Lecture 16 - Damage Theory

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schedule

- 12 Apr - Damage Theory
- 14 Apr - Project Work Day
- (19 Apr) - Class Canceled
- Project Work Days

outline

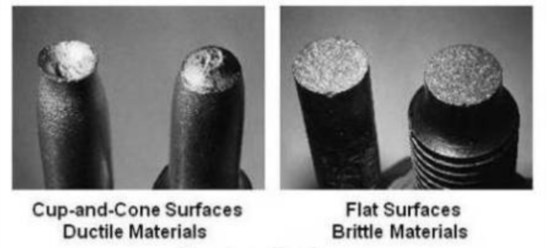
- failure
- spherical void growth
- cylindrical void growth
- micro cracks

failure

failure

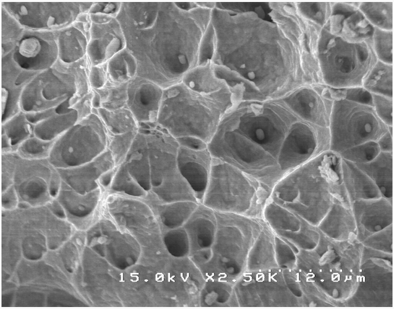
- Ductile fracture
  - plastic deformation prior to failure
  - dimpled, cup and cone fracture surface
- Brittle fracture
  - rapid crack propagation
  - generally flat fracture surface
  - common in glasses, thermoset polymers, brittle metals (BCC and HCP crystals)

fracture surface

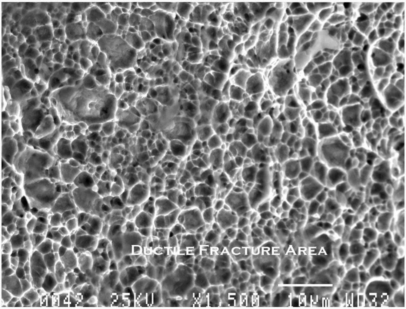


Fracture Surfaces

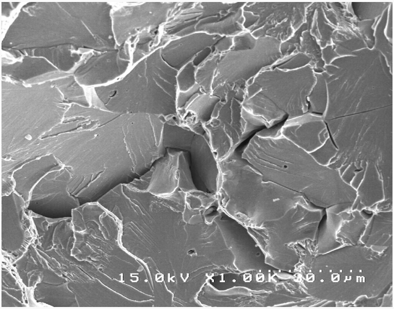
ductile fracture surface



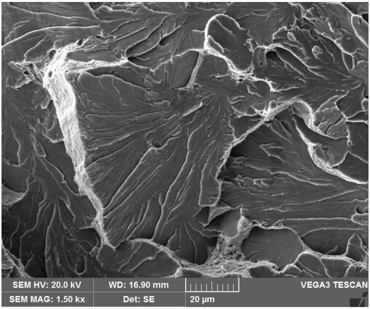
ductile fracture surface



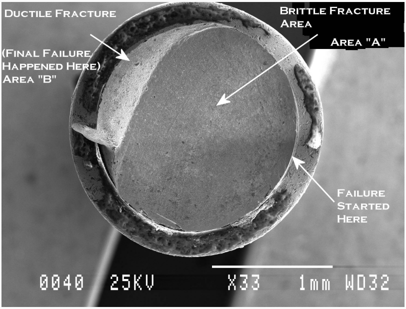
brittle fracture surface



brittle fracture surface



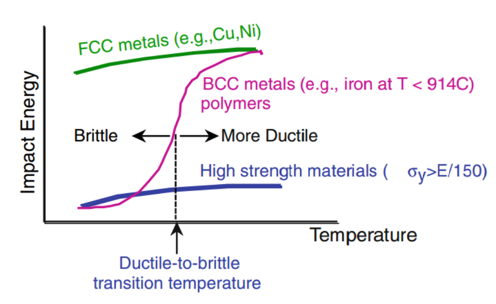
transition



what affects failure method

- While some materials are generally ductile or brittle, there are factors that can cause brittle failure in a ductile material
- Strain rate (materials are often more brittle at high strain rates)
- Temperature also affects ductility of many materials

temperature effects



spherical void growth

void growth

- From what we have observed on fracture surfaces, it appears that ductile materials fail due to void growth
- Some of the earliest and simplest micromechanical damage models are for spherical void growth
- Spherical voids are typical of uniaxial tension

spherical voids in viscous materials

- If we consider a spherical void in a linear, viscous RVE under some uniform remote stress, σ^{∞} the constitutive behavior is

$$\sigma_{ij} = L_{ijkl}\dot{\epsilon}_{kl}$$

- L is analogous to the stiffness tensor, but relates stress to strain-rate
- For an isotropic material, we can define L in terms of η and ν to give the familiar relationshiop

$$\sigma_{ij} = 2\eta\left(\dot{\epsilon}_{ij} + \frac{\nu}{1-2\nu}\dot{\epsilon}_{kk}\delta_{ij}\right)$$

spherical voids in viscous materials

- Eshelby's model holds true for a viscous material as well as a solid, so we can find the stress inside the void as

$$\sigma_{ij} = L_{ijkl}\left(\dot{\epsilon}_{kl}^{\infty} + \dot{\epsilon}_{kl}^{d} - \dot{\epsilon}_{kl}^{*}\right)$$

- But we know that there is no stress inside the void, thus we can say

$$\dot{\epsilon}_{kl}^{\infty} + \dot{\epsilon}_{kl}^{d} - \dot{\epsilon}_{kl}^{*} = 0$$

- Where, in this case, $\dot{\epsilon}_{kl}^{*}$ is the strain-rate of the void

spherical voids in viscous materials

- pp. 266-267 in the text show the details for calculating the Eshelby tensor with a spherical void
- However, when a non-uniform load is applied (uni-axial or biaxial tension) the void will no longer be spherical
- Also, there are not many solids that can be adequately described with a linearly viscous constitutive law

cylindrical void growth

mcclintock solution

- McClintock developed the first widely-accepted void growth model
- He assumed a cylindrical void shape (for tension along the cylinder axis)
- He assumed the material surrounding the void was incompressible, rigid-plastic
- In spite of the simplifications made, this model has served as a benchmark for many homogeneous schemes.

mcclintock solution

- To date, the McClintock solution is the only exact analytic solution for void growth in non-linear solids
- A full derivation, for some assumptions in yield criterion and plastic flow rule is in text pp. 268-271

mcclintock solution

- For the Von Mises (J2) yield criterion and the flow rule defined on p. 268, we find

$$\frac{\dot{a}}{a} = \frac{\sqrt{3}}{2}|\dot{\epsilon}_z| \sinh\left(\frac{\sqrt{3}\sigma^{\infty}}{\sigma_{YS}}\right) - \frac{1}{2}\dot{\epsilon}_z$$

- McClintock predicts that void growth increases exponentially with applied stress, while the linear viscous solution predicts a linear relationship between void growth and stress

mcclintock solution

- Many damage models use the volume fraction of voids
- In the McClintock solution, the matrix is considered incompressible
- This means we can write the rate of change of volume fraction as

$$\dot{f} = \sqrt{3}f(1-f)|\dot{\epsilon}_z| \sinh\left(\frac{\sqrt{3}\sigma_{11}}{|\sigma_{33}-\sigma_{11}|}\right)$$

gurson model

- Gurson's model builds on McClintock's solution
- He homegenizes the micro-stress to define a yield function entirely in terms of the macro-stresses
- A full derivation (for the same assumptions as McClintock) is on pp. 273-277

gurson model

- Gurson defines several intermediate stress calculations

$$\sigma_{eq} = \sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}}$$
$$\sigma'_{ij} = \sigma_{ij} - \sigma_m$$
$$\sigma_m = \frac{1}{3}\sigma_{ii}$$

gurson model

- He then finds the yield function as

$$\left(\frac{\sigma_{eq}}{\sigma_{YS}}\right)^2 + 2f \cosh\left(\frac{\sqrt{3}\sigma_{11}}{\sigma_{YS}}\right) - (1 + f^2) = 0$$

- Note: in Gurson's assumptions, the cylinder is along the 3 direction and an axi-symmetric state of stress with $\sigma_{11} = \sigma_{22}$ was assumed.
- Also, these stress quantities are volume averaged over the RVE
- Gurson has essentially used micromechanics to define a new constitutive relation

gurson tvergaard needleman

- Some moderate improvements were made to the Gurson model and are known as the Gurson-Tvergaard-Needleman model
- An elastic-plastic model with power-law hardening is used (instead of rigid plastic)

$$\bar{\sigma}_0 = \sigma_{YS}\left(1 - \frac{E}{\sigma_{YS}}\bar{\epsilon}_p\right)^N$$

- Tvergaard modified McClintock's void growth solution with a numerical analysis for a periodic array of voids
- Needleman introduced an equivalent damage parameter, f^* instead of volume fraction of voids.
- Equivalent damage includes void growth and nucleation of

needleman

- Needleman's contribution is to account for the rapid reduction in stiffness at some critical void volume fraction

$$f^*(f) = \begin{cases} f & \text{if } f \leq f_c \\ f_c + \frac{1/q_1 - f_c}{f_f - f_c}(f - f_c) & \text{if } f_c < f \leq f_f \\ 1/q - 1 & \text{if } f > f_f \end{cases}$$

- Where f_c is the void volume fraction at the incidence of coalescence
- f_f is the void volume fraction at failure

micro cracks

micro cracks

- There are many micro-crack damage models
- Some factors differentiating the models are whether they include plasticity
- Also whether they can handle anisotropy or heterogeneity
- Fracture mechanics becomes much more complicated in anisotropic or heterogeneous materials

micro cracks

- The Barenblatt-Dugdale model assumes micro-crack density is a measure of the damage state
- A key assumption is that the overall damage (due to permanent crack growth) is only associated with the hydrostatic stress
- Deviatoric stress has no effect
- This is essentially assuming cracks only grow in Mode I

fracture mechanics

- In fracture mechanics we consider three different modes
- Mode I is known as the “opening mode”
- Mode II is known as the “sliding mode”
- Mode III is known as the “tearing mode”

fracture mechanics

mixed-mode

- In fracture mechanics, we can consider the effect of the deviatoric stress on a crack
- Mixed-mode fracture analysis shows that cracks will always tend to open due to Mode I
- Shear stresses (i.e. deviatoric stress) can effect the principal stresses near a crack tip
- For many micro-cracks in a representative volume, we assume this effect is negligible

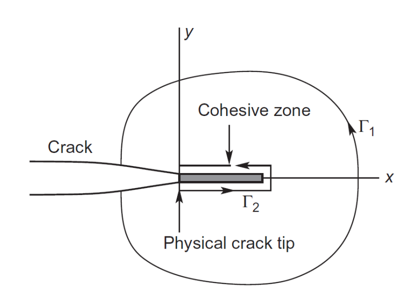
cohesive zone

- The Barenblatt-Dugdale model also assumes that there is a cohesive zone around the crack
- Cohesive zones are an alternate approach to modeling cracks

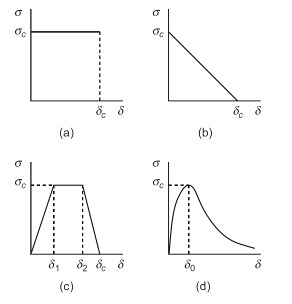
cohesive elements

- Cohesive elements are one way to model crack propagation
- We need to know the crack path in advance, we model the the crack growth using a traction-separation law
- The cohesive zone theory assumes stress can never reach infinity, the maximum allowable stress in a material is the stress required to separate atoms
- The stress required to separate the atoms changes as a function based on their Traction-Separation law, until the atomic bond is broken

cohesive zone



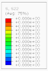
traction separation

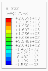


cohesive zone uses

- In practice, the cohesive zone can be used to model crack growth
- It is most often used to model de-bonding of adhesives
- Also commonly used to model delamination in composites

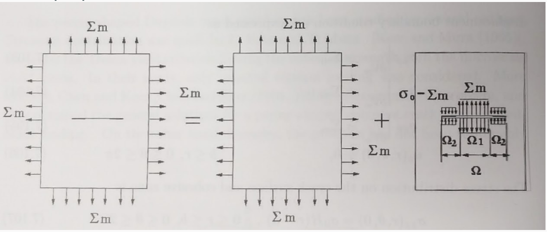
dcb





single crack

- To solve the problem of many cohesive cracks in an RVE, we first consider the case of a single crack
- For a crack under a uniform tri-axial stress, we consider the superposition



cohesive stress

- The cohesive stress, σ_0 can be found as

$$\frac{\sigma_0}{\Sigma_m} = \frac{1 + \sqrt{\left(\frac{4}{1-2\nu^*}\frac{\sigma_Y S}{\Sigma_m}\right)^2 - 3}}{4}$$

macro strain

- The macro strain tensor is not necessarily the volume average
- This is due to the discontinuities (cracks)
- The macro stress is the volume average (crack surfaces are traction free)

macro strain

- One technique for finding the macro strain involves finding some additional strain term

$$\varepsilon_{ij}\epsilon_{ij}^{0} + \epsilon_{ij}^{(}add)$$

- Where $\epsilon_{ij}^{0} = D_{ijkl}\sigma_{kl}$ and $\epsilon_{ij}^{(add)} = H_{ijkl}\sigma_{kl}$

additional strain

- The additional strain is given by

$$\epsilon_{ij}^{(add)} = \frac{4f(1-\nu^*)\Sigma_m \delta_{ij}}{3\beta\pi\mu^* \sqrt{1-\left(\frac{\Sigma_m}{\sigma_0}\right)^2}}$$

- Where β is the ratio between the volume of the physical crack and the volume of the cohesive crack and f is the effective volume fraction of cracks
- While cracks are assumed to be "penny-shaped" disks, their volume is treated as spherical for these purposes

interaction effect

- In the above calculations, properties with a * superscript an be calculated as either matrix or average properties
- We can capture the interaction effect by using average properties
- This is similar to the self-consistent model, and would need to be found iteratively
- Note: this does not model damage growth, which is still a field of active research, particularly in micromechanics