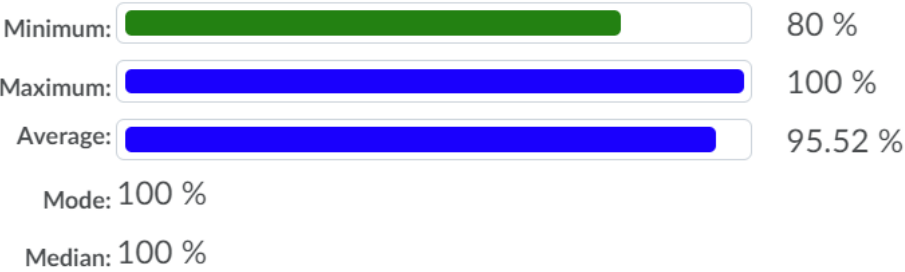


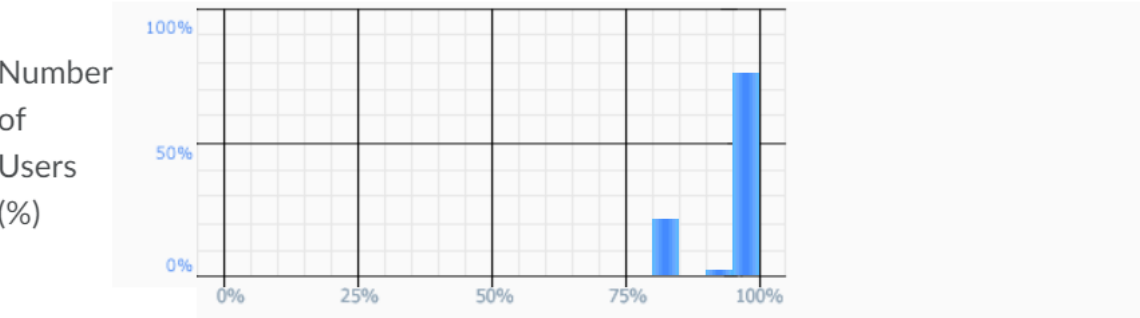
MQ4 (Mon Sep 20) Class Statistics

Number of submitted grades: 55 / 57



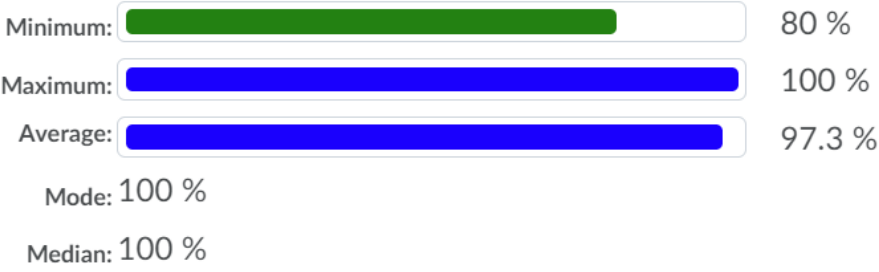
Standard Deviation: 8.24 % ?

Grade Distribution



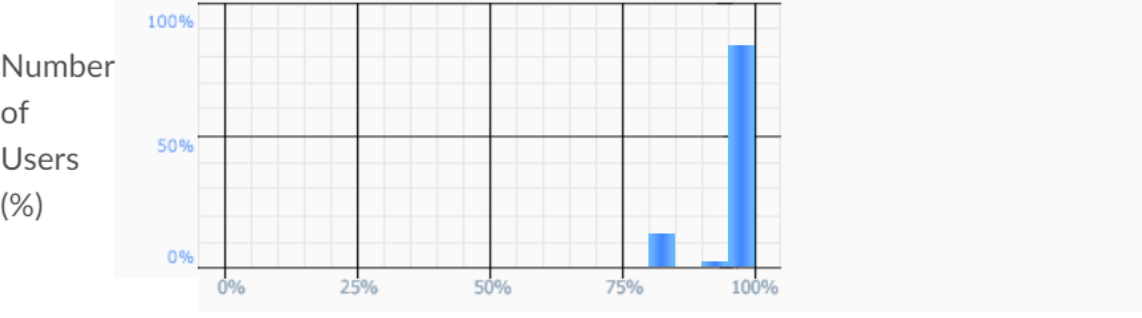
MQ4 (Mon Sep 20) Class Statistics

Number of submitted grades: 47 / 51



Standard Deviation: 6.69 % ?

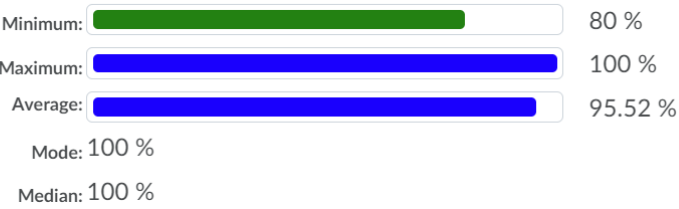
Grade Distribution



- You should be done reading up to Chapter 3.6 of the course notes. **Pages 35-57.**
- **Wednesday 22 September:**
 - **Complete Written Assignment 2: WA2**
- Wednesday 22 September:
 - Mobius Quiz 5
- Wednesday 22 September:
 - **Look at your WA01 results thoroughly! Where did you lose marks?**
- Thursday 23 September:
 - WA02 solutions will be posted, hopefully before 12pm: **Check the solutions in detail!**
- Friday 24 September before class:
 - Mobius Quiz 6
- Sunday 26 September:
 - Complete reading up to the end of Section 0.3 (Polynomials)
- Monday 27 September:
 - Mobius Quiz 7
- Tuesday 28 September:
 - Complete reading from Chapter 3.6 up to 4.4 of the course notes. **Pages 55-75.**

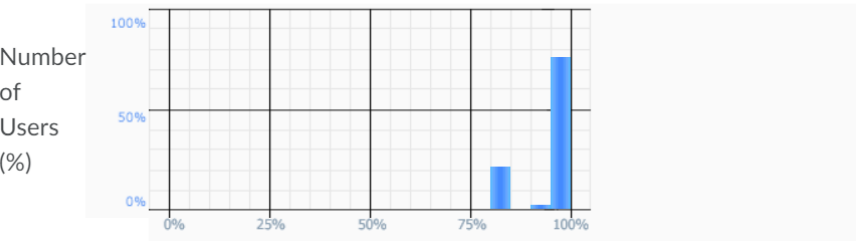
MQ4 (Mon Sep 20) Class Statistics

Number of submitted grades: 55 / 57



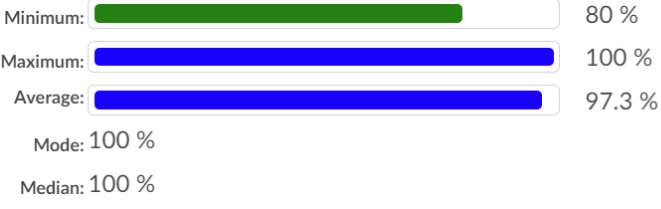
Standard Deviation: 8.24 %

Grade Distribution



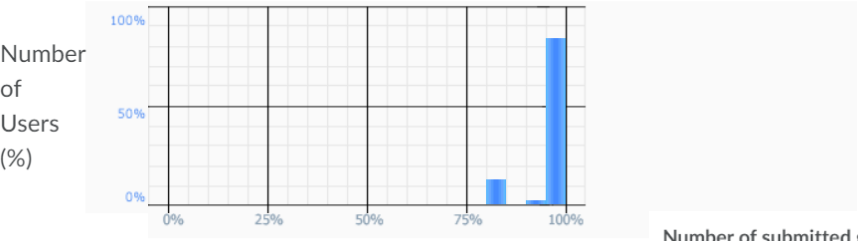
MQ4 (Mon Sep 20) Class Statistics

Number of submitted grades: 47 / 51

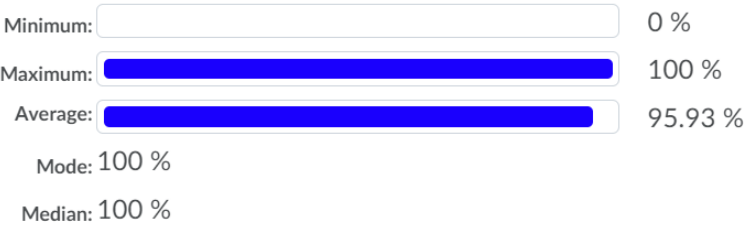


Standard Deviation: 6.69 %

Grade Distribution

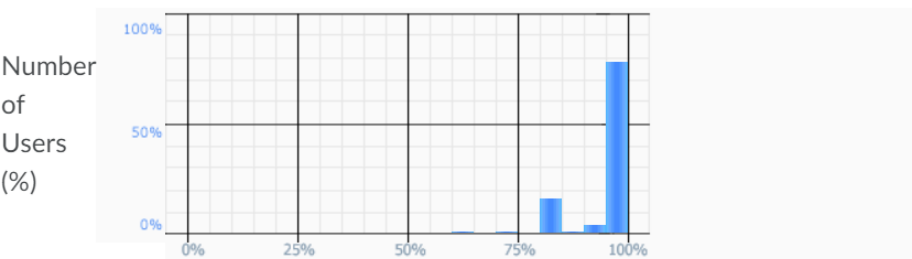


Number of submitted grades: 1,185 / 1,266



Standard Deviation: 8.58 %

Grade Distribution



Office hours: Mondays and Wednesdays 5-6pm.

MC 4059

Also: online tutorial center

MATH 135: Lecture 7

Dr. Nike Dattani

22 September 2021

Quick Review

If A then B. $(A \Rightarrow B)$

A if B. $(B \Rightarrow A)$

“Iff” is short for what?

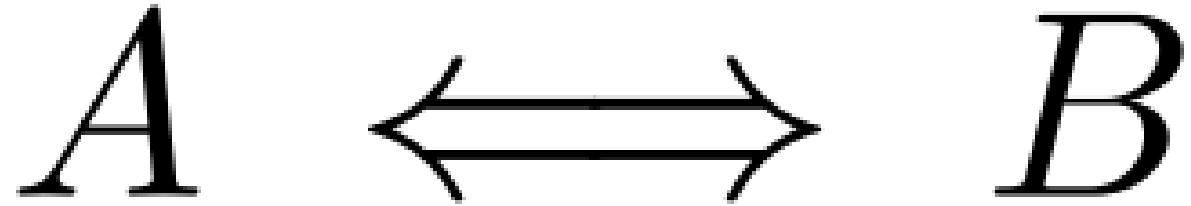
If A then B ***and*** only if B

If A then B ***and*** only if B then A

Don't consider iff to be an English term

Mathematically: $A \Rightarrow B$ and $B \Rightarrow A$ ($A \Leftrightarrow B$)

- A iff B
- A exactly when B
- A just if B
- A precisely when B
- A is true whenever B is true
- A is equivalent to B
- A is materially equivalent to B
- A is logically equivalent to B
- A exactly in case B
- A just in case B
- A XNOR B
- A is necessary and sufficient for B

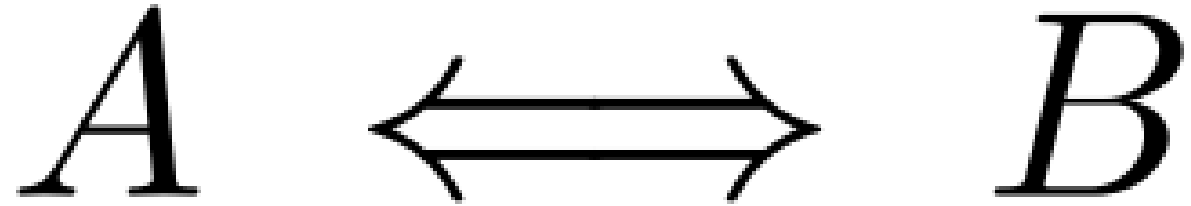


If and only if

From Wikipedia, the free encyclopedia

In writing, phrases commonly used as alternatives to P "if and only if" Q include: Q is *necessary and sufficient* for P, P is *equivalent* (or *materially equivalent*) to Q (compare with *material implication*), P *precisely if* Q, P *precisely* (or *exactly*) *when* Q, P *exactly in case* Q, and P *just in case* Q.^[3] Some authors regard "iff" as unsuitable in formal writing;^[4] others consider it a "borderline case" and tolerate its use.^[5]

- A iff B
- A exactly when B
- A just if B
- A precisely when B
- A is true whenever B is true
- A is equivalent to B
- A is materially equivalent to B
- A is logically equivalent to B
- A exactly in case B
- A just in case B
- A XNOR B



- **A is necessary and sufficient for B: A necessary for B ($B \Rightarrow A$), A sufficient for B ($A \Rightarrow B$)**

If and only if

From Wikipedia, the free encyclopedia

In writing, phrases commonly used as alternatives to P "if and only if" Q include: Q is *necessary and sufficient* for P, P is *equivalent* (or *materially equivalent*) to Q (compare with *material implication*), P *precisely if* Q, P *precisely* (or *exactly*) *when* Q, P *exactly in case* Q, and P *just in case* Q.^[3] Some authors regard "iff" as unsuitable in formal writing;^[4] others consider it a "borderline case" and tolerate its use.^[5]

- Converse
- Contrapositive
- Counter-example
- Contradiction

- Converse
- Contrapositive
- Counter-example
- Contradiction: $(A \wedge \neg A)$ is true

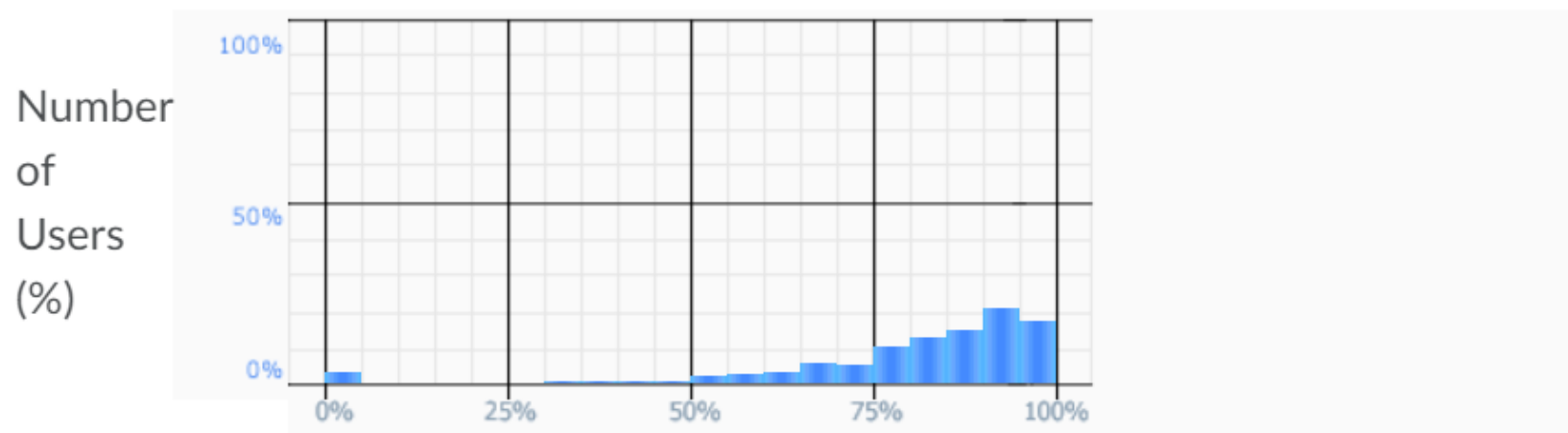
- Converse
- Contrapositive
- Counter-example
- Contradiction
- Corollary
- Conclusion
- Combinations

WA1 Class Statistics

Number of submitted grades: 1,235 / 1,249



Grade Distribution



WA1 Class Statistics

Number of submitted grades: 56 / 57

Minimum:

0 %

Maximum:

100 %

Average:

82.79 %

Mode:

94.44 %

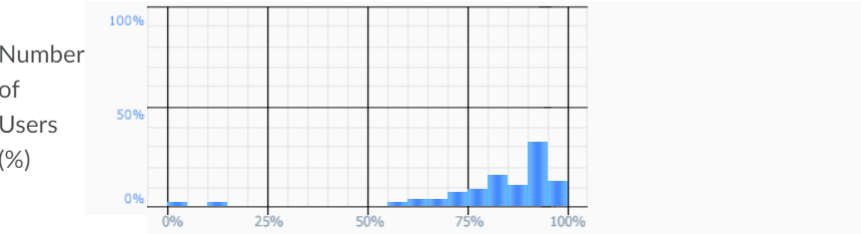
Median:

88.19 %

Standard Deviation:

18.01 %

Grade Distribution



WA1 Class Statistics

Number of submitted grades: 51 / 51

Minimum:

0 %

Maximum:

100 %

Average:

78.79 %

Mode:

94.44 %

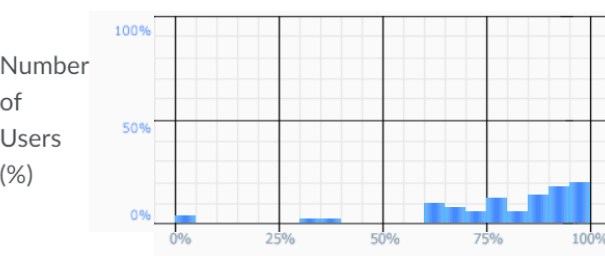
Median:

86.11 %

Standard Deviation:

21.97 %

Grade Distribution



WA1 Class Statistics

Number of submitted grades: 1,235 / 1,249

Minimum:

0 %

Maximum:

100 %

Average:

80.38 %

Mode:

94.44 %

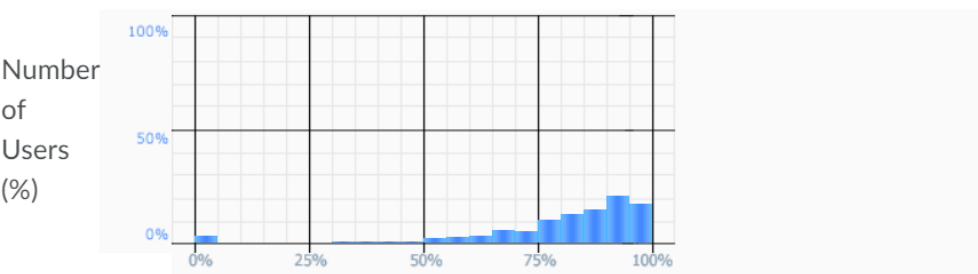
Median:

86.11 %

Standard Deviation:

20.73 %

Grade Distribution



For any real numbers $x, y \in \mathbb{R}$, $x^4 + x^2y + y^2 \geq 5x^2y - 3y^2$.

Proof: Let x and y be any real number. Then

$$\begin{aligned} LHS &= x^4 - 4x^2y + 4y^2 + 5x^2y - 3y^2 && \text{(Add and subtract } 5x^2y - 3y^2\text{)} \\ &= (x^2 - 2y)^2 + 5x^2y - 3y^2 \\ &\geq 5x^2y - 3y^2 \\ &= RHS. \end{aligned}$$

The following is not a valid proof:

$$\begin{aligned} x^4 + x^2y + y^2 &\geq 5x^2y - 3y^2 \\ x^4 - 4x^2y + 4y^2 &\geq 0 \\ (x^2 - 2y)^2 &\geq 0. \quad \checkmark \end{aligned}$$

In a proof, every line needs to be justified. The first line in the above "proof" is not justified.

IRRATIONALITY OF $\sqrt{2}$

Prove that $\sqrt{2}$ is irrational.

Proof: Suppose for a contradiction that there exists positive integers p and q such that $\sqrt{2} = p/q$. By replacing p and q by $p/2$ and $q/2$ if both p and q are even, we may assume without loss of generality that at least one of p or q is odd. Squaring both sides gives $p^2 = 2q^2$. Hence p^2 is even, which implies that p is even. Then $p = 2k$ for some integer k . Substituting $p = 2k$ gives $2k^2 = q^2$ which implies that q^2 , and thus q , is even. Hence both p and q are even. Contradiction.

Here is another, very slick, proof. Suppose for a contradiction that $\sqrt{2}$ is rational. Let k be the smallest positive integer such that $k\sqrt{2}$ is an integer. Then

$$(k\sqrt{2} - k)\sqrt{2} = 2k - k\sqrt{2} \in \mathbb{Z}.$$

However, $k\sqrt{2} - k$ is a positive integer smaller than k since $\sqrt{2} - 1 < 1$. Contradiction.