Mechanics of Materials

Lecture 12 - Bending

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schedule

- 4 Oct - Bending
- 6 Oct - Transverse Shear
- 8 Oct - Homework 5 Due, Homework 4 Self-grade due
- (11 Oct) - Fall Break
- 13 Oct - Transverse Shear
- 15 Oct - Homework 6 Due, Homework 5 Self-grade due
- 18 Oct - Exam 2 Review
- 20 Oct - Exam 2

outline

- flexure formula
- moment of inertia
- group problems
- compound centroids

flexure formula

hooke's law

- If the beam is a linear material that follows Hooke's Law, the stress must be linearly proportional to the strain
- Thus the stress will vary linearly through the beam, just like the strain does

neutral axis

- We have already hypothesized that a neutral axis must exist, we will now find its location
- To be in equilibrium, the resultant force(s) produced by the stress must sum to zero, which means

$$\begin{aligned}\sum F_x = 0 &= \int_A dF = \int_A \sigma dA \\ &= \int_A -\left(\frac{y}{c}\right)\sigma_{max} dA \\ &= -\frac{\sigma_{max}}{c}\int_A ydA\end{aligned}$$

neutral axis

- Since σ_{max} and c are both non-zero constants, we know that

$$\int_A y dA = 0$$

- Which can only be satisfied at the horizontal centroidal axis, so the neutral axis is the centroidal axis

bending moment

- The internal bending moment must be equal to the total moment produced by the stress distribution

$$\begin{aligned} M &= \int_A y dF = \int_A y(\sigma dA) \\ &= \int_A y \left(\frac{y}{c} \sigma_{max} \right) dA \\ &= \frac{\sigma_{max}}{c} \int_A y^2 dA \end{aligned}$$

bending moment

- We recognize that $\int_A y^2 dA = I$, and we re-arrange to write

$$\sigma_{max} = \frac{Mc}{I}$$

procedure

- Find the internal moment at the point of interest, or draw a moment diagram to find the maximum point (if needed)
- Determine the moment of inertia for the beam section
- Find the neutral axis and/or the distance from the neutral axis to the point of interest
- Use the flexure formula to find the stress

example 6.12

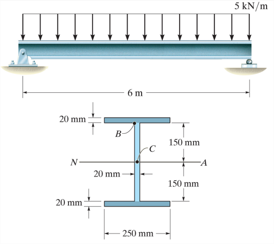

Figure 1: A 6 meter long beam is pinned at both ends and subjected to a uniformly distributed load of 5 kN/m.

Find the maximum bending stress and draw the stress distribution through the thickness at that point.

moment of inertia

moment of inertia

- We know that $I = \int_A y^2 dA$
- For common shapes, this integral has been pre-calculated (about the centroid of the shape)
- For compound shapes, we use the parallel axis theorem to combine inertias from multiple areas

parallel axis theorem

- The parallel axis theorem is used to find the moment about any axis parallel to an axis passing through the centroid
- If we consider an axis parallel to the x-axis, separated by some dy we have

$$I_X = \int_A (y + dy)^2 dA$$

- Which gives

$$I_x = \int_A y^2 dA + 2dy \int_A y dA + dy^2 \int_A dA$$

parallel axis theorem

- The first integral is the moment of inertia about the original x-axis, which we will call $\bar{I}_x$
- The second integral is zero since the x-axis passes through the centroid
- This gives the parallel axis theorem

$$I_x = \bar{I}_x + Ady^2$$

parallel axis theorem

- Similarly for the y-axis and polar moment of inertia we find

$$I_y = \bar{I}_y + Adx^2$$
$$J_O = \bar{J}_C + Ad^2$$

group problems

group one

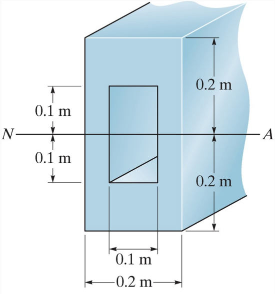

Figure 2: A hollow rectangular tube 0.4m tall and 0.2m wide on the outside and 0.2 m tall and 0.1 wide on the inside

Determine the moment of inertia about the neutral axis

group two

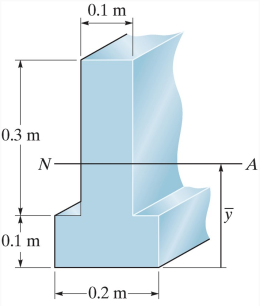

Figure 3: An upside-down t-section is 0.2m wide at the base, which is 0.1 m tall, the web is 0.1

Find the neutral axis and determine the moment of inertia about the neutral axis

group three

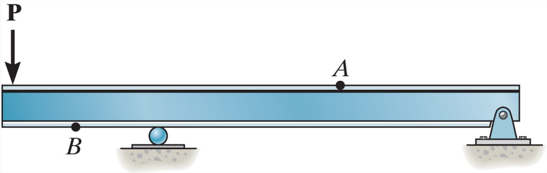

Figure 4: A beam is supported by pins at the right end and a point some distance away from the left end. A vertical load is applied at the left end, point B as it at the bottom side of the beam between the load and the first pin support, point A is between the two pin supports.

Show how the bending stress acts on a differential volume at

compound centroids

composite bodies

- Often we have to deal with bodies that are not described by a continuous function, but are made of different materials or different shapes
- We can use the same logic previously, but with a finite sum instead of an integral

$$\bar{x} \sum W = \sum \tilde{x} W$$
$$\bar{y} \sum W = \sum \tilde{y} W$$
$$\bar{z} \sum W = \sum \tilde{z} W$$

analysis procedure

- Use a sketch to divide the body into sub-bodies
- If a body has a hole, it may be easier to treat that volume as whole and then subtract the hole
- Take note of any symmetry (an object symmetric about any axis will have a centroid along that axis)