

Name:

Homework 1

Due 9 Feb 2021

- Composites material properties are usually calibrated in the material coordinate system, with the base vectors denoted as a_i . To carry out analysis, we usually have to set up problem coordinate system denoted as e_i . In the most general case, the material coordinate system can be obtained through three consecutive rotations from the problem coordinate system. Let us assume we rotate e_i about e_1 by θ_1 to become e'_i . This is followed by a rotation of e'_i about e'_2 by θ_2 to become e''_i . Finally, we rotate rotate e''_i about e''_3 by θ_3 to give a_i . Find the effective total direction cosine matrix Q_{ij} such that $a_i = Q_{ij}e_j$.

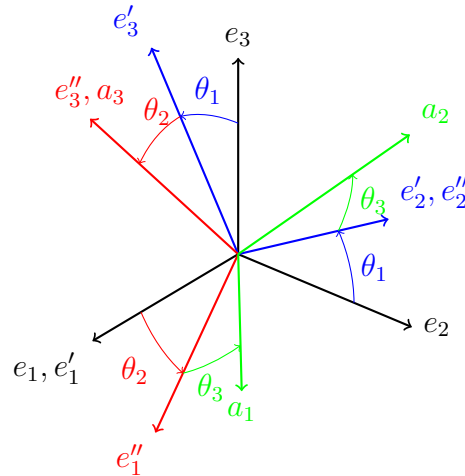


Figure 1: Illustration of rotations described in Problem 1

- Consider a unidirectional fiber-reinforced composite as a linearly elastic, orthotropic material with

Property	Value
E_1	20 Mpsi
E_2	1.4 Mpsi
E_3	2.5 Mpsi
G_{12}	0.8 Mpsi
G_{13}	2.0 Mpsi
G_{23}	1.5 Mpsi
ν_{12}	0.2
ν_{13}	0.3
ν_{23}	0.25

Where the 1-direction is in the fiber direction, the 2-direction is normal to the fibers in the plane, and the 3-direction is normal to the plane. Four 45° laminae are used to make a box beam (45° refers to the rotation about the outward normal at each wall). Compute the 6x6 stiffness matrix for each wall of the box beam in the global coordinate system. The cross-section of the box is shown in Figure 2.

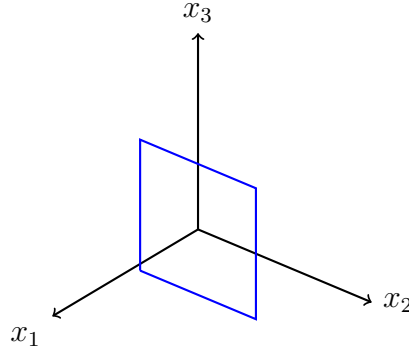


Figure 2: Cross-section of box beam for Problem 2

3. A plate mechanic uses an atypical arrangement of the stress and strain tensors in vector form. His preferred format is

$$\sigma = [\sigma_{11} \quad \sigma_{12} \quad \sigma_{22} \quad \sigma_{13} \quad \sigma_{23} \quad \sigma_{33}]^T$$

$$\epsilon = [\epsilon_{11} \quad \epsilon_{12} \quad \epsilon_{22} \quad \epsilon_{13} \quad \epsilon_{23} \quad \epsilon_{33}]^T$$

- (a) Find the rotation matrices for R_σ and R_ϵ for a general direction cosine matrix.
- (b) Verify that $(R_\epsilon)^{-1} = R_\sigma$ for the direction cosine matrix in Problem 1 with $\theta_1 = 20^\circ$, $\theta_2 = 45^\circ$, and $\theta_3 = 60^\circ$.
- (c) Compute the new 6x6 stiffness matrix (C') for the left wall of the box beam in this new notation.
- (d) Compare C' with the original stiffness matrix in the left wall, C and verify that $C' = BCB^T$ where

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

why is this the case?