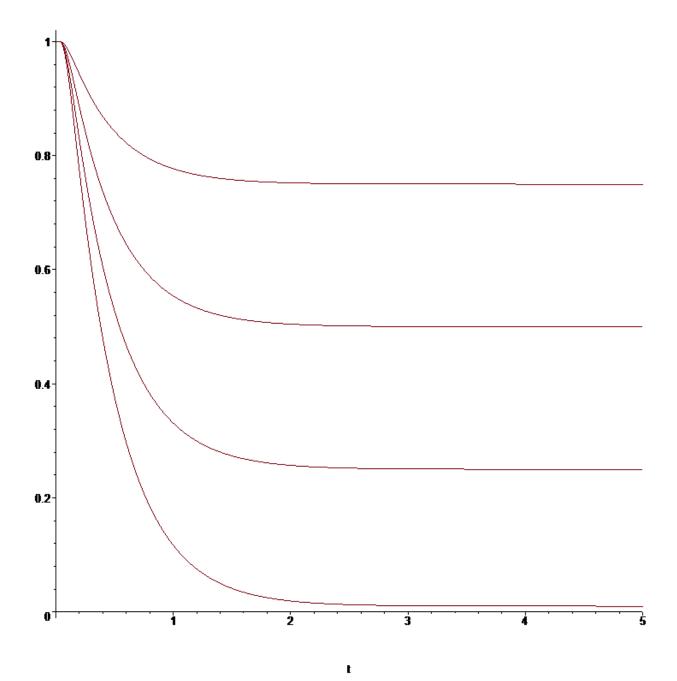
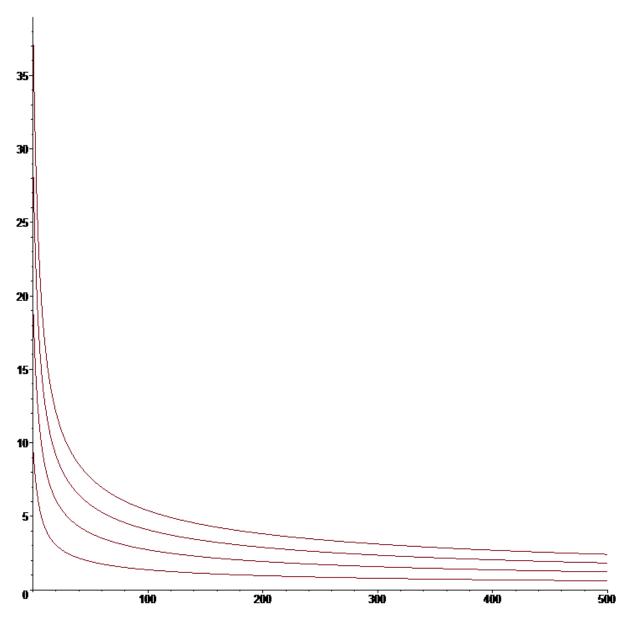
```
Neal Dawson-Elli
                                                                                   HW3
1)
> restart:
eq1:=diff(u(x,t),t)=diff(diff(u(x,t),x),x);
                               eq1 := \frac{\partial}{\partial t} \mathbf{u}(x, t) = \frac{\partial^2}{\partial r^2} \mathbf{u}(x, t)
> N := 20;
y := (2*n-1)/2*Pi;
An:=int((1-Ceq)*cos(y*x),x=0..1)/int(cos(y*x)^2,x=0..1);
C:=Ceq+sum(An*cos(y*x)*exp(-y^2*t),n=1..N):
dC:=diff(C,x):
                                     y \coloneqq \frac{(2n-1)\pi}{2}
                           An := \frac{4 \cos(\pi \, n) \, (-1 + Ceq)}{2 \, \pi \, n - 2 \sin(\pi \, n) \cos(\pi \, n) - \pi}
>
> Pi;
                                            \pi
>c 1:=subs(x=0.,C):
>################## varying Cea vs concentration
#################
with (plots):
>p1:=plot(subs(Ceq=0.01,c 1),t=0..5):
p2:=plot(subs(Ceq=0.25,c 1),t=0..5):
p3:=plot(subs(Ceq=0.50,c 1),t=0..5):
p4:=plot(subs(Ceq=0.75,c 1),t=0..5):
```

display(p1,p2,p3,p4);

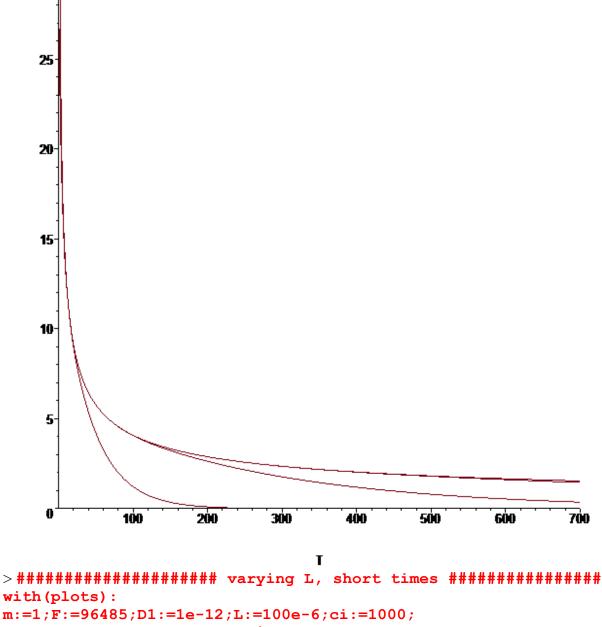


```
F := 96485
D1 := 0.1 \ 10^{-11}
L := 0.000100
ci := 1000
```



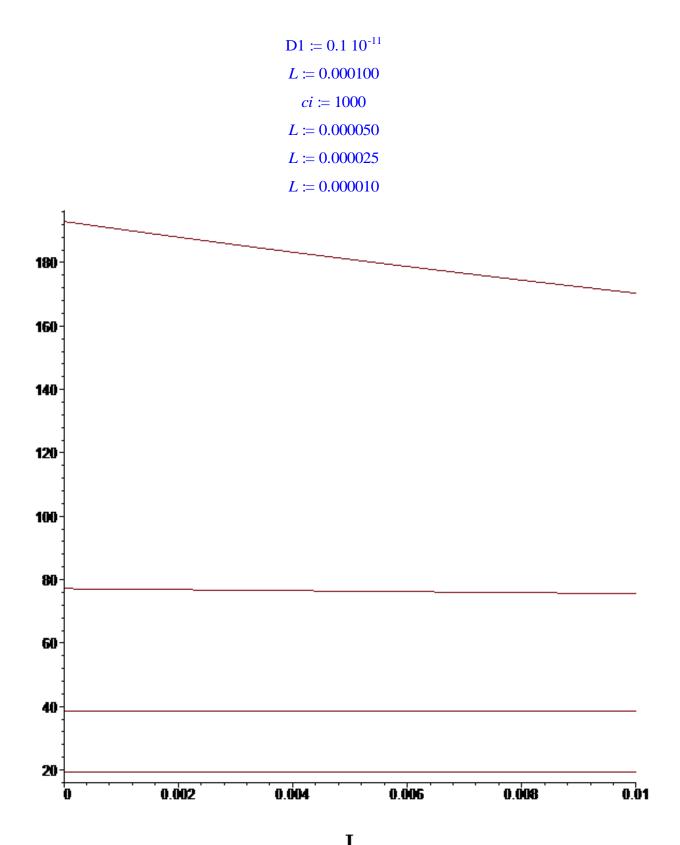
```
>############# long times, varying L ############
with(plots):
m:=1;F:=96485;D1:=1e-12;L:=100e-6;ci:=1000;
ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p21:=plot(subs([Ceq=0.25,t=D1/L^2*T],ibatt),T=0..700):
L:=50e-6;ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
```

```
 \begin{array}{l} \texttt{p22:=plot(subs([Ceq=0.25, t=D1/L^2*T], ibatt), T=0..700):} \\ \texttt{L:=25e-6; ibatt:=subs(x=1, -m*F*D1*ci/L*dC):} \\ \texttt{p23:=plot(subs([Ceq=0.25, t=D1/L^2*T], ibatt), T=0..700):} \\ \texttt{L:=10e-6; ibatt:=subs(x=1, -m*F*D1*ci/L*dC):} \\ \texttt{p24:=plot(subs([Ceq=0.25, t=D1/L^2*T], ibatt), T=0..700):} \\ \texttt{display(p21,p22,p23,p24);} \\ \\ m:=1 \\ F:=96485 \\ \texttt{D1}:=0.110^{-11} \\ L:=0.000100 \\ ci:=1000 \\ L:=0.000050 \\ L:=0.000050 \\ L:=0.000025 \\ L:=0.000010 \\ \end{array}
```

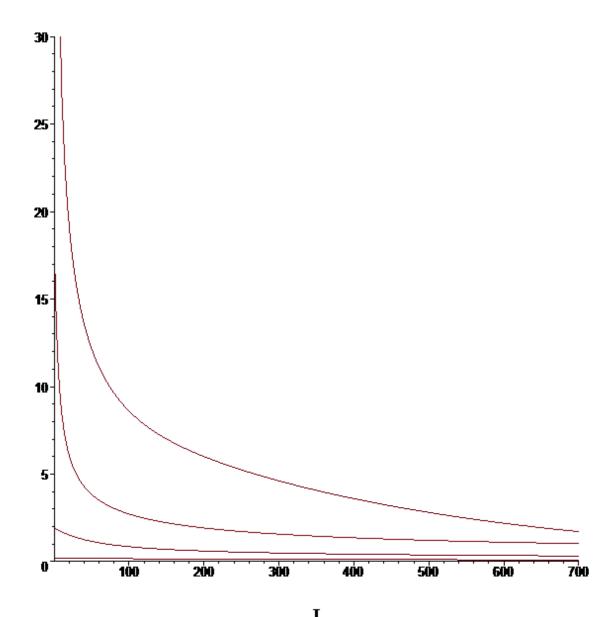


```
with(plots):
m:=1;F:=96485;D1:=1e-12;L:=100e-6;ci:=1000;
ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p21:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..0.01):
L:=50e-6;ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p22:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..0.01):
L:=25e-6;ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p23:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..0.01):
L:=10e-6;ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p24:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..0.01):
display(p21,p22,p23,p24);
m:=1
```

F := 96485



```
m:=1;F:=96485;D1:=1e-12;L:=100e-6;ci:=1000;
ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p41:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..700):
D1:=1e-11;ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p42:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..700):
D1:=1e-13; ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p43:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..700):
D1:=1e-14; ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p44:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..700):
display(p41,p42,p43,p44);
                                 m := 1
                                F = 96485
                              D1 := 0.1 \ 10^{-11}
                              L := 0.000100
                                ci := 1000
                              D1 := 0.1 \ 10^{-10}
                              D1 := 0.1 \ 10^{-12}
                              D1 := 0.1 \ 10^{-13}
```



F = 96485

```
D1 := 0.1 \ 10^{-11}

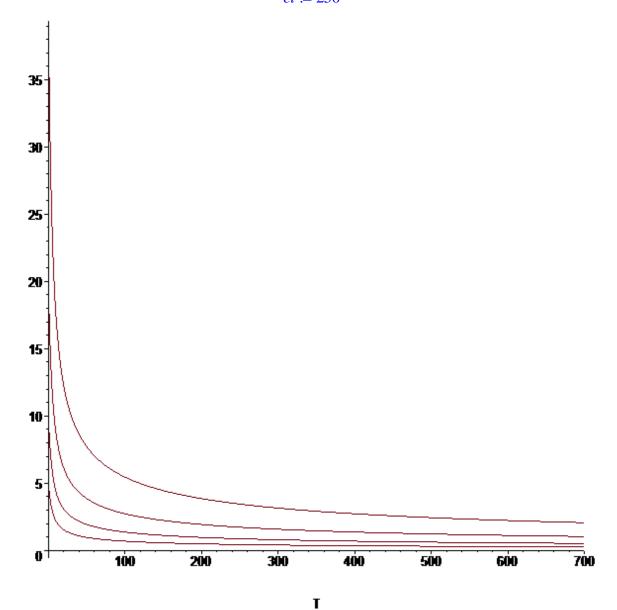
L := 0.000100

ci := 1000

ci := 2000

ci := 500

ci := 250
```

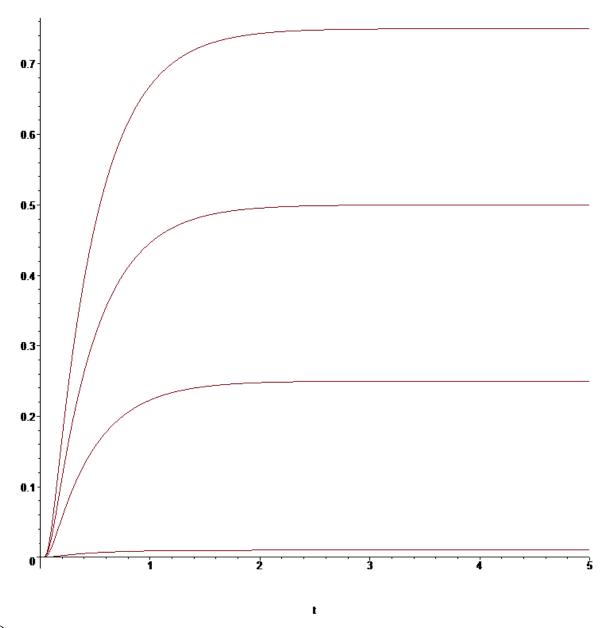


```
> SOC:=int(C,x=0..1):
p1:=plot(subs(Ceq=0.01,SOC),t=0..1):
p2:=plot(subs(Ceq=0.25,SOC),t=0..1):
p3:=plot(subs(Ceq=0.50,SOC),t=0..1):
p4:=plot(subs(Ceq=0.75,SOC),t=0..1):
```

```
0.8
            0.6
            0.4^{\circ}
            0.2
                             0.2
                                            0.4
                                                          0.6
                                                                         0.8
                                                    t
>
> restart:
eq1:=diff(u(x,t),t)=diff(diff(u(x,t),x),x);

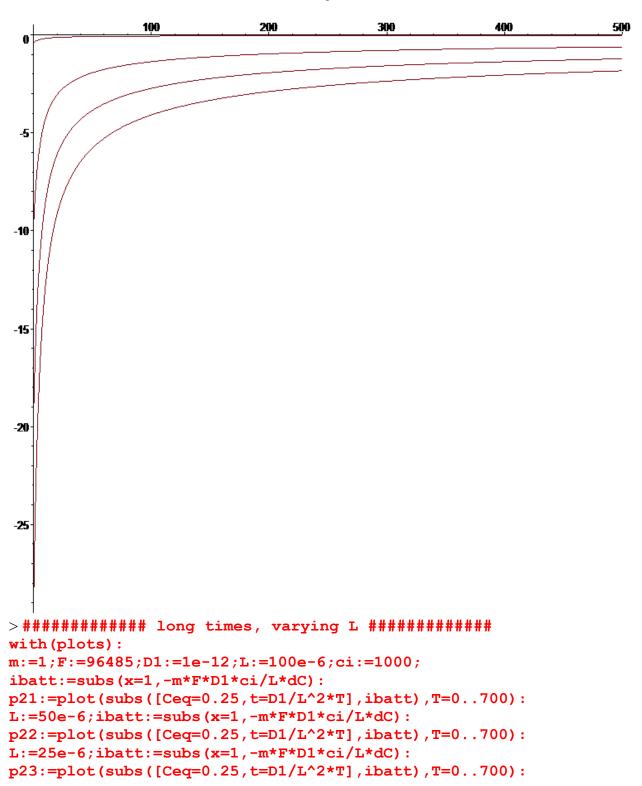
eq1 := \frac{\partial}{\partial t} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t)
> N := 20;
y := (2*n-1)*Pi/2;
An:=int((0-Ceq)*cos(y*X),X=0..1)/int((cos(y*X))^2,X=0..1);
C:=Ceq+sum(An*cos(y*x)*exp(-y^2*t),n=1..N):
dC:=diff(C,x):
                                               N = 20
                                          y \coloneqq \frac{(2 n - 1) \pi}{2}
                               An := \frac{4 \operatorname{Ceq} \cos(\pi n)}{2 \pi n - 2 \sin(\pi n) \cos(\pi n) - \pi}
```

display(p1,p2,p3,p4);



$L \coloneqq 0.000100$ $ci \coloneqq 1000$

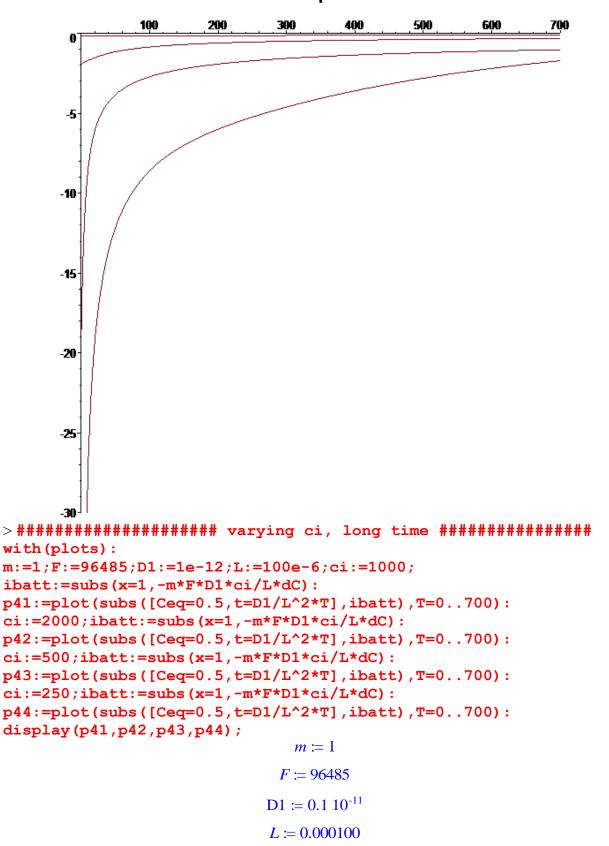
T



```
L:=10e-6; ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p24:=plot(subs([Ceq=0.25,t=D1/L^2*T],ibatt),T=0..700):
display(p21,p22,p23,p24);
                             m := 1
                            F := 96485
                           D1 := 0.1 \ 10^{-11}
                           L := 0.000100
                            ci := 1000
                           L := 0.000050
                           L := 0.000025
                           L := 0.000010
                                T
                100
                      200
                            300
                                  400
                                               600
                                        500
         -2
with (plots):
m:=1;F:=96485;D1:=1e-12;L:=100e-6;ci:=1000;
ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p21:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..0.01):
L:=50e-6; ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
```

```
p22:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..0.01):
L:=25e-6; ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p23:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..0.01):
L:=10e-6; ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p24:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..0.01):
display(p21,p22,p23,p24);
                                    m := 1
                                  F := 96485
                                 D1 := 0.1 \ 10^{-11}
                                 L := 0.000100
                                   ci = 1000
                                 L := 0.000050
                                 L := 0.000025
                                 L := 0.000010
           -20
           -40
           -60
           -80
          -100
          -120
          -140
          -160
          -180
                      0.002
                                 0.004
                                           0.006
                                                      0.008
                                                                 0.01
```

```
m:=1;F:=96485;D1:=1e-12;L:=100e-6;ci:=1000;
ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p41:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..700):
D1:=1e-11;ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p42:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..700):
D1:=1e-13; ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p43:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..700):
D1:=1e-14; ibatt:=subs(x=1,-m*F*D1*ci/L*dC):
p44:=plot(subs([Ceq=0.5,t=D1/L^2*T],ibatt),T=0..700):
display(p41,p42,p43,p44);
                                 m := 1
                                F = 96485
                              D1 := 0.1 \ 10^{-11}
                              L := 0.000100
                                ci := 1000
                              D1 := 0.1 \ 10^{-10}
                              D1 := 0.1 \ 10^{-12}
                              D1 := 0.1 \ 10^{-13}
```



```
ci := 1000

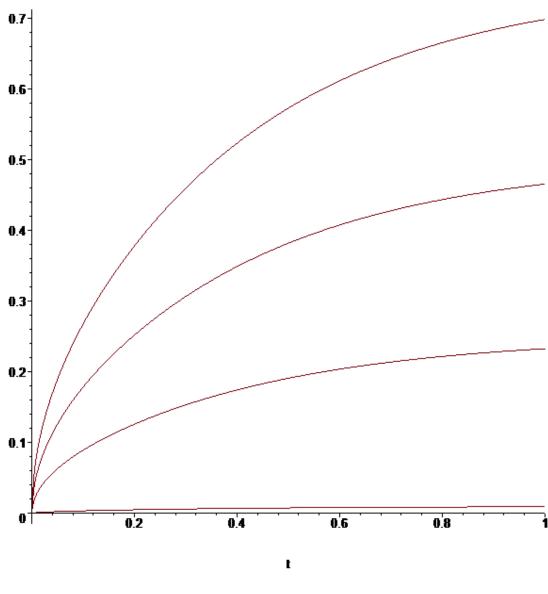
ci := 2000

ci := 500

ci := 250
```

T

```
100
                        200
                                300
                                        400
                                                500
                                                        600
                                                                700
        -5
       -10
       -15
       -20
       -25
       -30
       -35
> SOC:=int(C,x=0..1):
p1:=plot(subs(Ceq=0.01,SOC),t=0..1):
p2:=plot(subs(Ceq=0.25,SOC),t=0..1):
p3:=plot(subs(Ceq=0.50,SOC),t=0..1):
p4:=plot(subs(Ceq=0.75,SOC),t=0..1):
display(p1,p2,p3,p4);
```



> for i from 1 to N do eq[i]:=diff(u[i](t),t)=(u[i-1](t)-2*u[i](t)+u[i+1](t))/h^2;od;
$$eq_1 \coloneqq \frac{d}{dt} u_1(t) = 121 u_0(t) - 242 u_1(t) + 121 u_2(t)$$

$$eq_2 \coloneqq \frac{d}{dt} u_2(t) = 121 u_1(t) - 242 u_2(t) + 121 u_3(t)$$

$$eq_3 \coloneqq \frac{d}{dt} u_3(t) = 121 u_2(t) - 242 u_3(t) + 121 u_4(t)$$

$$eq_4 \coloneqq \frac{d}{dt} u_4(t) = 121 u_3(t) - 242 u_4(t) + 121 u_5(t)$$

$$eq_5 \coloneqq \frac{d}{dt} u_5(t) = 121 u_4(t) - 242 u_5(t) + 121 u_6(t)$$

$$eq_6 \coloneqq \frac{d}{dt} u_6(t) = 121 u_5(t) - 242 u_6(t) + 121 u_7(t)$$

$$eq_7 \coloneqq \frac{d}{dt} u_7(t) = 121 u_6(t) - 242 u_7(t) + 121 u_8(t)$$

$$eq_8 \coloneqq \frac{d}{dt} u_8(t) = 121 u_7(t) - 242 u_8(t) + 121 u_9(t)$$

$$eq_9 \coloneqq \frac{d}{dt} u_0(t) = 121 u_8(t) - 242 u_9(t) + 121 u_{10}(t)$$

$$eq_{10} \coloneqq \frac{d}{dt} u_{10}(t) = 121 u_9(t) - 242 u_{10}(t) + 121 u_{11}(t)$$

$$\ge eq_{11} \coloneqq u_{11}(t) = 0$$

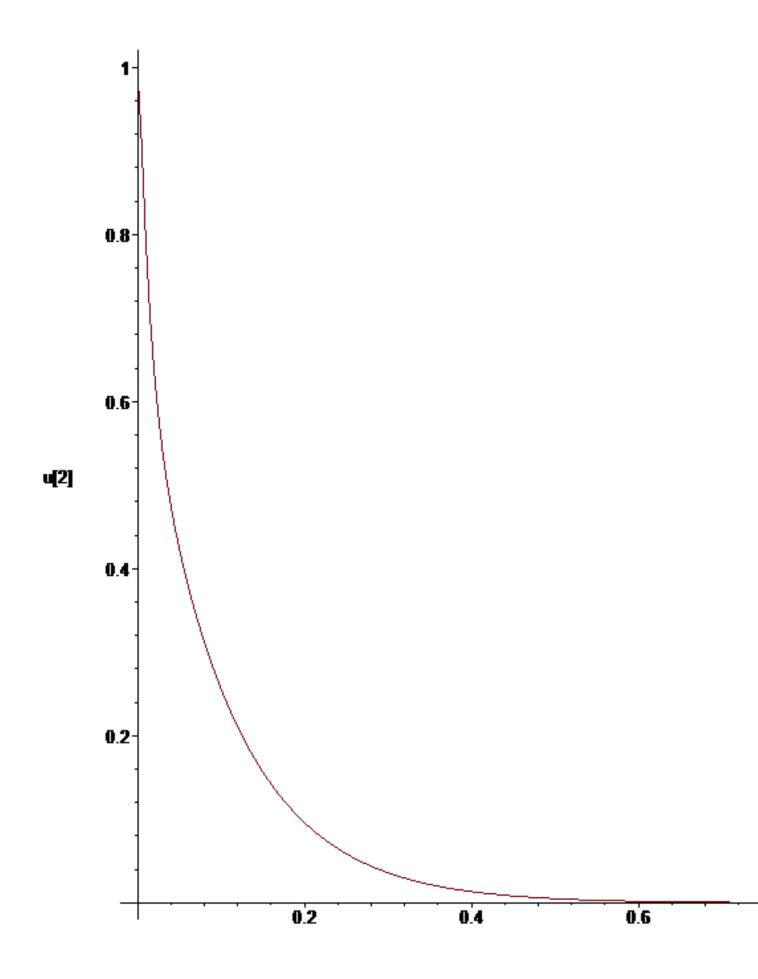
$$\ge ics := seq(u[i](0) = 1, u_1(0) = 1, u_2(0) = 1, u_3(0) = 1, u_4(0) = 1, u_5(0) = 1, u_6(0) = 1, u_7(0) = 1, u_8(0) = 1, u_9(0) = 1, u_{10}(0) = 1, u_{11}(0) = 1$$

$$\ge Eqs := seq(eq[i], i=0 ...N+1);$$

$$Eqs := seq(eq[i],$$

 $\frac{d}{dt}u_4(t) = 121 u_3(t) - 242 u_4(t) + 121 u_5(t),$

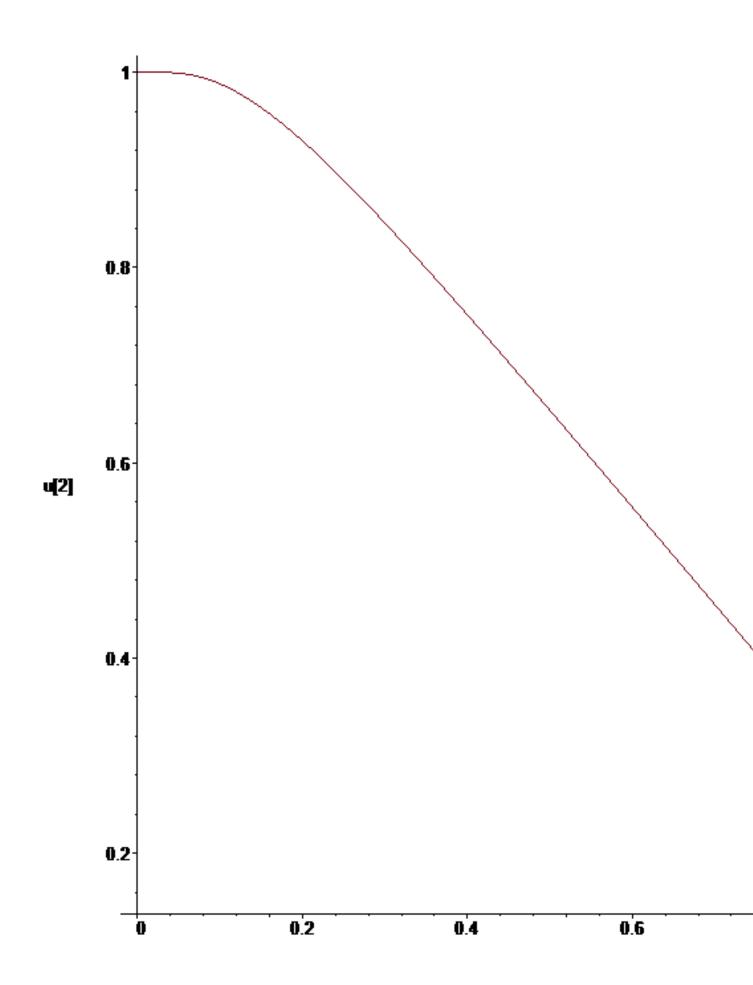
```
\frac{d}{dt}u_5(t) = 121 u_4(t) - 242 u_5(t) + 121 u_6(t),
     \frac{d}{dt}u_6(t) = 121 u_5(t) - 242 u_6(t) + 121 u_7(t),
     \frac{d}{dt}u_{\gamma}(t) = 121 u_{6}(t) - 242 u_{\gamma}(t) + 121 u_{8}(t),
     \frac{d}{dt}u_8(t) = 121 u_7(t) - 242 u_8(t) + 121 u_9(t),
     \frac{d}{dt}u_9(t) = 121 u_8(t) - 242 u_9(t) + 121 u_{10}(t),
     \frac{d}{dt}u_{10}(t) = 121 \ u_9(t) - 242 \ u_{10}(t) + 121 \ u_{11}(t), u_{11}(t) = 0
>infolevel[all]:=0;
                                             infolevel_{all} := 0
> sol:=dsolve({Eqs,ics},type=numeric,stiff=true):
>infolevel[all]:=0;
sol(1);
                                             infolevel_{all} := 0
[t = 1., u_0(t) = 0., u_1(t) = 0.0000197128030461880, u_2(t) = 0.0000378285920261276,
     u_{\scriptscriptstyle 3}(t) = 0.0000528797334554097, \, u_{\scriptscriptstyle 4}(t) = 0.0000636468733680186, \,
     u_5(t) = 0.0000692577221228813, u_6(t) = 0.0000692577221228813,
     u_7(t) = 0.0000636468733680186, \, u_8(t) = 0.0000528797334554097, \,
     u_{0}(t) = 0.0000378285920261276, u_{10}(t) = 0.0000197128030461880, u_{11}(t) = 0.
> with (plots):
> odeplot(sol,[t,u[2](t)],0..1);
```



```
> restart:
Digits:=15;
N := 10;
h:=1/N;
eq[0] := u[1](t) - u[0](t) = 0;
for i from 1 to N do eq[i]:=diff(u[i](t),t)=(u[i-1](t)-
2*u[i](t)+u[i+1](t))/h^2;od;
eq[N+1] := (u[N+1](t) - u[N](t))/h = -1;
ics:=seq(u[i](0)=1,i=0..N+1);
Eqs:=seq(eq[i], i=0..N+1);
sol:=dsolve({Eqs,ics},type=numeric,stiff=true):
with (plots):
odeplot(sol,[t,u[2](t)],0..1);
                                            Digits = 15
                                              N = 10
                                               h := \frac{1}{10}
                                      eq_0 := u_1(t) - u_0(t) = 0
                        eq_1 := \frac{d}{dt}u_1(t) = 100 u_0(t) - 200 u_1(t) + 100 u_2(t)
                        eq_2 := \frac{d}{dt} u_2(t) = 100 u_1(t) - 200 u_2(t) + 100 u_3(t)
                        eq_3 := \frac{d}{dt}u_3(t) = 100 u_2(t) - 200 u_3(t) + 100 u_4(t)
                        eq_4 := \frac{d}{dt} u_4(t) = 100 u_3(t) - 200 u_4(t) + 100 u_5(t)
                        eq_5 := \frac{d}{dt}u_5(t) = 100 u_4(t) - 200 u_5(t) + 100 u_6(t)
                        eq_6 := \frac{d}{dt}u_6(t) = 100 u_5(t) - 200 u_6(t) + 100 u_7(t)
                        eq_7 := \frac{d}{dt} u_7(t) = 100 u_6(t) - 200 u_7(t) + 100 u_8(t)
                        eq_8 := \frac{d}{dt} u_8(t) = 100 u_7(t) - 200 u_8(t) + 100 u_9(t)
                       eq_9 := \frac{d}{dt}u_9(t) = 100 u_8(t) - 200 u_9(t) + 100 u_{10}(t)
                      eq_{10} := \frac{d}{dt}u_{10}(t) = 100 \ u_9(t) - 200 \ u_{10}(t) + 100 \ u_{11}(t)
```

$$\begin{split} eq_{11} &\coloneqq 10\,u_{11}(t) - 10\,u_{10}(t) = -1 \\ ics &\coloneqq u_0(0) = 1,\,u_1(0) = 1,\,u_2(0) = 1,\,u_3(0) = 1,\,u_4(0) = 1,\,u_5(0) = 1,\,u_6(0) = 1, \\ u_7(0) &= 1,\,u_8(0) = 1,\,u_9(0) = 1,\,u_{10}(0) = 1,\,u_{11}(0) = 1 \end{split}$$

$$Eqs &\coloneqq u_1(t) - u_0(t) = 0,\,\frac{d}{dt}\,u_1(t) = 100\,u_0(t) - 200\,u_1(t) + 100\,u_2(t), \\ \frac{d}{dt}\,u_2(t) &= 100\,u_1(t) - 200\,u_2(t) + 100\,u_3(t), \\ \frac{d}{dt}\,u_3(t) &= 100\,u_2(t) - 200\,u_3(t) + 100\,u_4(t), \\ \frac{d}{dt}\,u_4(t) &= 100\,u_3(t) - 200\,u_4(t) + 100\,u_5(t), \\ \frac{d}{dt}\,u_5(t) &= 100\,u_4(t) - 200\,u_5(t) + 100\,u_6(t), \\ \frac{d}{dt}\,u_6(t) &= 100\,u_5(t) - 200\,u_6(t) + 100\,u_7(t), \\ \frac{d}{dt}\,u_7(t) &= 100\,u_6(t) - 200\,u_7(t) + 100\,u_8(t), \\ \frac{d}{dt}\,u_8(t) &= 100\,u_7(t) - 200\,u_8(t) + 100\,u_9(t), \\ \frac{d}{dt}\,u_9(t) &= 100\,u_8(t) - 200\,u_9(t) + 100\,u_{10}(t), \\ \frac{d}{dt}\,u_{10}(t) &= 100\,u_9(t) - 200\,u_{10}(t) + 100\,u_{11}(t), 10\,u_{11}(t) - 10\,u_{10}(t) = -1 \end{split}$$



```
> restart:
Digits:=15;
N := 10;
h:=1/N;
eq[0] := u[1](t) - u[0](t) = 0; # u0
for i from 1 to N do eq[i]:=diff(u[i](t),t)=(u[i-1](t)-
2*u[i](t)+u[i+1](t))/h^2+2/(i*h)*(u[i+1](t)-u[i-1](t))/(2*h)-1
u[i](t)^2;od; #eqn
eq[N+1] := (u[N+1](t)-u[N](t))/h=-1; #uN+1
ics:=seq(u[i](0)=1,i=0..N+1);
Eqs:=seq(eq[i], i=0..N+1);
sol:=dsolve({Eqs,ics},type=numeric,stiff=true):
with (plots):
odeplot(sol,[t,u[2](t)],0..1);
                                             Digits := 15
                                                N = 10
                                                h := \frac{1}{10}
                                       eq_0 := u_1(t) - u_0(t) = 0
                          eq_1 := \frac{d}{dt}u_1(t) = -200 u_1(t) + 200 u_2(t) - u_1(t)^2
                    eq_2 := \frac{d}{dt}u_2(t) = 50 u_1(t) - 200 u_2(t) + 150 u_3(t) - u_2(t)^2
                    eq_3 := \frac{d}{dt}u_3(t) = \frac{200}{3}u_2(t) - 200u_3(t) + \frac{400}{3}u_4(t) - u_3(t)^2
                    eq_4 := \frac{d}{dt}u_4(t) = 75 u_3(t) - 200 u_4(t) + 125 u_5(t) - u_4(t)^2
                    eq_5 := \frac{d}{dt} u_5(t) = 80 u_4(t) - 200 u_5(t) + 120 u_6(t) - u_5(t)^2
                    eq_6 := \frac{d}{dt}u_6(t) = \frac{250}{3}u_5(t) - 200u_6(t) + \frac{350}{3}u_7(t) - u_6(t)^2
                    eq_7 := \frac{d}{dt}u_7(t) = \frac{600}{7}u_6(t) - 200u_7(t) + \frac{800}{7}u_8(t) - u_7(t)^2
                    eq_8 := \frac{d}{dt}u_8(t) = \frac{175}{2}u_7(t) - 200u_8(t) + \frac{225}{2}u_9(t) - u_8(t)^2
                   eq_9 := \frac{d}{dt}u_9(t) = \frac{800}{9}u_8(t) - 200u_9(t) + \frac{1000}{9}u_{10}(t) - u_9(t)^2
```

$$\begin{split} eq_{10} \coloneqq \frac{d}{dt} u_{10}(t) &= 90 \ u_9(t) - 200 \ u_{10}(t) + 110 \ u_{11}(t) - u_{10}(t)^2 \\ eq_{11} &\coloneqq 10 \ u_{11}(t) - 10 \ u_{10}(t) = -1 \\ ics &\coloneqq u_0(0) = 1, \ u_1(0) = 1, \ u_2(0) = 1, \ u_3(0) = 1, \ u_4(0) = 1, \ u_5(0) = 1, \ u_6(0) = 1, \\ u_7(0) &= 1, \ u_8(0) = 1, \ u_9(0) = 1, \ u_{10}(0) = 1, \ u_{11}(0) = 1 \end{split}$$

$$Eqs \coloneqq u_1(t) - u_0(t) = 0, \frac{d}{dt} u_1(t) = -200 \ u_1(t) + 200 \ u_2(t) - u_1(t)^2, \\ \frac{d}{dt} u_2(t) &= 50 \ u_1(t) - 200 \ u_2(t) + 150 \ u_3(t) - u_2(t)^2, \\ \frac{d}{dt} u_3(t) &= \frac{200}{3} \ u_2(t) - 200 \ u_3(t) + \frac{400}{3} \ u_4(t) - u_3(t)^2, \\ \frac{d}{dt} u_4(t) &= 75 \ u_3(t) - 200 \ u_4(t) + 125 \ u_5(t) - u_4(t)^2, \\ \frac{d}{dt} u_5(t) &= 80 \ u_4(t) - 200 \ u_5(t) + 120 \ u_6(t) - u_5(t)^2, \\ \frac{d}{dt} u_6(t) &= \frac{250}{3} \ u_5(t) - 200 \ u_6(t) + \frac{350}{3} \ u_7(t) - u_6(t)^2, \\ \frac{d}{dt} u_7(t) &= \frac{600}{7} \ u_6(t) - 200 \ u_7(t) + \frac{800}{7} \ u_8(t) - u_7(t)^2, \\ \frac{d}{dt} u_8(t) &= \frac{175}{2} \ u_7(t) - 200 \ u_8(t) + \frac{225}{2} \ u_9(t) - u_8(t)^2, \\ \frac{d}{dt} u_9(t) &= \frac{800}{9} \ u_8(t) - 200 \ u_9(t) + \frac{1000}{9} \ u_{10}(t) - u_9(t)^2, \\ \frac{d}{dt} u_1(t) &= 90 \ u_9(t) - 200 \ u_{10}(t) + 110 \ u_{11}(t) - u_{10}(t)^2, 10 \ u_{11}(t) - 10 \ u_{10}(t) = -1 \end{split}$$

