Mechanics of Materials

Lecture 23 - Buckling

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

19 November, 2020

schedule

- 19 Nov - Buckling
- 20 Nov - Project 3 Due
- 1 Dec - Bucking, Final Review
- 3 Dec - Final Review, Problem Solving, HW 11 Due
- 8 Dec - Final Exam (comprehensive, same format as other exams)

outline

- buckling
- ideal pin-supported column
- columns with other supports

stability

- In engineering problems, stability and instability relate how an object behaves when it experiences some random perturbation
- A stable aircraft has aerodynamic features that tend to keep it flying level, small bumps of wind that would cause it to rotate will eventually get pushed back to level
- Some aircraft are designed to be unstable (can have a tighter turn radius), but they need to be actively controlled, as a small perturbation will cause them to spiral out of control

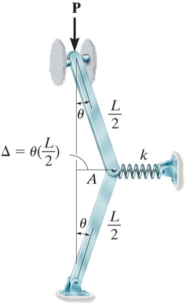
buckling

- For long and slender structures, stability comes into play in the form of buckling
- A structure that is subject to buckling is generally referred to as a column
- Buckling is usually a very sudden and drastic failure, so we need to design columns to avoid buckling

critical load

- The critical load is the maximum load a column can hold before buckling
- We can model the critical load by considering the column as a rigid truss with a spring force acting to maintain stability
- When the loading force overcomes the spring force, buckling occurs

critical load



critical load

- The balance of forces will be

$$F = k\Delta = P \tan\theta$$

- For small θ, we can further say that $\Delta = \theta(L/2)$ and `\tan \tehta = \theta`

critical load

- We find that, for stability, we need

$$P < \frac{kL}{4}$$

ideal column

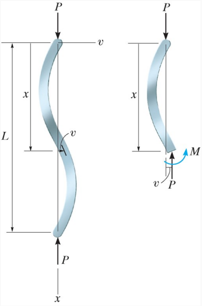
- Our previous analysis treated a column as a two-member truss with a spring, but we can be more precise
- An ideal column is made of homogeneous linear elastic material and is perfectly straight before loading
- The load is assumed to be applied through the centroid of the cross section

euler-bernoulli

- We can treat the column as a beam and use the familiar relationship

$$EI\frac{d^2 v}{dx^2} = M$$

euler-bernoulli



solution

- We see by equilibrium that $M = -Pv$, which gives the differential equation

$$EI\frac{d^2v}{dx^2} = -Pv \quad \frac{d^2v}{dx^2} + \frac{P}{EI}v \quad = 0$$

- Which has the solution

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right)$$

boundary conditions

- We know that for $v = 0$ at $x = 0$, $C2 = 0$
- We also know that $v = 0$ at $x = L$ which gives

$$C_1 \sin\left(\sqrt{\frac{P}{EI}}L\right) = 0$$

- $C1 = 0$ would give the trivial solution, or we can say that

$$\sqrt{\frac{P}{EI}}L = n\pi$$

critical load

- The smallest value where this occurs is when $n = 1$ and gives the critical load of

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

- This is sometimes called the "Euler Load"
- We can increase Pcr by decreasing L, increasing E, or increasing I

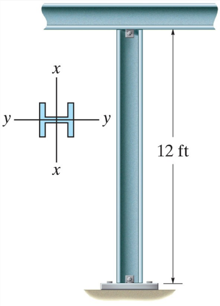
radius of gyration

- Sometimes we desire to find the critical stress instead of the critical load
- We re-formulate the equation with $I = Ar2$ (where r is the radius of gyration)
- This gives

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2}$$

- L/r is often called the slenderness ratio

example 13.1



Find the largest axial load the A992 steel member shown can support before it buckles or yields, use $\sigma_y = 50$ ksi.

other supports

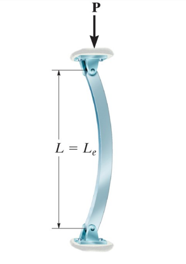
- we can still use Euler-Bernoulli beam theory when handling columns with other supports
- the general derivation is the same, but with different boundary conditions

effective length

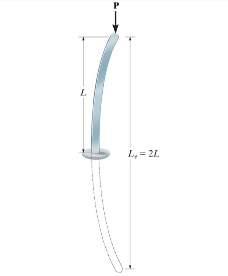
- One simple way to use the same formula for different supports is to modify the effective length of the column
- We can also use a length factor, K, to define the effective length

$$L_e = KL$$

length factors

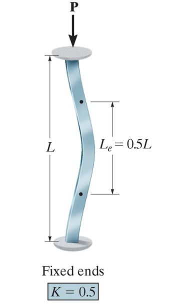


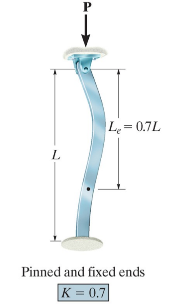
Pinned ends
$\boxed{K = 1}$



Fixed and free ends
$\boxed{K = 2}$

length factors





effective length

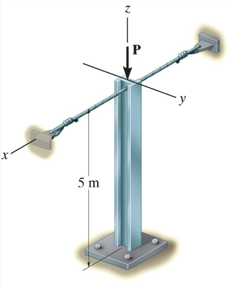
- The formulas now become

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

- or

$$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$$

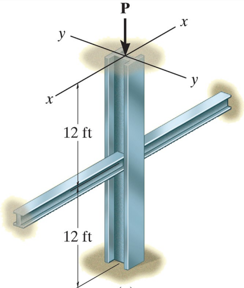
example 13.2



(a)

The column shown is braced by cables preventing movement in *x*. Determine the largest *P* that can be applied if $E = 70$ GPa, $\sigma_y = 215$ MPa, $A = 7.5$ (103) m2, Ix = 61.3 (10-6) m4 and Iy = 23.2(10-6) m4.

example 13.3



(a)

A W6 x 15 steel column is fixed at its ends and braced in the *y*-*y* axis assumed to be pinned at the midpoint. Determine the maximum load before buckling or yield with Est = 29 Msi and $\sigma_y = 60$ ksi.