Mechanics of Materials

Lecture 17 - Strain Transformation

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schedule

- 20 Oct - Strain Transformation, HW 7 Due
- 22 Oct - Beam Deflection
- 27 Oct - Beam Deflection, HW 8 Due, HW 7 Self-Grade Due
- 29 Oct - Beam Deflection

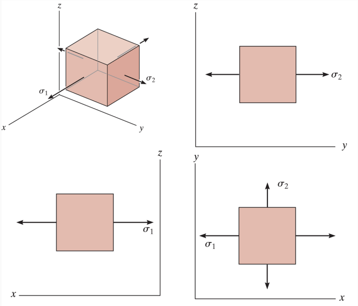
outline

- absolute maximum shear
- plane strain
- principal strains and mohr's circle
- strain rosettes
- material property relationships

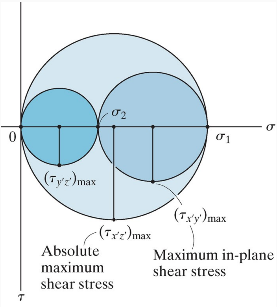
absolute maximum shear

- We already know how to find the maximum in-plane shear, but sometimes the maximum shear stress can occur in another plane
- We can do this (without treating it as a fully 3D problem) by treating each plane as its own plane stress problem

mohr's circle



mohr's circle



absolute max shear

- The maximum absolute shear will depend on whether σ_1 and σ_2 are in the same or opposite directions

$$\tau_{abs,max} = \frac{\sigma_1}{2} \qquad \text{same direction}$$
$$\tau_{abs,max} = \frac{\sigma_1 - \sigma_2}{2} \qquad \text{opposite directions}$$

- Which of the three mohr's circles the maximum occurs in will determine which plane the shear acts in

plane strain

- Under plane stress we assume no out-of-plane stresses are present
- This is typically a good assumption for very thin materials
- Under plane strain we assume no out-of-plane strains are present
- Typically a good assumption for very thick materials

sign convention

- Normal strains, ϵ_x and ϵ_y, are considered positive when they cause elongation, and negative when they cause contraction
- Shear strains, γ_{xy} are positive if the interior angle becomes smaller than 90°, and negative if the angles becomes larger than 90°

general equations

- We derive the general strain transformation equations by comparing infinitesimal elements before and after deformation
- To find $\gamma_{x'y'}$ we compare the angle between dx and dy before and after deformation

general equations

- The equations are nearly exactly the same as the stress transformation equations
- Pay attention to the difference, strain transformation equations are NOT on the equation sheet

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

- As with σ_y', we find ϵ_y' by letting $\theta_y = \theta_x + 90^\circ$

engineering strain

- Side note: there is another definition of shear strain known as tensorial shear strain, where $\gamma_{xy} = 2\epsilon_{xy}$
- Under tensorial strain, the transformation equations are exactly the same (as in this case both stress and strain are tensors)
- γ_{xy} is known as engineering strain, you will need to pay attention to which strain convention is used when extracting data from finite elements or other sources

principal strains

- As you might imagine, since the transformation equations are nearly identical so are the principal strain equations

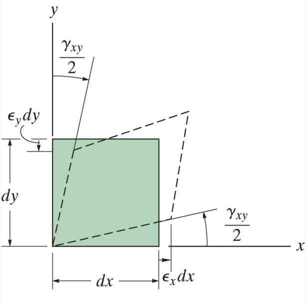
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

mohr's circle

- Mohr's circle can also be used in exactly the same way for strain as it is for stress
- The only difference is that the vertical axis is tensor strain, or $\gamma_{xy}/2$

example 10.4



The state of plane strain at a point has components of $\epsilon_x = 250\mu\epsilon$, $\epsilon_y = -150\mu\epsilon$, and $\gamma_{xy} = 120\mu\epsilon$. Determine the principal strains and the direction they act.

rosettes

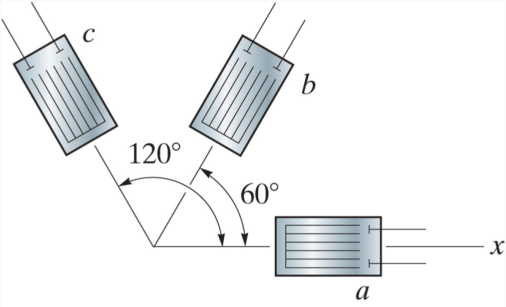
- Normal strain is fairly easy to measure using a strain gage
- Shear strain is more difficult to measure directly, so instead a "rosette" of normal strain gages is used
- We can use the strain transformation equations to determine τ_{xy}

rosettes

- Usually, we have $\theta_a = 0$, $\theta_b = 90$ and $\theta_c = 45$ OR $\theta_a = 0$, $\theta_b = 60$ and $\theta_c = 120$

$$
\begin{aligned}
\epsilon_a &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_a + \frac{\gamma_{xy}}{2}\sin 2\theta_a \\
\epsilon_b &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_b + \frac{\gamma_{xy}}{2}\sin 2\theta_b \\
\epsilon_c &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_c + \frac{\gamma_{xy}}{2}\sin 2\theta_c
\end{aligned}
$$

example 10.8



The readings from the rosette shown are $\epsilon_a = 60\mu\epsilon$, $\epsilon_b = 135\mu\epsilon$ and $\epsilon_c = 264\mu\epsilon$. Find the in-plane principal strains and their directions.

generalized hooke's law

- We have previously used Hooke's Law in 2D, in 3D we have

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$
$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

generalized hooke's law

- And in shear

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$
$$\gamma_{yz} = \frac{1}{G}\tau_{yz}$$
$$\gamma_{xz} = \frac{1}{G}\tau_{xz}$$

dilatation

- When a material deforms it often changes volume
- The change in volume per unit volume is called "volumetric strain" or dilatation

$$e = \frac{\partial V}{dV} = \epsilon_x + \epsilon_y + \epsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

hydrostatic pressure

- One way of characterizing volumetric response is to apply hydrostatic pressure (equal compression on all sides with no shear)
- Under this case, we have

$$\frac{p}{e} = -\frac{E}{3(1-2\nu)}$$

- We call the term on the right (with no negative sign) the bulk modulus, k