INVESTIGATIONS ON THE ADJOINT BATEMAN EQUATIONS

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- Introduction
- Presentation of Boltzmann equation and Bateman equation
- Presentation of Go Chiba's method for simplifying depletion chain
- A numerical test on reactor physics of xenon poisoning
- Conclusion

Boltzmann equation:

$$\frac{1}{v}\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -\left(\vec{\Omega}\cdot\vec{\nabla}\Phi\left(\vec{r},E,\vec{\Omega},t\right) + \Sigma_t\left(\vec{r},E,\vec{\Omega},t\right)\cdot\Phi\left(\vec{r},E,\vec{\Omega},t\right)\right) + \int_{4\pi} d^2\Omega' \int_0^\infty dE'\,\Sigma_s(r,E\leftarrow E',\mathbf{\Omega}\leftarrow\mathbf{\Omega}')\Phi(\mathbf{r},E,\mathbf{\Omega})$$

$$+\frac{1}{4\pi K_{eff}}\chi(E)\int_0^\infty dE'\,\nu\Sigma_f(\mathbf{r},E')\Phi(\mathbf{r},E')$$

Simplification:

Steady-state:	$\frac{1}{v}\frac{\mathrm{d}\Phi}{\mathrm{d}t}=0$
Infinite homogeneous medium	$\vec{\Omega}\cdot\vec{\nabla}\Phi\left(\vec{r},E,\vec{\Omega},t\right)=0$
isotropic medium	
Volume integrated	$\Phi(E)$

Final expression:

$$\Sigma(E)\Phi(E) = \int_0^\infty dE'\,\Sigma_{s0}(E\leftarrow E')\Phi(E') + \frac{\chi(E)}{K_{eff}}\int_0^\infty dE'\,\nu\Sigma_f(E')\Phi(E')$$

Operator form:

$$B\Phi = (M - \frac{1}{k_eff}F)\Phi = 0$$

Power normalization:

$$P = \sum_j \kappa_{f,j}N_j\langle\sigma_{f,j}\Phi\rangle + \sum_l \kappa_{c,l}N_l\langle\sigma_{c,l}\Phi\rangle$$

- Depletion chain: collection of isotopes undergoing modifications
- Variation of nuclide number density (NND):

Production/ Disappearance
- radioactive decay
- nucleus reaction: (n,n), (n,α)…
- neutron absorption
- fission reaction

For nuclide k:

$$\frac{\mathrm{d}N_k}{\mathrm{d}x} = \Delta\dot{N}_{generation} - \Delta\dot{N}_{generation}$$

$$= \sum_{l=1}^{N_{decay}} \lambda_{l\to k} N_l(t) + \sum_{l=1}^{N_{nucleus}} \langle \sigma_{n,l\to k}\phi(t)\rangle N_l(t)$$

$$+ \sum_{l=1}^{N_{fission}} Y_{l\to k}\langle\sigma_{f,l}\phi(t)\rangle N_l(t) - (\lambda_k + \langle\sigma_{a,k}\phi(t)\rangle)N_k(t)$$

For NND vector: $\mathbf{N} = [N_1, N_2, \dots, N_{N-1}]$

$$\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}t} = \mathbf{MN}$$

(Bateman equation)

- Isotope importance:

$$\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}t} = \mathbf{MN} \qquad t \in [t_i, t_{i+1}]$$

$\mathbf{N}(t_i)$ → $\mathbf{N}(t_{i+1})$

$+$ $\quad$ $+$

$\Delta \boldsymbol{N}_{\boldsymbol{k}}\,(t_i)$ $\quad$ $\Delta \boldsymbol{N}_{\boldsymbol{l}}\,(t_i)$??

Nuclide k $\quad$ Target Nuclide l

If:

for small $\Delta \boldsymbol{N}_{\boldsymbol{k}}\,(t_i)$, $\Delta \boldsymbol{N}_{\boldsymbol{l}}\,(t_i)$ big ⇒ k is important to l

for big $\Delta \boldsymbol{N}_{\boldsymbol{k}}\,(t_i)$, $\Delta \boldsymbol{N}_{\boldsymbol{l}}\,(t_i)$ small ⇒ k is not important to l

Nuclide sensitivity:

$$S_k = \frac{\dfrac{\Delta N_l(t_{i+1})}{N_l(t_{i+1})}}{\dfrac{\Delta N_k(t_i)}{N_k(t_i)}}$$

Key assumption(linearization): $\mathbf{M}(t) = \mathbf{M}(t_i)\ \ for\ t \in [t_i, t_{i+1}]$

$$\frac{\partial \mathbf{N}}{\partial t} = \mathbf{M}^i\mathbf{N}(t), \qquad t_i \le t \le t_{i+1} \xrightarrow{\text{New equation}} \frac{\partial \mathbf{N}'}{\partial t} = \mathbf{M}^{i'}\mathbf{N}'(t), \qquad t_i \le t \le t_{i+1}$$

At $t = t_i$: $\Delta\mathbf{N}(t_i)$

$$\mathbf{N}'(t_i) = \mathbf{N}(t_i) + \Delta\mathbf{N}(t_i) \xrightarrow[\text{Equation}]{\text{Boltzmann}} \phi'(t_i) = \phi(t_i) + \Delta\phi(t_i) \xrightarrow[\text{Equation}]{\text{Bateman}} \mathbf{M}^{i'} = \mathbf{M}^i + \Delta\mathbf{M}^i$$

$$\frac{\partial \Delta\mathbf{N}}{\partial t} = \Delta\mathbf{M}^i\mathbf{N}(t) + \mathbf{M}^i\Delta\mathbf{N}(t),\ \ t_i \le t \le t_{i+1} \Longrightarrow \int_{t_i}^{t_{i+1}} \mathbf{w}^T(t)\frac{\partial \Delta\mathbf{N}}{\partial t} = \int_{t_i}^{t_{i+1}} \mathbf{w}^T(t)\Delta\mathbf{M}^i\mathbf{N}(t)dt + \int_{t_i}^{t_{i+1}} \mathbf{w}^T(t)\mathbf{M}^i\Delta\mathbf{N}(t)dt$$

Final expression

$$\mathbf{w}^T(t_{i+1})\Delta\mathbf{N}(t_{i+1}) - \mathbf{w}^T(t_i)\Delta\mathbf{N}(t_i) = \int_{t_i}^{t_{i+1}} \Delta\mathbf{N}^T(t)\left(\frac{\partial \mathbf{w}}{\partial t} + \mathbf{M}^{iT}\mathbf{w}(t)\right)dt + \int_{t_i}^{t_{i+1}} \mathbf{w}^T(t)\Delta\mathbf{M}^i\mathbf{N}(t)dt$$

If: $\dfrac{\partial \mathbf{w}}{\partial t} = -\mathbf{M}^{iT}\mathbf{w}(t), t_i \le t \le t_{i+1}$ and final condition: $\mathbf{w}(t_{i+1}) = \mathbf{e}_l = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ (l-th element)

We have

$$\Delta N_l(t_{i+1}) = \mathbf{w}^T(t_i)\Delta\mathbf{N}(t_i) + \int_{t_i}^{t_{i+1}} \mathbf{w}^T(t)\Delta\mathbf{M}^i\mathbf{N}(t)dt$$

($\mathbf{w}(t_{i+1})$as a selection)

How to calculate $\Delta\mathbf{M}^i(\Delta\mathbf{N}(t_i))$?

Go Chiba's method:

$$\Delta\mathbf{M}^i = \frac{\mathrm{d}\mathbf{M}^i}{\mathrm{d}\overline{\phi}^i}\Delta\overline{\phi}^i = \frac{\mathrm{d}\mathbf{M}^i}{\mathrm{d}\overline{\phi}^i}\frac{\int_{r\in V_f}\Delta\phi^i(r)dr}{V_f}$$

Adjoint neutron transport equation:

$$\mathbf{B}^{i*}\mathbf{\Gamma}^{i*} = \mathbf{S}^{i*}$$

$$\mathbf{S}^{i*} = \begin{cases} \left(\frac{\int_{t_i}^{t_{i+1}} \mathbf{N}^{*T}\frac{\mathrm{d}\mathbf{M}^i}{\mathrm{d}\overline{\phi}^i}\mathbf{N}dt}{V_f}\right) - \left(\int_{t_i}^{t_{i+1}} \mathbf{N}^{*T}\overline{\mathbf{M}}^i\mathbf{N}dt\right)\frac{\sum_j \kappa_j N_j(t_i)\sigma_{f,j}^i}{p^i}, & \text{if } r\in V_f \\ 0, & \text{if } r\notin V_f \end{cases}$$

$$\left(\int_{t_i}^{t_{i+1}} \mathbf{N}^{*T}\frac{\mathrm{d}\mathbf{M}^i}{\mathrm{d}\overline{\phi}^i}\mathbf{N}dt\right)\frac{\int_{r\in V_f}\Delta\phi^i(r)dr}{V_f}$$

$$= \langle\mathbf{\Gamma}^{i*}\mathbf{B}^{i*}\Delta\phi^i\rangle + P^{i*}\cdot\sum_j \kappa_j N_j(t_i)\langle\sigma_{f,j}^i\Delta\phi^i\rangle$$

$$\begin{cases} \langle\mathbf{B}^{i*}\mathbf{\Gamma}^{i*}\Delta\phi^i\rangle = \langle\Delta\phi^i\mathbf{B}^{i*}\mathbf{\Gamma}^{i*}\rangle = \langle\mathbf{\Gamma}^{i*}\mathbf{B}^i\Delta\phi^i\rangle \\ B^i\Delta\phi^i + \Delta B^i\phi^i = 0 \\ \Delta P^i = \sum_j \kappa_j N_j(t_i)\langle\sigma_{f,j}^i\Delta\phi^i\rangle + \Delta\mathbf{N}^T(t_i)\mathbf{K}(t_i) = 0 \end{cases}$$

$$\Delta N_l(t_{i+1}) = \left(\mathbf{N}^{*T}(t_i) - \left\langle\mathbf{\Gamma}^{i*}\frac{\mathrm{d}B^i}{\mathrm{d}\mathbf{N}^T}\phi^i\right\rangle - P^{i*}\cdot\mathbf{K}^T(t_i)\right)\Delta\mathbf{N}(t_i)$$

$$= \widehat{\mathbf{N}}^{*T}(t_i)\Delta\mathbf{N}(t_i) = \sum_k \widehat{N_k}^*(t_i)\Delta N_k(t_i)$$

$$CF_k(t_i) = \frac{\frac{\Delta N_{k\to l}(t_{i+1})}{N_l(t_{i+1})}}{\frac{\Delta N_k(t_i)}{N_k(t_i)}} = \frac{\frac{\widehat{N_k}^*(t_i)\Delta N_k(t_i)}{N_l(t_{i+1})}}{\frac{\Delta N_k(t_i)}{N_k(t_i)}}$$

$$= \frac{\widehat{N_k}^*((t_i)N_k(t_i)}{N_l(t_i+1)}$$

Modification

Idea: $$\Delta N_l(t_{i+1}) = \widehat{\mathbf{N}}^{*T}(t_i)\Delta\mathbf{N}(t_i)$$

$$\Delta N_l(t_{i+1}) = {\color{red}??}\,\Delta\mathbf{N}(t_i)$$

From: $$\Delta\mathbf{M}^i = \frac{\mathrm{d}\mathbf{M}^i}{\mathrm{d}\overline{\phi}^i}\Delta\overline{\phi}^i$$

We have

$$\Delta N_l(t_{i+1}) = \mathbf{w}^T(t_i)\Delta\mathbf{N}(t_i) + \int_{t_i}^{t_{i+1}} \mathbf{w}^T(t)\Delta\mathbf{M}^i\mathbf{N}(t)dt$$

$$= \mathbf{w}^T(t_i)\Delta\mathbf{N}(t_i) + \sum_g \int_{t_i}^{t_{i+1}} \mathbf{w}^T(t)\frac{\partial\mathbf{M}^i}{\partial\phi_g^i}\mathbf{N}(t)dt\,\Delta\phi_g^i$$

$$= \mathbf{w}^T(t_i)\Delta\mathbf{N}(t_i) + \sum_g \int_{t_i}^{t_{i+1}} \mathbf{w}^T(t)\frac{\partial\mathbf{M}^i}{\partial\phi_g^i}\mathbf{N}(t)dt\,\frac{\mathrm{d}\phi_g^i}{\mathrm{d}\mathbf{N}^T}\Delta\mathbf{N}(t_i)$$

$$= \left(\mathbf{w}^T(t_i) + \sum_g \int_{t_i}^{t_{i+1}} \mathbf{w}^T(t)\frac{\partial\mathbf{M}^i}{\partial\phi_g^i}\mathbf{N}(t)dt\,\frac{\mathrm{d}\phi_g^i}{\mathrm{d}\mathbf{N}^T}\right)\Delta\mathbf{N}(t_i)$$

$$= \widehat{\mathbf{N}}^{*T}(t_i)\Delta\mathbf{N}(t_i)$$

$$\frac{\mathrm{d}N_k}{\mathrm{d}x} = \Delta \dot{N}_{generation} - \Delta \dot{N}_{generation}$$

$$= \sum_{l=1}^{N_{decay}} \lambda_{l\to k}N_l(t) + \sum_{l=1}^{N_{nucleus}} \langle\sigma_{n,l\to k}\phi(t)\rangle N_l(t)$$

$$+ \sum_{l=1}^{N_{fission}} Y_{l\to k}\langle\sigma_{f,l}\phi(t)\rangle N_l(t) - (\lambda_k + \langle\sigma_{a,k}\phi(t)\rangle)N_k(t)$$

$$\frac{\partial\mathbf{M}^i}{\partial\phi_g^i}$$

$$\frac{\mathrm{d}\phi_g^i}{\mathrm{d}\mathbf{N}^T} = [\frac{\partial\phi_g^i}{\partial N_1}\cdots\frac{\partial\phi_g^i}{\partial N_{N_{fissile}}}]$$

For N_k:

$$\frac{\partial\phi_g^i}{\partial N_k} = \lim_{\varepsilon\to 0}\frac{\phi_g^i\left(N_k(t_i)+\varepsilon N_k(t_i)\right) - \phi_g^i\left(N_k(t_i)\right)}{\varepsilon N_k(t_i)}$$

$$[t_i, t_{i+1}] \Rightarrow \left[t_{initial}, t_{initial+1}, \dots, t_{final-1}, t_{final}\right]$$

Method 1: conserving final adjoint condition

For $[t_i, t_{i+1}] \in \left[t_{initial}, t_{initial+1}, \dots, t_{final-1}, t_{final}\right]$:

$$\frac{\partial \mathbf{w}}{\partial t} = -\mathbf{M}^{iT}\mathbf{w}(t), t_i \le t \le t_{i+1} \qquad \mathbf{w}(t_{i+1}) = \mathbf{e}_l$$

$$\Delta N_l(t_{i+1}) = \widehat{\mathbf{N}}^{*T}(t_i)\Delta\mathbf{N}(t_i) = \sum_k \widehat{N_k}^*(t_i)\Delta N_k(t_i)$$

$$CF_k(t_i) == \frac{\widehat{N_k}^*(t_i)N_k(t_i)}{N_l(t_{i+1})}$$

Analogue to derivation:

$$\frac{dy}{dx}(t_i) = \frac{y(t_{i+1}) - y(t_i)}{\Delta t}$$

Method 2: conserving final adjoint condition

$$\frac{\partial \mathbf{w}}{\partial t} = -\mathbf{M}^{iT}\mathbf{w}(t), t_i \le t \le t_{i+1} \quad \mathbf{w}(t_{i+1}) = \begin{cases} \widehat{N_k}^* \text{ for } i+1 = f \\ \mathbf{e}_l \text{ for other steps} \end{cases}$$

$$\begin{aligned} \widehat{N_k}^*(t_{i+1})\Delta\mathbf{N}(t_{i+1}) &= \mathbf{w}^T(t_{i+1})\Delta\mathbf{N}(t_{i+1}) \\ &= \mathbf{w}^T(t_i)\Delta\mathbf{N}(t_i) + \int_{t_i}^{t_{i+1}} \mathbf{w}^T(t)\Delta\mathbf{M}^i\mathbf{N}(t)dt \\ &= \widehat{\mathbf{N}}^{*T}(t_i)\Delta\mathbf{N}(t_i) \end{aligned}$$

$$\Delta N_l(t_{final}) = \widehat{\mathbf{N}}^{*T}(t_i)\Delta\mathbf{N}(t_i) \; for\, i \in [initial, final-1]$$

$$CF_k(t_i) == \frac{\widehat{N_k}^*(t_i)N_k(t_i)}{N_l(t_{final})}$$

Analogue to integration:

$$y(t_{final}) = y(t_i) + \int_{t_i}^{t_{final}} y'\,dt$$

Simplified neutron transport equation:

$$\Sigma(E)\Phi(E) = \int_0^\infty dE' \Sigma_{s0}(E \leftarrow E')\Phi(E') + \frac{\chi(E)}{K_{eff}} \int_0^\infty dE' \nu\Sigma_f(E')\Phi(E')$$

Multiple energy groups notations:

$$\Phi_g = \int_{E_g}^{E_{g-1}} dE\Phi(E) \qquad \Sigma_g = \frac{1}{\Phi_g}\int_{E_g}^{E_{g-1}} dE\Sigma(E)\Phi(E)$$

$$\Sigma_{s0,g\leftarrow h} = \frac{1}{\Phi_g}\int_{E_g}^{E_{g-1}} dE \int_{E_h}^{E_{h-1}} dE'\Sigma_{s0}(E \leftarrow E')\Phi(E')$$

$$\chi_g = \int_{E_g}^{E_{g-1}} dE\chi(E) \qquad \Sigma_{f,g} = \frac{1}{\nu\Phi_g}\int_{E_g}^{E_{g-1}} dE\nu\Sigma_f(E)\Phi(E)$$

$$\Rightarrow \quad \Sigma_g\Phi_g = \sum_{h=1}^{G} \Sigma_{s0,g\leftarrow h}\Phi_h + \frac{\chi_g}{K_{eff}} \sum_{h=1}^{G} \nu\Phi_h\Sigma_{f,h}$$

Matrix form:

$$\mathbf{\Sigma} = \begin{bmatrix} \Sigma_1 & 0 & \cdots & 0 \\ 0 & \Sigma_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & \Sigma_G \end{bmatrix} \qquad \mathbf{\Phi} = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_G \end{bmatrix} \qquad \mathbf{\Sigma}_{s0} = \begin{bmatrix} \Sigma_{1,s0} & 0 & \cdots & 0 \\ 0 & \Sigma_{2,s0} & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & \Sigma_{G,s0} \end{bmatrix}$$

Single fission spectrum:

$$\mathbf{F} = [\nu\Sigma_{f,1} \cdots \nu\Sigma_{f,G}] \qquad \chi = \begin{bmatrix} \chi_1 \\ \vdots \\ \chi_G \end{bmatrix}$$

Multiple fission spectrum:

$$\chi = \begin{bmatrix} \chi_{1,1} & \chi_{2,1} & \cdots & \chi_{N_{fisile},1} \\ \chi_{1,2} & \chi_{2,2} & \cdots & \chi_{N_{fisile},2} \\ \vdots & \vdots & \ddots & \vdots \\ \chi_{1,G} & \chi_{2,G} & \cdots & \chi_{N_{fisile},G} \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} \nu\Sigma_{f,1,1} & \nu\Sigma_{f,1,2} & \cdots & \nu\Sigma_{f,1,G} \\ \nu\Sigma_{f,2,1} & \nu\Sigma_{f,2,2} & \cdots & \nu\Sigma_{f,2,G} \\ \vdots & \vdots & \ddots & \vdots \\ \nu\Sigma_{f,N_{fisile},1} & \nu\Sigma_{f,N_{fisile},2} & \cdots & \nu\Sigma_{f,N_{fisile},G} \end{bmatrix}$$

Matrix equation:

$$\mathbf{\Sigma\Phi} - \mathbf{\Sigma_{s0}\Phi} = \frac{1}{K_{eff}} \chi \otimes (\mathbf{F\Phi})$$

$$\mathbf{\Sigma\Phi} - \mathbf{\Sigma_{s0}\Phi} = \frac{1}{K_{eff}} \chi\mathbf{F\Phi}$$

Single fissionspectrum :

$$\mathbf{\Sigma\Phi} - \mathbf{\Sigma_{s0}\Phi} = \frac{1}{K_{eff}}\chi \otimes (\mathbf{F\Phi})$$

Normalization: $K_{eff} = \mathbf{F\Phi}$

Then:

$$[\mathbf{\Sigma} - \mathbf{\Sigma_{s0}}]\mathbf{\Phi} = \chi$$

$$\mathbf{\Phi} = [\mathbf{\Sigma} - \mathbf{\Sigma_{s0}}]^{-1}\,\chi$$

Multiple fission spectrum:

$$\mathbf{\Sigma\Phi} - \mathbf{\Sigma_{s0}\Phi} = \frac{1}{K_{eff}}\chi\mathbf{F\Phi}$$

$$\mathbf{R\Phi} = \frac{1}{K_{eff}}\chi\mathbf{F\Phi} \qquad K_{eff}\mathbf{\Phi} = \mathbf{R}^{-1}\chi\mathbf{F\Phi} \qquad (\mathbf{R} = \mathbf{\Sigma} - \mathbf{\Sigma_{s0}})$$

$$K_{eff}\xi = \mathbf{M}\xi \qquad (\mathbf{M} = \mathbf{FR}^{-1}\chi \text{ and } \xi = \mathbf{F\Phi})$$

$$\mathbf{K} = \begin{bmatrix} k_1 & 0 & \cdots & 0 \\ 0 & k_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & k_{N_{fisile}} \end{bmatrix} \qquad \begin{matrix} \zeta = [\xi_1 \cdots \xi_{N_{fisile}}] \\ \\ \bar{\mathbf{\Phi}} = [\mathbf{\Phi}_1 \cdots \mathbf{\Phi}_{N_{fisile}}] \end{matrix}$$

$$\mathbf{M}\zeta = \zeta\mathbf{K}$$

$$\mathbf{F}\bar{\mathbf{\Phi}} = \zeta = \mathbf{M}\zeta\mathbf{K}^{-1} = \mathbf{FR}^{-1}\chi\zeta\mathbf{K}^{-1} \qquad \bar{\mathbf{\Phi}} = \mathbf{R}^{-1}\chi\zeta\mathbf{K}^{-1}$$

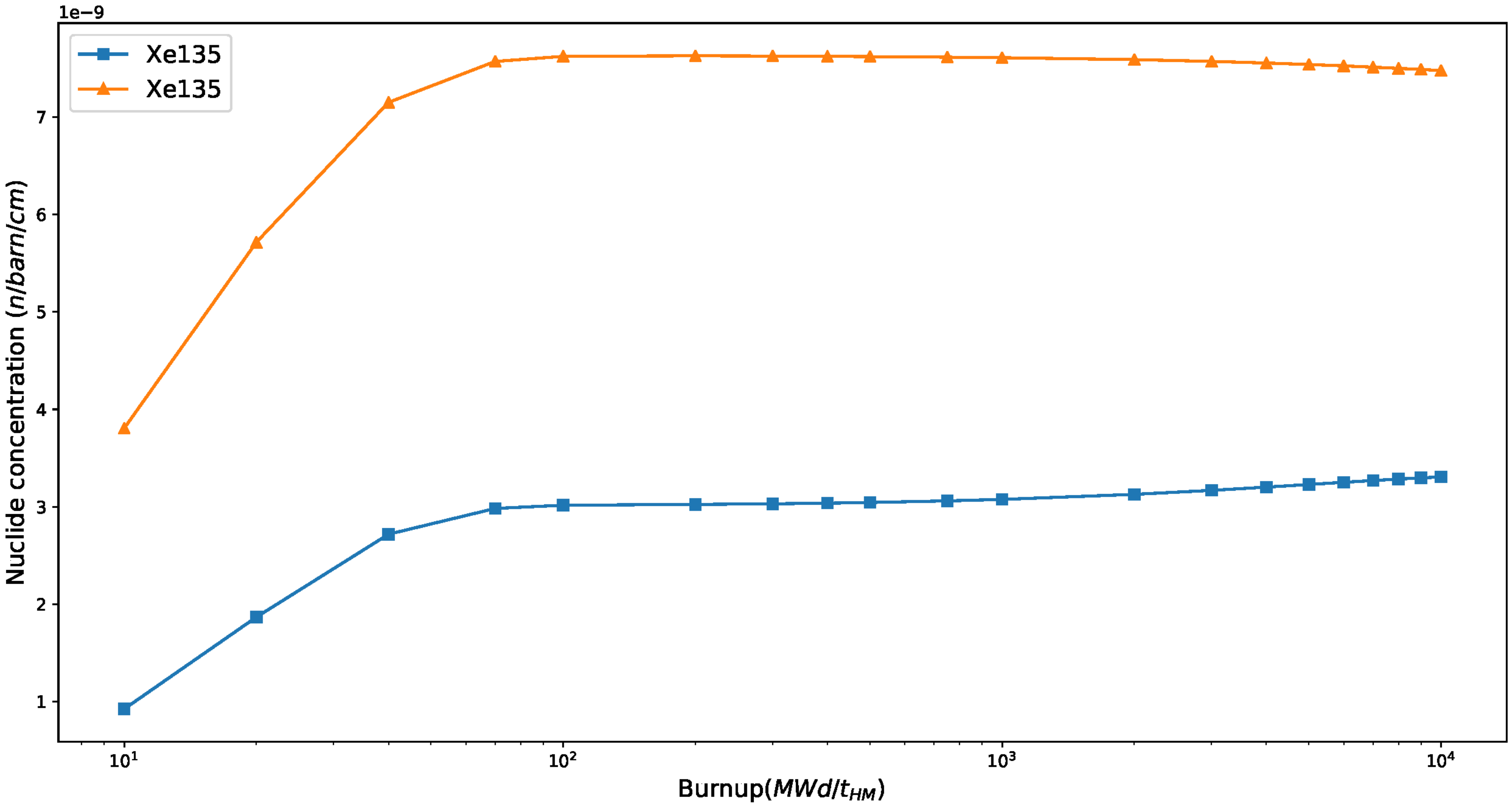
Number of equations to resolve:

Number of energy groups

Number of fissile isotopes

Result and analysis: system evolution analysis

Figure 1: Burnup evolution of ^{135}Xe and ^{135}I for different power level



Result and analysis: system evolution analysis

Figure 2: Time evolution of ^{135}I for Burnup= $300.0MWd/t_{HM}$

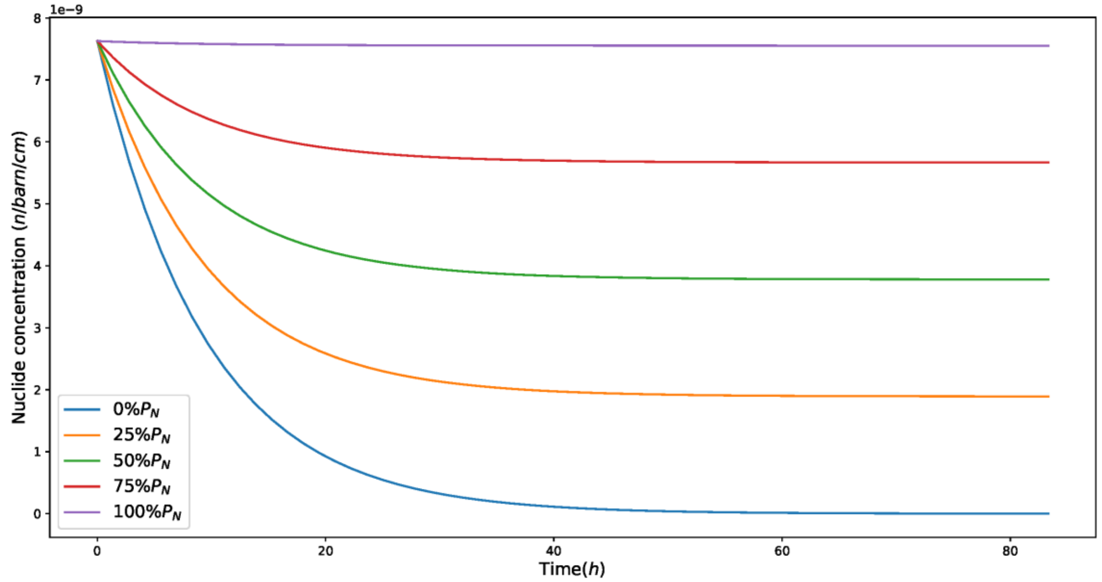
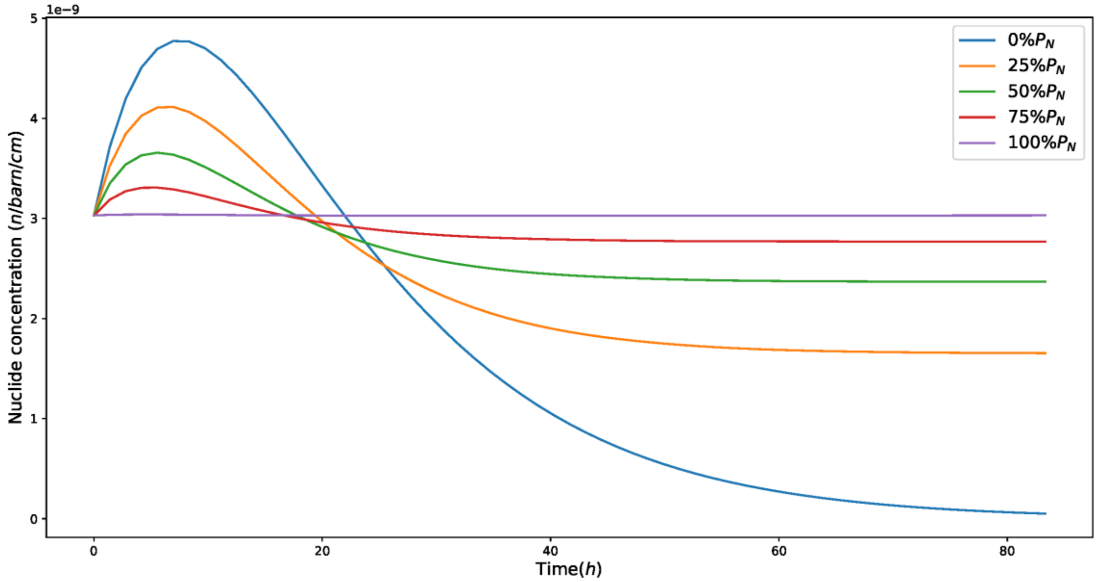


Figure 3: Time evolution of ^{135}Xe for Burnup= $300.0MWd/t_{HM}$



Equilibrium concentration:

$$I_{eq} = \frac{\gamma_I \Sigma_f \phi}{\lambda_I}$$

$$Xe_{eq} = \frac{\lambda_I I_{eq} + \gamma_{Xe} \Sigma_f \phi}{\lambda_{Xe} + \Sigma_{Xe,a} \phi} = \frac{(\gamma_I + \gamma_{Xe}) \Sigma_f \phi}{\lambda_{Xe} + \Sigma_{Xe,a} \phi}$$

Figure 4: Time evolution of ^{135}I
from full power level to a lower power level
for Burnup= $300.0MWd/t_{HM}$

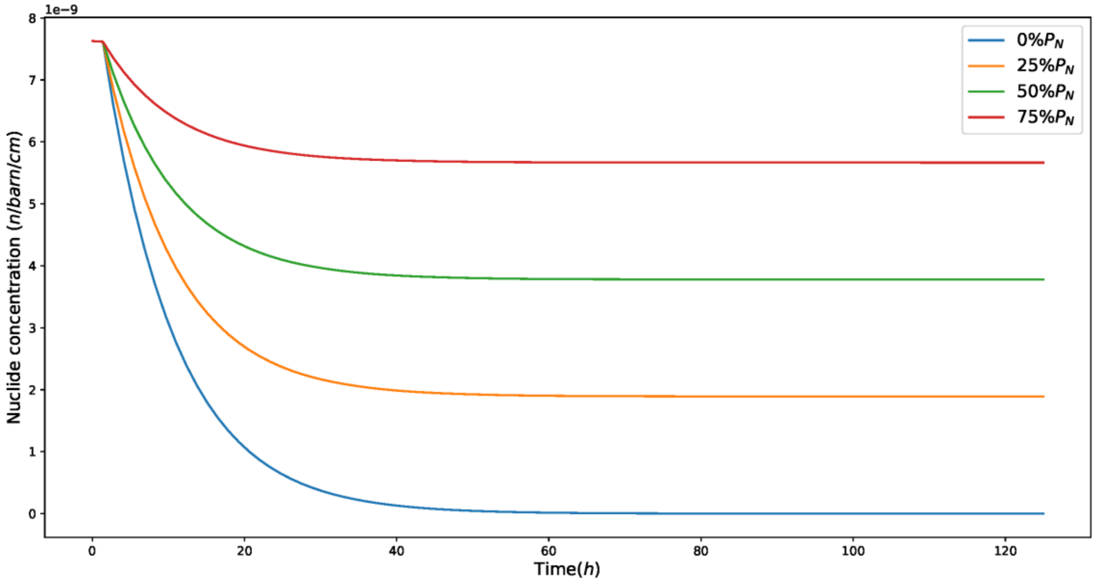
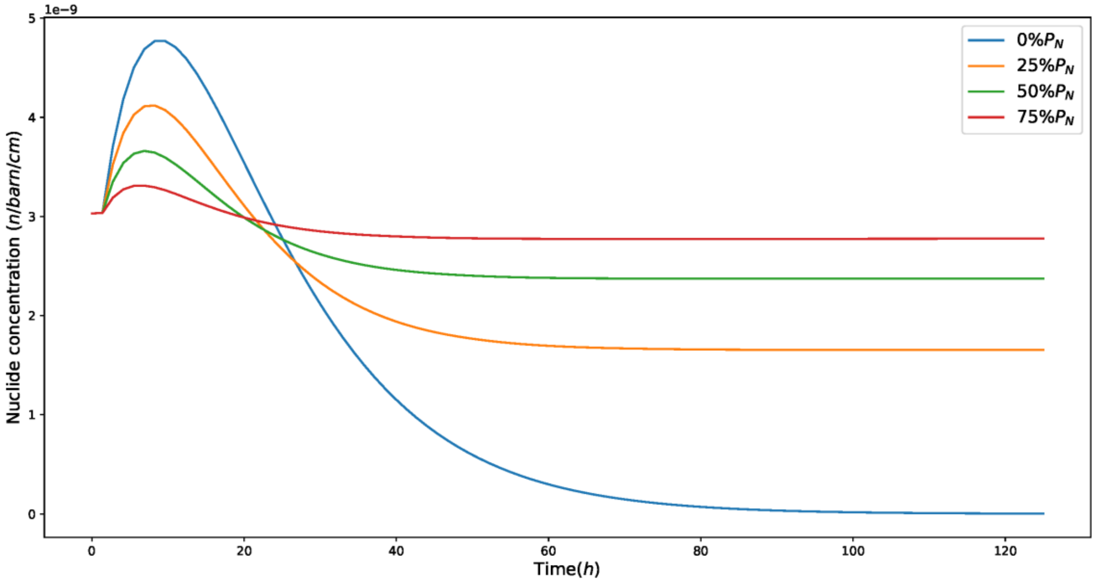


Figure 5: Time evolution of ^{135}Xe
from full power level to a lower power level
for Burnup= $300.0MWd/t_{HM}$

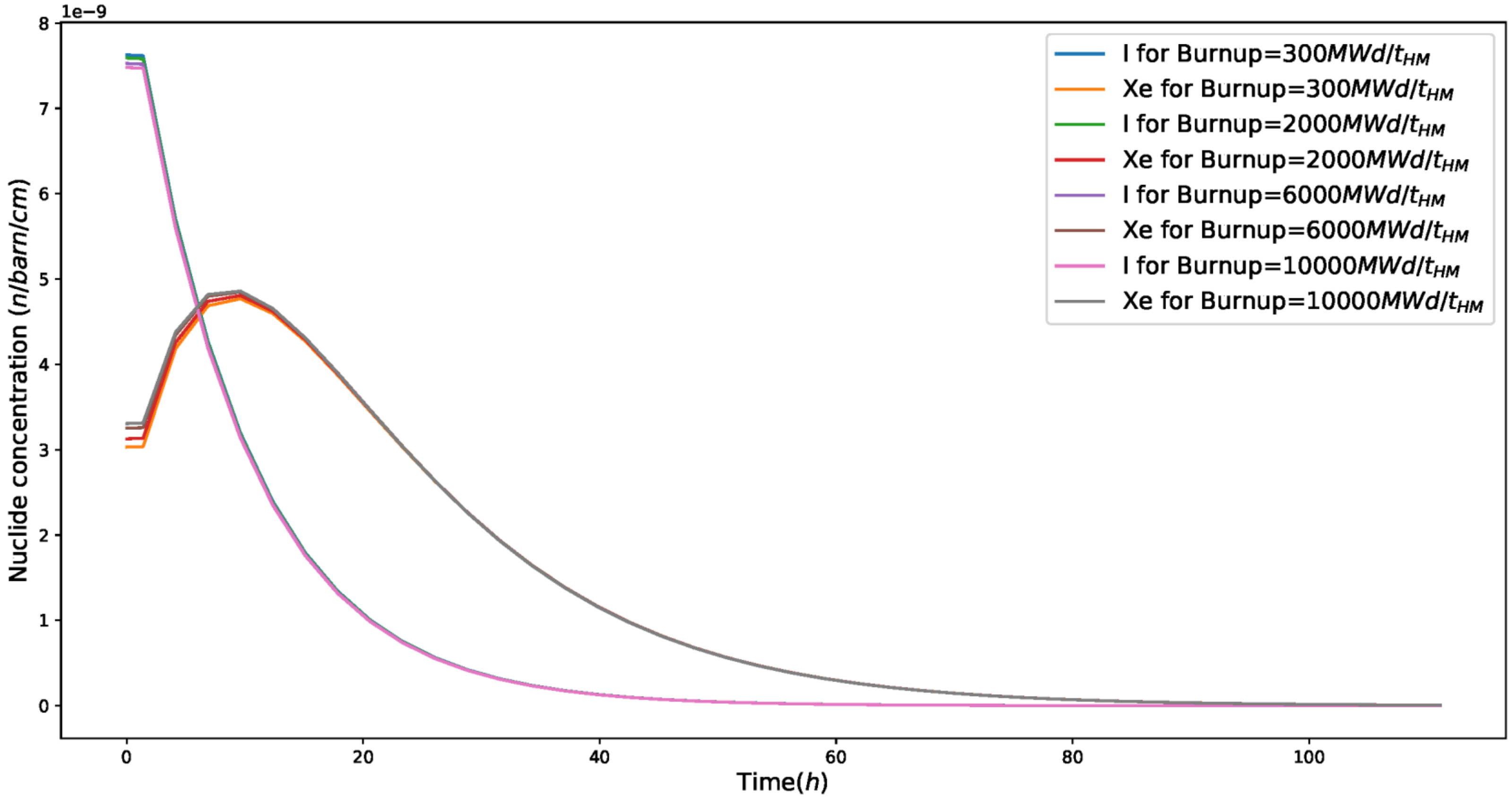


Equilibrium concentration:

$$I_{eq} = \frac{\gamma_I \Sigma_f \phi}{\lambda_I}$$

$$Xe_{eq} = \frac{\lambda_I I_{eq} + \gamma_{Xe} \Sigma_f \phi}{\lambda_{Xe} + \Sigma_{Xe,a} \phi} = \frac{(\gamma_I + \gamma_{Xe}) \Sigma_f \phi}{\lambda_{Xe} + \Sigma_{Xe,a} \phi}$$

Figure 6: Time evolution of ^{135}I and ^{135}Xe from $100\%P_N$ to $0\%P_N$ for different burnup points



Result and analysis: system evolution analysis

Figure 7: Time evolution of ^{135}I from lower power level to full power level for Burnup=2000MWd/t_{HM}

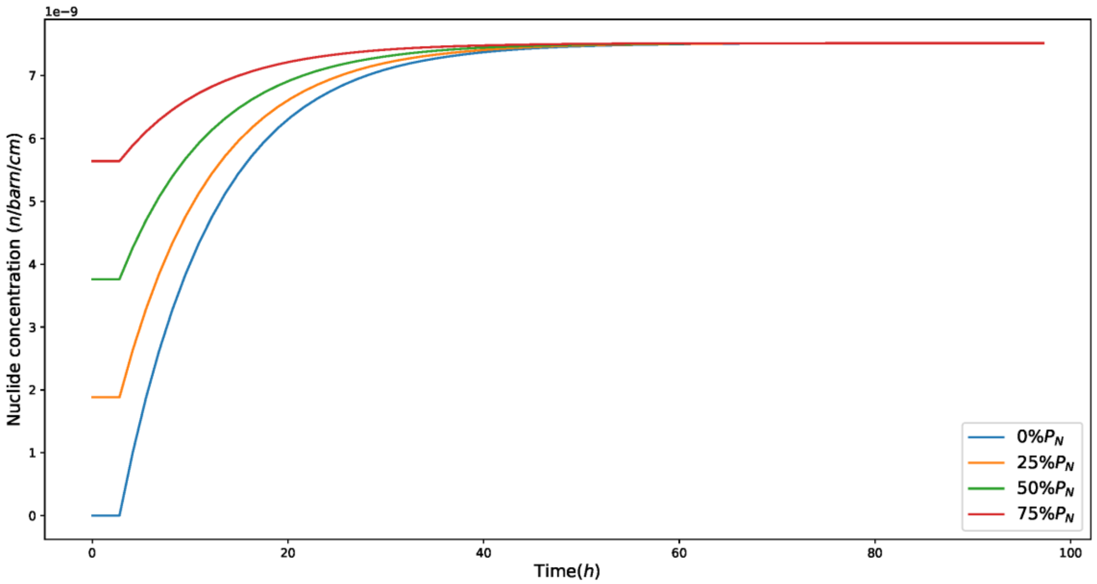
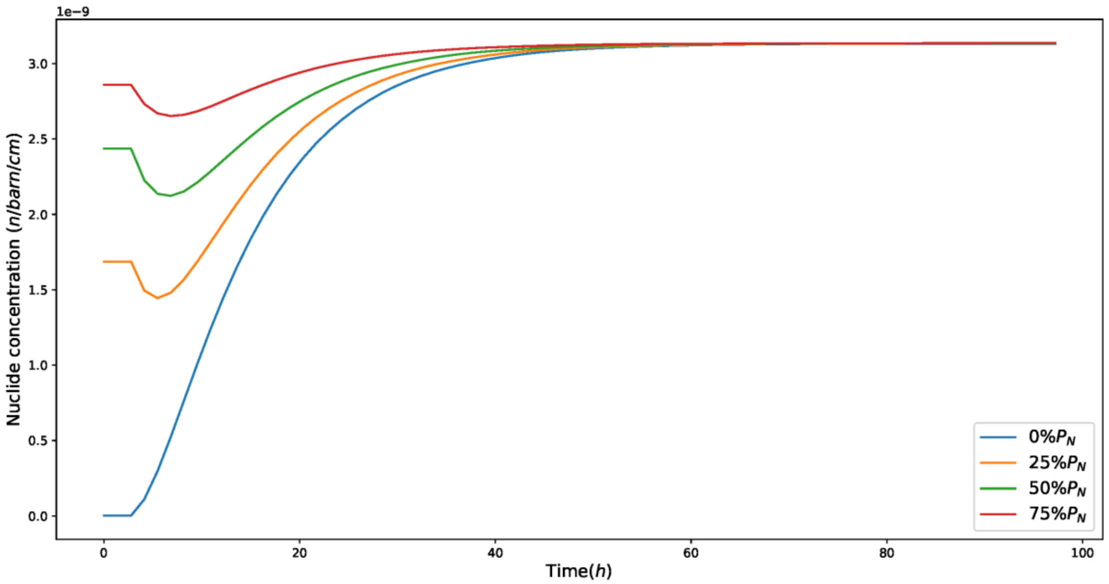


Figure 8: Time evolution of ^{135}Xe from lower power level to full power level for Burnup=2000MWd/t_{HM}



Equilibrium concentration:

$$I_{eq} = \frac{\gamma_I \Sigma_f \phi}{\lambda_I}$$

$$Xe_{eq} = \frac{\lambda_I I_{eq} + \gamma_{Xe}\Sigma_f \phi}{\lambda_{Xe} + \Sigma_{Xe,a}\phi} = \frac{(\gamma_I + \gamma_{Xe})\Sigma_f \phi}{\lambda_{Xe} + \Sigma_{Xe,a}\phi}$$

Figure 9: Time evolution of ^{135}I
from different power levels to zero power level
for Burnup=6000MWd/t_{HM}

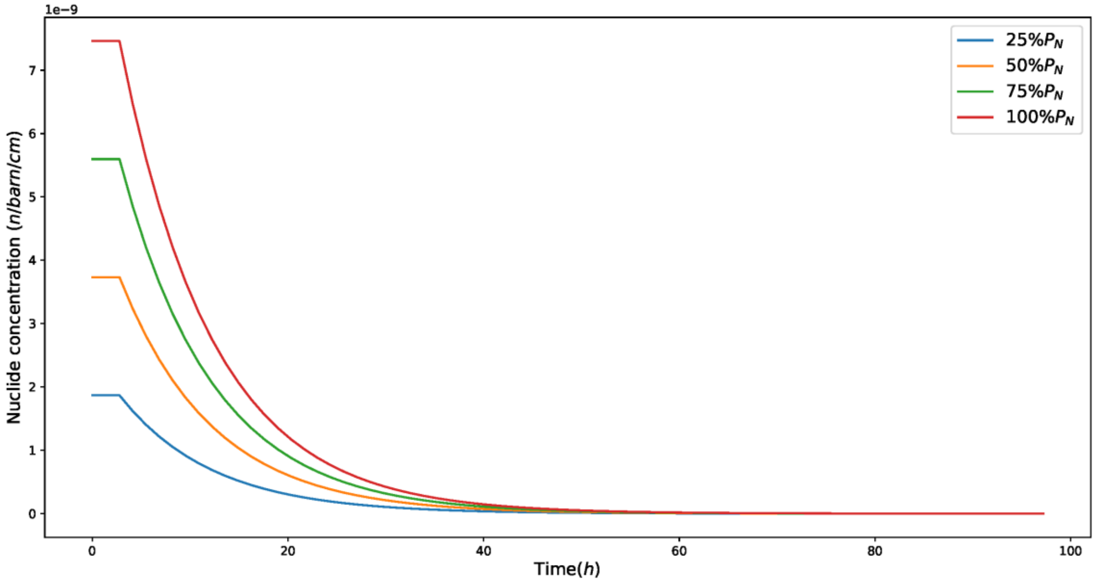
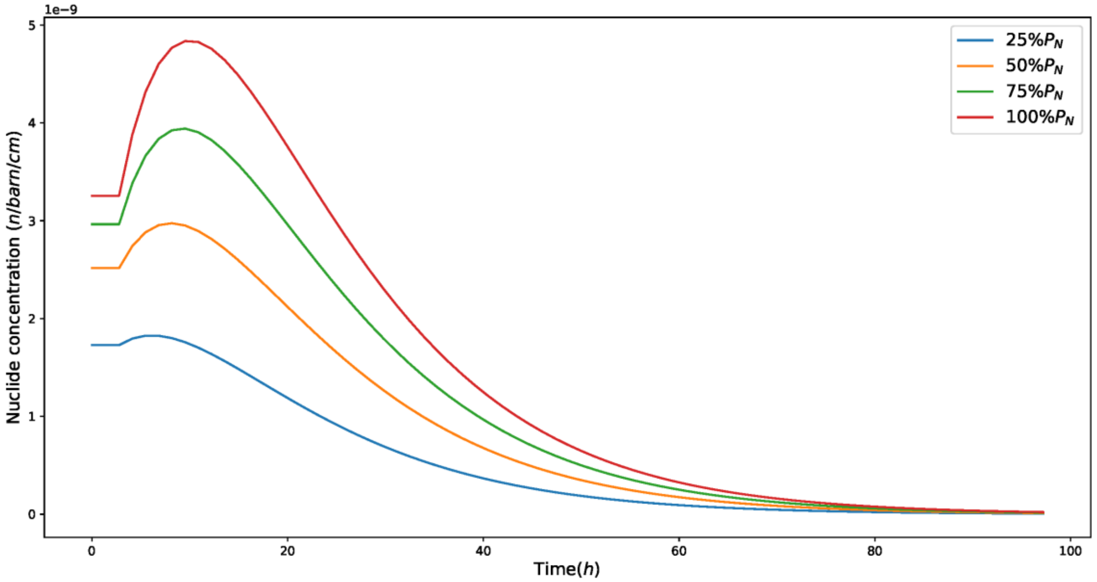


Figure 10: Time evolution of ^{135}Xe
from different power levels to zero power level
for Burnup=6000MWd/t_{HM}



Equilibrium concentration:

$$I_{eq} = \frac{\gamma_I \Sigma_f \phi}{\lambda_I}$$

$$Xe_{eq} = \frac{\lambda_I I_{eq} + \gamma_{Xe} \Sigma_f \phi}{\lambda_{Xe} + \Sigma_{Xe,a} \phi} = \frac{(\gamma_I + \gamma_{Xe}) \Sigma_f \phi}{\lambda_{Xe} + \Sigma_{Xe,a} \phi}$$

Contribution function: steady-state

Figure 11: Time evolution of ^{135}Xe and ^{135}I for different perturbation

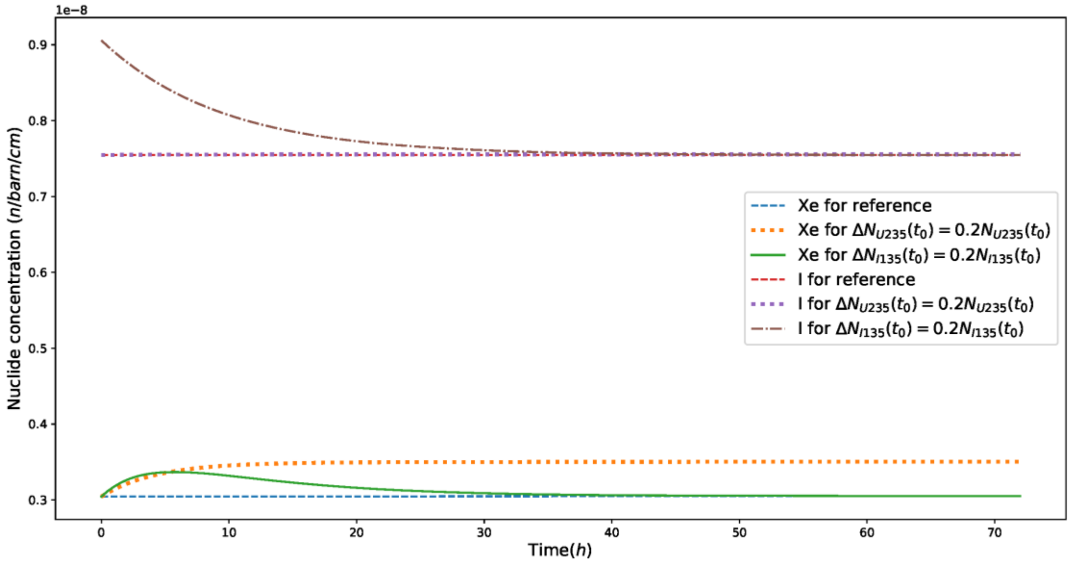


Figure 11: Time evolution of contribution function for selected important isotopes conserving final adjoint condition

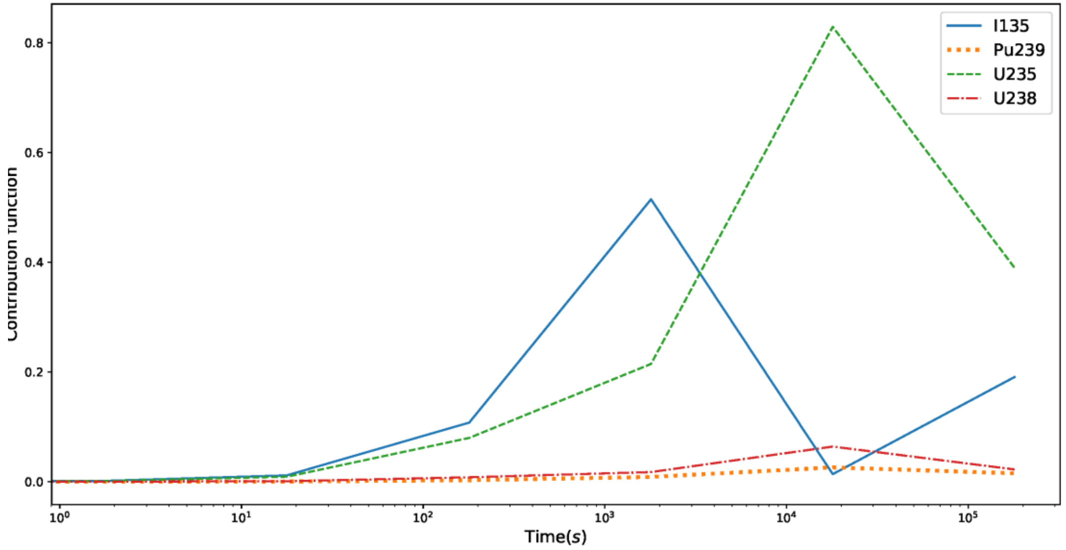


Figure 11: Time evolution of ^{135}Xe and ^{135}I for different perturbation

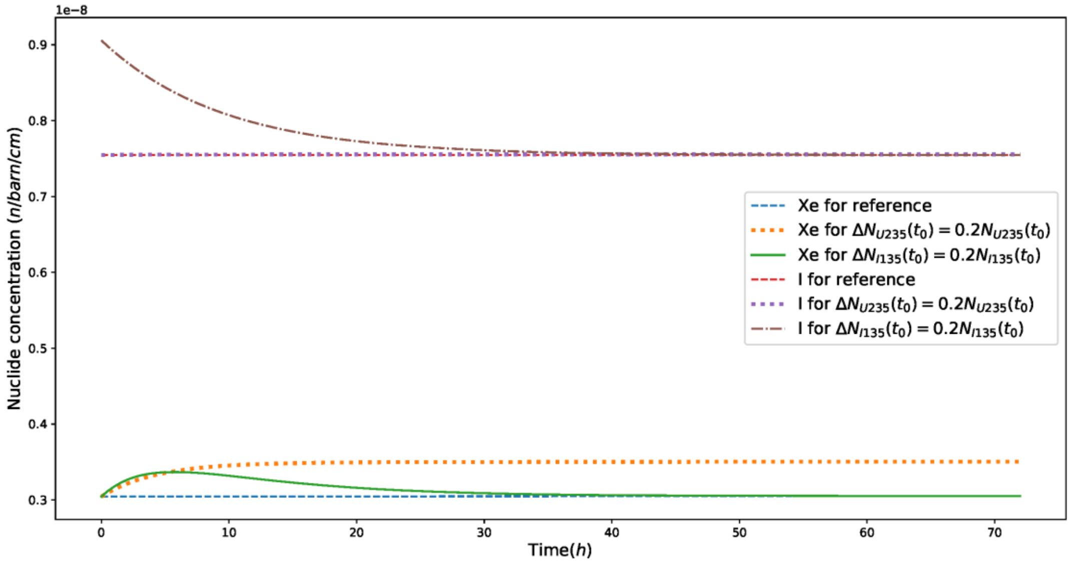
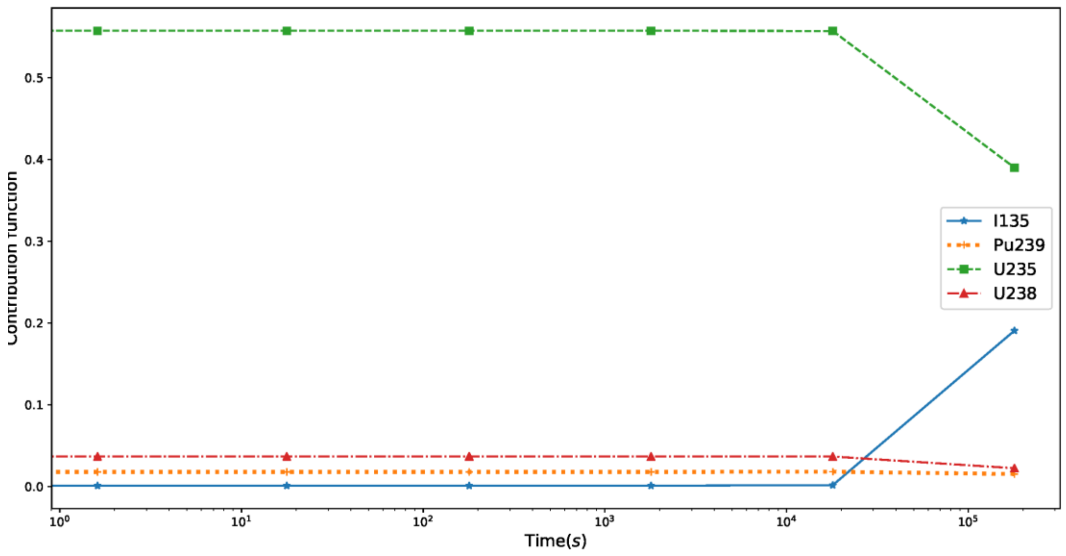


Figure 13: Time evolution of contribution function for selected important isotopes conserving final ^{135}Xe perturbation



Contribution function: transient-state

Figure 14: Time evolution of ^{135}Xe and ^{135}I for different perturbation

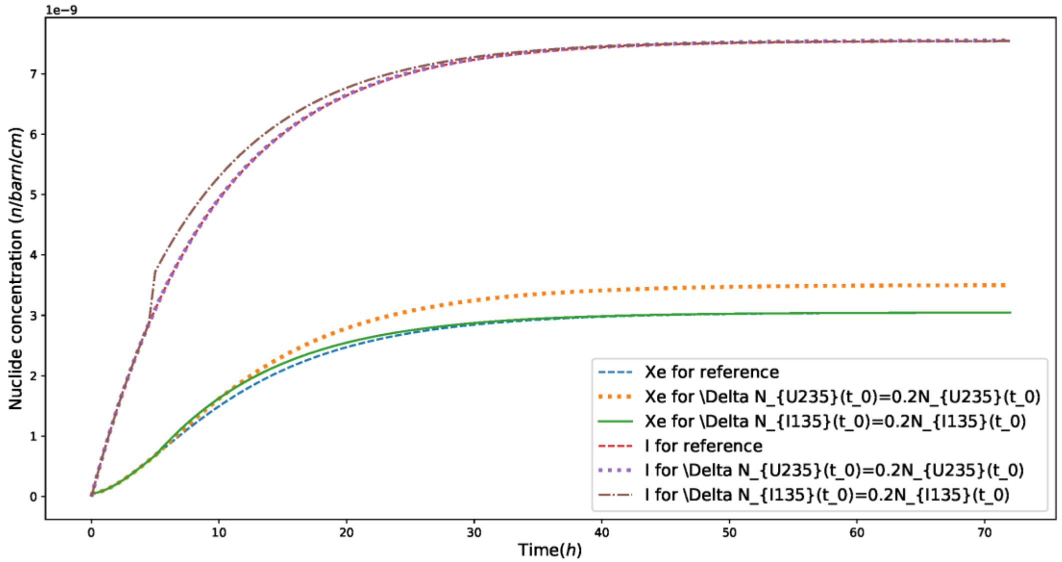
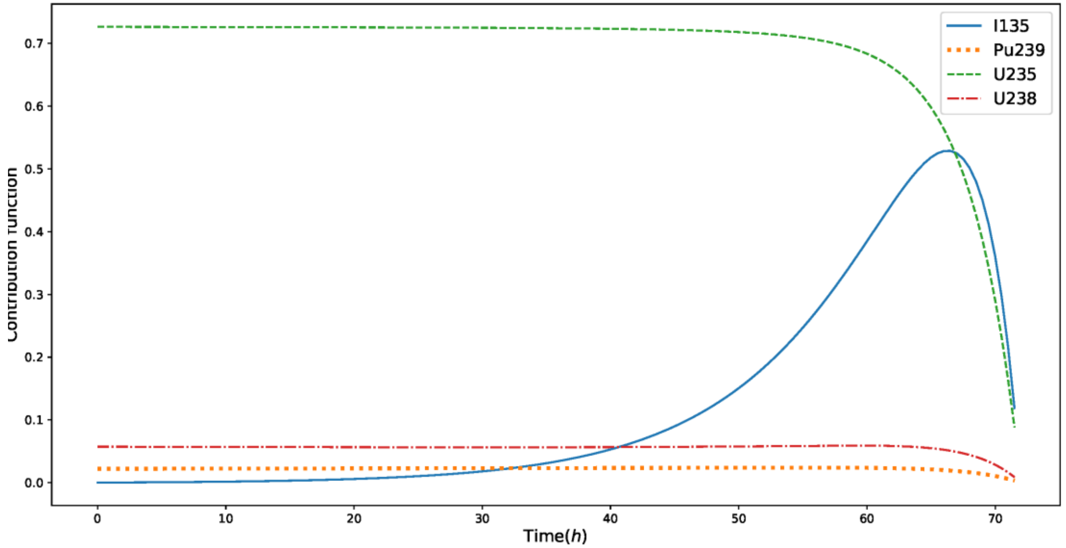


Figure 15: Time evolution of contribution function for selected important isotopes conserving final ^{135}Xe perturbation



- Modification of existing method
- Implementation for solving neutron flux
- Implementation for solving direct and adjoint Bateman equation
- A numerical test on reactor physics of xenon poisoning

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