If a number’s last digit is 5,
then is it *always* 5 times an odd number?

Prove it.

MATH 135: Lecture 4

Dr. Nike Dattani

15 September 2021

Upcoming responsibilities!

- Thursday 16 September:
  - WA01 solutions will be posted, hopefully before 12pm: Check the solutions in detail!
- Friday 17 September before class:
  - Complete reading Chapter 2 of the course notes
- Friday 17 September:
  - Complete Mobius Quiz 3: MQ3
- Sunday 19 September:
  - Complete reading up to the end of Section 0.2 (Polynomials)
  - There's a question on next MQ based on the polynomials reading!
- Monday 20 September:
  - Complete Mobius Quiz 4: MQ4
- **Wednesday 22 September:**
  - **Complete Written Assignment 2: WA2**
- Wednesday 22 September:
  - Complete Mobius Quiz 5: MQ5

Divisibility by 5 [edit]

Divisibility by 5 is easily determined by checking the last digit in the number (475), and seeing if it is either 0 or 5. If the last number is either 0 or 5, the entire number is divisible by 5.[2][3]

If the last digit in the number is 0, then the result will be the remaining digits multiplied by 2. For example, the number 40 ends in a zero, so take the remaining digits (4) and multiply that by two (4 × 2 = 8). The result is the same as the result of 40 divided by 5(40/5 = 8).

If the last digit in the number is 5, then the result will be the remaining digits multiplied by two, plus one. For example, the number 125 ends in a 5, so take the remaining digits (12), multiply them by two (12 × 2 = 24), then add one (24 + 1 = 25). The result is the same as the result of 125 divided by 5 (125/5=25).

Why is every number which ends in 5 divisible by 5?

Asked 7 years, 1 month ago Active 7 years, 1 month ago Viewed 172 times

0

Is there more of an answer to this which is more than just 'it does'?

arithmetic

Share Cite Follow

asked Aug 17 '14 at 7:55
BenjaminJB
123 1 5

math.stackexchange.com/questions/898096/... – lab bhattacharjee Aug 17 '14 at 7:58

It's because 5 divides 10, and every number which ends in 5 is 5 plus powers of 10. – David H Aug 17 '14 at 7:59

Add a comment

1 Answer

Active Oldest Votes

1

Well, it's only true in decimal ;)

That is the hint of observing this property: all digits that are in front add factors of 10, which is divisible by 10 and therefor also by 5. And that doesn't change anything in divisibility.

Share Cite Follow

answered Aug 17 '14 at 8:01
Pieter21
3,292 1 9 14

Divisibility rules/Rule for 5 and powers of 5 proof

A number N is divisible by 5^n if the last n digits are divisible by that power of 5.

Proof

An understanding of basic modular arithmetic is necessary for this proof.

Let the base-ten representation of N be $\underline{a_k a_{k-1} \cdots a_1 a_0}$ where the a_i are digits for each i and the underline is simply to note that this is a base-10 expression rather than a product. If N has no more than n digits, then the last n digits of N make up N itself, so the test is trivially true. If N has more than n digits, we note that:

$$N = 10^k a_k + 10^{k-1} a_{k-1} + \cdots + 10 a_1 + a_0.$$

Taking this $\mod 5^n$ we have

$$\begin{array}{|l|l|}\hline N & = 10^k a_k + 10^{k-1} a_{k-1} + \cdots + 10 a_1 + a_0 \\ \hline & \equiv 10^{n-1} a_{n-1} + 10^{n-2} a_{n-2} + \cdots + 10 a_1 + a_0 \pmod{5^n} \\ \hline \end{array}$$

because for $i \geq n$, $10^i \equiv 0 \pmod{5^n}$. Thus, N is divisible by 5^n if and only if

$$10^{n-1} a_{n-1} + 10^{n-2} a_{n-2} + \cdots + 10 a_1 + a_0 = \underline{a_{n-1} a_{n-2} \cdots a_1 a_0}$$

is. But this says exactly what we claimed: the last n digits of N are divisible by 5^n if and only if N is divisible by 5^n.

If a number's last digit is 5, then is it *always* 5 times an odd number?

$S = \{\text{Integers for which the last digit is } 5\}$

$\forall\, N \in S\ ,\ \exists\, m \in \mathbb{Z}\ ,\ N = 5(2m+1)\ ?$

$\forall\, N \in S,\ \exists\, a \in \mathbb{Z},\quad N = 10a + 5$

$N = 5(2a+1)$

$(2a+1)$ is odd!

$\therefore\ N = 5 \times (\text{odd number}),\ \forall\, N \in S$

Class Statistics | User Statistics

View By: Groups | Groups: 1219-MATH.135.019.1.LEC | Apply

MQ1 (Mon Sep 13) Class Statistics

Number of submitted grades: 59 / 61

Minimum: 40 %

Maximum: 100 %

Average: 86.41 %

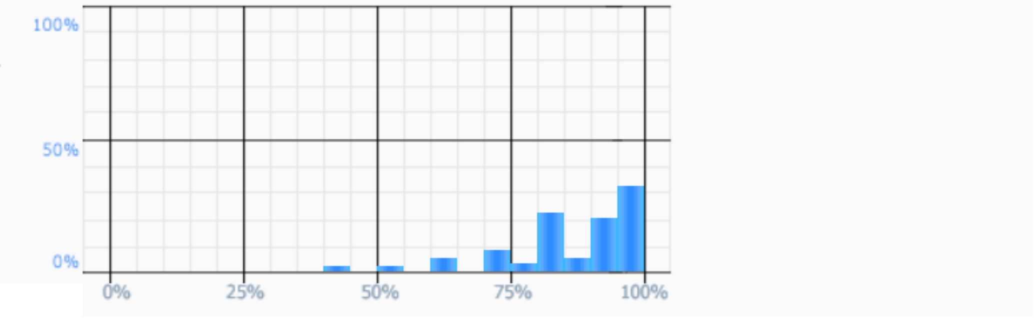
Mode: 100 %

Median: 90 %

Standard Deviation: 13.48 %

Grade Distribution

Number of Users (%)



Class Statistics | User Statistics

View By: Groups | Groups: 1219-MATH.135.016.1.LEC | Apply

MQ1 (Mon Sep 13) Class Statistics

Number of submitted grades: 51 / 56

Minimum: 20 %

Maximum: 100 %

Average: 85.23 %

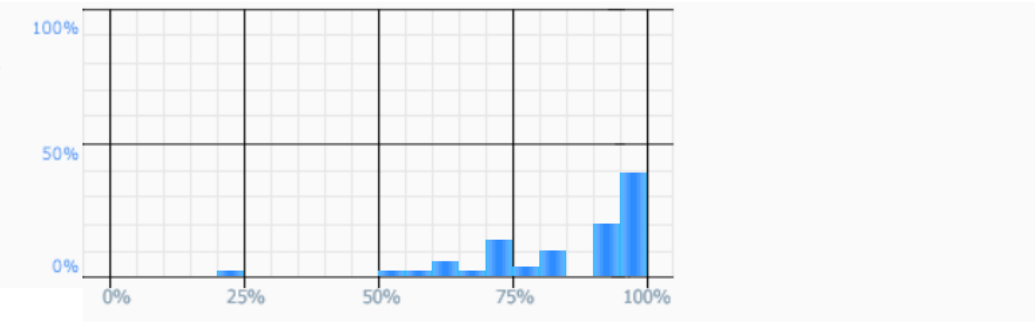
Mode: 100 %

Median: 90 %

Standard Deviation: 16.51 %

Grade Distribution

Number of Users (%)



Tips for MATH 135!

- Do not use "s.t." but you can instead use a comma.

- **How many people here use GitHub?**

Lecture Notes:

- Github → ndattani → Lecture_Notes
- https://github.com/ndattani/Lecture_Notes
- Private repository: send me your username and I'll give you access!
- Tips for username: Just use your actual name!
  - GitHub is a portfolio of all of your academic work, and later career work

Logic

$\wedge$ = AND   ($\wedge$ looks like A)
$\vee$ = OR

A	B	$A \wedge B$	$A \vee B$	$\neg A$	$\neg B$	$(\neg A) \wedge (\neg B)$	$(\neg A) \vee (\neg B)$
T	T						
T	F						
F	T						
F	F						

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F	T	F	T	T	F	F	T
F	F	F	F	T	T	T	T

$$\forall x \in \mathbb{R}, ((x > 2) \Rightarrow (x^2 \geq 4)).$$

x	$x > 2$	$x^2 \geq 4$	$(x > 2) \Rightarrow (x^2 \geq 4)$
$x > 2$			
$x = 2$			
$-2 < x < 2$			
$x \leq -2$			

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$x > 2$	T	T	T
$x = 2$	F	T	T
$-2 < x < 2$	F	F	T
$x \leq -2$	F	T	T

- We will use the convention that (A => B) is ***true*** if A is ***false***. In this case we say it's "***vacuously true***".
- This way we don't have to spend time checking cases that ***do not impact*** the open sentence.

- This convention *might* not be followed in some types of non-classical logic (click for link to Wikipedia page!).