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Name:

Homework 3

Due 3 March 2021

1. For a functional $I[y] = \int_0^1 (\dot{y}^2 + 12xy)dx$ with $y(0) = 0$ and $y(1) = 0$, find the function y which corresponds to the stationary value of I.

2. Use two different approaches to find the maximum area of a rectangular of given perimeter L.

3. Find the stationary curve of the functional $I[y] = \int_{-1}^1 \sqrt{y(1+\dot{y}^2)}dx$ with boundary conditions $y(-1) = 1$ and $y(1) = 1$

4. Find the natural conditions to minimize the functional
$$I[x, y, z] = \int_{t_0}^{t_1} \left[\frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \frac{1}{2}k(x^2 + y^2 + z^2)\right] dt \tag{1}$$

5. Does the following functional have stationary points. If so, under which conditions, if not, why not?
$$I[y] = \int_0^{\pi/2} \left[x \sin y + \left(\frac{x^2}{2} \cos y\right) \dot{y}\right] dx \tag{2}$$
with $y(0) = 0$ and $y(\pi/2) = \pi/2$

6. Find the curve corresponding to the stationary value of the functional
$$I[y, z] = \int_0^1 (\dot{y}\dot{z} + \dot{y}^2 + \dot{z}^2)dx \tag{3}$$
with $y(0) = z(0) = 0$ and $y(1) = z(1) = 1$

7. The potential energy of a circular plate with radius R under axisymmetric distributed load, $q(r)$, with $r \in [0, R]$ can be expressed in terms of deflection, $w(r)$ as
$$I[w] = \int_0^R \left(r\ddot{w}^2 + \frac{\dot{w}^2}{r} + 2\mu\dot{w}\ddot{w} - \frac{2q}{D}rw\right) dr \tag{4}$$
where D and μ are elastic constants. Show that $w(r)$ must satisfy the following equation of equilibrium
$$r\,\ddddot{w} + 2\,\dddot{w} - \frac{\ddot{w}}{r} + \frac{\dot{w}}{r^2} = \frac{qr}{D} \tag{5}$$

8. Find the Euler-Lagrange equation for the following functional
$$J[u(x, y, z)] = \int_G \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + 2uf(x, y, z)\right] dxdydz \tag{6}$$
where $f(x, y, z)$ is a given known function.