Assignment 4:

Q4. (5 points) Alice and Bob play a game starting with a pile of n sticks. Each player on their turn can remove 1, 2 or 3 sticks from the pile. The last player to remove a stick wins. Alice goes first. Prove by induction that Alice has a winning strategy if and only if $4 \nmid n$.

- A winning strategy is a rule that a player can follow which guarantees they will win no matter what decisions their opponent makes. You may assume, without proof, that either Alice or Bob has a winning strategy.
- You may also assume without proof that any integer n can be written as $n = 4q + r$ where $q \in \mathbb{Z}$ and $r \in \{0, 1, 2, 3\}$.

MATH 135: Lecture 11

Dr. Nike Dattani

1 October 2021

- Thursday 30 September:
  - Look at WA4 **!!!**
- Thursday 30 September:
  - WA03 solutions will be posted, hopefully before 12pm: Check the solutions in detail!
- Thursday 30 September:
  - Complete reading from Chapter 3.6 up to 5 of the course notes. **Pages 55-81.**
- Friday 1 October:
  - Mobius Quiz 9
- Sunday 3 October:
  - You'll need to know more before 0.4 (Polynomials), so use this time to review **Pages 55-81**, and do **practice problems!**
- Monday 4 October:
  - Mobius Quiz 10
- Tuesday 28 September:
  - **Look at your WA02 results thoroughly! Where did you lose marks?**
- **Wednesday 29 September:**
  - **Complete Written Assignment 3: WA3**
- Wednesday 29 September:
  - Mobius Quiz 8

View By: Groups | Groups: 1219-MATH.135.019.1.LEC | Apply

MQ7 (Mon Sep 27) Class Statistics

Number of submitted grades: 53 / 56

Minimum: 40 %

Maximum: 100 %

Average: 80.75 %

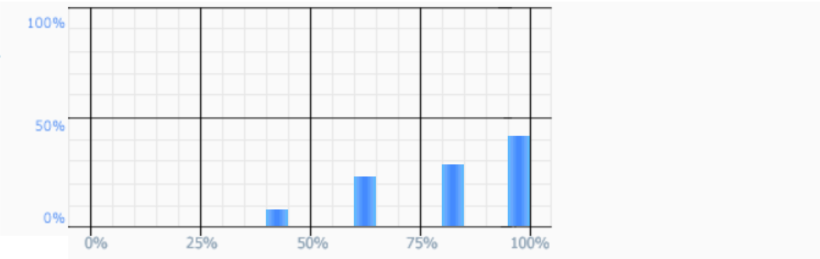
Mode: 100 %

Median: 80 %

Standard Deviation: 19.41 %

Grade Distribution

Number of Users (%)



View By: Groups | Groups: 1219-MATH.135.016.1.LEC | Apply

MQ7 (Mon Sep 27) Class Statistics

Number of submitted grades: 46 / 50

Minimum: 40 %

Maximum: 100 %

Average: 85.65 %

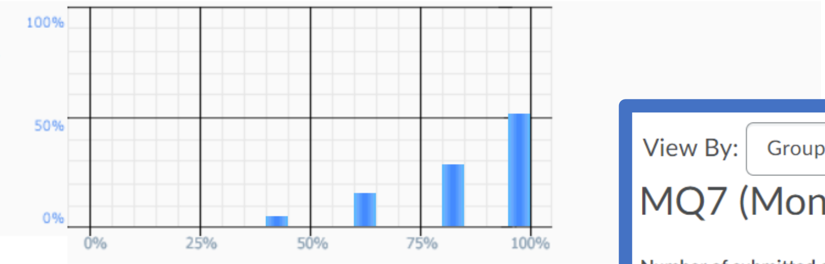
Mode: 100 %

Median: 100 %

Standard Deviation: 17.53 %

Grade Distribution

Number of Users (%)



View By: Groups | Groups: MATH135_1219_LEC | Apply

MQ7 (Mon Sep 27) Class Statistics

Number of submitted grades: 1,168 / 1,258

Minimum: 0 %

Maximum: 100 %

Average: 79.71 %

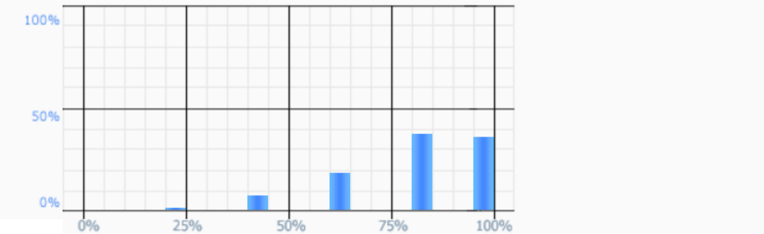
Mode: 80 %

Median: 80 %

Standard Deviation: 19.9 %

Grade Distribution

Number of Users (%)



View By: Groups Groups: 1219-MATH.135.019.1.LEC Apply

MQ8 (Wed Sep 29) Class Statistics

Number of submitted grades: 55 / 56

Minimum: 40 %

Maximum: 100 %

Average: 84 %

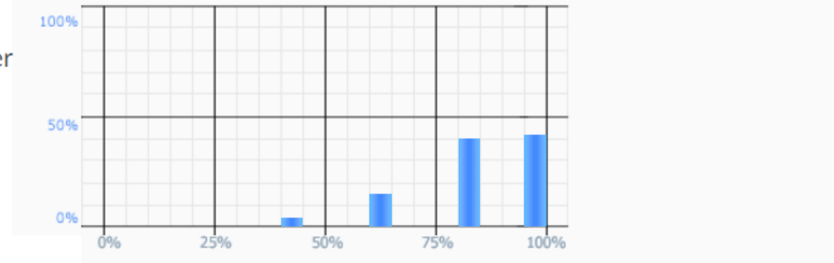
Mode: 100 %

Median: 80 %

Standard Deviation: 16.36 %

Grade Distribution

Number of Users (%)



View By: Groups Groups: 1219-MATH.135.016.1.LEC Apply

MQ8 (Wed Sep 29) Class Statistics

Number of submitted grades: 48 / 50

Minimum: 20 %

Maximum: 100 %

Average: 79.17 %

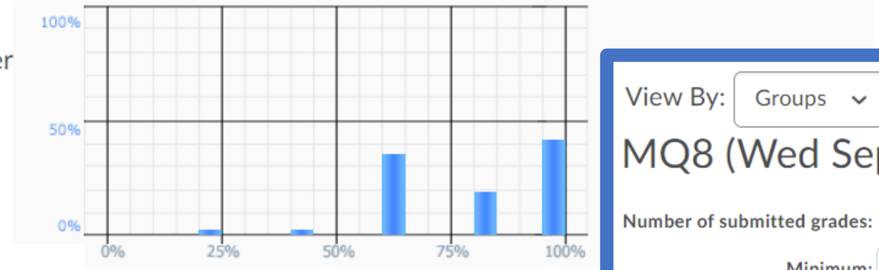
Mode: 100 %

Median: 80 %

Standard Deviation: 20.4 %

Grade Distribution

Number of Users (%)



View By: Groups Groups: MATH135_1219_LEC Apply

MQ8 (Wed Sep 29) Class Statistics

Number of submitted grades: 1,187 / 1,258

Minimum: 0 %

Maximum: 100 %

Average: 78.97 %

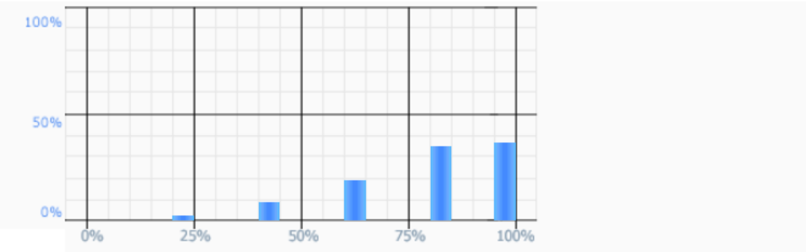
Mode: 100 %

Median: 80 %

Standard Deviation: 20.82 %

Grade Distribution

Number of Users (%)



Week 3	Chapter 3: Proving Mathematical Statements [Chapter 0: Polynomials Over **R**]	3.2 Proving Existentially Quantified Statements 3.3 Proving Implications 3.4 Divisibility of Integers 3.5 Proof by Contrapositive 0.3 Polynomial Divisibility	Mobius Quizzes 4, 5, 6 **Available:** Mon Sep 20, Wed Sep 22, Fri Sep 24 WA2 **Due:** Wed Sep 22 at 5 PM EDT	**MQ:** 0.4% each **WA2:** 2.22%	Assessments cover the material from Weeks 1–2
Week 4	Chapter 3: Proving Mathematical Statements	3.6 Proof by Contradiction 3.7 Proving If and Only If Statements 4.1 Notation for Summations, Products and Recurrences 4.2 Proof by Induction 4.4 Proof by Strong Induction	Mobius Quizzes 7, 8, 9 **Available:** Mon Sep 27, Wed Sep 29, Fri Oct 1 **Due:** midnight WA3 **Due:** Wed Sep 29 at 5 PM EDT	**MQ:** 0.4% each **WA3:** 2.22%	Assessments cover the material from Weeks 1–3

Q1) We want to prove something fall all integers n.

What method?

Induction on n.

Inductive step:
Case 1: $k\alpha/\pi$ an integer
Case 2: $k\alpha/\pi$ not an integer

e.g. of $k\alpha/\pi$ an integer?

Q1) We want to prove something fall all integers n.

What method?

Induction on n.

Inductive step:
Case 1: $k\alpha/\pi$ an integer
Case 2: $k\alpha/\pi$ not an integer

e.g. of $k\alpha/\pi$ an integer? $\alpha = \pi$
e.g. of $k\alpha/\pi$ not an integer?

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What method?

Induction on n.

Inductive step:
Case 1: $k\alpha/\pi$ an integer
Case 2: $k\alpha/\pi$ not an integer

e.g. of $k\alpha/\pi$ an integer? $\alpha = \pi$
e.g. of $k\alpha/\pi$ not an integer? $\alpha = 1$

Q1) We want to prove something fall all integers n.

What method?

Induction on n.

Inductive step:
Case 1: $k\alpha/\pi$ an integer
Case 2: $k\alpha/\pi$ not an integer

e.g. of $k\alpha/\pi$ an integer? $\alpha = \pi$
e.g. of $k\alpha/\pi$ not an integer? $\alpha = 1$

If $(k+1)\alpha/\pi$ is not an integer, then is $k\alpha/\pi$ not an integer?
$k+1 = 17$, $\alpha = \pi/16$, $(k+1)\alpha/\pi = 17/16$
$k = 16$, $\alpha = \pi/16$, $k\alpha/\pi = 1$

Q1) We want to prove something fall all integers n.

What method?

Induction on n.

Inductive step:
Case 1: $k\alpha/\pi$ an integer
Case 2: $k\alpha/\pi$ not an integer

Case 2 is the harder one (need to use the hint).

Another hint: can denominator be 0?

Can this happen when $(k+1)\alpha/\pi$ is not an integer?

b) What type of proof?
Contradiction! Assume $\cot(1^o)$ is rational!
Try some values of n. If $\cot(1^o)$ is rational and $n\alpha/\pi$ is non-integer, then $\cot(n\alpha)$ must be rational (because of part a).
If we violate what we proved in part (a), then $\cot(1^o)$ is not rational.

Q2)

First show that m exists.

<u>Marks lost if you assume it exists!</u>

Figure out an m that works, then plug it into $n^2 = 8m + 1$.

Then show it's unique!!

Example of proving uniqueness. If we have:

$(n^4 = 9m_1) \wedge (n^4 = 9m_2)$, then:

$9m_1 = n^4 = 9m_2$

$m_1 = m_2$ (So they are the same. You can also try with m_3, m_4, etc.)

Q3) Strong induction.

Induction hypothesis:
Make sure you give the correct range for k.
Make sure induction hypothesis is not “for all k”.

Assume ***P(r)*** is true for ***r = 1,2,…,k*** and ***k*** ≥ last base case.

- Inductive step begins with the definition of the sequence,
- then you use the induction hypothesis to prove that the definition of the sequence gives you what you want to prove, for the k+1 case.

Q5)

Use induction