

Lecture 8 - Axial Load, Torsion

Dr. Nicholas Smith

Wichita State University, Department of Aerospace Engineering

13 September, 2021

1

## schedule

- 13 Sep - Project 1 Due
- 13 Sep - Axial Load
- 15 Sep - Torsion
- 17 Sep - Homework 2 Self-Grade Due
- 20 Sep - Torsion
- 22 Sep - Bending
- 24 Sep - Homework 3 Due, Project 1 Recovery Due

2

- superposition
- statically indeterminate
- force method
- thermal stress
- torsion

## superposition

---

- Some problems are too complicated to solve all at once
- Instead, we break them up into two simpler problems
- Each “sub-problem” must still satisfy equilibrium
- Problem must be linear and the deformation should be small enough that it does not cause moment-equilibrium issues

## statically indeterminate

---

## statically indeterminate

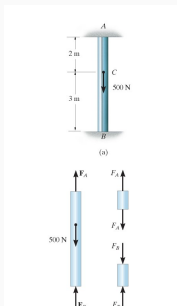
- There are many problems that are at least slightly over-constrained
- While this is common engineering practice, it creates too many variables for statics analysis
- These problems are called “statically indeterminate”

5

## statically indeterminate

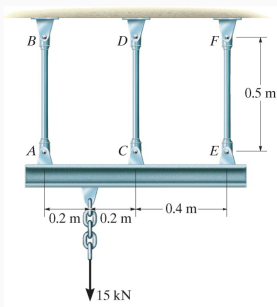
- One extra equation we can use is called “compatibility” or the “kinematic condition”
- We know that at the displacement must be equal on both sides of any arbitrary section we make in a member
- We can separate a member into two parts, then use compatibility to relate the two unknown forces

6



7

## example 4.7



Assuming the bottom bar is rigid, find the force developed in each bar. AB and EF have cross-sectional areas of  $50 \text{ mm}^2$  while CD has a cross-sectional area of  $30 \text{ mm}^2$ .

**Figure 1:** A 0.8 m long rigid horizontal bar is supported by hanging from 3 vertical rods. Rod

8

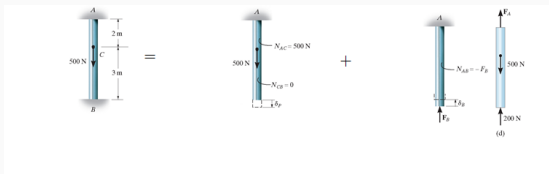
## force method

---

### force method

- One way to solve statically indeterminate problems is using the principle of superposition
- We choose one redundant support and remove it
- We then add it back as a force separately (without the other forces in the problem)

## force method



**Figure 2:** An illustration of the force method, we have the same statically indeterminate problem as before, a 5 m long, vertically-oriented bar is fixed at both ends, with a 500 N downward load applied 2 m from the top. We set this equivalent to a bar with the same load, but no support on the bottom end. We then add a force which will provide enough displacement to cancel out the displacement introduced by removing the load.

10

## force method

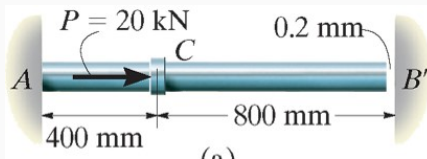
- We connect the two problems by requiring that the displacement in both frames adds to 0 to meet the support requirements
- This is referred to as the equation of compatibility

11

- Choose one support as redundant, write the equation of compatibility
- Express the external load and redundant displacements in terms of load-displacement relationship
- Draw free body diagrams and use the equations of equilibrium to solve

12

### example 4.9



**Figure 3:** A 1200 mm long horizontal rod is fixed at its left end and has a fixed support 0.2 mm away from its right end. A 20 kN load is applied to the rod 400 mm away from its left end.

The steel rod shown has a diameter of 10 mm. Determine the reactions at A and B'.

13



## thermal stress

---

### thermal stress

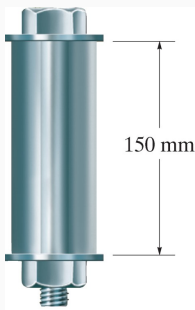
- A change in temperature causes a material to either expand or contract
- For most materials this is linear and can be described using the coefficient of linear expansion

$$\delta_T = \alpha \Delta T L$$

- When a body is free to expand, the deformation can be readily calculated using
- If it is not free to expand, however, thermal stresses develop
- We can use the force method described previously to find the thermal stresses developed

15

### example 4.12



An aluminum tube with cross-section of  $600 \text{ mm}^2$  is used as a sleeve for a steel bolt with cross-sectional area of  $400 \text{ mm}^2$ . When  $T=15$  degrees Celsius there is negligible force and a snug fit, find the force in the bolt and sleeve when  $T=80$  degrees Celsius.

**Figure 4:** An aluminum tube used as a sleeve for a steel bolt. The tube is 150 mm long.

16

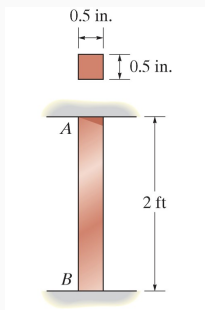
## group problems

---

### problem 1

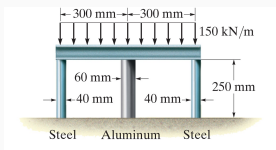
The A36 Steel bar is constrained to just fit between two fixed supports when  $T_1 = 60^\circ\text{F}$ . If the temperature is raised to  $T_2 = 120^\circ\text{F}$  determine the average normal stress developed in the bar.

Note: you may use  $\alpha = 6.5 \times 10^{-6} \text{ } ^\circ\text{F}^{-1}$



**Figure 5:** steel bar for thermal expansion example

## problem 2

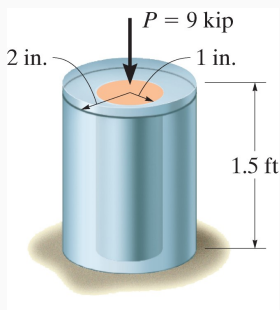


**Figure 6:** figure four group problem 2

The rigid beam is fixed to the top of three posts made from A992 steel and 2014 T-6 aluminum. The posts each have a length of  $250 \text{ mm}$  when no load is applied to the beam and the temperature is  $T_1 = 20^\circ\text{C}$ . Determine the force supported by each post if the bar is subjected to a uniformly distributed load of  $150 \text{ kN/m}$  and the temperature is raised to  $T_2 = 80^\circ\text{C}$ . Use  $E_{st} = 200 \text{ GPa}$ ,  $E_{al} = 70 \text{ GPa}$ , <sup>18</sup>

## problem 3

The aluminum post shown is reinforced with a brass core. If this assembly supports an axial compressive load of  $P = 9 \text{ kip}$  determine the average normal stress in the aluminum and the brass. Use  $E_{al} = 10 \text{ Msi}$  and  $E_{br} = 15 \text{ Msi}$



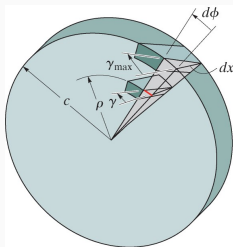
**Figure 7:** figure for group problem 3

## torsion

---

### torsion

- Torque is a moment that tends to twist a member about its axis
- For small deformation problems, we assume that the length and radius do not change significantly under torsion
- The primary deformation we are concerned with in torsion is the angle of twist, denoted with  $\phi$



The shear strain at points on the cross section increases linearly with  $\rho$ , i.e.,  $\gamma = (\rho/c)\gamma_{\max}$ .

21

## torsion formula

- For a linearly elastic material, Hooke's Law in shear will hold ( $\tau = G\gamma$ )
- This means that, like shear strain, shear stress will vary linearly along the radius

22

## torsion formula

- We can find the total force on an element,  $dA$  by multiplying the shear stress by the area

$$dF = \tau dA$$

- The torque ( $dT = \rho dF$ ) produced by this force is then

$$dT = \rho(\tau dA)$$

23

## torsion formula

- Integrating over the whole cross-section gives

$$T = \int_A \rho(\tau dA) = \frac{\tau_{max}}{c} \int_A \rho^2 dA$$

- The integral  $\int_A \rho^2 dA$  is also called the Polar Moment of Inertia,  $J$ , so we can write

$$\tau_{max} = \frac{Tc}{J}$$

24

- We know that  $J = \int_A \rho^2 dA$ , so we can compute this for some common shapes
- For a solid circular cross-section, we have

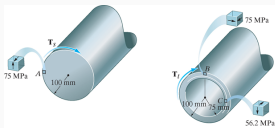
$$J = \int_0^c \rho^2 (2\pi \rho d\rho) = \frac{\pi}{2} c^4$$

- For a circular tube we have

$$J = \int_{c_1}^{c_2} \rho^2 (2\pi \rho d\rho) = \frac{\pi}{2} (c_2^4 - c_1^4)$$

25

## example 5.1



**Figure 8:** On left is a solid 100 mm radius tube, while on the right is a hollow tube with outer radius of 100 mm and inner radius of 75 mm. Element A is on the surface of the solid tube on the left, element B is on the outer surface of the hollow tube on the right and Element C is on the inner surface of the hollow tube

The allowable shear stress is 75 MPa. Determine the maximum torque that can be applied to each of the cross-sections shown and find the stress acting on a small element at A, B and C.

26