POSSESION OF MOBILES IN EXAM IS UFM PRACTICE.

Name _____ **Enrollment No.** 37

Jaypee Institute of Information Technology, Noida
End Semester Examination, Even 2023-24
B.Tech IV Semester

Course Title : Probability and Random Process **Maximum Time : 2 Hr**
Course Code : 15B11MA301 **Maximum Marks : 35**

After pursuing this course, students will be able to:

CO1	**recall the concepts of probability theory and probability distributions.**
CO2	**explain random variables, probability distributions and reliability models.**
CO3	**solve the problems concerning random variables, their distributions, reliability models and random process.**
CO4	**examine random process models and solve the related problems.**

Note: Attempt all questions (Non programmable calculator will be allowed).

Q1. A doctor is called to see a sick child. The doctor has prior information that 90% of sick children in that neighbourhood have the flu, while the other 10% are sick with measles. Also, there is no other maladies in that neighbourhood. A well-known symptom of measles is a rash. Assume that the probability of having a rash if one has measles is 0.95. However, occasionally children with flu also develop rash, and the probability of having a rash if one has flue is 0.08. Upon examining the child, the doctor observes a rash. Find the probability that the child has measles. **[CO1 (Remembering), 3 Marks]**

Q2. The probability density function of a random variable X is given by $f(x) = \frac{e^{-2|x|}}{3}, -\infty < x < \infty$. If $Y = 2X + 5$, then demonstrate the moment generating function of Y and hence find the mean and variance of Y. **[CO2 (Understanding), 4 Marks]**

Q3. A manufacturer does not know the mean and standard deviation of the diameters of ball bearings, he is producing. However, a sieving system rejects all bearing larger than 2.4 cm and those under 1.8 cm in diameter. Out of 1000 ball bearings, 8% are rejected as too small and 5.5% as too big. Apply the Normal distribution, to find the mean and standard deviation of ball bearings produced. **[CO3 (Applying), 4 Marks]**

Q4. Let $\{X(t)\}$ be a WSS (wide sense stationary) process. Consider a random process $\{Y(t)\}$ such that $Y(t) = X(t)\cos(\lambda t + \theta)$, where λ is a constant and θ is a random variable independent of $X(t)$, uniformly distributed in $(-\pi, \pi)$. Identify whether $\{Y(t)\}$ is WSS process or not. **[CO3 (Applying), 4 Marks]**

Q5. Let $\{X(t)\}$ be a stationary random process with zero mean and auto-correlation function as $R(\tau) = 10e^{-|\tau|}$. Test for the mean ergodicity of the process over $(-T, T)$. **[CO4 (Analyzing), 4 Marks]**

Q6. Assume that a random process $\{X(t)\}$ has the auto-correlation function $R(\tau) = e^{-\alpha|\tau|}(1 + \alpha|\tau|)$, where α is a positive constant. Find the power spectral density function of the random process $\{X(t)\}$. **[CO4 (Analyzing), 4 Marks]**

Q7. Identify whether the Poisson process $\{X(t)\}$ is a Markov process or not. Justify your answer. **[CO3 (Applying), 4 Marks]**

Q8. The density function of time to failure (in years) of an appliance is $f(t) = \frac{16}{(t+1)^3}; t > 0$. Construct the reliability function $R(t)$ and failure rate function $\lambda(t)$. Also, find the mean Time to Failure (MTTF) and design life for 0.99 reliability. **[CO3 (Applying), 4 Marks]**

Q9. The transition probability matrix of a three state Markov chain is given by $\begin{bmatrix} 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$.

Classify the states of the chain and find the steady state distribution of it. **[CO4 (Analyzing), 4 Marks]**

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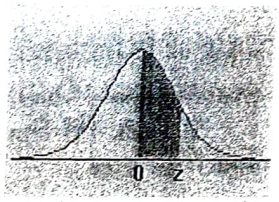
Standard Normal (Z) Table
Area between 0 and z



1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817