Earth's Energy Balance and Climate Change: 2-Box Model

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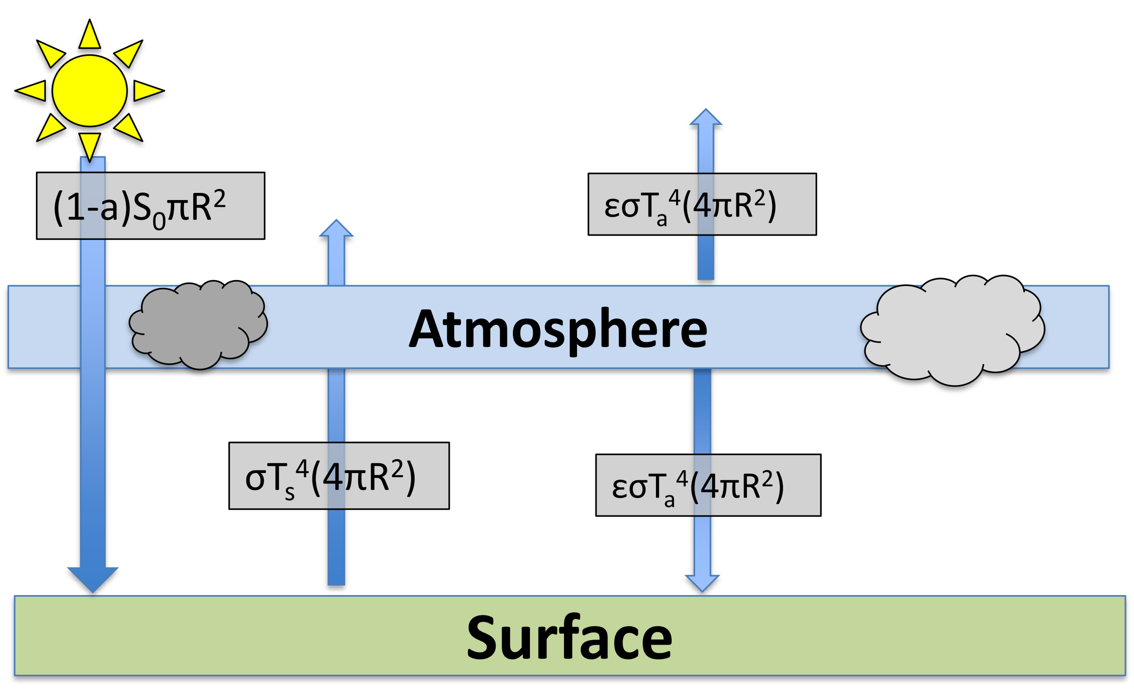
Introduction

In this model the earth is represented as a perfect blackbody, and the atmosphere is an imperfect blackbody with absorptivity and emissivity equal to a constant. (Absorptivity and emissivity are equal according to Kirchoff's law of thermal radiation).
Where:
S_0 is the solar constant
a is albedo (proportion of reflected radiation)
σ is the Stephan-Boltzmann constant
R is the radius of the Earth
ε is the emissivity/absorptivity of the atmosphere.



Notice that no incoming solar radiation is absorbed by the atmosphere in this model. In reality this is not true, but it is a fair approximation because the atmosphere absorbs longer wavelength radiation emitted by the Earth at much higher rates than it absorbs shorter wavelength radiation emitted by the Sun.

The above equations can be used to find the rate of change of the temperatures of the surface and atmosphere:

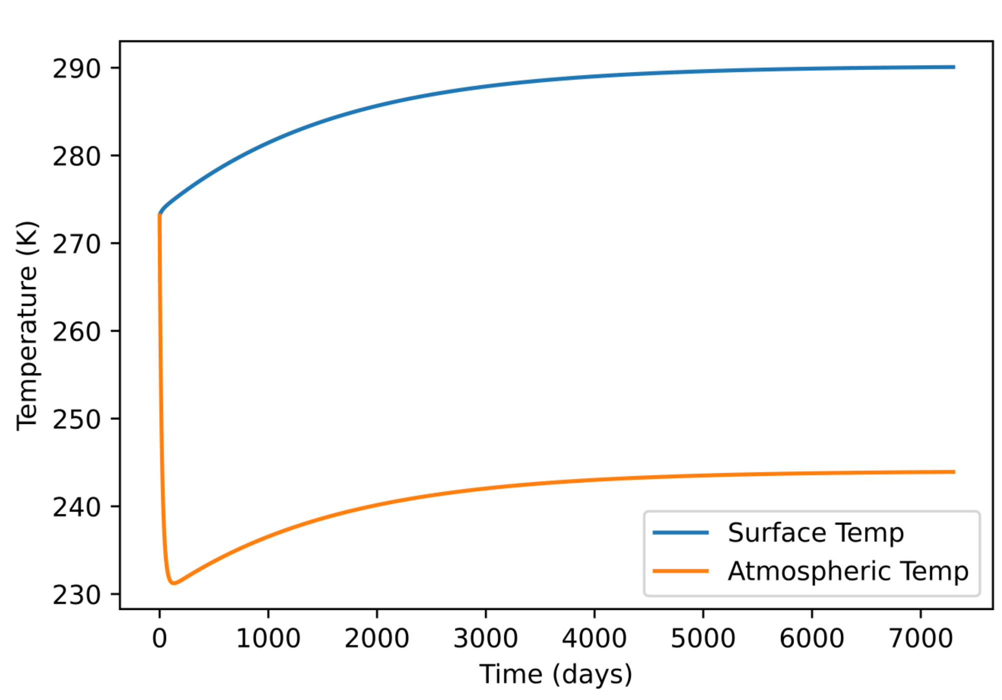
$$\frac{dT_S}{dt} = \pi R^2 \, \frac{(S_0(1-a)-4\sigma(T_S^4-\varepsilon T_a^4))}{C_S}$$

$$\frac{dT_a}{dt} = \frac{4\sigma\varepsilon\pi R^2}{C_a}(T_S^4 - 2T_a^4)$$

Where C_a and C_S are the heat capacity of the atmosphere and surface, respectively.

Simulation over Time

As an initial demonstration of this model, set both the atmosphere and surface to 0 degrees C initially and observe how the system evolves over time



The surface and atmosphere both approach an equilibrium temperature

Equilibrium Temperatures

The values used in the prior simulation for albedo (0.3) and atmospheric absorptivity/emissivity (0.8) are approximations of their real values. However, humans have changed the composition of the atmosphere, so these values are subject to change.

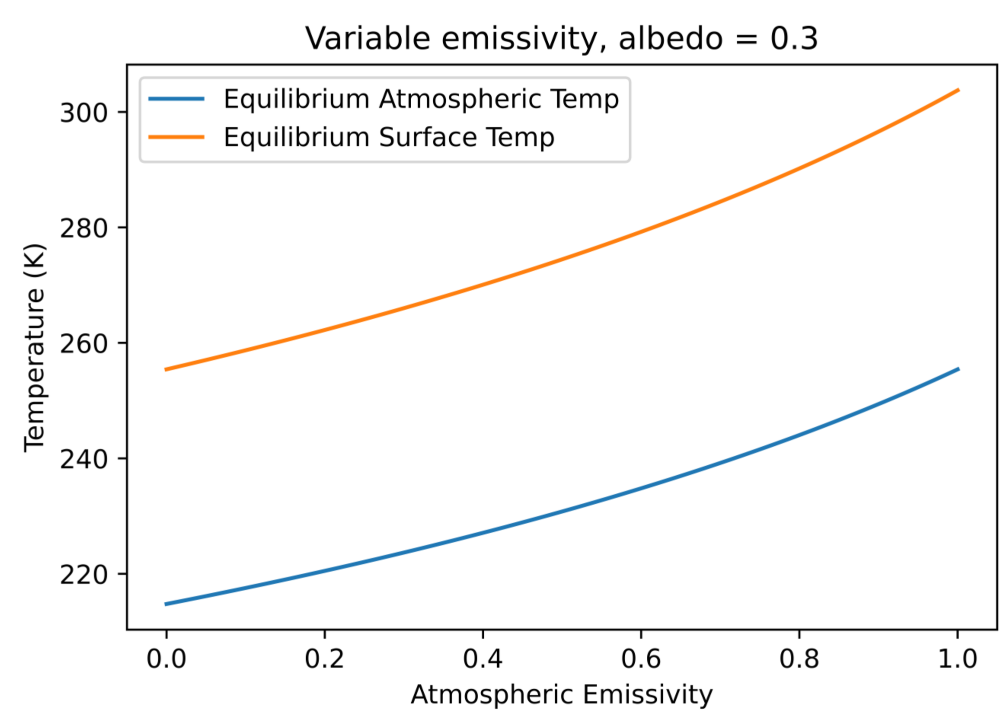
CO2 absorbs radiation emitted by the Earth at high rates, and so increasing the concentration of CO2 in the atmosphere effectively increases atmospheric absorptivity/emissivity.

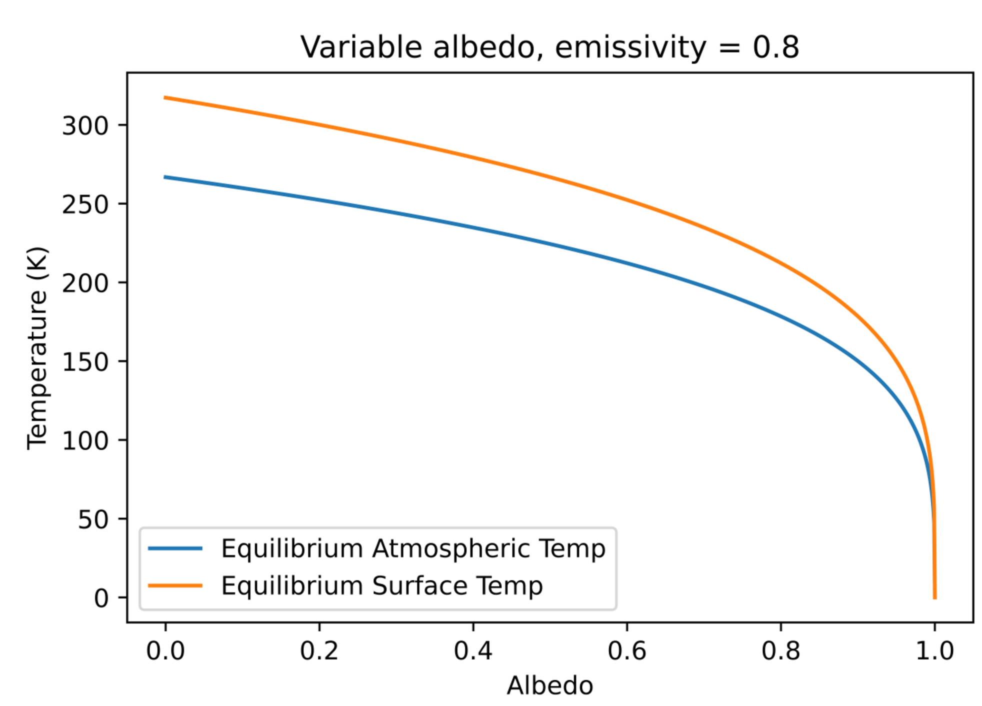
One common geoengineering proposal to counteract the negative effects of climate change is to intentionally emit sulfate aerosols into the stratosphere. These aerosols are reflective towards the shortwave radiation incoming from the sun, but do not significantly interact with outgoing shortwave radiation leaving the earth. Therefore, these aerosols increase the Earth's albedo.

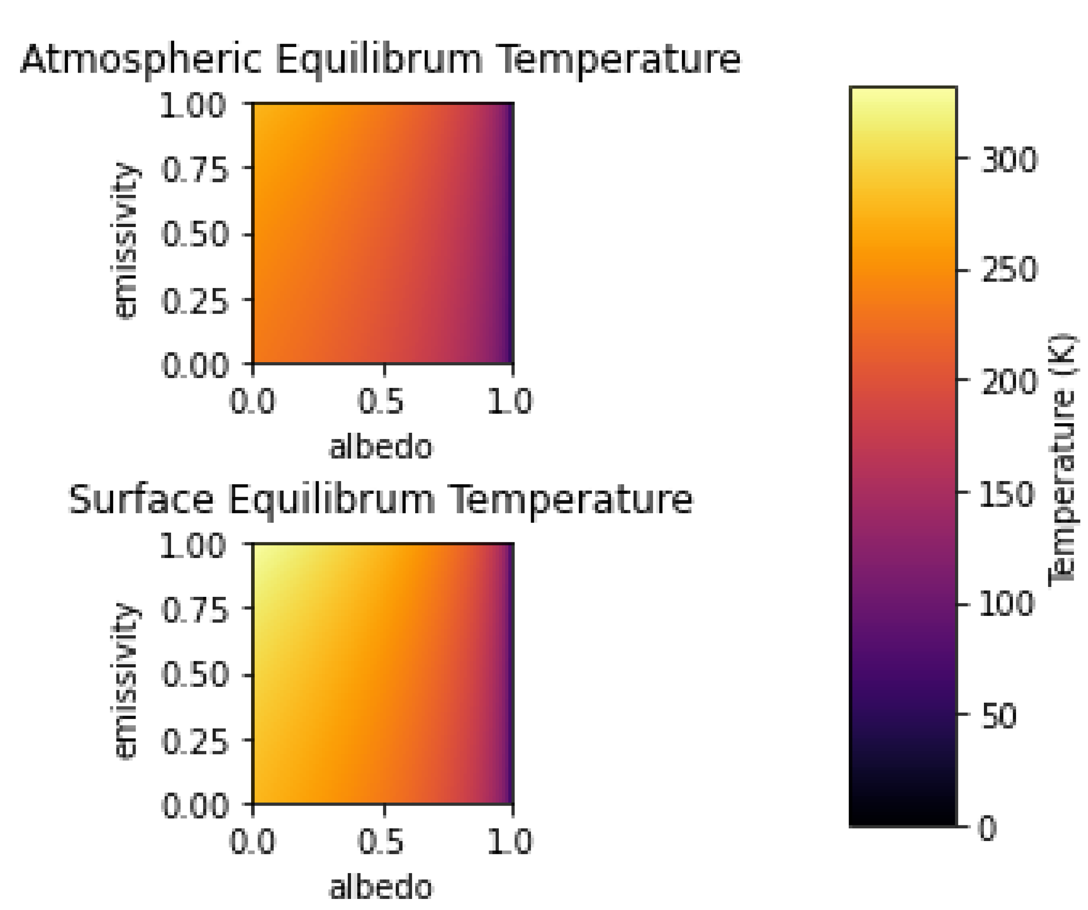
Equations for equilibrium temperatures depending on these parameters can be found by setting the derivatives of temperature equal to 0 and solving for Ta and Ts as functions of a and ε:

$$T_a = \frac{S_0(1-a)\pi R^2}{C_S \sqrt[4]{\frac{4\sigma\pi R^2(2-\varepsilon)}{C_S}}}$$

$$T_S = \sqrt{2}T_a$$



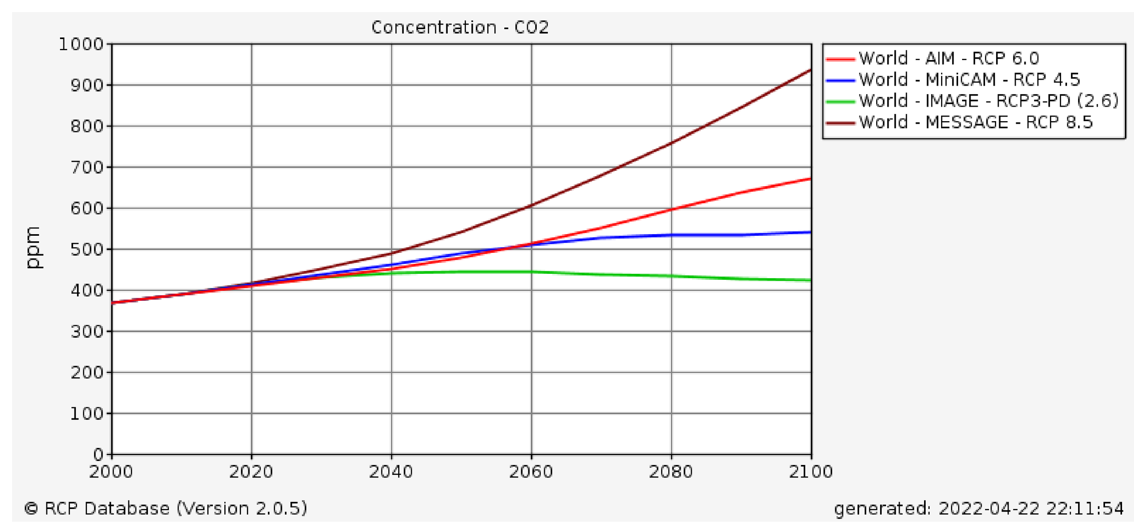




From these graphs, we can observe that increasing emissivity increases equilibrium temperatures, and increasing albedo decreases equilibrium temperatures. If the Earth reflected all incoming radiation, its equilibrium temperature would be absolute 0.

Emissivity and CO_2

The Intergovernmental Panel on Climate Change has adopted several "Representative Concentration Pathways" each of which represents a different possible scenario of human response to climate change. CO_2 concentrations over time have been calculated for these RCPs:



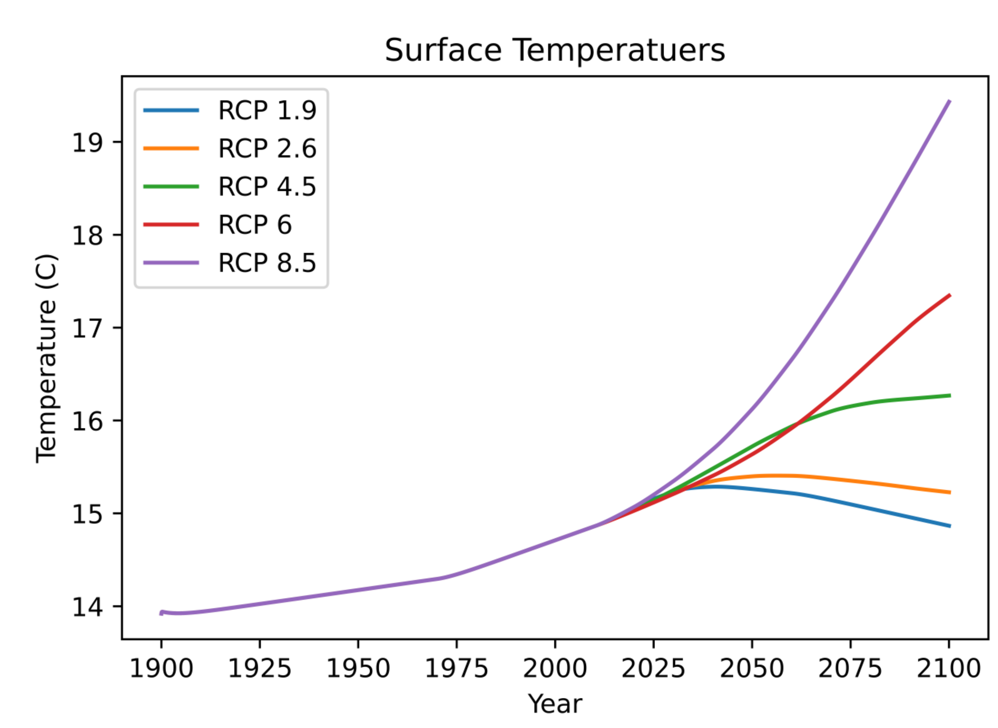
By relating emissivity/absorptivity to CO_2 in the atmosphere we can observe how temperatures change as a result of CO_2 concentrations. To do this, use the function:

$$\varepsilon = 0.3\tanh\left(\frac{c}{1800}\right) + 0.7$$

Where ε is emissivity and c is the concentration of CO_2 in the atmosphere in ppm

Climate Change

Plugging the CO_2 concentrations of the RCPs into this model results in the following plot of temperatures over time:



Compared to official estimates, this model slightly underestimates the impact that small changes in CO2 concentration have on global temperatures and overestimates the impact that large changes have.

Official sources estimate that the RCP 8.5 scenario would lead to 4.3 degrees of warming compared to pre-industrial temperatures by the end of the century. This model shows about 5.5 degrees of warming under that scenario.

The RCP1.9 scenario should lead to about 1.5 degrees of warming; however, this model shows about 1 degree of warming.

Given the simplicity of this model, however, this is a surprising degree of accuracy.