

POLYNOMIAL REGRESSION 1.

Taking data for a quadratic equation:

$$X = [1, 2, 3, 4, 5]$$

$$Y = [10, 19, 32, 49, 70]$$

We will try to fit with a quadratic curve [polynomial degree = 2]

Let's assume the quadratic equation,

$$y = ax^2 + bx + c$$

We need to solve for the coefficients a, b, c .

$$y = ax^2 + bx + c$$

$$xy = ax^3 + bx^2 + cx$$

$$x^2y = ax^4 + bx^3 + cx^2$$

$$\begin{bmatrix} x^2 & x & 1 \\ x^3 & x^2 & x \\ x^4 & x^3 & x^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y \\ xy \\ x^2y \end{bmatrix}$$

$$\begin{bmatrix} \sum x^4 & \sum x^3 & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum x \\ \sum x^2 & \sum x & n \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum x^2y \\ \sum xy \\ \sum y \end{bmatrix}$$

$$\sum x = 1 + 2 + 3 + 4 + 5 = 15$$

$$\sum y = 10 + 19 + 32 + 49 + 70 = 180$$

$$\sum x^2 = 1 + 4 + 9 + 16 + 25 = 55$$

$$\sum x^3 = 1 + 8 + 27 + 64 + 125 = 225$$

$$\sum x^4 = 1 + 16 + 81 + 256 + 625 = 979$$

$$\sum xy = 1 \times 10 + 2 \times 19 + 3 \times 32 + 4 \times 49 + 5 \times 70 = 690$$

$$\sum x^2y = 1 \times 10 + 4 \times 19 + 9 \times 32 + 16 \times 49 + 25 \times 70 = 2908$$

$$\begin{bmatrix} 979 & 225 & 55 \\ 225 & 55 & 15 \\ 55 & 15 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2908 \\ 690 \\ 180 \end{bmatrix}$$

$$55a + 15b + 5c = 180 \quad \text{--- (1)}$$

$$225a + 55b + 15c = 690 \quad \text{--- (2)}$$

$$979a + 225b + 55c = 2908 \quad \text{--- (3)}$$

$$\textcircled{1} \times 3 \Rightarrow 165a + 45b + 15c = 540$$

$$\textcircled{2} \times 1 \Rightarrow \begin{array}{cccc} 225a & + & 55b & + & 15c & = & 690 \\ \text{---} & & & & & & \end{array}$$

$$-60a - 10b = -150$$

$$60a + 10b = 150 \quad \text{--- (A)}$$

$$\textcircled{1} \times 11 \Rightarrow 605a + 165b + 55c = 1980$$

$$\textcircled{3} \times 1 \Rightarrow \begin{array}{cccc} 979a & + & 225b & + & 55c & = & 2908 \\ \text{---} & & & & & & \end{array}$$

$$-374a - 60b = -928$$

$$374a + 60b = 928 \quad \text{--- (B)}$$

$$\textcircled{A} \times 6 \Rightarrow 360a + 60b = 900$$

$$\textcircled{B} \times 1 \Rightarrow \begin{array}{cccc} 374a & + & 60b & = & 928 \\ \text{---} & & & & \end{array}$$

$$-14a = -28$$

$$\boxed{a = 2}$$

$$a = 2 \text{ in (A)}$$

$$120 + 10b = 150 \Rightarrow 10b = 30 \Rightarrow \boxed{b = 3}$$

Sub a & b in ①

$$55 \times 2 + 15 \times 3 + 5c = 180$$

$$5c = 180 - 110 - 45$$

$$5c = 25$$

$$\boxed{c = 5}$$

The Equation

$$y = ax^2 + bx + c$$

$$\boxed{y = 2x^2 + 3x + 5}$$