Number Systems

Denary Number System

Definition

A number system that is made up of **10 unique digits**.

- Uses place values of powers of 10.

Binary Number System

Definition

A number system that is made up of **2 unique digits**.

- Uses place values of powers of 2.

Notation

To distinguish binary numbers from denary numbers, they can be written in any of the following ways:

- 1101
- $(1101)_2$
- $0b1101$

Leading zeros are sometimes also shown when using binary numbers in computer systems to show all 8 binary bits in a byte:

e.g. $0000 \space 1101$

Denary to Binary

Algorithm 1: Dividing by 2

1. Draw a table with three columns - one column for denary numbers, one column for the quotients and one column for the remainders.
2. Fill in the denary number in the first row.
3. Divide the denary number by 2 and fill in its quotient and remainder in the same row.
4. If the quotient is 0, proceed to step 5. Otherwise, copy the quotient to the denary number column of the next row and repeat step 3.
5. The equivalent binary number is the remainder column read from the bottom up.

Example: Converting 135 to binary

Denary	Quotient	Remainder

Denary	Quotient	Remainder
135	67	1
67	33	1
33	16	1
16	8	0
8	4	0
4	2	0
2	1	0
1	0	1

∴ $(135)_{10} = (10000111)_2$

Algorithm 2: Sum of Place Values

E.g. Convert 135 to binary

Place value	256	128	64	32	16	8	4	2	1
	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Binary digit		1	0	0	0	0	1	1	1

Hexadecimal Number System

Definition

Number system that is made up of 16 unique digits.

Denary equivalents of the hexadecimal

Hexadecimal digit	Denary equivalent
0	0
1	1
2	2
3	3
4	4
5	5
6	6

Hexadecimal digit	Denary equivalent
7	7
8	8
9	9
A	10
B	11
C	12
D	13
E	14
F	15

Example of Hexadecimal Number

$$1C6A$$

$$1C6A_{16} = 1 \times 16^3 + 12 \times 16^2 + 6 \times 16^1 + 10 \times 16^0$$

To distinguish hexadecimal numbers from denary numbers, they can be written in any of the following ways:

- $1C6A_{16}$
- $(1C6A)_{16}$
- $0x1C6A$

Denary to Hexadecimal

Algorithm 1: Divide by 16

1. Draw a table with three columns - one column for denary numbers, one column for the quotients and one column for the remainders.
2. Fill in the denary number in the first row.
3. Divide the denary number by 16 and fill in its quotient and remainder in the same row.
4. If the quotient is 0, proceed to step 5. Otherwise, copy the quotient to the denary number column of the next row and repeat step 3.
5. The equivalent denary number is the remainder column read from the bottom up.

Example

Convert 1899 to hexadecimal

Denary	Quotient	Remainder
1899	118	$11 = B_{16}$
118	7	$6 = 6_{16}$

Denary	Quotient	Remainder
7	0	$7 = 7_{16}$

Hexadecimal to Binary, or Vice Versa

Hexadecimal digit	0	1	2	3	4	5	6	7
Denary equivalent	0	1	2	3	4	5	6	7
Binary equivalent	0000	0001	0010	0011	0100	0101	0110	0111
Hexadecimal digit	8	9	A	B	C	D	E	F
Denary equivalent	8	9	10	11	12	13	14	15
Binary equivalent	1000	1001	1010	1011	1100	1101	1110	1111