Introduction

How does a snowball melt when hit with an infinitesimal ray of sunlight? How does an ice cube grow from a seed crystal? How do sheets of metal deform when pierced by lasers? What will these objects and their heat distributions look like in 10 minutes, and how did they appear 10 seconds ago? While we know what happens generally through common logic, I aim to answer these and other questions with precise and feasible models and computations. I'm most interested in numerical and computational mathematics applied to engineering and industrial topics; specifically, I'm interested in quantifying the behavior of specific physical processes governed by partial differential equations, with interdisciplinary roots and industrial applications using methods of computational mathematics.

Work So Far

When I began studying under Dr. Tausch, I started by studying lecture notes from Dr. Tausch's Computational Electromagnetics graduate course and his most recent PhD student's (Elizabeth Case) thesis[1]. I learned about Green's function and identities, and their applications to the boundary element method in electrostatics, as well as computational solutions to the boundary element method, such as the collocation method and the Galerkin method. We then applied these examples to the problem of the heat equation on an interface with a moving geometry. Our main goal is to build upon work done by Elizabeth using the Galerkin method[2]. We hope that using the Galerkin method, and reworked singularity-removing transformations, we could improve upon the results of Dr. Tausch's previous student Elizabeth, who employed the Nystrom method, and achieve better numerical stability for problems with more interesting geometries. While we are currently testing error plots for our model and hope to begin writing our paper shortly, but this will not be the end for us. We're interested in expanding this topic from the two dimensional spatial case we are currently working on, to the three dimensional spatial case and creating more interesting and practical test problems and geometries before moving on to other topics in computational mathematics.

Intellectual Merit

Our current topic of integral equations applied to moving geometries has interesting contributions to the body of mathematics. There are numerical methods other than boundary element methods for melting and cooling problems. The benefit to using the boundary element method, for time dependent problems especially, is that only the boundary needs to be discretized, and not the interior. This makes the problem computationally easier since it isn't necessary to remesh or rediscretize the entire domain with each timestep. This also makes the boundary element method particularly suitable for challenging geometries with complicated interfaces and boundaries. The Galerkin method is also inherently stable. When this problem was solved using the Nystrom method[1], stability was an issue for complicated geometries, although numerical stability is an issue in general when solving partial differential equations using timestepping methods. In the time discretization, very small timesteps were needed for convergence in order to avoid blow up. This limits the complexity of the interface geometry. Researchers have solved interface problems, but using the Galerkin method with the boundary element method presents a new way of thinking about the numerical solutions of the interface problem. The boundary element method is also a hot topic as a solution of time dependent problems, but as of now no one is doing work on the time dependent problem with a moving geometry. Another totally new element of the problem stemmed from handling singularities which arose from using the Galerkin method. Because this is a new problem, finding transformations to remove these singularities and ensure stability presented a new challenge.

Broader Impact

Boundary element methods have been employed in thermal simulations with very particular industrial applications, such as determining working temperatures of hot forming tools[2] and in the heating of batteries, which is a good example of how boundary element methods can be applied to hot topics in industry. In order to properly cool or insulate objects such as electric car or cellphone batteries, a good model of heat flow in the battery is necessary. While solutions for modeling these instances exist, batteries can have very complicated geometries. In the case of the moving geometry, some interesting applications of our model would be in manufacturing, applied to the deformation of metal sheeting after the application of point heat sources such as laser cutting tools. The ability to model the numerical solutions of complicated heat flow problems is very useful, leading to new, safer and more efficient technology.

Future Goals

After analyzing the heat problem with a moving interface using the boundary element method and the Galerkin method, we hope to expand our model to three spatial dimensions. We will then analyze the problem using the fast multipole method to see if we can improve convergence of our numerical solution and the stability of the problem (if stability needs improving). We are also interested in Stefan problems, which describe how the field moves the surface forward. Our current work is based on a known, physically motivated closed form for the movement of the interface, and it's currently unclear how to extend boundary element methods to time dependent problems to describe how these unknown interfaces advance in the case of the Stefan problem. We need to do more research on this topic, but there isn't currently a tested variational formulation, such as the Galerkin method, for this type of problem. So, finding the correct variational formulation for the Stefan interface condition would be an interesting extension of our model. Our current and future work will contribute to the knowledge on numerical methods for handling free interface problems. Our methodology is novel and will contribute to the body of numerical methods, while our research has several interesting industrial applications.

References:

(1) Case, E. (2013). Numerical integral equations in solidification and melting problems (Doctoral Dissertation). Retrieved from ProQuest Dissertations & Theses- Gradworks. (Publication No. 3566369)

(2) Messner, M., Tausch, J., Schanz, M. (2015). An Efficient Galerkin Boundary Element Method for the Transient Heat Equation. SIAM Journal on Scientific Computing, 37(3), pp. A1554-A1576.