Warm up!

I want to spend exactly \$100 on buying 49¢ and 53¢ stamps.

In how many ways can I do it?

I want to spend exactly \$100 on buying 49¢ and 53¢ stamps.

$$49x + 53y = 10000,\ x,y \in \mathbb{N}$$

Does $\gcd(49,53) \mid 10000$?

Answer!

49x + 53y = 10000, all integer solutions

NATURAL LANGUAGE MATH INPUT EXTENDED KEYBOARD

Input interpretation

solve	$49x + 53y = 10000$	over the integers

Result

$x = 203 - 53n$ and $y = 49n + 1$ and $n \in \mathbb{Z}$

49*(203-53*3)

↳ = 2156

49*(203-53*4)

↳ = -441

53*(49*(-1)+1)

↳ = -2544

MATH 135: Lecture 23

Dr. Nike Dattani

5 November 2021

Results from survey

Question 8

If you had to choose one, which one would you like more in my classes:

Practice problems from previous exams	45	(55.56 %)
Going through material from the course notes	22	(27.16 %)
This class is too easy, I already learned what's in the course notes, and did the practice problems: Now give me my money's worth by teaching me more cool things	1	(1.23 %)
Going over Möbius quiz questions	2	(2.47 %)
Real life advice	6	(7.41 %)
Advice on how to get through university	5	(6.17 %)

I will do previous exam questions!

Course notes: Try Piazza, office hours, tutorial centre, etc.

…or let me know in advance what part of the reading is giving you trouble

Office Hours: For real-life advice, advice for getting through university, etc.

Question 8

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Advice on how to get through university	5	(6.17 %)

18 second video:

Former student of mine I taught at Oxford University

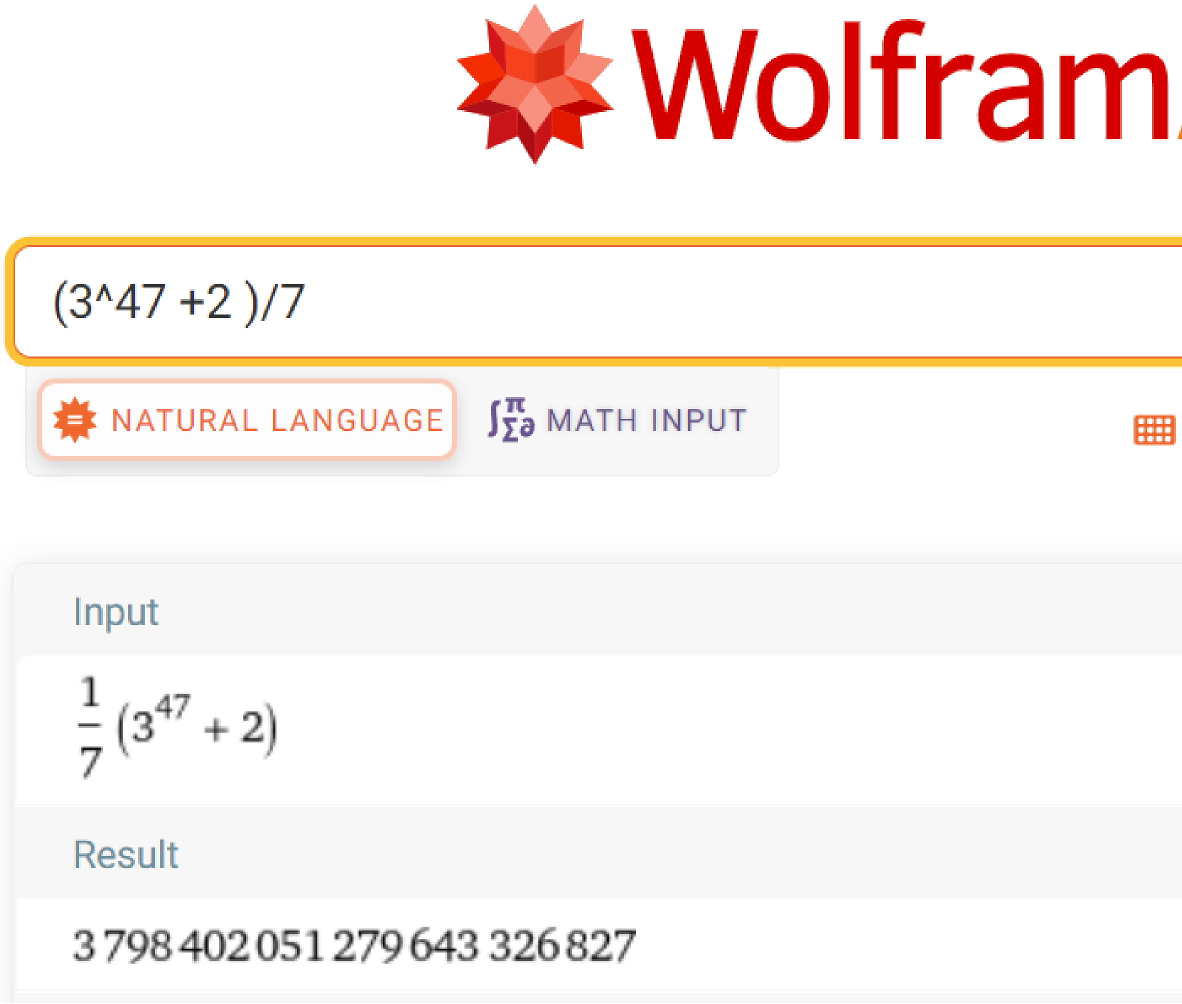
Host: How did you know that?

Hugh Binnie: "Modular arithmetic"

$$28 \equiv 0 \pmod 7$$
$$28 - 1 \equiv -1 \pmod 7$$
$$27 \equiv -1 \pmod 7$$

$$\begin{aligned} 3^{47} &\equiv \,? &&\pmod 7\\ &\equiv 3^2 3^{45} &&\pmod 7\\ &\equiv 3^2 (3^3)^{15} &&\pmod 7\\ &\equiv 9(27)^{15} &&\pmod 7\\ &\equiv 2(-1)^{15} &&\pmod 7\\ &\equiv -2 &&\pmod 7 \end{aligned}$$

$$3^{47} + 2 \equiv 0 \pmod 7.$$



$$7 \mid 3^{47} + 2$$

[10] 3. Solve the following system of linear congruences.

$$x \equiv 12 \pmod{20}$$
$$x \equiv 11 \pmod{39}$$

$x = 20n + 12$

$(20n + 12) \equiv 11 \pmod{39}$

$20n \equiv -1 \pmod{39}$

$20n = 39y - 1$

$1 = 39y - 20n$ [now solve the Diophantine eqn]

$2^{22}3^{33}5^{55} \pmod{11}$

Powers of 2:

2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768,

$2^{22}3^{33}5^{55} \pmod{11}$

Powers of 2:

2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768,

Powers of 3:

3, 9, 27, 81, 243, 729, 2187, 6561

$2^{22}3^{33}5^{55} \pmod{11}$

Powers of 2:

2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768,

Powers of 3:

3, 9, 27, 81, 243, 729, 2187, 6561.

$3^2 = 9 \equiv -2 \pmod{11}$

Powers of 5:

5, 25, 125, 625, 3125, 15625, 78125.

$2^{22}3^{33}5^{55} \pmod{11}$

Powers of 2:

2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768,

Powers of 3:

3, 9, 27, 81, 243, 729, 2187, 6561.

$3^2 = 9 \equiv -2 \pmod{11}$

Powers of 5:

5, 25, 125, 625, 3125, 15625, 78125.

$5^5 = 3125 \equiv 1 \pmod{11}$

$$2^{22}3^{33}5^{55} \pmod{11}$$

$2^5 \equiv -1 \pmod{11}$, $3^2 = 9 \equiv -2 \pmod{11}$, $5^5 = 3125 \equiv 1 \pmod{11}$

$$5^{55} = \left(5^5\right)^{11}$$

$2^{22}3^{33}5^{55} \pmod{11}$

$2^5 \equiv -1 \pmod{11}$, $3^2 = 9 \equiv -2 \pmod{11}$, $5^5 = 3125 \equiv 1 \pmod{11}$

$$5^{55} = \left(5^5\right)^{11} \equiv (1)^{11} \equiv 1 \pmod{11}$$

$$3^{33} = \left(3^2\right)^{15} 3^3$$

$2^{22}3^{33}5^{55}$ (mod 11)

$2^5 \equiv -1 \pmod{11}$, $3^2 = 9 \equiv -2 \pmod{11}$, $5^5 = 3125 \equiv 1 \pmod{11}$

$$5^{55} = \left(5^5\right)^{11} \equiv (1)^{11} \equiv 1 \pmod{11}$$

$$3^{33} = \left(3^2\right)^{15} 3^3 \equiv (-2)^{15}\, 27$$

$2^{22}3^{33}5^{55} \pmod{11}$

$2^5 \equiv -1 \pmod{11}$, $3^2 = 9 \equiv -2 \pmod{11}$, $5^5 = 3125 \equiv 1 \pmod{11}$

$$5^{55} = \left(5^5\right)^{11} \equiv (1)^{11} \equiv 1 \pmod{11}$$

$$3^{33} = \left(3^2\right)^{15} 3^3 \equiv (-2)^{15}\, 27 \equiv \left((-2)^5\right)^3 27$$

$2^{22}3^{33}5^{55} \pmod{11}$

$2^5 \equiv -1 \pmod{11}$, $3^2 = 9 \equiv -2 \pmod{11}$, $5^5 = 3125 \equiv 1 \pmod{11}$

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$2^{22}3^{33}5^{55} \pmod{11}$

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$$2^{22} = \left(2^5\right)^4 2^2$$

$2^{22}3^{33}5^{55} \pmod{11}$

$2^5 \equiv -1 \pmod{11}$, $3^2 = 9 \equiv -2 \pmod{11}$, $5^5 = 3125 \equiv 1 \pmod{11}$

$$5^{55} = \left(5^5\right)^{11} \equiv (1)^{11} \equiv 1 \pmod{11}$$

$$3^{33} = \left(3^2\right)^{15} 3^3 \equiv (-2)^{15}\, 27 \equiv \left((-2)^5\right)^3 27 \equiv 1^3 5 \equiv 5 \pmod{11}$$

$$2^{22} = \left(2^5\right)^4 2^2 \equiv (-1)^4\, 4 \equiv 4 \pmod{11}$$

$$2^{22}3^{33}5^{55} \equiv 4 \cdot 5 \cdot 1 \equiv 20 \equiv 9 \pmod{11}$$

6. Let a, b and c be non-zero integers. Their *greatest common divisor* $\gcd(a, b, c)$ is the largest positive integer that divides all of them.

(a) If $d = \gcd(a, b, c)$, prove that d is a common divisor of a and $\gcd(b, c)$.

(b) If f is a common divisor of a and $\gcd(b, c)$, prove that f is a common divisor of a, b and c.

(c) Prove that $\gcd(a, b, c) = \gcd(a, \gcd(b, c))$.

This tests whether you understand how the course notes proved the basic GCD theorems

- Regrade requests:
  - If you disagree with your regrade, let me know by end of Tuesday

- Friday 5 November:
  - Mobius quiz tonight! (covers up to middle of page 109)

- **Wednesday 10 November:**
  - **Submit Written Assignment 7: WA7 (covers up to page 121)**

Thank you!