AE333

Mechanics of Materials

Lecture 7 - Axial Load

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schedule

- 7 Feb - Axial Load
- 10 Feb - Exam 1
- 12 Feb - Torsion
- 14 Feb - Torsion, HW3 Due

outline

- saint venant's principle
- elastic axial deformation
- superposition
- statically indeterminate

saint venant's principle

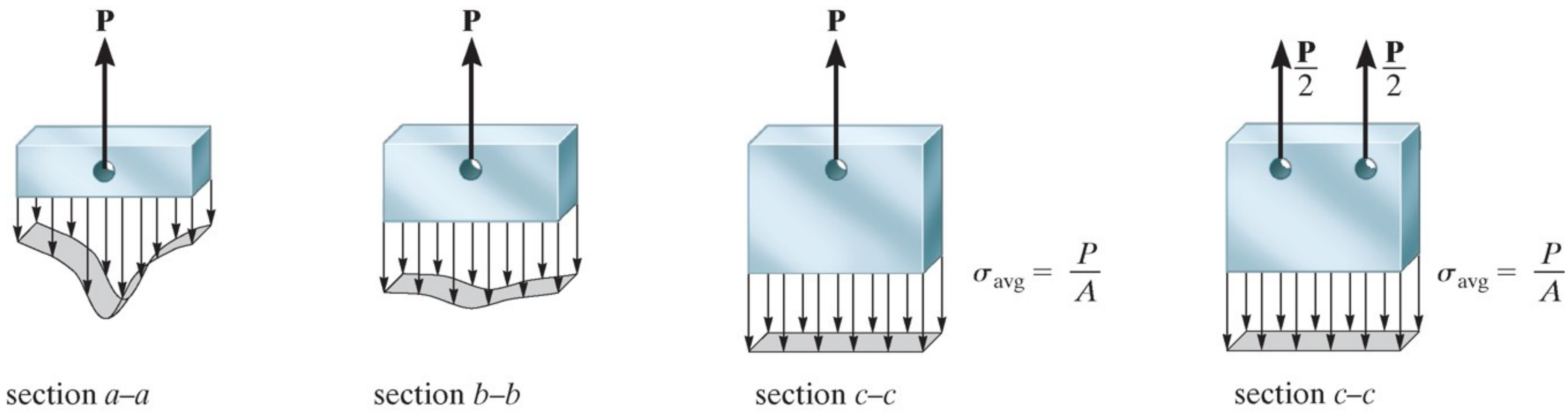
saint venant's principle

- We use Saint Venant's principle to generalize various loading applications
- If we apply a concentrated force, near where we apply it (for example, along a pin), the stress will not be very uniform
- Far away from that point, however, the stess will be uniform, whether we apply the force with 1 pin, 2 pins, or via a uniform grip

saint venant's principle

- We use *saint venant's principle* to replace difficult to model loads with easier ones
- There are two conditions
    1. The load must be statically equivalent
    2. Our region of interest must be far enough away from the point where the load was applied

saint venant's principle



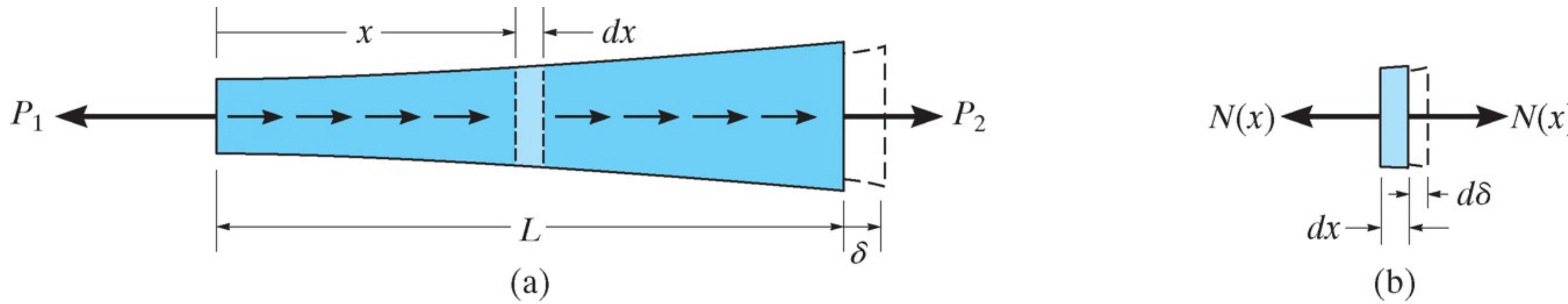
(b)

(c)

elastic axial deformation

axially loaded member

- We can use Hooke's Law to find the deformation of a general body under axial loading (below the elastic limit)



axially loaded member

- For some differential element, we can consider the internal forces and stresses

$$\sigma = \frac{N(x)}{A(x)} = E(x)\epsilon(x) = E(x)\left(\frac{d\delta}{dx}\right)$$

- We can solve this for $d\delta$ to find

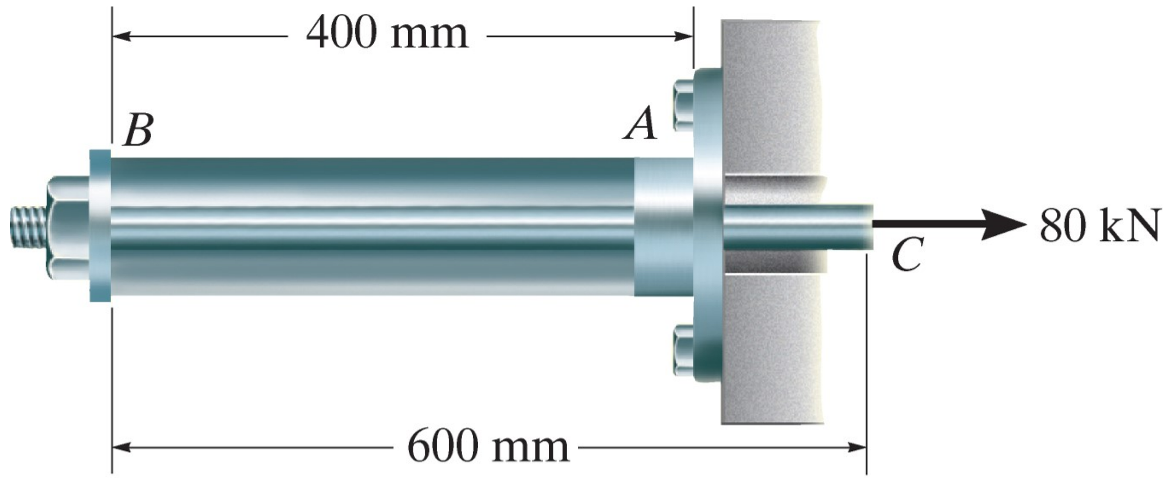
$$d\delta = \frac{N(x)dx}{A(x)E(x)}$$

- We integrate this over the length of the bar to find the total displacement

sign convention

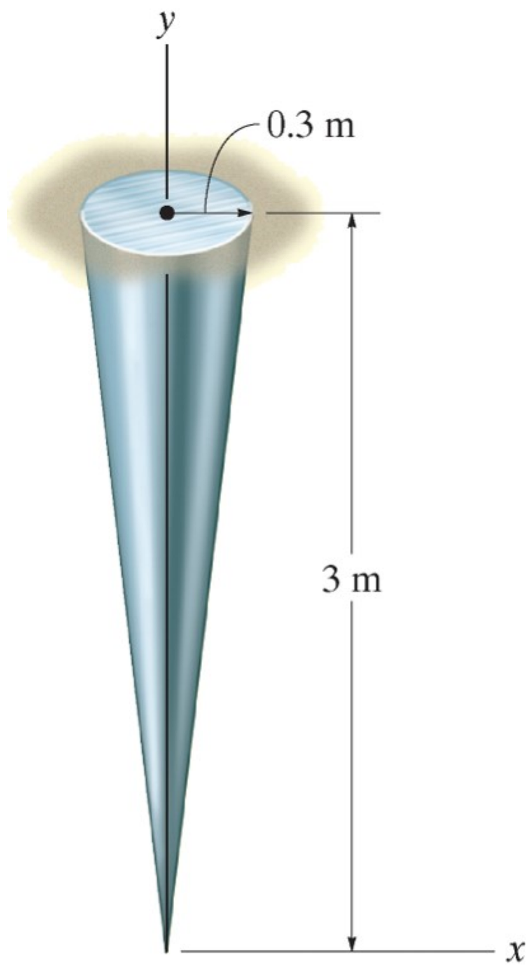
- In general, we consider a force or stress to be positive if it causes tension and elongation
- It is negative if it causes compression and contraction

example 4.2



A steel rod with a 10mm diameter is attached to a rigid collar passing through an aluminum tube with cross-sectional area of 400 mm². Find the displacement at C if $E_{st} = 200$ GPa and $E_{al} = 70$ GPa.

example 4.4



The cone shown has a specific weight of $\gamma = 6\ \mathrm{kN/m^3}$ and $E = 9\ \mathrm{GPa}$. Determine how far the end is displaced due to gravity.

superposition

superposition

- Some problems are too complicated to solve all at once
- Instead, we break them up into two simpler problems
- Each "sub-problem" must still satisfy equilibrium
- Problem must be linear and the deformation should be small enough that it does not cause moment-equilibrium issues

statically indeterminate

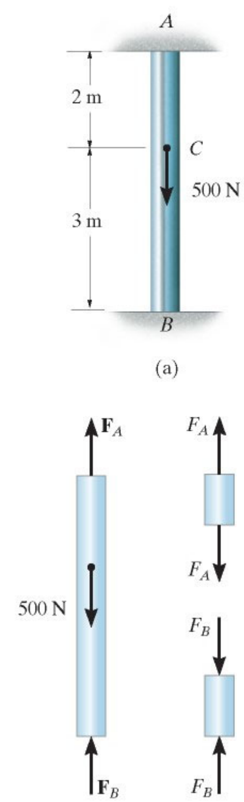
statically indeterminate

- There are many problems that are at least slightly over-constrained
- While this is common engineering practice, it creates too many variables for statics analysis
- These problems are called “statically indeterminate”

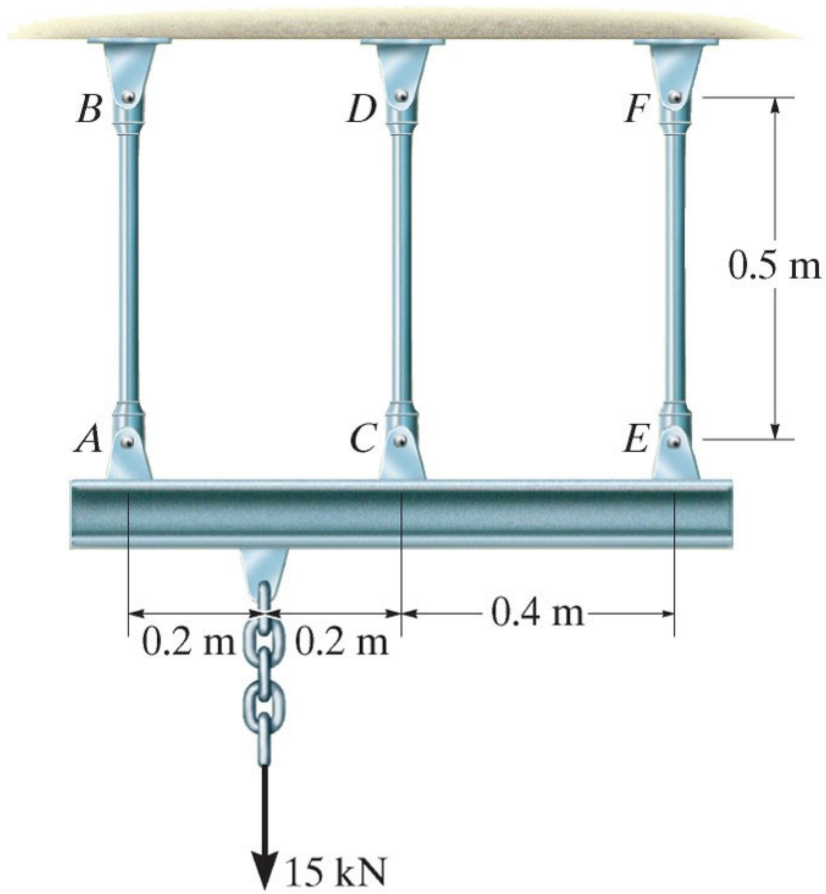
statically indeterminate

- One extra equation we can use is called “compatibility” or the “kinematic condition”
- We know that at the displacement must be equal on both sides of any arbitrary section we make in a member
- We can separate a member into two parts, then use compatibility to relate the two unknown forces

statically indeterminate



example 4.7



Assuming the bottom bar is rigid, find the force developed in each bar. AB and EF have cross-sectional areas of 50 mm^2 while CD has a cross-sectional area of 30 mm^2.