Mechanics of Materials

Lecture 22 - Stress Concentration

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schedule

- 3 May - Stress concentration, buckling
- 5 May - Final exam review
- 6 May - Project 3 Due
- Homeworks 9-11 (posted to blackboard) are optional and provide some practice for exam

outline

- stress concentration factors

stress concentration factors

stress concentration

- Our textbook splits the idea of concentration factors across multiple chapters
- 4.7, 5.8, 6.9
- The basic idea of a stress concentration factor is that some geometry causes the maximum stress to be greater than the 'nominal' stress

stress concentration

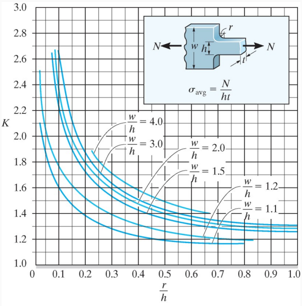
- Stress concentrations occur when there is a sudden change in cross-sectional area
- Features such as holes and fillets will have a stress concentration factor

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

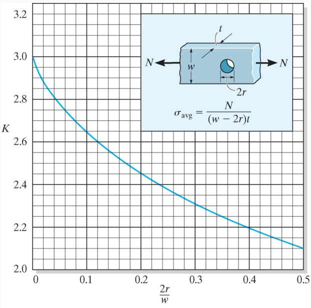
stress concentration

- The exact value of the stress concentration factor can be derived for simple shapes, but in practice it is usually looked up on a chart or figure
- The value of K depends on the ratio of the radius and depth of the feature relative to the total object depth

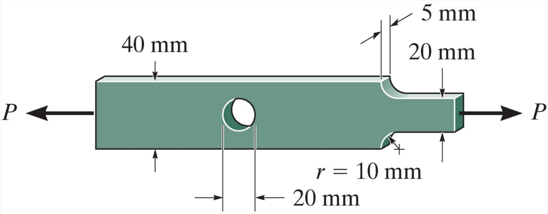
fillets



holes



example



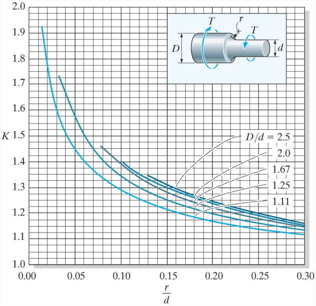
If $\sigma_{allow} = 120$ MPa, find the maximum axial force, P.

stress concentration in torsion

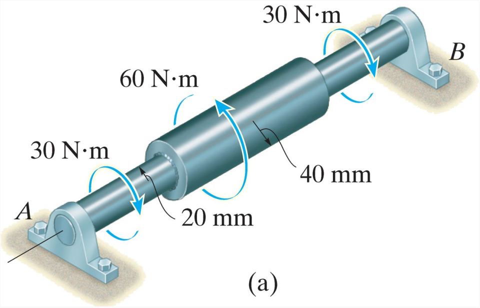
- We can also have stress concentration in torsion
- For circular shafts, this is usually around a filleted shaft as shown in the next slide
- The maximum shear can be found with

$$\tau_{max} = K\frac{Tc}{J}$$

fillet



example 5.14



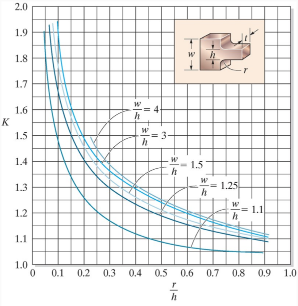
Determine the maximum stress in the shaft due to the applied torques. The shoulder fillet has a radius of r=6 mm

beams

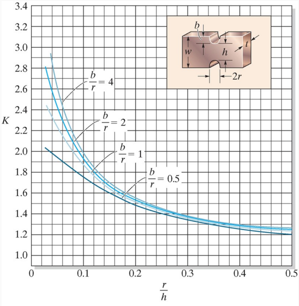
- We can also have a stress concentration in a beam
- The maximum stress can be found with

$$\sigma_{max} = K\frac{Mc}{I}$$

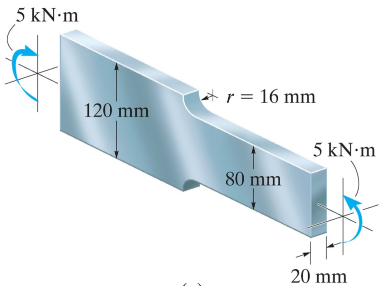
fillet



notch



example 6.20



(a)

Determine the maximum normal stress for a steel bar with a shoulder fillet as shown.

buckling

stability

- In engineering problems, stability and instability relate how an object behaves when it experiences some random perturbation
- A stable aircraft has aerodynamic features that tend to keep it flying level, small bumps of wind that would cause it to rotate will eventually get pushed back to level
- Some aircraft are designed to be unstable (can have a tighter turn radius), but they need to be actively controlled, as a small perturbation will cause them to spiral out of control

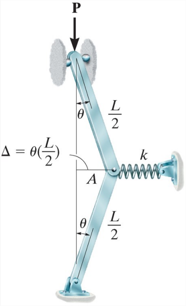
buckling

- For long and slender structures, stability comes into play in the form of buckling
- A structure that is subject to buckling is generally referred to as a column
- Buckling is usually a very sudden and drastic failure, so we need to design columns to avoid buckling

critical load

- The critical load is the maximum load a column can hold before buckling
- We can model the critical load by considering the column as a rigid truss with a spring force acting to maintain stability
- When the loading force overcomes the spring force, buckling occurs

critical load



critical load

- The balance of forces will be

$$F = k\Delta = P \tan\theta$$

- For small θ, we can further say that $\Delta = \theta(L/2)$ and $\tan\theta = \theta$

critical load

- We find that, for stability, we need

$$P < \frac{kL}{4}$$

ideal pin-supported column

ideal column

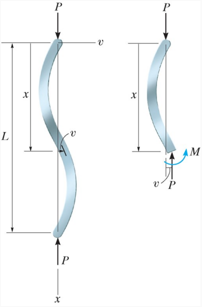
- Our previous analysis treated a column as a two-member truss with a spring, but we can be more precise
- An ideal column is made of homogeneous linear elastic material and is perfectly straight before loading
- The load is assumed to be applied through the centroid of the cross section

euler-bernoulli

- We can treat the column as a beam and use the familiar relationship

$$EI\frac{d^2v}{dx^2} = M$$

euler-bernoulli



solution

- We see by equilibrium that $M =$ -Pv, which gives the differential equation

$$EI\frac{d^2v}{dx^2} = -Pv$$
$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

- Which has the solution

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right)$$

boundary conditions

- We know that for $v = 0$ at $x = 0$, $C_2 = 0$
- We also know that $v = 0$ at $x = L$ which gives

$$C_1 \sin\left(\sqrt{\frac{P}{EI}}L\right) = 0$$

- $C_1 = 0$ would give the trivial solution, or we can say that

$$\sqrt{\frac{P}{EI}}L = n\pi$$

critical load

- The smallest value where this occurs is when $n = 1$ and gives the critical load of

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

- This is sometimes called the "Euler Load"
- We can increase P_{cr} by decreasing L, increasing E, or increasing I

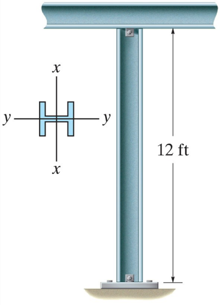
radius of gyration

- Sometimes we desire to find the critical stress instead of the critical load
- We re-formulate the equation with $I = Ar^2$ (where r is the radius of gyration)
- This gives

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2}$$

- L/r is often called the slenderness ratio

example 13.1



Find the largest axial load the A992 steel member shown can support before it buckles or yields, use $\sigma_y = 50$ ksi.

columns with other supports

other supports

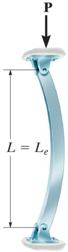
- we can still use Euler-Bernoulli beam theory when handling columns with other supports
- the general derivation is the same, but with different boundary conditions

effective length

- One simple way to use the same formula for different supports is to modify the effective length of the column
- We can also use a length factor, K, to define the effective length

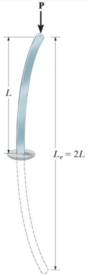
$$L_e = KL$$

length factors



Pinned ends

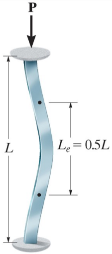
$\boxed{K = 1}$



Fixed and free ends

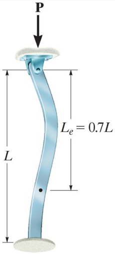
$\boxed{K = 2}$

length factors



Fixed ends

$\boxed{K = 0.5}$



Pinned and fixed ends

$\boxed{K = 0.7}$

effective length

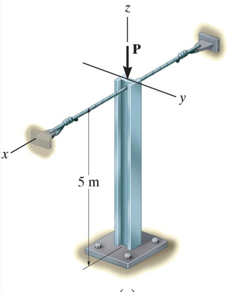
- The formulas now become

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

- or

$$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$$

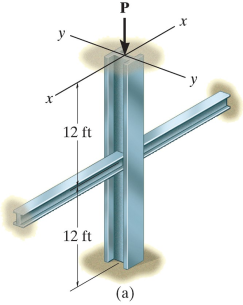
example 13.2



(a)

The column shown is braced by cables preventing movement in x. Determine the largest P that can be applied if E=70 GPa, $\sigma_y = 215$ MPa, $A = 7.5$ (103) m2, Ix = 61.3 (10-6) m4 and Iy

example 13.3



A W6 x 15 steel column is fixed at its ends and braced in the y-y axis assumed to be pinned at the midpoint. Determine the maximum load before buckling or yield with Est = 29 Msi and $\sigma_y = 60$ ksi.