If a number's last digit is 5,
then is the last digit of its square *always* 5?

Prove it.

MATH 135: Lecture 3

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Upcoming responsibilities!

- Monday 13 September:
  - Complete Mobius Quiz 1: MQ1
- Wednesday 15 September:
  - Complete Mobius Quiz 2: MQ2
- **Wednesday 15 September 5PM:**
  - **Complete first Written Assignment (WA)**
- Friday 17 September 2PM:
  - Complete reading Chapter 2 of the course notes
- Friday 17 September:
  - Complete Mobius Quiz 3: MQ3
- Sunday 19 September:
  - Complete reading up to the end of Section 0.2 (Polynomials)

If a number's last digit is 5, then is the last digit of its square *always* 5?

$S = \{\text{Integers for which the last digit is } 5\}$

$\forall\, N \in S\ ,\ N^2 \in S\ ?$

$\forall\, N \in S,\ \exists\, a \in \mathbb{Z}$ s.t. $N = 10a + 5$

$$\begin{aligned}(10a+5)^2 &= 100a^2 + 100a + 25 \\ &= 100a(a+1) + 25\end{aligned}$$

$\exists\, b \in \mathbb{Z}$ s.t. $N^2 = 100b + 25$

$\forall\, b \in \mathbb{Z},\ \ 100b + 25 \in S$

$\therefore\ N^2 \in S\ ,\ \forall\, N \in S$

Assignment 1!

- Don't use these:

$$\nexists$$

$$L \not\ni x \qquad x \notin L$$

- Everything can be done using what's in the course notes (e.g. **§** 1.4.3 & 1.5.3):

$$\neg\ \big(\forall\ x \in S,\ P(x)\big) \equiv \big(\exists\ x \in S,\ \neg\ P(x)\big)$$

$$\neg\ \big(\exists\ x \in S,\ P(x)\big) \equiv \big(\forall\ x \in S,\ \neg\ P(x)\big)$$

Assignment 1!

If S is unknown, and/or if P(x) is unknown:

$$\forall\, x \in S,\ \ P(x)$$

cannot be true or false until we specify S or P(x).

Technically, the above is an open sentence in S and an open sentence in P, let's say Q(S,P)

What's an example of P(x)?
Now that we've chosen P(x), the above is still an open sentence in S, let's say: Q(S).

Question 4

Open sentence or statement ?

$$\forall\, x \in S, \;\; \exists \;\; y \in S \;,\; P(x,y)$$

What's a property of an infinite set of integers like:

$$S = \{\ldots\ldots,-3,-2,-1,0,1,2,3,4\}$$

For all x in S, what do we know about the other elements y?

Extra problems

Determine whether the following statements are universally quantified or existentially quantified.

1. One can find an integer k that is even.
2. Not every integer is odd.
3. No matter what integer n we take, $n(n+1)/2$ is an integer.
4. There is an integer m such that $4m^2 + 4m - 3 = 0$.
5. There does not exist an integer k such that $k^2 = 2$.
6. A square of any real number is non-negative.
7. The number $n(n+1)$ is negative for some integer n.
8. No matter what integer n we take, the number $n^2 + n + 1$ will be odd.