Lecture 13 - SwiftComp

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schedule

- 22 Mar - SwiftComp
- 24 Mar - Fourier Analysis, HW 5 Due, Project Abstract Due
- 29 Mar - Method of Cells
- 31 Mar - Workday

outline

- variational asymptotic method
- swiftcomp
- asymptotic homogenization
- moose

variational asymptotic method

variational asymptotic method

- For more details see the paper:
- Variational asymptotic method for unit cell homogenization of periodically heterogeneous materials
- Wenbin Yu, Tian Tang

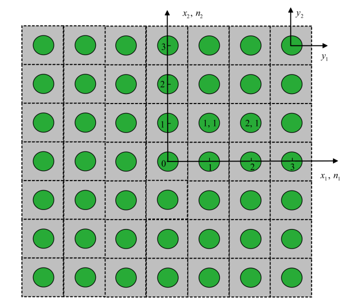
assumptions

- exact solutions of field variables have volume averages over the unit cell

$$v_i = \frac{1}{\Omega}\int_{\Omega} u_i d\Omega \equiv \langle u_i \rangle$$

- effective material properties of the unit cell are independent of geometry, boundary conditions, and loading
- homogenization is also only appropriate when $h/l \ll 1$ where h is the characteristic size of the unit cell and l is the characteristic wavelength of deformation

variational statement



variational statement

- coordinates are set up as shown, with two sets of cartesian coordinates and one set of integer coordinates
- x_i denote the global coordinate system
- y_i denote the local coordinate system
- n_i denote each individual unit cell

variational statement

- since any geometry would produce the same effective stiffness, this work considers an infinite space covered by periodic repititions of the unit cell
- for an elastic material, the total potential energy is given as

$$\Pi = \sum_{n=-\infty}^{\infty} \int_{\Omega} \frac{1}{2} C_{ijkl}(y_1, y_2, y_3) \epsilon_{ij} \epsilon_{kl} d\Omega$$

periodicity

- the strain field in each unit cell is related to the displacement
- displacements in adjacent unit cells must be continuous, for example

$$u_i(n_1, n_2, n_3; d_1/2, y_2, y_3) = u_i(n_1 + 1, n_2, n_3; -d_1/2, y_2, y_3)$$

- this constraint can be imposed in the other two directions as well
- using the lagrange multiplier technique, these constraints can be included in a functional with the total potential energy

substitution

- it is convenient to write

$$u_i(x_i; y_i) = v_i(x_i) + w_i(x_i; y_i)$$

- where $\langle w_i \rangle = 0$
- a further substitution is convenient for the solution

$$w_i(x_i; y_i) = y_j \frac{\partial v_i}{\partial x_j} + \chi(x_i; y_j)$$

implementation

- There are more details in the paper as well as some applications and results
- This is the method used by SwiftComp

swiftcomp

swiftcomp

- SwiftComp is a software built on the Variational Asymptotic Method, applied in particular to composites
- You are not required to use SwiftComp in your project (we will also discuss Fourier and Method of Cells methods), but it may be the easiest
- SwiftComp itself is a command-line tool, but Dr. Yu has merged it with a couple other software tools to give some form of GUI
- gmsh4sc - modifies gmesh to work build mesh for SwiftComp, runs SwiftComp from the gmesh gui
- texgen4sc - uses a textile software (for composite weaves) and runs swiftcomp from the texgen gui
- plugins for Ansys and ABAQUS - if you use either of these software programs, you can run Swiftcomp from them as a

swiftcomp

- SwiftComp can either be run in the cloud or downloaded to run locally
- Right now Dr. Yu only has the linux executables for download, I contacted him to get the Windows files
- We will run through a few demos, but before we get lost in some of the software details, it is important to remember the big picture

micromechanics

- In micromechanics, we are trying to represent a periodic structure with some effective property
- For example, if we have a beam with a very complex cross-section, we can calculate the inertia of that cross-section and then model the beam as a straight line
- We may, however, need to know the local stresses at certain points in the beam, the ability to recover local stresses is what SwiftComp calls "dehomogenization"

workflow

- Thus the general workflow in Swiftcomp is
  1. Run SwiftComp to homogenize some unit cell (beam cross-section, fiber weave, etc.)
  2. Run FEA to get displacements/stresses using homogenized stiffness
  3. Run SwiftComp with FEA displacement/stress data to find the local stresses

links

- You should be able to run SwiftComp in the cloud (requires an account at cdmhub.org)
- For gmesh[1] (arbitrary shapes)
- For texgen[2] (woven composites)

[1] https://cdmhub.org/tools/scstandard
[2] https://cdmhub.org/tools/texgen4sc/

asymptotic homogenization

asymptotic homogenization

- another technique, which does not use variational calculus, is described in:
- Asymptotic homogenisation in linear elasticity. Part I: Mathematical formulation and finite element modelling
- J. Pinho-da-Cruz, J.A. Oliveira, F. Teixeira-Dias
- This approach is implemented in the open-source MOOSE framework[3]

[3] https://mooseframework.inl.gov/

asymptotic homogenization

- the displacement field solution to an elasticity solution is approximated by asymptotic expansion

$$u_i^{\epsilon}(x) = u_i^{(0)}(x,y) + \epsilon u_i^{(1)}(x,y) + \epsilon^2 u_i^{(2)}(x,y)$$

such that

$$\varepsilon_{ij}^{(0)} = \frac{1}{2}\left(\frac{\partial u_i^{(0)}}{\partial y_j} + \frac{\partial u_j^{(0)}}{\partial y_i}\right)$$

$$\varepsilon_{ij}^{(r)} = \frac{1}{2}\left(\frac{\partial u_i^{(r-1)}}{\partial x_j} + \frac{\partial u_j^{(r-1)}}{\partial x_i} + \frac{\partial u_i^{(r)}}{\partial y_j} + \frac{\partial u_j^{(r)}}{\partial y_i}\right)$$

first-order

- When $\epsilon \ll 1$ a first-order expansion is often sufficient, which simplifies the homogenization and localization
- The homogenized stiffness can be found as

$$C_{ijmn}^{H} = \frac{1}{|Y|}\int_Y C_{ijkl}(y)\left[\delta_{km}\delta_{ln} - \frac{\partial \chi_k^{mn}}{\partial y_l}\right]$$

- And localized stresses and strains can be found as

$$\sigma_{ij}^{(1)}(x,y) = C_{ijkl}(y)\left(\delta_{km}\delta_{ln} - \frac{\partial \chi_k^{mn}}{\partial y_l}\right)\frac{\partial u_m^{(0)}}{\partial x_n}$$

MOOSE

moose framework

- Asymptotic expansion homogenization is built-in to the open-source MOOSE framework
- Although installing it locally can be a little bit tricky, this might be easier than fussing with the cloud-based SwiftComp implementation, and I am personally interested in comparing results from the two methodologies
- Another advantage to the MOOSE AEH approach is that a multi-scale model can be done simultaneously (compared to SwiftComp, where you would input a stiffness into some other FEA code, then take those stresses/strains and input them for a localization study)

installation

- MOOSE now provides Docker containers, this is probably the easiest way to get started
- Will essentially install a virtual linux environment with the source code already set up and compiled
- Install Docker Desktop[4] for your environment
- After the installation has completed, you can download, install, and run initial tests by typing

```
docker run -ti idaholab/moose:latest /bin/bash -c
'cd test; ./run_tests'
```

Into a terminal

[4] https://www.docker.com/get-started/