Berlin Mathematics Research Center MATH+

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Modeling opinion dynamics under the impact of influencer and media strategies

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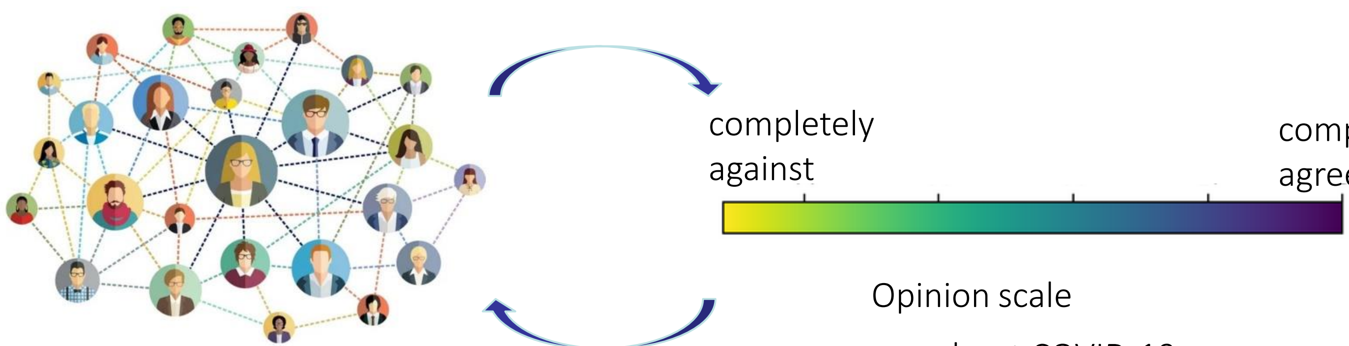
ASU/MATH+ Spring School

4.03.2024

Understanding social&opinion dynamics

Main questions:

- How does our **social environment** influence our **opinion formation**?
- How do our **opinions** shape our **social environment**?



Opinion scale

e.g. about COVID-19 measures

Social environment

https://medium.com/

Understanding social&opinion dynamics

Social environment
https://medium.com/

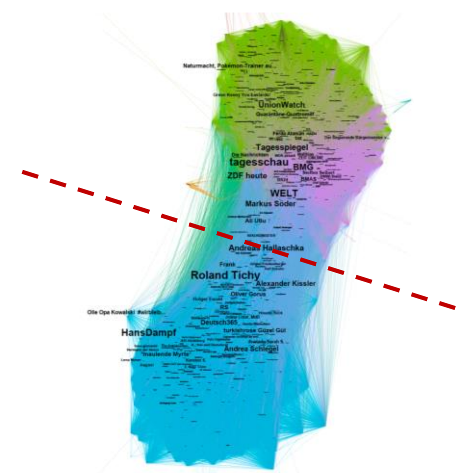
completely against

completely agree

Opinion scale e.g. about COVID-19 measures

Application examples:

- understanding **online discourse** on e.g. COVID-19:
  - How do opinions (co)-evolve in a population?
  - How do different actors influence the opinion dynamics?
- studying **polarization** on e.g. the German Twitter:
  - Why/how/when does a society polarize?
  - Can polarization be prevented/reduced?



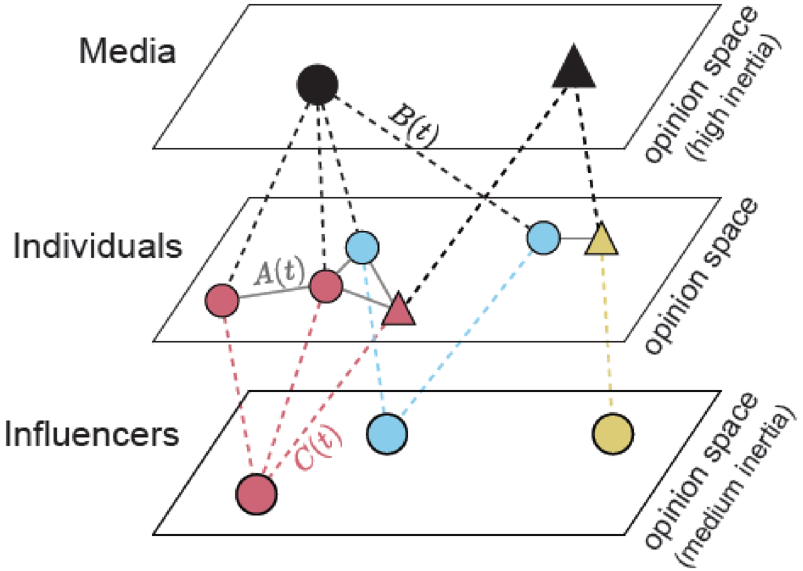
German retweet network (most active accounts)
keyword: “COVID”, spring 2020, **[Gidel, Lorenz-Spreen]**

Modeling opinion dynamics under the impact of influencer and media strategies

Model features:

- Three types of agents:
  - individuals $i = 1, \dots, N$,
  - media $m = 1, \dots, M$,
  - influencers $l = 1, \dots, L$.

  We assume $M < L \ll N$.

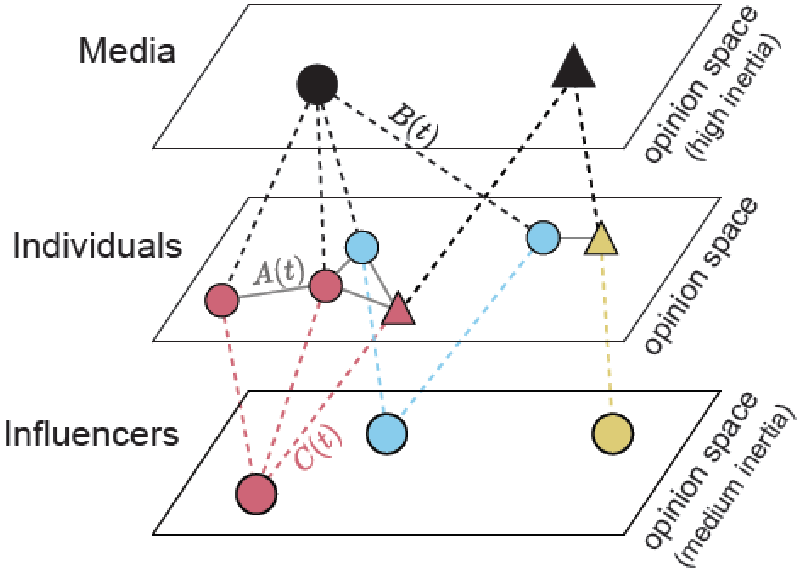


- Social environment at time t is given by social networks:
  - network between individuals $A(t) \in \{0,1\}^{NxN}$,
  - medium-follower network $B(t) \in \{0,1\}^{NxM}$,
  - influencer-follower network $C(t) \in \{0,1\}^{NxL}$.

Helfmann, Dj Conrad, Lorenz-Spreen, Schütte, 2023

Modeling opinion dynamics under the impact of influencer and media strategies

Original setting:
A,B are constant in time;
Each agent follows exactly 1 media and
Each agent follows exactly 1 influencer.



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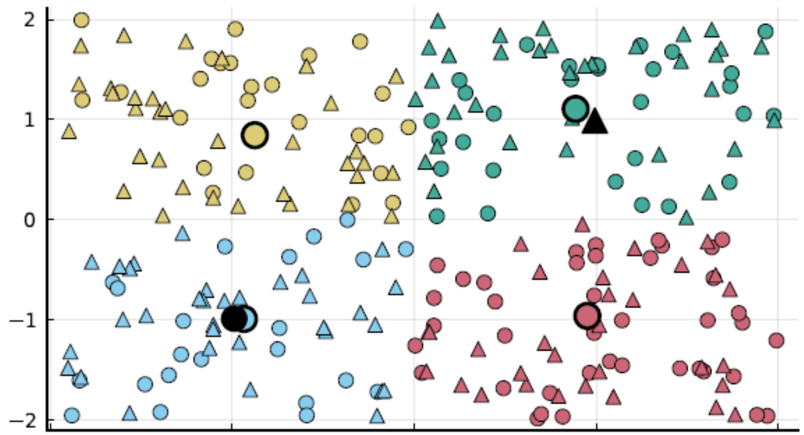
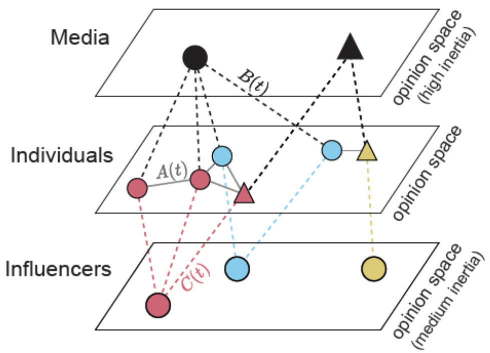
Helfmann, Dj Conrad, Lorenz-Spreen, Schütte, 2023

Modeling opinion dynamics under the impact of influencer and media strategies

- All agents have continuous opinions in an opinion space $D \subset \mathbb{R}^2$:
  - opinions $x_i(t)$ of N individuals,
  - opinions $y_m(t)$ of M media,
  - opinions $z_l(t)$ of L influencers.



- Agents change their opinions on different time scales: individuals (fast) - influencers, media (slow) influencers (γ) - media ($\gamma < \Gamma$).



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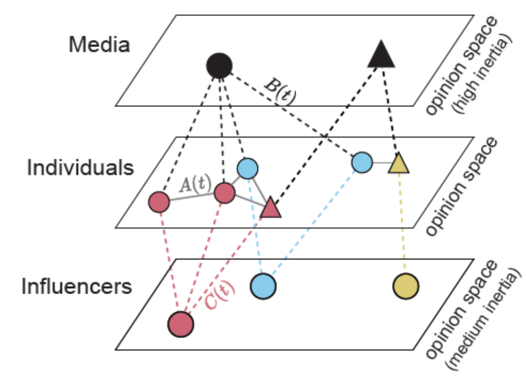
Modeling opinion dynamics

Opinion changes of individuals:



- opinion change of individual i given by

$$\frac{dx_i}{dt}(t) = \underbrace{F_i(x, y, z)}_{\text{interaction force on i}} \underbrace{+\sigma \frac{dW_i}{dt}(t)}_{\text{noise}}$$

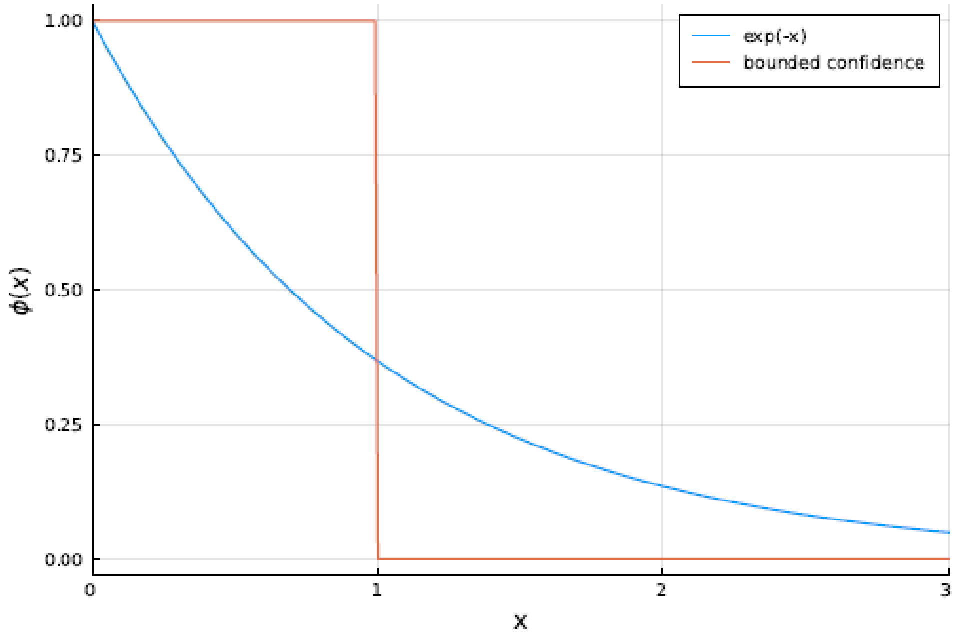
- with (attractive) interaction force on individual i

$$\begin{aligned} F_i(x, y, z) =& a \sum_{j=1}^{N} \frac{A_{ij}}{Z_i} \phi(|x_j(t) - x_i(t)|)(x_j(t) - x_i(t)) \\ &+ b \sum_{m=1}^{M} B_{im}(y_m(t) - x_i(t)) \\ &+ c \sum_{l=1}^{L} C_{il}(t)(z_l(t) - x_i(t)). \end{aligned}$$

Pairwise interaction function

possible pair functions $\phi(|x_j(t) - x_i(t)|)$:

- $\phi(x) = \exp(-x)$ (exponentially decaying in distance)
- $\phi(x) = 1_{[0,d]}(x)$ (bounded confidence)
- $\phi(x) = 1$ (DeGroot model)

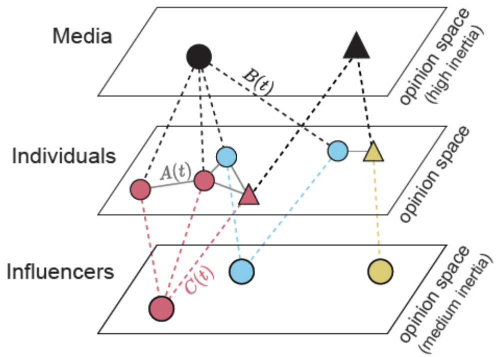


Slide L.Helfmann

Modeling opinion dynamics

Opinion changes of individuals:

$$\frac{dx_i}{dt}(t) = \underbrace{F_i(x,y,z)}_{\text{interaction force on i}} + \sigma \underbrace{\frac{dW_i}{dt}(t)}_{\text{noise}}$$



Opinion changes of media:

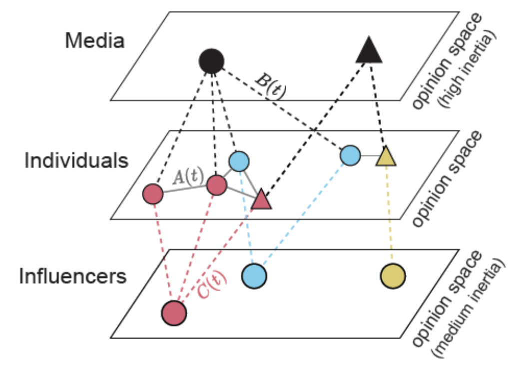
$$\Gamma dy_m(t) = f(\tilde{x}_m(t) - y_m(t))dt + \tilde{\sigma} d\tilde{W}_m(t),$$

where the force function f can be used to model nonlinear influence effects but is set to $f(x) = x$ subsequently, i.e., media agents are drawn in the direction of the average opinion of their followers

$$\tilde{x}_m(t) = \frac{1}{\sum_k B_{km}(t)} \sum_{i=1}^{N} B_{im}(t) x_i(t).$$

Note: media and influencers adapt their opinions on a much slower timescale compared to individuals.

Modeling opinion dynamics



Opinion changes of individuals:

$$\frac{dx_i}{dt}(t) = \underbrace{F_i(x,y,z)}_{\text{interaction force on i}} + \underbrace{\sigma\frac{dW_i}{dt}(t)}_{\text{noise}}$$

Opinion changes of media:

$$\underbrace{\Gamma}_{\text{resistance/inertia}} \frac{dy_m}{dt}(t) = \underbrace{\sum_{i=1}^{N} \frac{B_{im}}{\sum_k B_{km}}(x_i(t) - y_m(t))}_{\text{attraction force to average follower}} + \underbrace{\tilde{\sigma}\frac{d\tilde{W}_m}{dt}(t)}_{\text{noise}}$$

Opinion changes of influencers:

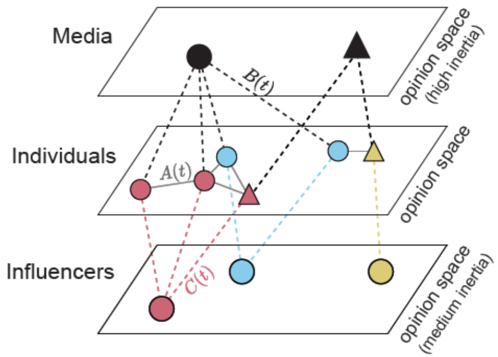
$$\gamma dz_l(t) = g(\hat{x}_l(t) - z_l(t))dt + \hat{\sigma} d\hat{W}_l(t),$$

where the average opinion of followers is given by

$$\hat{x}_l(t) = \frac{1}{\sum_k C_{kl}(t)} \sum_{i=1}^{N} C_{il}(t)\, x_i(t).$$

Note: media and influencers adapt their opinions on a much slower timescale compared to individuals.

Modeling opinion dynamics



Individuals:

$$\frac{dx_i}{dt}(t) = \underbrace{F_i(x,y,z)}_{\text{interaction force on i}} + \underbrace{\sigma \frac{dW_i}{dt}(t)}_{\text{noise}}$$

Media:

$$\underbrace{\Gamma}_{\text{resistance/inertia}} \frac{dy_m}{dt}(t) = \underbrace{\sum_{i=1}^{N} \frac{B_{im}}{\sum_k B_{km}}(x_i(t) - y_m(t))}_{\text{attraction force to average follower}} + \underbrace{\tilde{\sigma} \frac{d\tilde{W}_m}{dt}(t)}_{\text{noise}}$$

Influencers:

$$\gamma \frac{dz_l}{dt}(t) = \sum_{i=1}^{N} \frac{C_{il}(t)}{\sum_k C_{kl}(t)}(x_i(t) - z_l(t)) + \hat{\sigma} \frac{d\hat{W}_l}{dt}(t)$$

How can individuals switch the influencer?

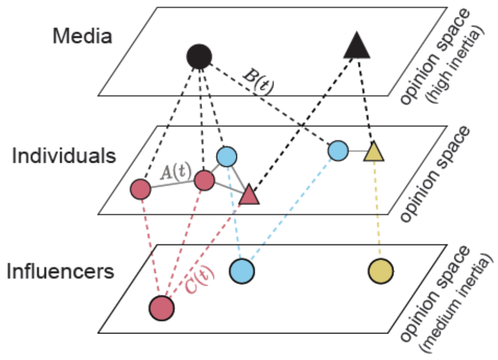
Each individual, following a media m, can at time t switch to influencer l with a given rate

$$\Lambda_m^{\to l}(x,t) = \eta\, \psi(|z_l - x|)\, r\left(\frac{n_{m,l}(t)}{\sum_{m'=1}^{M} n_{m',l}(t)}\right)$$

Scaling parameter

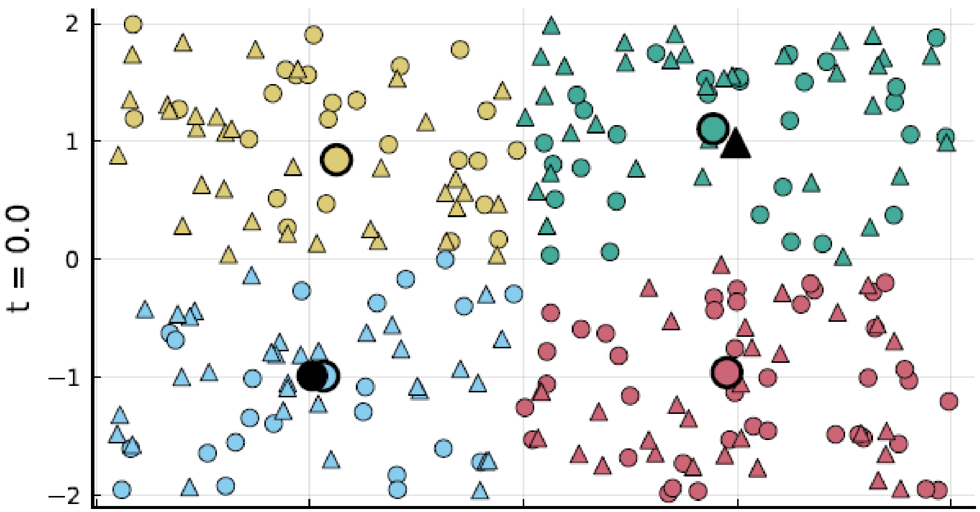
Influencer homophily: determines the rate of switching based on similarity of opinion z_l

Link recommendation function: individuals have a higher chance of switching to an influencer with a structurally similar followership

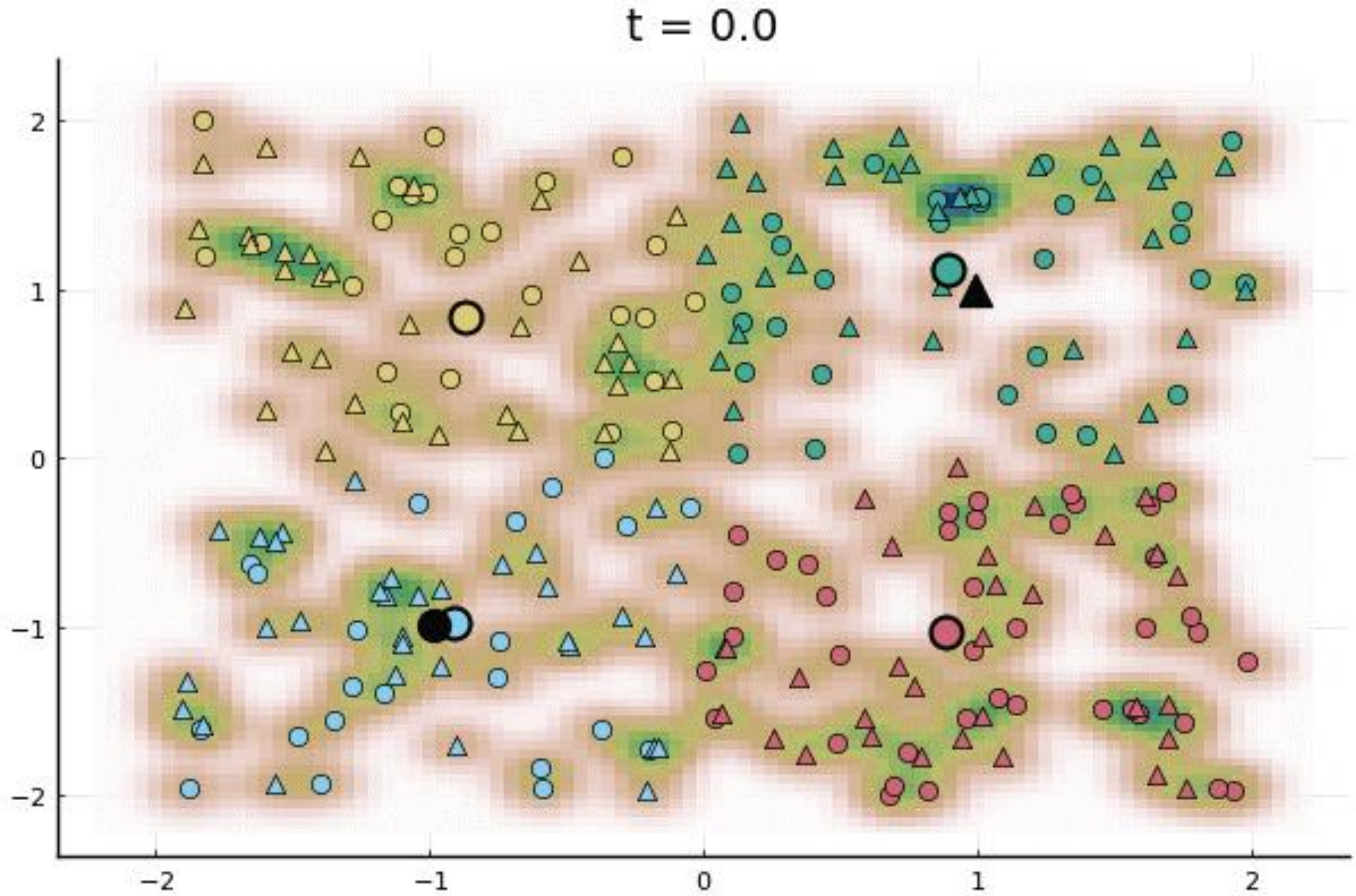


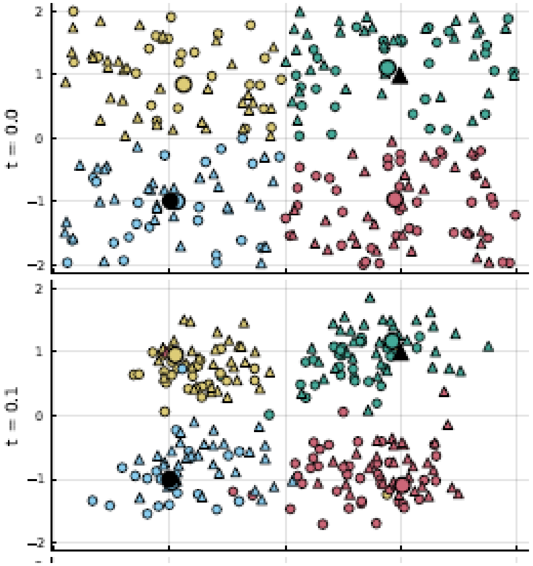
Example:

- 250 individuals, 2 media and 4 influencers.
- Initially individuals are **randomly distributed in opinion space** and uniformly at **random assigned to the 2 media**.
- The network A is **fully-connected**.
- Individuals in each of the 4 quadrants are **assigned to a different influencer.**
- Initial opinion of the influencer is set to the mean opinion of its followers.

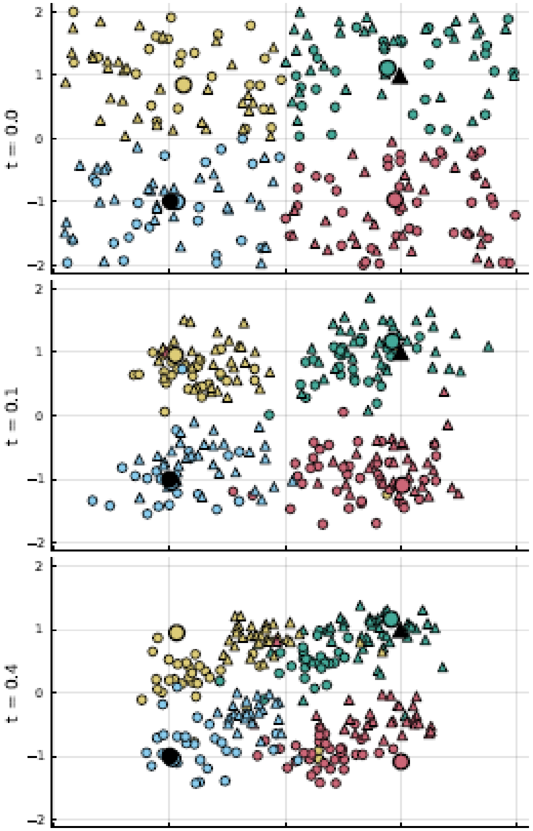


$$a = 1,\ b = 2,\ c = 4,\ \sigma = 0.5,\ \tilde{\sigma} = 0,\ \hat{\sigma} = 0,\ \Gamma = 100,\ \gamma = 10,$$
$$A_{ij} = 1 \text{ for all } (i,j),\ \phi(x) = \psi(x) = \exp(-x),\ \eta = 15.$$


t = 0.0
2
1
0
−1
−2
−2
−1
0
1
2

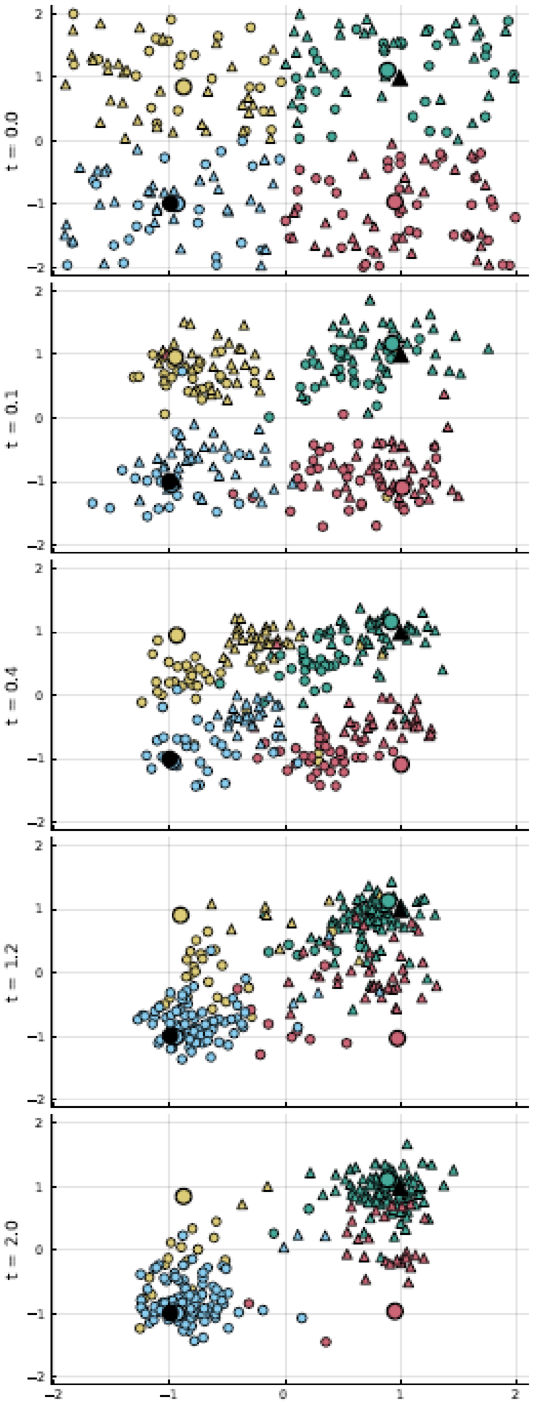


Individuals are quickly being attracted by their respective **influencer** and forming **4 clusters**



Individuals are quickly being attracted by their respective **influencer** and forming **4 clusters**

After some time, the 4 clusters split further because individuals are also **attracted to their medium**, s.t. individuals now form **roughly 8 groups.**



Individuals are quickly being attracted by their respective **influencer** and forming **4 clusters**

After some time, the 4 clusters split further because individuals are also **attracted to their medium**, s.t. individuals now form **roughly 8 groups**.

Some **individuals switch the influencer** to a more suitable influencer, i.e., one that is closer in opinion space and whose majority of followers are connected to the same medium as the individual. They then get attracted to the new influencer (t = 1.2), until finally (t = 2) **individuals have formed 2 mixed clusters near the 2 media opinions**.

More details

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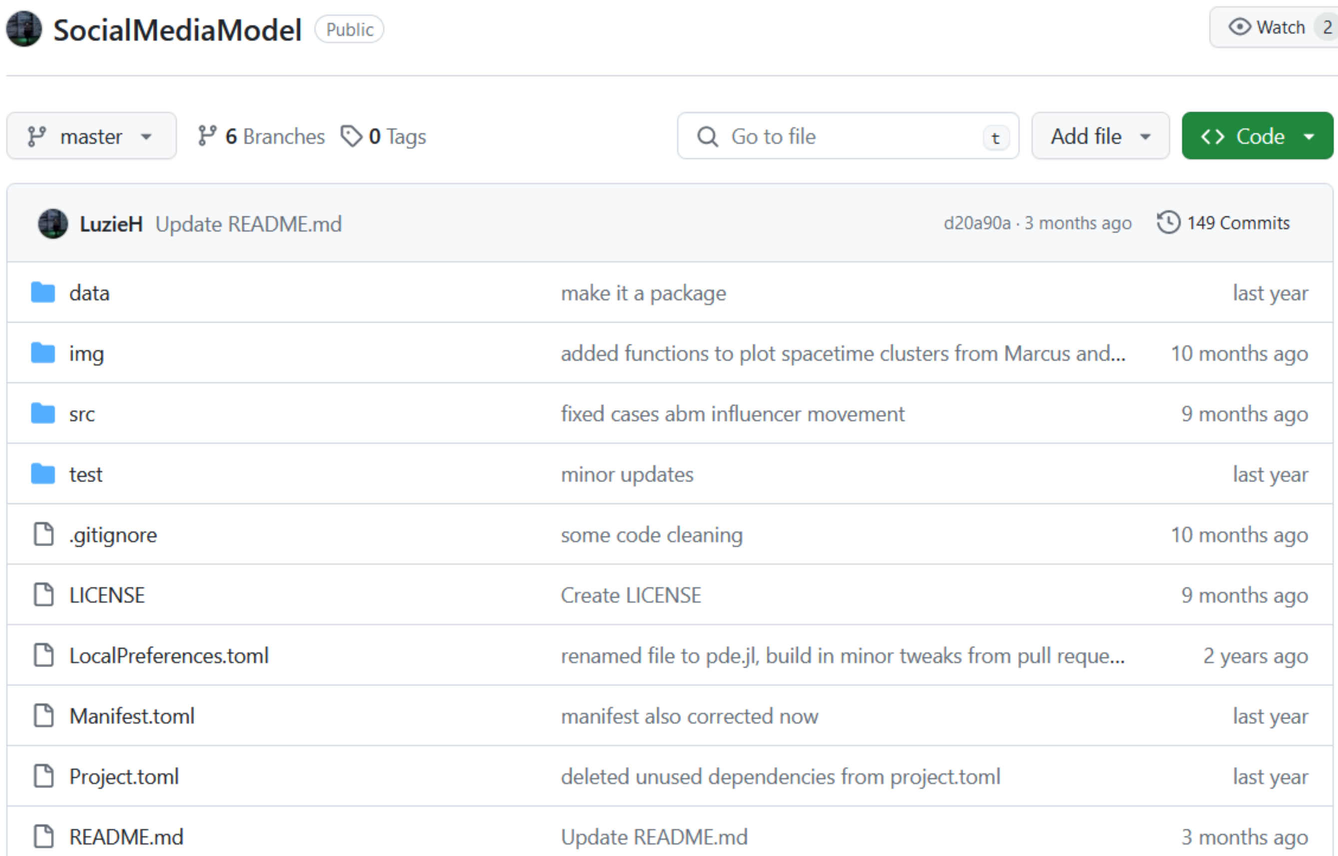
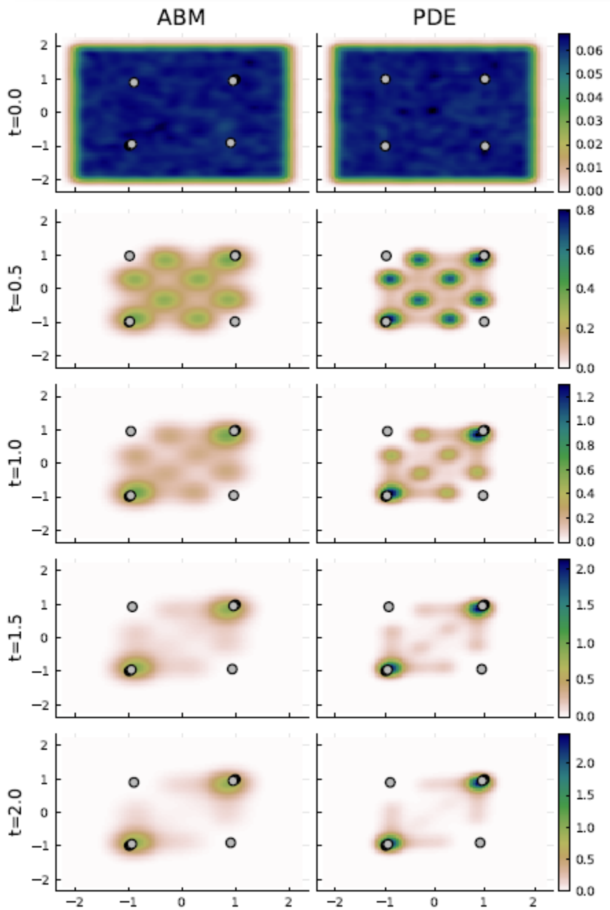
Scientific Reports 13, Article number: 19375 (2023) | Cite this article

Code in Julia-by L.Helfmann



github.com/LuzieH/SocialMediaModel/tree/master

Additional aspects



ABM: mean distribution over 1000 realizations with 250 agents;
PDE: discretized using a Finite Difference scheme.

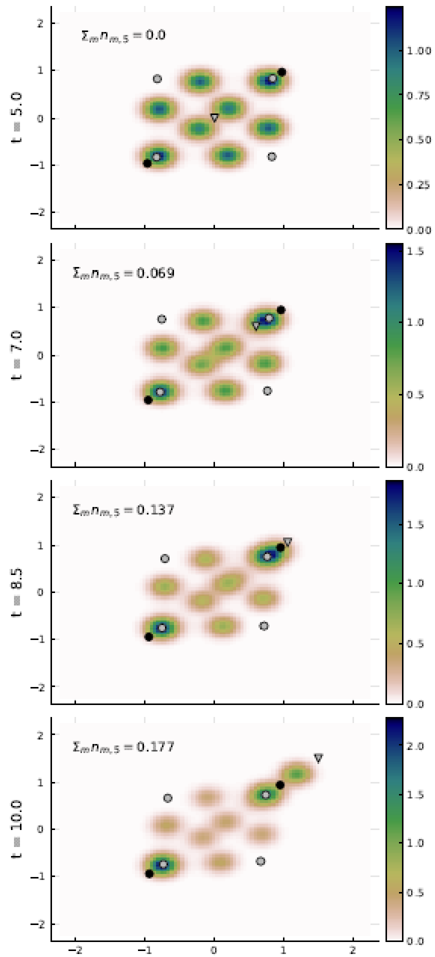
We derived a partial mean-field model
many agents, few influencers and media.

Advantages of the mean-field model:

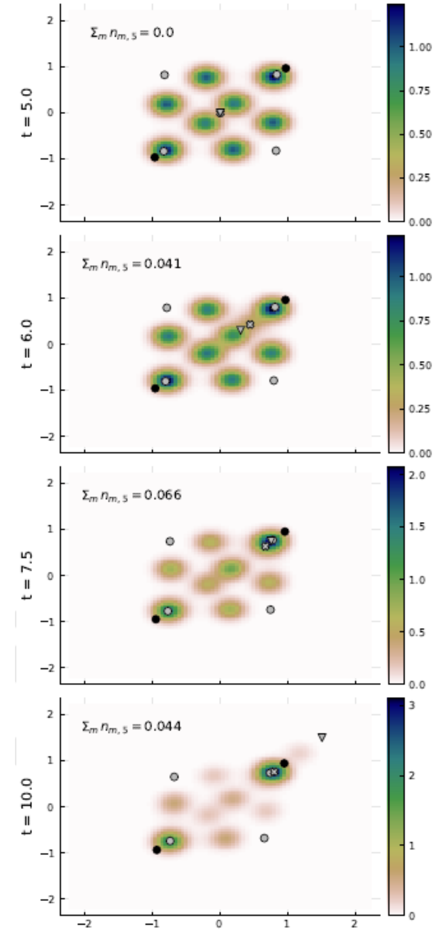
- Reduced computational cost that is not dependant on N.
- Deterministic model.
- Easier to study, e.g. the effect of influencer strategies in the attention economy.
- Allows for deriving optimal control schemes for counteracting actions that influence opinion distribution.

Strategies in the attention economy

Strategy of an influencer (marked by a triangle to increase followership:



Strategy of an influencer (marked by a star) to optimally counteract the goal of another agent (marked by a triangle)



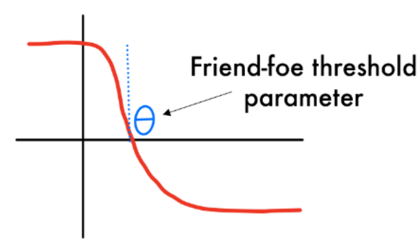
Project from the ASU/MATH+ School 2023

One More Lonely Girl

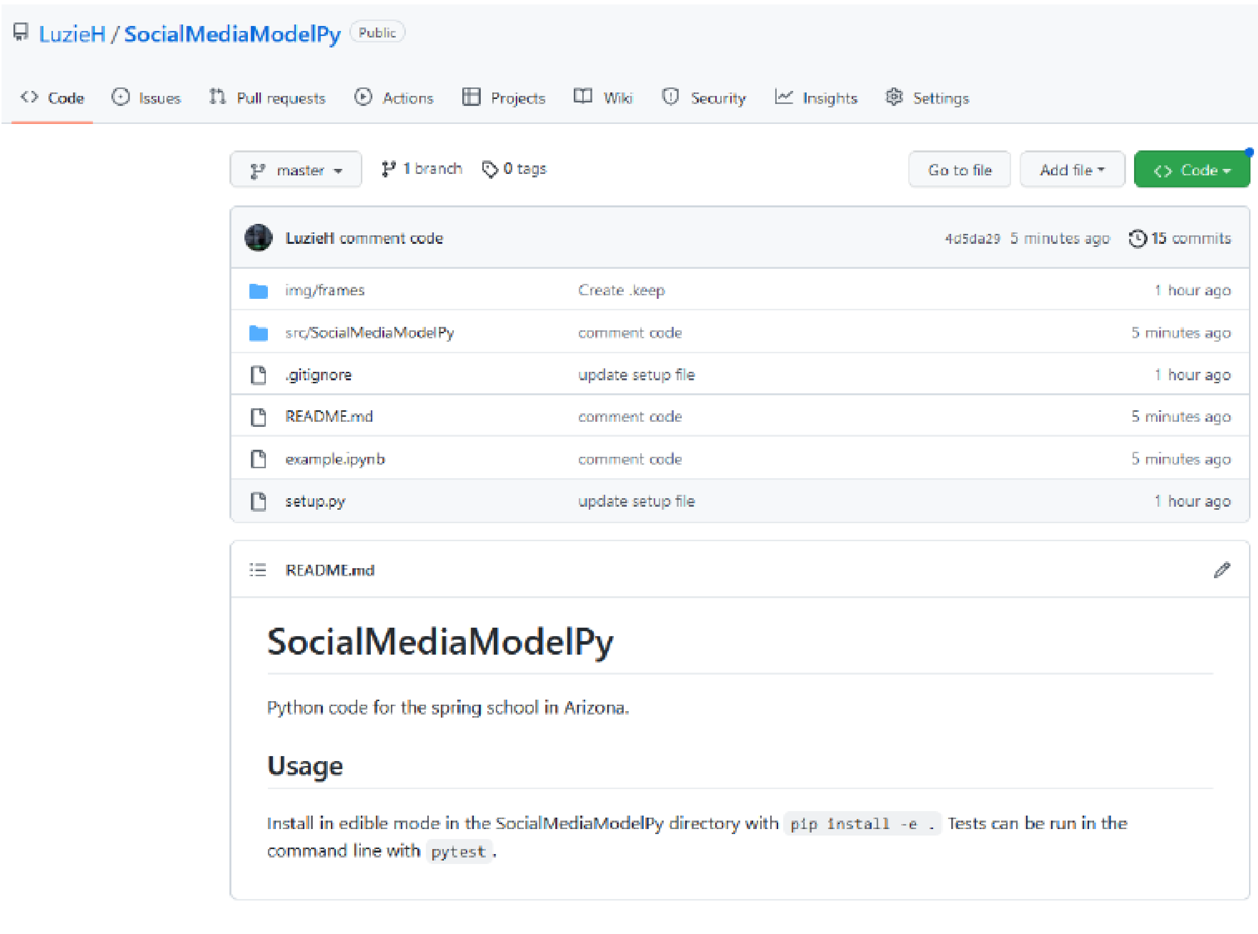
Inan Bostanci, Luzie Helfmann, Kristina Maier,
Nayely Vélez-Cruz,
and Adam Wiechman

- Influencer feuds create polarized social networks
- Introduce influencer-influencer dynamics and repulsion mechanism driven by feuds

Attraction-Repulsion Function ϕ



Code in Python



github.com/LuzieH/SocialMediaModelPy

Open questions:

- How can we measure the clusters in the opinion distribution?
  - Number/size/diversity of clusters.
  - How clusters evolve: split and merge?
- How do the opinion distribution and the opinion clusters change in different settings?
  - for different parameters, networks, interaction functions, influencer/media dynamics, ...
- Realistic influencer+individual dynamics:
  - Individuals can follow more than 1 influencer/media,
  - adapt the influencer dynamics (e.g. stubborn agents), adapt the switching dynamics.
- Include an underlying network and that is changing in time (e.g. driven by homophily or "transitive homophily").
- Neighbourhood effect = include the effect of friends of a friend: the influence of the opinion of a neighbour is proportional to the similarity of the mean opinions of the neighbours neighbours to their own opinion.
- In this model we consider only dyadic influences, study the effect of a 3-body interactions.

Thank you

for your attention

Questions?

Email: natasa.conrad@zib.de