

Alcuni esercizi sulla trasformata di Fourier

Calcolare la trasformata di Fourier delle seguenti funzioni

1. $g(t) = te^{-t^2}$

2. $g(t) = e^{-2t^2+4t}$

3. $g(t) = \cos t$

4. $g(t) = \cos(2t + 1)$

5. $g(t) = \cos te^{-3t}H(t)$

6. $g(t) = \frac{x^2}{1+x^2}$

7. $g(t) = |t|e^{-|t|}$

8. $g(t) = p_T(t - t_0)$ (dove $t_0 \in \mathbb{R}$ e $T > 0$)

9. $g(t) = \mathbf{1}_{[a,b]}$ (dove $a, b \in \mathbb{R}$ e $a < b$)

10. $g(t) = \begin{cases} 2 & \text{se } -1 < t < 0 \\ -1 & \text{se } 0 \leq t < 2 \\ 0 & \text{altrimenti} \end{cases}$

(suggerimento: scrivere g come somma di opportune funzioni porta).

11. $g(t) = te^{-|t+2|/2}$

12. $g(t) = \frac{t \sin(2t)}{(t^2 + 4)^2}$ (suggerimento: $\frac{d}{dt} \frac{1}{t^2+4} = ?$)

13. $g(t) = tH(t)$

14. $g(t) = |t|$

15. $g(t) = H(t - 2)e^{-2t}$

16. $g(t) = p_{2\pi}(t) \sin t$

17. $g(t) = e^{-5t} \sin t H(t)$

18. $g(t) = p_{\pi}(t) \cos t$

19. $g(t) = \mathbf{1}_{[1,2]}(t) \sin t$

Risposte:

1. $-i\pi\sqrt{\pi}\nu e^{-\pi^2\nu^2}$
2. $\sqrt{\frac{\pi}{2}}e^2e^{\pi\nu(2i-\pi\nu/2)}$
3. $\frac{1}{2}(\delta_{1/2\pi} + \delta_{-1/2\pi})$
4. $\frac{1}{2}(e^i\delta_{1/\pi} + e^{-i}\delta_{-1/\pi})$
5. $\frac{3 + 2\pi i\nu}{(3 + 2\pi i\nu)^2 + 1}$
6. $-\pi e^{-2\pi|\nu|} + \delta_0$
7. $\frac{-8\pi i\nu}{(1 + 4\pi^2\nu^2)^2}$
8. $e^{-2\pi i\nu t_0} \frac{\sin(\pi\nu T)}{\pi\nu}$
9. $e^{-\pi i\nu(a+b)} \frac{\sin(\pi\nu(b-a))}{\pi\nu}$
10. $\frac{1}{\pi\nu}(2e^{\pi i\nu} \sin(\pi\nu) - e^{-2\pi i\nu} \sin(2\pi\nu))$
11. $8ie^{4\pi i\nu} \frac{i(1 + 16\pi^2\nu^2) - 8\pi\nu}{(1 + 16\pi^2\nu^2)^2}$
12. $\frac{\pi}{4} [(1 - \pi\nu)e^{-4\pi|\nu-1/\pi|} + (1 + \pi\nu)e^{-4\pi|\nu+1/\pi|}]$
13. $\frac{1}{4\pi} \left[\frac{1}{\pi} (\text{v.p. } \frac{1}{\nu})' + i\delta_0' \right]$
14. $\frac{1}{2\pi^2} (\text{v.p. } \frac{1}{\nu})'$
15. $\frac{e^{-4(1+\pi i\nu)}}{2 + 2\pi i\nu}$
16. $\frac{1}{2i} \left[\frac{\sin(2\pi^2(\nu - 1/2\pi))}{\pi(\nu - 1/2\pi)} - \frac{\sin(2\pi^2(\nu + 1/2\pi))}{\pi(\nu + 1/2\pi)} \right]$
17. $\frac{1}{2i} \left[\frac{1}{5 + 2\pi i(\nu - 1/2\pi)} - \frac{1}{5 + 2\pi i(\nu + 1/2\pi)} \right]$
18. $\frac{1}{2} \left[\frac{\sin(\pi^2(\nu - 1/2\pi))}{\pi(\nu - 1/2\pi)} + \frac{\sin(\pi^2(\nu + 1/2\pi))}{\pi(\nu + 1/2\pi)} \right]$
19. $\frac{1}{2i} \left[e^{-3\pi i(\nu-1/2\pi)} \frac{\sin(\pi(\nu - 1/2\pi))}{\pi(\nu - 1/2\pi)} - e^{-3\pi i(\nu+1/2\pi)} \frac{\sin(\pi(\nu + 1/2\pi))}{\pi(\nu + 1/2\pi)} \right]$