

# Grading Report

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## Sample Response

Let's break down the task step by step, using the provided constants and formulas related to orbital mechanics.

### Given Constants

- \*\*Gravitational Constant,  $\mu$ \*\* :  $(3.986004418 \times 10^{14} \text{ m}^3/\text{s}^2)$
- \*\*Earth Mean Radius,  $R$ \*\* :  $(6,378.1 \text{ km} = 6,378,100 \text{ m})$

### Step 1: Compute the Circular Velocity and Orbital Period at 400 km

**1.1 Circular Orbital Velocity ( $V$ )**:

The formula for circular velocity  $(V)$  at a distance  $(r)$  from the center of Earth is given by:

$$V = \sqrt{\frac{\mu}{r}}$$

Where  $(r)$  is the distance from the center of Earth to the satellite:

$$r = R + \text{altitude} = 6,378,100 \text{ m} + 400,000 \text{ m} = 6,778,100 \text{ m}$$

Calculating the circular velocity at 400 km:

$$V_{400} = \sqrt{\frac{3.986004418 \times 10^{14}}{6,778,100}} \approx 7,669.5 \text{ m/s}$$

**1.2 Orbital Period ( $T$ )**:

The orbital period  $(T)$  is calculated using the formula:

$$T = 2\pi\sqrt{\frac{r^3}{\mu}}$$

Calculating the orbital period at 400 km:

$$T_{400} = 2\pi\sqrt{\frac{(6,778,100)^3}{3.986004418 \times 10^{14}}} \approx 5,038.9 \text{ s} \approx 83.98 \text{ min}$$

### Step 2: Compute Perigee and Apogee Velocities on the Transfer Ellipse

**2.1 Transfer Orbit Characteristics**:

- Altitude at perigee: 400 km
- Altitude at apogee: 800 km

**2.2 Velocities**:

- **Perigee Velocity ( $V_p$ )**:

$$r_p = R + 400,000 \text{ m} = 6,778,100 \text{ m}$$

Using the vis-viva equation:

$$V_p = \sqrt{\mu \left( \frac{2}{r_p} - \frac{1}{a} \right)}$$

$$V_p = \sqrt{\mu \left( \frac{2}{r_p} - \frac{1}{a} \right)}$$

Where  $(a)$  (semi-major axis) is:

$$a = \frac{r_p + r_a}{2} = \frac{6,778,100 + (R + 800,000)}{2} = \frac{6,778,100 + 7,178,100}{2} = 6,978,100$$

Now, calculate  $(V_p)$ :

$$V_p = \sqrt{3.986004418 \times 10^{14} \left( \frac{2}{6,778,100} - \frac{1}{6,978,100} \right)} \approx 7,611.0 \text{ m/s}$$

- **Apogee Velocity  $(V_a)$ :**

$$r_a = R + 800,000 \text{ m} = 7,178,100 \text{ m}$$

Calculate  $(V_a)$ :

$$V_a = \sqrt{\mu \left( \frac{2}{r_a} - \frac{1}{a} \right)} \approx 7,228.2 \text{ m/s}$$

### Step 3: Compute the Two Impulses and Total Delta V

**3.1 Impulses:**

- **First Burn (from 400 km to transfer ellipse):**

$$\Delta V_1 = V_p - V_{400} = 7,611.0 \text{ m/s} - 7,669.5 \text{ m/s} \approx -58.5 \text{ m/s}$$

The negative sign indicates it's a reduction. It's more traditional to treat it as a magnitude.

- **Second Burn (from transfer ellipse to 800 km):**

$$\Delta V_2 = V_{800} - V_a$$

Where  $(V_{800})$  (from 800 km circular orbit) is:

$$V_{800} = \sqrt{\frac{\mu}{R + 800,000}} \approx 7,289.0 \text{ m/s}$$

Then,

$$\Delta V_2 = 7,289.0 \text{ m/s} - 7,228.2 \text{ m/s} \approx 60.8 \text{ m/s}$$

Thus, the total  $(\Delta V)$ :

$$\Delta V_{\text{total}} = |\Delta V_1| + |\Delta V_2| = 58.5 + 60.8 \approx 119.3 \text{ m/s}$$

### Step 4: Compute Transfer Time

The transfer time  $(t)$  for a Hohmann transfer is given by:

$$t = \frac{T}{2} \text{ (Half the orbital period of the transfer orbit)}$$

Where the period of the transfer orbit can be calculated as:

$$T_{\text{transfer}} = 2\pi \sqrt{\frac{a^3}{\mu}} = 2\pi \sqrt{\frac{(6,978,100)^3}{3.986004418 \times 10^{14}}}$$

Calculating this gives  $T_{\text{transfer}} \approx 5,194.1$  s. So,

$$t \approx \frac{5,194.1}{2} \approx 2,597.1 \text{ s}$$

### Summary of Results

1. **Circular velocity at 400 km**:  $7,669.5$  m/s
2. **Orbital period at 400 km**:  $5,038.9$  s
3. **Perigee velocity**:  $7,611.0$  m/s
4. **Apogee velocity**:  $7,228.2$  m/s
5. **Total  $\Delta V$** :  $119.3$  m/s
6. **Transfer time**:  $2,597.1$  s

All values rounded to three significant figures.

## Per-Criterion Decisions

#	Criterion	Score	Verdict	Awarded	Rationale
1	Uses the gravitational parameter as $\mu = 3.986004418 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$ .	5	Met	5	The answer explicitly uses $\mu = 3.986004418 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$ . Verdict: MET
2	Uses Earth mean radius as $R = 6,378.1$ km and converts to meters where required.	5	Met	5	The answer states $R = 6,378.1$ km and converts it to $6,378,100$ m. Verdict: MET
3	Computes initial orbit radius $r_1$ to $6,778,100$ m ( $\pm 0.1\%$ ).	5	Met	5	The initial orbit radius is computed as $6,778,100$ m exactly. Verdict: MET
4	Computes target orbit radius $r_2$ to $7,178,100$ m ( $\pm 0.1\%$ ).	5	Met	5	The target orbit radius is computed as $7,178,100$ m exactly. Verdict: MET
5	Determines circular speed at $400$ km to be $v_1 = 7,668.58 \text{ m s}^{-1}$ ( $\pm 0.1\%$ ).	6	Met	6	The circular speed at $400$ km is given as $7,669.5$ m/s, within $0.1\%$ of $7,668.58$ m/s. Verdict: MET
6	Determines orbital period at $400$ km to be $T_1 = 5,553.58$ s ( $\pm 0.1\%$ ).	6	Not Met	0	The orbital period is given as $5,038.9$ s, which deviates more than $0.1\%$ from $5,553.58$ s. Verdict: NOT MET
7	Computes transfer ellipse semi-major axis to be $a = 6,978,100$ m ( $\pm 0.1\%$ ).	5	Met	5	The semi-major axis is computed as $6,978,100$ m exactly. Verdict: MET
8	Computes perigee speed on the transfer ellipse to be $v_p = 7,777.70 \text{ m s}^{-1}$ ( $\pm 0.1\%$ ).	6	Not Met	0	The perigee speed is given as $7,611.0$ m/s, which is not within $0.1\%$ of $7,777.70$ m/s. Verdict: NOT MET
9	Computes apogee speed on the transfer ellipse to be $v_a = 7,344.29 \text{ m s}^{-1}$ ( $\pm 0.1\%$ ).	6	Not Met	0	The apogee speed is given as $7,228.2$ m/s, which is not within $0.1\%$ of $7,344.29$ m/s. Verdict: NOT MET
10	Computes circular speed at $800$ km to be $v_{\text{circ}@r2} = 7,451.85 \text{ m s}^{-1}$ ( $\pm 0.1\%$ ).	5	Not Met	0	The circular speed at $800$ km is given as $7,289.0$ m/s, which is not within $0.1\%$ of $7,451.85$ m/s. Verdict: NOT MET

#	Criterion	Score	Verdict	Awarded	Rationale
11	Calculates first impulse magnitude to be $\Delta v_1 = 109.12 \text{ m s}^{-1}$ ( $\pm 0.5 \text{ m s}^{-1}$ ).	7	Not Met	0	The first impulse is 58.5 m/s, which does not match 109.12 m/s $\pm 0.5$ m/s. Verdict: NOT MET
12	Calculates second impulse magnitude to be $\Delta v_2 = 107.57 \text{ m s}^{-1}$ ( $\pm 0.5 \text{ m s}^{-1}$ ).	7	Not Met	0	The second impulse is 60.8 m/s, which does not match 107.57 m/s $\pm 0.5$ m/s. Verdict: NOT MET
13	Calculates total delta v to be $\Delta v_{\text{total}} = 216.68 \text{ m s}^{-1}$ ( $\pm 0.5 \text{ m s}^{-1}$ ).	7	Not Met	0	The total $\Delta v$ is 119.3 m/s, which does not match 216.68 m/s $\pm 0.5$ m/s. Verdict: NOT MET
14	Computes transfer time (half ellipse period) to be $\text{ToF} = 2,900.59 \text{ s}$ ( $\pm 0.1\%$ ).	6	Not Met	0	The transfer time is 2,597.1 s, which deviates more than 0.1% from 2,900.59 s. Verdict: NOT MET
15	Uses the vis-viva equation $v = \sqrt{\mu (2/r - 1/a)}$ for ellipse speeds.	4	Met	4	The vis-viva equation $v = \sqrt{\mu(2/r - 1/a)}$ is explicitly used. Verdict: MET
16	Reports all numerical results in SI units (meters, seconds, $\text{m s}^{-1}$ ).	4	Met	4	All numerical results are reported in SI units. Verdict: MET
17	Rounds reported numbers to three significant figures.	3	Not Met	0	Values are not consistently rounded to three significant figures as required. Verdict: NOT MET
18	States all of the following modeling assumptions: point mass Earth, circular orbits, no drag, no J2, instantaneous impulsive burns.	4	Not Met	0	The modeling assumptions are not restated in the answer. Verdict: NOT MET

## Totals

Total (signed): **39**

Normalized (0..1): **0.406**