

Biến đổi Fourier

BIẾN ĐỔI FOURIER:

$f(t)$	\rightarrow	$F(w)$
$\frac{1}{t}$	\rightarrow	$\begin{cases} -i & \text{nếu } w < 0 \\ 0 & \text{nếu } w = 0 \\ i & \text{nếu } w > 0 \end{cases}$
$e^{-a t } \quad (a > 0)$	\rightarrow	$\frac{2a}{a^2 + w^2}$
$te^{-a t } \quad (a > 0)$	\rightarrow	$\frac{-4aiw}{(w^2 + a^2)^2}$
$ t e^{-a t } \quad (a > 0)$	\rightarrow	$\frac{2(a^2 - w^2)}{a^2 + w^2}$
$e^{-a^2t^2} \quad (a > 0)$	\rightarrow	$\frac{\sqrt{\pi}}{a} \exp\left(-\frac{w^2}{4a^2}\right)$
$\frac{1}{a^2 + t^2} \quad (a > 0)$	\rightarrow	$\frac{\pi}{a} e^{-a w }$
$\frac{t}{a^2 + t^2} \quad (a > 0)$	\rightarrow	$\frac{\pi}{a} e^{-a w } \cdot \frac{-iw}{2}$
$H(t + a) - H(t - a)$	\rightarrow	$\frac{2 \sin(aw)}{w}$
$\delta(t)$	\rightarrow	1
$H(t)e^{-at}$	\rightarrow	$\frac{1}{a + iw}$

CÁC TÍNH CHẤT:

$af(t) + bg(t)$	\rightarrow	$aF(w) + bG(w)$
$F(t)$	\rightarrow	$2\pi f(-w)$
$f(kt)$	\rightarrow	$\frac{1}{ k } F\left(\frac{w}{k}\right)$
$\frac{1}{ k } f\left(\frac{t}{k}\right)$	\rightarrow	$F(kw)$
$f(t - a)$	\rightarrow	$F(w)e^{-iaw}$
$f(t)e^{iat}$	\rightarrow	$F(w - a)$

$\frac{f(t+a) + f(t-a)}{2}$	\rightarrow	$\cos(aw)F(w)$
$\frac{f(t+a) - f(t-a)}{2i}$	\rightarrow	$\sin(aw)F(w)$
$\cos(at)f(t)$	\rightarrow	$\frac{F(w+a) + F(w-a)}{2}$
$\sin(at)f(t)$	\rightarrow	$-\frac{F(w+a) - F(w-a)}{2i}$
$\frac{d^n}{dt^n}f(t)$	\rightarrow	$(iw)^n F(w)$
$(-it)^n f(t)$	\rightarrow	$\frac{d^n}{dw^n}F(w)$
$f(t) \otimes g(t)$	\rightarrow	$F(w) \cdot G(w)$
$f(t) \cdot g(t)$	\rightarrow	$\frac{1}{2\pi}F(w) \otimes G(w)$

MỘT SỐ TÍNH CHẤT KHÁC

Định lý Parseval:

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(w)|^2 dw$$

Nếu $f(t) \in \mathbb{R}$:

$$F(-w) = \overline{F(w)}$$

Nếu $f(t) \in \mathbb{R}$, $f(t)$ chẵn:

$$\operatorname{Re}(F(w)) = 0$$

Nếu $f(t) \in \mathbb{R}$, $f(t)$ lẻ:

$$\operatorname{Im}(F(w)) = 0$$

END