Math 3323 Exam 1 Practice

(1) For each equation below, say which method is well suited to find its solutions (there could be more than one!), and then use it to find a formula for the general solution to the equation

(a) $y' = (2+\cos(x))y$	(e) $y' = (1+x)y(100-y)$
(b) $y' = \cos(x)y$	(f) $y' = 3xy + x$
(c) $y' = \frac{\sin(t)}{2+\cos(t)}y$	(g) $y' = 2y^2$
(d) $y' = -5y^2$	(h) $y' = 9y + 4x$

(2) Consider the differential equation

$$\frac{dy}{dx} = (y-1)(y-2)$$

Then,
(a) Use separation of variables to find a general formula for a solution $y(x)$ that is written in terms of the initial value $y(0)$.
(b) Provide two examples of solutions to differential equation which are constant.
(c) Determine what happens with $y(x)$ as $x \to \infty$ in these two cases 1) when $1 < y(0) < 2$ and when $y(0) > 2$.

(3) Consider the differential equation

$$\dot{x} = \frac{x^2}{(x+2)t^2}$$

and find a solution such that $x(0) = 0$ and find a solution such that $x(0) = 2$.

(4) Consider the first order (non-homogeneous) linear differential equation

$$\dot{x} - \tfrac{2}{t}x = \sin(t)$$

(a) Find all solutions to the equation

$$\dot{x} - \tfrac{2}{t}x = 0,$$

and express the undetermined parameter C in terms of the initial value, $x(0)$.
(b) Solve the original equation, taking the initial condition of zero.
(c) Find a solution to the original equation taking the value 3 at $t = 0$, check that this solution is a sum of one of the solutions from part a) with the solution from part b).

(5) Pair each function on the left column with the linear system it solves on the right column

i) $\mathbf{x}(t) = \begin{pmatrix} e^{4t} \\ e^{4t} \end{pmatrix}$	a) $\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} x$
ii) $\mathbf{x}(t) = \begin{pmatrix} 4e^{-t} \\ 3 \end{pmatrix}$	b) $\dot{x} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$
iii) $\mathbf{x}(t) = \begin{pmatrix} t + \frac{1}{2}(e^{2t} - 1) \\ \frac{1}{3}(e^{3t} - 1) \end{pmatrix}$	d) $\dot{x} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} x$
iv) $\mathbf{x}(t) = \begin{pmatrix} \cos(t+1) \\ \sin(t+1) \end{pmatrix}$	c) $\dot{x} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} x + \begin{pmatrix} 2 - 2t \\ 1 \end{pmatrix}$

(6) The functions

$$x_1(t) = e^{9t} \text{ and } x_2(t) = e^{-3t}$$

both solve the second order linear differential equation

$$\ddot{x} - 6\dot{x} - 27x = 0.$$

Using this information, find a solution x to the above equation for each of the following initial conditions

(a) $x(0) = 0,\ \dot{x}(0) = 1$	(d) $x(0) = -11,\ \dot{x}(0) = 8$
(b) $x(0) = 3,\ \dot{x}(0) = 1$	(e) $x(0) = 10,\ \dot{x}(0) = 23$
(c) $x(0) = -13,\ \dot{x}(0) = 59$	(f) $x(0) = 8,\ \dot{x}(0) = 4$