Problem Set 7

(1) Solve the IVP for each linear second order equation given bellow
$$\text{(a) } 3\ddot{x} - \dot{x} + 2u = 0, \; x(0) = 2, \; \dot{x}(0) = 0$$
$$\text{(b) } 5\ddot{x} + 2\dot{x} + 7x = 0, \; x(0) = 2, \; \dot{x}(0) = 1$$
$$\text{(c) } \ddot{x} + 2\dot{x} + 6\dot{x} = 0, \; x(0) = 2, \dot{x}(0) = -1$$

(2) Find the general solution to the following two systems
$$\text{(a) } \dot{x} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 1 & 4 \end{pmatrix} x \quad \text{(b) } \dot{x} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -4 & -1 \\ 0 & 1 & -4 \end{pmatrix} x$$

Don't forget to provide the final answer in terms of solutions that are real-valued only.

(3) For each pair of matrices A and $\Psi(t)$ given bellow compute $\dot{\Psi}(t)$ and $A\Psi(t)$.
$$\text{(a) } A = \begin{pmatrix} -10 & 0 \\ 0 & 5 \end{pmatrix}, \quad \Psi(t) = \begin{pmatrix} e^{-10t} & 0 \\ 0 & e^{5t} \end{pmatrix}$$
$$\text{(b) } A = \begin{pmatrix} -10 & 0 \\ 0 & 5 \end{pmatrix}, \quad \Psi(t) = \begin{pmatrix} e^{-10t} & -e^{-10t} \\ 3e^{5t} & -2e^{5t} \end{pmatrix}$$
$$\text{(c) } A = \begin{pmatrix} 0 & -8 & 0 \\ 8 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad \Psi(t) = \begin{pmatrix} \cos(8t) & -\sin(8t) & 0 \\ \sin(8t) & \cos(8t) & 0 \\ 0 & 0 & e^{-2t} \end{pmatrix}$$

(4) (BONUS) Solve problems #3 (d),(e), and (f) from the Exam 2 Practice.

(5) (BONUS) Find the general solution to the system $\dot{x} = Ax$, where
$$A = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & -4 & 7 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}$$