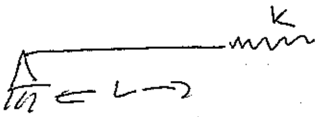
BC EXAMPLE (2.5)

~~POWER LINE (2.12)~~



$$I[u(x)] = \underbrace{\int_0^L \tfrac{1}{2} EA (u')^2 dx}_{\text{STRAIN ENERGY OF BAR}} + \underbrace{\tfrac{1}{2} k [u(L)]^2}_{\text{STRAIN ENERGY OF SPRING}} - \underbrace{\int_0^L p(x)\, u(x)\, dx}_{\text{WORK DONE}}$$

$$0 = \delta I = \int_0^L EA u' \delta u' dx + k u(L) \delta u|_L - \int_0^L p(x)\, \delta u(x)\, dx$$

$u = EAu'$ $\quad v = \delta u$

$du = (EAu')'$ $\quad dv = \delta u' dx$

$$EA u' \delta u|_0^L + k u(L) \delta u|_L - \underbrace{\int_0^L (EAu')' \delta u dx - \int_0^L p\, \delta u\, dx}$$

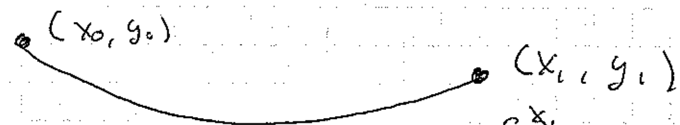
BOUNDARY CONDITIONS

$EAu' + ku = 0 \; @ x = L$

GOVERNING DIFF. EQ'S

$-(p + (EAu')') = 0$

POWER LINE (2.12)



$$I[y] = \underbrace{\rho \int_{x_0}^{x_1} y \sqrt{1+\dot{y}^2}\, dx}_{\text{POTENTIAL ENERGY}} \quad \text{SUBJECT TO} \quad J = \underbrace{\int_{x_0}^{x_1} \sqrt{1+\dot{y}^2}\, dx - C}_{\text{LENGTH OF POWER LINE IS CONSTANT}} = 0$$

$y(x_0) = y_0$

$y(x_1) = y_1$

~~BEAM DEFLECTION (B.8)~~

$$I^* = I + \lambda J = \int_{x_0}^{x_1} \left[\rho y \sqrt{1+\dot{y}^2} + \lambda \left(\sqrt{1+\dot{y}^2} - \frac{C}{x_1 - x_0} \right) \right] dx$$

THIS MOVES C INSIDE INTEGRAL

$F - \dot{y} \dfrac{\partial F}{\partial \dot{y}} = C \Leftarrow$ USE THIS EULER-LAGRANGE ~~[illegible] FORM~~

$$\underbrace{(\lambda + \rho y)\sqrt{1+\dot{y}^2}}_{F} - \dot{y}(\lambda + \rho y) \frac{2\dot{y}(\frac{1}{2})}{\sqrt{1+\dot{y}^2}} = C_1$$

CANCEL

$$= \frac{(\lambda+\rho y)(1+\dot{y}^2)}{\sqrt{1+\dot{y}^2}} - \frac{\dot{y}^2(\lambda+\rho y)}{\sqrt{1+\dot{y}^2}} \Rightarrow \frac{\lambda + \rho y}{\sqrt{1+\dot{y}^2}} = C_1$$

$$\frac{\lambda + \rho y}{C_1} = \sqrt{1+\dot{y}^2} \Rightarrow \left(\frac{\lambda+\rho y}{C_1}\right)^2 = 1 + \dot{y}^2 \Rightarrow \dot{y} = \frac{dy}{dx} = \sqrt{\left(\frac{\lambda+\rho y}{C_1}\right)^2 - 1}$$

HAS SOLUTION $\dfrac{\rho x}{C_1} + C_2 = \cosh^{-1}\left(\dfrac{\rho y + \lambda}{C_1}\right)$

C_1, C_2, λ CAN BE FOUND FROM $J = \int_{x_0}^{x_1} \sqrt{1+\dot{y}^2}\, dx = C$

AND $y(x_0) = y_0$ $\quad y(x_1) = y_1$

BEAM (3.6)

$$U = \frac{1}{2}\int \frac{M^2}{EI} = \underline{\frac{1}{2}\int EI\ddot{y}^2}$$ (STRAIN ENERGY OF BEAM)

$$I[y] = \int_0^L \Big[\underbrace{\tfrac{1}{2}EI(\ddot{y})^2}_{\text{STRAIN ENERGY}} - \underbrace{f(x)\,y}_{\text{WORK DONE}}\Big]\,dx$$

$$\delta I = \int_0^L EI\ddot{y}\,\delta\ddot{y}\,dx - \int_0^L f\,\delta y\,dx$$

$$= EI\ddot{y}\,\delta\dot{y}\Big|_0^L - \int_0^L \tfrac{d}{dx}(EI\ddot{y})\,\delta\dot{y}\,dx - \int_0^L f\,\delta y\,dx$$

$$= \underbrace{EI\ddot{y}\,\delta\dot{y}\Big|_0^L - \tfrac{d}{dx}(EI\ddot{y})\,\delta y\Big|_0^L}_{\text{BOUNDARY CONDITIONS}} + \int_0^L \underbrace{\tfrac{d^2}{dx^2}(EI\ddot{y} - f)}_{\text{EULER-LAGRANGE}}\,\delta y\,dx$$

MINIMIZE FUNCTIONAL (3.7)

$$I^* = \int_{x_0}^{x_1} \left[y^2 - z^2 + \lambda(x)(\dot{y} - y + z)\right] dx$$

EULER-LAGRANGE $\Rightarrow$ $\dfrac{\partial F}{\partial y} - \dfrac{d}{dx}\left(\dfrac{\partial F}{\partial \dot{y}}\right) = 0$ $\qquad$ $\dfrac{\partial F}{\partial z} - \dfrac{d}{dx}\left(\partial F/\partial \dot{z}\right) = 0$

$$2y - \lambda + \dot{\lambda} = 0 \qquad\qquad -2z + \lambda = 0$$

AND $\dot{y} - y + z = 0$ $\qquad\qquad$ $\underline{z = \lambda/2}$

$\dot{y} - y = 0$ (HOMOGENEOUS)

$y_H = C_1 e^x$

$y_p = z = \lambda/2$

CAN BE SOLVED FOR BOUNDARY CONDITIONS AS PROVIDED

$2(C_1 e^x + \lambda/2)$