Warm-up!

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underline{?}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \underline{?}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Circuit-Based Quantum Computing

Lecture 1

Nike Dattani

nike@hpqc.org

HPQC Labs

2022 11 28

Getting to know you!

How many of you have taken a university-level quantum computing course before?

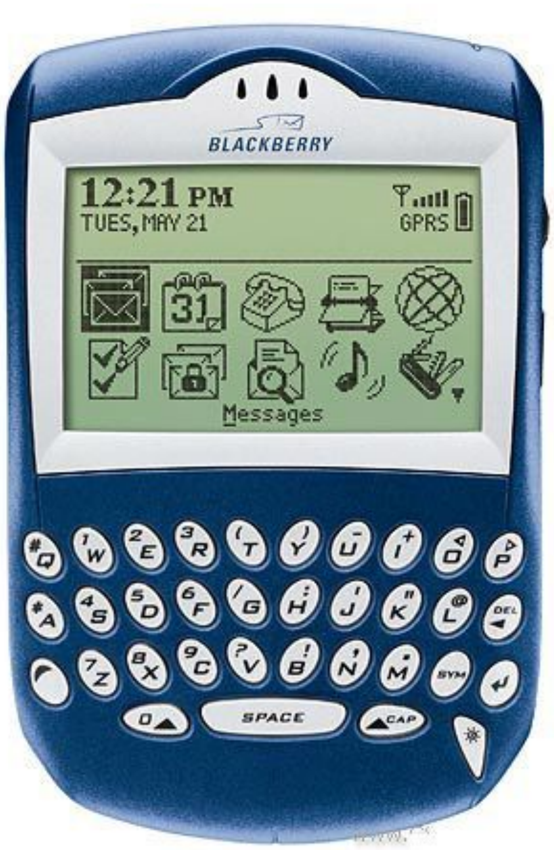
Getting to know me!

- From Waterloo, Canada

Trivia:

What does Waterloo have in common with the Helsinki area?

Waterloo



Helsinki / Espoo

Blackberry

Founder (Mike Laziridis) **loved basic science**.

1st company outside Scandinavia to develop products for Mobitex networks (1990s).

Profits from that were **plowed** into wireless science **research** (leading to their first smartphone in 1999) and into basic science research (Perimeter Institute, $170 million since 1999, **Institute for Quantum Computing**, $100 million since 2002)

Nokia 6810,6820,9300,9300i,9500,E-series used BB's email client

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Nokia 6810,6820,9300,9300i,9500,E-series used BB's email client

September 2016: BlackBerry stops making phones (focus on software)
January 2022: BlackBerry phones no longer work (including 911 calls!)

Getting to know me!

- Undergrad from University of Waterloo (Math, Physics & Biology)
- Took my first quantum computing course decades ago
- Undergrad thesis was with Ray Laflamme (Director of IQC)
- Also undergrad research with Bob Le Roy (quantum chem)

Getting to know me!

- Undergrad research also with Bob Le Roy (quantum chem)

A DPF data analysis yields accurate analytic potentials for $Li_2(a^3\Sigma_u^+)$ and $Li_2(1^3\Sigma_g^+)$ that incorporate 3-state mixing near the $1^3\Sigma_g^+$ state asymptote

Nikesh S. Dattani *, Robert J. Le Roy

Department of Chemistry, University of Waterloo, Waterloo, ON, Canada N2L 3G1

Morse/Long-range potential

From Wikipedia, the free encyclopedia

The **Morse/Long-range potential** (**MLR potential**) is an interatomic interaction model for the potential energy of a diatomic molecule.

Getting to know me!

- Undergrad from University of Waterloo (physics, math, biology)
- PhD from Oxford University (chemistry)

Getting to know me!

- Undergrad from University of Waterloo (physics, math, biology)
- PhD from Oxford University (chemistry)
- Post-PhD work:
  - Kyoto University, **Japan** (JSPS Fellow)
  - Nanyang Technological University, **Singapore**
  - Harvard University / Smithsonian Institution, **USA**
  - Max Planck Institute for Solid-State Research, **Germany**
  - Jilin University, **China**

  - McMaster University, **Canada** (Banting Fellow)
  - National Research Council, **Canada**
  - University of Waterloo, **Canada**
- Director of HPQC Labs in Waterloo, and UW council member

Ein neuer Interferenzrefraktor.

Von

Dr. **L. Zehnder** in Basel.

Die Brewster'sche Entdeckung der Farben dicker Platten[1]) wurde von Herrn Jamin in glücklichster Weise zur Konstruktion seiner Interferenzrefraktoren[2])

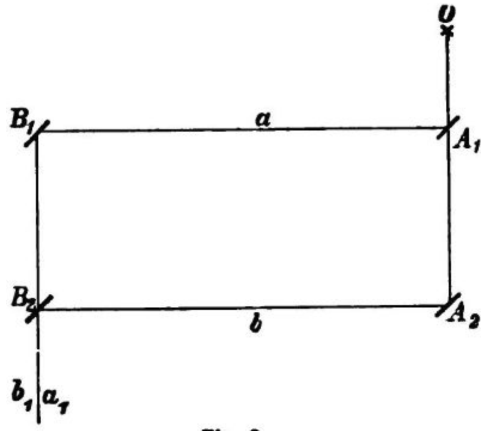

Fig. 2.

XI. Jahrgang. **August 1891.** Achtes Heft.

Ein neuer Interferenzrefraktor.

Von
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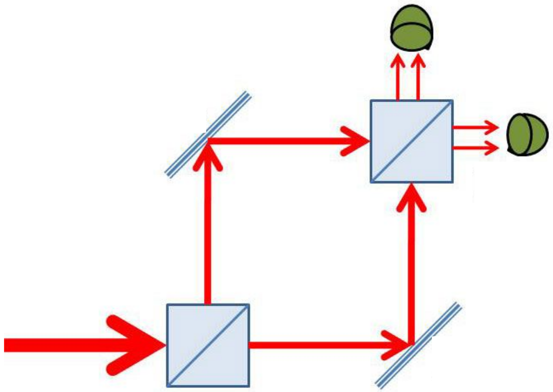
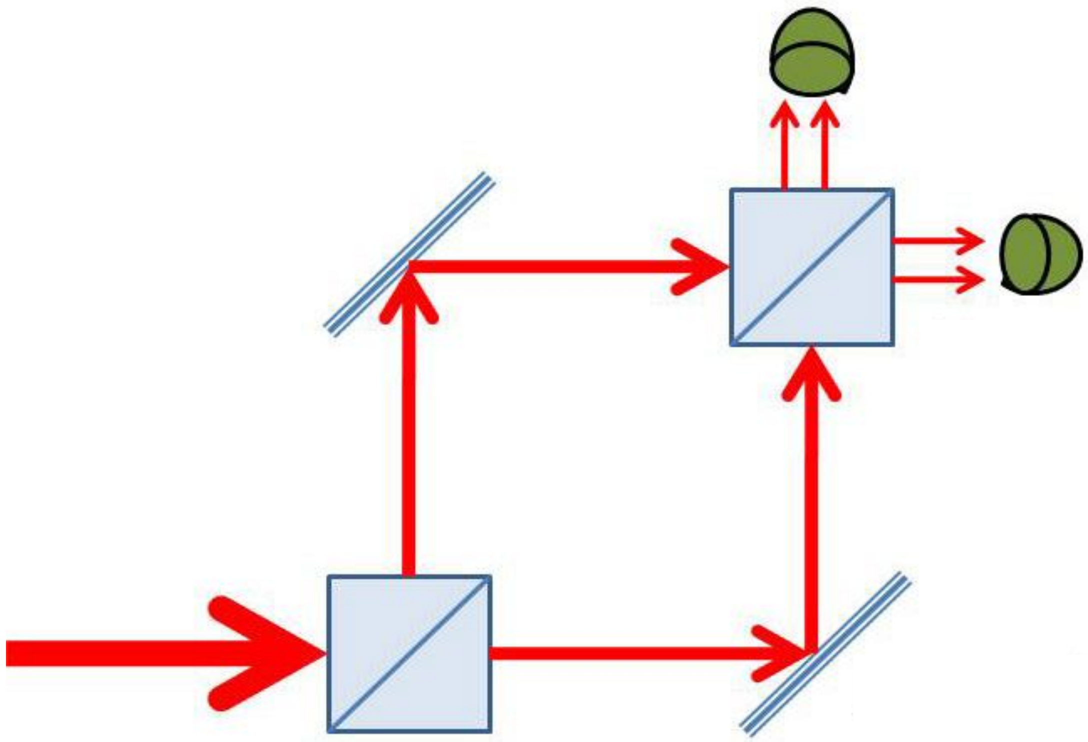


Mach-Zehnder Experiment



Qubits and quantum gates

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad , \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Qubits and quantum gates

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Qubits and quantum gates

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$$X|0\rangle = \ ? \qquad X|1\rangle = \ ?$$

Qubits and quantum gates

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Classical Computer Bits

0 and 1 represent any distinct classical states!

- CPU processing
  0 = Low voltage (0 mV)
  1 = High voltage (5 mV)

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- CPU processing
  - 0 = Low voltage (0 mV)
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- Barcodes
  - 0 = Thin line
  - 1 = Thick line

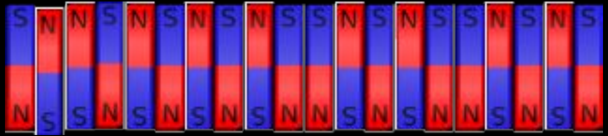
Classical Computer Bits

0 and 1 represent any distinct classical states!

- CPU processing
  - 0 = Low voltage (0 mV)
  - 1 = High voltage (5 mV)
- Barcodes
  - 0 = Thin line
  - 1 = Thick line
- Optical disks
  - 0 = Absence of pit
  - 1 = Presence of pit

Hard Drive

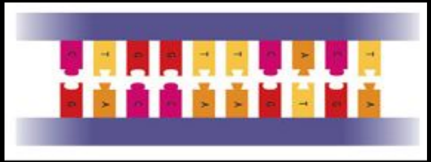
01101010101001010010101



DNA Storage

0 : CG

1 : AT

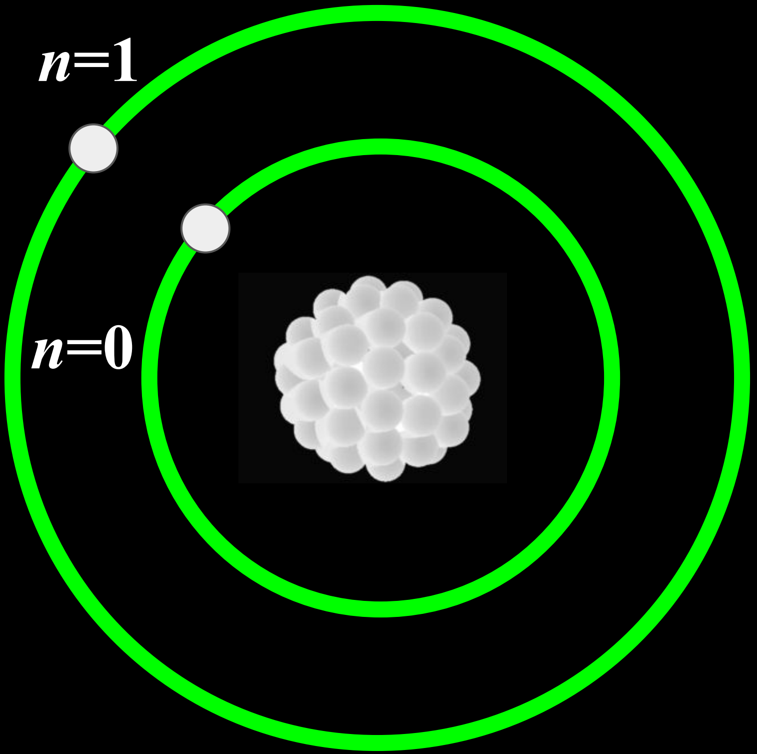


dna inside

Quantum computer bits (qubits)

0 and 1: two quantum mechanically allowed states

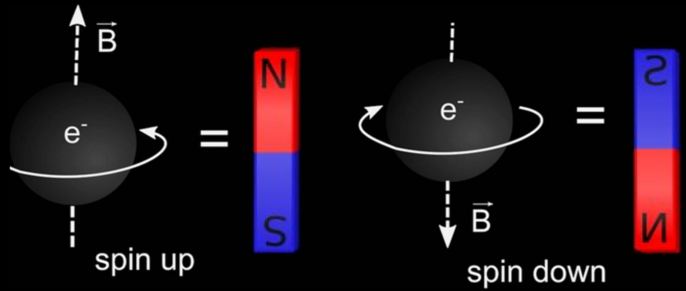
- Atomic levels
  - 0 = Ground state
  - 1 = Excited state

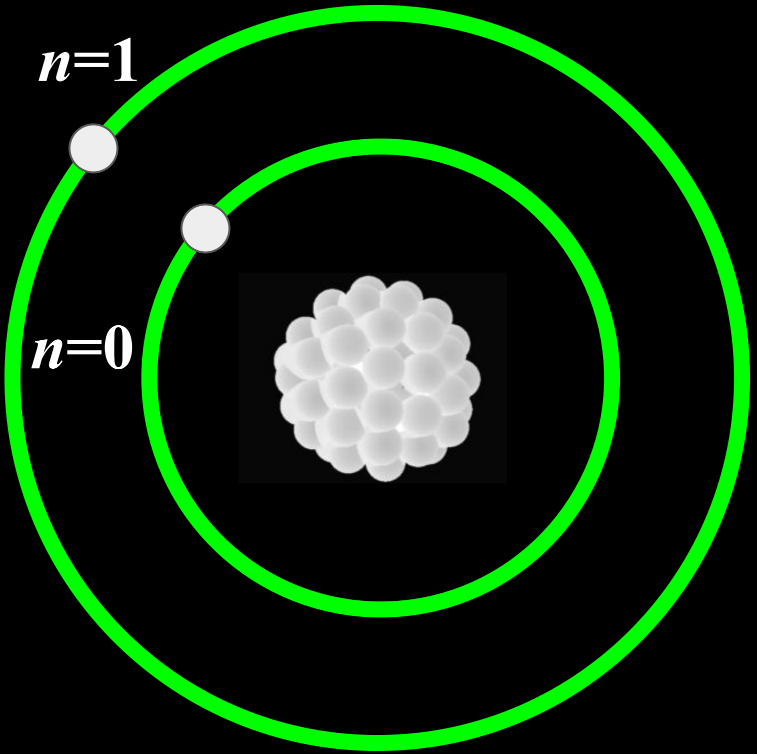


Quantum computer bits (qubits)

0 and 1: two quantum mechanically allowed states

- Atomic levels
  - 0 = Ground state
  - 1 = Excited state
- Spin
  - 0 = Up
  - 1 = Down

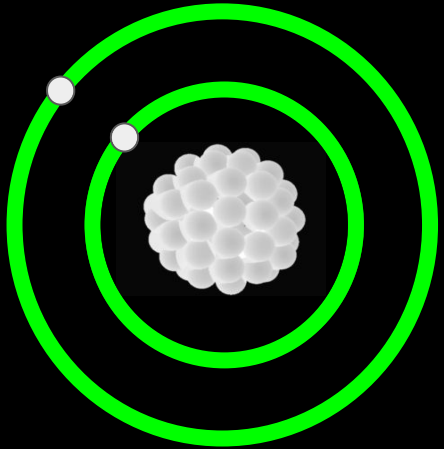


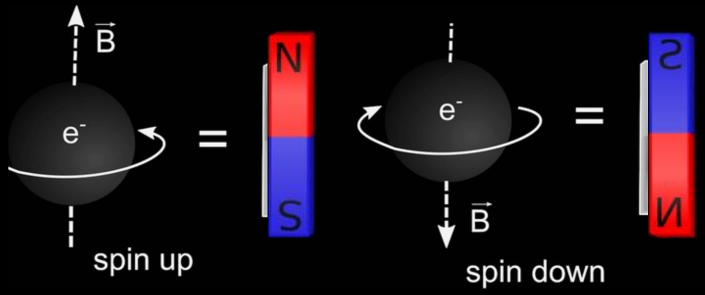


Quantum computer bits (qubits)

0 and 1: two quantum mechanically allowed states

- Atomic levels
  - 0 = Ground state
  - 1 = Excited state
- Spin
  - 0 = Up
  - 1 = Down
- Photons
  - 0 = Horizontal Polarization
  - 1 = Vertical Polarization
- **Many more possibilities!**





Schrödinger tells us:

$$e^{-\frac{i}{\hbar}Ht}|\psi\rangle = |\psi_t\rangle$$

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$$e^{-\frac{\mathrm{i}}{\hbar}Ht}|\psi\rangle = |\psi_t\rangle$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = e^{-\frac{\mathrm{i}}{\hbar}Ht}$$

$$X|0\rangle = |1\rangle \qquad X|1\rangle = |0\rangle$$

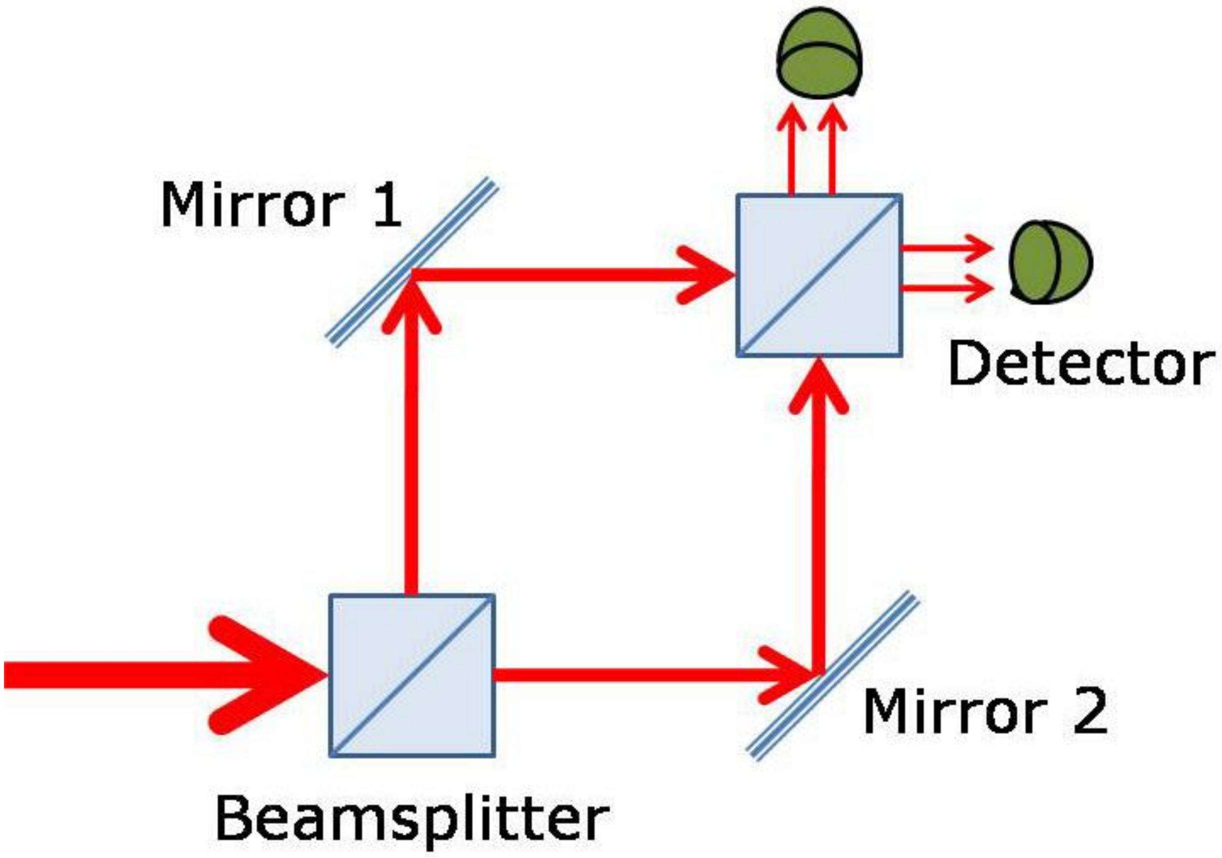
More logic gates than classical computers!

$$e^{-\frac{i}{\hbar}Ht}|\psi\rangle = |\psi_t\rangle$$

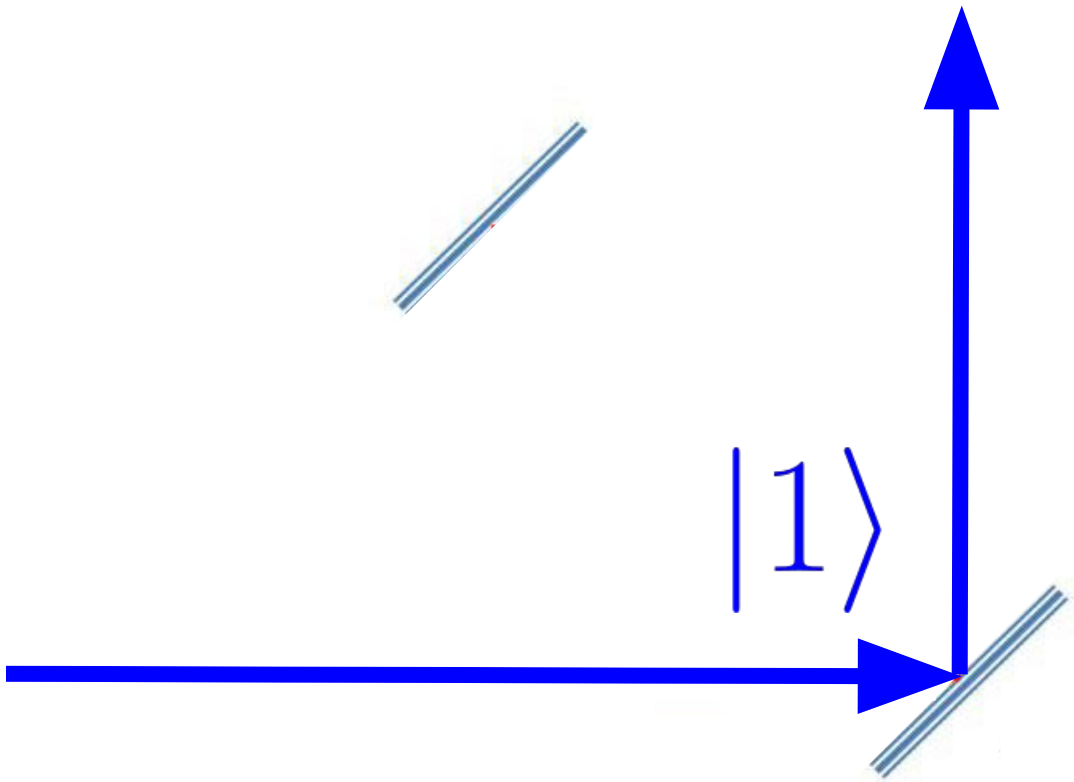
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad M = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

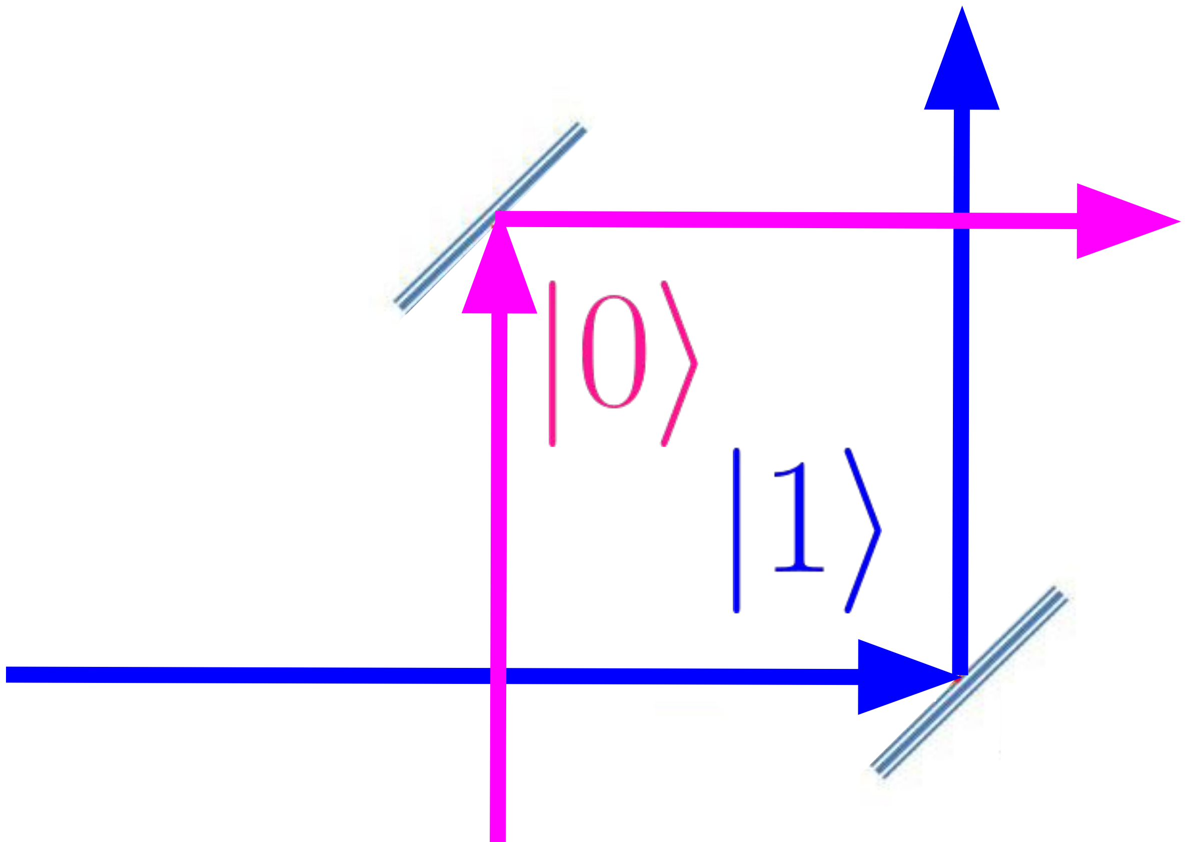
Demystifying the Mach-Zehnder Experiment!



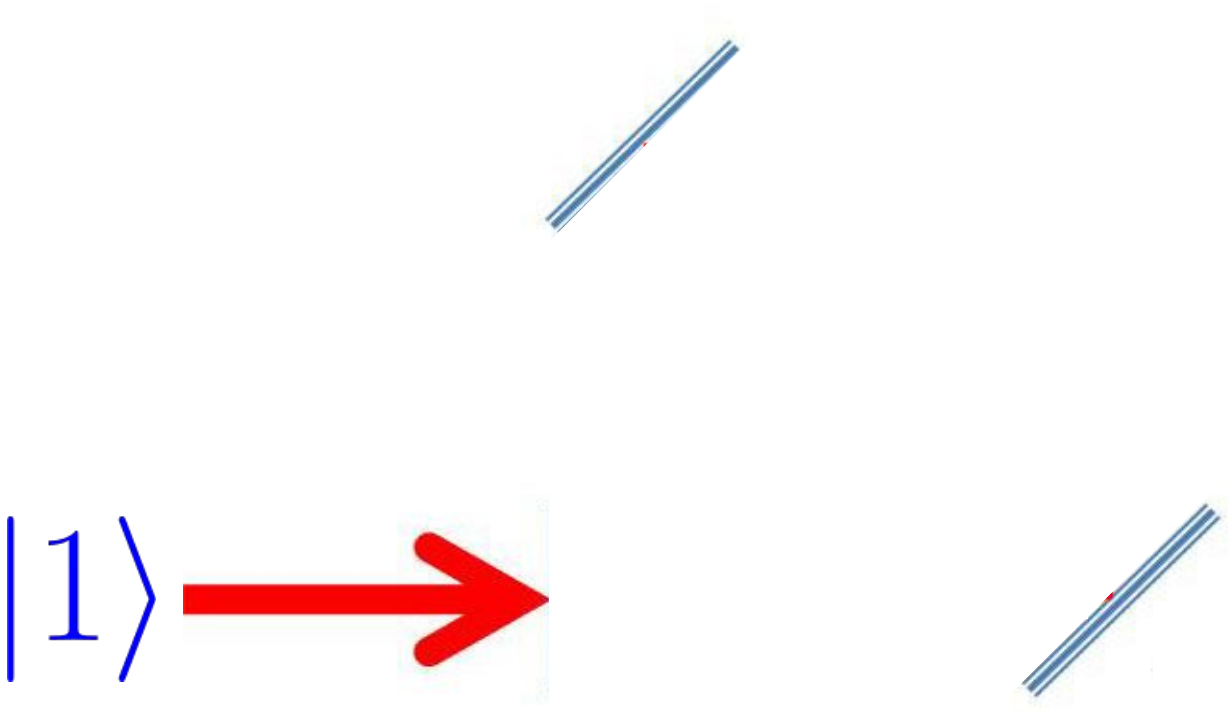
Demystifying the Mach-Zehnder Experiment!



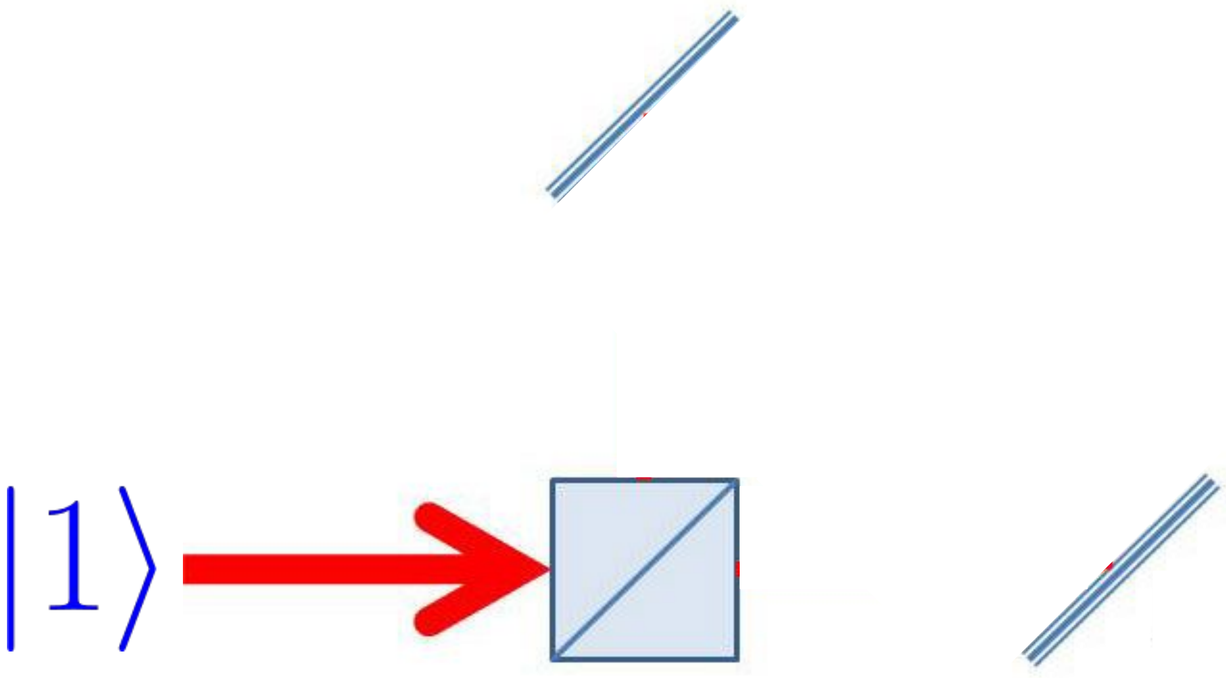
Demystifying the Mach-Zehnder Experiment!



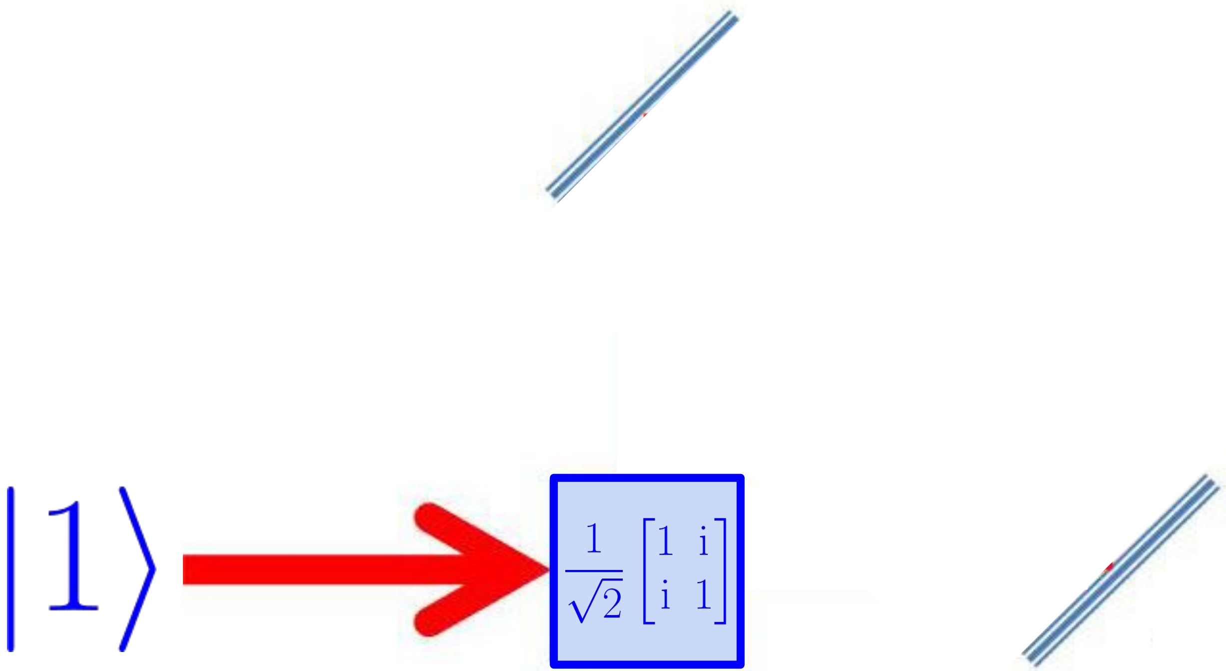
Demystifying the Mach-Zehnder Experiment!



Demystifying the Mach-Zehnder Experiment!

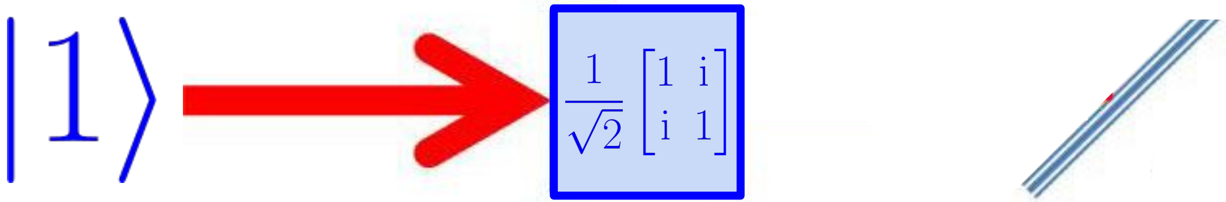


Demystifying the Mach-Zehnder Experiment!



Demystifying the Mach-Zehnder Experiment!

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}$$

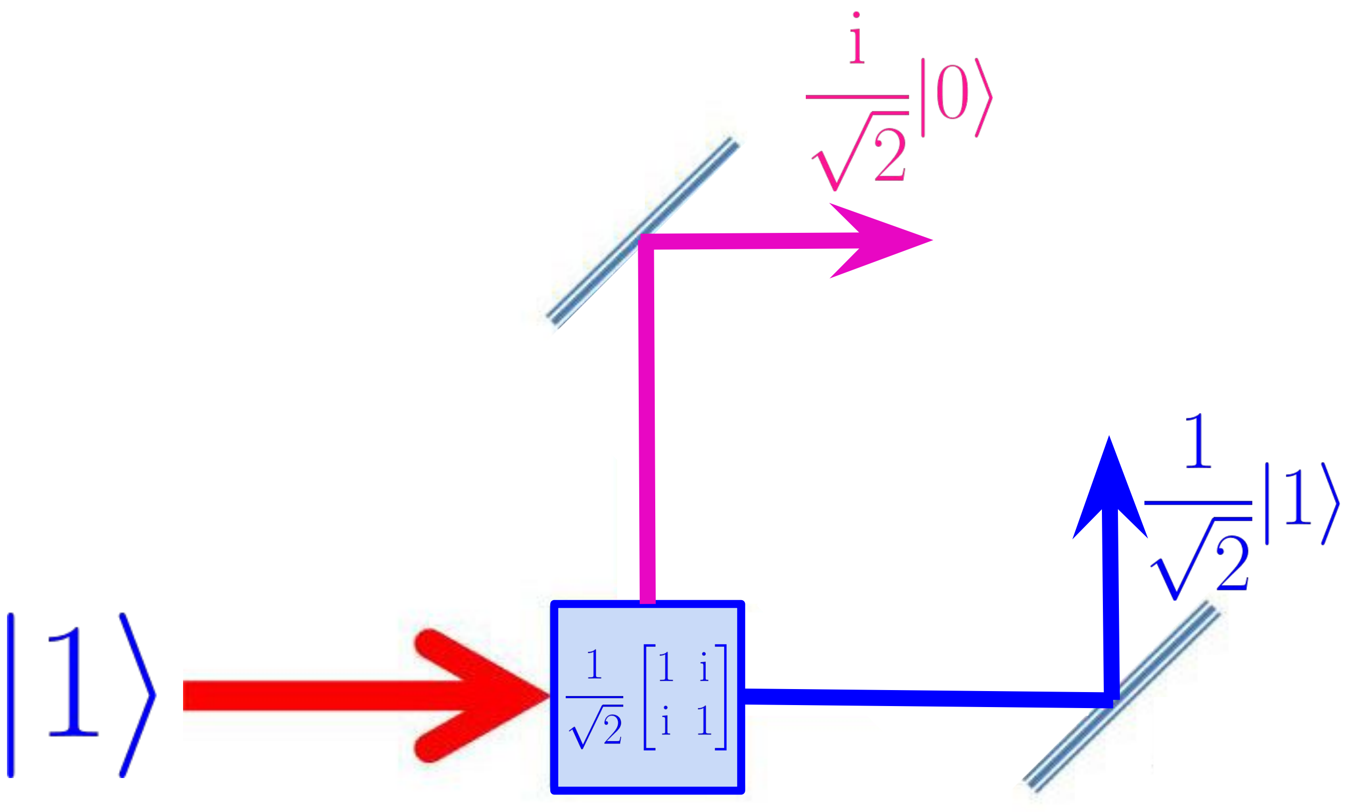


Demystifying the Mach-Zehnder Experiment!

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix} = \frac{\mathrm{i}}{\sqrt{\mathbf{2}}}|\mathbf{0}\rangle + \frac{1}{\sqrt{\mathbf{2}}}|\mathbf{1}\rangle$$

$$|1\rangle \longrightarrow \boxed{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \mathrm{i} \\ \mathrm{i} & 1 \end{bmatrix}}$$

Demystifying the Mach-Zehnder Experiment!



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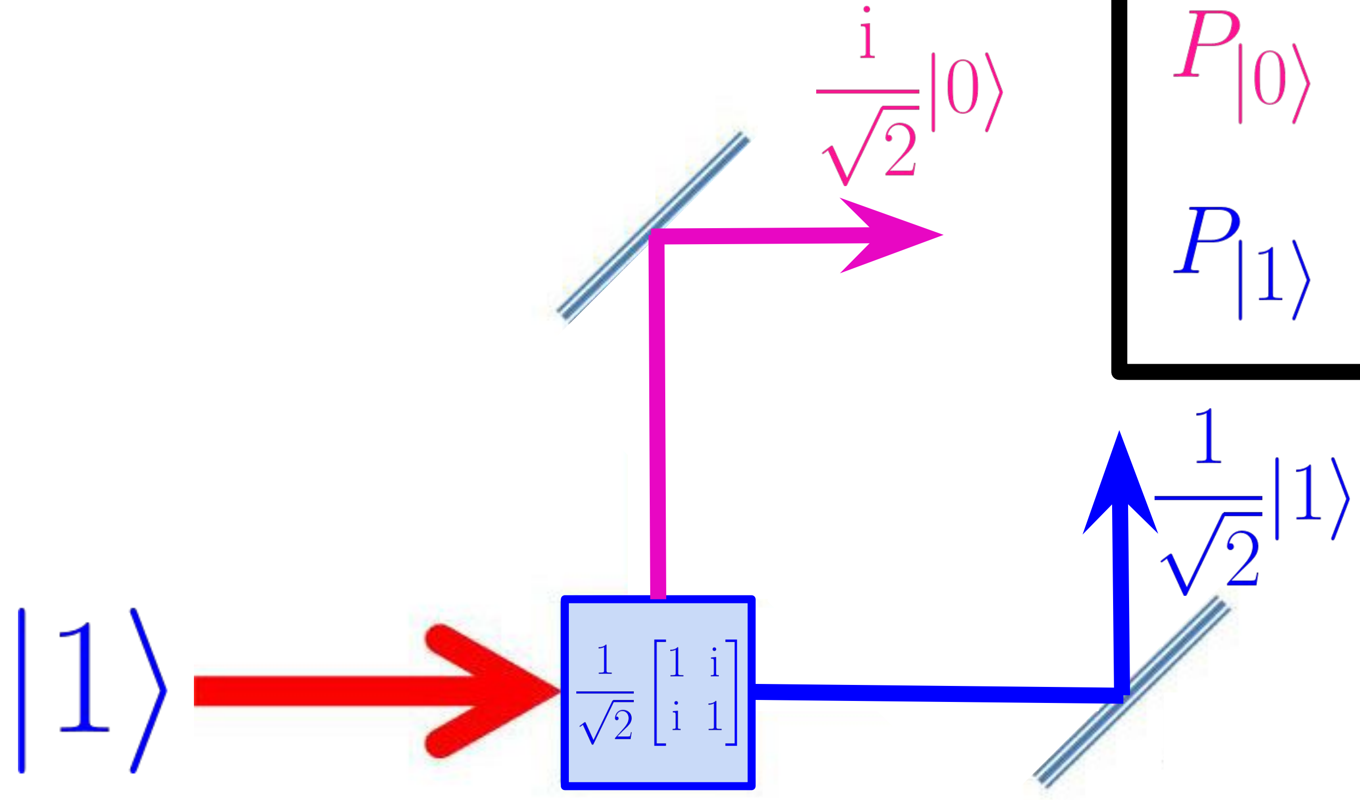
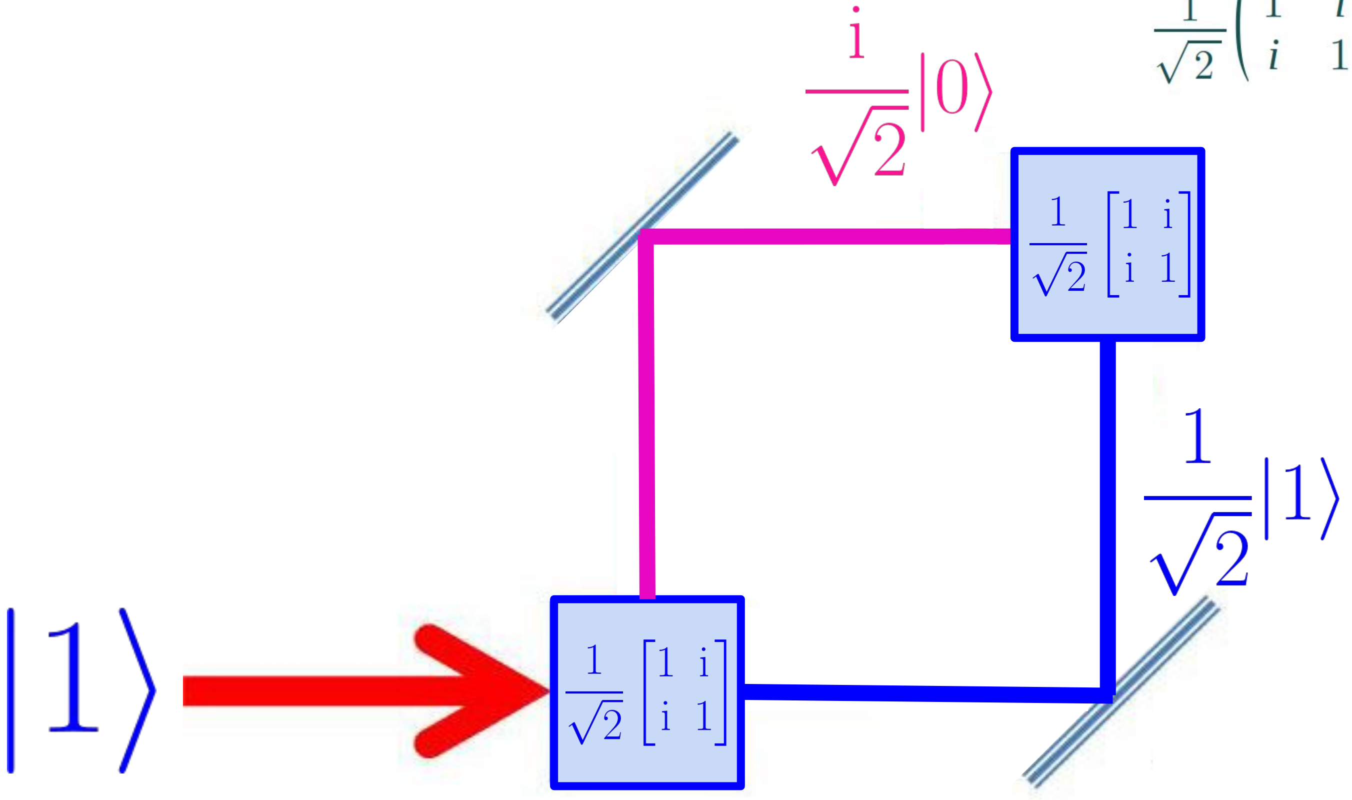


$$P_{|0\rangle} = \left|\frac{i}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

$$P_{|1\rangle} = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

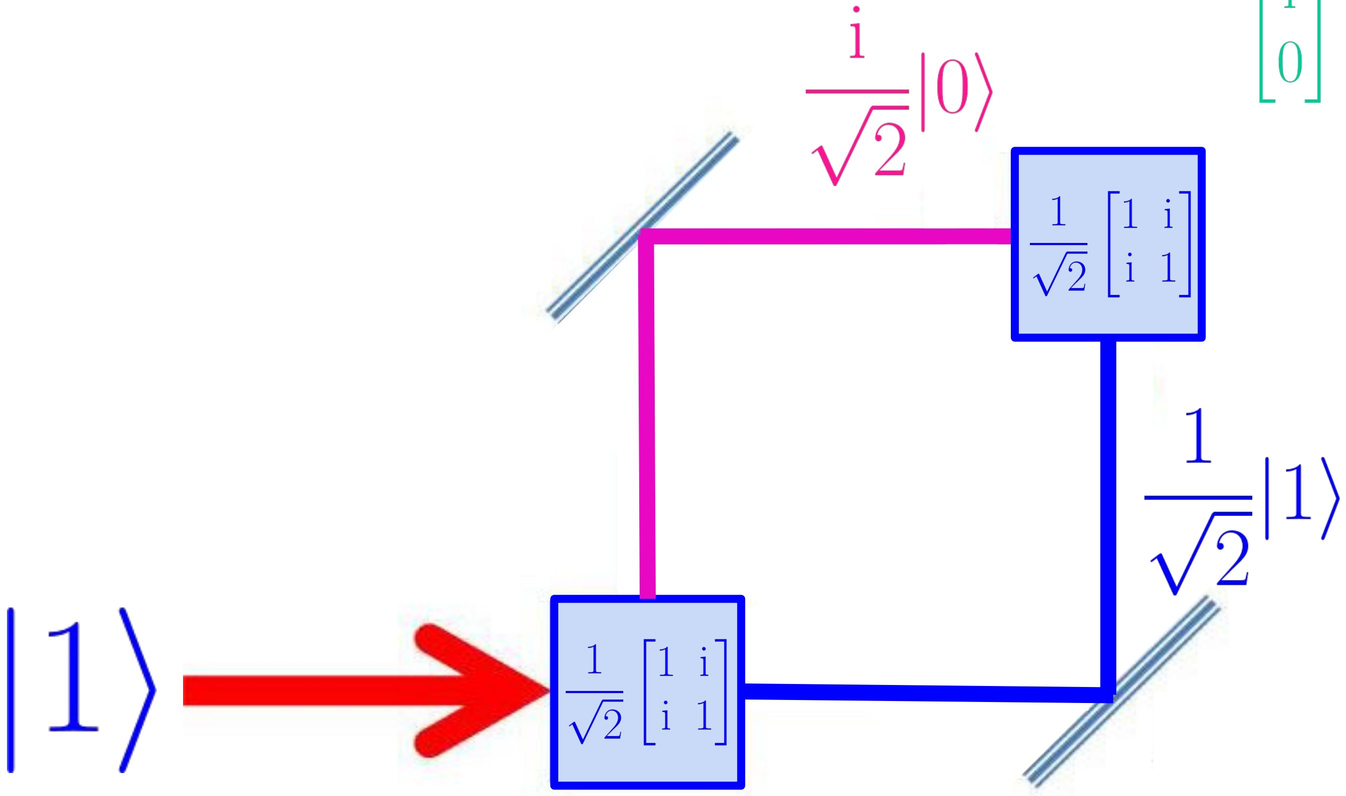
Demystifying the Mach-Zehnder Experiment!

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} i \\ 0 \end{pmatrix}$$



Demystifying the Mach-Zehnder Experiment!

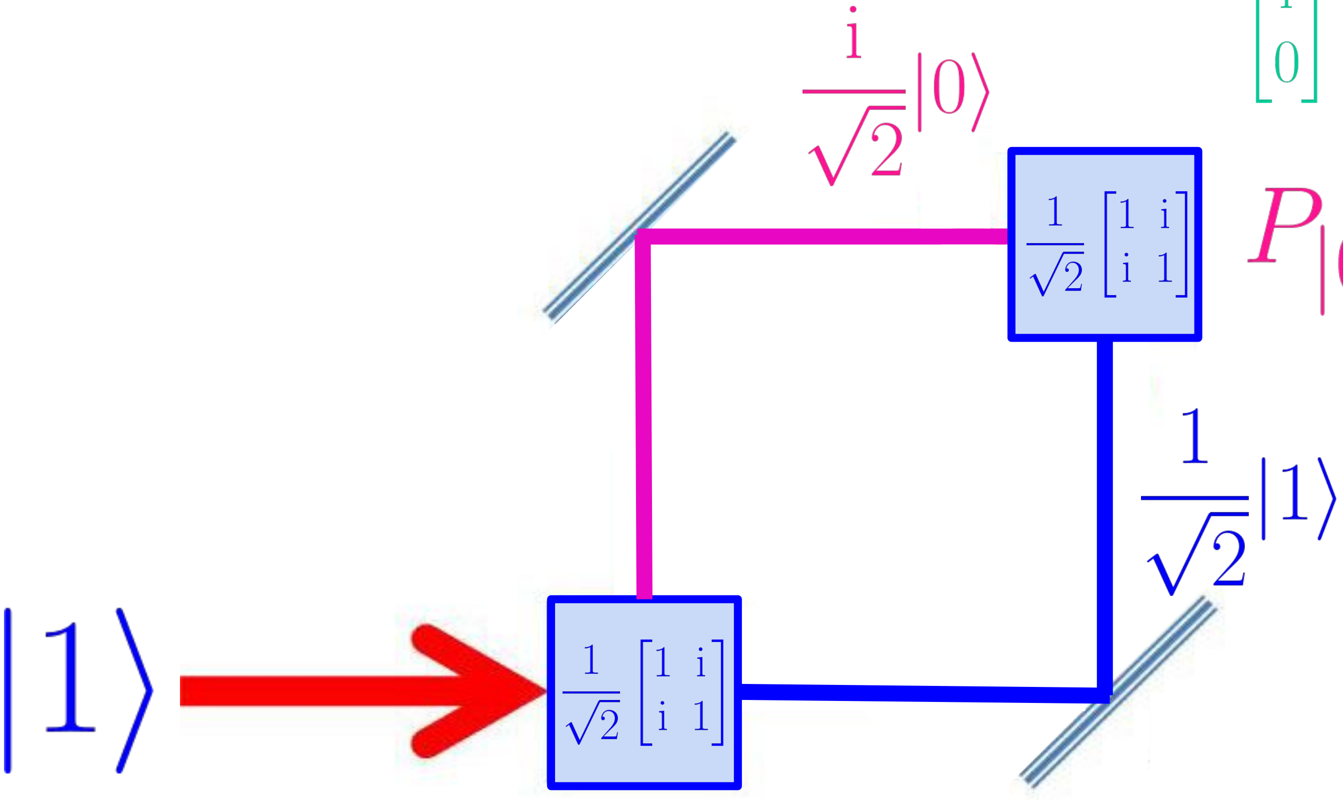
$$\begin{bmatrix} i \\ 0 \end{bmatrix} = i \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = i|0\rangle$$



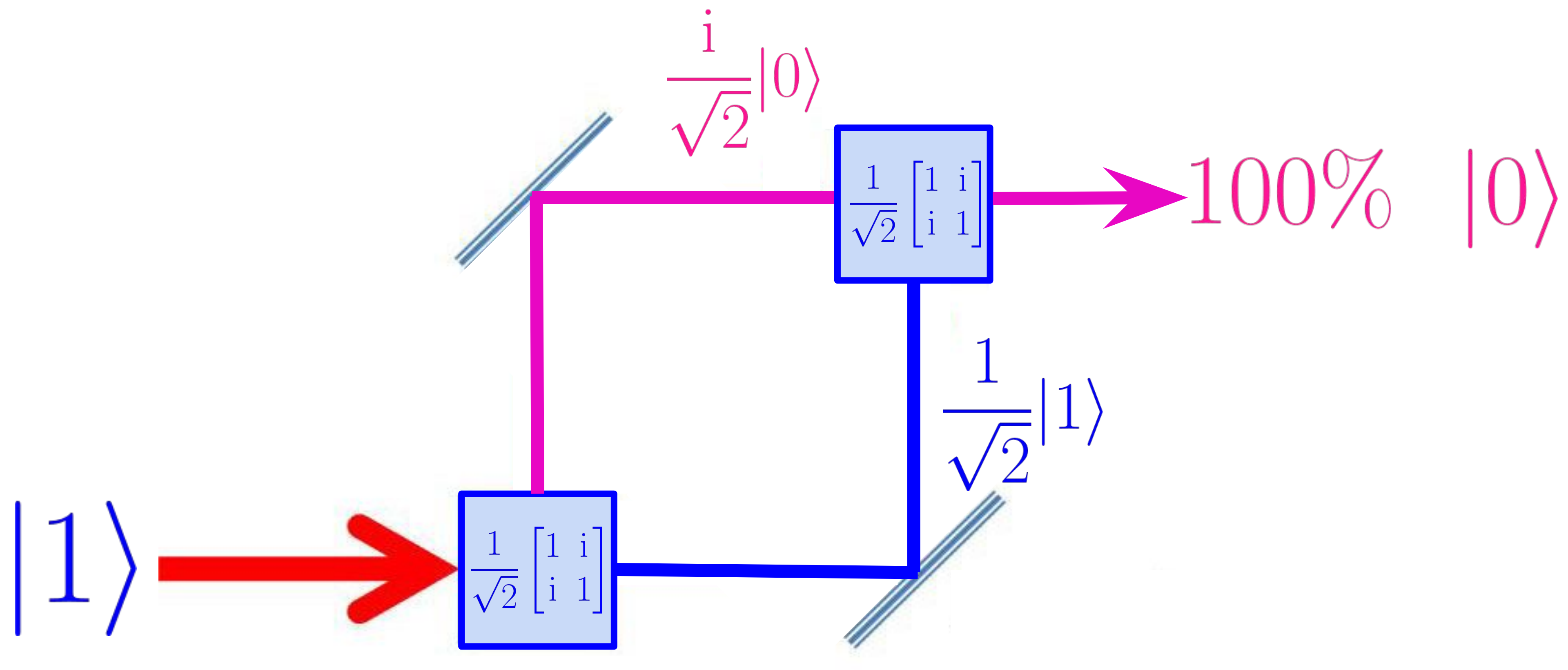
Demystifying the Mach-Zehnder Experiment!

$$\begin{bmatrix} i \\ 0 \end{bmatrix} = i \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = i|0\rangle$$

$$P_{|0\rangle} = |i|^2 = 1$$



Demystifying the Mach-Zehnder Experiment!



The Deutsch Problem (1985)

$$x = 0 \text{ or } 1.$$

How many times do we need to evaluate $f(x)$ in order to know if $f(0) + f(1) = 1$?

The Deutsch Problem (1985)

$$x = 0 \text{ or } 1.$$

How many times do we need to evaluate $f(x)$ in order to know if $f(0) + f(1) = 1$?

If $f(0) = f(1) = 0,$ $\qquad f(0) + f(1) = 0$

The Deutsch Problem (1985)

$x = 0$ or 1.

How many times do we need to evaluate $f(x)$ in order to know if $f(0) + f(1) = 1$?

If $f(0) = f(1) = 0, \qquad f(0) + f(1) = 0$

If $f(0) = 0, f(1) = 1, \quad f(0) + f(1) = 1$

The Deutsch Problem (1985)

$x = 0$ or 1.

How many times do we need to evaluate $f(x)$ in order to know if $f(0) + f(1) = 1$?

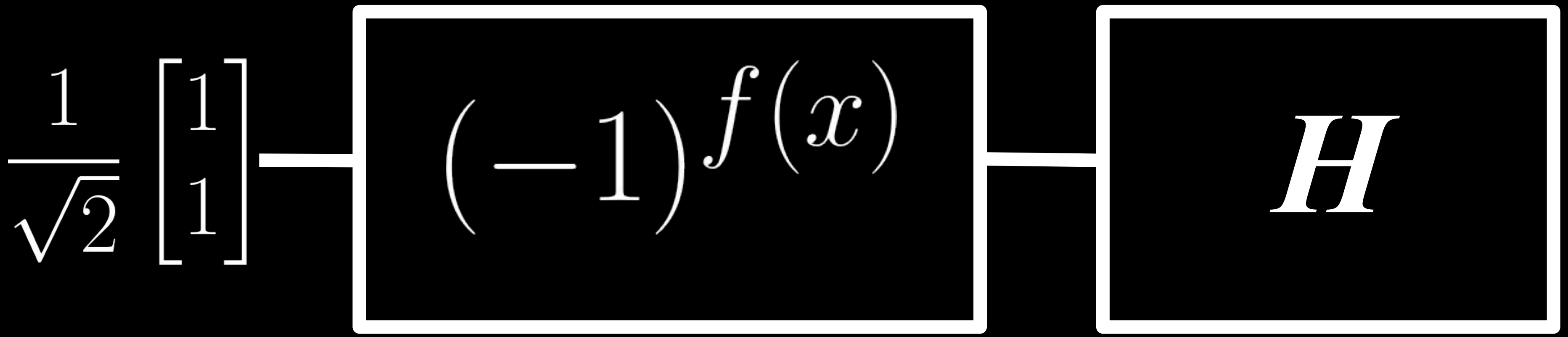
If $f(0) = f(1) = 0,\qquad f(0) + f(1) = 0$

If $f(0) = 0, f(1) = 1,\qquad f(0) + f(1) = 1$

If $f(0) = f(1) = 1,\qquad f(0) + f(1) = 2$

The Deutsch Problem (1985)

These simple gates are enough to determine *f(0) + f(1)* with <u>one</u> evaluation of *f(x)*:



The Deutsch Problem (1985)

These simple gates are enough to determine $f(0) + f(1)$ **with <u>one</u> evaluation of** $f(x)$**:**

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} - \boxed{\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}} - \boxed{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}}$$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} \quad \boxed{\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}} \quad \boxed{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}}$$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)}&0\\0&(-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\\frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\;\boxed{\begin{bmatrix}(-1)^{f(0)}&0\\0&(-1)^{f(1)}\end{bmatrix}}\;\boxed{\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}}$$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} — \boxed{\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}} — \boxed{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}}$$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

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The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \underbrace{\boxed{\begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}}}_{\psi}$$

If $f(0) = f(1) = 0$, $\psi = ?$

If $f(0) = 0$, $f(1) = 1$, $\psi = ?$

If $f(0) = f(1) = 1$, $\psi = ?$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

If $f(0) = f(1) = 0,$ $\quad \psi = \begin{bmatrix}1\\ 0\end{bmatrix} = |0\rangle$

If $f(0) = 0,\ f(1) = 1,$ $\quad \psi = \begin{bmatrix}0\\ 1\end{bmatrix} = |1\rangle$

If $f(0) = f(1) = 1,$ $\quad \psi = -\begin{bmatrix}1\\ 0\end{bmatrix} = -|0\rangle$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

If $f(0) = f(1) = 0,$ $\quad \psi = \begin{bmatrix}1\\ 0\end{bmatrix} = |0\rangle$

If $f(0) = 0,\ f(1) = 1,$ $\quad \psi = \begin{bmatrix}0\\ 1\end{bmatrix} = |1\rangle$ $\quad f(0) + f(1) = 1$

If $f(0) = f(1) = 1,$ $\quad \psi = -\begin{bmatrix}1\\ 0\end{bmatrix} = -|0\rangle$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

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If $f(0) = 0,\ f(1) = 1,$ $\quad \psi = \begin{bmatrix}0\\ 1\end{bmatrix} = |1\rangle$

If $f(0) = f(1) = 1,$ $\quad \psi = -\begin{bmatrix}1\\ 0\end{bmatrix} = -|0\rangle$

Done in a real experiment in 1998!

The Deutsch-Jozsa Problem (1992)

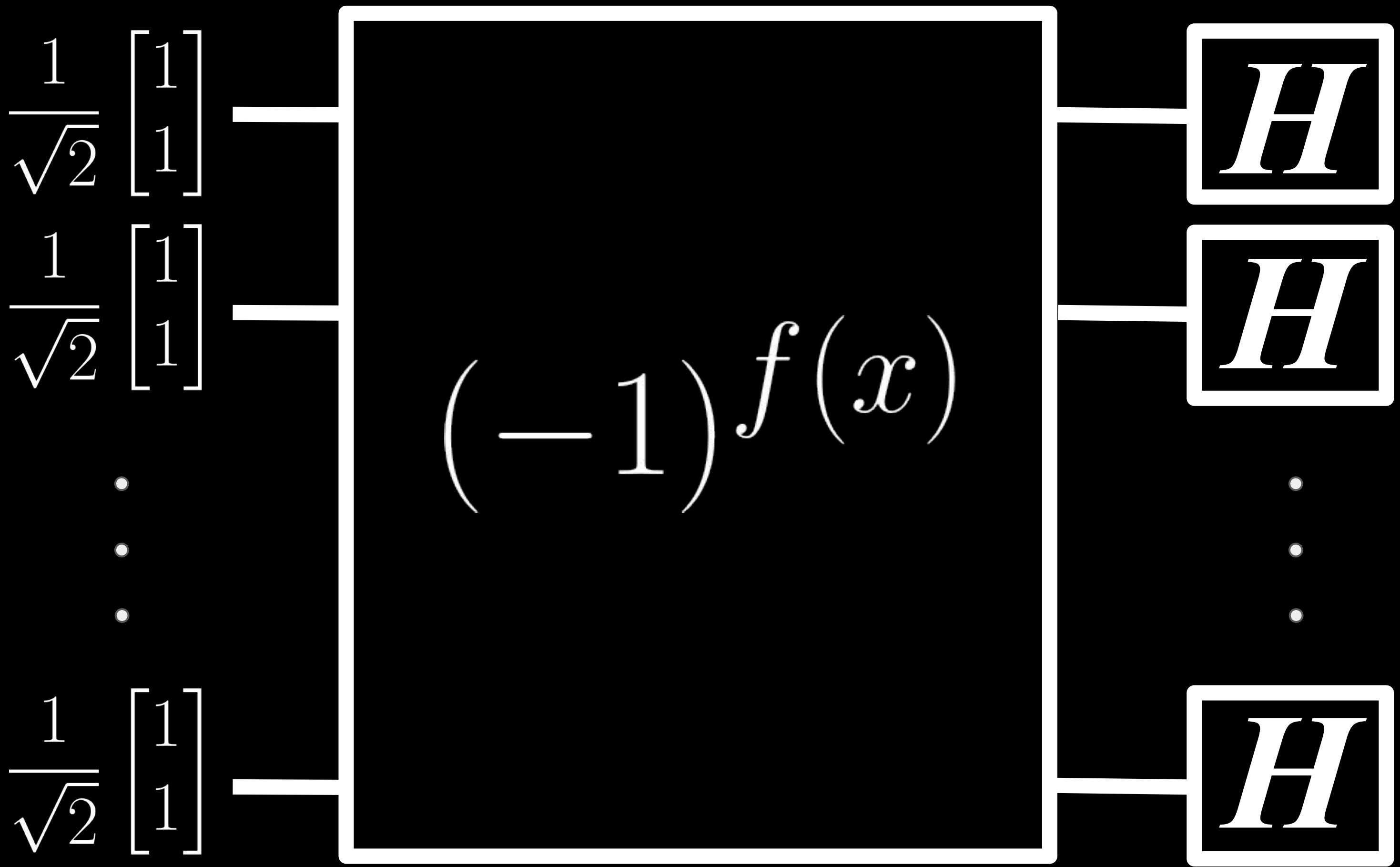
$x_i = 0$ or 1.

How many times do we need to evaluate $f(x_1, x_2, \ldots, x_n)$ in order to know if it's constant?

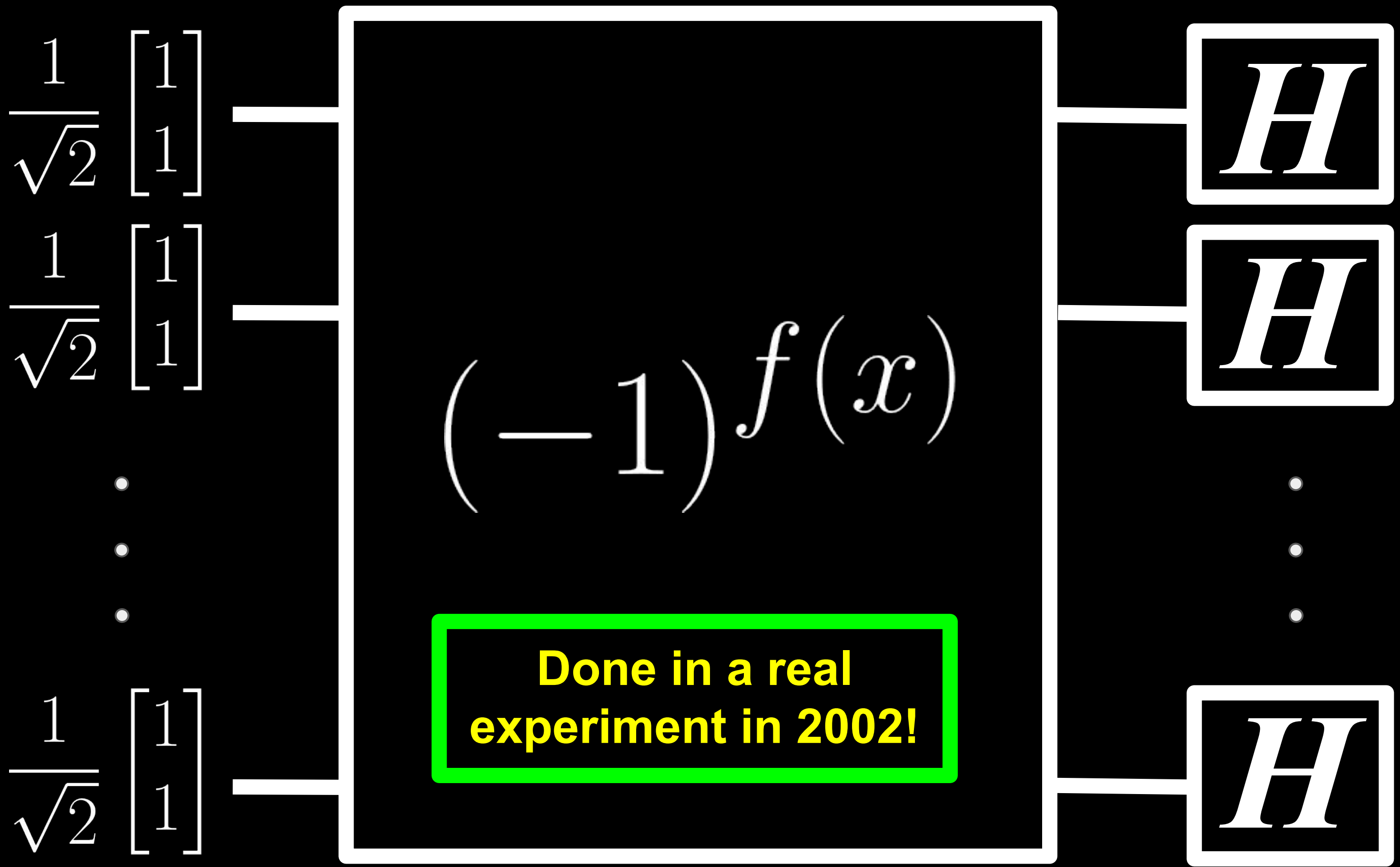
The Deutsch-Jozsa Problem (1992)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \text{ — } \boxed{(-1)^{f(x)}} \text{ — } \boxed{\boldsymbol{H}}$$

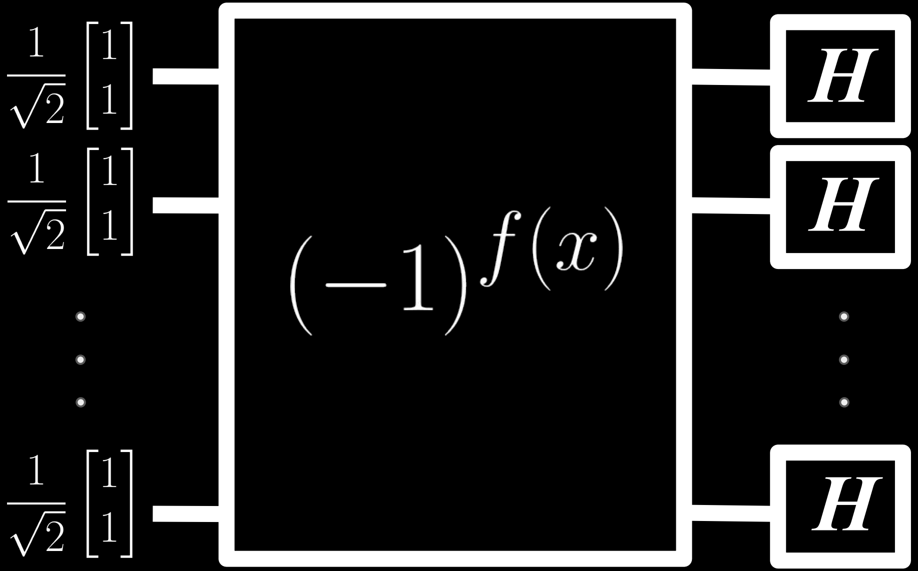
The Deutsch-Jozsa Problem (1992)



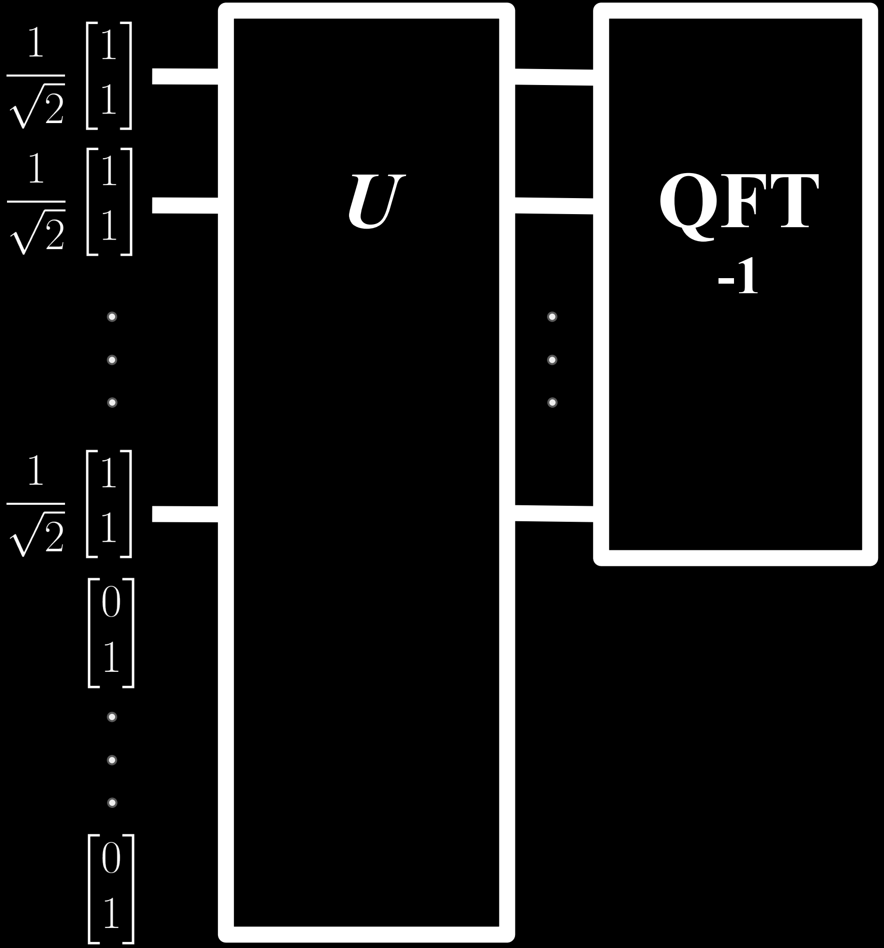
The Deutsch-Jozsa Problem (1992)



The Deutsch-Jozsa Problem (1992)



Shor's Algorithm (1994)



Full course on circuit-based quantum computing?

- Quantum machine learning
- Quantum computing for finance
- Quantum money / Quantum cryptocurrencies
- Quantum communication / Quantum internet
- Quantum security
- Quantum decoherence

- How to actually implement quantum gates:
    - Superconducting qubits
    - Photonic qubits
    - Spin-based qubits (NMR / NV centres)
    - Ion traps, Rydberg atoms, ultracold molecules, etc.

nike@hpqc.org, info@hpqc.org (online, in-person, or where you live)

Thank you!