TP3 - éléments de correction

Exercice 2

1

Si $X \sim \mathcal{B}(n, p)$:

$$\mathbb{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

```r
n <- 10
p <- 1/3
k <- 1

choose(n, k)*(p**k)*((1-p)**(n-k))
```

```
## [1] 0.08670765
```

```r
dbinom(k, n, p)
```

```
## [1] 0.08670765
```

2

```r
n <- 10
p <- 1/3
k <- 0:10

choose(n, k)*(p**k)*((1-p)**(n-k))
```

```
##  [1] 1.734153e-02 8.670765e-02 1.950922e-01 2.601229e-01 2.276076e-01
##  [6] 1.365645e-01 5.690190e-02 1.625768e-02 3.048316e-03 3.387018e-04
## [11] 1.693509e-05
```

```r
dbinom(k, n, p)
```

```
##  [1] 1.734153e-02 8.670765e-02 1.950922e-01 2.601229e-01 2.276076e-01
##  [6] 1.365645e-01 5.690190e-02 1.625768e-02 3.048316e-03 3.387018e-04
## [11] 1.693509e-05
```

3

$$\mathbb{P}(45 \leqslant X < 55) = \sum_{k=45}^{54} \mathbb{P}(X = k) = \sum_{k=0}^{54} \mathbb{P}(X = k) - \sum_{k=0}^{44} \mathbb{P}(X = k) = F_X(54) - F_X(44)$$

```
n <- 100
p <- 1/2
k <- 45:54

sum(dbinom(k, n, p))
```

```
## [1] 0.6802727
```

```
pbinom(54, n, p) - pbinom(44, n, p)
```

```
## [1] 0.6802727
```

4

$$\mathbb{P}(X = 4) = \left(\frac{6}{10}\right)^3 \frac{4}{10} = \left(\frac{3}{5}\right)^3 \frac{2}{5}$$

```
6*6*6*4/10000
```

```
## [1] 0.0864
```

```
dgeom(3, 4/10)
```

```
## [1] 0.0864
```

5, 6

```
1 - ppois(4, 2.7)
```

```
## [1] 0.1370921
```

```
rpois(10, 2.7)
```

```
##  [1] 2 1 1 3 2 1 3 3 2 1
```

3

##1

```
lambda <- 0.7
x <- 1.6
lambda*exp(-lambda*x)
```

```
## [1] 0.2283959
```

```
dexp(x, lambda)
```

```
## [1] 0.2283959
```

2

Pour $X \sim \mathcal{E}(\lambda)$, $F_X(x) = 1 - e^{-\lambda x}$.

```
1-exp(-lambda*x)
```

```
## [1] 0.6737202
```

```
ppois(x, lambda)
```

```
## [1] 0.844195
```

3

Pour $X \sim \mathcal{N}(0; 1)$:

$$\mathbb{P}(X > 1.96) = 1 - \mathbb{P}(X \leqslant 1.96)$$

```
1-pnorm(1.96)
```

```
## [1] 0.0249979
```

Cette probabilité vaut environ 0,025.

4

Le quantile q_α de niveau α associé à la loi d'une variable aléatoire X est défini par :

$$\mathbb{P}(X \leqslant q_\alpha) = \alpha$$

```
qnorm(0.975)
```

```
## [1] 1.959964
```

Ce quantile de niveau $0,975 = 1 - 0.025$ vaut approximativement 1,96.

Exercice 4

En se demandant quelles situations permettent de réaliser la situation $X = k$ et en exprimant $\mathbb{P}(X = k)$, on trouve aisément que la loi utilisée dans cet exercice est la loi binomiale.

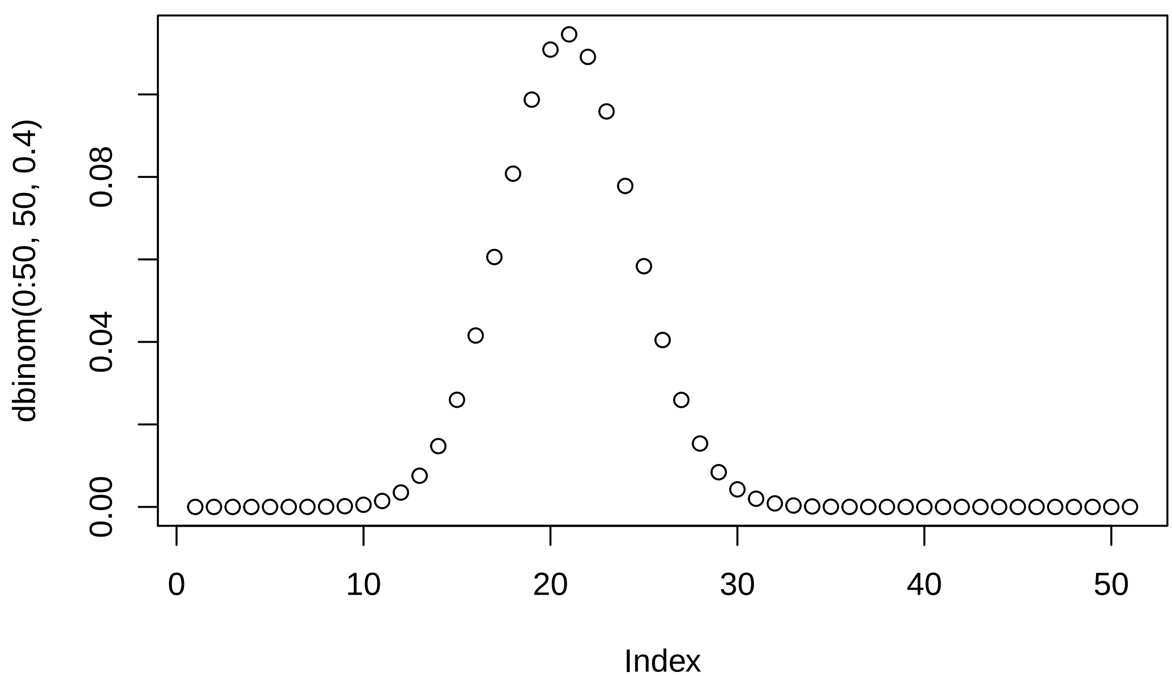
1

$X \sim \mathbb{B}(0,4; 50)$

```
dbinom(0:50, 50, 0.4)
```

```
##  [1] 8.082813e-12 2.694271e-10 4.400643e-09 4.694019e-08 3.676981e-07
##  [6] 2.255215e-06 1.127608e-05 4.725213e-05 1.693201e-04 5.267737e-04
## [11] 1.439848e-03 3.490541e-03 7.562839e-03 1.473784e-02 2.596667e-02
## [16] 4.154667e-02 6.058890e-02 8.078520e-02 9.873746e-02 1.108631e-01
## [21] 1.145586e-01 1.091034e-01 9.587873e-02 7.781462e-02 5.836097e-02
## [26] 4.046360e-02 2.593821e-02 1.537079e-02 8.417337e-03 4.257044e-03
## [31] 1.986621e-03 8.544605e-04 3.382239e-04 1.229905e-04 4.099684e-05
## [36] 1.249428e-05 3.470632e-06 8.754747e-07 1.996697e-07 4.095788e-08
## [41] 7.508945e-09 1.220967e-09 1.744238e-10 2.163396e-11 2.294511e-12
## [46] 2.039565e-13 1.477946e-14 8.385509e-16 3.493962e-17 9.507380e-19
## [51] 1.267651e-20
```

```
plot(dbinom(0:50, 50, 0.4))
```



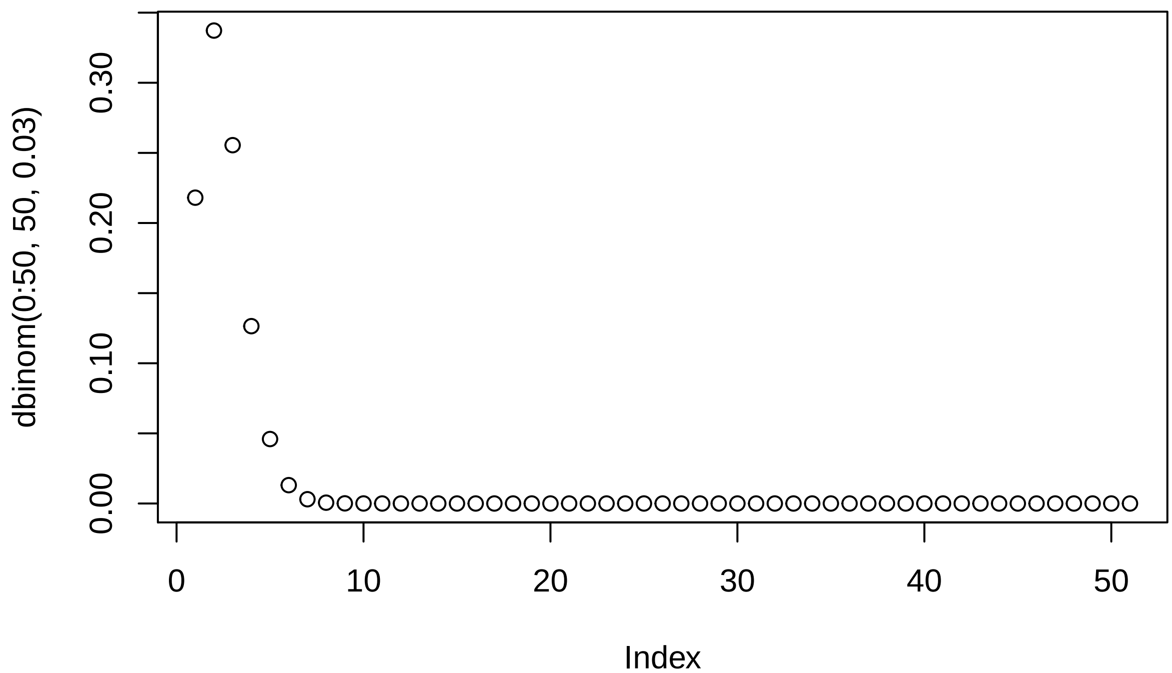
2

$X \sim \mathbb{B}(0,03; 50)$

```r
dbinom(0:50, 50, 0.03)
```

```
##  [1] 2.180654e-01 3.372145e-01 2.555182e-01 1.264420e-01 4.594928e-02
##  [6] 1.307423e-02 3.032682e-03 5.895641e-04 9.800743e-05 1.414540e-05
## [11] 1.793695e-06 2.017277e-07 2.027675e-08 1.833108e-09 1.498343e-10
## [16] 1.112172e-11 7.524363e-13 4.654245e-14 2.639005e-15 1.374631e-16
## [21] 6.589725e-18 2.911513e-19 1.186981e-20 4.469134e-22 1.554982e-23
## [26] 5.001592e-25 1.487389e-26 4.089041e-28 1.038821e-29 2.437333e-31
## [31] 5.276701e-33 1.052884e-34 1.933452e-36 3.261682e-38 5.043837e-40
## [36] 7.131199e-42 9.189689e-44 1.075416e-45 1.137852e-47 1.082810e-49
## [41] 9.209463e-52 6.947043e-54 4.604078e-56 2.649194e-58 1.303493e-60
## [46] 5.375228e-63 1.807002e-65 4.756311e-68 9.193900e-71 1.160602e-73
## [51] 7.178980e-77
```

```r
plot(dbinom(0:50, 50, 0.03))
```



3

```r
pbinom(4, 50, 0.03)
```

```
## [1] 0.9831894
```

```r
1-pbinom(2, 50, 0.03)
```

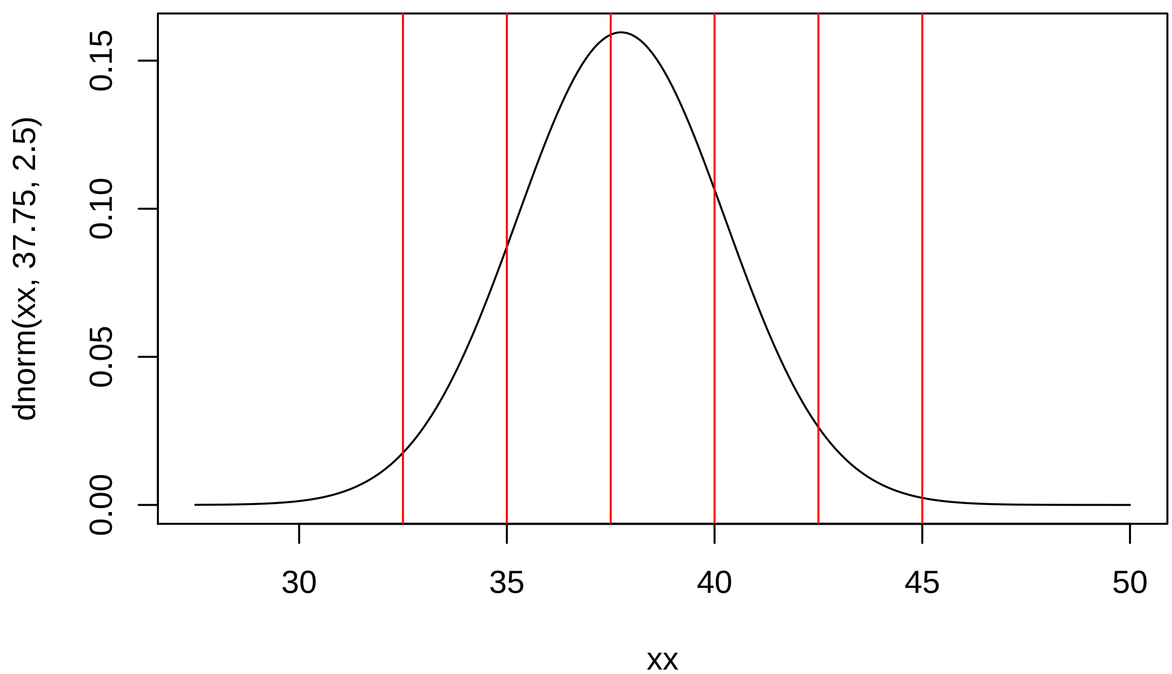
```
## [1] 0.1892019
```

```
n <- 50
p <- 0.03
mu <- n*p
sig2 <- n*p*(1-p)
```

Exercice 5

On commence par faire un dessin pour fixer les idées.

```
bornes <- seq(32.5,45,2.5)
xx <- seq(27.5,50,0.1)
plot(xx, dnorm(xx, 37.75, 2.5), type='l')
abline(v=bornes, col='red')
```



1

On a besoin de calculer les probas de se trouver dans chaque intervalle.

```
probas <- diff(pnorm(bornes, 37.75, 2.5))
```

On remarque toutefois que leur somme ne vaut pas 1 (c'est normal, on ne recouvre pas l'ensemble des évènements)

```
sum(probas)
```

```
## [1] 0.9802698
```

On ne dispose donc pas des proportions des tailles, mais seulement la probabilité qu'un client au hasard rentre dans la grille de tailles. Pour se ramener à des proportions, il faut normaliser ces probabilités.

```
propor <- probas/sum(probas)
round(propor, 2)
```

```
## [1] 0.12 0.33 0.36 0.16 0.03
```

2

On a la proba qu'un client trouve sa taille. Pour connaître la proba qu'un client ne trouve pas sa taille, c'est très simple :

```
1-sum(probas)
```

```
## [1] 0.01973023
```

Environ 2% des clients ne trouveront pas de chemise à leur taille.