# A Comparison of Time Domain VS Frequency Domain Delay and Sum Beamforming for Underwater 3D Acoustical Imaging


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*Abstract*—**Digital delay and sum beamforming is the promising technique for underwater 3D acoustical imaging. Even though time domain delay and sum beamforming is widely used for medical ultrasound imaging, it is hardly used for underwater acoustical 3D imaging. The conventional narrowband transmit coupled with the computational complexity required for the implementation of interpolation filter is the major reason for that. However, with the increasing popularity of wideband and ultrawideband transmission for achieving high resolution imaging and with the availability of efficient software interpolation libraries in modern computing systems, the computational time required for frequency domain beamforming appears to be more than that of time domain beamforming. Thus, through this work, we have attempted to compare the frequency domain vs time domain delay and sum beamforming for wideband underwater 3D acoustical imaging. The simulation findings demonstrated that time domain DAS beamforming is faster, computationally effective, memory-efficient, and offers somewhat superior image quality.**

***Keywords— Time domain delay and sum beamforming, Frequency domain beamforming, Underwater 3D acoustical imaging.***

## I. Introduction

In the recent past, underwater 3D acoustical imaging is being popular for ocean exploration as it gives almost all the geometrical information about the target by analysing different slices of the reconstructed 3D image. However, the computational complexity, large memory and hardware requirements has dented the popularity of underwater 3D imaging among ocean researchers. In order to mitigate the computational complexity, memory and hardware requirements for underwater acoustic 3D imaging, many fast-beamforming algorithms [1-5] have been proposed. However, most of the algorithms are performed in frequency domain. To the best of authors knowledge, we were the first group to implement a fast-beamforming algorithm in time domain for underwater acoustical 3D imaging [6].

A fast-beamforming method with dynamic focusing utilising the Fast Fourier transform for underwater 3D imaging has been proposed by Murino et al. in [7]. Only narrowband transmission and uniform 2D arrays or sparse arrays thinned from uniform 2D arrays can be used with this technology. Although most fast-beamforming techniques are employed for narrowband transmission, two fast beamforming approaches have been presented in [5] and [2] for wideband transmission. A thorough comparison of time domain vs frequency domain 3D beamforming has not yet been done, despite the fact that all of the proposed fast-beamforming approaches are in the frequency domain.

A relatively small number of articles [8-9] have attempted to contrast 3D beamforming in the time and frequency domains. However, they only compared the hardware implementation difficulty of the two approaches for the narrowband transmission. To the best of our knowledge, there has been no comparison on the software implementation complexities. In this paper, we have implemented and compared the time domain and frequency domain delay and sum beamforming for underwater 3D acoustical imaging.

The paper is organized as follows. The time and frequency domain delay and sum 3D beamforming are described in Section II and III respectively. The computation and memory requirements for the approaches are thoroughly examined in Section IV. The simulation results are discussed in Section V. Finally, Section VI concludes the paper.

## II. Time Domain Delay and Sum Beamforming

The block diagram of the 3D image reconstruction pipeline is shown in Fig.1. The promising method for image reconstruction in underwater acoustical 3D imaging is delay and sum (DAS) beamforming [10]. In the time domain, DAS beamforming essentially consist of two procedures: delay compensation and summation. Towards this, the imaging grid is considered as a collection of large number of point scatterers.





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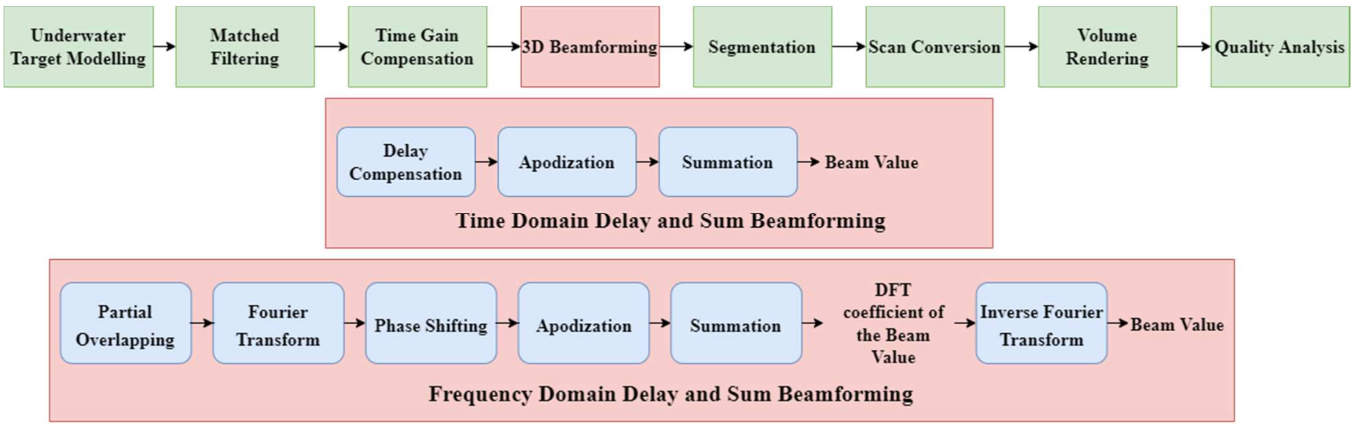


Fig.1. Block diagram of the 3D image reconstruction pipeline.

A scatterer can be assumed as a pixel for 2D imaging and a voxel for 3D imaging. In order to find the beam value corresponding to the scatterer, the array of sensors should be focused at the scatterer by delaying the received signals at each sensor element accordingly. The process of making all the signals from the scatterer aligned with time is known as delay compensation. Each delay-compensated signal is multiplied with a suitable weight before being added together to attenuate the signals from the unwanted direction. Apodization is the term used to describe this weight-based multiplication procedure. In this work, Hanning window apodization has been used. The time domain beamforming for underwater imaging is represented in Fig.2.

Consider a 2D receiving array of $M$ omnidirectional and point sensors, indexed by $m$ and placed on the plane $Z = 0$. The time domain beamformed signal, $b_k(r_0, \hat{u}, t)$ for the $k^{th}$ scatterer at a focusing distance $r_0$ in the steering direction $\hat{u}$ can be represented as in (1).

$$b_k(r_0, \hat{u}, t) = \sum_{m=1}^{M} W_{m,k}\ s_m\left(t - \tau\left(m, r_0, \hat{u}\right)\right) \qquad (1)$$

Where $s_m(t)$ and $W_{m,k}$ represent the total received signal for the $m^{th}$ sensor element and the apodization weight for the corresponding sensor element and $k^{th}$ scatterer respectively. The unit vector in the steering direction $(\alpha, \beta)$, $\hat{u}$ is expressed in (2)

$$\hat{u} = (\sin\alpha, \sin\beta, \sqrt{\cos^2\alpha - \sin^2\beta}\,) \qquad (2)$$

Where the azimuth angle, $\alpha$ is the angle between the vector, $\hat{u}$ and its projection on the plane $yz$, and the elevation angle, $\beta$ is the angle between the vector $\hat{u}$ and its projection on the plane $xz$. The delay required to steer the beam to the direction of $(\alpha, \beta)$, at the focusing distance $r_0$ for the $m^{th}$ sensor element at can be expressed as [11]

$$\tau(m, r_0, \hat{u}) = \frac{r_0 - |P_m - r_0\hat{u}|}{c} \qquad (3)$$

Where $P_m$ is the position vector of $m^{th}$ sensor element. The far-field approximation condition is $r_0 > D^2/2\lambda$ [12] Where $D$ is the array width of the 2D receiving array and $\lambda$ is the wavelength. The far-field approximation delay [13] can be expressed as

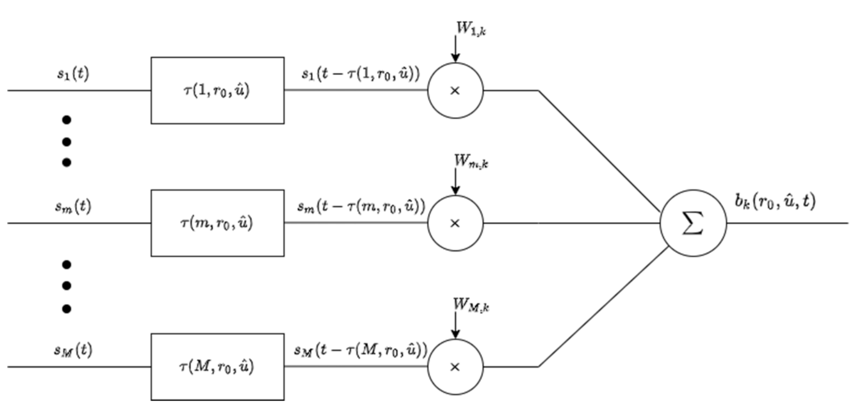


Fig.2. Time Domain Delay and Sum Beamforming

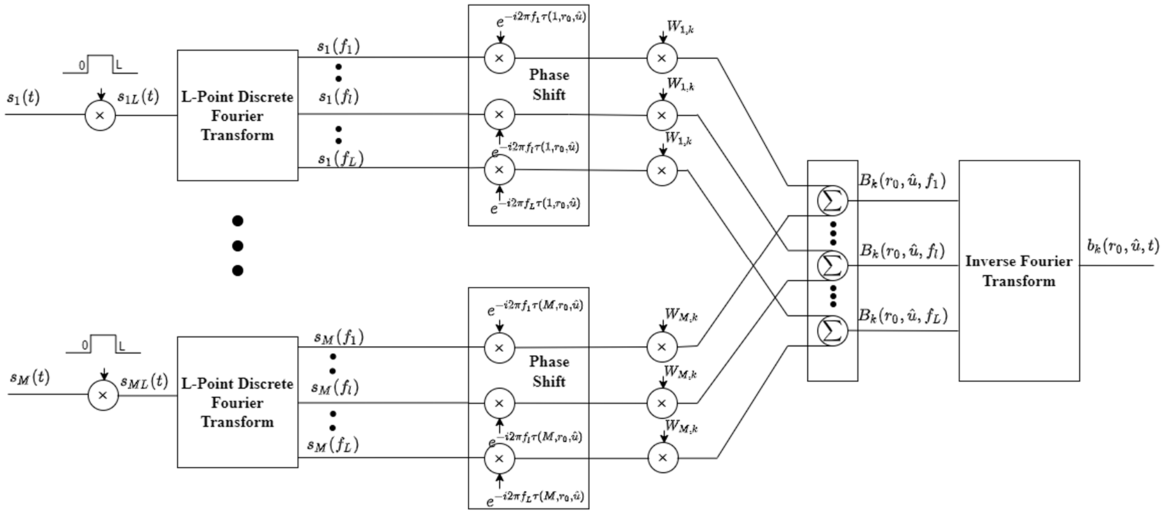

Fig.3. Frequency Domain Delay and Sum Beamforming

$$\tau\,(m, r_0, \alpha, \beta\,) = \frac{x_m sin\alpha + y_n sin\beta}{c} \tag{4}$$

Where, $c$ is the acoustic speed of the medium.

## III. Frequency Domain Delay and Sum Beamforming

Beamforming can also be performed in frequency domain. The Frequency domain beamforming is illustrated in Fig.3. The discrete Fourier transform coefficient of the beamformed signal in (1) for the frequency bin $f_l$, $B_k(r_0, \hat{u}\,, f_l)$ can be written as

$$B_k(r_0, \hat{u}\,, f_l) = \sum_{m=1}^{M} W_{m,k} \;\; s_m(f_l) \exp\left(-i2\pi f_l \tau\,(m, r_0, \hat{u}\,)\right) \tag{5}$$

Where $s_m(f_l)$ is the $l^{th}$ DFT coefficient of the partially overlapped signal, $s_{mL}(t)$ and the discrete frequency $f_l$ is expressed as in (6)

$$f_l = \frac{l f_s}{L} \tag{6}$$

Where, $f_s$ is the sampling frequency and $L$ is the length of the signal. The frequency index, $l$ varies from 0 to $L/2$. The received signal, $s_m(t)$ is divided into different overlapping blocks of length $L$ to range localize the DFT. The number of samples, $o$, which should be overlapped for computing all the beam signal samples correctly is given by (8) [5]

$$o = ceil(f_s \tau_{max}) - ceil(f_s \tau_{min}) \tag{8}$$

Where the function $ceil(a)$ round a to the nearest integer towards infinity [5]. The process of diving the received signals into overlapping blocks is known as partial overlapping. Each block after partial overlapping is represented as $s_{mL}(t)$ in Fig.3.

## IV. Computation and Memory Analysis

*Time Domain beamforming:* The number of additions and multiplications required to implement the conventional time domain beamforming in (1) are $M-1$ and $M$ respectively. Assuming linear interpolation is used to find the value corresponding to the delay in time domain beamforming, the number of additions and multiplications required are $2M$ and $M$ respectively.

*Frequency Domain Beamforming:* The number of additions and multiplications required to implement (5) to find the DFT coefficient of the beamformed signal for a single frequency bin are $M-1$ and $M$ respectively. For $L$ point DFT, $L/2$ frequency bins should be considered. Thus, the number of real additions and multiplications required to find $L/2$ DFT coefficients of the beam value are $L/2(M-1)$ and $L/2 \times M$ respectively. Besides, the $L$ point DFT of the received signal for $M$ elements should be computed by using FFT algorithm. The number of additions and multiplications required for computing FFT of $M$ signals respectively are $Llog_2L \times M$ and $\frac{L}{2} log_2L \times M$. Finally, $L$ point IFFT should be performed to get the final beam value. The number of additions and multiplications required for computing $L$ point IFFT are $Llog_2L$ and $\frac{L}{2} log_2L$ respectively.

*Computation Memory Analysis:* The memory required to store the apodization weights, delays and other additional parameters are considered for comparison. All other memory requirements which are common for all the methods are omitted

TABLE I
THE COMPUTATION AND MEMORY ANALYSIS

| **Method** | **Number of additions** | **Number of multiplications** | **Memory (bytes)** |
|---|---|---|---|
| Time domain DAS | $3M-1$ | $2M$ | $2M$ |
| Frequency domain DAS | $L/2(M-1) + (M+1)Llog_2L$ | $\frac{L}{2} \times M + (M+1)L/2log_2L$ | $2M + \mathrm{L}/2$ |

TABLE II

THE SIMULATION PARAMETERS

| Parameter | Value |
|---|---|
| Frequency | $500KHz$ |
| Bandwidth | $218KHz$ |
| Sampling frequency | $1.67MHz$ |
| Signal strength | $100 \times 10^{20}\ Pa$ |
| Acoustic speed of medium | $1500m/s$ |
| Density of medium | $1000Kg/m^3$ |
| Attenuation coefficient | $0.0022\ dB/MHz\ cm$ |
| Acoustic speed of target | $3235m/s$ |
| Density of target | $7860Kg/m^3$ |
| Aperture size | $36mm \times 36mm$ |
| Number of sensor elements | $24 \times 24$ |
| Element aperture | $1.5mm$ |
| Element spacing | $1.5mm$ |
| Field of view | $24^0 \times 24^0$ |

from the comparison. Each memory location is considered as a size of 1 $byte$ for the comparison. Since the 3D beamforming considers all the elements in the 2D receiving array, the memory required to store apodization weights and delays for frequency domain beamforming and time domain beamforming are $2M\ bytes$**.** In addition to that, the frequency domain direct method requires $\frac{L}{2}\ bytes$ to store $\frac{L}{2}$ DFT coefficients of the beam signal. The computation and memory analysis of the two methods are summarized in TABLE I.

## V. SIMULATION RESULTS AND DISCUSSION

The time domain and frequency domain 3D beamforming were compared by reconstructing 3D images of three different underwater targets using the aforementioned methods. The simulation is done using MATLAB [15] and k-Wave toolbox [16]. The underwater target is modelled using k-Wave. For 3D imaging, a 3D grid of size $283mm \times 123mm \times 123mm$ was defined in k-Wave. The water medium with some random scatterers in normal distribution and the target were defined inside the grid by defining their acoustic speed and density at the required positions inside the grid. A rigid mild steel ball of diameter 50mm, a mild steel cube of side length and a mild steel cross of length 60mm were defined as targets. They were placed at a range of 100mm from the sensor array in three different grids. The sensor array is placed at the center of the $yz$ plane in the k-Wave grid. The half wavelength element spacing was used in order to avoid ambiguities due to the grating lobes. Since the spherical wavefront is more like plane wave in the far field [8], all the grid points in $yz$ plane were used as the source with planewave excitation. The Source resembles an omni directional transmitter placed at the center of the receiving 2D array. The wide band transmission was used by limiting the

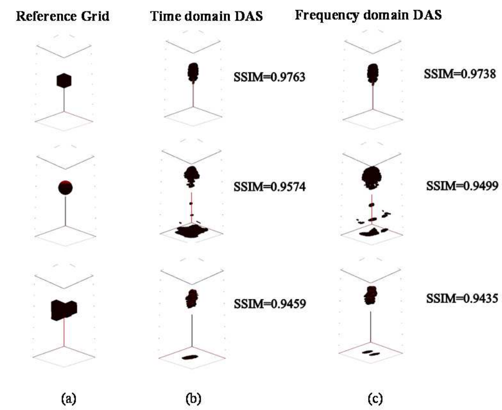


Fig.4(a) Reference grid (b) Reconstructed volume by time domain DAS (c) Reconstructed volume by frequency domain DAS

transmitted pulse of frequency $500KHz$ with 3 cycles which provides a bandwidth of $218KHz$ and range resolution of 3mm. The sampling rate of the signal obtained by k-Wave is always several times greater than the Nyquist sampling frequency. Thus, the signal has been down sampled by a factor of six to save memory and computation time. All the simulation parameters are listed in TABLE II.

The backscattered echoes obtained from k-Wave simulation were used to reconstruct the 3D image of the target using time domain and frequency domain delay and sum beamforming. After segmentation and scan conversion, the quality of the images was analyzed by computing structural similarity index (SSIM) [16] between reference and reconstructed 3D images. The average time required to reconstruct one voxel of the 3D image using AMD Ryzen 7 4700G with Radeon Graphics, 3600 $Mhz$, 8 Core, 16 Logical Processors was also observed. The reference and reconstructed 3D images obtained by time domain and frequency domain beamforming for three different targets and their corresponding SSIM values are shown in Fig.4.

The computational complexity, memory and time requirements and average SSIM value are tabulated in TABLE III for comparison. Time domain DAS beamforming requires 96.69% and 96.19% less additions and multiplications than frequency domain DAS beamforming, respectively. In addition, frequency domain beamforming requires more memory than time domain beamforming does. It is based on the signal length after partial overlapping. If beamforming is carried out using software, time domain DAS beamforming is 54.5 times faster than frequency domain beamforming. The slightly higher SSIM value for time domain beamforming

TABLE III

TABLE OF COMPARISON

| Method | Number of additions | Number of multiplications | Memory (bytes) | Average Time (Seconds) | Average SSIM |
|---|---|---|---|---|---|
| Time domain DAS | 1727 | 1152 | 1152 | 0.0262 | 0.9598 |
| Frequency domain DAS | 52320 | 30270 | 1162 | 1.4279 | 0.9557 |

shows that image quality of time domain DAS beamforming is equivalent to and superior to that of frequency domain DAS, despite the fact that it is computationally more efficient and uses less memory.

## VI. CONCLUSION

The software implementation of time domain and frequency domain beamforming were compared in terms of computational complexity, memory and time requirements and the image quality. DAS beamforming in the time domain speeds up computing by 54.5 times and decreases computation by more than 96%. The system will run more quickly by estimating all the delay compensated values simultaneously using software interpolation (such as Interp1 in MATLAB). In comparison to frequency domain, time domain DAS implementation uses less memory. The length of the overlapped block of signals will determine how much memory is saved. Although the time domain implementation uses less memory and is computationally more efficient than the frequency domain implementation, the image quality is practically same.


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