

Q1

a) Convert unsigned decimal \rightarrow binary value (w=8)
binary value \rightarrow hexadecimal value.

I) 157_{10} : To binary:

$$\begin{array}{rcl} 157 / 2 & = & 78 \text{ (r=1)} \\ 78 / 2 & = & 39 \text{ (0)} \\ 39 / 2 & = & 19 \text{ (1)} \\ 19 / 2 & = & 9 \text{ (1)} \\ 9 / 2 & = & 4 \text{ (1)} \\ 4 / 2 & = & 2 \text{ (0)} \\ 2 / 2 & = & 1 \text{ (0)} \\ 1 / 2 & = & 0 \text{ (1)} \end{array}$$

$$\rightarrow 157_{10} = 10011101_2$$

$$\rightarrow = (9D)_{16}$$

II) 248_{10} : To binary:

$$\begin{array}{rcl} 248 - 128 & = & 120 \text{ (128 = } 2^7) \\ 120 - 64 & = & 56 \text{ (64 = } 2^6) \\ 56 - 32 & = & 24 \text{ (32 = } 2^5) \\ 24 - 16 & = & 8 \text{ (16 = } 2^4) \\ 8 - 8 & = & 0 \text{ (8 = } 2^3) \end{array}$$

$$\rightarrow 248_{10} = 11111000_2$$

$$= (F8)_{16}$$

b) Convert signed decimal \rightarrow T's C binary (w=8)
 \rightarrow hexadecimal.

I) $123_{10} \Rightarrow$

$$\begin{array}{rcl} 123 - 64 & = & 59 \text{ (64 = } 2^6) \\ 59 - 32 & = & 27 \text{ (32 = } 2^5) \\ 27 - 16 & = & 11 \text{ (16 = } 2^4) \\ 11 - 8 & = & 3 \text{ (8 = } 2^3) \\ 3 - 2 & = & 1 \text{ (2 = } 2^1) \\ 1 - 1 & = & 0 \text{ (1 = } 2^0) \end{array}$$

$$\Rightarrow 123_{10} = 01111011_2$$

$$(7B)_{16}$$

$$\begin{aligned} \text{II) } -74_{10} &\Rightarrow 74 - 64 = 10 \quad (64 = 2^6) \\ & \quad (10) \quad 10 - 8 = 2 \quad (8 = 2^3) \\ & \quad 2 - 2 = 0 \quad (2 = 2^1) \end{aligned}$$

$$74_{10} = 01001010_2$$

$$\begin{aligned} \text{C) } -74_{10} &= 10110110_2 \\ &= (B6)_{16} \end{aligned}$$

e) Interpret binary val as [unsigned
T's (signed)

$$\begin{aligned} \text{2) } 11101001_2 &\rightarrow \text{unsigned: } = 2^7 + 2^6 + 2^5 + 2^3 + 2^0 \\ &= 128 + 64 + 32 + 8 + 1 \\ &= (233)_{10} \\ &\rightarrow \text{signed: } = -2^7 + 2^6 + 2^5 + 2^3 + 2^0 \\ &= (-23)_{10} \end{aligned}$$

$$\begin{aligned} \text{II) } 10010110_2 &\rightarrow \text{unsigned: } = 2^7 + 2^4 + 2^2 + 2^1 \\ &= 128 + 16 + 4 + 2 \\ &= (150)_{10} \\ &\rightarrow \text{signed: } = -2^7 + 2^4 + 2^2 + 2^1 \\ &= (-106)_{10} \end{aligned}$$

$$\text{d) } \overset{247}{\underset{-112}{V}} \text{ Signed val: } 247 - 256 = -9 \quad (256 = 2^8)$$

$$\text{e) } \overset{-112}{\underset{247}{V}} \text{ Unsigned val: } -112 + 256 = 144 \quad (\text{---})$$

Q2

a) Convert to binary & compare true vs actual sum ($w=8$)

$$\begin{aligned} \text{I) } 74_{10} &\Rightarrow 74 - 64 = 10 \quad (64 = 2^6) & 63_{10} &\Rightarrow 63 - 32 = 31 \quad (32 = 2^5) \\ &10 - 8 = 2 \quad (8 = 2^3) & 31 - 16 &= 15 \quad (16 = 2^4) \\ &2 - 2 = 0 \quad (2 = 2^1) & 15 - 8 &= 7 \quad (8 = 2^3) \end{aligned}$$

$$\therefore 74_{10} = 01001010_2$$

$$7 - 4 = 3 \quad (4 = 2^2)$$

$$+ 74$$

$$\overset{1111}{01001010} \rightarrow \text{carry in}$$

$$3 - 2 = 1 \quad (2 = 2^1)$$

$$+ 63$$

$$+ 00111111$$

$$1 - 01 = 0 \quad (1 = 2^0)$$

$$137$$

$$10001001$$

$$(True = Actual Sum)$$

$$63_{10} = 00111111$$

3) No overflow

$$\begin{array}{l} \text{II) } 123_{10} \Rightarrow 123 - 64 = 59 \quad (64 = 2^6) \\ 59 - 32 = 27 \quad (32 = 2^5) \\ 27 - 16 = 11 \quad (16 = 2^4) \\ 11 - 8 = 3 \quad (8 = 2^3) \\ 3 - 2 = 1 \quad (2 = 2^1) \\ 1 - 1 = 0 \quad (1 = 2^0) \end{array} \quad \begin{array}{l} 157_{10} \Rightarrow 157 - 128 = 29 \quad (128 = 2^7) \\ 29 - 16 = 13 \quad (16 = 2^4) \\ 13 - 8 = 5 \quad (8 = 2^3) \\ 5 - 4 = 1 \quad (4 = 2^2) \\ 1 - 1 = 0 \quad (1 = 2^0) \end{array}$$

$$\begin{array}{r} 123_{10} = 01111011 \\ 157_{10} = 10011101 \\ \hline 280 \end{array} \quad \begin{array}{r} 01111011 \\ 10011101 \\ \hline 10001000 \end{array} \quad \begin{array}{l} \text{True Sum} \\ \text{Actual Sum} \end{array}$$

carry out \Rightarrow Overflow (280 out of range)

\Rightarrow Detecting overflow: Overflow

(=) 1) At bit level: carry out bit = 1

(=) 2) Decimal operands: decimal sum $> 2^w - 1$ (max unsigned)

b) Convert to binary & compare True vs Actual Sum ($w=8$)

$$\begin{array}{l} \text{I) } 28_{10} \Rightarrow 28 - 16 = 12 \quad (16 = 2^4) \\ 12 - 8 = 4 \quad (8 = 2^3) \\ 4 - 4 = 0 \quad (4 = 2^2) \end{array} \quad \begin{array}{l} -74_{10} = 10110110_2 \\ \text{(see Q1)} \end{array}$$

$$\begin{array}{r} 28_{10} = 00011100_2 \\ -74_{10} = 10110110_2 \\ \hline -46 \end{array} \quad \begin{array}{r} 00011100 \\ 10110110 \\ \hline 11010010 \end{array} \quad \begin{array}{l} \text{True = Actual Sum} \end{array}$$

$$\begin{array}{l} \text{II) } -117_{10} \Rightarrow -117 + 2^8 = 139 \\ 139 - 128 = 11 \quad (128 = 2^7) \\ 11 - 8 = 3 \quad (8 = 2^3) \\ 3 - 2 = 1 \quad (2 = 2^1) \\ 1 - 1 = 0 \quad (1 = 2^0) \\ -117_{10} = 10001011_2 \end{array} \quad \begin{array}{l} 126_{10} \Rightarrow 126 - 64 = 62 \quad (64 = 2^6) \\ 62 - 32 = 30 \quad (32 = 2^5) \\ 30 - 16 = 14 \quad (16 = 2^4) \\ 14 - 8 = 6 \quad (8 = 2^3) \\ 6 - 4 = 2 \quad (4 = 2^2) \\ 2 - 2 = 0 \quad (2 = 2^1) \\ -126_{10} = 01111110_2 \end{array}$$

$$\begin{array}{r}
 126 \quad 1111111 \rightarrow \text{carry in} \\
 01111110 \\
 + \\
 -117 \quad 10001011 \\
 \hline
 9 \quad 100001001 \\
 \checkmark 100001001
 \end{array}$$
 True Sum \neq Actual Sum \Rightarrow no arithmetic overflow (carry out discarded)

III) $74_{10} = 0100\ 1010_2$; $63_{10} = 0011\ 1111_2$
 74 $0100\ 1010$ Carry in
 63 $0011\ 1111$
 137 $10001\ 001$ \Rightarrow Positive Overflow (137 out of range)

$\text{IV) } -119_{10} \Rightarrow -119 + 2^8 = 137$	$-105_{10} \Rightarrow -105 + 2^8 = 151$
$137 - 128 = 9 \quad (128 = 2^7)$	$151 - 128 = 23 \quad (128 = 2^7)$
$9 - 8 = 1 \quad (8 = 2^3)$	$23 - 16 = 7 \quad (16 = 2^4)$
$1 - 1 = 0 \quad (1 = 2^0)$	$7 - 4 = 3 \quad (4 = 2^2)$
$\Rightarrow -119_{10} = 10001001_2$	$3 - 2 = 1 \quad (2 = 2^1)$
	$1 - 1 = 0 \quad (1 = 2^0)$
	$\Rightarrow -105_{10} = 10010111_2$

⇒ Detect overflow:

1) At bit level: let s = actual sum of u & v

$$MSB(s) \neq MSB(u) = MSB(v) \Rightarrow \text{overflow}$$

(negative if $MSB(u) = 1$
positive if $MSB(u) = 0$)

2) Using decimal operands:

if decimal sum $> 2^{w-1} - 1$ → positive overflow

$< -2^{w-1}$ -) negative overflow

Q.3 (b) \Rightarrow CISC computers are Little Endian.

$0 \times 711c4599845e \mid 0 \times 39 \text{ (LSB)} \mid \text{val} = 12345 \text{ (} 0 \times 00003039 \text{)}$

d	0×30	↓
e	0×00	
f	0×00 (MSB)	

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