Lecture 16 - Damage Theory

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schedule

- Apr 1 - Damage Theory
- Apr 6 - Damage Theory
- Apr 8 - Dislocation Theory
- Project Work Days

outline

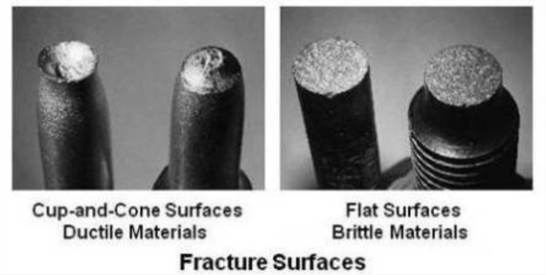
- failure
- spherical void growth
- cylindrical void growth
- micro cracks

failure

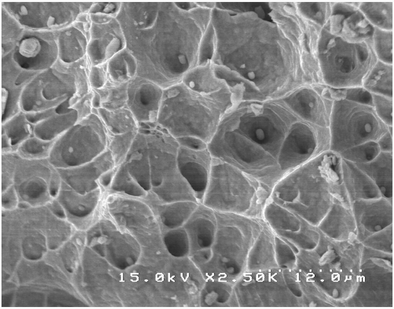
failure

- Ductile fracture
  - plastic deformation prior to failure
  - dimpled, cup and cone fracture surface
- Brittle fracture
  - rapid crack propagation
  - generally flat fracture surface
  - common in glasses, thermoset polymers, brittle metals (BCC and HCP crystals)

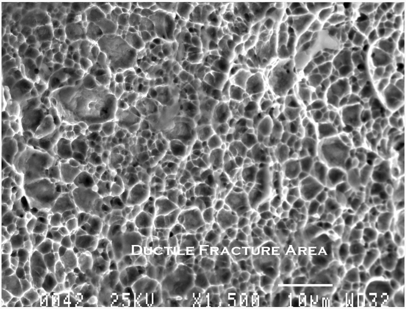
fracture surface



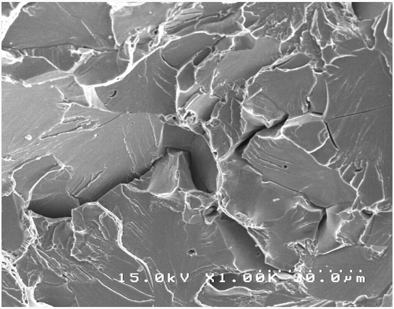
ductile fracture surface



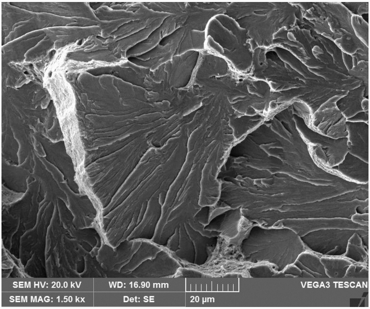
ductile fracture surface



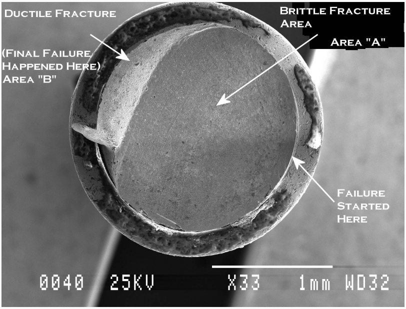
brittle fracture surface



brittle fracture surface



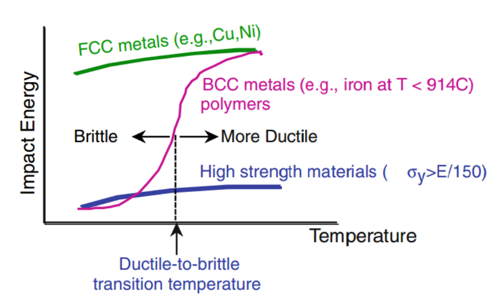
transition



what affects failure method

- While some materials are generally ductile or brittle, there are factors that can cause brittle failure in a ductile material
- Strain rate (materials are often more brittle at high strain rates)
- Temperature also affects ductility of many materials

temperature effects



spherical void growth

void growth

- From what we have observed on fracture surfaces, it appears that ductile materials fail due to void growth
- Some of the earliest and simplest micromechanical damage models are for spherical void growth
- Spherical voids are typical of uniaxial tension

spherical voids in viscous materials

- If we consider a spherical void in a linear, viscous RVE under some uniform remote stress, σ^{∞} the constitutive behavior is

$$\sigma_{ij} = L_{ijkl}\dot{\epsilon}_{kl}$$

- L is analogous to the stiffness tensor, but relates stress to strain-rate
- For an isotropic material, we can define L in terms of η and ν to give the familiar relationshiop

$$\sigma_{ij} = 2\eta\left(\dot{\epsilon}_{ij} + \frac{\nu}{1-2\nu}\dot{\epsilon}_{kk}\delta_{ij}\right)$$

spherical voids in viscous materials

- Eshelby's model holds true for a viscous material as well as a solid, so we can find the stress inside the void as

$$\sigma_{ij} = L_{ijkl} \left(\dot{\epsilon}^{\infty}_{kl} + \dot{\epsilon}^{d}_{kl} - \dot{\epsilon}^{*}_{kl} \right)$$

- But we know that there is no stress inside the void, thus we can say

$$\dot{\epsilon}^{\infty}_{kl} + \dot{\epsilon}^{d}_{kl} - \dot{\epsilon}^{*}_{kl} = 0$$

- Where, in this case, $\dot{\epsilon}^{*}_{kl}$ is the strain-rate of the void

spherical voids in viscous materials

- pp. 266-267 in the text show the details for calculating the Eshelby tensor with a spherical void
- However, when a non-uniform load is applied (uni-axial or biaxial tension) the void will no longer be spherical
- Also, there are not many solids that can be adequately described with a linearly viscous constitutive law

cylindrical void growth

mcclintock solution

- McClintock developed the first widely-accepted void growth model
- He assumed a cylindrical void shape (for tension along the cylinder axis)
- He assumed the material surrounding the void was incompressible, rigid-plastic
- In spite of the simplifications made, this model has served as a benchmark for many homogeneous schemes.

mcclintock solution

- To date, the mcClintock solution is the only exact analytic solution for void growth in non-linear solids
- A full derivation, for some assumptions in yield criterion and plastic flow rule is in text pp. 268-271

mcclintock solution

- For the Von Mises (J2) yield criterion and the flow rule defined on p. 268, we find

$$\frac{\dot{a}}{a} = \frac{\sqrt{3}}{2}|\dot{\epsilon}_z| \sinh\left(\frac{\sqrt{3}\sigma^{\infty}}{\sigma_{YS}}\right) - \frac{1}{2}\dot{\epsilon}_z$$

- McClintock predicts that void growth increases exponentially with applied stress, while the linear viscous solution predicts a linear relationship between void growth and stress

mcclintock solution

- Many damage models use the volume fraction of voids
- In the McClintock solution, the matrix is considered incompressible
- This means we can write the rate of change of volume fraction as

$$\dot{f} = \sqrt{3}f(1-f)|\dot{\epsilon}_z| \sinh\left(\frac{\sqrt{3}\sigma_{11}}{|\sigma_{33}-\sigma_{11}|}\right)$$

gurson model

- Gurson's model builds on McClintock's solution
- He homegenizes the micro-stress to define a yield function entirely in terms of the macro-stresses
- A full derivation (for the same assumptions as McClintock) is on pp. 273-277

gurson model

- Gurson defines several intermediate stress calculations

$$\sigma_{eq} = \sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}}$$
$$\sigma'_{ij} = \sigma_{ij} - \sigma_m$$
$$\sigma_m = \frac{1}{3}\sigma_{ii}$$

gurson model

- He then finds the yield function as

$$\left(\frac{\sigma_{eq}}{\sigma_{YS}}\right)^2 + 2f \cosh\left(\frac{\sqrt{3}\sigma_{11}}{\sigma_{YS}}\right) - (1 + f^2) = 0$$

- Note: in Gurson's assumptions, the cylinder is along the 3 direction and an axi-symmetric state of stress with $\sigma_{11} = \sigma_{22}$ was assumed.
- Also, these stress quantities are volume averaged over the RVE
- Gurson has essentially used micromechanics to define a new constitutive relation

gurson tvergaard needleman

- Some moderate improvements were made to the Gurson model and are known as the Gurson-Tvergaard-Needleman model
- An elastic-plastic model with power-law hardening is used (instead of rigid plastic)

$$\bar{\sigma}_0 = \sigma_{YS} \left(1 - \frac{E}{\sigma_{YS}} \bar{\epsilon}_p\right)^N$$

- Tvergaard modified McClintock's void growth solution with a numerical analysis for a periodic array of voids
- Needleman introduced an equivalent damage parameter, f^* instead of volume fraction of voids.
- Equivalent damage includes void growth and nucleation of

needleman

- Needleman's contribution is to account for the rapid reduction in stiffness at some critical void volume fraction

$$f^*(f) = \begin{cases} f & \text{if } f \le f_c \\ f_c + \frac{1/q_1 - f_c}{f_f - f_c}(f - f_c) & \text{if } f_c < f \le f_f \\ 1/q - 1 & \text{if } f > f_f \end{cases}$$

- Where f_c is the void volume fraction at the incidence of coalescence
- f_f is the void volume fraction at failure

micro cracks

micro cracks

- There are many micro-crack damage models
- Some factors differentiating the models are whether they include plasticity
- Also whether they can handle anisotropy or heterogeneity
- Fracture mechanics becomes much more complicated in anisotropic or heterogeneous materials

micro cracks

- The Barenblatt-Dugdale model assumes micro-crack density is a measure of the damage state
- A key assumption is that the overall damage (due to permanent crack growth) is only associated with the hydrostatic stress
- Deviatoric stress has no effect
- This is essentially assuming cracks only grow in Mode I

fracture mechanics

- In fracture mechanics we consider three different modes
- Mode I is known as the “opening mode”
- Mode II is known as the “sliding mode”
- Mode III is known as the “tearing mode”

fracture mechanics

mixed-mode

- In fracture mechanics, we can consider the effect of the deviatoric stress on a crack
- Mixed-mode fracture analysis shows that cracks will always tend to open due to Mode I
- Shear stresses (i.e. deviatoric stress) can effect the principal stresses near a crack tip
- For many micro-cracks in a representative volume, we assume this effect is negligible

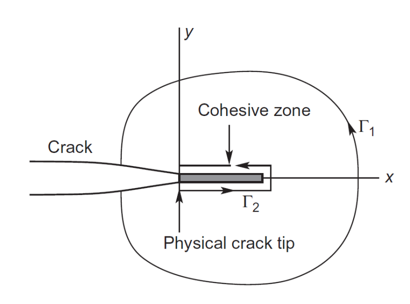
cohesive zone

- The Barenblatt-Dugdale model also assumes that there is a cohesive zone around the crack
- Cohesive zones are an alternate approach to modeling cracks

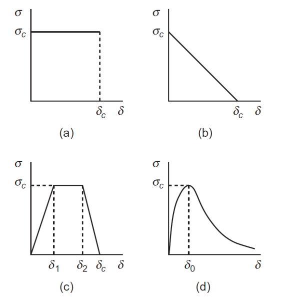
cohesive elements

- Cohesive elements are one way to model crack propagation
- We need to know the crack path in advance, we model the the crack growth using a traction-separation law
- The cohesive zone theory assumes stress can never reach infinity, the maximum allowable stress in a material is the stress required to separate atoms
- The stress required to separate the atoms changes as a function based on their Traction-Separation law, until the atomic bond is broken

cohesive zone



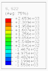
traction separation



cohesive zone uses

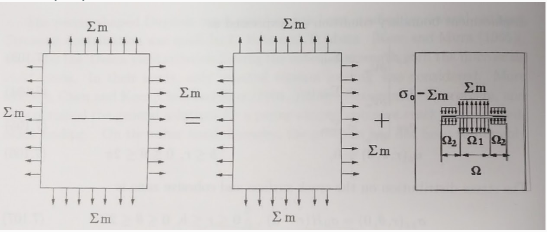
- In practice, the cohesive zone can be used to model crack growth
- It is most often used to model de-bonding of adhesives
- Also commonly used to model delamination in composites

dcb



single crack

- To solve the problem of many cohesive cracks in an RVE, we first consider the case of a single crack
- For a crack under a uniform tri-axial stress, we consider the superposition



cohesive stress

- The cohesive stress, σ_0 can be found as

$$\frac{\sigma_0}{\Sigma_m} = \frac{1 + \sqrt{\left(\frac{4}{1-2\nu^*}\frac{\sigma_Y S}{\Sigma_m}\right)^2 - 3}}{4}$$

macro strain

- The macro strain tensor is not necessarily the volume average
- This is due to the discontinuities (cracks)
- The macro stress is the volume average (crack surfaces are traction free)

macro strain

- One technique for finding the macro strain involves finding some additional strain term

$$\varepsilon_{ij}\epsilon_{ij}^{0} + \epsilon_{ij}^{(}add)$$

- Where $\epsilon_{ij}^{0} = D_{ijkl}\sigma_{kl}$ and $\epsilon_{ij}^{(add)} = H_{ijkl}\sigma_{kl}$

additional strain

- The additional strain is given by

$$\epsilon_{ij}^{(add)} = \frac{4f(1-\nu^*)\Sigma_m \delta_{ij}}{3\beta\pi\mu^* \sqrt{1-\left(\frac{\Sigma_m}{\sigma_0}\right)^2}}$$

- Where β is the ratio between the volume of the physical crack and the volume of the cohesive crack and f is the effective volume fraction of cracks
- While cracks are assumed to be "penny-shaped" disks, their volume is treated as spherical for these purposes

interaction effect

- In the above calculations, properties with a * superscript an be calculated as either matrix or average properties
- We can capture the interaction effect by using average properties
- This is similar to the self-consistent model, and would need to be found iteratively
- Note: this does not model damage growth, which is still a field of active research, particularly in micromechanics