Biến đổi Fourier

BIẾN ĐỔI FOURIER:

$f(t)$		$\to$	$F(w)$
$\dfrac{1}{t}$		$\to$	$\begin{cases} -i & \text{nếu } w<0 \\ 0 & \text{nếu } w=0 \\ i & \text{nếu } w>0 \end{cases}$
$e^{-a\lvert t\rvert}$	$(a>0)$	$\to$	$\dfrac{2a}{a^2+w^2}$
$te^{-a\lvert t\rvert}$	$(a>0)$	$\to$	$\dfrac{-4aiw}{(w^2+a^2)^2}$
$\lvert t\rvert\, e^{-a\lvert t\rvert}$	$(a>0)$	$\to$	$\dfrac{2(a^2-w^2)}{a^2+w^2}$
$e^{-a^2t^2}$	$(a>0)$	$\to$	$\dfrac{\sqrt{\pi}}{a}\exp\left(-\dfrac{w^2}{4a^2}\right)$
$\dfrac{1}{a^2+t^2}$	$(a>0)$	$\to$	$\dfrac{\pi}{a}e^{-a\lvert w\rvert}$
$\dfrac{t}{a^2+t^2}$	$(a>0)$	$\to$	$\dfrac{\pi}{a}e^{-a\lvert w\rvert}\cdot\dfrac{-iw}{2}$
$H(t+a)-H(t-a)$		$\to$	$\dfrac{2\sin(aw)}{w}$
$\delta(t)$		$\to$	1
$H(t)e^{-at}$		$\to$	$\dfrac{1}{a+iw}$

CÁC TÍNH CHẤT:

$af(t)+bg(t)$	$\to$	$aF(w)+bG(w)$
$F(t)$	$\to$	$2\pi f(-w)$
$f(kt)$	$\to$	$\dfrac{1}{\lvert k\rvert}F(\dfrac{w}{k})$
$\dfrac{1}{\lvert k\rvert}f(\dfrac{t}{k})$	$\to$	$F(kw)$
$f(t-a)$	$\to$	$F(w)e^{-iaw}$
$f(t)e^{iat}$	$\to$	$F(w-a)$

$$\frac{f(t+a)+f(t-a)}{2} \quad \to \quad \cos(aw)F(w)$$

$$\frac{f(t+a)-f(t-a)}{2i} \quad \to \quad \sin(aw)F(w)$$

$$\cos(at)f(t) \quad \to \quad \frac{F(w+a)+F(w-a)}{2}$$

$$\sin(at)f(t) \quad \to \quad -\frac{F(w+a)-F(w-a)}{2i}$$

$$\frac{d^n}{dt^n}f(t) \quad \to \quad (iw)^n F(w)$$

$$(-it)^n f(t) \quad \to \quad \frac{d^n}{dw^n}F(w)$$

$$f(t) \circledast g(t) \quad \to \quad F(w) \cdot G(w)$$

$$f(t) \cdot g(t) \quad \to \quad \frac{1}{2\pi}F(w) \circledast G(w)$$

MỘT SỐ TÍNH CHẤT KHÁC

Định lý Parseval:

$$\int_{-\infty}^{\infty} |f(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(w)|^2 \, dw$$

Nếu $f(t) \in \mathbb{R}$:

$$F(-w) = \overline{F(w)}$$

Nếu $f(t) \in \mathbb{R}$, $f(t)$ chẵn:

$$\mathrm{Re}(F(w)) = 0$$

Nếu $f(t) \in \mathbb{R}$, $f(t)$ lẻ:

$$\mathrm{Im}(F(w)) = 0$$

END