Warm-up!

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underline{?}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \underline{?}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Introduction to Quantum Computing

Lecture 1

Nike Dattani

Outline

- A bizarre experiment!
- Qubits and quantum gates
- Your first quantum computation

  (demystifying the above experiment)
- Your second quantum computation

  (2 x more efficient than the best classical algorithm)
- Your third quantum computation

  (**exponentially** more efficient than the best classical algorithm)

XI. Jahrgang. **August 1891.** Achtes Heft.

Ein neuer Interferenzrefraktor.

Von

Dr. **L. Zehnder** in Basel.

Die Brewster'sche Entdeckung der Farben dicker Platten[1]) wurde von Herrn Jamin in glücklichster Weise zur Konstruktion seiner Interferenzrefraktoren[2])

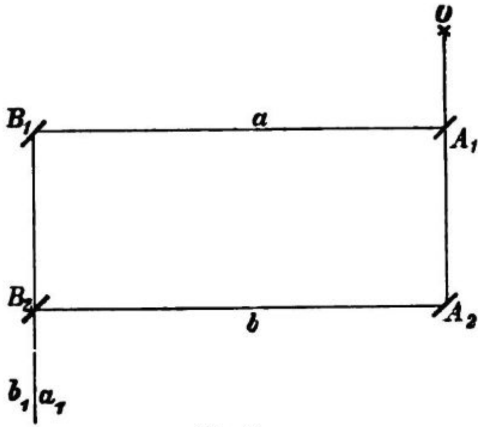

Fig. 2.

Ein neuer Interferenzrefraktor.

Von
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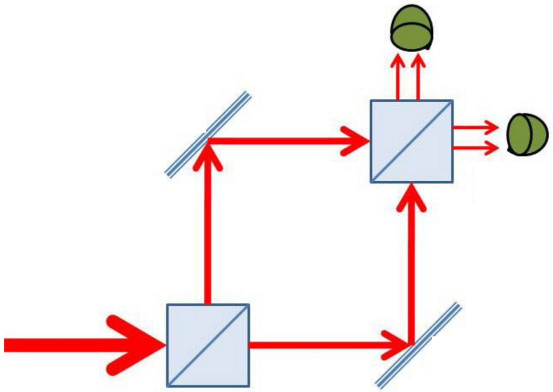
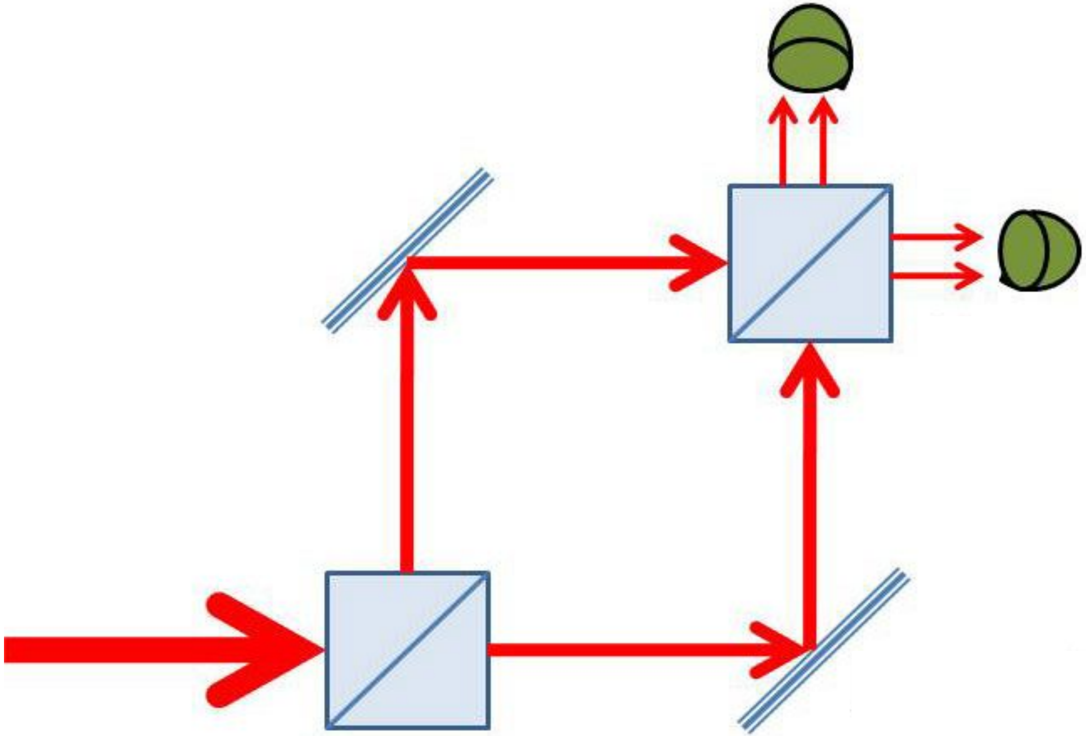


Mach-Zehnder Experiment



Qubits and quantum gates

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad , \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Qubits and quantum gates

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$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Qubits and quantum gates

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$$X|0\rangle = \ ? \qquad X|1\rangle = \ ?$$

Qubits and quantum gates

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad , \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X|0\rangle = |1\rangle \qquad X|1\rangle = |0\rangle$$

Classical Computer Bits

0 and 1 represent any distinct classical states!

- CPU processing
  0 = Low voltage (0 mV)
  1 = High voltage (5 mV)

Classical Computer Bits

0 and 1 represent any distinct classical states!

- CPU processing
  - 0 = Low voltage (0 mV)
  - 1 = High voltage (5 mV)
- Barcodes
  - 0 = Thin line
  - 1 = Thick line

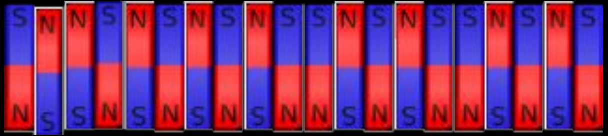
Classical Computer Bits

0 and 1 represent any distinct classical states!

- CPU processing
  - 0 = Low voltage (0 mV)
  - 1 = High voltage (5 mV)
- Barcodes
  - 0 = Thin line
  - 1 = Thick line
- Optical disks
  - 0 = Absence of pit
  - 1 = Presence of pit

Hard Drive

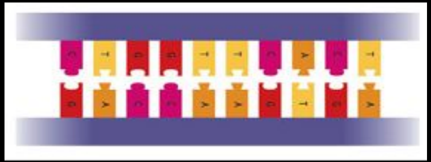
0110101010100101001010101



DNA Storage

0 : CG

1 : AT

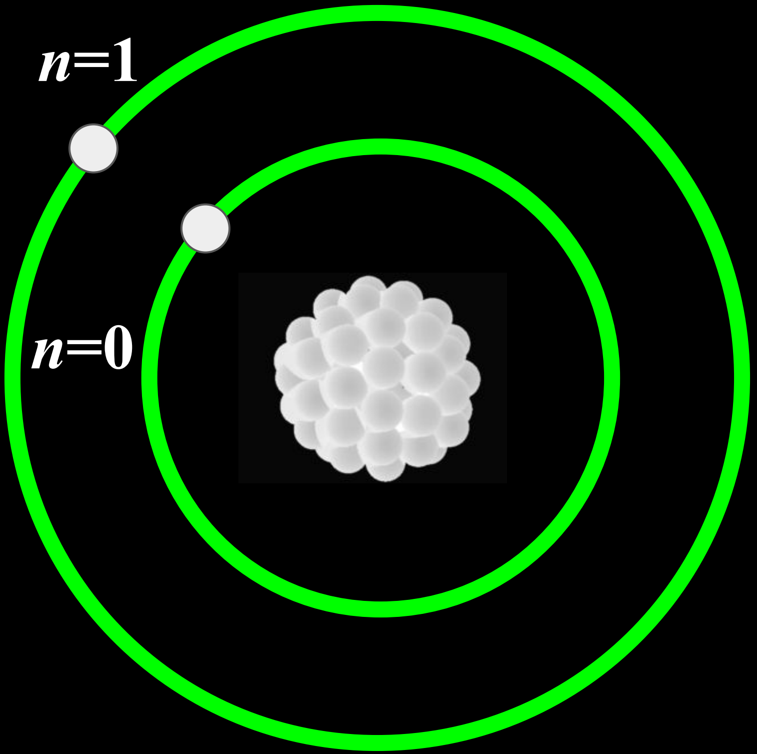


dna inside

Quantum computer bits (qubits)

0 and 1: two quantum mechanically allowed states

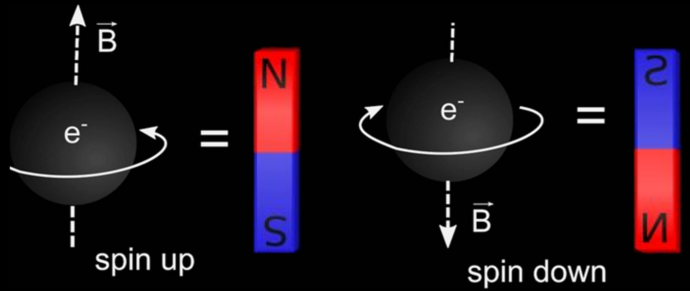
- Atomic levels
  - 0 = Ground state
  - 1 = Excited state

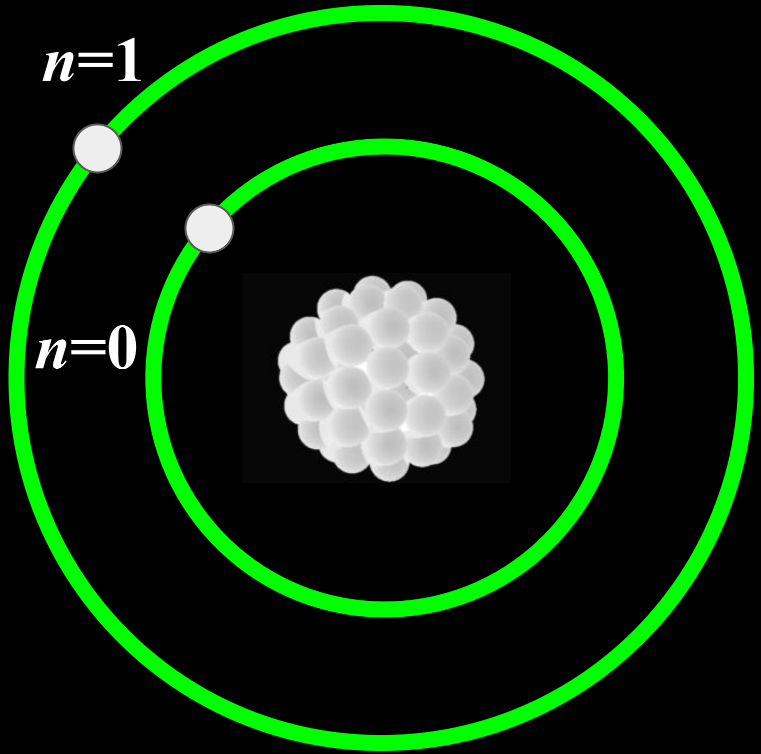


Quantum computer bits (qubits)

0 and 1: two quantum mechanically allowed states

- Atomic levels
  - 0 = Ground state
  - 1 = Excited state
- Spin
  - 0 = Up
  - 1 = Down

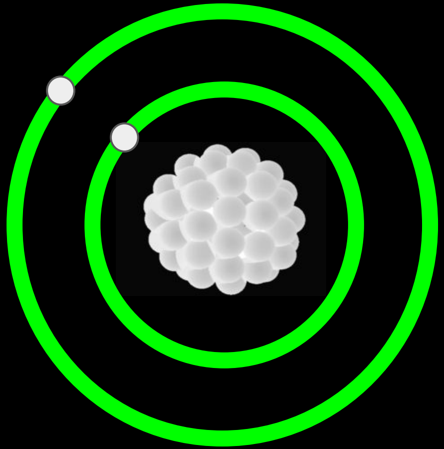


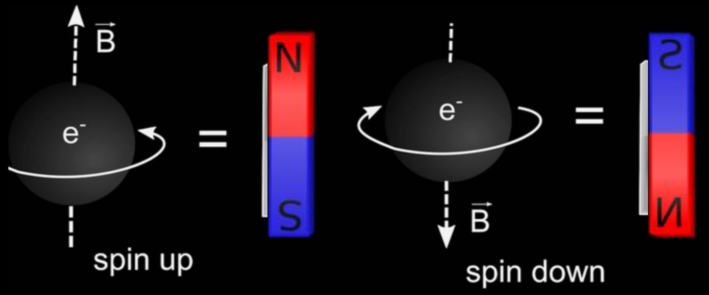


Quantum computer bits (qubits)

0 and 1: two quantum mechanically allowed states

- Atomic levels
  - 0 = Ground state
  - 1 = Excited state
- Spin
  - 0 = Up
  - 1 = Down
- Photons
  - 0 = Horizontal Polarization
  - 1 = Vertical Polarization
- **Many more possibilities!**





Schrödinger tells us:

$$e^{-\frac{i}{\hbar}Ht}|\psi\rangle = |\psi_t\rangle$$

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$$e^{-\frac{i}{\hbar}Ht}|\psi\rangle = |\psi_t\rangle$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = e^{-\frac{i}{\hbar}Ht}$$

$$X|0\rangle = |1\rangle \qquad X|1\rangle = |0\rangle$$

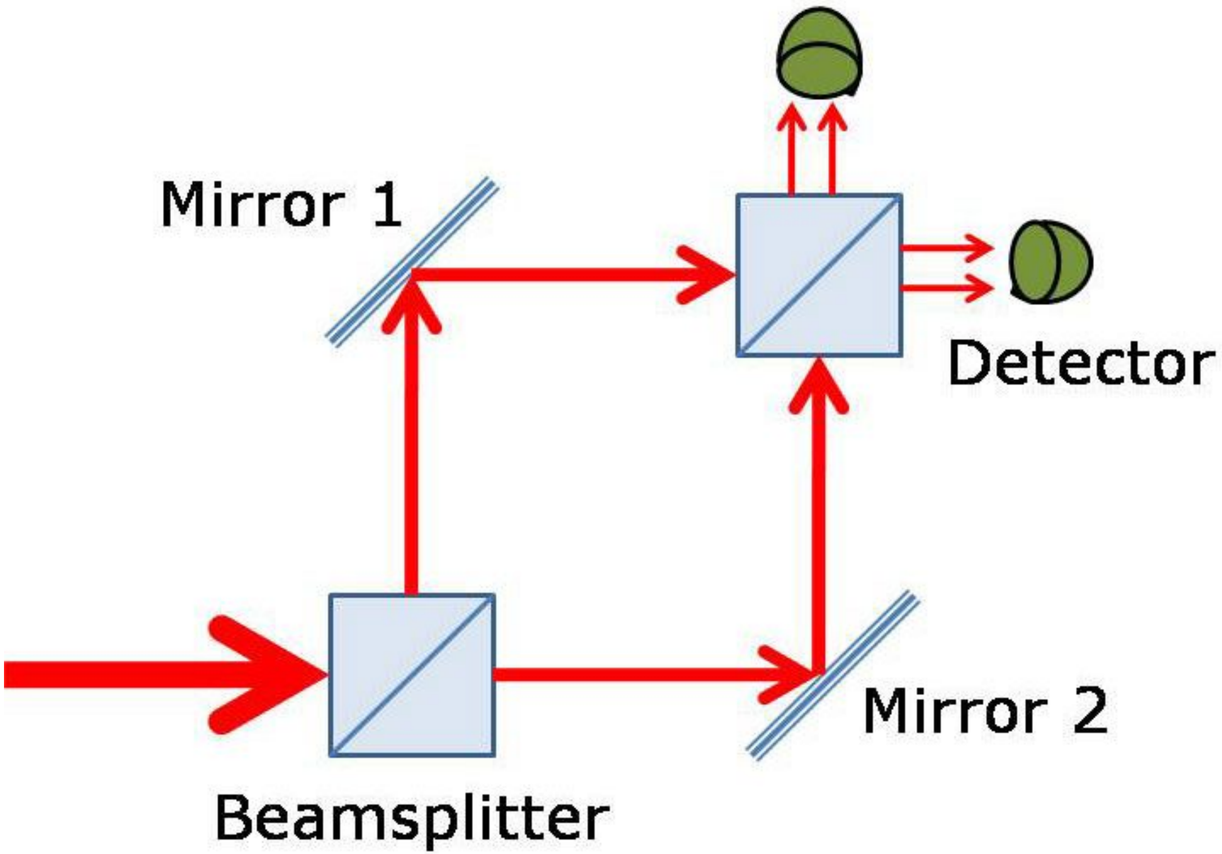
More logic gates than classical computers!

$$e^{-\frac{\mathrm{i}}{\hbar}Ht}|\psi\rangle = |\psi_t\rangle$$

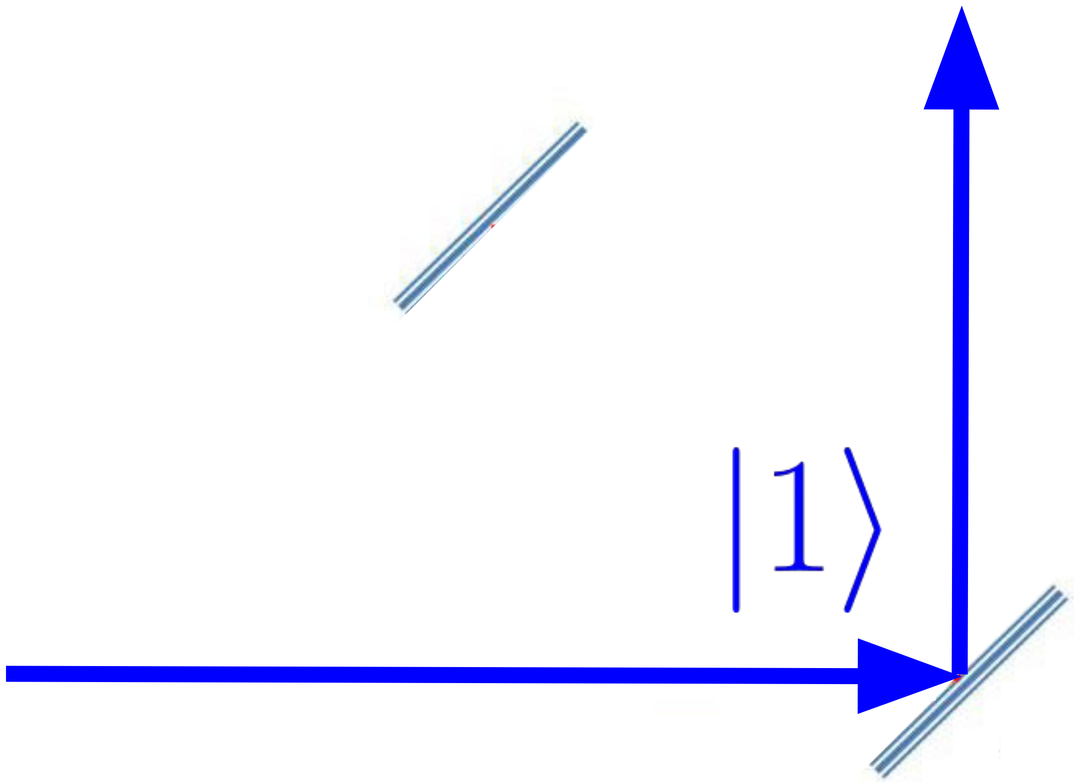
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad M = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \mathrm{i} \\ \mathrm{i} & 1 \end{bmatrix}$$

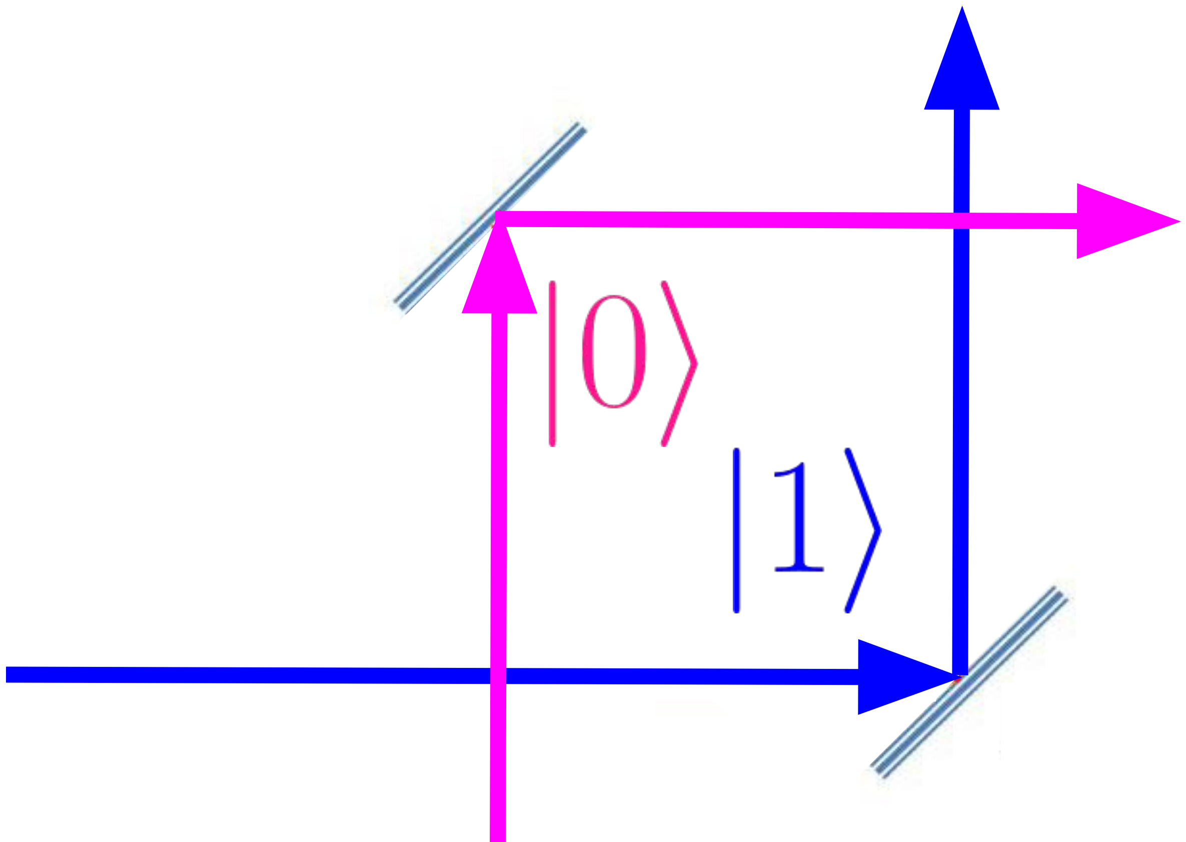
Demystifying the Mach-Zehnder Experiment!



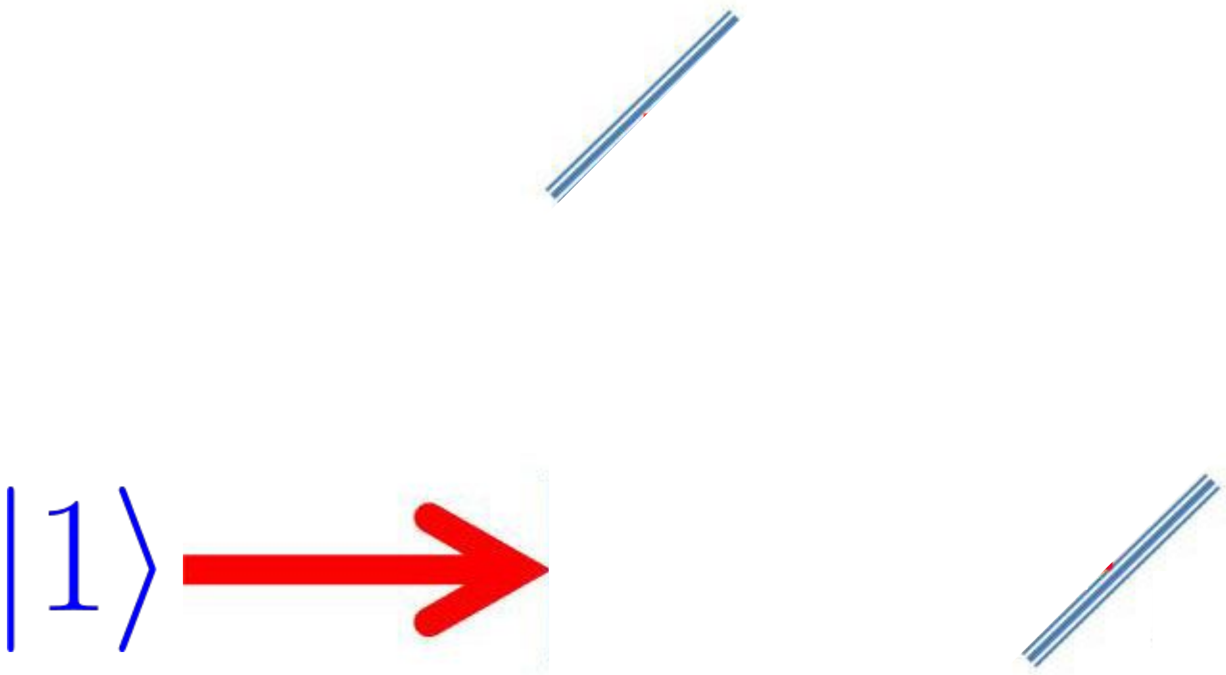
Demystifying the Mach-Zehnder Experiment!



Demystifying the Mach-Zehnder Experiment!



Demystifying the Mach-Zehnder Experiment!



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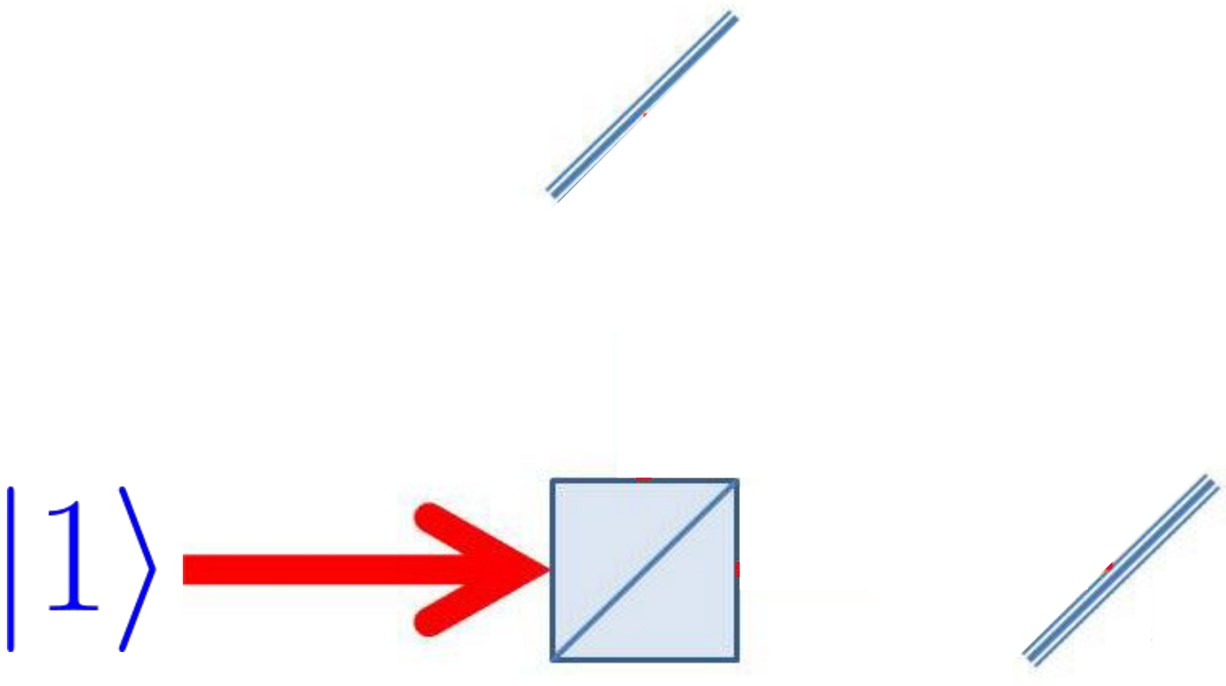
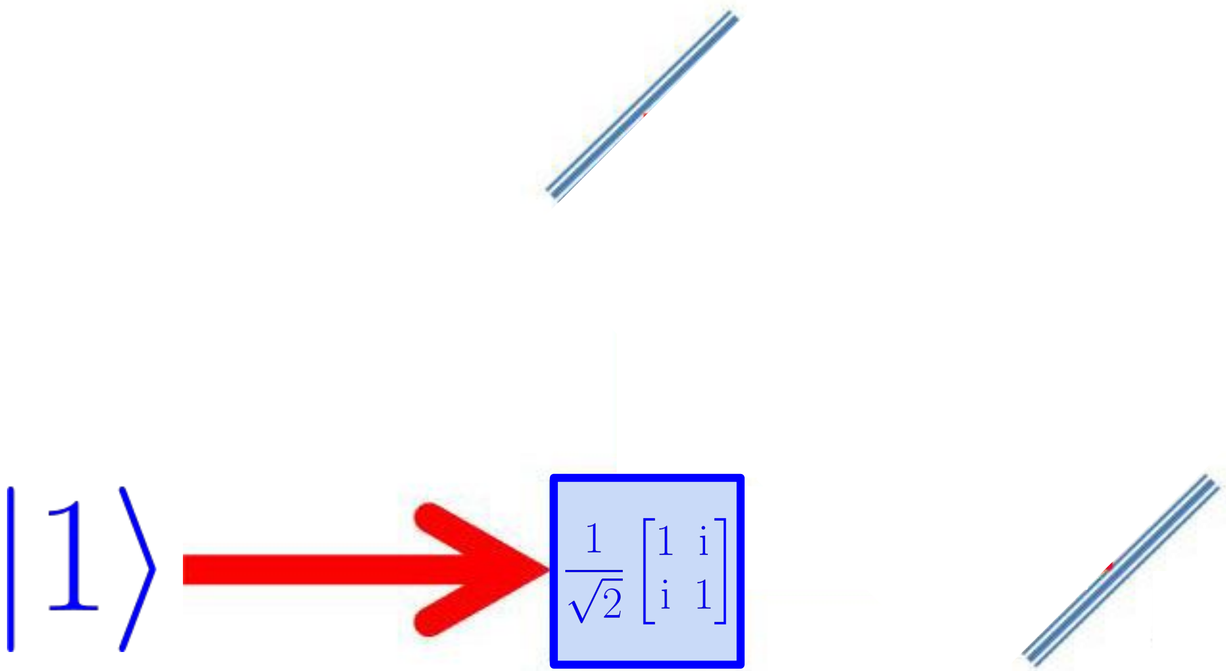


Demystifying the Mach-Zehnder Experiment!



Demystifying the Mach-Zehnder Experiment!

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$|1\rangle \longrightarrow \boxed{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \mathrm{i} \\ \mathrm{i} & 1 \end{bmatrix}}$$

Demystifying the Mach-Zehnder Experiment!

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix} = \frac{\mathrm{i}}{\sqrt{\mathbf{2}}}|\mathbf{0}\rangle + \frac{1}{\sqrt{\mathbf{2}}}|\mathbf{1}\rangle$$

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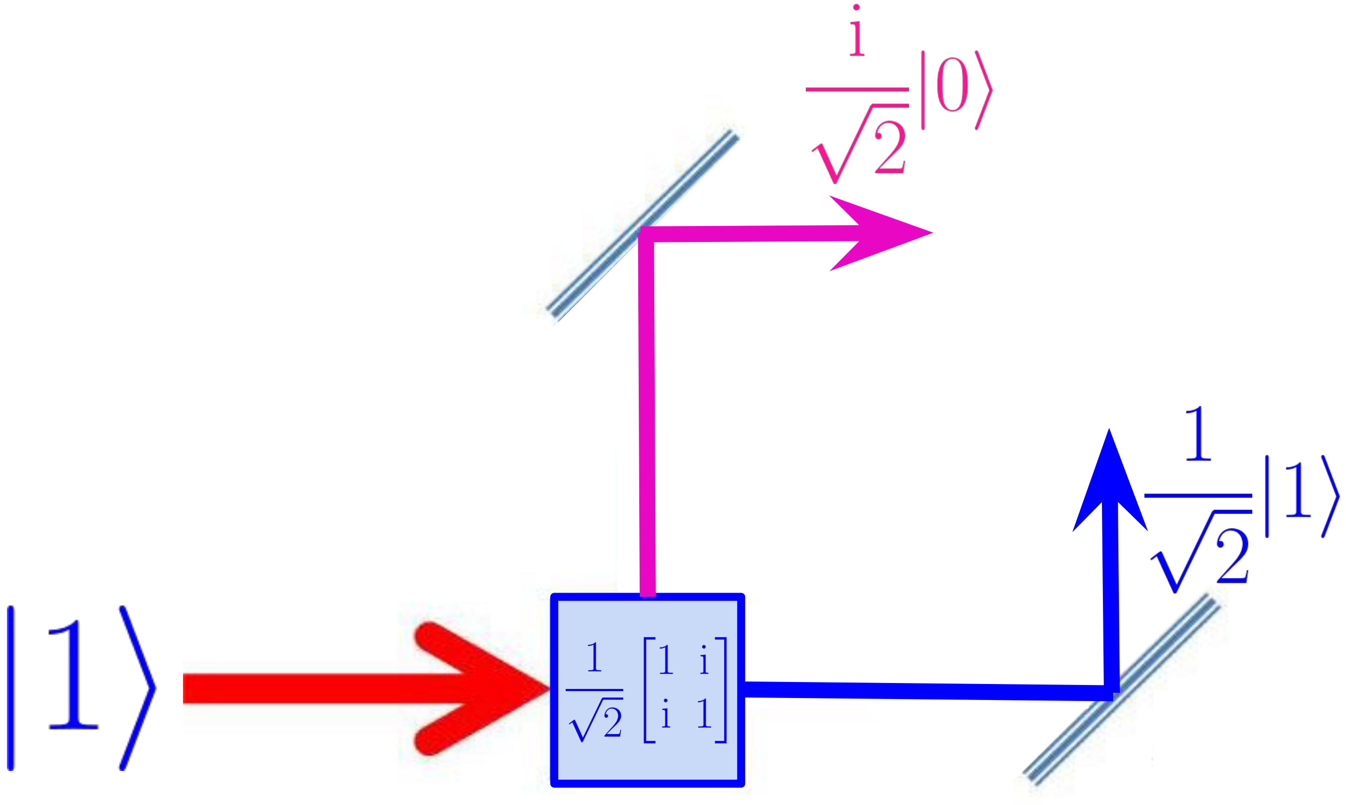
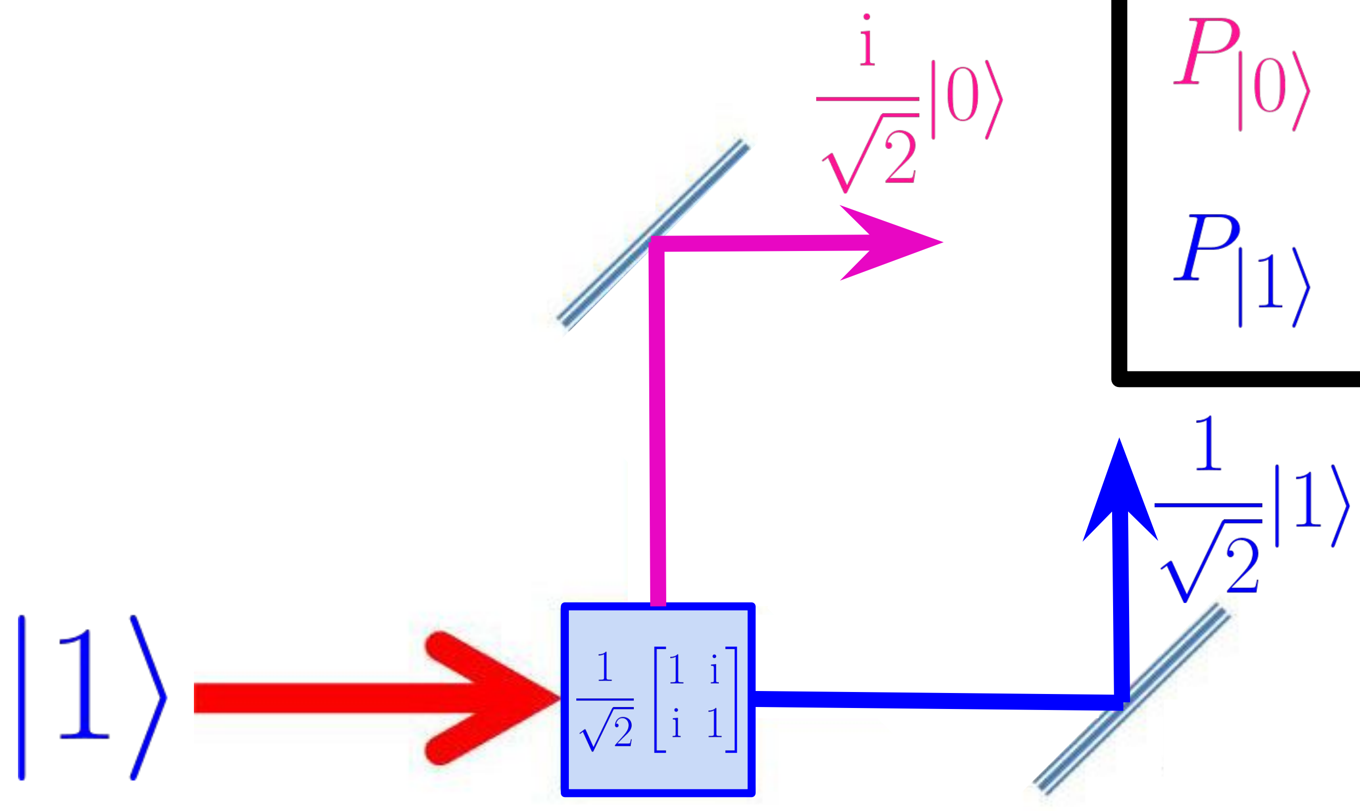
Demystifying the Mach-Zehnder Experiment!



Demystifying the Mach-Zehnder Experiment!

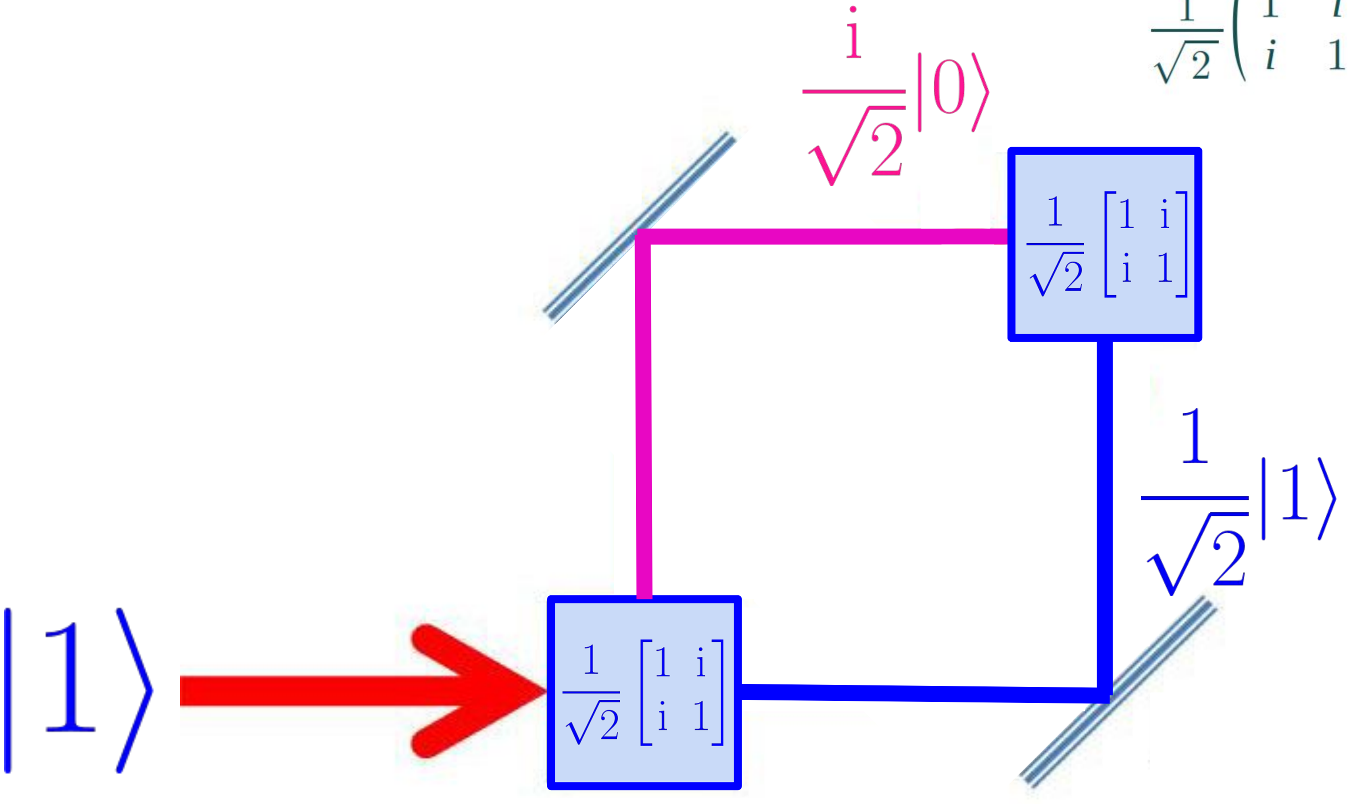
$$P_{|0\rangle} = \left|\frac{i}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

$$P_{|1\rangle} = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$



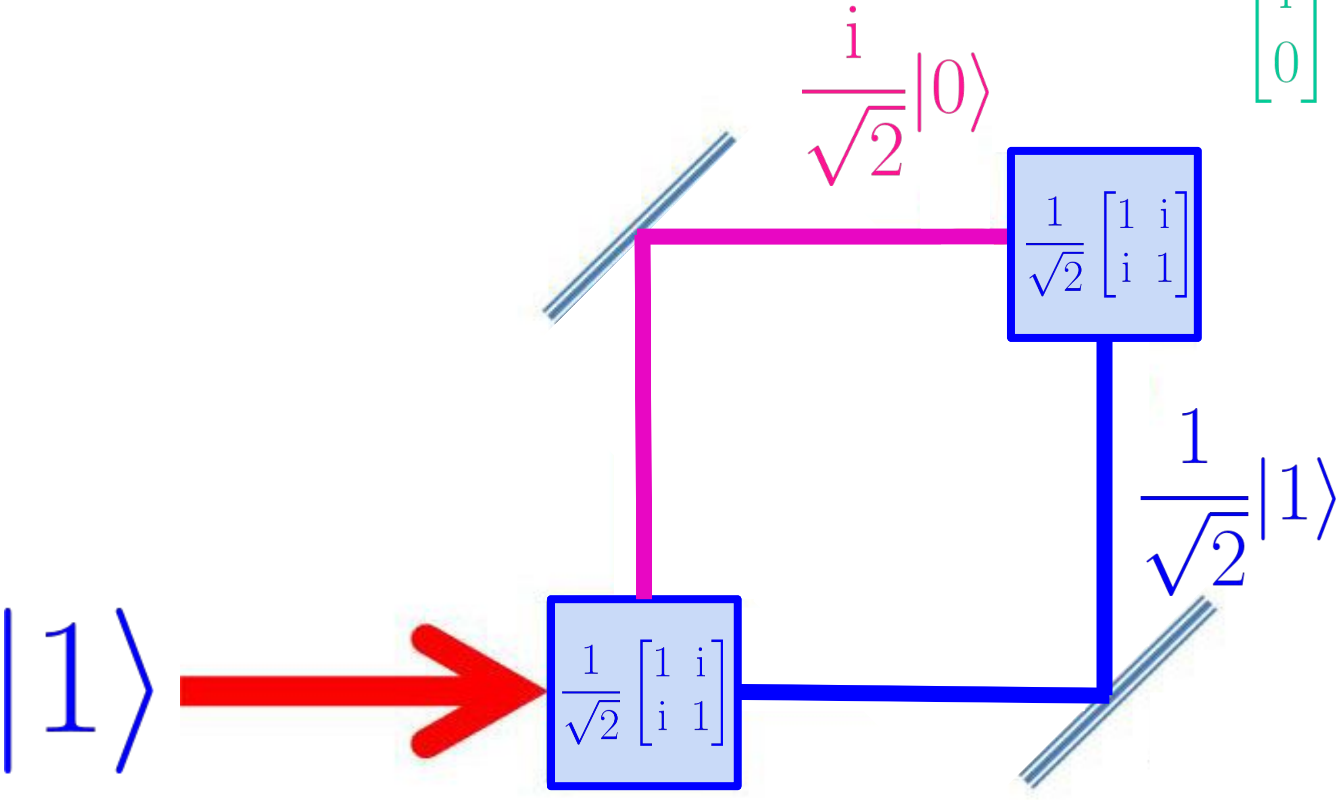
Demystifying the Mach-Zehnder Experiment!

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} i \\ 0 \end{pmatrix}$$



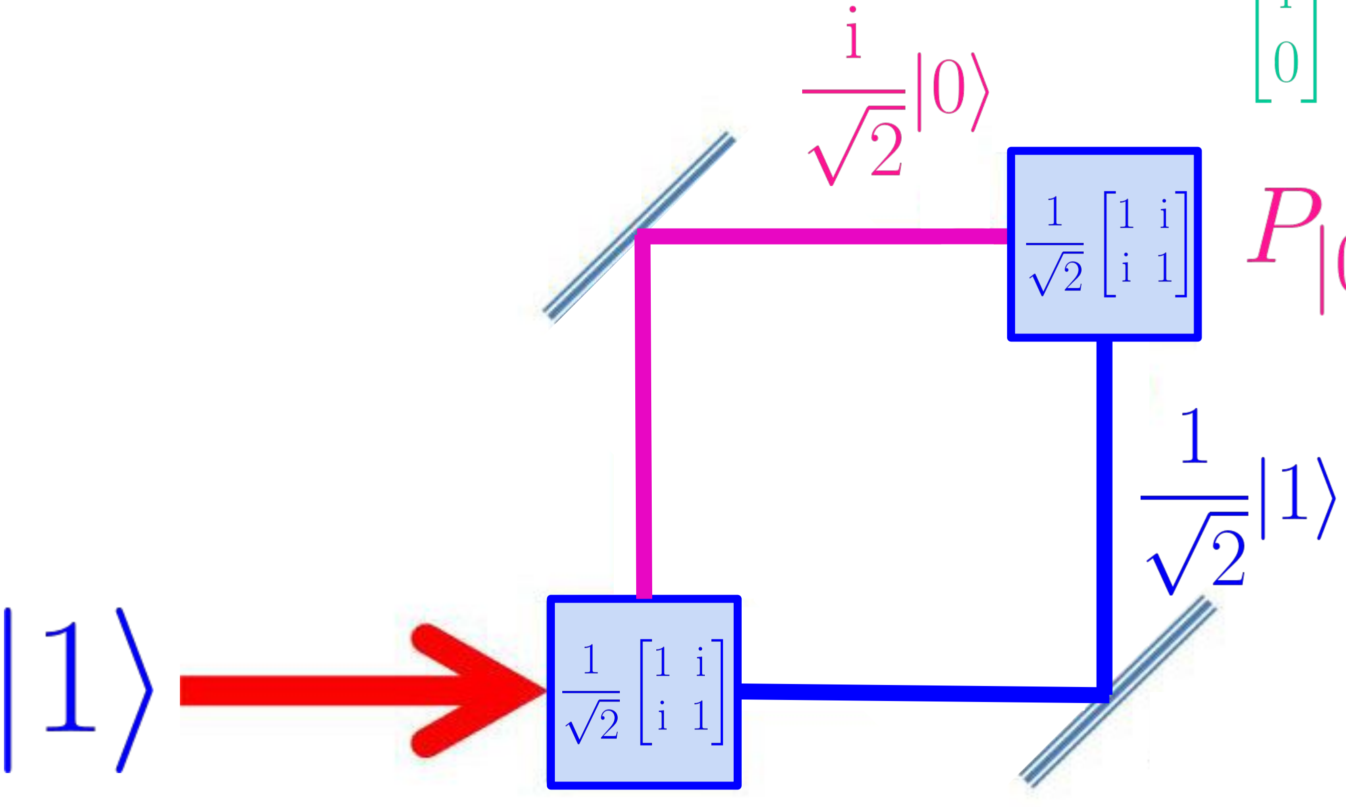
Demystifying the Mach-Zehnder Experiment!

$$\begin{bmatrix} i \\ 0 \end{bmatrix} = i \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = i|0\rangle$$



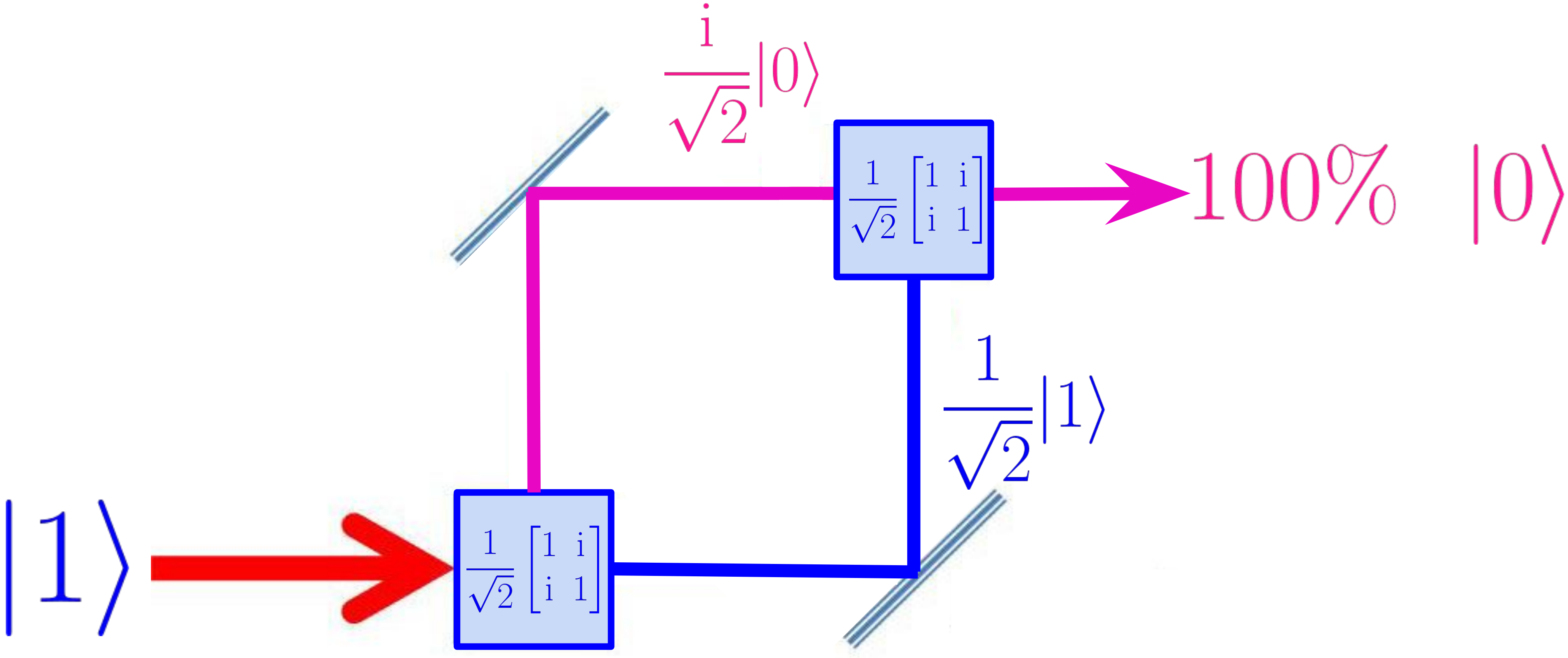
Demystifying the Mach-Zehnder Experiment!

$$\begin{bmatrix} i \\ 0 \end{bmatrix} = i \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = i|0\rangle$$



$$P_{|0\rangle} = |i|^2 = 1$$

Demystifying the Mach-Zehnder Experiment!



The Deutsch Problem (1985)

$x = 0$ or 1.

How many times do we need to evaluate $f(x)$ in order to know if $f(0) + f(1) = 1$?

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If $f(0) = f(1) = 0, \qquad f(0) + f(1) = 0$

The Deutsch Problem (1985)

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How many times do we need to evaluate $f(x)$ in order to know if $f(0)+f(1)=1$?

If $f(0)=f(1)=0,\quad f(0)+f(1)=0$

If $f(0)=0, f(1)=1,\quad f(0)+f(1)=1$

The Deutsch Problem (1985)

$x = 0$ or 1.

How many times do we need to evaluate $f(x)$ in order to know if $f(0) + f(1) = 1$?

If $f(0) = f(1) = 0,$ $\quad f(0) + f(1) = 0$

If $f(0) = 0, f(1) = 1,$ $\quad f(0) + f(1) = 1$

If $f(0) = f(1) = 1,$ $\quad f(0) + f(1) = 2$

The Deutsch Problem (1985)

These simple gates are enough to determine *f(0) + f(1)* with <u>one</u> evaluation of *f(x)*:

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} - \boxed{(-1)^{f(x)}} - \boxed{\boldsymbol{H}}$$

The Deutsch Problem (1985)

These simple gates are enough to determine *f(0)* + *f(1)* with one evaluation of *f(x)*:

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \;-\; \boxed{\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}} \;-\; \boxed{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}}$$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} \text{---} \boxed{\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}} \text{---} \boxed{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}}$$

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$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} - \boxed{\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}} - \boxed{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}}$$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

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The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

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$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \boxed{\begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}}$$

ψ

If $f(0) = f(1) = 0,$ ψ = ?

If $f(0) = 0,\ f(1) = 1,$ ψ = ?

If $f(0) = f(1) = 1,$ ψ = ?

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

If $f(0) = f(1) = 0,$ $\quad \psi = \begin{bmatrix}1\\ 0\end{bmatrix} = |0\rangle$

If $f(0) = 0,\ f(1) = 1,$ $\quad \psi = \begin{bmatrix}0\\ 1\end{bmatrix} = |1\rangle$

If $f(0) = f(1) = 1,$ $\quad \psi = -\begin{bmatrix}1\\ 0\end{bmatrix} = -|0\rangle$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

If $f(0) = f(1) = 0,$ $\quad \psi = \begin{bmatrix}1\\ 0\end{bmatrix} = |0\rangle$

If $f(0) = 0,\ f(1) = 1,$ $\quad \psi = \begin{bmatrix}0\\ 1\end{bmatrix} = |1\rangle$ $\quad f(0) + f(1) = 1$

If $f(0) = f(1) = 1,$ $\quad \psi = -\begin{bmatrix}1\\ 0\end{bmatrix} = -|0\rangle$

The Deutsch Problem (1985)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}(-1)^{f(0)} & 0\\ 0 & (-1)^{f(1)}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\\ \frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\end{bmatrix}$$

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If $f(0) = f(1) = 1,\quad \psi = -\begin{bmatrix}1\\ 0\end{bmatrix} = -|0\rangle$

Done in a real experiment in 1998!

The Deutsch-Jozsa Problem (1992)

x_i = 0 or 1.

How many times do we need to evaluate *$f(x_1, x_2, \ldots, x_n)$* in order to know if it's constant?

The Deutsch-Jozsa Problem (1992)

- 1992 algorithm by Deutsch-Jozsa required:
  2 function evaluations
- 1997 algorithm by Cleve *et al.* requires:
  1 function evaluation!

BY R. CLEVE[1], A. EKERT[2], C. MACCHIAVELLO[2,3] AND M. MOSCA[2,4]

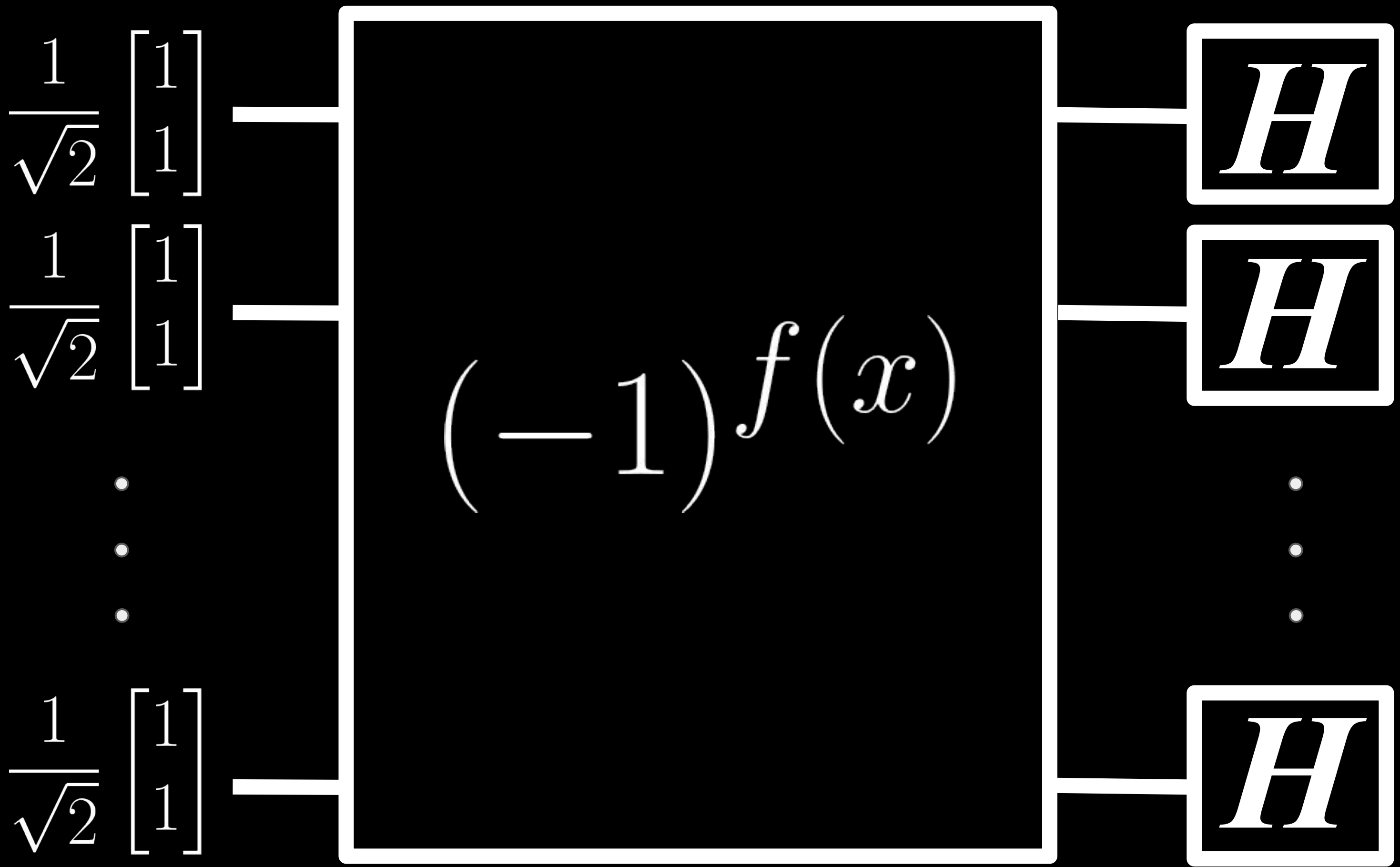
[2] *Clarendon Laboratory, Department of Physics, University of Oxford, Parks Road, Oxford OX1 3PU, U.K.*

[1] *Department of Computer Science, University of Calgary Calgary, Alberta, Canada T2N 1N4.*

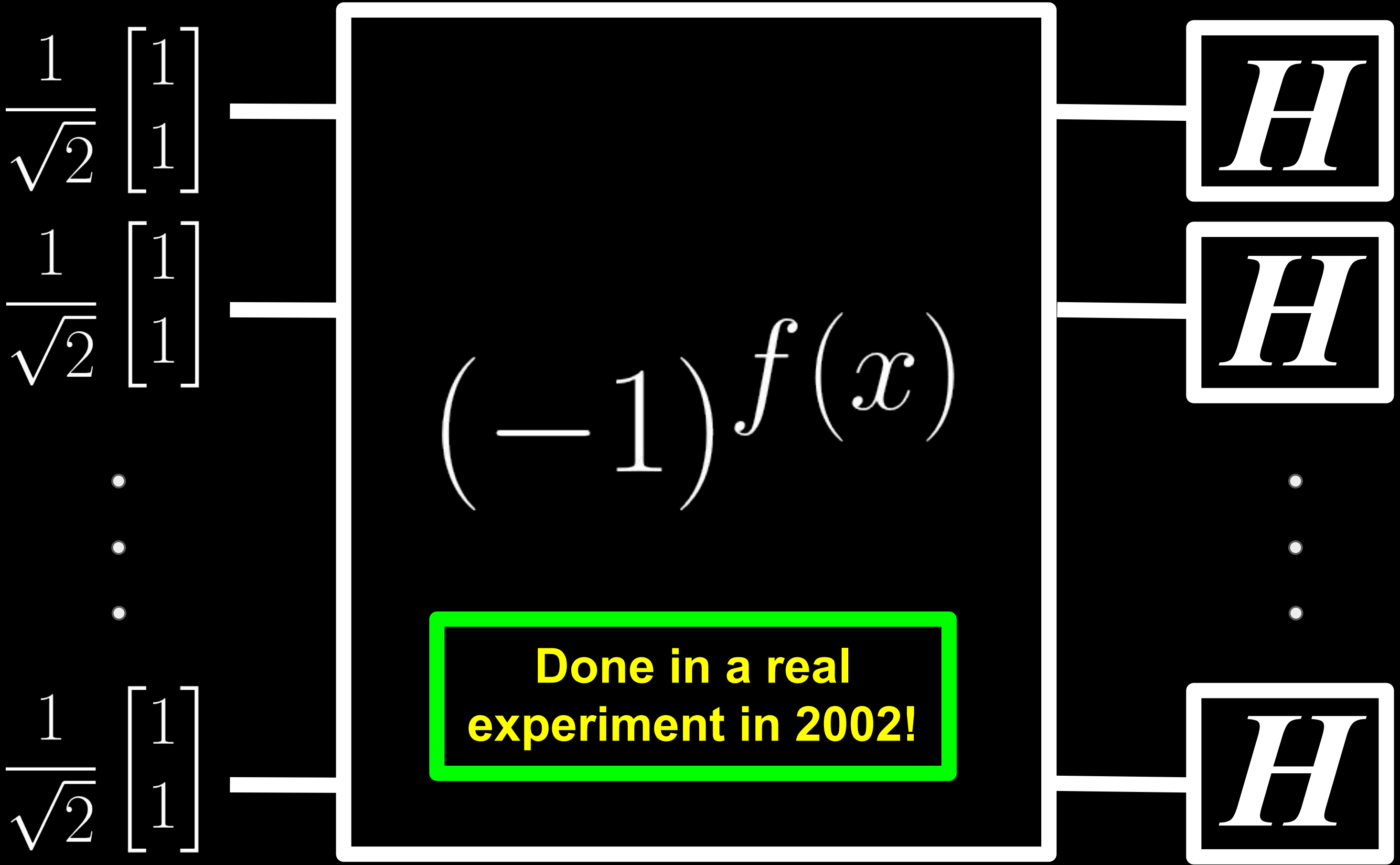
The Deutsch-Jozsa Problem (1992)

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \longrightarrow \boxed{(-1)^{f(x)}} \longrightarrow \boxed{\boldsymbol{H}}$$

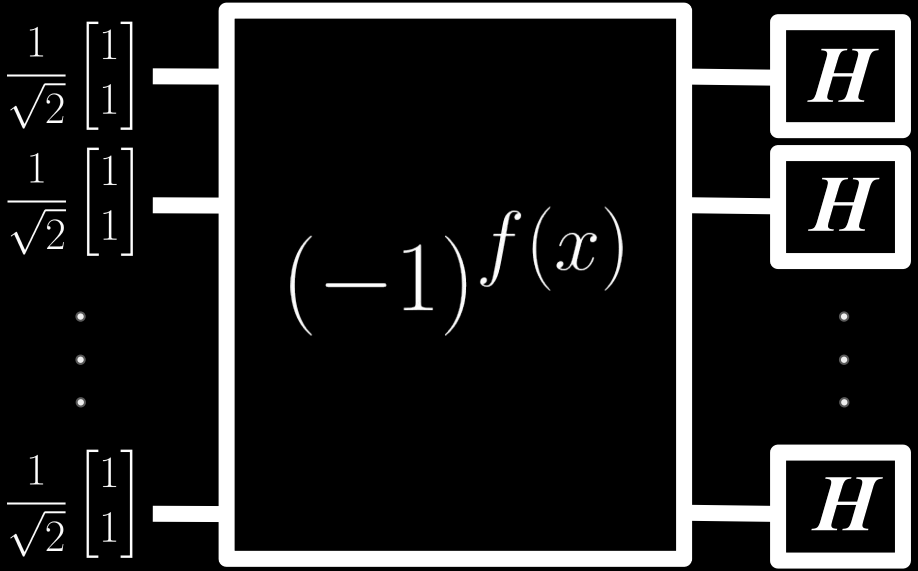
The Deutsch-Jozsa Problem (1992)



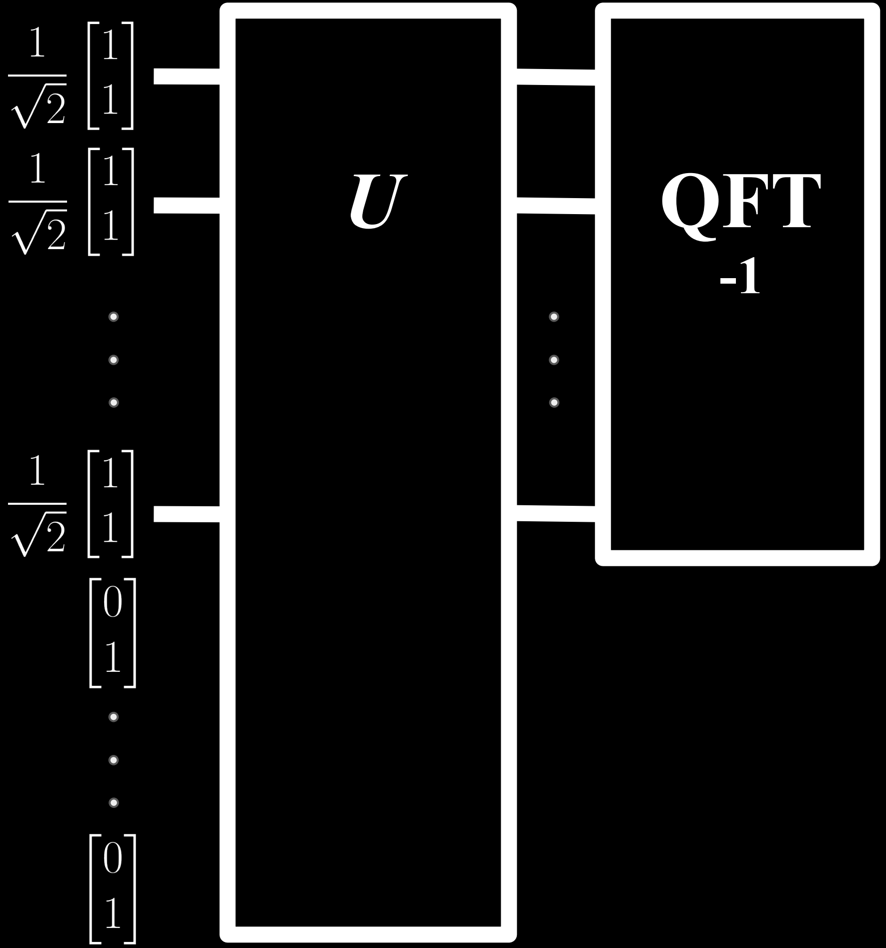
The Deutsch-Jozsa Problem (1992)



The Deutsch-Jozsa Problem (1992)



Shor's Algorithm (1994)



Recap

- Classical vs Quantum bits and gates
- Mach-Zehnder Experiment
  - Explained using qubits and quantum gates
- Deutsch algorithm
  - $f(0) + f(1)$ by only evaluating $f(x)$ once rather than twice!
- Deutsch-Jozsa algorithm
  - Determined if $f(x_1, x_2, \ldots, x_n)$ is constant by only evaluating it 1 time rather than 2^n times!
- Shor's algorithm preview
  - The most famous quantum algorithm

Thank you!

Upcoming lectures

- Quantum chemistry on a quantum computer $\langle \mathrm{QC} | \mathrm{QC} \rangle$
- BB84 protocol (quantum communication security)
- Grover's algorithm
- HHL algorithm
- Quantum decoherence
- How to actually implement quantum gates:
  - Superconducting qubits
  - Photonic qubits
  - Spin-based qubits (NMR / NV centres)
  - Ion traps, Rydberg atoms, ultracold molecules, etc.