

Lecture 7 - Axial Load

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schedule

- 24 Feb - Axial Load
- 26 Feb - Project 1 Due, Homework 2 Self-grade Due
- 1 Mar - Torsion
- 3 Mar - Torsion
- 5 Mar - Homework 3 Due

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- saint venant's principle
- elastic axial deformation
- superposition
- statically indeterminate

saint venant's principle

saint venant's principle

- We use Saint Venant's principle to generalize various loading applications
- If we apply a concentrated force, near where we apply it (for example, along a pin), the stress will not be very uniform
- Far away from that point, however, the stress will be uniform, whether we apply the force with 1 pin, 2 pins, or via a uniform grip

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saint venant's principle

- We use *saint venant's principle* to replace difficult to model loads with easier ones
- There are two conditions
 1. The load must be statically equivalent
 2. Our region of interest must be far enough away from the point where the load was applied

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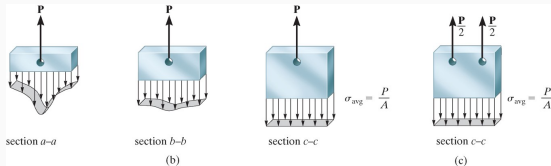


Figure 1: An image showing what the stress field looks like in a body both near to an applied load and far away.

elastic axial deformation

axially loaded member

- We can use Hooke's Law to find the deformation of a general body under axial loading (below the elastic limit)

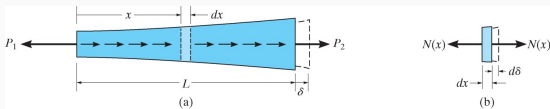


Figure 2: If we consider some homogeneous body with an applied load, we can look at a small section of this body with an applied load of $N(x)$ which is initially dx wide, but under load stretches an additional $d\delta$.

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axially loaded member

- For some differential element, we can consider the internal forces and stresses

$$\sigma = \frac{N(x)}{A(x)} = E(x)\epsilon(x) = E(x) \left(\frac{d\delta}{dx} \right)$$

- We can solve this for $d\delta$ to find

$$d\delta = \frac{N(x)dx}{A(x)E(x)}$$

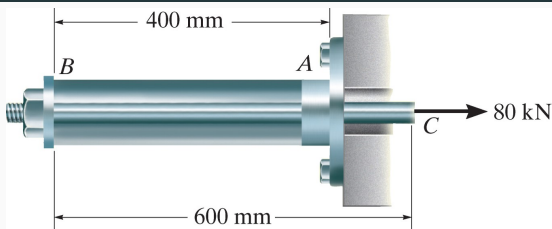
- We integrate this over the length of the bar to find the total displacement

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- In general, we consider a force or stress to be positive if it causes tension and elongation
- It is negative if it causes compression and contraction

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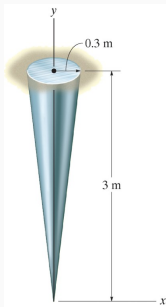
example 4.2



A steel rod with a 10mm diameter is attached to a rigid collar passing through an aluminum tube with cross-sectional area of 400 mm². Find the displacement at *C* if $E_{st} = 200$ GPa and $E_{al} = 70$ GPa.

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example 4.4



The cone shown has a specific weight of $\gamma = 6 \text{ kN/m}^3$ and $E = 9 \text{ GPa}$. Determine how far the end is displaced due to gravity.

superposition

- Some problems are too complicated to solve all at once
- Instead, we break them up into two simpler problems
- Each “sub-problem” must still satisfy equilibrium
- Problem must be linear and the deformation should be small enough that it does not cause moment-equilibrium issues

statically indeterminate

statically indeterminate

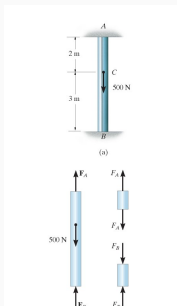
- There are many problems that are at least slightly over-constrained
- While this is common engineering practice, it creates too many variables for statics analysis
- These problems are called “statically indeterminate”

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statically indeterminate

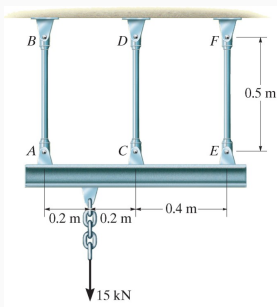
- One extra equation we can use is called “compatibility” or the “kinematic condition”
- We know that at the displacement must be equal on both sides of any arbitrary section we make in a member
- We can separate a member into two parts, then use compatibility to relate the two unknown forces

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example 4.7



Assuming the bottom bar is rigid, find the force developed in each bar. AB and EF have cross-sectional areas of 50 mm^2 while CD has a cross-sectional area of 30 mm^2 .

Figure 3: A 0.8 m long rigid horizontal bar is supported by hanging from 3 vertical rods. Rod

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