Texas State University
MATH 3323: Differential Equations
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Problem Set 4

This problem set covers a bit more of integrating factor and then shifts to systems, the relevant sections are 2.2, 7.1, and 7.2.

(1) Consider the differential equation

$$\frac{dy}{dx} = 4y + 3e^{8x}(x+1)$$

Find a general formula for solutions of the equation. Then, find a solution with initial value $y(0) = 0$.

(2) Solve problems 1, 2, and 3 at the end of Section 7.1 –remember you may use the answers at the end of the book to check your solution, but as always, you must show your work to get credit for the question.

(3) Transform each second order equation into an equivalent first order system

(a) $\ddot{x} - 2t\dot{x} + 7x = \sin(t)$,
(b) $\ddot{x} - 2\dot{x} + 2x = 0$,
(c) $\ddot{x} + \dfrac{9}{x^2} = 0$,
(d) $\ddot{x} + y = 0,\ \ddot{y} + \sin(x) = 1$.

(4) For each matrix, find all the eigenvalues, and provide an eigenvector for each eigenvalue

a) $\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$

(5) (BONUS) Find the value α such that if $x(t)$ solves the initial value problem

$$\dot{x} = -\tfrac{2}{3}x + 1 - \tfrac{1}{2}t,\ x(0) = \alpha$$

then $x(t)$ does not change sign but takes the value $x(t) = 0$ for at least some t.

(6) (BONUS) Consider the nonlinear differential equation

$$\dot{x} = \tfrac{1}{3}x(\tfrac{7}{23} - x)$$

Then
(a) Find the general formula for the solution (in terms of the initial value $x(0)$).
(b) When $x(0) = 1$, what happens with $x(t)$ as $t \to \infty$?
(c) Find a solution $x(t)$ of the equation which is constant in time.
(d) Give an example of initial value $x(0)$ so that $\lim_{t\to\infty} x(t) = 0$.