$$V_c = \frac{m}{m+M}\sqrt{2gh}$$

$$\Delta E = E_f - E_i$$

$$= \frac{1}{\cancel{2}}\cancel{(m+M)}\left(\frac{m^{2}}{m+M}\right)^{\cancel{2}}\cancel{2}gh - \frac{1}{\cancel{2}}m\cancel{2}gh$$

$$= mgh\left(\frac{m}{m+M} - 1\right) = mgh\left(\frac{\cancel{m}}{m+M} - \frac{\overset{-M}{\cancel{m+M}}}{m+M}\right) = mgh\left(-\frac{M}{m+M}\right)$$

c) $V_{top} = \sqrt{2g\left(\frac{m}{m+M}\right)^2 h - 4gr}$

Solution:

$$\cancel{\frac{1}{2}}(\cancel{m+M})\left(\frac{m}{m+M}\right)^2 \cancel{2}gh = \frac{1}{2}(\cancel{m+M})V_t^2 + (\cancel{m+M})g2r$$

$$2\left(\frac{m^2 gh}{m+M} - g2r\right) = \cancel{\frac{1}{2}}V_t^2$$

$$\sqrt{\frac{2gh\,m^2}{m+M} - 4gr} = V_t$$

d)

$$\vec{F} = m\vec{a}$$

$$\cancel{m}\vec{g} = \cancel{m}\vec{a}$$

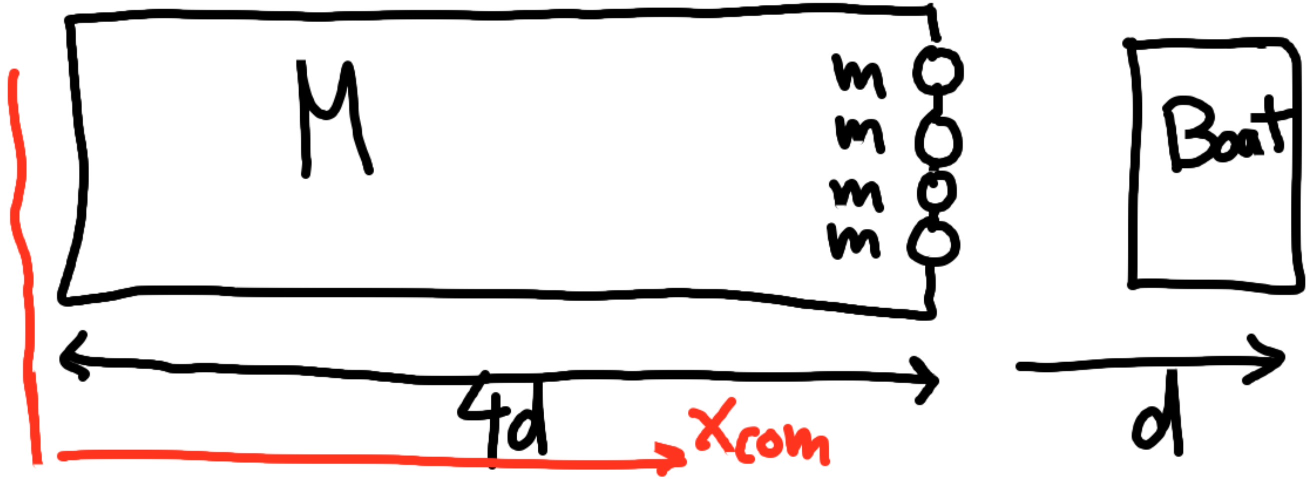
$$V_{top}^2 = gr$$

$$2g\left(\frac{m}{m+M}\right)^2 h - 4gr = gr$$

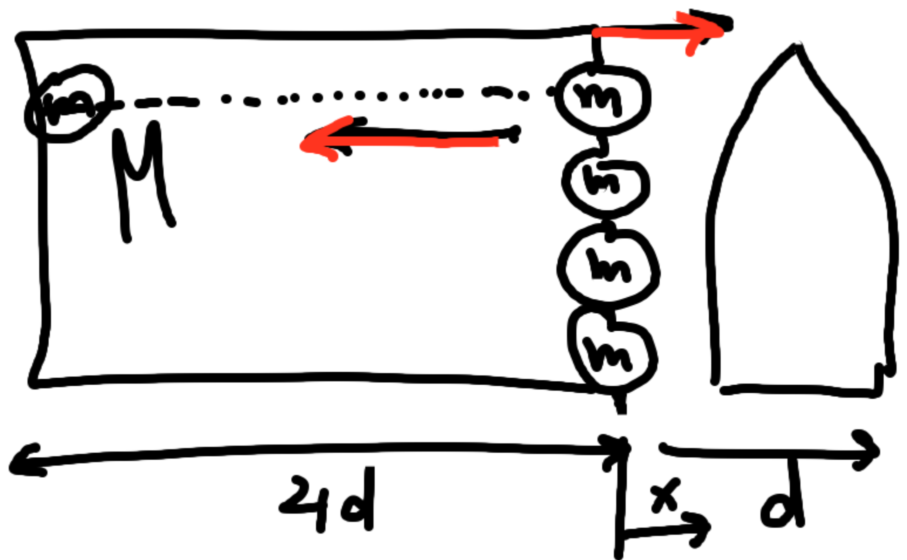
$$h_{min} = \frac{5}{2}\left(\frac{m+M}{m}\right)^2 r$$

e) $\lim_{M \to 0} h_{min} = \frac{5r}{2}$

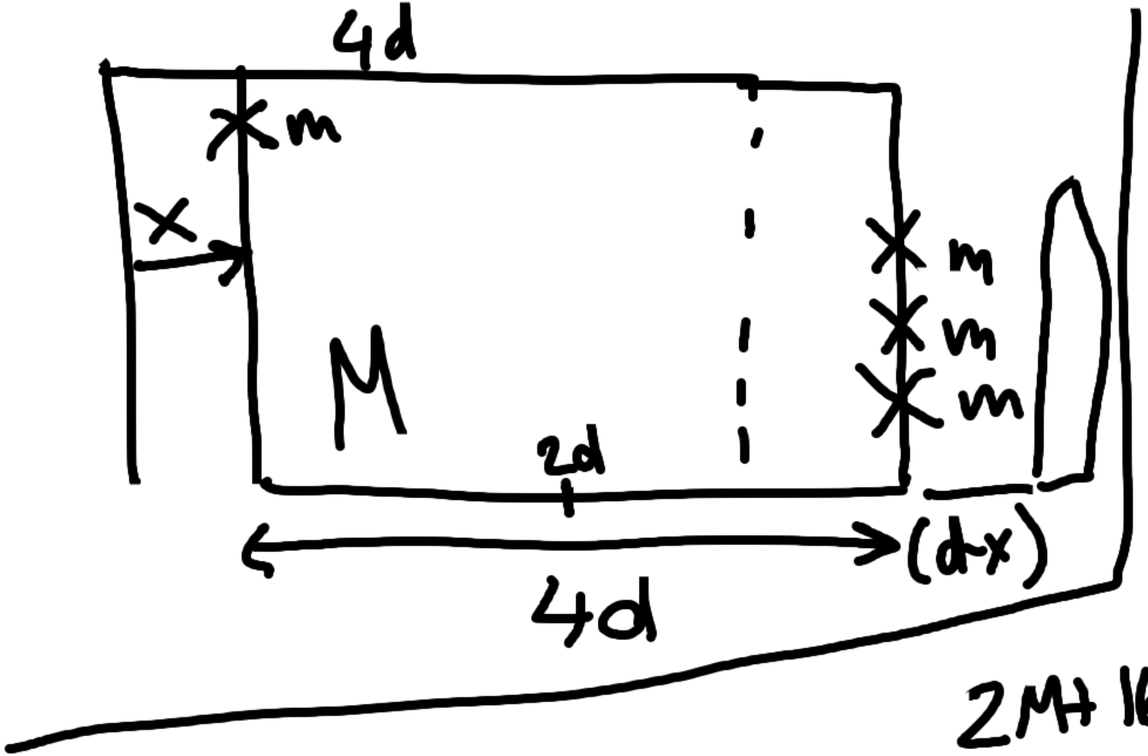
2) Four boys, each of mass m, are standing on a swimming platform of width 4d, and of mass M and frictionless bottom. Boat is distance d from the platform.



$$X_{com} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{4m\,4d + M\,2d}{4m+M} = \frac{2M+16m}{M+4m}d$$

b) 

How far x, has the edge of the platform moved towards the boat?



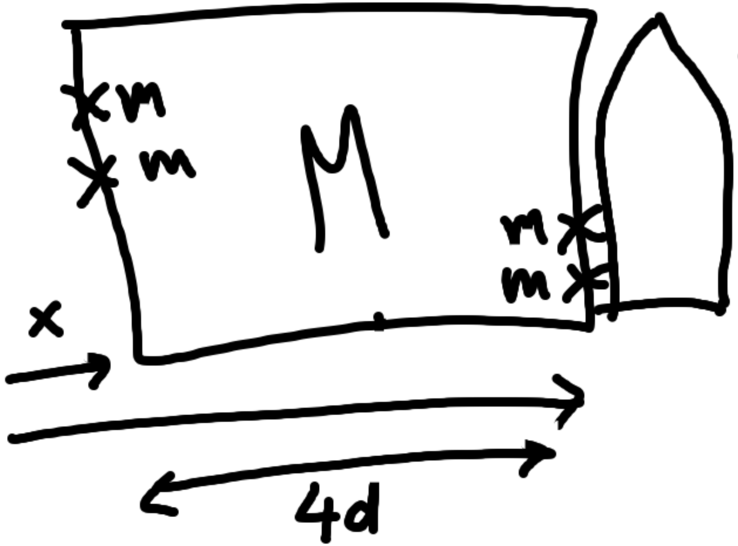
$$X_{com} = \frac{M(2d+x)+3m(4d+x)+mx}{4m+M}$$

$$= \frac{2dM+Mx+12md+3mx+mx}{4m+M}$$

$$\frac{2M+16m}{\cancel{M+4m}}d = \frac{x(M+4m)+d(2M+12m)}{\cancel{4m+M}}$$

$$x = \frac{d(\cancel{2M}+16m)-d(\cancel{2M}+12m)}{M+4m} = \frac{4dm}{M+4m} \xrightarrow{M\to 0} d$$

d) two boys walk to opposite side and platform touches boat.



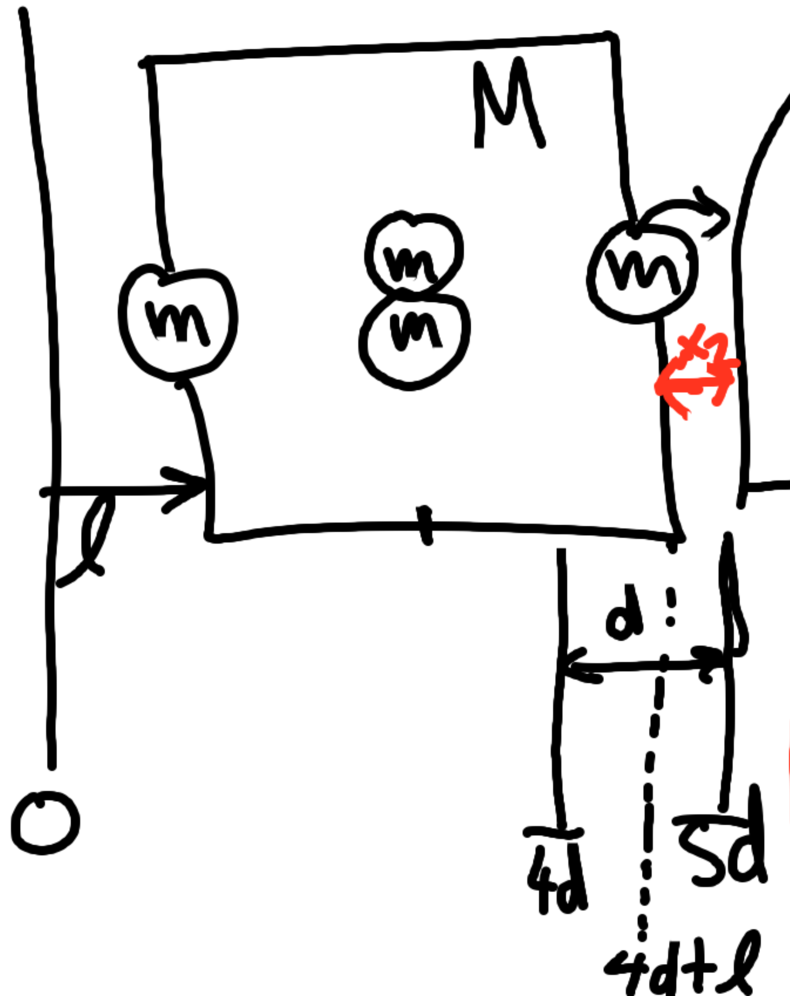
$\underline{\underline{x=d}}:$

$$X_{com} = \frac{2md + 2m\,5d + M(3d)}{4m+M}$$

$$\frac{2M+16m}{\cancel{4m+M}}\cancel{d} = \frac{12m\cancel{d} + 3M\cancel{d}}{\cancel{4m+M}}$$

$$4m = M$$

e)

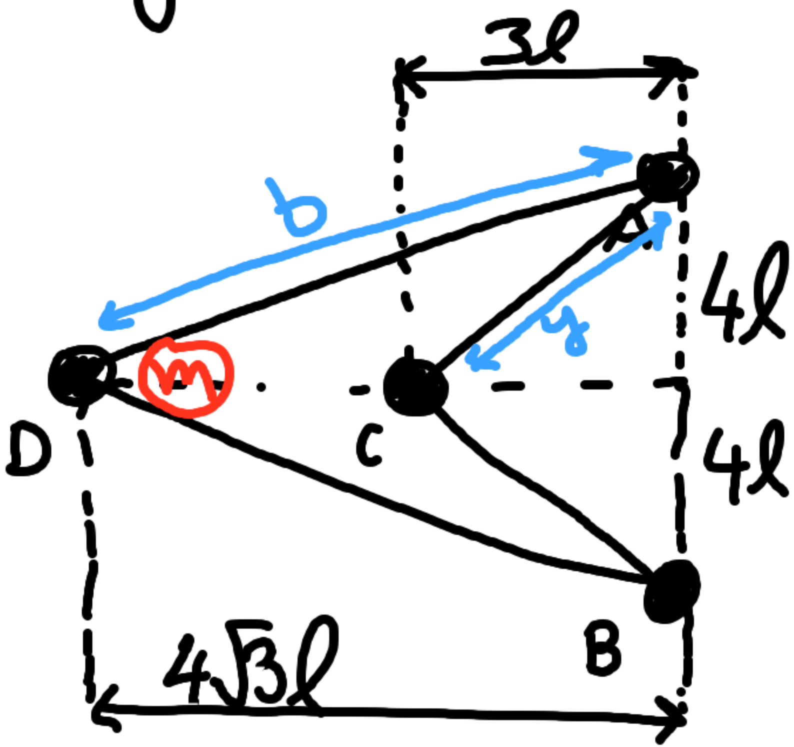


$$x_{com} = \frac{m(5d - x_2) + 2m(5d - x_2 - 2d) + m(d - x_2) + M(5d - x_2 - 2d)}{4m + M}$$

$$d\,\frac{2M+16m}{\cancel{M+4m}} = \frac{x_2(-m - 2m - M - m) + d(5m + 6m + m + 3M)}{\cancel{4m+M}}$$

$$x_2 = \frac{d(2M + 16m - 3M - 12m)}{(-M - 4m)} = \frac{d(-M + 4m)}{-(M + 4m)}$$

4) Unstretched rubber band has length 10ℓ, and obeys Hooke's law with spring constant k.



a) Use dimensional analysis to find an expression for v_1, the speed of stone after it leaves. Answer depends on m, k, ℓ and a multiplicative constant.

$$v_1 = C m^a k^b \ell^c$$

$$[v_1] = [C][m]^a[k]^b[\ell]^c$$

$$LT^{-1} = 1 \cdot M^a(MT^{-2})^b L^c$$

$$= M^a M^b T^{-2b} L^c$$

$$L^1 T^{-1} = M^{a+b} T^{-2b} L^c$$

$$F = -kx$$

$$k \propto \frac{F}{x}$$

$$[k] = \frac{[MLT^{-2}]}{[L]}$$

$$= MT^{-2}$$

$$c = 1, \quad a+b = 0, \quad -2b = -1 \Rightarrow b = \frac{1}{2} \Longrightarrow v_1 \propto \sqrt{\frac{k}{m}}\,\ell$$

b) Speed v_1 of stone after it leaves catapult.

$$\Delta E = \Delta K + \Delta E_{spring}$$

$$= \left(\frac{1}{2}mv_1^2 - 0\right) + \left(0 - \frac{1}{2}kx^2\right)$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv_1^2$$

$$y = \sqrt{(3\ell)^2 + (4\ell)^2}, \quad b = \sqrt{(4\ell)^2 + (4\sqrt{3}\ell)^2}$$

Total stretched length: $2y + 2b$. $x = 2y + 2b - 10\ell$