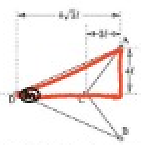
Student ID:

PHYS121 2009F
Test #2
17:00-19:00 Saturday 2009-11-12

Surname: O'Donnell

Given Name:

4. A light rubber band has an unstretched length 10ℓ and obeys Hooke's Law with a spring constant k. Its ends are attached, as shown, to a horizontal, frictionless desktop with pins at A and B a distance 8ℓ apart, forming a catapult. A small stone of mass m is placed at the middle of the rubber band and it is pulled back from the unstretched position C to the position D (both of which are on the desktop) where it is released.



(2) (a) Use dimensional analysis to find an expression, to within a dimensionless multiplicative constant, for the speed, v_1, of the stone after it leaves the catapult. Your result should depend upon the given quantities, m, k and ℓ.

(3) (b) Calculate the speed, v_1, of the stone after it leaves the catapult.

(2) (c) The stone strikes a piece of putty of mass M balanced at the edge of the desk and becomes embedded in it. Calculate the speed of the compound object, v_2, after the collision.

(3) (d) What is the speed, v_3, of the compound object when it lands on the ground if the height of the desk is h?

(1) (e) What is the $M \to \infty$ limit of your answer to part (d)?

a) $v_1 \propto \sqrt{\frac{k}{m}}\,\ell$

b) $\frac{1}{2} m v_1^2 = \frac{1}{2} k x^2$

$\cancel{\frac{1}{2}} m v_1^2 = \cancel{\frac{1}{2}} k (16\ell - 10\ell)^2$

$v_1 = \sqrt{\frac{k}{m}(6\ell)^2}$

$= 6\ell\sqrt{\frac{k}{m}}$

c)

$p_{x_f} = p_{x_i}$

$(m+M)v_2 = m v_1$

$v_2 = \frac{m v_1}{m+M}$

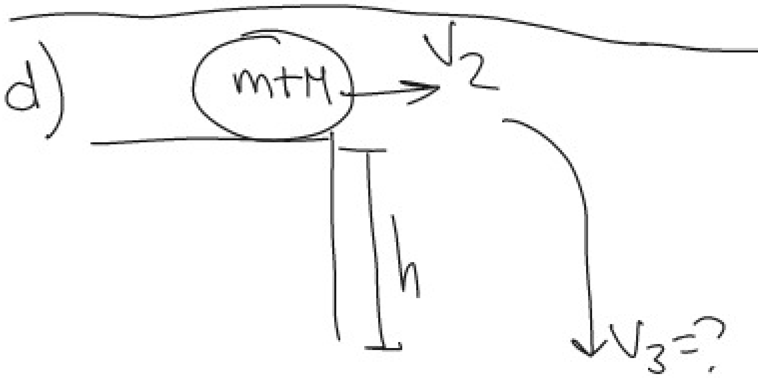
$= \frac{m\, 6\ell\sqrt{\frac{k}{m}}}{m+M} \leftarrow$

$= \frac{6\ell\sqrt{km}}{m+M}$

$m = \sqrt{m}\sqrt{m}$

$\frac{m}{\sqrt{m}} = \frac{\cancel{\sqrt{m}}\sqrt{m}}{\cancel{\sqrt{m}}} = \sqrt{m}$

d)



$\Delta E = \Delta K + \Delta U_g$

$0 = \left(\frac{1}{2}(m+M)v_3^2 - \frac{1}{2}(m+M)v_2^2\right) + \left(0 - (m+M)gh\right)$

$\cancel{(m+M)}gh + \frac{1}{2}\cancel{(m+M)}\left(\frac{6\ell\sqrt{km}}{m+M}\right)^2 = \frac{1}{2}\cancel{(m+M)}v_3^2$

$v_3^2 = 2gh + \frac{36\ell^2 km}{(m+M)^2}$

$v_3 = \sqrt{2gh + \frac{36\ell^2 km}{(m+M)^2}}$

e) $\lim_{M\to\infty} v_3 = \sqrt{2gh}$