Analysis of Algorithms
CS 312 - HBD1
Department of Physics and Computer Science
Medgar Evers College
Exam 2

Instructions:

- **The exam requires completing a set of tasks within 80 minutes.**
- **Write your solutions in the blue book provided.**
- **Use the pseudocode guidelines when requested.**
- **Notes are not allowed.**
- **Cheating of any kind is prohibited and will not be tolerated.**
- **Violating and/or failing to follow any of the rules will result in an automatic zero (0) for the exam.**

TO ACKNOWLEDGE THAT YOU HAVE READ AND UNDERSTOOD THE INSTRUCTIONS ABOVE, PRINT YOUR NAME AND THE DATE ON YOUR SUBMISSIONS

Grading

Section	Maximum Points	Points Earned
1	4	
2	4	
3	4	
4	4	
5	4	
Total	20	

1. **For each of the following recurrences, use the master theorem to find their theta boundary. You must show work to receive full credit.**

   a. $\mathbf{T(n) = \frac{1}{2}T(n/8) + n}$

   b. $\mathbf{T(n) = 4T(n/2) + 3\lg^2(n)}$

   c. $\mathbf{T(n) = 27T(n/3) + 20n^3}$

   d. $\mathbf{T(n) = 32T(n/4) + n^2\lg(n)}$

2. **Construct the pseudocode for the `Partition()` procedure used in the `QuickSort()` algorithm, such that the lower index is used as the pivot instead of the upper index.**

3. **Simulate the execution of the max-heap `BuildHeap()` algorithm for each of the given input arrays. For each simulation, begin with the input, and then illustrate the array after each swap in the heap-building process.**

   - `[48,31,92,62,96]`
   - `[3,12,21,30,18,27]`
   - `[27,65,94,80,32,11,98,70]`
   - `[38,35,86,28,72,40,39,81,10,56]`

4. **Construct the pseudocode of the `CountSort()` algorithm that only takes an array consisting only of digits (0 -9) as an input, and terminates with the array input sorted.**

5. **Calculate the runtime function of the pseudocode from Question 4 by constructing a detailed runtime table. Then, using this function, prove that the algorithm has $\Theta(n)$ time-complexity.**

Extra Credit

6. **Prove that the recurrence**

$$T(n) = 5T(n/25) + 2\sqrt{n}$$

is $\Theta(\sqrt{n}\lg(n))$ using the substitution method (mathematical induction). (2 points)