Mech-3000
Single-Stage Gearbox Project
Fall 2020

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Professor Duva,

Enclosed is our report regarding the "Single-Stage Gearbox" with specifications for team 3. The report documents our process of calculating the necessary parameters for selection of components necessary to construct a gearbox that will meet the given design requirements. We have completed our excel calculator and used the data it generated to research parts across various manufacturers to be used for the design.

Our team assumed that gears would be symmetrical with a pitch angle of 20 degrees and would require hubs. We also began the project assuming that the gears' baseline bore would be required to fit our calculated minimum diameter based on our static analysis. This led to the formulation of a gearbox with an unnecessarily large but nonetheless functional shaft.

The FEA tests of the shaft with the keyseat recommended by Boston Gear indicate that the shaft's critical speed far exceeds the maximum speeds it will endure under the design conditions. All other selected components exceed the required loading and fatigue requirements and therefore will conduct the intended procedures without failure.

We look forward to your review of the report.

Sincerely,
Nicolas Deguglielmo, Collin Killiany, Ricky Amin

Reduction Gear Box

Design of Machine Elements Final Project

Professor Anthony Duva– Fall 2020

WENTWORTH

INSTITUTE OF TECHNOLOGY

Prepared by

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Abstract:

Through the completion of the MECH3000 Major Design project, a single stage gearbox was developed to the following specifications: an input speed of 1800 rpm, three horsepower, a gear ratio of 7.2. and the gearbox must be designed to withstand torque fluctuations of up to 5%. Based on the calculated minimum gear teeth to prevent interference and the recommendation of Boston Gear's catalog, a twenty teeth pinion with a diametral pitch of twelve teeth per inch were selected. A driven gear with 144 teeth with the same diametral pitch and pressure angle was chosen to supplement the pinion gear. Using the calculated pitch radii of these gears and the required input speed and power, we designed input and output shafts out of AISI 1020 CR steel to withstand the forces generated by the gears. With the shaft diameter selected, a set of bearings could be selected based on their inner diameter and loading parameters. A key could also be selected, allowing the team to conduct FEA analysis on its key seat.

Contents:

Introduction:

Our team was assigned the set of specifications for which we must design a gearbox. From these specs, preliminary parameters must be calculated to be used in research and selection of parts sold by manufacturers. Most of the governing equations used to calculate these parameters are taken from Shigley's Mechanical Engineering Design textbook. These equations will be programed into an excel calculator to allow dimensional changes in this iterative design process. The calculations begin with the equation for the minimum pinion gear count required to prevent interference [1]. When the closest combination of whole-number teeth above the calculated minimum values is found, component research began in manufacturing catalogs to determine the required diametral pitch and face width of the gear and pinion. Boston Gear's catalog tabulates the maximum power and torque that can be experienced by a gear of a given tooth count and speed [2]. The material strength of these gears must exceed a safe stress in order to prevent tooth breakage. This stress can be calculated by solving the Lewis equation for safe stress. So long as the gears sold by the manufacturer are made of a material of greater shear strength than the safe stress, the gear will not fail.

The shaft will require a length that supports the gear's face width as well as their opposing hubs. The gearbox will be symmetric, in which the gears will be aligned at the center of the shaft. As shown in figures 15-16, the hubs will interface with the walls of the housing while shaft steps hold the gear's opposite face. When the necessary shaft length is found, the bending moment must be calculated for the purpose of finding the minimum shaft diameter. The shaft diameter is calculated first through static analysis followed by fatigue analysis. The design specifications present the goal to conserve material; therefore, the distortion energy theory must be employed to calculate the fatigue shaft diameter. With a minimum shaft diameter obtained, various accessories to the shaft can be researched and selected.

Bearings on either end of each shaft will be used to allow free rotation of the shaft. They must be able to withstand the static and dynamic loading under the operating conditions and support an inner diameter large enough to fit the final diameter of the shaft. Lastly, a key to bind the gear to the shaft will be selected based on the shaft diameter. The shaft along with the appropriate key seat will be modeled in SolidWorks for FEA analysis to verify that the shaft will not fail under the design rotation speed.

Design Specifications:

The gearbox must be designed for a light-shock input power of 3 hp at 1800 RPM, a load factor of 1.5, and a gear ratio of 7.2 within 1% variation. The torque is expected to fluctuate by 5%. The gearbox will operate for 4000 hrs with a factor of safety of 2 and a fatigue factor of 2.5. Fatigue calculations must be done with the objective of conserving material; therefore, distortion energy theory must be used in this application.

Design and Analysis

Phase 1: Gears

The process began with the calculation of the minimum teeth on the pinion required for the gearbox to operate without interference. A typical pressure angle of 20 degrees and gear factor of 1 was used for this gear.

$$Np = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{(m^2 + (1+2m)\sin^2\phi)}\right)$$

$$Np = \frac{2*1}{(1+2*7.2)\sin^2(20)}\left(7.2 + \sqrt{(7.2^2 + (1+2*7.2)\sin^2(20))}\right) = 16.9\ teeth$$

$$N_p * m_g = N_g$$

$$16.9 * 7.2 = 121.68\ teeth$$

Equation 1- Pinion teeth

The rotational speed of the gear can be found by dividing the speed of the pinion by the gear ratio

$$n_g = \frac{n_p}{m_g} = \frac{1800\ RPM}{7.2} = 250\ RPM$$

Equation 2- Output speed

Boston Gear's catalogs feature gears of 12 diametral pitch and the desired tooth count that can withstand the design power

$$P_{dg} = P_{dp} = 12\frac{teeth}{in}$$

The circular pitch of the pinion and gear can then be calculated.

$$p_{cp} = \frac{\pi}{P_p} = 1.886\ in$$

Equation 3- Circular pitch pinion

$$p_{cg} = \frac{\pi}{P_{dg}} = .26in$$

Equation 4- Circular pitch gear

The nearest whole number pinion available is:

$$N_p = 17\ teeth$$

This would theoretically require a gear of the following tooth count:

$$N_g = N_p * m = 122.4\ teeth$$

To propose whole number quantities of gear teeth, the pinion's tooth count was increased to 20 teeth resulting in a gear tooth count of 144 teeth. Boston Gear offers cast iron gears of 144 teeth for our diametral pitch of 12. Since the gear ratio was left unchanged, the variation remains at zero percent.

$$N_p = 20\ teeth$$

$$N_g = N_p * m = 144\ teeth$$

% variation: 0%

A pinion of the updated tooth count was selected from a catalog in Boston gears which recommended a face width of 1"

12 DIAMETRAL PITCH STEEL **20° PRESSURE ANGLE** **1" FACE** **REFERENCE PAGE 43.**

No. Teeth	25 RPM		50 RPM		100 RPM		200 RPM		300 RPM		600 RPM		900 RPM		1200 RPM		1800 RPM		3600 RPM	
	H.P.	Torque	H.P.	Torque	H.P.	Torque	H.P.	Torque	H.P.	Torque	H.P.	Torque	H.P.	Torque	H.P.	Torque	H.P.	Torque	H.P.	Torque
12	.08	202	.16	200	.31	196	.60	188	.86	181	1.54	162	2.09	147	2.55	134	3.27	114	4.53	79.4
13	.09	233	.18	230	.36	225	.68	215	.98	206	1.75	183	2.36	165	2.86	150	3.63	127	4.97	87.1
14	.11	265	.21	262	.41	256	.77	244	1.11	233	1.96	206	2.63	184	3.18	167	4.00	140	5.42	94.8
15	.12	297	.23	293	.45	285	.86	271	1.23	259	2.16	227	2.88	202	3.46	182	4.34	152	5.80	102
16	.13	323	.25	319	.49	310	.93	294	1.33	279	2.31	243	3.07	215	3.68	193	4.58	160	6.05	106
18	.15	379	.30	373	.57	361	1.08	340	1.53	322	2.63	276	3.46	242	4.10	215	5.04	177	6.55	115
20	.17	437	.34	429	.66	415	1.23	389	1.74	365	2.95	310	3.84	269	4.52	238	5.50	193	7.02	123
21	.19	468	.36	459	.70	443	1.31	413	1.84	388	3.11	327	4.03	282	4.73	249	5.73	201	7.26	127
24	.22	548	.43	537	.82	515	1.51	477	2.11	444	3.50	368	4.48	314	5.21	274	6.22	218	7.72	135
28	.26	667	.52	651	.99	621	1.80	568	2.49	524	4.04	425	5.10	357	5.86	308	6.89	241		
30	.29	720	.56	707	1.07	673	1.94	612	2.68	562	4.29	451	5.37	376	6.15	323	7.19	252		
36	.37	928	.71	899	1.34	847	2.41	759	3.27	688	5.11	537	6.28	440	7.09	373	8.15	285		
42	.44	1112	.85	1073	1.59	1001	2.81	884	3.77	792	5.73	602	6.94	486	7.76	407				
48	.52	1304	.99	1252	1.84	1159	3.20	1009	4.25	893	6.33	665	7.56	529	8.37	440				
54	.60	1505	1.14	1437	2.09	1319	3.60	1133	4.73	993	6.90	724	8.14	570						

Figure 1(Diametral Pitch Chart)

Diametral Pitch: 12

Face Width: 1”

Addendum: a=1/12”

This information allowed us to calculate the pitch and outer diameters of the pinion and gear.

Pinion:

- Pitch diameter:

$$D_{pitch,p} = \frac{N_p}{P} = \frac{20}{12} = 1.666in$$

- Pitch radius:

$$r_p = \frac{D_{pitch,p}}{2} = .833in$$

- Outside diameter

$$D_{out,p} = 1.666 + \frac{1}{12} = 1\frac{3}{4}\ in$$

Gear:

- Pitch diameter:

$$D_{pitch,g} = \frac{N_p}{P} = \frac{144}{12} = 12in$$

- Pitch radius:

$$r_g = \frac{D_{pitch,g}}{2} = \frac{12in}{2} = 6in$$

- Outside diameter

$$D_{out,g} = 12 + \frac{1}{12} = 12\frac{1}{12}in$$

- Nominal center distance

$$C = r_p + r_g = 6.833in$$

Transmitted torque on both shafts (to be used in shaft design calculations):

- Input shaft

RPM = 1800

$$T_p = 4.5 * \frac{63025}{1800} = 157.5625 \, in - lb$$

- Output shaft

RPM = 250

$$T_g = 4.5 * \frac{63025}{250} = 1134.45 \, in - lb$$

The synonymous tangential force of the gears can be found by dividing the maximum torque of the pinion by its radius.

$$F_t = \frac{T_p}{r_p} = \frac{159}{.833} = 190.8N$$

The tangential force is horizontal component of the resultant load on either shaft separated by the pitch angle, therefore:

$$F_r = \frac{F_t}{\cos(\phi)} = \frac{190.8}{\cos(20)} = 203N$$

The Lewis bending equation is included in Boston Gear's Catalog

METALLIC SPUR GEARS

$$W = \frac{SFY}{P}\left(\frac{600}{600 + V}\right)$$

W = Tooth Load, Lbs. (along the Pitch Line)
S = Safe Material Stress (static) Lbs. per Sq. In. (Table II)
F = Face Width, In.
Y = Tooth Form Factor (Table I)
P = Diametral Pitch
D = Pitch Diameter
V = Pitch Line Velocity, Ft. per Min. = .262 x D x RPM

Figure 2 (Lewis Equation)

The pitch line velocity as described in Figure 2:

$$V = .262 * D * RPM = .262 * 1.666 \, in * 1800 \, rpm = 786 \frac{ft}{min}$$

From the included table we find the tooth form factor of .32 in

TABLE I TOOTH FORM FACTOR (Y)

Number of Teeth	14-1/2° Full Depth Involute	20° Full Depth Involute
10	0.176	0.201
11	0.192	0.226
12	0.210	0.245
13	0.223	0.264
14	0.236	0.276
15	0.245	0.289
16	0.255	0.295
17	0.264	0.302
18	0.270	0.308
19	0.277	0.314
20	0.283	0.320
22	0.292	0.330
24	0.302	0.337
26	0.308	0.344

Figure 3: Tooth form factor chart

The Lewis equation can then be solved for the safe stress of the gears:

$$S_{safe} = \left(\frac{PW}{FY}\right) * \frac{600 + V}{600} = \frac{12" * 188.9lb}{1" * .32} * \frac{600 + 786ft/min}{600} = 16366psi$$

The Boston Gear recommended materials of the gears exceed this safe stress in strength.

Gear

Material: Cast Iron; Bending strength: 40 kpsi

Pinion

Material: Steel; Bending strength: 50.8 kpsi

Preliminary Shaft diameter

Based on the recommended dimensions of Boston Gear’s catalog [2].

Pinion Bore = .75 in

Gear Boar = .875 in

Tolerance:
Boston gears lists a tolerance of .0005 in for all dimensions based on the hub limits.

Key style:
A square key will be used for the design

Phase 2: Bearings

Assuming the bearing loads are symmetrical, the static loading of the bearings are as follows.

$$R_1 = R_2 = \frac{F_r}{2} = \frac{203 lb}{2} = 101.5\ lb$$

Pinion:

R_1=101.5 lb R_2=101.5 lb

Gear:

R_1=101.5 lb R_2=101.5 lb

Pinion Bearing:

The R12 ball bearings available on AST bearings match the bore of the selected pinion and exceed the required static and dynamic load rating [3].

Gear Bearings:

National Precision Bearing offers R18 bearings of a suitable bore for the gear shaft which meet our inner diameter and load requirements [4].

R SERIES BALL BEARINGS | EXTRA LIGHT INCH													
NPB Part No.	Bore			Outside Diameter			Width		Max. Fillet Radius	Balls		Static Load Lbs.	Dynamic Load Lbs.
	Frac.	Inch	Tol.	Frac.	Inch	Tol.	Tol.	+.0000 -.0050		No.	Diam.		
							Open	Seal/Shield					
R2	1/8	.1250	-.0003	3/8	.3750	-.0004	.1562	.1562	.012	7	1/16	49	113
R3	3/16	.1875	-.0003	1/2	.5000	-.0004	.1562	.1960	.012	7	3/32	70	145
R4	1/4	.2500	-.0003	5/8	.6250	-.0004	.1960	.1960	.012	7	1/8	195	385
R4A	1/4	.2500	-.0003	3/4	.7500	-.0004	.2188	.2812	.016	6	9/54	150	320
R6	3/8	.3750	-.0003	7/8	.8750	-.0004	.2188	.2812	.016	7	5/32	305	570
R8	1/2	.5000	-.0003	1 1/8	1.1250	-.0004	.2500	.3125	.016	8	3/16	500	885
R10	5/8	.6250	-.0003	1 3/8	1.3750	-.0005	.2812	.3438	.031	9	3/16	565	970
R12	3/4	.7500	-.0004	1 5/8	1.6250	-.0005	.3125	.4375	.031	10	7/32	850	1265
R14	7/8	.8750	-.0004	1 7/8	1.8750	-.0005	.3750	.5000	.031	10	1/4	1110	1735
R16	1	1.0000	-.0004	2	2.0000	-.0005	.3750	.5000	.031	10	1/4	1110	1735
R18	1 1/8	1.1250	-.0004	2 1/8	2.1250	-.0005	.3750	.5000	.031	11	1/4	1225	1840
R20	1 1/4	1.2500	-.0005	2 1/4	2.2500	-.0005	.3750	.5000	.031	12	1/4	1335	1920

Figure 3 (National Precision Bearing Catalog Table for R Series Ball Bearings)

Pinion Bearings:

a. Static Load
   1177 lb
b. Dynamic Load
   2477 lb
c. Bore and tolerances
   .75 -.0004 in
d. OD and tolerances
   1.625-.0005 in
e. Width and tolerances
   .2812 -.0047 in

Gear Bearings:

a. Static Load
   1225 lb
b. Dynamic Load
   1840 lb
c. Bore and tolerances
   1.125 -.0004 in
d. OD and tolerances
   2.125-.0005 in
e. Width and tolerances
   .375 -.005 in

The Dynamic load factor for the given reliability can be found with the equation below.

$$C_{10} \doteq a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a} \qquad R \geq 0.90$$

Where a_f is the load factor, F_D is the load on the bearing, R_D is the reliability factor, and $a = 3$ for ball bearings. $\theta, b,$ and x_0 are the Weibull parameters recommended by the manufacturer. x_D is the product of the time life of the bearing ($\mathcal{L}_D$) and the rotational speed of the shaft (n_d) divided by 10^6 revolutions of the shaft.

$$x_D = \frac{\mathcal{L}_D n_D 60}{10^6}$$

$$C_{10} \doteq a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a} \qquad R \geq 0.90 \qquad (11\text{–}7)$$

Pinion:

$$x_D = \frac{4000\ hr * 1800\ RPM * 60 \frac{min}{hr}}{10^6\ rotations} = 432$$

$$C_{10} = (1.5)(101.68)\left(\frac{432}{.02+(4.459-.02)(1-.975)^{\frac{1}{1.482}}}\right)\text{^}\left(\frac{1}{3}\right) = 1578.399 \text{ lb}$$

Gear:

$$x_D = \frac{4000\ hr * 250\ RPM * 60 \frac{min}{hr}}{10^6\ rotations} = 60$$

$$C_{10} = (1.5)(101.68)\left(\frac{60}{.02+(4.459-.02)(1-.975)^{\frac{1}{1.482}}}\right)\text{^}\left(\frac{1}{3}\right) = 817.40 \text{ lb}$$

Phase 3: Shaft Design

Assuming a symmetric shaft:

$$R_1 = R_2 = \frac{F_r}{2} = \frac{203lb}{2} = 101.5\ lb$$

Pinion:
R_1=101.5 lb R_2=101.5 lb

Gear:
R_1=101.5 lb R_2=101.5 lb

The bearing-bearing shaft length can be estimated as the sum of the face width, total width of the gear hub projection and a shaft step equal to the hub projection to ensure symmetry.

$$L = 1{+}1 * 2 + .34 = 3.34in$$

Bending moment diagrams for both pinion and gear shafts.

Assuming a mean moment of 0, the moment and torque parameters are:

$$M_a = \frac{F_r}{2}(\frac{l}{2})$$

M_m input= 0 M_a input = 169.7 lb-in

M_m output= 0 M_a output = 169.7 lb-in

Given that the torque will fluctuate at 5%:

$$T_a = Tm * .05$$

T_m input = 157.56 in-lb T_a input 7.878 in-lb

T_m output = 1134 in-lb T_a output 56.7 in-lb

The shaft concentrations can be found on table 7-1

	Bending	Torsional	Axial
Shoulder fillet—sharp ($r/d = 0.02$)	2.7	2.2	3.0
Shoulder fillet—well rounded ($r/d = 0.1$)	1.7	1.5	1.9
End-mill keyseat ($r/d = 0.02$)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove	5.0	3.0	5.0

Figure 4 (Stress Concentration Factor Table)

The stress concentration factors of an end-milled keyseat will be used. For the preliminary shaft calculation, we must assume that the fatigue concentrations will be synonymous with the stress concentration factors.

$$K_t = K_f = 2.14 \quad K_{ts} = K_{fs} = 3$$

Knowing that the shaft material, AISI 1020 steel has a yield strength of 50700 psi [5] and the load factor is said to be 1.5, the minimum shaft diameter is:

$$d = \left(\frac{16 * (n)}{\pi S_y}\left\{4 * \left[K_f(M_m + M_a)\right]^2 + 3[K_{fs}(T_m + T_a)]^2\right\}^{1/2}\right)^{1/3}$$

$$d_{pinion} = 0.609\ in\ \rightarrow 0.75\ in$$

$$d_{pinion} = 1.077\ in\ \rightarrow 1.08\ in$$

The endurance limit of the AISI 1020 CR steel used for the shaft can be calculated with the following piecewise function.

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200\text{ kpsi (1400 MPa)} \\ 100\text{ kpsi} & S_{ut} > 200\text{ kpsi} \\ 700\text{ MPa} & S_{ut} > 1400\text{ MPa} \end{cases}$$

$$S_{ut} = 60.9kpsi < 200\ kpsi$$
$$S_e{}' = .5(60.9) = 30.45\ kpsi$$

A series of factors must be calculated to determine the true endurance stress, beginning with the surface factor:

$$k_a = aS_{ut}^b \tag{6–19}$$

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis*, vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook*, McGraw-Hill, New York. Copyright © 1953 by The McGraw-Hill Companies, Inc. Reprinted by permission.

Figure 5 (Parameters for Marin Surface Modification: Table 6-2)

$$k_{a\ pinion} = .91$$

$$k_{a\ gear} = .91$$

The size factor for the preliminary diameter:

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \tag{6–20}$$

$$k_{b\ pinion} = 0.1421$$

$$k_{b\ gear} = 0.13369$$

For bending stress, equation 6-26 indicates a loading factor of 1.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases} \tag{6–26}$$

$$k_{c\ pinion} = 1$$

$$k_{c\ gear} = 1$$

Assume the temperature of the gearbox will be under 250 C to avoid oil breakdown, k_d=1;

From table 6-5 we find the reliability factor for 99% reliability

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Figure 6 (Reliability Factor Table)

$$k_{e\ pinion} = .814$$
$$k_{e\ gear} = .814$$

1. The modified endurance strength as a function of the diameter becomes:
   $S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S_e'$
$$S_{e\ pinion} = 3.2\ kpsi$$
$$S_{e\ gear} = 3.01\ kpsi$$

   The notch sensitivity can be calculated by the following equation:
$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$
   For the bending stress:
   $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$

   Therefore,
$$q = .536$$

   We can then calculate the design fatigue concentration factors.
$$K_f = 1 + \frac{K_t - 1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = 1.61$$

$$K_{fs} = 1 + \frac{K_{ts} - 1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = 2.07$$

Where K_t = 2.14 and K_{ts}=3 for the end milled keyseat.

The equation for the fatigue factor of safety in terms of the stress concentrations is:

$$n = \frac{\pi d^3}{16}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{-1/2}$$

Reworking the equation, we can conclude that the design value of the diameter must be:

$$d = \left\{\frac{16n}{\pi}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}\right\}^{1/3}$$

$$d_{pinion} = 1.3" \quad d_{gear} = 1.4"$$

These hole diameters can be drilled into the 1.32” and 2.5” hubs of the pinion and gear respectively. To satisfy the fatigue factor of safety of 2.5, the shaft of the pinion and gear must be increased to these diameters. Fortunately, both design diameters fit within the limits of the hub diameter, allowing us to drill the necessary holes without changing the pitch diameter of the gears. This design change does however demand bearings with larger inner diameters. AST offers R22 and R24 bearings with the necessary inner diameters for this updated shaft.

Product	Bearing Type	Bore Dia (d) (in)	Outer Dia (D) (in)	Width (B) (in)	Outer Width (Bo) (in)	Inner Width (Bi) (in)	Flange Dia (Df) (in)	Flange Width (Bf) (in)	Dynamic Load Rating (Cr) (lbs)	Static Load Rating (Cor) (lbs)
R22	Open	1.3750	2.5000	0.4375					4,067	2,247
R24	Open	1.5000	2.6250	0.4375					4,426	2,446

Figure 8: Properties of larger bearings available on AST Bearings.

Both class 1 bearings fall within the same range that which AST bearings recommends the following tolerances:

Table IA

Tolerance Class ABEC-1, RBEC-1
(ISO Class Normal)
Inner Ring

English — Tolerance values in 0.0001 inch

Bore Diameter (d)				Bore Tolerance (Δd_{mp})		Radial Runout (K_{ia})	Width Tolerance (ΔB_{S})		
mm		inch					all	single bearing	paired bearings
over	incl.	over	incl.	high	low	max.	high	low	
0.6	2.5	0.0236	0.0984	0	-3	4	0	-16	-
2.5	10	0.0984	0.3937	0	-3	4	0	-47	-98
10	18	0.3937	0.7087	0	-3	4	0	-47	-98
18	30	0.7087	1.1811	0	-4	5	0	-47	-98
30	50	1.1811	1.9685	0	-4.5	6	0	-47	-98

Figure 7(Tolerance Table for AST Bearings)

$$Bore\ tolerance:\ -4.5*10^{-4}\ in$$
$$Width\ tolerance:\ -48*10^{-4}\ in$$

Phase 4: Key Frequency

The required length of the square key used for each shaft can be calculated with the following equation.

$$L_{key} = \frac{\frac{F_{key}n}{Sy}}{W}$$

Where F_{key} is the tangential force enacted on the key, n is the factor of safety, Sy is the shear yield strength of the material and W is the key width. The force on the key is determined by the torque divided by the radius of the fatigue shaft diameter.

Pinion:

$$F_{key} = \frac{Tmax}{d/2} = 240lb$$

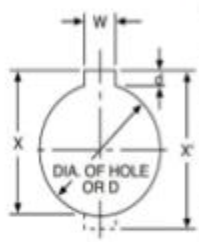
Gear:

$$F_{key} = \frac{Tmax}{d/2} = 1623lb$$

The key width can be estimated based on the recommendations of Boston Gear's catalog.

STANDARD KEYWAYS & SETSCREW

Diam. of Hole	Standard Keyway W	Standard Keyway D	Recom-mended Setscrew
5/16 to 7/16"	3/32"	3/64"	10–32
1/2 to 9/16	1/8	1/16	1/4–20
5/8 to 7/8	3/16	3/32	5/16–18
15/16 to 1-1/4	1/4	1/8	3/8–16
1-5/16 to 1-3/8	5/16	5/32	7/16–14
1-7/16 to 1-3/4	3/8	3/16	1/2–13
1-13/16 to2-1/4	1/2	1/4	9/16–12
2-5/16 to 20-3/4	5/8	5/16	5/8–11
2-13/16 to 3-1/4	3/4	3/8	3/4–10
3-5/16 to 3-3/4	7/8	7/16	7/8–9
3-13/16 to 4-1/2	1	1/2	1–8
4-9/16 to 5-1/2	1-1/4	7/16	1-1/8–7
5-9/16 to 6-1/2	1-1/2	1/2	1-1/4–6



FORMULA:

$X = \sqrt{(D/2)^2 - (W/2)^2} + D + D/2$

$X' = 2X - D$

EXAMPLE:

Hole 1"; Keyway 1/4" wide by 1/8" deep.

$X = \sqrt{(1/2)^2 - (1/8)^2} + 1/8 + 1/2 = 1.109"$

$X' = 2.218 - 1.000 = \mathbf{1.218"}$

Figure 8 (Boston Gear Recommended Key Dimensions)

Both the gear and pinion boars fall within the range for which a 5/16" key width is recommended. Therefore:

Pinion:

$$L_{key} = \frac{\frac{(240lb)(2)}{25100psi}}{.3125in} = .06\ in$$

Gear:

$$L_{key} = \frac{\frac{(1632lb)(2)}{25100psi}}{.3125in} = .4\ in$$

Both of these lengths fit within our hub lengths.

The final steps to complete the analysis on the input and output shafts that have been designed up to this point is to complete a Finite Element Analysis, to confirm the calculations conducted throughout the completion of the project, as well as to confirm that the shafts did not reach their natural frequency through their rotation and fail.

The first two figures below indicate the mesh that was created to implement the Finite Element Analysis, to allow the most accurate results.

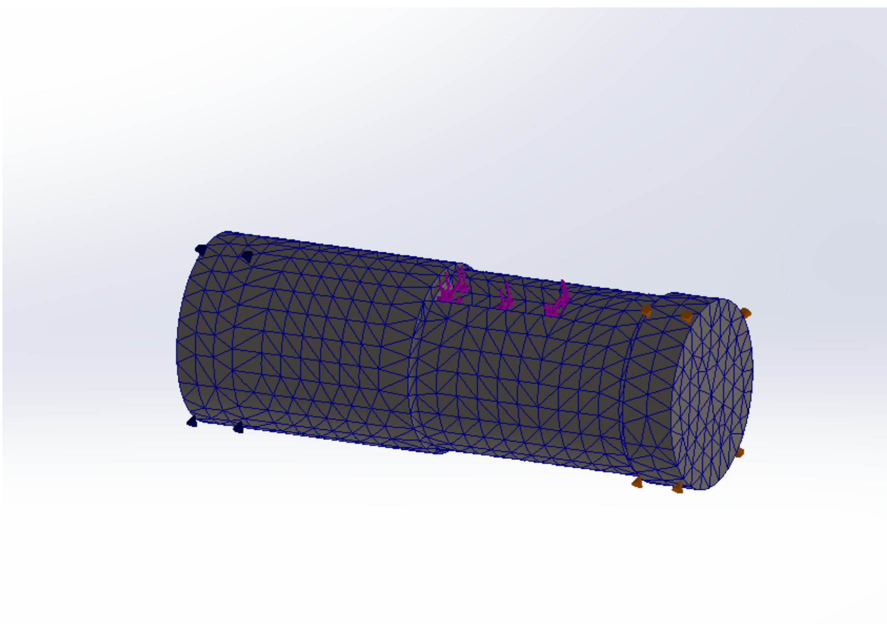


Figure 9 (FEA Mesh for Input Shaft)

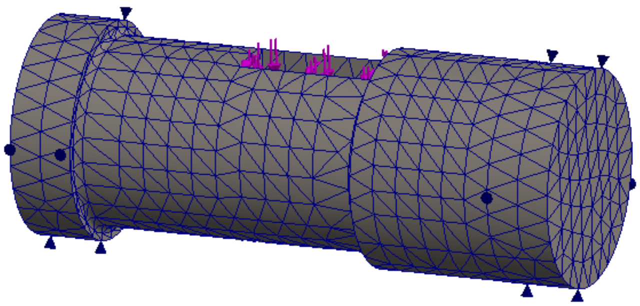


Figure 10 (FEA Mesh for Output Shaft)

Once the mesh for the shafts had been rendered individually, the Finite Element Analysis could be run. From the Finite Element Analysis, it was crucial that we examined the deflection of the shafts to ensure that they were not being pushed past their elastic limit in terms of strain. The following deflection plots are visual representations of the deflection encountered throughout the length of the beam.

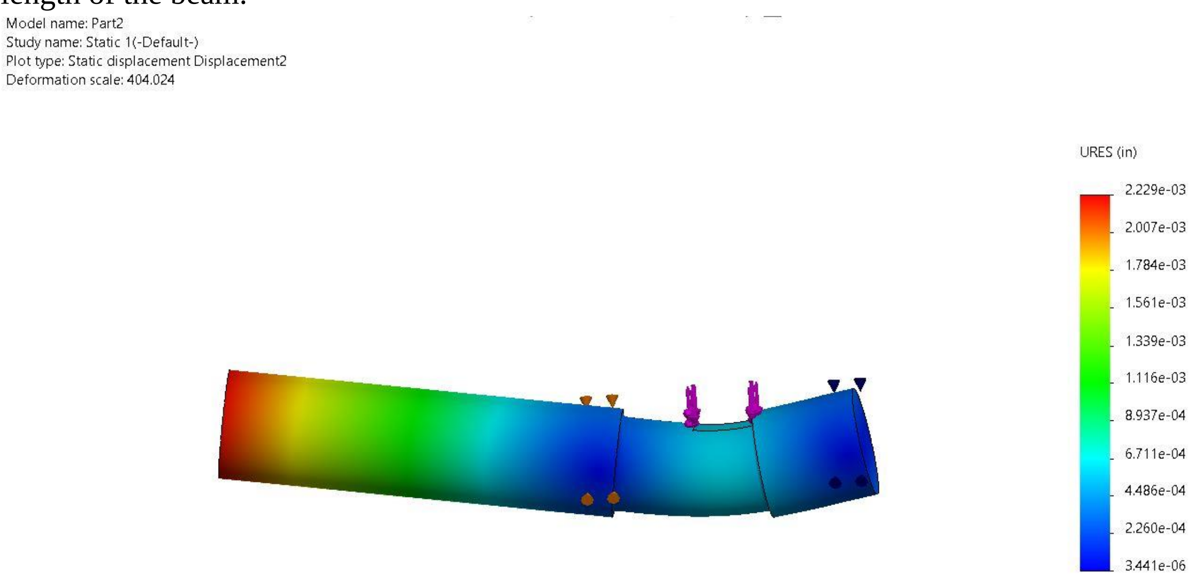


Figure 11 FEA Deflection Plot for Input Shaft

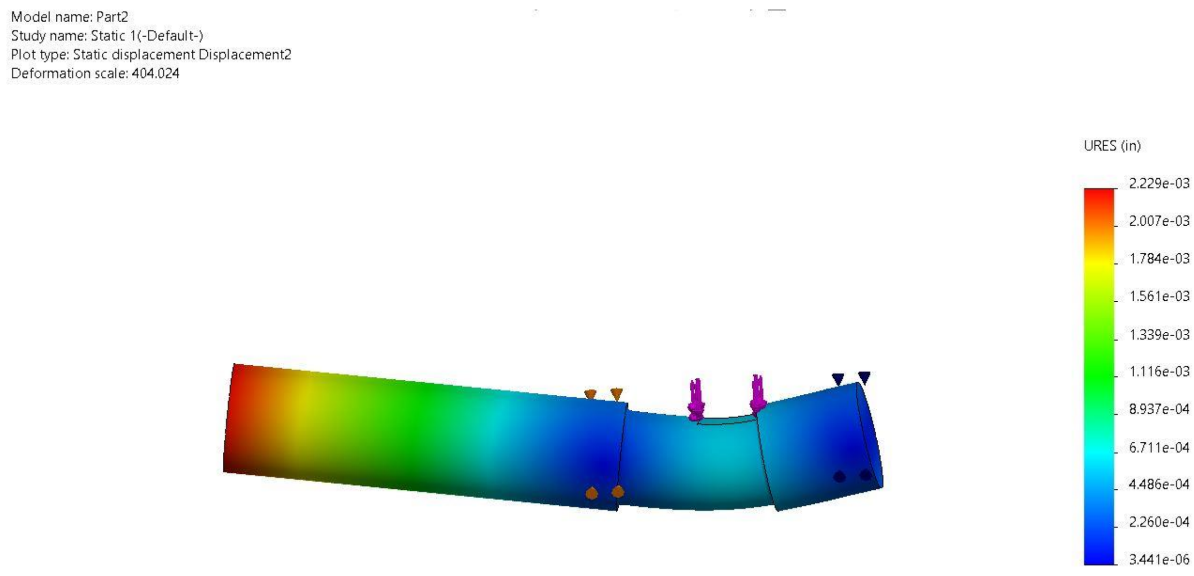


Figure 12 FEA Deflection Plot for Output Shaft

Following the static loading Finite Element Analysis, a frequency Finite Element Analysis was conducted to determine the Critical speeds at which the shaft would fail. This was done by taking the frequency of the rotating shaft in Hz and converting that value in Hz to RPM to determine if our design shaft speed would cause the shaft to spin at its natural frequency and fail. The following two plot indicate the frequency of the shaft vs the mode number of the shaft.

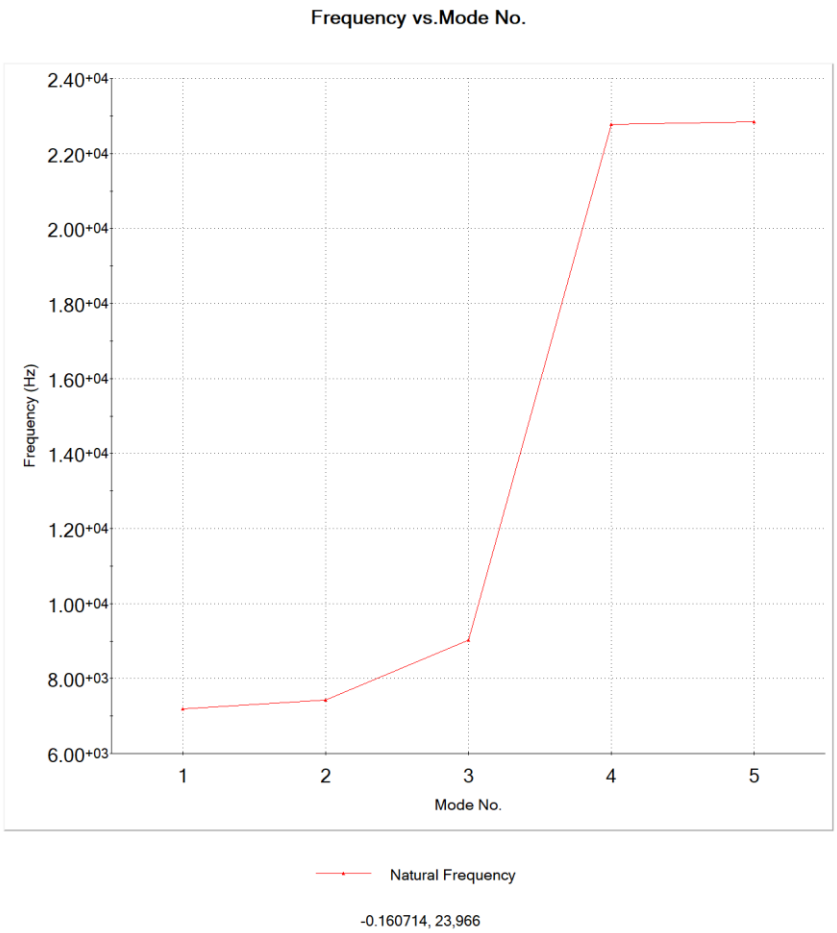


Figure 13 (Plot of Input Shaft Key Frequency)

Frequency at node 1 = 7172.8 Hz

Critical speed:

$$7172.8\,Hz\ =\ 45068.03152\,rad/s\ =\ 430368.0\,RPM$$

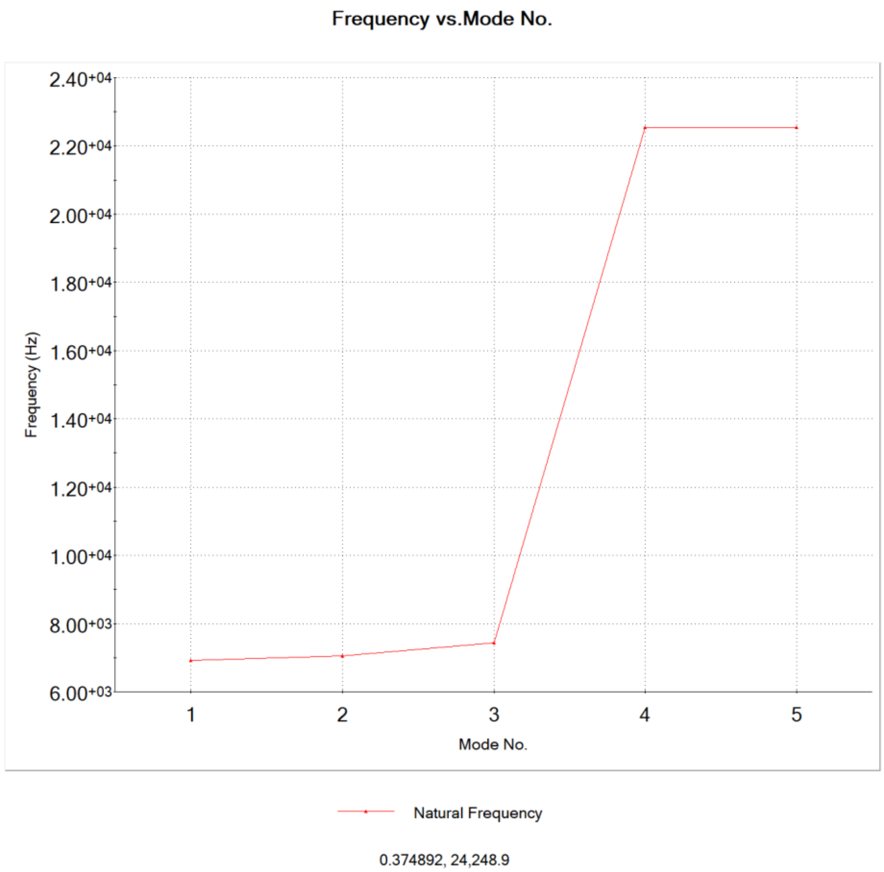


Figure 14 (Plot of The Output Shaft Key Frequency)

Frequency at node 1 = 6916.85 Hz

Critical speed:

$$6916.85\,Hz = 43459.85\,rad/s = 415010.99\,RPM$$

Neither the input nor output speeds exceed the critical speeds estimated for them, therefore, the shafts will not fail.

Drawings for Components and Assemblies

Below are the orthographic drawings for the shafts and their respective bearings, from AST. All dimensions are up to date with the final calculated values, designed to withstand the

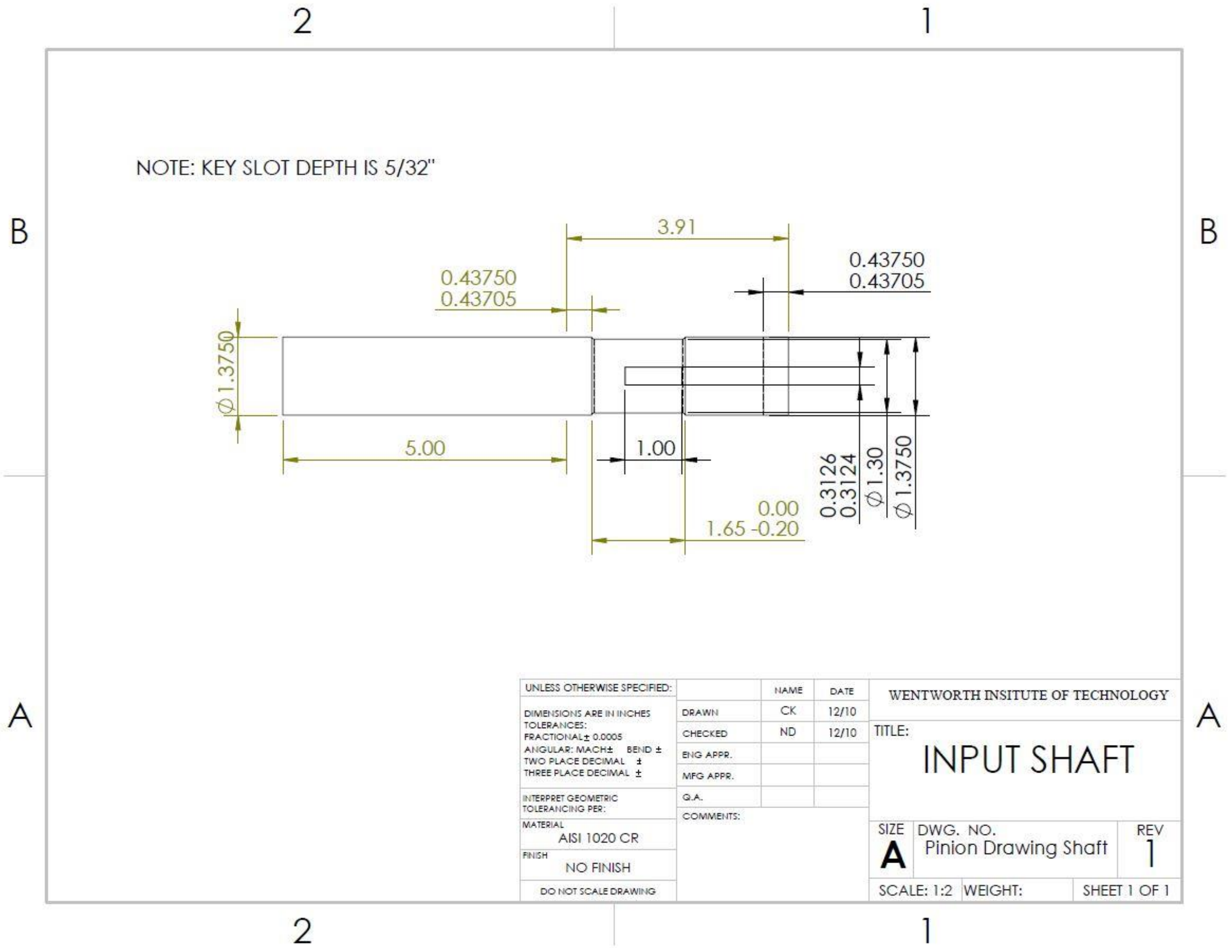


Figure 15 (Engineering Drawing of Input Shaft)

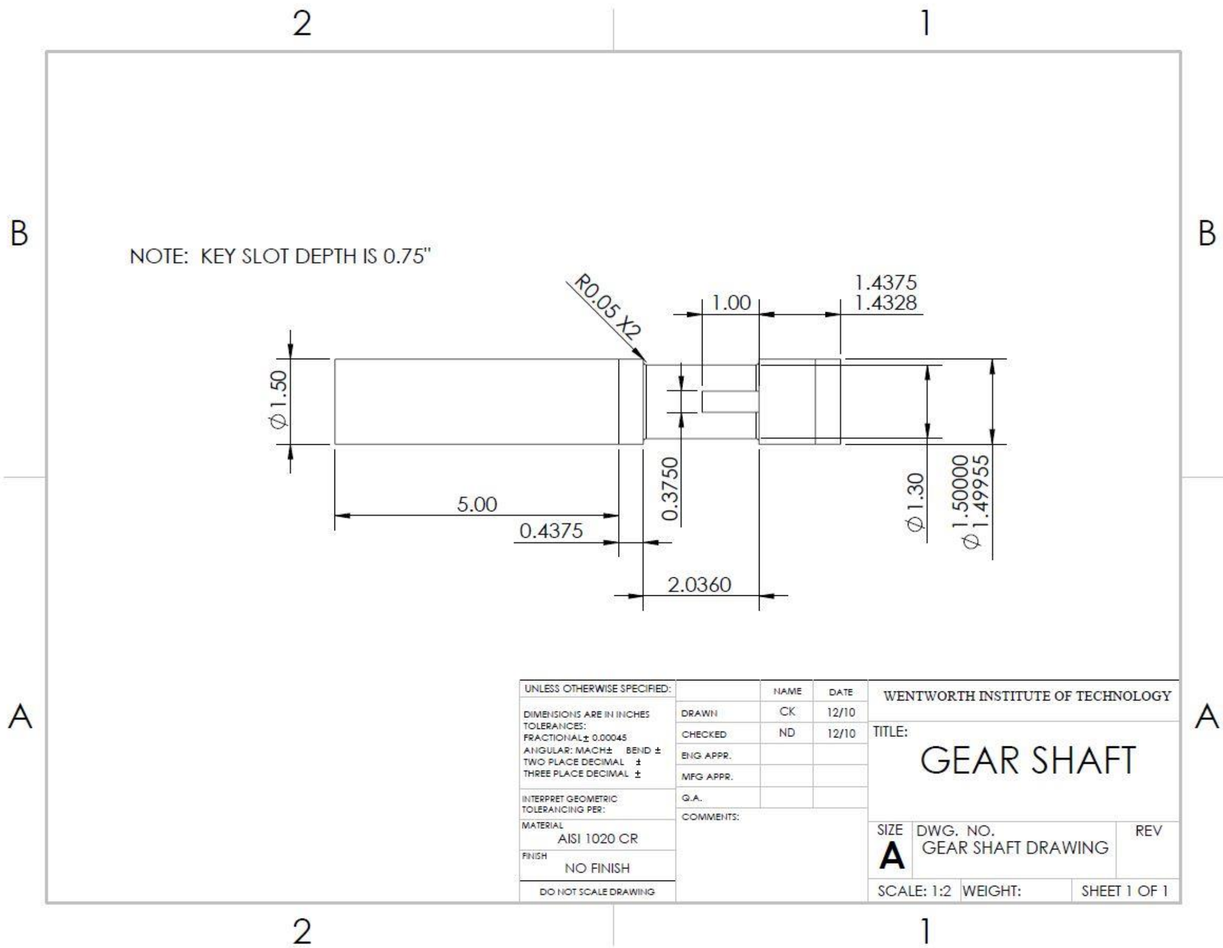


Figure 16 (Engineering Drawing of Output Shaft)

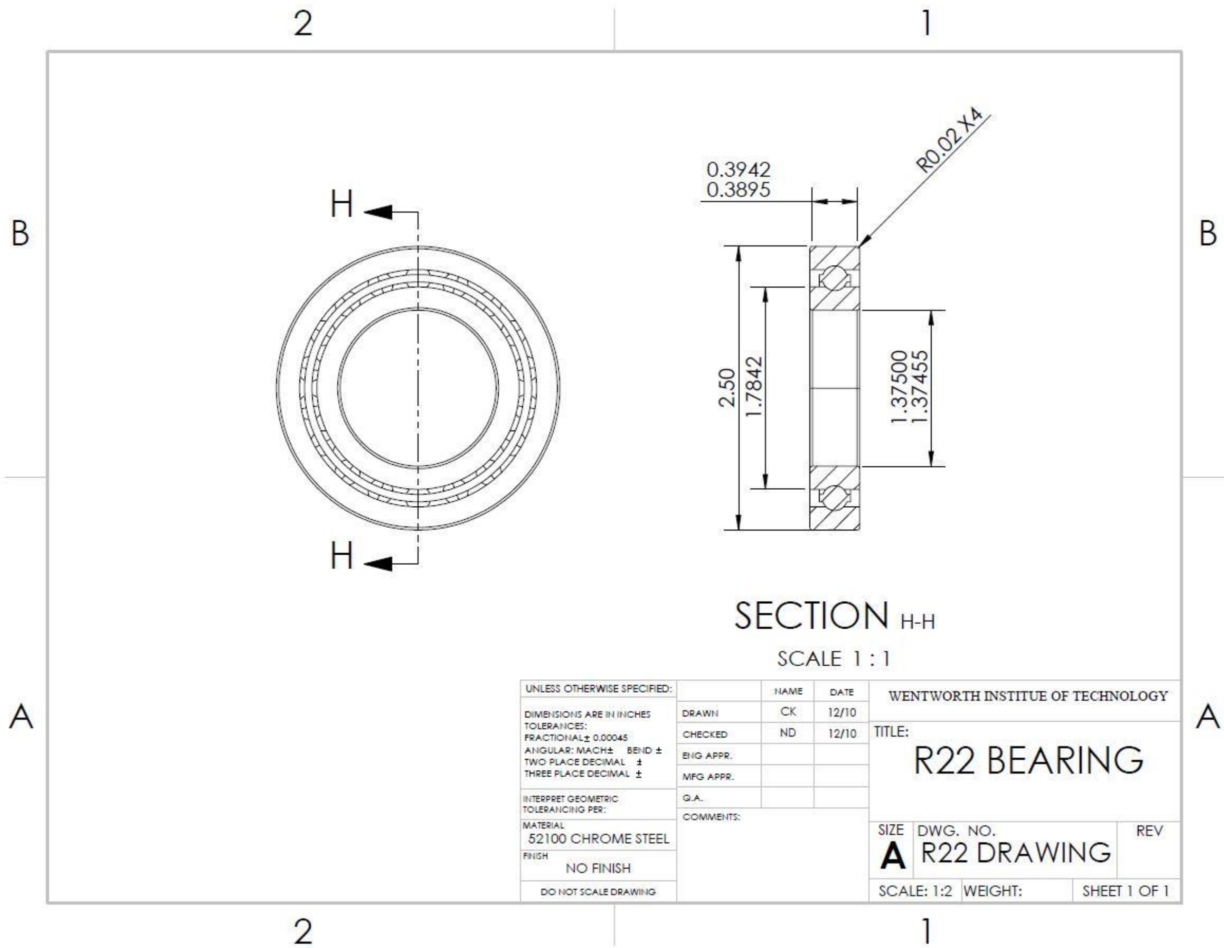


Figure 17 (Engineering Drawing of Final Input Bearing)

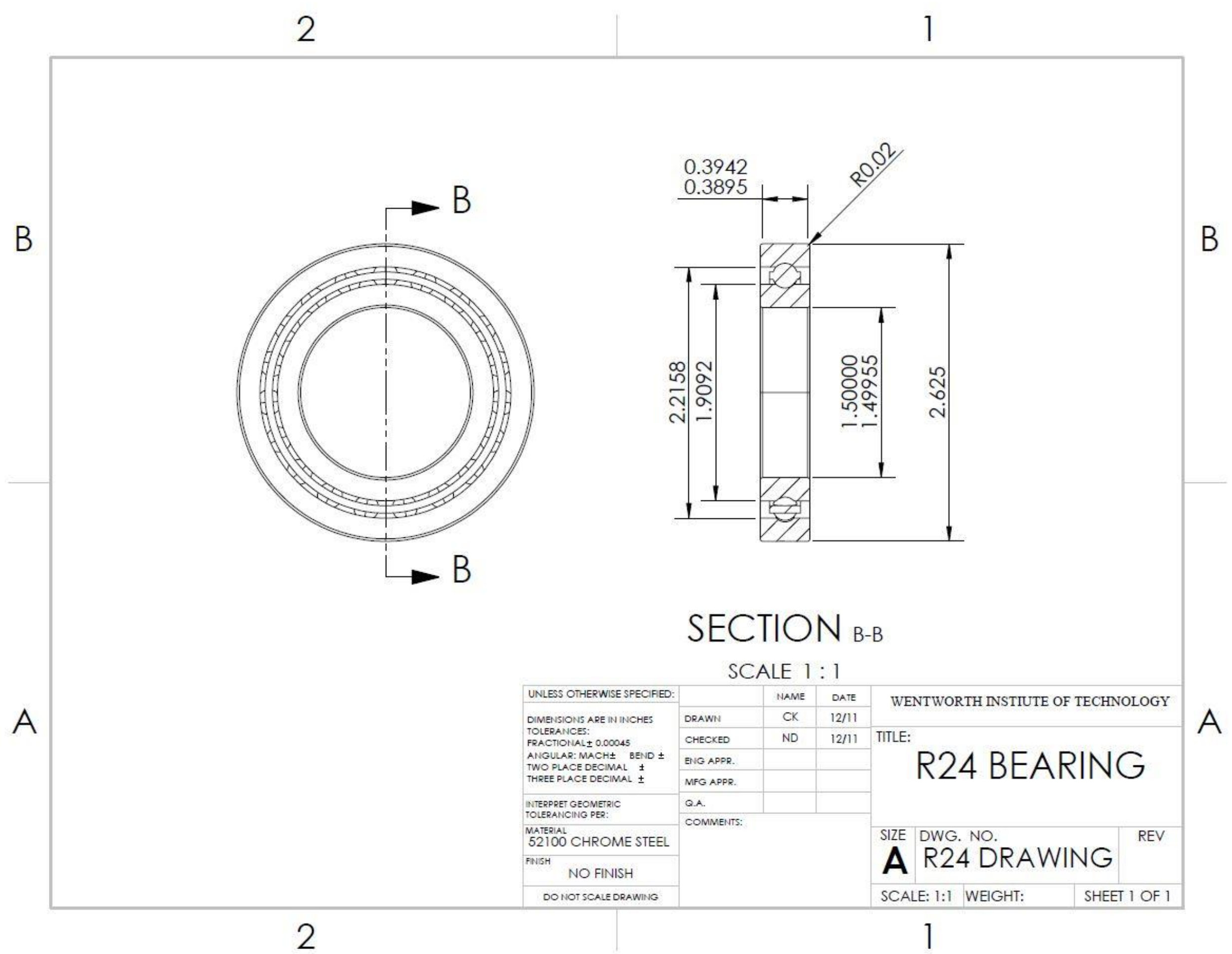


Figure 18 (Engineering Drawing of Final Output Bearing)

Case Design & Bolt Selection

Due to the nature of the project the design of the gear box case was of foremost importance, however ensuring that the bolts selected would withstand the forces enacted by the shafts on the case. To determine this we looked at the bearing reactionary loading to determine the force that each bolt would be subjected to. It was determined that the selected bolt would be required to withstand up to 500lbf of shear force. Using this information, the correct bolt could be selected from a bolt load table. From the table it was determined that the bolts that would be selected were a USS/SAE Grade 5 ¼-20 Hex-bolt.

These bolts would be placed in the eight threaded fastener locations located on the sides of the prototype gearbox housing as shown in the figure below.

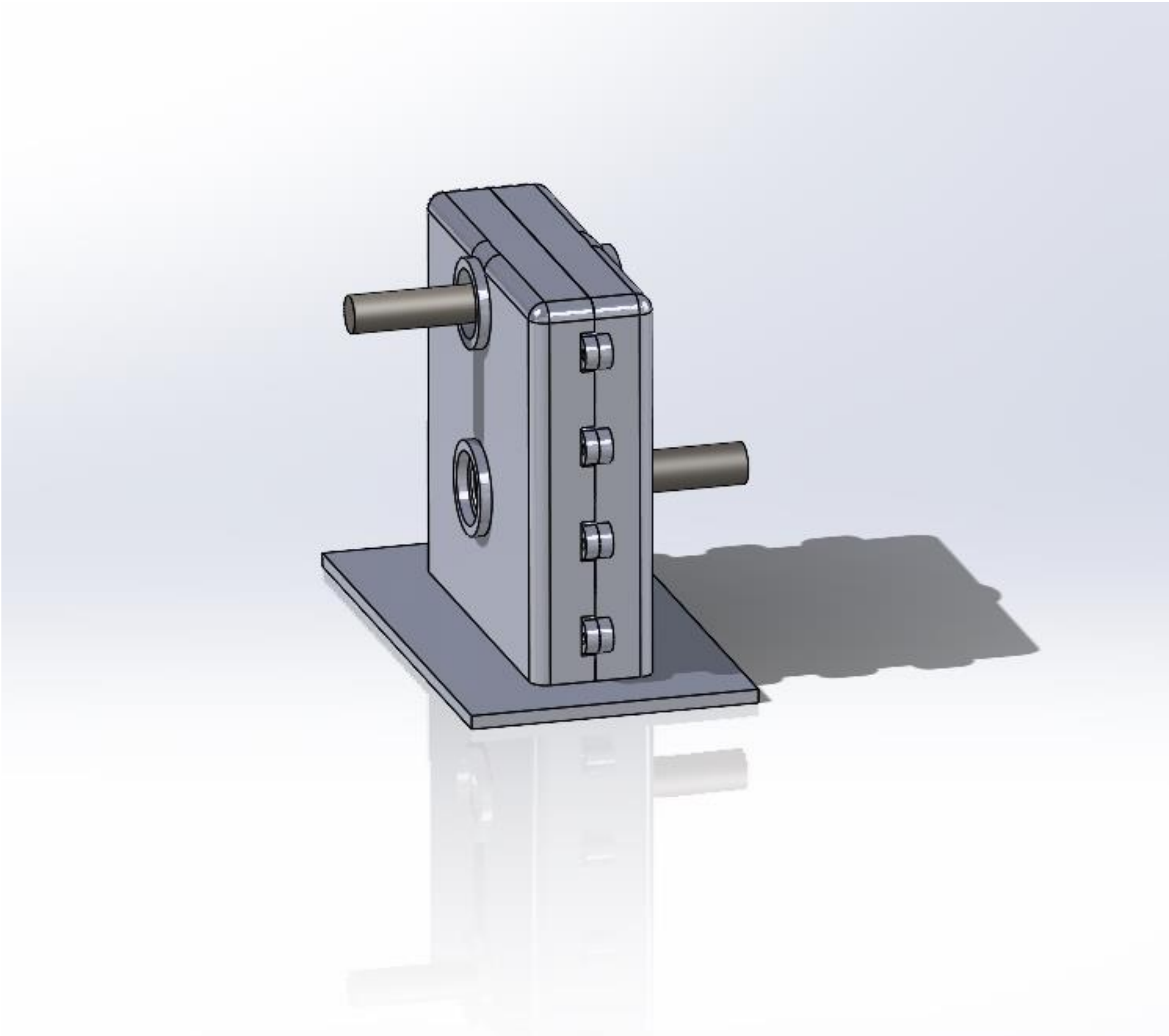


Figure 19 (Proposed Case Design)

Discussion and conclusions

The parts selected from the manufacturers' catalogs meet all of our loading and fatigue requirements and have bores that will fit our minimum shaft diameter. While the bores listed for the gear and pinion on Boston Gear's catalog are too small for our shaft diameter, these bores can be drilled to the proper diameter provided that the shaft does not exceed the diameter of the hub. Upon further investigation of the calculated key length, the key required to hold the gear without failure does not exceed the gear's face width. This means that the hubs of the pinion and gear were not necessarily required and could have been removed. This would have reduced the minimum shaft diameter as it would be subjected to a lower bending moment. In addition, there were combinations of gears with less collective teeth and a ratio that satisfies the 1% variation limit. Teeth of lower tooth count and the same diametral pitch reduces the pitch radius, which would then reduce the minimum shaft diameter by reducing the torque brought on by the tangential force. Our large shaft diameter demands bearings and keys designed for large shafts which exceed our loading and fatigue requirements. Therefore, despite not being the ideal design, our gearbox will meet the design requirements without part failure.

References:

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List of Figures