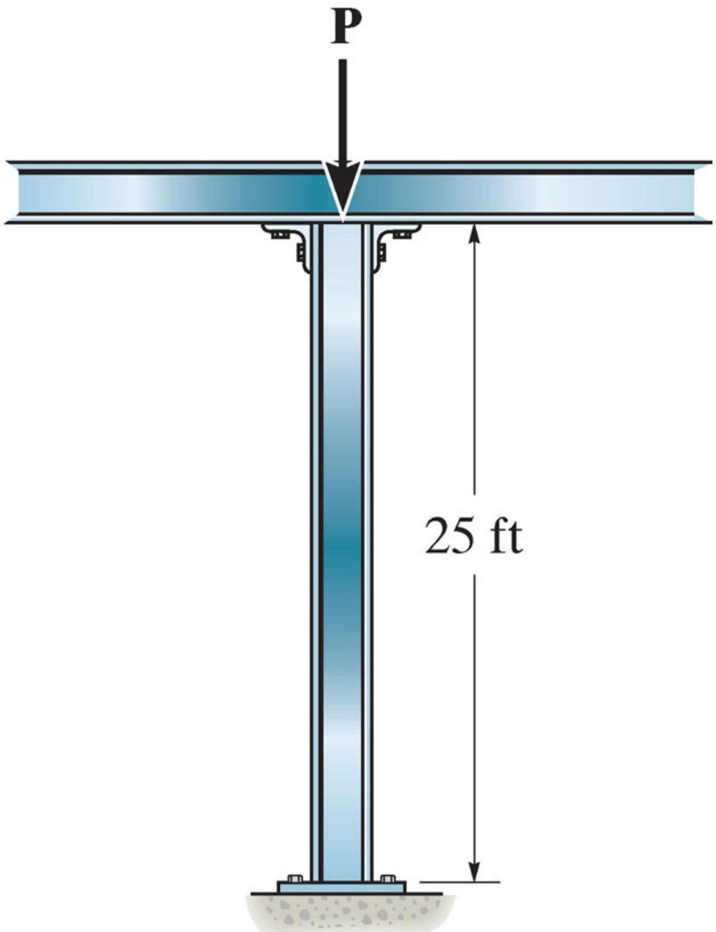


Figure 1: 13-14

Solution:

- The buckling formula in terms of load is $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$
- For the given column we have $E = 10.6\,\text{Msi}$ and $I = 272\,\text{in}^4$ about the $x - x$ axis and $I = 88.6\,\text{in}^4$ about the $y - y$ axis. We can clearly see that the buckling load will be lower about the $y - y$ axis, so that will be our limiting case.
- For a fixed-pinned beam, the length factor $K = 0.7$
- We can substitute these values to find $P_{cr} = 210\,\text{klb}$

6. Solve the previous problem assuming that it is fixed at its base and free at its top **Solution:**

   - For a fixed-free column we change the length factor to $K = 2.0$ which gives a critical load of $P_{cr} = 25.6\,\text{klb}$

7. Repeat the previous problems assuming that it is fixed at both the base and top. Which of these cases is the best for buckling, which is the worst? **Solution:**

- A fixed-fixed column has a length factor of $K = 0.5$ which gives $P_{cr} = 412\,\text{klb}$
- It is not surprising that the fixed-fixed beam is the best and the fixed-free is the worst in buckling, but it may be somewhat surprising how strong the differences are.
- The fixed-fixed column can support almost double the load of the fixed-pinned column without buckling, while supporting almost 20 times the load of the fixed-free column.