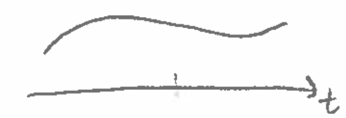
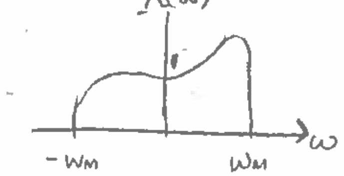
PS08 Nitya Dhanushkodi

1. $x(t)$



$X(\omega)$



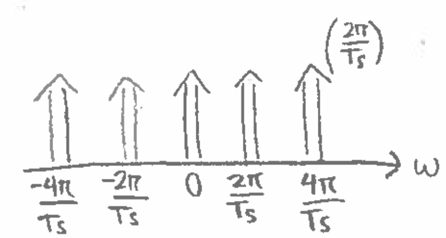
$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT_s)$ train of impulses

$x_p(t) = x(t)\,p(t)$

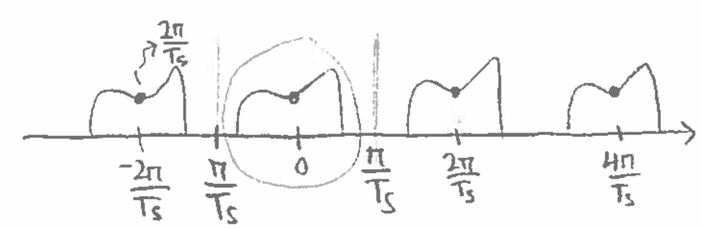
e) scale by $\frac{T_s}{2\pi}$, low pass filter $X_p(\omega)$

a) $x_p(t)$

b) $P(\omega)$



c) $X_p(\omega)$



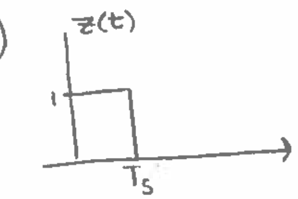
d) Make sure they don't overlap

$$\boxed{\frac{2\pi}{T_s} > 2\omega_M}$$

b/c $\frac{2\pi}{T_s} - \omega_m > \omega_m$

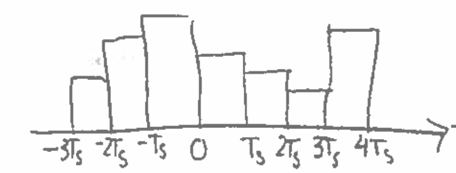
$\frac{2\pi}{T_s} > 2\omega_m$

f) $z(t)$



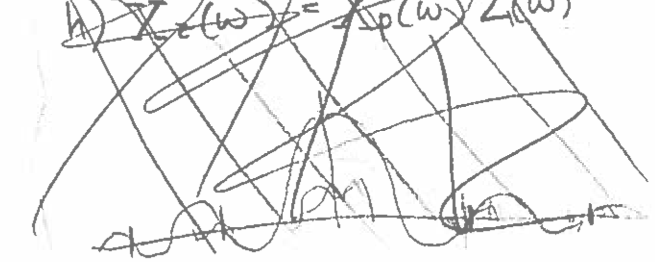
rectangular pulse

g) $x_z(t) = x_p * z(t)$



This is a zero-order hold.

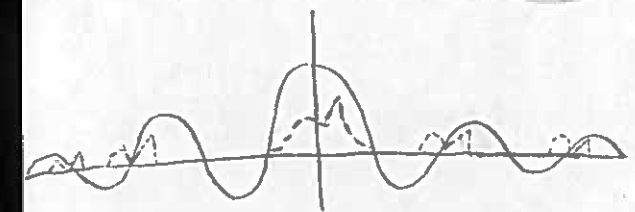
~~h) $X_z(\omega) = X_p(\omega) Z(\omega)$~~



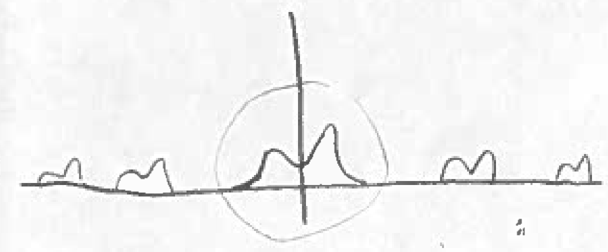
h) $X_z(\omega) = X_p(\omega)\underbrace{Z_1(\omega)}$ → $\underbrace{\frac{2\sin(\omega\frac{T_s}{2})}{\omega} \cdot \underbrace{e^{j\frac{T_s}{2}}}_{\text{offset of box}}}_{\text{This is the FT of } z(t)}$

$Z(\omega)$

$--- = X_z(\omega)$

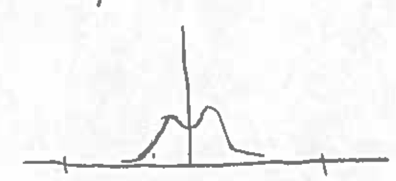


$X_z(\omega)$

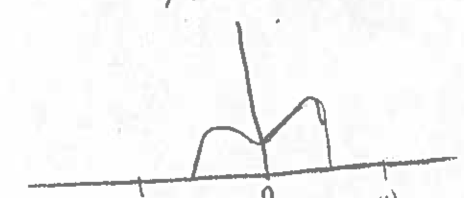


i) $-\omega_c$ to ω_c encompasses the parts circled in parts c) and h). This part is kept because we multiply it by 1 but the rest is cut off by $H(\omega)$

$\bar{X}(\omega)$



$\hat{X}(\omega)$



j) The sinc function that is multiplied by $X_p(\omega)$ to make $X_z(\omega)$ attenuates higher frequencies in $\bar{X}(\omega)$, while $\hat{X}(\omega)$ stays the same.

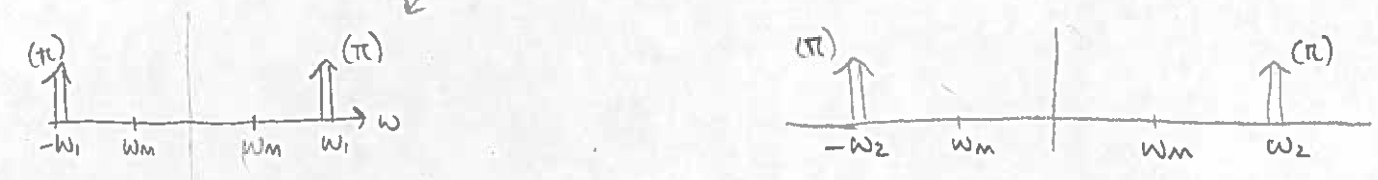
k) ratio $\bar{X}(\omega_m)$ to $\hat{X}(\omega_m)$ when $\omega_m = \frac{\pi}{T_s}$

$$\frac{\bar{X}(\omega_m)}{\hat{X}(\omega_m)} = \frac{\cancel{X_p(\omega_m)}Z_1(\omega_m)}{\cancel{X_p(\omega_m)}} = Z_1(\omega_m) = \frac{2\sin(\omega\frac{T_s}{2})}{\omega} \cdot e^{j\frac{T_s}{2}}$$

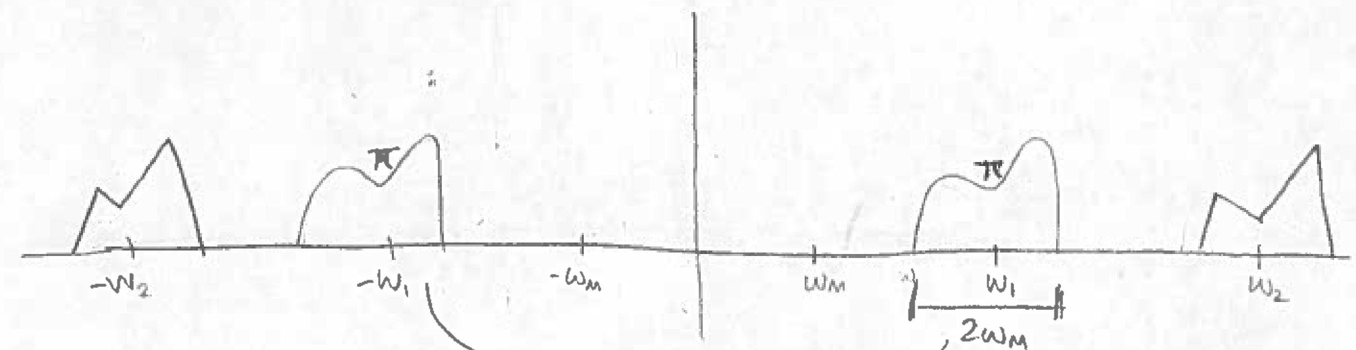
plug in $\frac{\pi}{T_s}$ → $\frac{2\sin(\frac{\pi}{T_s}\cdot\frac{T_s}{2})}{\frac{\pi}{T_s}} = \frac{2\sin(\frac{\pi}{2})}{\frac{\pi}{T_s}} = \frac{2}{\frac{\pi}{T_s}} = \boxed{\frac{2T_s}{\pi}\cdot e^{j\frac{T_s}{2}}}$

2. $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$

$$Y(\omega) = X_1(\omega) * \underbrace{\left[\pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1)\right]} + X_2(\omega) * \underbrace{\left[\pi\delta(\omega-\omega_2) + \pi\delta(\omega+\omega_2)\right]}$$

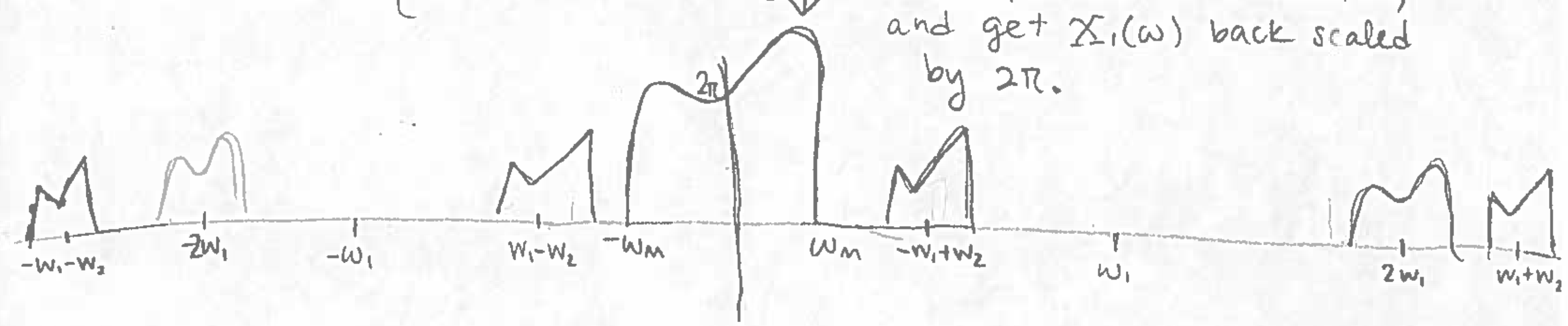


a) $Y(\omega)$

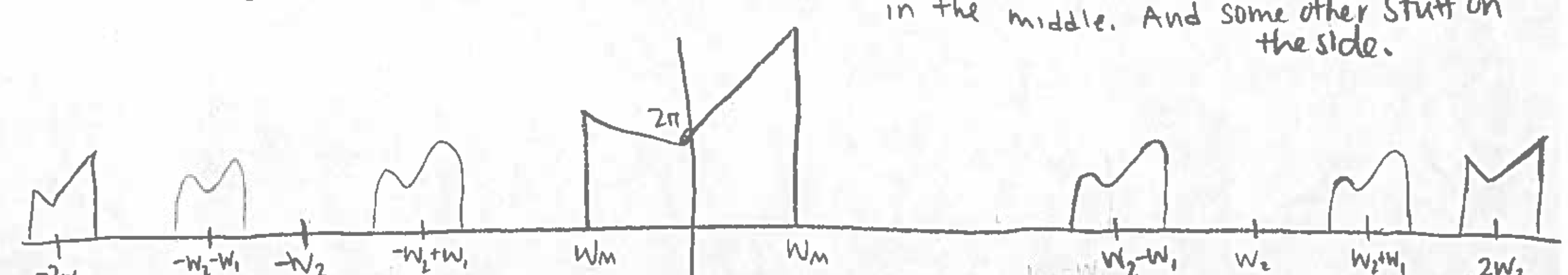


b) FT of $y(t)\cos(\omega_1 t)$ ⟶ all of $Y(\omega)$ centered at ω_1 means that you move ω_1 away from ω_1 back to the center, and get $X_1(\omega)$ back scaled by 2π.

$= Y(\omega) * \left[\pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1)\right]$

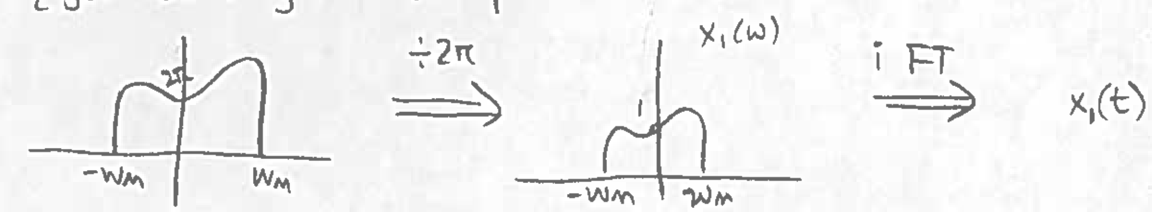


FT of $y(t)\cos(\omega_2 t)$ ⟶ now, I'll get back $X_2(\omega)$ scaled by 2π in the middle. And some other stuff on the side.

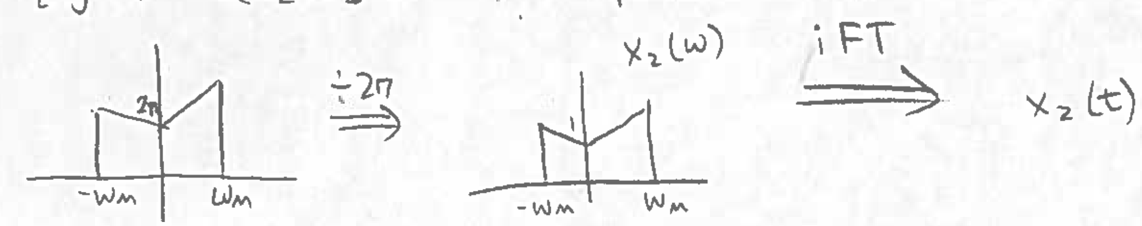


2. c) We want just the middle part (low frequencies) of FT$\{y(t)\cos(\omega_1 t)\}$ and FT$\{y(t)\cos(\omega_2 t)\}$. So low pass filter the FT $\{y(t)\cos(\omega_1 t)\}$ and FT$\{y(t)\cos(\omega_2 t)\}$ with a cutoff of ω_M to get $X_1(\omega)$ and $X_2(\omega)$ scaled by 2π. Divide by 2π, and take the inverse FT to get $x_1(t)$ and $x_2(t)$.

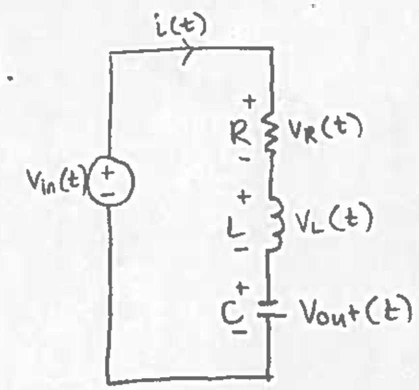
FT$\{y(t)\cos(\omega_1 t)\}$ $\cdot$ low pass filter



FT$\{y(t)\cos(\omega_2 t)\}$ $\cdot$ low pass filter



3.



(3) $i(t) = C\frac{d}{dt} V_{out}(t)$

(4) $v_L(t) = L\frac{d}{dt} i(t)$

a) $V_{in}(t) = v_R(t) + v_L(t) + v_{out}(t)$

$v_R(t) = Ri(t)$

$v_L(t) = L\frac{d}{dt} i(t)$

$i(t) = C\frac{d}{dt} V_{out}(t)$

$$\boxed{V_{in}(t) = RC\frac{d}{dt} V_{out}(t) + LC\frac{d^2}{dt^2} V_{out}(t) + V_{out}(t)}$$

b) Take FT of expression from a)

$$V_{in}(\omega) = j\omega RC V_{out}(\omega) + \overset{\to -1}{j^2}\omega^2 LC V_{out}(\omega) + V_{out}(\omega)$$

$$\boxed{H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{j\omega RC - \omega^2 LC + 1}}$$

c) $$|H(\omega)| = \frac{1}{|\underbrace{j\omega RC}_{} \underbrace{- \omega^2 LC + 1}_{real}|} = \boxed{\frac{1}{\sqrt{\omega^2(RC)^2 + (1-\omega^2 LC)^2}}}$$

d) Minimize bottom half of fraction to maximize H(ω)!

- then max

$$\frac{d}{d\omega}\left(\sqrt{(\omega RC)^2 + (1-\omega^2 LC)^2}\right) = 0$$

can ignore $\sqrt{\ }$

$$\frac{d}{d\omega}\left((\omega RC)^2 + (1-\omega^2 LC)^2\right) = 0$$

$$2\omega RC \cdot RC + 2(1-\omega^2 LC) \cdot (0 - 2\omega LC) = 0$$

$$2\omega R^2C^2 + -4\omega LC(1-\omega^2 LC) = 0$$

$$2\omega R^2C^2 - 4\omega LC + 4\omega^3 L^2C^2 = 0$$

$$(2\omega C)(R^2C - 2L + 2\omega^2 L^2 C) = 0$$

$$R^2C - 2L + 2\omega^2 L^2 C = 0$$

$$2\omega^2 L^2 C = 2L - R^2 C$$

$$\omega^2 = \frac{2L - R^2C}{2L^2C}$$

$$\omega = \pm\sqrt{\frac{2L - R^2C}{2L^2C}} \quad \text{or} \quad \omega = 0$$

d) continued

I need to make sure that particular ω minimizes the bottom of the fraction from part c), so I need to take the derivative again and check if it's positive to indicate a minimum.

$$\frac{d}{d\omega} = 2\omega R^2C^2 - 4\omega LC + 4\omega^3 L^2C^2 = 0$$

$$\frac{d}{d\omega}\left(\frac{d}{d\omega}\right) = 2R^2C^2 - 4LC + 4L^2C^2 \cdot 3\omega^2$$

$$= 2R^2C^2 - 4LC + 12L^2C^2\omega^2$$

$$= 2C(R^2C - 2L) + \cancel{12}^{6}\cancel{L^2}\cancel{C^2}\left(\frac{2L - R^2C}{\cancel{2L^2C}}\right)$$

$$= \underbrace{2C(R^2C - 2L)}_{\text{comes out negative}} + 6C\underbrace{(2L - R^2C)}_{\text{comes out positive}}$$

$2L > R^2C$ for ω to be real

$=$

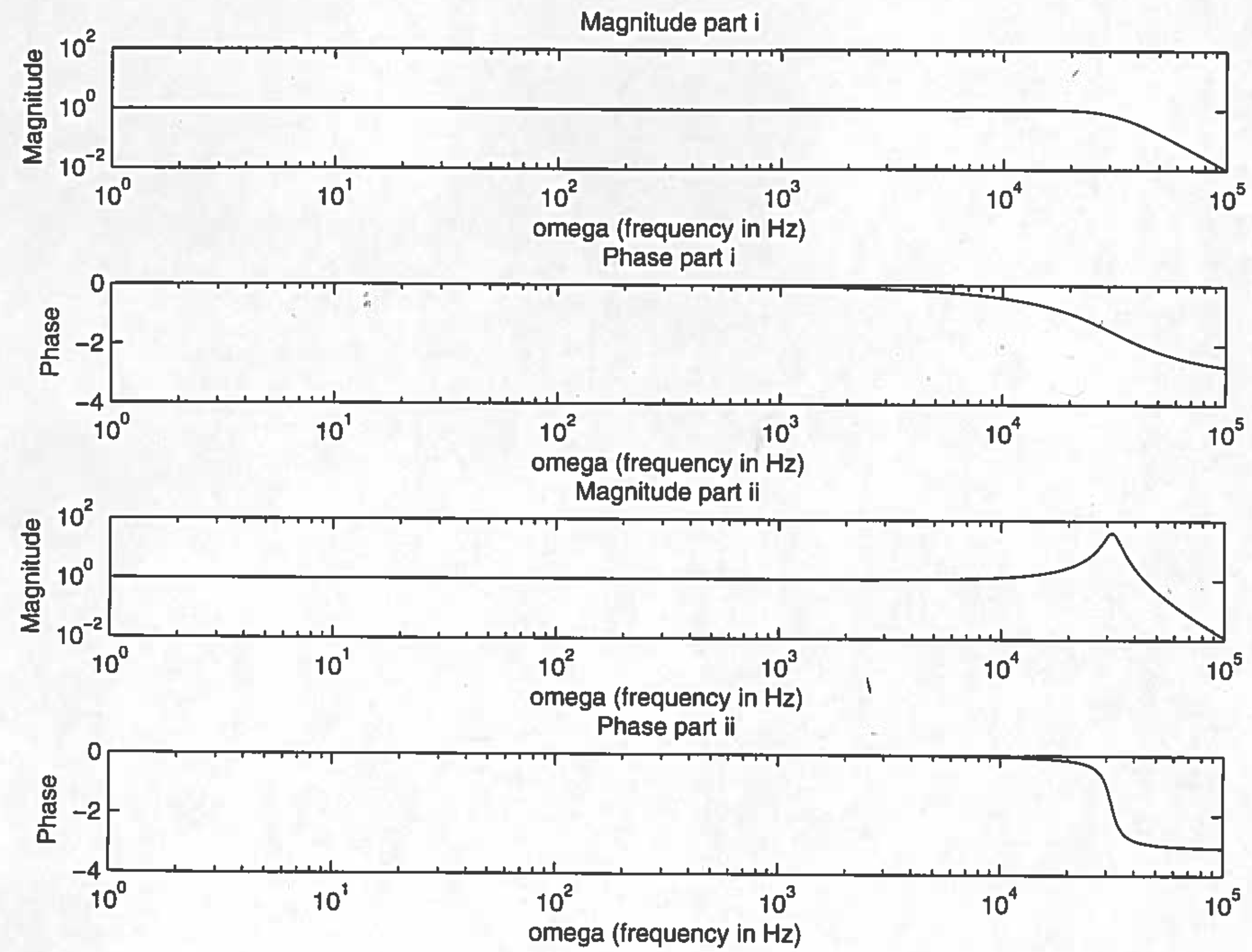
but this is bigger than $2C$, so this is positive for $\omega = \pm\sqrt{\frac{2L - R^2C}{2L^2C}}$

So now I know that $\boxed{\omega = \pm\sqrt{\frac{2L - R^2C}{2L^2C}}}$ to maximize $|H(\omega)|$.

This is the resonant frequency.

e) phase $\frac{1}{j\omega RC - \omega^2 LC + 1} \cdot \frac{j\omega RC + \omega^2 LC - 1}{j\omega RC + \omega^2 LC - 1} = \frac{j\omega RC + \omega^2 LC - 1}{-(\omega RC)^2 + -\omega^4 L^2C^2 + 2\omega^2 LC - 1}$

plots on next page


Magnitude part i
Magnitude
omega (frequency in Hz)
Phase part i
Phase
omega (frequency in Hz)
Magnitude part ii
Magnitude
omega (frequency in Hz)
Phase part ii
Phase
omega (frequency in Hz)

```
C = 10^-7;
L = 10^-2;
R1 = 400;
R2 = 50;

%atan imag/real

%mag 1
w = 1:1:100000;
Hmag=(1./((w.^2*R1.^2*C.^2)+(1-w.^2*L*C).^2));
subplot(4,1,1);
loglog(w,Hmag);
title('Magnitude part i');
xlabel('omega (frequency in Hz)');
ylabel('Magnitude');


%phase 1
w = 1:1:100000;
% real = 1./(1-(w.^2*L*C));
% imag = 1./(w*R1*C);

% imag = (w*R1*C)./(-(w.^2*R1^2*C^2) + -(w.^4*L^2*C^2) + 2*w.^2*L*C-1);
% real = (w.^2*L*C-1)./(-w.^2*R1^2*C^2 + -w.^4*L^2*C^2 + 2*w.^2*L*C-1);
% Hphase=atan(imag./real);
Hphase = angle(1./(j*w*R1*C-w.^2*L*C+1));
subplot(4,1,2);
semilogx(w,Hphase);
title('Phase part i');
xlabel('omega (frequency in Hz)');
ylabel('Phase');



%mag 2
w = 1:1:100000;
Hmag2=(1./((w.^2*R2.^2*C.^2)+(1-w.^2*L*C).^2));
subplot(4,1,3);
loglog(w,Hmag2);
title('Magnitude part ii');
xlabel('omega (frequency in Hz)');
ylabel('Magnitude');

%phase 2
w = 1:1:100000;
% real = 1./(1-(w.^2*L*C));
% imag = 1./(w*R2*C);
% imag = (w*R2*C)./(-(w.^2*R2^2*C^2) + -(w.^4*L^2*C^2) + 2*w.^2*L*C-1);
% real = (w.^2*L*C-1)./(-w.^2*R2^2*C^2 + -w.^4*L^2*C^2 + 2*w.^2*L*C-1);
%
% Hphase=atan(imag./real);
Hphase2 = angle(1./(j*w*R2*C-w.^2*L*C+1));
subplot(4,1,4);
semilogx(w,Hphase2);
title('Phase part ii');
xlabel('omega (frequency in Hz)');
ylabel('Phase');
```