Last Updated: January 13, 2022 at 3:35pm

Name:

Homework 1

Due 4 Feb 2022

1. Composites material properties are usually calibrated in the material coordinate system, with the base vectors denoted as a_i. To carry out analysis, we usually have to set up problem coordinate system denoted as e_i. In the most general case, the material coordinate system can be obtained through three consecutive rotations from the problem coordinate system. Let us assume we rotate e_i about e_1 by θ_1 to become e_i'. This is followed by a rotation of e_i' about e_2' by θ_2 to become e_i''. Finally, we rotate rotate e_i'' about e_3'' by θ_3 to give a_i. Find the effective total direction cosine matrix Q_{ij} such that $a_i = Q_{ij}e_j$.

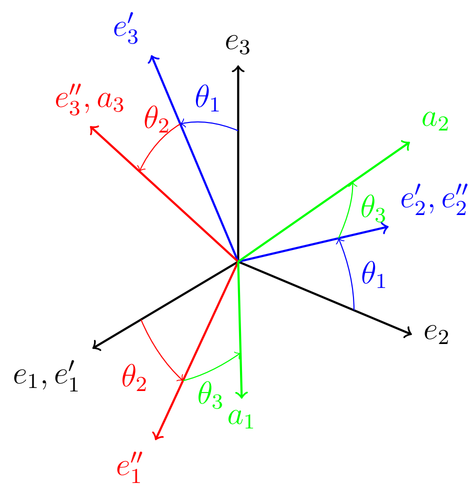

Figure 1: Illustration of rotations described in Problem 1

2. Consider a unidirectional fiber-reinforced composite as a linearly elastic, orthotropic material with

Property	Value
E_1	20 Mpsi
E_2	1.4 Mpsi
E_3	2.5 Mpsi
G_{12}	0.8 Mpsi
G_{13}	2.0 Mpsi
G_{23}	1.5 Mpsi
ν_{12}	0.2
ν_{13}	0.3
ν_{23}	0.25

Where the 1-direction is in the fiber direction, the 2-direction is normal to the fibers in the plane, and the 3-direction is normal to the plane. Four 45° laminae are used to make a box beam (45° refers to the rotation about the outward normal at each wall). Compute the 6x6 stiffness matrix for each wall of the box beam in the global coordinate system. The cross-section of the box is shown in Figure 2.

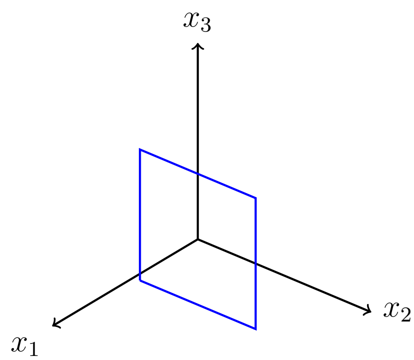

Figure 2: Cross-section of box beam for Problem 2

3. A plate mechanician uses an atypical arrangement of the stress and strain tensors in vector form. His preferred format is

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{22} & \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}^T$$
$$\epsilon = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{22} & \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{bmatrix}^T$$

   (a) Find the rotation matrices for R_σ and R_ϵ for a general direction cosine matrix.

   (b) Verify that $(R_\epsilon)^{-1} = R_\sigma^T$ for the direction cosine matrix in Problem 1 with $\theta_1 = 20°$, $\theta_2 = 45°$, and $\theta_3 = 60°$.

   (c) Compute the new 6x6 stiffness matrix (C') for the left wall of the box beam in this new notation.

   (d) Compare C' with the original stiffness matrix in the left wall, C and verify that $C' = BCB^T$ where

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

   why is this the case?