$P_i = P_f$

$\cancel{m}v_{1i} = \cancel{m}v_{1f} + \cancel{m}v_{2f}\cos\theta_2 + \cancel{m}v_{3f}\cos\theta_3$

$v_{1i} = v_{1f} + v_{2f}\frac{\sqrt{3}}{2} + v_{3f}\frac{\sqrt{3}}{2}$

$\boxed{v_{1i} = v_{1f} + \cancel{2}v_{2f}\frac{\sqrt{3}}{\cancel{2}}} \longrightarrow \boxed{v_{1i} - v_{1f} = v_{2f}\sqrt{3}}$

$E_i = E_f$

$\cancel{\tfrac{1}{2}}mv_{1i}^2 = \cancel{\tfrac{1}{2}}mv_{1f}^2 + \cancel{\tfrac{1}{2}}mv_{2f}^2 + \cancel{\tfrac{1}{2}}mv_{3f}^2$

$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + v_{3f}^2$

$\boxed{v_{1i}^2 = v_{1f}^2 + 2v_{2f}^2} \longrightarrow \boxed{v_{1i}^2 - v_{1f}^2 = 2v_{2f}^2}$

$(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = 2v_{2f}^2$

$\cancel{v_{2f}}\sqrt{3}\,(v_{1i} + v_{1f}) = 2v_{2f}^{\cancel{2}}$

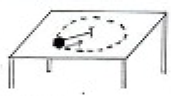
$v_{1i} + v_{1f} = \frac{2}{\sqrt{3}}\left(\frac{v_{1i} - v_{1f}}{\sqrt{3}}\right)$

$3(v_{1i} + v_{1f}) = \frac{2}{3}(v_{1i} - v_{1f})$

$3v_{1f} + 2v_{1f} = 2v_{1i} - 3v_{1i}$

$\boxed{v_{1f} = -\frac{1}{5}v_i}$

10. A 0.500 kg particle is connected by an ideal elastic bungee cord to a fixed vertical nail stuck into the centre of a frictionless horizontal table top. The elastic cord has an unstretched length of 0.400 m and a force constant of 650 N/m. If the particle is made to travel in a horizontal circular path at a uniform rate of 2.50 revolutions per second, find the tension in the cord. [Answer in N.]



$\ell = 0.400\text{m} + x \implies F_{spring} = -kx$

$\vec{N}$, $\vec{mg}$, $\vec{T} = kx \rightsquigarrow a = \frac{v^2}{r}$

$F_{Net} = kx = ma = m\frac{v^2}{r}$

$x = \frac{mv^2}{kr} = \frac{m(2\pi r f)^2}{kr} = \frac{4m\pi^2 r f^2}{k} = \frac{4(0.5\text{kg})\pi^2\, 0.4\text{m}\,(2.5^2\,\frac{\text{rev}^2}{\text{s}^2})}{650\,\frac{\text{N}}{\text{m}}}$

12.5 rev in 5 seconds.

12.5 rev = how many metres of travel?

$2\pi r\frac{\text{m}}{\cancel{\text{rev}}}\left(12.5\,\frac{\text{rev}}{5\text{s}}\right) = 2\pi r f\left[\frac{\text{m}}{\text{s}}\right]$

$x = 0.09571\text{m}$

$T = kx = 60.9\text{N}$

9. A light rigid rod of length L has a ball of mass m at one end. The other end is pivoted so that the rod and ball can travel in a vertical circle. Starting from the horizontal position as shown, the ball is given an initial downward velocity v_0 such that later it just barely makes it over the top of the circle. Under those conditions, what was the tension in the rod when the ball was at its lowest point? (A negative tension corresponds to compression).

Between beginning and top:

$\Delta E = \Delta K + \Delta U_g$

$0 = (0 - \tfrac{1}{2}mv_0^2) + \underbrace{(2Lmg - mgL)}_{gL}$

$v_0 = \sqrt{2gL}$

Between beginning and bottom

$\Delta E = \Delta K + \Delta U_g$

$= (\tfrac{1}{2}\cancel{m}v_f^2 - \tfrac{1}{2}\cancel{m}v_0^2) + (0 - mgL)$

$v_f = \sqrt{4gL} = 2\sqrt{gL} = v_{bottom}$

$F_{net} = T_{bottom} - mg = m\frac{v_{bottom}^2}{r}$

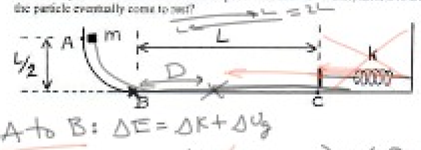
$T_{bottom} = \frac{mv_{bottom}^2}{L} + mg$

$= \frac{m\,4g\cancel{L}}{\cancel{L}} + mg$

$= 5mg$

8. A particle of mass m slides along a track in a vertical plane as shown. The upward curved part of the track is frictionless and the flat horizontal part from B to C of length L has a coefficient of kinetic friction μ_k = 0.250. The flat section to the right of C is frictionless and the ideal spring has k = 500 N/m. The particle is released from rest at point A as shown. Where, measured from point B does the particle eventually come to rest?

$L = 2L$



$\frac{1}{2}kx^2$

A to B: $\Delta E = \Delta K + \Delta U_g$

$= (\tfrac{1}{2}\cancel{m}v_B^2 - 0) + (0 - \cancel{m}g\tfrac{L}{2})$

$v_B = \sqrt{gL}$

B to C: $W_{friction} = \Delta K$

$|\vec{F}_{\mu_k} \cdot \vec{d}| = \tfrac{1}{2}mv_C^2 - \tfrac{1}{2}mv_B^2$

$= |\vec{F}_{\mu}||\vec{d}|\cos\theta =$

$= |\mu_k \vec{N}||\vec{d}|\,180$

$= \mu_k mg L\cos(180°)$

$= -\mu_k \cancel{m}gL = \tfrac{1}{2}\cancel{m}v_C^2 - \tfrac{1}{2}\cancel{m}v_B^2$

$v_C = \sqrt{v_B^2 + 2\mu_k gL}$

$= \sqrt{gL + 2\mu_k gL}$

$= \sqrt{gL(1 + 2\mu_k)}$

$W_{friction} = \Delta K$

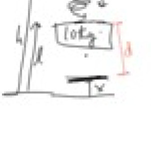
$-\mu_k \cancel{mg}x = 0 - \tfrac{1}{2}\cancel{m}(\cancel{g}L(1 + 2\mu_k))$

$2\mu_k x = L(1 + 2\mu_k)$

$x = \frac{L(1+2\mu_k)}{2\mu_k} = \frac{L}{2\mu_k} + L$

$D = L - x = \frac{L}{2\mu_k} = \frac{L}{2(\frac{1}{4})} = 2L$

7. Prof. Brandon's first grandson, Matthew James (MJ), was too young to go trick-or-treating on his first Halloween this year. So his sister, Caitlyn Elizabeth (CE), and his cousin Sydney Jean (SJ), decided to suspend MJ in his jolly-jumper from the ceiling of their front porch where he could watch them trick-or-treat around the neighbourhood. The jolly jumper acted like an ideal spring with an unstretched length of 1.20 m and with a spring constant of 98.0 N/m. The girls attached MJ, of total mass 10.0 kg in his costume, to the lower end of the jolly-jumper and released him from rest. If the ceiling to floor distance was 3.70 m, how close to the floor did MJ come after the girls released him? (Answers in m.)



1.2m, $k = 98.0\frac{\text{N}}{\text{m}}$, $h = 3.7\text{m}$

$\Delta E = \Delta K + \Delta U_g + \Delta U_s$

$0 = \cancel{(0 - 0)} + \underbrace{(mgx - mg\ell)}_{-mgd} + (\tfrac{1}{2}kd^2 - 0)$

$= -\cancel{2}mg\cancel{d} + \tfrac{1}{\cancel{2}}kd^{\cancel{2}}$

$\frac{2mg}{k} = kd$

$d = 2.00\text{m}$, $h = 3.7\text{m} \implies h = 1.20\text{m} + 2.00\text{m} + x$

$\boxed{x = 3.70\text{m} - 1.20\text{m} - 2.00\text{m} = 0.50\text{m}.}$