Corrigé loi de Poisson

1) On a $p(x|\omega_1) = \dfrac{e^{-\lambda_1}\lambda_1{}^x}{x!}$ et $p(x|\omega_2) = \dfrac{e^{-\lambda_2}\lambda_2{}^x}{x!}$

on décide la classe 1 si $P(\omega_1|x) > P(\omega_2|x) \Leftrightarrow \dfrac{p(x|\omega_1)P(\omega_1)}{p(x)} > \dfrac{p(x|\omega_2)P(\omega_2)}{p(x)}$

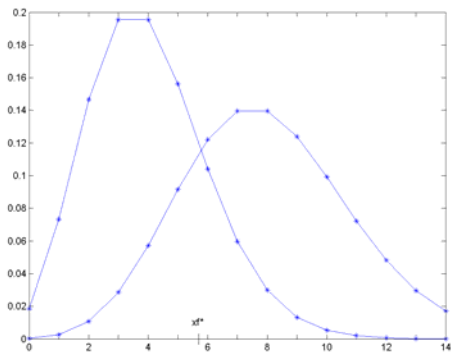
$$\frac{e^{-\lambda_1}\lambda_1{}^x}{x!}P(\omega_1) > \frac{e^{-\lambda_2}\lambda_2{}^x}{x!}P(\omega_2) \Leftrightarrow \left(\frac{\lambda_1}{\lambda_2}\right)^x > e^{\lambda 1-\lambda 2}\frac{P(\omega_2)}{P(\omega_1)}$$

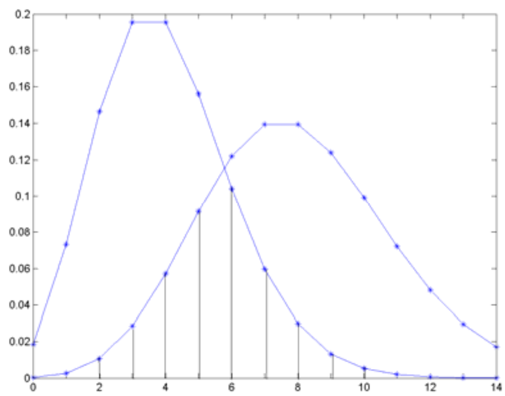
Le point frontière est : $x^* = \dfrac{\lambda_1 - \lambda_2 + LogP(\omega 2) - LogP(\omega 1)}{Log(\lambda_1) - Log(\lambda_2)}$

application : $\lambda 1=8$, $\lambda 2=4$, classes équiprobables

xf*= 5.7708

<u>complément (sous matlab)</u>





2) Probabilité d'erreur :
Pe= P ($x \in R_1, \omega_2$) + P ($x \in R_2, \omega_1$)= P ($x \in R_1 |\omega_2$). P (ω_2) + P ($x \in R_2 |\omega_1$). P (ω_1)

PE1= P ($x \in R_2 |\omega_1$). P (ω_1)= $\displaystyle\sum_0^{x^*} P(\omega_1)\frac{e^{-\lambda_1}\lambda_1{}^x}{x!}$

PE= $\displaystyle\sum_0^{x^*} P(\omega_1)\frac{e^{-\lambda_1}\lambda_1{}^x}{x!} + \sum_{x^*}^{+\infty} P(\omega_2)\frac{e^{-\lambda_2}\lambda_2{}^x}{x!}$

complément
PE = 0.2812