Lecture 15 - Method of Cells

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schedule

- Mar 30 - Method of Cells
- Apr 1 - Damage Theory
- Apr 6 - Damage Theory
- Apr 8 - Dislocation Theory

outline

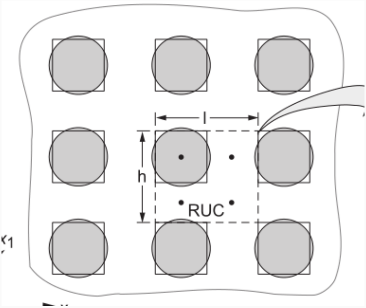
- method of cells
- results

method of cells

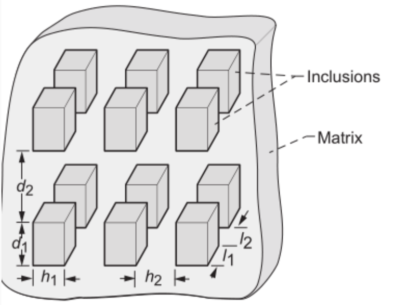
method of cells

- The methodology referred to as the "method of cells" can refer to a few different variations of the same general theory
- Some terminology the author uses are "doubly periodic" and "triply periodic"

doubly periodic



triply periodic

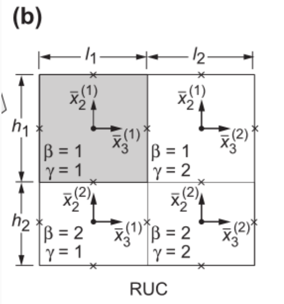


periodicity

- While both models are actually triply periodic, a continuous fiber cross-section can be modeled in 2D, while discontinuous fibers need to be modeled in 3D
- We will follow the derivation for the 2D model

2D problems

- For 2D problems (i.e. continuous fibers) the Repeating Unit Cell is divided up into 4 sub-cells



2D problems

- As a first-order approximation (which should be sufficient to determine stiffness properties) we assume a displacement function which varies linearly from the center of the cell

$$u_i = w_i(x) + \bar{x}_2\phi_i + \bar{x}_3\psi_i$$

- where w_i are the displacements at the centroid and ψ and ϕ relate the local displacement to the distance from the centroid
- Note that due to the linearity of this equation, the equilibrium equations will be automatically satisfied

continuity

- Displacements must be continuous across adjacent cells
- In the method of cells, this continuity is enforced in the average sense, thus the integral over adjacent boundaries of displacement components must be equal and opposite
- Due to this method of enforcing continuity, there is no actual shape implied in the RUC, so although we have drawn it as a rectangle, it could be any shape

$$\int_{-l/2}^{l/2} u_i^{1\gamma}|_{\bar{x}_2=-h/2} d\bar{x}_3 = \int_{-l/2}^{l/2} u_i^{2\gamma}|_{\bar{x}_2=h/2} d\bar{x}_3$$

$$\int_{-h/2}^{l/2} u_i^{\beta 1}|_{\bar{x}_3=l/2} d\bar{x}_2 = \int_{-h/2}^{h/2} u_i^{\beta 2}|_{\bar{x}_3=-l/2} d\bar{x}_2$$

continuity

- The next steps are to
- Perform the integration
- Enforce periodicty over the entire RUC (instead of the sub-cells)
- Represent functions of x (w_i) with a Taylor Series approximation

sub-cell strain field

$$
\begin{aligned}
\epsilon_{11} &= \frac{\partial}{\partial x_1} w_1 \\
\epsilon_{22} &= \phi_2 \\
\epsilon_{33} &= \psi_3 \\
2\epsilon_{12} &= \phi_1 + \frac{\partial}{\partial x_1} w_2 \\
2\epsilon_{13} &= \psi_1 + \frac{\partial}{\partial x_1} w_3 \\
2\epsilon_{23} &= \phi_3 + \psi_2
\end{aligned}
$$

effective stiffness

- To find the effective stiffness we still need to solve for ϕ_i and ψ_i in terms of the constituent properties
- By enforcing continuity of tractions along the cell and sub-cell boundaries we obtain enough equations to solve

$$
\begin{bmatrix} 0 & a_1 & a_2 & a_3 \\ a_4 & 0 & a_5 & a_6 \\ a_7 & a_8 & a_9 & 0 \\ a_{10} & a_{11} & 0 & a_{12} \end{bmatrix}
\begin{bmatrix} \phi_2^{11} \\ \phi_2^{22} \\ \psi_3^{11} \\ \psi_3^{22} \end{bmatrix}
=
\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix}
$$

local field equations

- The method of cells can also be used to relate local strains to macro strains
- Similar to the Eshelby and Mori-Tanaka discussion, we will use a strain concentration tensor to relate these fields

$$\epsilon = A\bar{\epsilon}$$

strain concentration tensor

- The general form of the strain concentratio tensor from the method of cells is

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 & 0 & 0 \\ A_{31} & A_{32} & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix}$$

imperfect bonding

- Up to now, we have assumed a perfect bond between constituents
- This is often not the case, and similar to other methods, we can incorporate the effects of imperfect bonding by considering an interphase between the fiber and matrix
- In this model, we can modify the continuity requirement at the interface between fiber and matrix

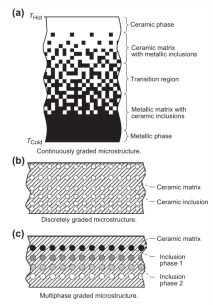
generalized method of cells

- For 3D problems (i.e. discontinuous composites) the same procedure can be used as described above, but in three dimensions
- The generalized method of cells provides the framework for subdividing an RUC into many smaller sub-cells
- This begins to look very similar to finite elements, but there are a few key differences
  - it automatically ensures periodicity
  - displacement continuity is enforced in only an average sense (as opposed to point-wise)
  - solution is not mesh-dependent (no shape functions)

high fidelity method of cells

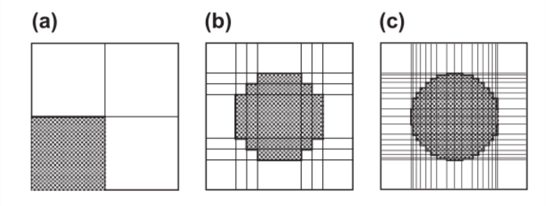
- The authors have made further modifications to the generalized method of cells
- Most analytic models (Mori-Tanaka) as well as the Method of Cells predict no shear coupling
- In most cases, while there is some shear coupling it is negligible
- In short fiber composites and woven composites, however, the shear coupling effects are not negligible
- The High Fidelity Method of Cells addresses this issue
- Assumes second-order displacement field, equilibrium is not automatically satisfied, is required to be satisfied in an average sense in each sub-cell
- Even higher order displacement fields can be considered for functionally graded composites

functionally graded composites

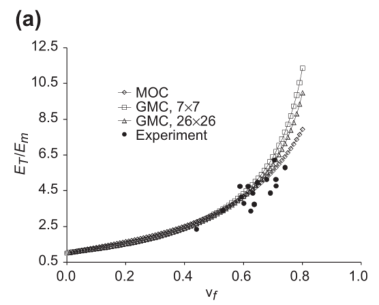


results

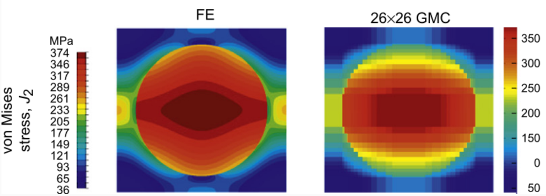
continuous composite



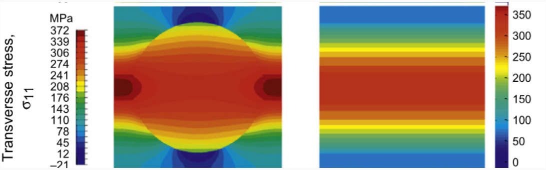
results



mises stress



tensile stress



shear stress