All talks will take place in the Mathematics building, Room 312.

May 23

8:45 – 9:15am Breakfast, Cantor Lounge

9:30 – 10:20am *Donatella Danielli*, Purdue

The obstacle problem for the fractional heat equation: properties of the free boundary.

In this talk we will discuss the structure of the free boundary in the obstacle problem for fractional powers of the heat operator. First introduced by M. Riesz in 1938, this nonlocal operator represents a basic model of the continuous time random walks introduced by Montroll and Weiss. Our results are derived from the study of a lower-dimensional obstacle problem for a class of local, but degenerate, parabolic operators. Its analysis, in turn, hinges on the monotone character of functionals of Almgren, Weiss, and Monneau type, and the properties of the associated blow-ups. This is joint work with A. Banerjee, N. Garofalo, and A. Petrosyan.

10:30-10:55am Coffee Break, Cantor Lounge

11:00–11:50am *Ricardo Nochetto*, U of Maryland

Finite element approximation of fractional diffusion.

We approximate the integral formulation of the fractional Laplacian in bounded polyhedral domains. We discuss how to deal numerically with the singular non-integrable kernel, derive optimal energy estimates, and discuss preconditioning with emphasis on the boundary behavior and how to cope with the reduced boundary Sobolev regularity. We further examine fractional obstacle problems and fractional minimal graphs in light of the newly developed linear theory, with emphasis on the development of boundary discontinuities.

2:00–2:50pm *Luis Silvestre*, U of Chicago

Regularity and structure of scalar conservation laws.

Scalar conservation law equations develop jump discontinuities even when the initial data is smooth. Ideally, we would expect these discontinuities to be confined to a collection of codimension-one surfaces, and the solution to be relatively smoother away from these jumps. The picture is less clear for rough initial data which is merely bounded. While a linear transport equation may have arbitrarily rough solutions, genuinely nonlinear conservation laws have a subtle regularization effect. We prove that the entropy solution will become immediately continuous outside of a codimension-one rectifiable set, that all entropy dissipation is concentrated on the closure of this set, and that the $\mathcal{L}^\infty$ norm of the solution decays at a certain rate as t goes to infinity.

3:00-3:25pm Coffee Break, Cantor Lounge

3:30-4:20pm *Felix Otto*, Max-Planck Institute

A variational approach to the regularity theory for the Monge Ampere equation.

We present a purely variational approach to the regularity theory for the Monge-Ampère equation, or rather optimal transportation, introduced with M. Goldman. Following the general strategy in the regularity theory of minimal surfaces, it is based on the approximation of the displacement by a harmonic gradient, which leads to a One-Step Improvement Lemma, and feeds into a Campanato iteration on the $C^{1,\alpha}$-level for the displacement, capitalizing on affine invariance.
On the one hand, this allows to reprove the ϵ-regularity result (Figalli-Kim, De Philippis-Figalli) bypassing Caffarelli's celebrated theory. This also extends to boundary regularity (Chen-Figalli), which is joint work in progress with T. Miura. On the other hand, it can be used as a large-scale regularity theory for the problem of matching the Lebesgue measure to the Poisson measure in the thermodynamic limit. More precisely, it can be shown that in the critical dimension two (Ambrosio-Stra-Trevisan), increments of the displacement are still stationary. This is joint work with M. Goldman and M. Huesmann.

May 24

9:00 – 9:30am Breakfast, Cantor Lounge

9:45–10:35am *Alessio Figalli*, ETH Zurich

Generic regularity in obstacle problems.

The so-called Stefan problem describes the temperature distribution in a homogeneous medium undergoing a phase change, for example ice melting to water. An important goal is to describe the structure of the interface separating the two phases.

In its stationary version, the Stefan problem can be reduced to the classical obstacle problem, which consists in finding the equilibrium position of an elastic membrane whose boundary is held fixed and which is constrained to lie above a given obstacle.

The aim of this talk is to discuss sone recent developments on the generic regularity of the free boundary in both problems.

10:45-11:00am Coffee Break, Cantor Lounge

11:10 – 12:00am *Juan Luis Vázquez*, Universidad Autónoma de Madrid

Nonlocal porous medium and nonlocal thin film flows. results and open problems.

Nonlocality plays an important role in the modelling and theory of PDEs related to diffusion phenomena. The talk presents work on the existence, regularity and typical behaviour of solutions of nonlinear parabolic equations driven by fractional operators, which introduce nonlocal effects into classical settings. The models we discuss are related to porous medium and thin film equations. The connection between both equations is stressed. The problems in bounded domains offer new challenges.

1:45–2:35pm *Inwon Kim,* UCLA

Free boundary regularity for Porous Medium Equation with Drift.

I will discuss qualitative behavior of the free boundary for solutions of the porous medium equation with the presence of drift, starting with a general initial data. We will discuss relevant results for the problem with and without the drift. If the initial data has super-quadratic growth at the free boundary, then we show that the support of the solution strictly expands relative to the streamline, suggesting regularization. If in addition the solution is directionally monotone in a local neighborhood, then we derive non-degeneracy of solutions and regularity of the free boundary. This is joint work with Yuming Zhang.

2:45-3:00pm Coffee Break, Cantor Lounge

3:10–4:00pm *Sandro Salsa,* Politecnico di Milano

Recent progress in free boundary problems with distributed sources: higher regularity.

We describe some recent results on two-phase free boundary problems governed by inhomogeneous elliptic equations with particular emphasis on higher regularity. Joint work with Daniela De Silva and Fausto Ferrari.