HelioFocus.ltd dish (square) array mutual shading calculation / algorithm

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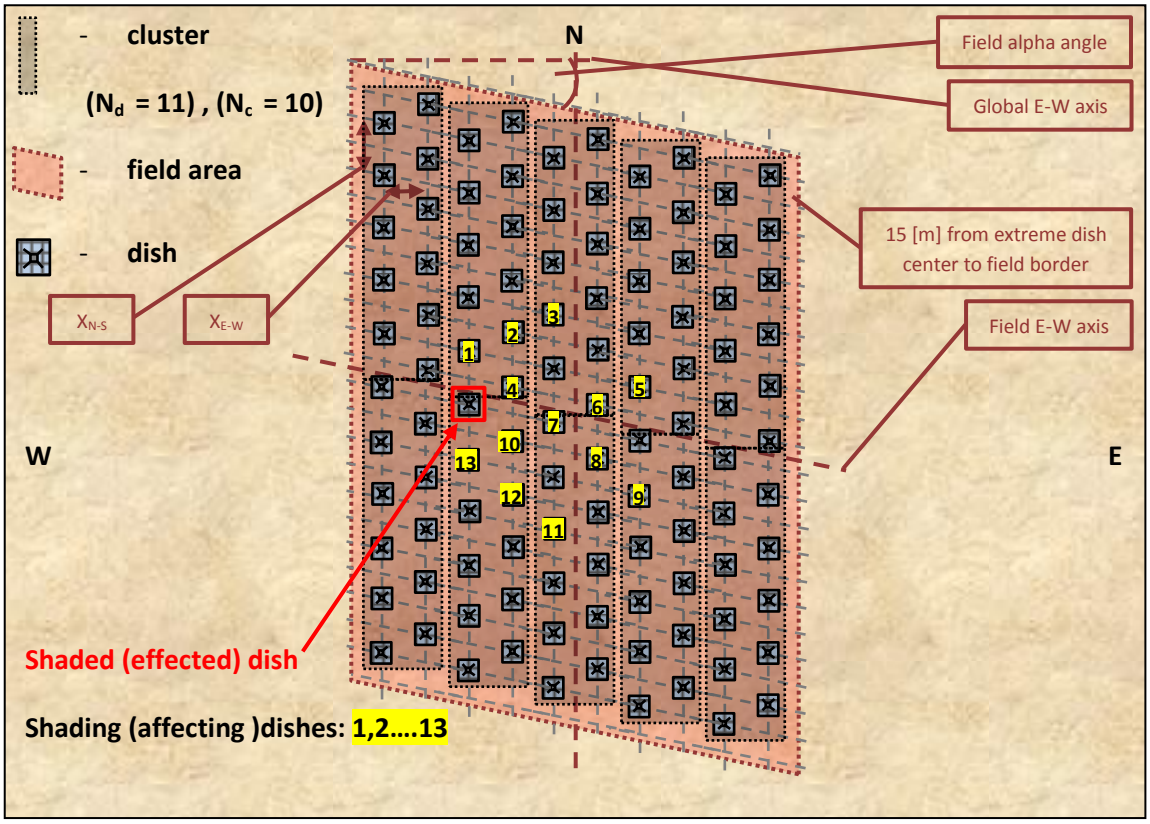

Base assumption: *dishes in the field always follow the sun across the sky.*

$\psi = sun\ elevation\ (off\ horizon)\ [deg]: 0° < \psi < 90°$

$$If\ \psi \leq 0 \rightarrow No\ shading\ and\ no\ calculation \rightarrow \boldsymbol{shading = 0\%}$$

$$If\ \psi > 0 \rightarrow$$

$\varphi = sun\ azimuth\ (off\ north)\ [deg]: 0° < \varphi < 360°$

$\alpha = field\ alpha\ angle\ (parallelogram\ angle\ from\ E-W\ line\ (clock\ wise)\ [deg]: 0° < \alpha < 45°\ allowed$

$Faz = field\ azimuth\ orientation\ angle\ (clock\ wise\ from\ north)\ [deg]: 0° < \alpha < 90°\ allowed$

$Fel = field\ elevation\ orientation\ angle\ (@\ Faz\ line)\ [deg]: 0° < \alpha < 10°\ allowed$

$az = \varphi + Faz = actual\ azimuth\ of\ sun\ relating\ dishes\ array\ [deg]$

$$If\ az > 180° \rightarrow az = az - 180°$$

$el = \psi + Fel = actual\ elevation\ of\ sun\ relating\ dishes\ array\ [deg]$

$a = dish\ width\ [m]$

$S = a^2 = dish\ area\ (sail)\ [m^2]$

$D = a/sin(el) = one\ dish\ shadow\ length\ on\ the\ ground\ /\ at\ elevation\ axis\ plane\ [m]$

$atg = tan(\alpha)$

$X_{N-S} = north\ to\ south\ distance\ of\ dish\ rows,$
$parallel\ to\ field\ (not\ global)\ N-S\ axis\ [m]$

$X_{E-W} = east\ to\ west\ distance\ of\ dish\ columns, at\ the\ same\ row,$
$parallel\ to\ field\ (not\ global)\ E-W\ axis\ [m]$

$N_{N-S} = number\ of\ dishes\ rows\ along\ north - south\ axis,$
$parallel\ to\ field\ (not\ global)\ N-S\ axis$

$N_{E-W} = number\ of\ dishes\ columns\ along\ east-west\ axis,$
$parallel\ to\ field\ (not\ global)\ E-W\ axis$

surrounding dishes location coordinates for 4th rank distance:

designating field $E-W$ coordinates (***X axis***):

$$B\ [m] = X_{E-W} \cdot [0\ ,0.5\ ,1\ ,0.5\ ,2\ ,1.5\ ,1\ ,1.5\ ,2\ ,0.5\ ,1\ ,0.5\ ,0]$$

designating field $N-S$ coordinates (***Y axis***):

$$C1\ [m] = X_{N-S} \cdot [1\ ,1.5\ ,2\ ,0.5\ ,1\ ,0.5\ ,0\ ,-0.5\ ,-1\ ,-0.5\ ,-2\ ,-1.5\ ,-1]$$

Designated field N – S alteration due to alpha angle:

$$C2 = atg \cdot [-1\ ,-1\ ,-1\ ,-1\ ,-1\ ,-1\ ,-1\ ,1\ ,1\ ,1\ ,1\ ,1]$$

designating field $N-S$ actual coordinates (***Y axis***):

$$A\ [m] = C1 + C2$$

designating affecting dishes distance from effected (representing)dish at field center:

$$Dto_i\,[m] \;=\; \sqrt{A_i^2 + B_i^2}$$

designating hand (radius)angle from field north to affecting dishesradial location:

$$Azto_i\,[deg] \;=\; \tan^{-1}\left(\frac{B}{A}\right)$$

$$If\ Azto_i < 0 \;\rightarrow\; Azto_i = Azto_i + 180°$$

$$Azto_N \;=\; 180°$$

shade on representing dish by elevation:

$$El.shade_i\,[m] \;=\; (D - Dto_i)\cdot \sin(el)$$

$$if\ El.shade_i < 0 \;\rightarrow\; El.shade_i = 0$$

shade on representing dish by azimuth:

$$Az.shade_i\,[m] \;=\; a - Dto_i \cdot |\sin(Azto_i - az)|$$

$$if\ |Azto_i - az| \geq 90° \;\rightarrow\; Az.shade_i = 0$$

$$if\ Az.shade_i < 0 \;\rightarrow\; Az.shade_i = 0$$

each shading dish azimuthial / width span on target dish:

$if\ \ Az.shade_i > 0 \;\rightarrow$

$$if\ \{sign[sin(Azto_i - azimuth)] > 0°\}\ AND\ \{|Azto_i - azimuth| \geq 90°\} \rightarrow$$
$$Az.shade.span_{1,i} \;=\; \frac{a}{2}$$
$$Az.shade.span_{2,i} \;=\; \frac{a}{2} - Az.shade_i$$

$$else,\quad if\ \{sign[sin(Azto_i - azimuth)] < 0°\}\ AND\ \{|Azto_i - azimuth| \geq 90°\} \rightarrow$$
$$Az.shade.span_{1,i} \;=\; Az.shade_i - \frac{a}{2}$$
$$Az.shade.span_{2,i} \;=\; -\frac{a}{2}$$

$$\textit{else,} \quad \textit{if } \{sign[sin(Azto_i - azimuth)] = 0°\} \textit{ AND } \{|Azto_i - azimuth| \geq 90°\} \rightarrow$$

$$Az.shade.span_{1,i} = \frac{a}{2}$$

$$Az.shade.span_{2,i} = -\frac{a}{2}$$

$$\textit{else} \rightarrow$$

$$Az.shade.span_{1,i} = 0$$

$$Az.shade.span_{2,i} = 0$$

$$\textit{else} \rightarrow$$

$$Az.shade.span_{1,i} = 0$$

$$Az.shade.span_{2,i} = 0$$

1^{st} *order shade overlap*:

$$\textit{if } \{Az.shade_1 > 0\} \textit{ AND } \{Az.shade_2 > 0\} \rightarrow$$

$$p1 = Az.shade.span_{1,1}$$

$$p2 = Az.shade.span_{2,1}$$

$$s1 = Az.shade.span_{1,2}$$

$$s2 = Az.shade.span_{2,2}$$

$$\textit{if } s1 < p2 \rightarrow$$

$$s1 = p2$$

$$L = s1 - s2$$

$$\textit{else,} \quad \textit{if } s2 < p2 \rightarrow$$

$$s2 = p1$$

$$L = s1 - s2$$

$$\textit{else} \rightarrow$$

$$L = Az.shade_2$$

$$Az.shade_2 = L$$

$$\textit{if } \{Az.shade_2 > 0\} \textit{ AND } \{Az.shade_3 > 0\} \rightarrow$$

$$p1 = Az.shade.span_{1,2}$$

$$p2 = Az.shade.span_{2,2}$$

$$s1 = Az.shade.span_{1,3}$$

$$s2 = Az.shade.span_{2,3}$$

$$\textit{if } s1 > p2 \rightarrow$$

$$s1 = p2$$

$$L = s1 - s2$$

$$\textit{else,} \quad \textit{if } s2 > p2 \rightarrow$$

$$s2 = p1$$

$$L = s1 - s2$$

$$\textit{else} \rightarrow$$

$$L = Az.shade_3$$

$$Az.shade_3 = L$$

$$\textit{if } \{Az.shade_4 > 0\} \textit{ AND } \{Az.shade_3 > 0\} \rightarrow$$

$$p1 = Az.shade.span_{1,4}$$

$p2 = Az.shade.span_{2,4}$
$s1 = Az.shade.span_{1,3}$
$s2 = Az.shade.span_{2,3}$
if $s1 > p2$ →
$s1 = p2$
$L = s1 - s2$
else, *if* $s2 > p2$ →
$s2 = p1$
$L = s1 - s2$
else →
$L = Az.shade_4$
$Az.shade_4 = L$

if $\{Az.shade_4 > 0\}$ *AND* $\{Az.shade_5 > 0\}$ →
$p1 = Az.shade.span_{1,4}$
$p2 = Az.shade.span_{2,4}$
$s1 = Az.shade.span_{1,5}$
$s2 = Az.shade.span_{2,5}$
if $s1 > p2$ →
$s1 = p2$
$L = s1 - s2$
else, *if* $s2 > p2$ →
$s2 = p1$
$L = s1 - s2$
else →
$L = Az.shade_5$
$Az.shade_5 = L$

if $\{Az.shade_6 > 0\}$ *AND* $\{Az.shade_5 > 0\}$ →
$p1 = Az.shade.span_{1,6}$
$p2 = Az.shade.span_{2,6}$
$s1 = Az.shade.span_{1,5}$
$s2 = Az.shade.span_{2,5}$
if $s1 > p2$ →
$s1 = p2$
$L = s1 - s2$
else, *if* $s2 > p2$ →
$s2 = p1$
$L = s1 - s2$
else →
$L = Az.shade_6$
$Az.shade_6 = L$

if $\{Az.shade_7 > 0\}$ *AND* $\{Az.shade_6 > 0\}$ →
$p1 = Az.shade.span_{1,7}$
$p2 = Az.shade.span_{2,7}$
$s1 = Az.shade.span_{1,6}$
$s2 = Az.shade.span_{2,6}$
if $s1 > p2$ →

$$
\begin{aligned}
&\quad s1 = p2 \\
&\quad L = s1 - s2 \\
&\textit{else,} \quad \textit{if } s2 > p2 \rightarrow \\
&\quad s2 = p1 \\
&\quad L = s1 - s2 \\
&\textit{else} \rightarrow \\
&\quad L = Az.shade_7 \\
&Az.shade_7 = L
\end{aligned}
$$

$$
\begin{aligned}
&\textit{if } \{Az.shade_7 > 0\} \textit{ AND } \{Az.shade_8 > 0\} \rightarrow \\
&\quad p1 = Az.shade.span_{1,7} \\
&\quad p2 = Az.shade.span_{2,7} \\
&\quad s1 = Az.shade.span_{1,8} \\
&\quad s2 = Az.shade.span_{2,8} \\
&\quad \textit{if } s1 > p2 \rightarrow \\
&\quad\quad s1 = p2 \\
&\quad\quad L = s1 - s2 \\
&\quad \textit{else,} \quad \textit{if } s2 > p2 \rightarrow \\
&\quad\quad s2 = p1 \\
&\quad\quad L = s1 - s2 \\
&\quad \textit{else} \rightarrow \\
&\quad\quad L = Az.shade_8 \\
&Az.shade_8 = L
\end{aligned}
$$

$$
\begin{aligned}
&\textit{if } \{Az.shade_8 > 0\} \textit{ AND } \{Az.shade_9 > 0\} \rightarrow \\
&\quad p1 = Az.shade.span_{1,8} \\
&\quad p2 = Az.shade.span_{2,8} \\
&\quad s1 = Az.shade.span_{1,9} \\
&\quad s2 = Az.shade.span_{2,9} \\
&\quad \textit{if } s1 > p2 \rightarrow \\
&\quad\quad s1 = p2 \\
&\quad\quad L = s1 - s2 \\
&\quad \textit{else,} \quad \textit{if } s2 > p2 \rightarrow \\
&\quad\quad s2 = p1 \\
&\quad\quad L = s1 - s2 \\
&\quad \textit{else} \rightarrow \\
&\quad\quad L = Az.shade_9 \\
&Az.shade_9 = L
\end{aligned}
$$

$$
\begin{aligned}
&\textit{if } \{Az.shade_{10} > 0\} \textit{ AND } \{Az.shade_9 > 0\} \rightarrow \\
&\quad p1 = Az.shade.span_{1,10} \\
&\quad p2 = Az.shade.span_{2,10} \\
&\quad s1 = Az.shade.span_{1,9} \\
&\quad s2 = Az.shade.span_{2,9} \\
&\quad \textit{if } s1 > p2 \rightarrow \\
&\quad\quad s1 = p2 \\
&\quad\quad L = s1 - s2 \\
&\quad \textit{else,} \quad \textit{if } s2 > p2 \rightarrow \\
&\quad\quad s2 = p1
\end{aligned}
$$

$L = s1 - s2$

else →

$L = Az.shade_{10}$

$Az.shade_{10} = L$

if $\{Az.shade_{10} > 0\}$ *AND* $\{Az.shade_{11} > 0\}$ →

$p1 = Az.shade.span_{1,10}$

$p2 = Az.shade.span_{2,10}$

$s1 = Az.shade.span_{1,11}$

$s2 = Az.shade.span_{2,11}$

if $s1 > p2$ →

$s1 = p2$

$L = s1 - s2$

else, if $s2 > p2$ →

$s2 = p1$

$L = s1 - s2$

else →

$L = Az.shade_{11}$

$Az.shade_{11} = L$

if $\{Az.shade_{12} > 0\}$ *AND* $\{Az.shade_{11} > 0\}$ →

$p1 = Az.shade.span_{1,12}$

$p2 = Az.shade.span_{2,12}$

$s1 = Az.shade.span_{1,11}$

$s2 = Az.shade.span_{2,11}$

if $s1 > p2$ →

$s1 = p2$

$L = s1 - s2$

else, if $s2 > p2$ →

$s2 = p1$

$L = s1 - s2$

else →

$L = Az.shade_{12}$

$Az.shade_{12} = L$

if $\{Az.shade_{13} > 0\}$ *AND* $\{Az.shade_{11} > 0\}$ →

$p1 = Az.shade.span_{1,12}$

$p2 = Az.shade.span_{2,12}$

$s1 = Az.shade.span_{1,13}$

$s2 = Az.shade.span_{2,13}$

if $s1 > p2$ →

$s1 = p2$

$L = s1 - s2$

else, if $s2 > p2$ →

$s2 = p1$

$L = s1 - s2$

else →

$L = Az.shade_{13}$

$Az.shade_{13} = L$

2^{nd} *order shade overlap*:

$$
\begin{aligned}
&\textit{if } \{Az.shade_1 > 0\} \textit{ AND } \{Az.shade_3 > 0\} \rightarrow \\
&\quad p1 = Az.shade.span_{1,1} \\
&\quad p2 = Az.shade.span_{2,1} \\
&\quad s1 = Az.shade.span_{1,3} \\
&\quad s2 = Az.shade.span_{2,3} \\
&\quad \textit{if } s1 > p2 \rightarrow \\
&\qquad s1 = p2 \\
&\qquad L = s1 - s2 \\
&\quad \textit{else, } \quad \textit{if } s2 > p2 \rightarrow \\
&\qquad s2 = p1 \\
&\qquad L = s1 - s2 \\
&\quad \textit{else} \rightarrow \\
&\qquad L = Az.shade_3 \\
&Az.shade_3 = L
\end{aligned}
$$

$$
\begin{aligned}
&\textit{if } \{Az.shade_4 > 0\} \textit{ AND } \{Az.shade_2 > 0\} \rightarrow \\
&\quad p1 = Az.shade.span_{1,4} \\
&\quad p2 = Az.shade.span_{2,4} \\
&\quad s1 = Az.shade.span_{1,2} \\
&\quad s2 = Az.shade.span_{2,2} \\
&\quad \textit{if } s1 > p2 \rightarrow \\
&\qquad s1 = p2 \\
&\qquad L = s1 - s2 \\
&\quad \textit{else, } \quad \textit{if } s2 > p2 \rightarrow \\
&\qquad s2 = p1 \\
&\qquad L = s1 - s2 \\
&\quad \textit{else} \rightarrow \\
&\qquad L = Az.shade_2 \\
&Az.shade_2 = L
\end{aligned}
$$

$$
\begin{aligned}
&\textit{if } \{Az.shade_4 > 0\} \textit{ AND } \{Az.shade_6 > 0\} \rightarrow \\
&\quad p1 = Az.shade.span_{1,4} \\
&\quad p2 = Az.shade.span_{2,4} \\
&\quad s1 = Az.shade.span_{1,6} \\
&\quad s2 = Az.shade.span_{2,6} \\
&\quad \textit{if } s1 > p2 \rightarrow \\
&\qquad s1 = p2 \\
&\qquad L = s1 - s2 \\
&\quad \textit{else, } \quad \textit{if } s2 > p2 \rightarrow \\
&\qquad s2 = p1 \\
&\qquad L = s1 - s2 \\
&\quad \textit{else} \rightarrow \\
&\qquad L = Az.shade_6
\end{aligned}
$$

$Az.shade_6 = L$

$if\ \{Az.shade_7 > 0\}\ AND\ \{Az.shade_5 > 0\}\ \rightarrow$
$p1 = Az.shade.span_{1,7}$
$p2 = Az.shade.span_{2,7}$
$s1 = Az.shade.span_{1,5}$
$s2 = Az.shade.span_{2,5}$
$if\ s1 > p2\ \rightarrow$
$s1 = p2$
$L = s1 - s2$
$else,\quad if\ s2 > p2\ \rightarrow$
$s2 = p1$
$L = s1 - s2$
$else\ \rightarrow$
$L = Az.shade_5$
$Az.shade_5 = L$

$if\ \{Az.shade_7 > 0\}\ AND\ \{Az.shade_9 > 0\}\ \rightarrow$
$p1 = Az.shade.span_{1,7}$
$p2 = Az.shade.span_{2,7}$
$s1 = Az.shade.span_{1,9}$
$s2 = Az.shade.span_{2,9}$
$if\ s1 > p2\ \rightarrow$
$s1 = p2$
$L = s1 - s2$
$else,\quad if\ s2 > p2\ \rightarrow$
$s2 = p1$
$L = s1 - s2$
$else\ \rightarrow$
$L = Az.shade_9$
$Az.shade_9 = L$

$if\ \{Az.shade_{10} > 0\}\ AND\ \{Az.shade_8 > 0\}\ \rightarrow$
$p1 = Az.shade.span_{1,10}$
$p2 = Az.shade.span_{2,10}$
$s1 = Az.shade.span_{1,8}$
$s2 = Az.shade.span_{2,8}$
$if\ s1 > p2\ \rightarrow$
$s1 = p2$
$L = s1 - s2$
$else,\quad if\ s2 > p2\ \rightarrow$
$s2 = p1$
$L = s1 - s2$
$else\ \rightarrow$
$L = Az.shade_8$
$Az.shade_8 = L$

$$
\begin{aligned}
&\textit{if } \{Az.shade_{10} > 0\} \textit{ AND } \{Az.shade_{12} > 0\} \rightarrow \\
&\quad p1 = Az.shade.span_{1,10} \\
&\quad p2 = Az.shade.span_{2,10} \\
&\quad s1 = Az.shade.span_{1,12} \\
&\quad s2 = Az.shade.span_{2,12} \\
&\quad \textit{if } s1 > p2 \rightarrow \\
&\qquad s1 = p2 \\
&\qquad L = s1 - s2 \\
&\quad \textit{else, } \quad \textit{if } s2 > p2 \rightarrow \\
&\qquad s2 = p1 \\
&\qquad L = s1 - s2 \\
&\quad \textit{else} \rightarrow \\
&\qquad L = Az.shade_{12} \\
&Az.shade_{12} = L
\end{aligned}
$$

$$
\begin{aligned}
&\textit{if } \{Az.shade_{13} > 0\} \textit{ AND } \{Az.shade_{11} > 0\} \rightarrow \\
&\quad p1 = Az.shade.span_{1,13} \\
&\quad p2 = Az.shade.span_{2,13} \\
&\quad s1 = Az.shade.span_{1,11} \\
&\quad s2 = Az.shade.span_{2,11} \\
&\quad \textit{if } s1 > p2 \rightarrow \\
&\qquad s1 = p2 \\
&\qquad L = s1 - s2 \\
&\quad \textit{else, } \quad \textit{if } s2 > p2 \rightarrow \\
&\qquad s2 = p1 \\
&\qquad L = s1 - s2 \\
&\quad \textit{else} \rightarrow \\
&\qquad L = Az.shade_{11} \\
&Az.shade_{11} = L
\end{aligned}
$$

$$
\textit{if } \; Az.shade_i < 0 \rightarrow \quad Az.shade_i = 0
$$

average shading with respect to exterior rows:

$$
\begin{aligned}
&\textit{if } \{0° \leq az < 90°\} \textit{ OR } \{180° \leq az < 270°\} \rightarrow \\
&\quad A.outer.rim = (N_{N-S} - 1) \cdot (Az.shade_1 \cdot El.shade_1) \\
&\qquad\qquad + (N_{E-W} - 1) \cdot (Az.shade_7 \cdot El.shade_7) \\
&\quad A.secondary.rim = [(N_{N-S} - 2) + (N_{E-W} - 2)] \cdot (Az.shade_4 \cdot El.shade_4) \\
&\textit{else} \rightarrow \\
&\quad A.outer.rim = (N_{N-S} - 1) \cdot (Az.shade_{13} \cdot El.shade_{13}) \\
&\qquad\qquad + (N_{E-W} - 1) \cdot (Az.shade_7 \cdot El.shade_7) \\
&\quad A.secondary.rim = [(N_{N-S} - 2) + (N_{E-W} - 2)] \cdot (Az.shade_{10} \cdot El.shade_{10})
\end{aligned}
$$

$$S.shade = \sum_{i=1}^{13} Az.shade_i \cdot El.shade_i$$

$$A.inner.field.shade = S.shade \cdot (N_{N-S} - 2) \cdot (N_{E-W} - 2)$$

$$A.shade = |A.outer.rim + A.secondary.rim + A.inner.field.shade|$$

$$\boldsymbol{shading} = \mathbf{1} - \frac{\boldsymbol{A.shade}}{\boldsymbol{S \cdot N_{N-S} \cdot N_{E-W}}} \; [.\%]$$