Texas State University
MATH 3323: Differential Equations
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Problem Set 9

(1) For the differential equation and pairs of numbers λ_1 and λ_2 given in each case bellow, find numbers A and B such that the function

$$Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

is a solution to the inhomogeneous equation.

(a) $\ddot{x} + \dot{x} = e^{2t} + e^{-2t},\ \lambda_1 = 2,\ \lambda_2 = -2$

(b) $\ddot{x} + x = 3e^{2t} - e^{3t},\ \lambda_1 = 1,\ \lambda_2 = 3$

(c) $\ddot{x} + 2\dot{x} + 6x = \sqrt{5}e^{-t} + 1,\ \lambda_1 = -1,\ \lambda_2 = 0.$

(2) For the differential equation and frequency ω given in each case bellow, find numbers A and B such that the function

$$A\cos(\omega t) + B\sin(\omega t)$$

is a solution to the inhomogeneous equation.

(a) $3\ddot{x} - \dot{x} + 2x = 3\cos(2t),\quad \omega = 2$

(b) $5\ddot{x} + 2\dot{x} + 7x = \sin(3t) + \cos(3t),\quad \omega = 3$

(c) $\ddot{x} + 2\dot{x} + 6x = -11\cos(4t) + 3\sin(4t),\quad \omega = 4.$

(3) Use variation of parameters to find the solution to each IVP

(a) $\ddot{x} - 2x = 3t,\ x(0) = 0, \dot{x}(0) = 1$

(b) $\ddot{x} - 2\dot{x} - x = 2e^{-5t} - e^{-7t},\ x(0) = 3, \dot{x}(0) = -1.$

(c) $\ddot{x} - 2\dot{x} - x = 2t - 1,\ x(0) = -3, \dot{x}(0) = 1$

(4) (BONUS) For each equation bellow, find the solution to the problem with initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$,

(a) $\ddot{x} + \dot{x} + x = e^{t}$

(b) $\ddot{x} + \dot{x} + x = e^{2t}$

(c) $\ddot{x} + \dot{x} + x = e^{3t}$

(d) $\ddot{x} + \dot{x} = e^{3t} + e^{2t} + e^{t}$

(e) $\ddot{x} + \dot{x} = 7e^{3t} - 2e^{2t} + 3e^{t}$

(5) (BONUS) For each equation bellow, find the solution to the problem with initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$,

(a) $\ddot{x} + \dot{x} + x = \cos(t)$

(b) $\ddot{x} + \dot{x} + x = \sin(t)$

(c) $\ddot{x} + \dot{x} + x = \cos(2t)$

(d) $\ddot{x} + \dot{x} = 2\cos(t)^2 + 4\sin(t) - 1$

(e) $\ddot{x} + \dot{x} = 4\cos(t) + 7\sin(t)$