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Name:

Homework 4

Due 1 April 2019

1. The differential statement of one-dimensional heat transfer with ϕ as the unknown function is

$$\frac{d^2\phi}{dx^2} + Q(x) = 0 \qquad \text{for} \qquad (0 \leq x \leq L) \tag{1}$$

with boundary conditions of $\phi(0) = \phi_0$ and $\phi(L) = \phi_0$

   (a) Find the corresponding variational statement for this problem

   (b) If

$$Q(x) = \begin{cases} \phi_0/L^2 & 0 \leq x \leq L/2 \\ 0.1\phi_0/L^2 & L/2 \leq x \leq L \end{cases} \tag{2}$$

   Solve the differential statement exactly

   (c) Use the Ritz method to find the approximate solution for $n = 2, 3, 4$ where

$$\phi = \sum_{i=1}^{n} a_i x^{i-1} \tag{3}$$

   (d) Plot a comparison between the exact solution and the Ritz method solutions

2. Find the third-order variational asymptotic approximation of the stationary points for

$$f(u, \epsilon) = u^2 + u^3 + 2\epsilon u + \epsilon u^2 + \epsilon^2 u \tag{4}$$

3. Use the variational asymptotic method to find an approximate solution for the stationary points of

$$f(u, \epsilon) = u + u^2 + \epsilon \sin u \tag{5}$$

accurate to the second order. Compare the approximate solution with the exact solution when $\epsilon = 0.1$ and $\epsilon = 0.2$