

Project 5

Nicholas Davidson

5/3/2023

(a)

Assume the single index model holds. Use only the stocks with positive betas in your data. Choose a value of R_f and find the optimal portfolio (point of tangency) using the optimization procedure as discussed in handout #12:

http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/statc183c283_tangent.pdf

(http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/statc183c283_tangent.pdf) . The approach here is based on $Z = \Sigma^{-1}R$.

```
a <- read.csv("stockData.csv", sep=",", header=TRUE)

#Convert adjusted close prices into returns:
r <- (a[-1,3:ncol(a)]-a[-nrow(a),3:ncol(a)])/(a[-nrow(a),3:ncol(a)])

#Compute the betas:
covmat <- var(r)
beta <- covmat[1,-1] / covmat[1,1]

#Keep only the stocks with positive betas:
rrr <- r[,-c(1,which(beta<0)+1)]
#Note: which(beta<0) gives the element in the beta vector with negative beta and add 1 because
#the first column in the initial data set is the index. We also remove column 1 (index) from the initial data #see
#t.

#Initialize
beta <- rep(0,ncol(rrr))
alpha <- rep(0,ncol(rrr))
mse <- rep(0,ncol(rrr))
Ribar <- rep(0,ncol(rrr))
Ratio <- rep(0,ncol(rrr))
stock <- rep(0,ncol(rrr))

#Risk free asset:
rf <- 0.001

#This for loop computes the required inputs:
for(i in 1:ncol(rrr)){
  q <- lm(data=rrr, formula=rrr[,i] ~ r[,1])
  beta[i] <- q$coefficients[2]
  alpha[i] <- q$coefficients[1]
  mse[i] <- summary(q)$sigma^2
  Ribar[i] <- q$coefficients[1]+q$coefficients[2]*mean(r[,1])
  Ratio[i] <- (Ribar[i]-rf)/beta[i]
  stock[i] <- i
}

#So far we have this table:
xx <- (cbind(stock,alpha, beta, Ribar, mse, Ratio))

#Order the table based on the excess return to beta ratio:
A <- xx[order(-xx[,6]),]

col1 <- rep(0,nrow(A))
col2 <- rep(0,nrow(A))
col3 <- rep(0,nrow(A))
col4 <- rep(0,nrow(A))
col5 <- rep(0,nrow(A))

#Create the last 5 columns of the table:
col1 <- (A[,4]-rf)*A[,3]/A[,5]
col3 <- A[,3]^2/A[,5]
for(i in 1:nrow(A)) {
  col2[i] <- sum(col1[1:i])
  col4[i] <- sum(col3[1:i])
}

#Compute the Ci (col5):
for(i in 1:nrow(A)) {
  col5[i] <- var(r[,1])*col2[i]/(1+var(r[,1])*col4[i])
}

#SHORT SALES ALLOWED:
#Compute the Zi:
z_short <- (A[,3]/A[,5])*(A[,6]-col5[nrow(A)])
#Compute the xi:
x_short <- z_short/sum(z_short)

#The final table when short sales allowed:
Weights_with_short <- cbind(A, col1, col2, col3, col4, col5, z_short, x_short)
Weights_with_short
```

```

##      stock      alpha      beta      Ribar      mse      Ratio
## [1,] 19 0.0128568768 0.09599006 0.0147787917 0.002830250 0.143543942
## [2,] 22 0.0226860588 0.18454304 0.0263809830 0.010159741 0.137534223
## [3,] 13 0.0072257172 0.06077196 0.0084424946 0.001284923 0.122465925
## [4,] 26 0.0084721243 0.07960230 0.0100659230 0.002403734 0.113890219
## [5,] 27 0.0086981865 0.08347342 0.0103694930 0.002601563 0.112245224
## [6,] 17 0.0118998199 0.14818837 0.0148668503 0.005325364 0.093575833
## [7,] 15 0.0281211613 0.41177943 0.0363658169 0.012500750 0.085885342
## [8,] 14 0.0127562873 0.18214885 0.0164032750 0.001927513 0.084564220
## [9,] 18 0.0241431532 0.37177830 0.0315869055 0.006022497 0.082271895
## [10,] 8 0.0151869219 0.24021645 0.0199965403 0.003119281 0.079080930
## [11,] 23 0.0121520026 0.19456093 0.0160475052 0.003166575 0.077340839
## [12,] 16 0.0077663648 0.11894350 0.0101478539 0.001494667 0.076909237
## [13,] 1 0.0190084306 0.39265863 0.0268702493 0.002617901 0.065884835
## [14,] 7 0.0140563654 0.29930404 0.0200490365 0.001472136 0.063644436
## [15,] 12 0.0107431558 0.24844569 0.0157175403 0.002284329 0.059238460
## [16,] 29 0.0072426892 0.17508675 0.0107482796 0.003599384 0.055676854
## [17,] 2 0.0333136619 0.91530123 0.0516398407 0.011141031 0.055325874
## [18,] 24 0.0110070363 0.31258927 0.0172657046 0.003030013 0.052035391
## [19,] 25 0.0118782444 0.37457870 0.0193780662 0.004303695 0.049063298
## [20,] 6 0.0053835627 0.15477605 0.0084824917 0.001451472 0.048343990
## [21,] 28 0.0074172992 0.24121647 0.0122469399 0.003187635 0.046625921
## [22,] 4 0.0067417375 0.38568112 0.0144638523 0.003202944 0.034909285
## [23,] 3 0.0067090522 0.40667227 0.0148514521 0.003091125 0.034060479
## [24,] 11 0.0066637834 0.41088872 0.0148906053 0.003998211 0.033806246
## [25,] 10 0.0038248103 0.34604510 0.0107533320 0.003554502 0.028185147
## [26,] 5 0.0027538205 0.75913460 0.0179532280 0.011914407 0.022332308
## [27,] 9 0.0008876338 0.39325187 0.0087613303 0.004547327 0.019736283
## [28,] 21 0.0002518150 0.38974154 0.0080552275 0.004814547 0.018102324
## [29,] 20 -0.0062142341 0.27585055 -0.0006911491 0.003809642 -0.006130671
##      col1      col2      col3      col4      col5      z_short
## [1,] 0.4673181 0.4673181 3.255575 3.255575 0.002668395 3.5170821
## [2,] 0.4610239 0.9283420 3.352067 6.607642 0.005201296 1.7744706
## [3,] 0.3520018 1.2803437 2.874283 9.481925 0.007059794 3.9077325
## [4,] 0.3002280 1.5805718 2.636118 12.118043 0.008590381 2.4521423
## [5,] 0.3006284 1.8812002 2.678318 14.796361 0.010077595 2.3230764
## [6,] 0.3858715 2.2670716 4.123623 18.919984 0.011882229 1.4952074
## [7,] 1.1649634 3.4320350 13.564170 32.484154 0.016794120 1.5166393
## [8,] 1.4556002 4.8876352 17.212956 49.697110 0.022058875 4.2260915
## [9,] 1.8881781 6.7758133 22.950463 72.647573 0.027710360 2.6191788
## [10,] 1.4629272 8.2387405 18.499115 91.146688 0.031323406 3.0216906
## [11,] 0.9245499 9.1632904 11.954227 103.100915 0.033323954 2.3039211
## [12,] 0.7279731 9.8912636 9.465354 112.566269 0.034774341 2.9496472
## [13,] 3.8802746 13.7715382 58.894807 171.461075 0.040110936 3.9059524
## [14,] 3.8729113 17.6444495 60.852316 232.313392 0.043654005 4.8390553
## [15,] 1.6006928 19.2451424 27.021176 259.334567 0.044630584 2.1094253
## [16,] 0.4741907 19.7193331 8.516837 267.851405 0.044844533 0.7701956
## [17,] 4.1603608 23.8796939 75.197381 343.048786 0.046375185 1.2719772
## [18,] 1.6780403 25.5577343 32.248059 375.296844 0.046708775 1.2577795
## [19,] 1.5995633 27.1572976 32.602034 407.898879 0.046841175 0.8024690
## [20,] 0.7978867 27.9551842 16.504361 424.403239 0.046882771 0.9064515
## [21,] 0.8510847 28.8062689 18.253467 442.656706 0.046875142 0.5132515
## [22,] 1.6212440 30.4275129 46.441627 489.098333 0.046034391 -0.5941380
## [23,] 1.8223143 32.2498272 53.502309 542.600643 0.045137745 -0.7608069
## [24,] 1.4275116 33.6773388 42.226266 584.826909 0.044505413 -0.6204259
## [25,] 0.9495263 34.6268651 33.688890 618.515799 0.043809793 -1.1349771
## [26,] 1.0801865 35.7070516 48.368779 666.884578 0.042571251 -1.1157305
## [27,] 0.6711982 36.3782498 34.008337 700.892915 0.041681462 -1.7388582
## [28,] 0.5711265 36.9493762 31.549899 732.442814 0.040858832 -1.7599572
## [29,] -0.1224536 36.8269226 19.973933 752.416747 0.039843389 -3.3289141
##      x_short
## [1,] 0.09396519
## [2,] 0.04740818
## [3,] 0.10440212
## [4,] 0.06551340
## [5,] 0.06206517
## [6,] 0.03994716
## [7,] 0.04051975
## [8,] 0.11290765
## [9,] 0.06997608
## [10,] 0.08072991
## [11,] 0.06155340
## [12,] 0.07880514
## [13,] 0.10435456
## [14,] 0.12928408
## [15,] 0.05635710
## [16,] 0.02057716
## [17,] 0.03398316
## [18,] 0.03360385
## [19,] 0.02143940
## [20,] 0.02421749
## [21,] 0.01371244
## [22,] -0.01587347
## [23,] -0.02032633
## [24,] -0.01657580
## [25,] -0.03032296
## [26,] -0.02980875
## [27,] -0.04645673
## [28,] -0.04702043
## [29,] -0.08893794

```

```
cat("The final weights for the optimal portfolio with a Rf of 0.001 with short sales allowed is:", x_short)
```

```
## The final weights for the optimal portfolio with a Rf of 0.001 with short sales allowed is: 0.09396519 0.04740
818 0.1044021 0.0655134 0.06206517 0.03994716 0.04051975 0.1129077 0.06997608 0.08072991 0.0615534 0.07880514 0.1
043546 0.1292841 0.0563571 0.02057716 0.03398316 0.03360385 0.0214394 0.02421749 0.01371244 -0.01587347 -0.020326
33 -0.0165758 -0.03032296 -0.02980875 -0.04645673 -0.04702043 -0.08893794
```

```
cat("Check that the sum of the weights is 1:", sum(x_short))
```

```
## Check that the sum of the weights is 1: 1
```

(b)

Adjust the betas using Blume's and Vasicek's techniques. For the Blume technique use the two periods: 01-Jan-2015 to 01-Jan-2020 and 01-Jan-2020 to 31-Mar-2023. For the Vasicek technique use only the period 01-Jan-2014 to 01-Jan-2019

```
a1 <- read.csv("stockData2015.csv", sep=",", header=TRUE)

a2 <- read.csv("stockData2023.csv", sep=",", header=TRUE)

spy <- read.csv("SPY2.csv", sep=",", header=TRUE)

#Convert adjusted close prices into returns:
r1 <- (a1[-1,3:ncol(a1)]-a1[-nrow(a1),3:ncol(a1)])/(a1[-nrow(a1),3:ncol(a1)])

r2 <- (a2[-1,3:ncol(a2)]-a2[-nrow(a2),3:ncol(a2)])/(a2[-nrow(a2),3:ncol(a2)])

rm <- (spy[-1,3:ncol(spy)]-spy[-nrow(spy),3:ncol(spy)])/(spy[-nrow(spy),3:ncol(spy)])

#Compute the variance covariance matrix of the returns for each period:
covmat1 <- var(r1)
covmat2 <- var(r2)

#Compute the betas in each period:
beta1 <- covmat1[1,-1] / covmat1[1,1]
beta2 <- covmat2[1,-1] / covmat2[1,1]

#Adjust betas using the Blume's technique:
q1 <- lm(beta2 ~ beta1)
beta3adj_blume <- q1$coef[1] + q1$coef[2]*beta2

#Vasicek's method:
beta2 <- rep(0,30)

alpha2 <- rep(0,30)

sigma_e2 <- rep(0,30)

var_beta2 <- rep(0,30)

for(i in 1:30){
  q <- lm(r1[,i] ~ rm)
  beta2[i] <- q$coefficients[2]
  alpha2[i] <- q$coefficients[1]
  sigma_e2[i] <- summary(q)$sigma^2
  var_beta2[i] <- vcov(q)[2,2]
}

#Adjusting the betas using the Vasicek's technique:
beta2adj_vasicek <- var_beta2*mean(beta2)/(var(beta2)+var_beta2) +
var(beta2)*beta2/(var(beta2)+var_beta2)

cat("The adjusted beta's using Blume's method:", beta3adj_blume, "\n")
```

```
## The adjusted beta's using Blume's method: 0.6513148 0.892809 0.5711414 0.5688403 0.61895 0.5640408 0.6213842
0.5097284 0.6305051 0.6490074 0.7178222 0.6165097 0.4907285 0.6308642 0.6193694 0.4756154 0.6932448 0.8260563 0.5
747907 0.7773759 0.695182 0.786382 0.6409142 0.6555884 0.6968731 0.401755 0.4501105 0.5787541 0.7510426
```

```
cat("The adjusted beta's using Vasicek's method:", beta2adj_vasicek)
```

```
## The adjusted beta's using Vasicek's method: 1.187177 1.133133 1.501654 1.127137 0.8474486 1.461058 0.8106808
0.9248067 1.121134 1.250505 1.466978 1.353996 0.848751 0.4517128 0.5293814 0.9976449 0.3722865 0.6375478 1.467466
0.6108178 1.001254 1.323176 0.8431415 0.8120955 1.04748 1.06334 0.5153818 0.5742489 1.02091 0.9936535
```

(c)

Compute PRESS only for the Vasicek technique. (You can compute the PRESS only for the Vasicek technique because you have the actual betas in the period 01-Jan-2020 to 31-Mar-2023.)

```
PRESS <- sum((beta2adj_vasicek-beta2)^2) / 30
PRESS
```

```
## [1] 0.01498543
```