Project 3: Uncertainty Report

1. Group members

- Luka Avni: Lavni@u.rochester.edu
- Ngoc Dao: Ndao2@u.rochester.edu

2. Work distribution

We both work on how to construct the Bayes Network and figure out how to read the commands from a txt file. We also collaborated on figuring out the exact inference part. Then, Luka worked on implementing rejection sampling, and Ngoc wrote the code for Gibbs Sampling. We then each experimented on our own to see which number of samples and burn-in factors produce the best result. The experiments below are conducted by both of us using the same code we have completed together.

3. Justification of AI Use

We made minimal use of AI to refine existing algorithms. This helped improve our formatting and overall organization.

4. Experiments

To cover cases with both evidence and without evidence, we will pick the queries on the network chain.bn and carry out experiments based on the example inputs

a. Rejection Sampling

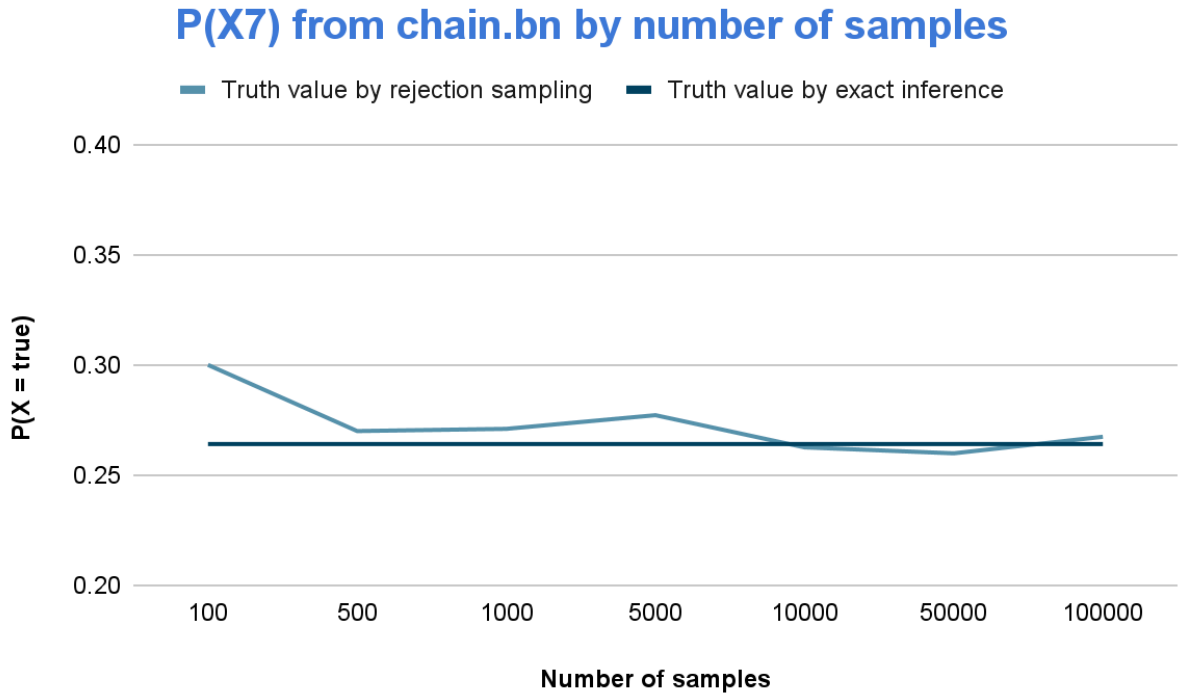
Rejection sampling proved to be very consistent, especially for higher sample sizes. Sample sizes of 500,000 ran somewhat on the slower end but with near-perfect accuracy. Smaller sample sizes

of around 10,000 ran instantly and provided very close estimates no more than plus or minus 0.02 away. We decided to be in the middle ground and set the sample size to 100,000 for accuracy and speed.

We have collected the example runs and shown as below:

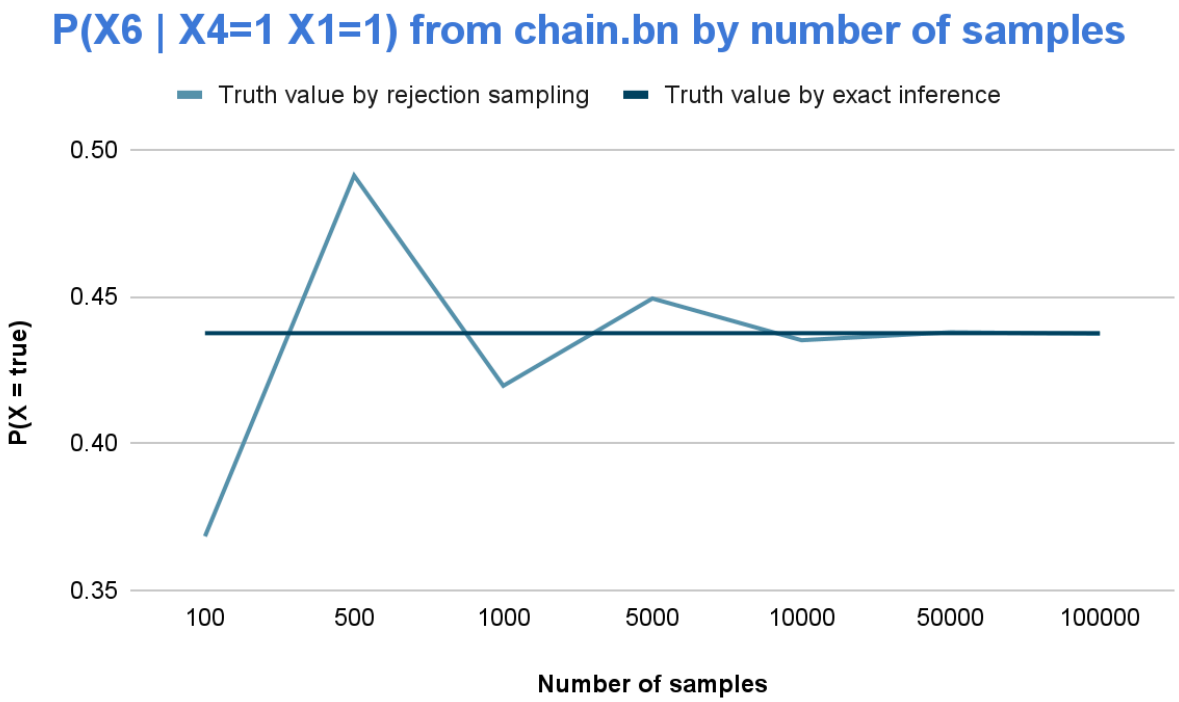
P(X7) from chain.bn by number of samples

	Truth value by rejection sampling	Truth value by exact inference
100	0.3000	0.2641
500	0.2700	0.2641
1000	0.2710	0.2641
5000	0.2772	0.2641
10000	0.2626	0.2641
50000	0.2599	0.2641
100000	0.2674	0.2641



P(X6 | X4=1 X1=1) from chain.bn by number of samples

	Truth value by rejection sampling	Truth value by exact inference
100	0.3684	0.4375
500	0.4911	0.4375
1000	0.4196	0.4375
5000	0.4493	0.4375
10000	0.4351	0.4375
50000	0.4378	0.4375
100000	0.4373	0.4375



b. Gibbs Sampling

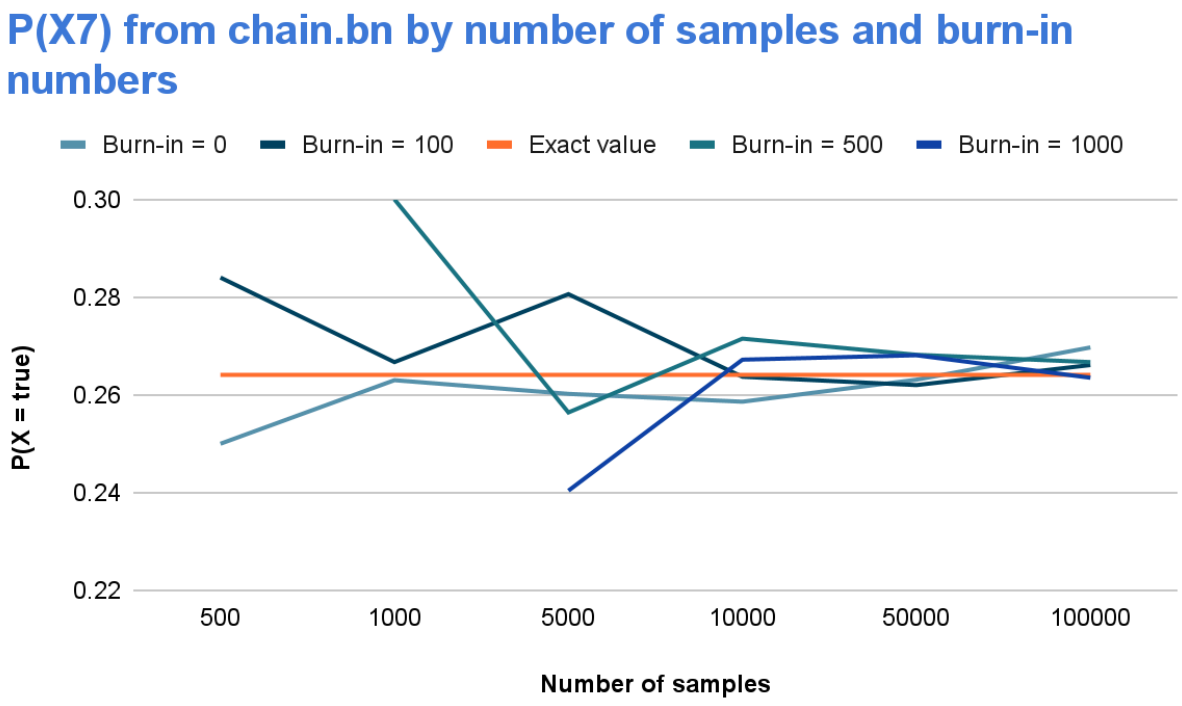
Compared to rejection sampling, Gibbs sampling has proved to be increasingly more accurate and time effective, even for small samples, with an error of only 0.01 for smallest samples (500 samples). However, the smaller the number of samples is, the less effective it gets when burn-in

increases. We have concluded that small amount of samples (<1000) would be better off without burn-in, and large samples perform the best with burn-in =100 (for number of samples = 100000, burn-in = 1000 performs the best)

P(X7) from chain.bn by number of samples and burn-in numbers

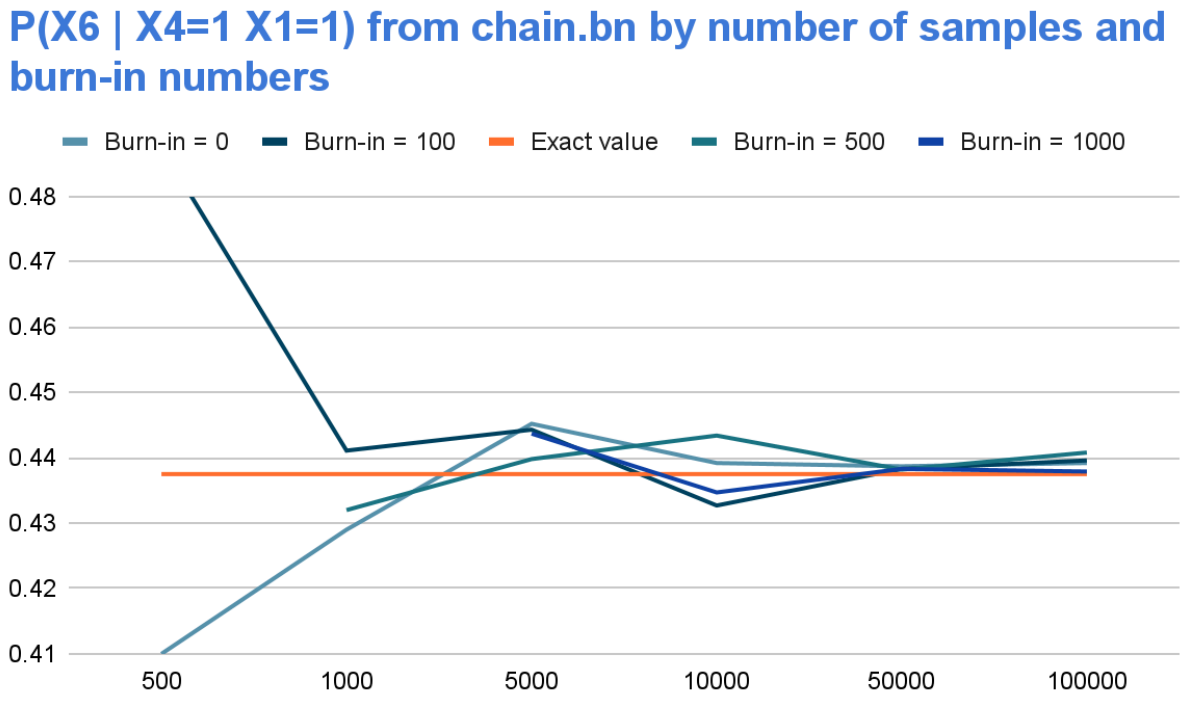
	Burn-in = 0	Burn-in = 100	Burn-in = 500	Burn-in = 1000	Exact value
500	0.2500	0.2840			0.2641
1000	0.2630	0.2667	0.3000		0.2641
5000	0.2602	0.2806	0.2564	0.2404	0.2641
10000	0.2586	0.2637	0.2715	0.2672	0.2641
50000	0.2631	0.2620	0.2682	0.2681	0.2641
100000	0.2697	0.2661	0.2667	0.2635	0.2641

*Not including cases where burn-in = number of samples



P(X6 | X4=1 X1=1) from chain.bn by number of samples and burn-in numbers

	Burn-in = 0	Burn-in = 100	Burn-in = 500	Burn-in = 1000	Exact value
500	0.4100	0.4875			0.4375
1000	0.4290	0.4411	0.4320		0.4375
5000	0.4452	0.4443	0.4398	0.4437	0.4375
10000	0.4392	0.4327	0.4434	0.4347	0.4375
50000	0.4387	0.4383	0.4382	0.4383	0.4375
100000	0.4392	0.4396	0.4408	0.4379	0.4375



c. Special networks or specific queries which have interesting properties

- Chain Network (chain.bn)
  - P(X7):

        - Easy for both rejection and Gibbs sampling
        - Converges quickly even with low sample counts

    - P(X6 | X4=1 X1=1):
        - Harder for rejection sampling—takes many samples to match evidence.
        - Gibbs sampling performs well with fewer samples and burn-in.

- Rare Evidence Queries
    - Rejection sampling performs poorly when evidence is unlikely.
    - Gibbs sampling remains effective, especially with burn-in.
- Overall Observation
    - Chain structures favor Gibbs over rejection for evidence-based queries.
    - Rejection works well only when evidence is shallow or absent.