Phys 121, Fall 2004, Final.

6) $m_1 = 120\,kg.$

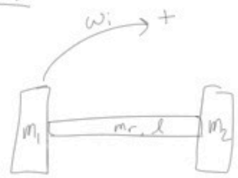
$m_2 = 120\,kg.$

$m_r = 60.0\,kg.$

$\ell = 8.00\,m$

$V_i = 6.00\,m/s$

$I_{c(rod)} = \frac{m_r \ell^2}{12}$



a) $\omega_i = ?$

$$\omega_i = \frac{V_i}{R} = \frac{2v_i}{\ell} = 1.50\,\frac{rad}{s}$$

b) $L_i = ?$

$$L_i = I\omega_i$$

$$= \left(\frac{m_r \ell^2}{12} + m_1 R^2 + m_2 R^2\right)\omega_i$$

$m_1 = m_2 = m$

$$L_i = \left(\frac{m_r \ell^2}{12} + 2m\left(\frac{\ell}{2}\right)^2\right)\omega_i$$

$$L_i = \left(\frac{m_r \ell^2}{12} + \frac{m\ell^2}{2}\right)\omega_i$$

$$= 6240\,kg\,\frac{m^2}{s}$$

$\theta = 2\pi$

$c = 2\pi R$

$$\boxed{s = R\theta}$$

$$V = \frac{ds}{dt} = \frac{d}{dt}\left(R\theta(t)\right)$$

$$= R\frac{d}{dt}\theta$$

$$\boxed{\begin{aligned} V &= R\omega \\ a &= R\alpha \end{aligned}}$$

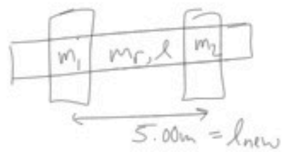
$$\frac{d}{dx}\,cf(x) = c\frac{d}{dx}f(x)$$

c) $K_i = ?$

$K_i = \frac{1}{2} I_i \omega_i^2$ <u>or</u> $K_i = \frac{1}{2} I_{rod} \omega_i^2 + 2\left(\frac{1}{2} m v_i^2\right)$

$= 4680\,J$

d) $v_f = ?$



$V_f = R\,\omega_f$

$= \frac{\omega_f l_{new}}{2}$

$L_i = L_f$

$L_f = I_f \omega_f$ $\longrightarrow l_{new} = 5.00m$

$= \left(\frac{m_r l^2}{12} + \frac{m l_{new}^2}{2}\right) \omega_f$

$\omega_f = \frac{L_i}{\left(\frac{m_r l^2}{12} + \frac{m l_{new}^2}{2}\right)}$

$V_f = \frac{L_i l_{new}}{2\left(\frac{m_r l^2}{12} + \frac{m l_{new}^2}{2}\right)}$

$= 8.57 \frac{m}{s}$

e) $W = ?$

$W = \Delta K$

$= K_f - K_i$

$= \frac{1}{2} I_f \omega_f^2 - K_i$

$= \frac{1}{2} L_f \omega_f - K_i$

$= 6017\,J$

$= 6020\,J$ (3 sig figs).

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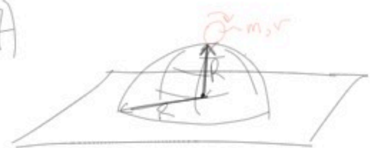
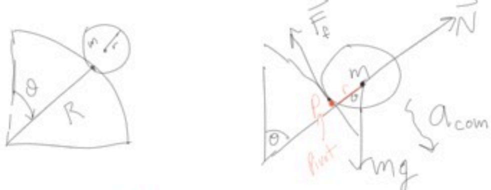
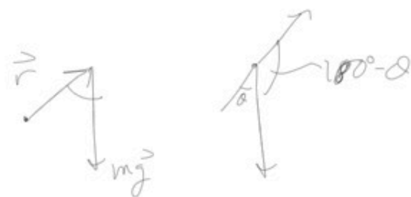
7)



$R = 2.00\,m$

a)





b)

$a_T(\theta) = ?$

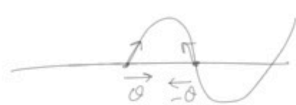
$$\tau_{net} = I\alpha$$

$$\tau_{mg} = \tau_{net} = |F||r_c|\sin\theta$$
$$= mg\,r\,\sin(\pi - \theta)$$
$$= mg\,r\,\sin\theta$$

$$\tau_{net} = I\alpha$$
$$= \frac{7}{5} mr^2 \alpha$$

$$mg\,r\sin\theta = \frac{7}{5} mr^2 \underbrace{\alpha}_{\frac{a}{r}}$$

$$a(\theta) = \frac{(5\cancel{m}g\cancel{r}\sin\theta)\cancel{r}}{7\cancel{mr^2}}$$
$$= \frac{5g\sin\theta}{7}$$



$$I_{com} = \frac{2}{5} mr^2$$

$$I_{at\,r} = \frac{2}{5} mr^2 + mr^2 = \frac{7}{5} mr^2$$

$$I = I_{com} + mr^2$$

r = distance from C.O.M. to Pivot

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c) $V_{cm}(\theta) = ?$ Neglect small r vs R of hemisphere

$$\Delta E = \Delta K + \Delta U_g$$

$$0 = \left(\tfrac{1}{2}mv_c^2 + \tfrac{1}{2}I\omega^2\right) - (0) \quad + \left(R\cos\theta\, mg - mgR\right)$$

$$2mgR - 2R\cos\theta\, mg - I\underbrace{\omega^2}_{\frac{v_c}{r}} = mv_c^2$$

$$mv_c^2 + I\frac{v_c^2}{r^2} = 2mgR - 2Rmg\cos\theta$$

$$v_c^2\left(m + \frac{I}{r^2}\right) = 2mgR(1-\cos\theta)$$

$$v_c^2 = \frac{2mgR(1-\cos\theta)}{m + \frac{I}{r^2}} = \frac{10}{7}gR(1-\cos\theta)$$

$$v_c = 5.29\sqrt{1-\cos\theta}$$

d) $\theta_s = ?$

$$F_{net} = ma_c$$

$$\underbrace{mg\cos\theta}_{\text{gravitational force parallel to } N} - N = m\frac{v_c^2}{R}$$

$$v_c^2 = \frac{10}{7}gR(1-\cos\theta)$$

$$mg\cos\theta = \frac{mv_c^2}{R} + \cancel{N}^{\to 0 \text{ when ball flies off}}$$

$$mg\cos\theta = \frac{m\,10g\cancel{R}(1-\cos\theta)}{7\cancel{R}}$$

$$\cancel{mg}\cos\theta = \frac{10\cancel{mg}}{7} - \frac{10\cancel{mg}}{7}\cos\theta$$

$$\cos\theta = \frac{10}{7}(1-\cos\theta)$$

$$\frac{7}{7}\cos\theta = \frac{10}{7} - \frac{10}{7}\cos\theta$$

$$\frac{17}{\cancel{7}}\cos\theta = \frac{10}{\cancel{7}}$$

$$\cos\theta = \frac{10}{17}$$

$$\theta = \arccos\left(\frac{10}{17}\right)$$

$$\theta = 53.97°$$

$$= 54.0° \text{ [3 sig figs]}$$

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