Regression Models: Assignment

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Executive Summary

When considering fuel efficiency (mpg) and transmission type (automatic or manual) alone, i.e. without accounting for other variables, the manual transmission cars in the `mtcars` data set are, on average, 7 mpg more efficient than the automatics. This does not necessarily mean manual transmission cars are more efficient as there are other variables that must be accounted for. The analysis is complicated by an interaction between transmission type and weight; the lighter cars are predominantly manual while the heavier cars are predominantly automatic. Since light automatics and heavy manuals are not represented in the data set it is difficult to make a fair comparison between the transmission types.

Introduction

Much of the R code in this document has been hidden to aid readability. Plots and model summaries can be found in the appendix below. The full R markdown source for this document can be found in my github repository. The first step in the analysis is to load the data, convert binary or categorical variables to factor variables, and then assign meaningful factor names.

Expoloratory Analysis

Once the data has been loaded the data can be inspected for patterns. The relationship of interest is that between fuel efficiency (`mpg`) and transmission type (`am`). The boxplot (appendix 1, fig. 1) shows a clear increase in the median mpg for manual transmission over automatic transmission. The normal q-q plots (appendix 1, fig. 2) show that the mpg of automatic and manual transmission cars in the data set is approximately normally distributed. The Welch Two Sample t-test suggests a significant difference between the means of the two groups (P = 0.0014, two sided).

Simple Model

The relationship shown above can be expressed as a simple linear model `mpg ~ am`. This model results in an intercept of 17.147, which corresponds to the mean mpg of the reference group (automatic). The second coefficient, 7.245 corresponds to the increase in mean of the manual group compared to the reference group. This result is not surprising considering the obvious difference between the boxplots and the statistically significant difference between the two means. The simple model however only accounts for 36.0% of the variation in mpg, and has a residual sum of squares of 720.90. This is not particularly surprising as the scatterplot matrix (appendix 1, fig. 3) shows mpg to be influenced by a range of variables besides transmission type.

Full Model

A linear model of the form `mpg ~ .` adds terms for every variable in the data set. After throwing every predictor variable into the model, it now accounts for 86.9% of the variation in mpg. When considering multiple variate regression the adjusted R^2 can be used to provide a better indication of explanatory power (as R^2 will increase for every predictor added to the model, regardless of the model's explanatory power). The adjusted R^2 for the full model is 0.807. The values of the coefficients represent the change

in response variable to a unit change in the relevant predictor variable, given the other variables are held constant. The magnitude of the values themselves depend on the measurement unit and therefore are not a reliable indication of correlation strength. The t-values and associated p-values indicate the significance of the effect. The most significant coefficient is weight (`wt`), with a p-value of 0.0633, followed by transmission type (`am`), acceleration (`qsec`) and engine power (`hp`).

Stepwise Regression

Stepwise regression uses an automatic procedure for choosing predictor variables on the basis of AIC (a measure of the trade-off between model fit and complexity). The forwards stepwise regression starts with the simple model and adds variables, resulting in the model `mpg ~ am + hp + wt + qsec`. The four predictor variables in this model correspond to the four most significant coefficients in the full model. The adjusted R^2 is 0.837. The backwards stepwise regression starts with the full model and removes variables, resulting in the model `mpg ~ wt + qsec + am`. The adjusted R^2 for the backward stepwise model is 0.834, slightly lower than that of the forward stepwise model.

Interaction Terms

The scatterplot matrix (appendix 1, fig. 2) shows that many of the data set variables are correlated. Appendix 1, fig. 4 illustrates the individual relationships between the three most significant additional variables. The weight plot shows that the automatic cars tend to be heavier than the manual cars. This may be a reason why the difference between automatic and manual varies according to weight, i.e. there is an interaction between weight and transmission type. The quarter mile time plot shows that automatic cars have a tendency towards lower mpg than the manual cars. The difference increases as quarter mile time increases, i.e. there is also an interaction between quarter mile time and transmission type. The horsepower plot shows that across the range of horsepower, the automatic cars have a consistent tendency towards lower mpg than the manual cars, i.e there is no interaction between transmission type and horsepower.

Model Selection

The final model was selected on the basis of the most significant variables in the full model, as confirmed by the stepwise regression. Since there is a clear interaction between weight and transmission type an interaction term `am:wt` was added to the model. This reduced the significance of the `hp` variable, which was removed from the model. The final model `mpg ~ am + wt + qsec + am:wt` has an adjusted R^2 of 0.880 and a residual sum of squares of 117.28. The model's overall p-value is $7.1679528 \times 10^{-13}$, suggesting that the model is statistically significant.

Interpretation

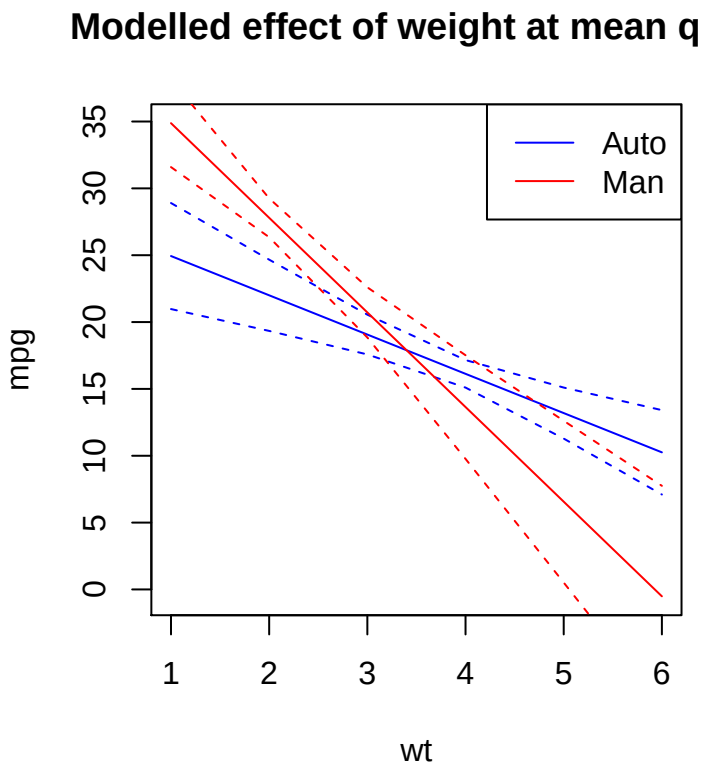
The presence of the interaction term somewhat complicates the interpretation of coefficients in the final model; the effect on mpg of weight is dependent on transmission type. The model can be expressed as the following relationship:

$$mpg \approx 9.7 + 14.1amManual - 2.9wt + 1.0qsec - 4.1(amManual * wt)$$

Since the variable `amManual` is zero for automatic transmission and one for manual transmission, the model can also be expressed as the following pair of relationships:

$$mpg_{auto} \approx 9.7 - 2.9wt + 1.0qsec$$
$$mpg_{manual} \approx 23.8 - 7.0wt + 1.0qsec$$

This means that `mpg` decreases with increasing weight for both transmission types, but the effect is more pronounced with manual transmissions (a reduction of 7 mpg per unit increase in weight). The intercept term is much larger for manual transmission so, at lower weights, the mpg will be higher than for automatics. The plot below shows the modelled effect of weight for both transmission types at the mean quarter mile time. The dotted lines indicate confidence intervals. On account of the lack of lighter automatics and heavier manuals in the data set, the model may be unreliable when extrapolating the relationship for automatic transmission cars of weights below about 2.5 (i.e. 2500 lb) and for manual transmission cars of weights above about 3.6 (i.e. 3600 lb). Within this fairly narrow range, the degree of overlap between the confidence intervals suggests it is not possible to make a firm conclusion regarding the effect of transmission type when accounting for weight.



Diagnostics

Appendix 1, fig. 5 includes four diagnostic plots. There are no noticeable patterns in the residuals, which appear to be approximately normally distributed. The scale-location plot does show a small increase in variance at the larger fitted values. The residuals vs. leverage plot shows that none of the individual data points are highly influential. In general there appear to be no obvious issues with the validity of the model.

Appendix 1: Plots

Simple comparison

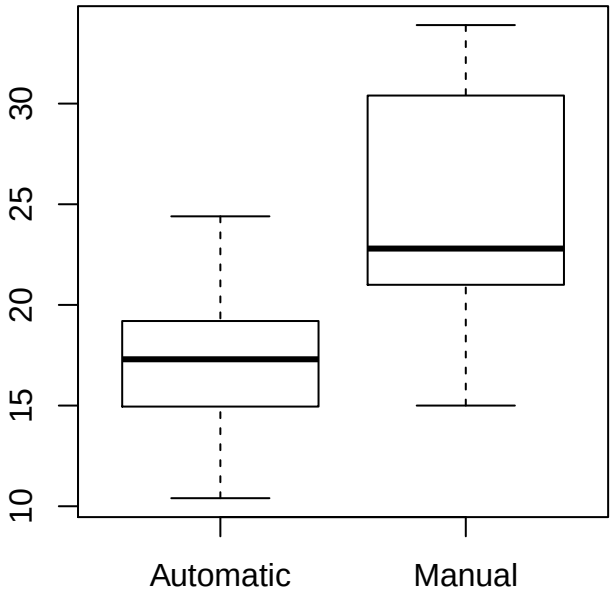


Figure 1: Boxplot for MPG by transmission type

Normal q-q plot

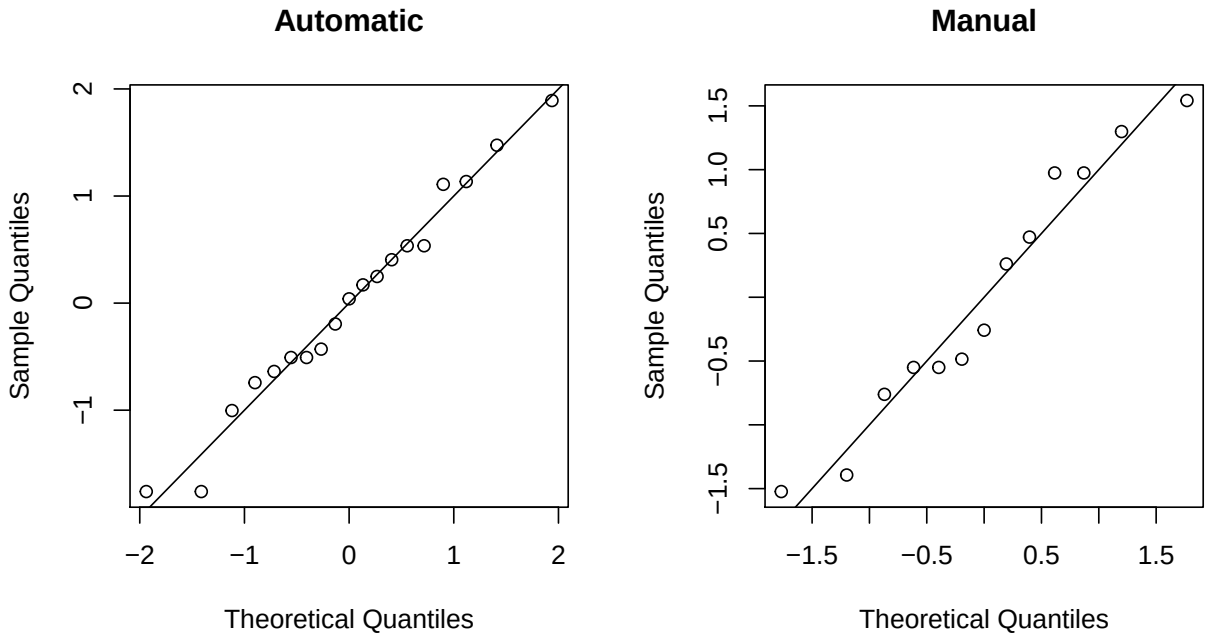


Figure 2: Normal q-q plot for MPG by transmission type

Scatterplot matrix

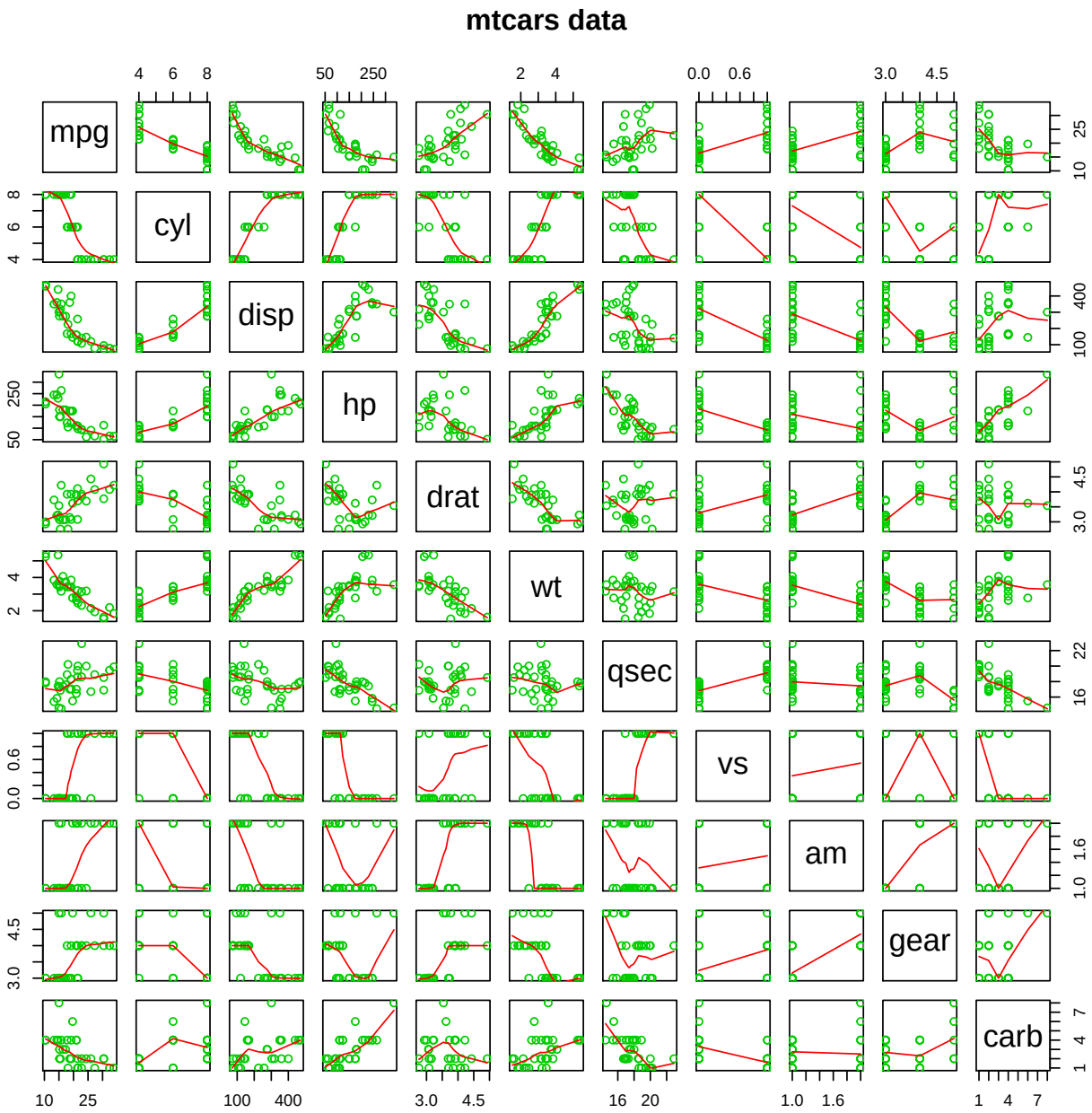


Figure 3: Scatterplot matrix for the mtcars data set

Individual iteractions

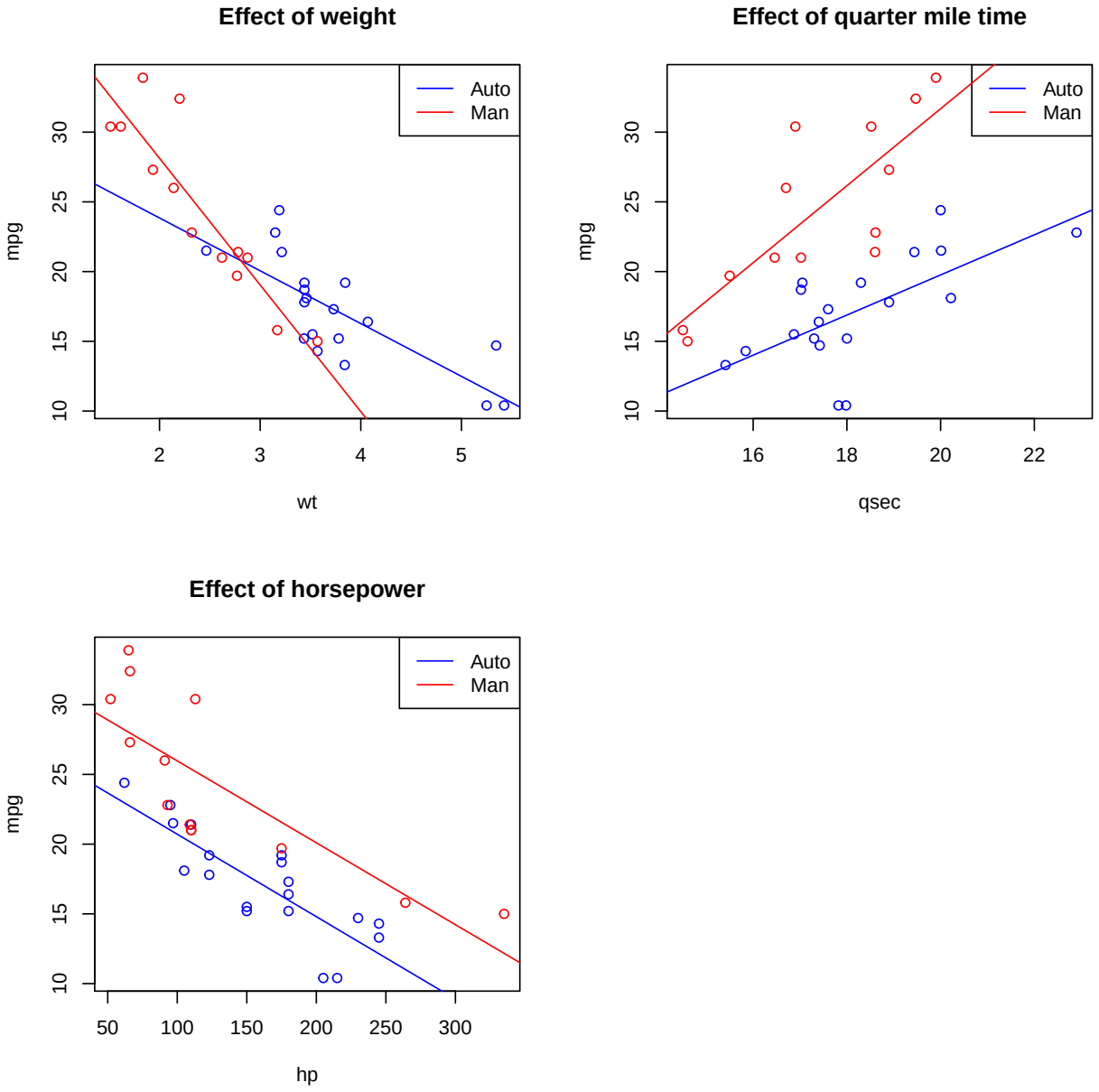


Figure 4: Individual interactions

Diagnostic plots

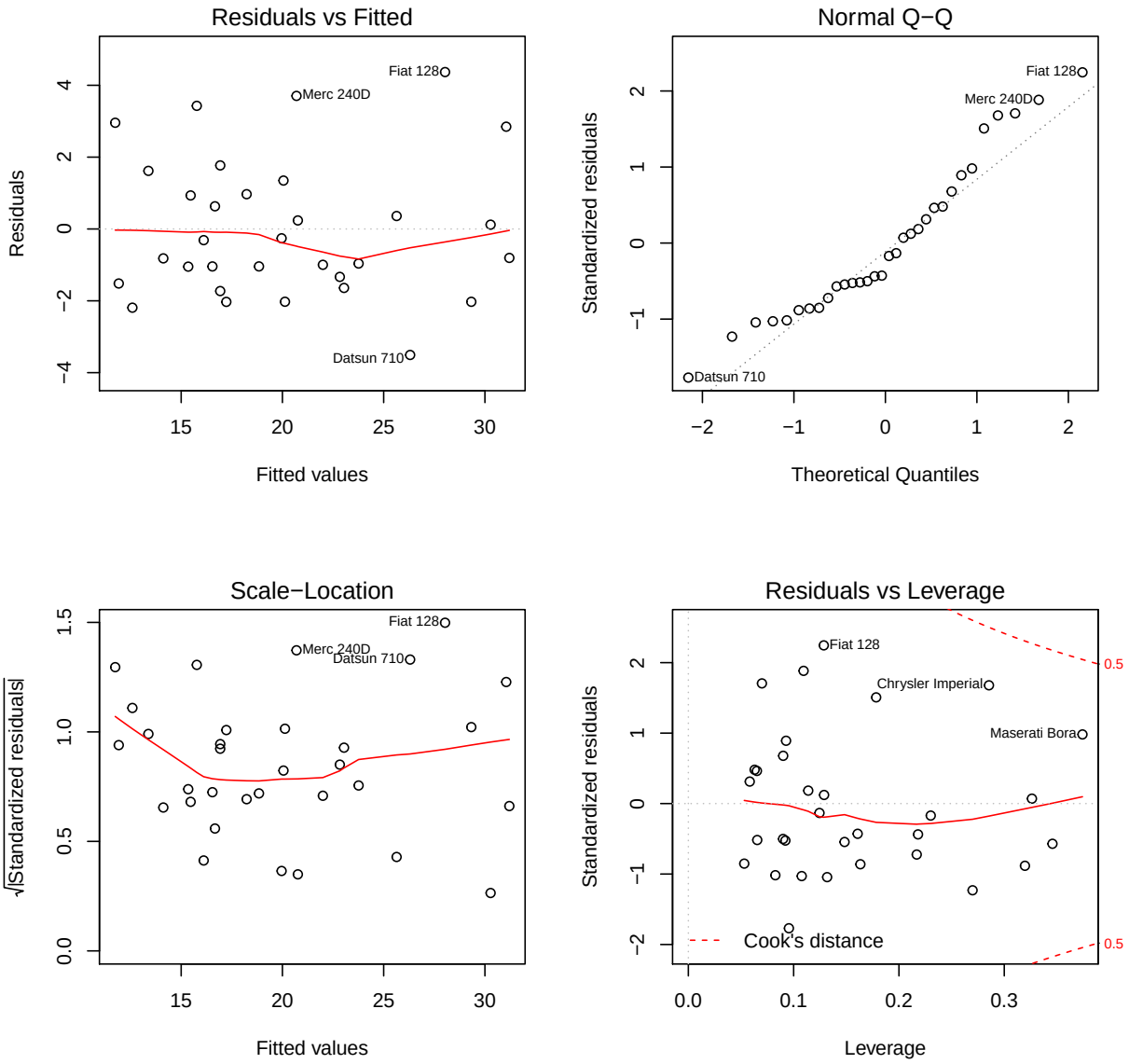


Figure 5: Model diagnostics plots

Appendix 2: Model Summaries

```
summary(mSimple) # Simple model
```

```
##
## Call:
## lm(formula = mpg ~ am, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.3923 -3.0923 -0.2974  3.2439  9.5077
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   17.147      1.125  15.247 1.13e-15 ***
## amManual       7.245      1.764   4.106 0.000285 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.902 on 30 degrees of freedom
## Multiple R-squared:  0.3598, Adjusted R-squared:  0.3385
## F-statistic: 16.86 on 1 and 30 DF,  p-value: 0.000285
```

```
summary(mFull) # Full model
```

```
##
## Call:
## lm(formula = mpg ~ ., data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.4506 -1.6044 -0.1196  1.2193  4.6271
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 12.30337   18.71788   0.657   0.5181
## cyl         -0.11144    1.04502  -0.107   0.9161
## disp         0.01334    0.01786   0.747   0.4635
## hp          -0.02148    0.02177  -0.987   0.3350
## drat         0.78711    1.63537   0.481   0.6353
## wt          -3.71530    1.89441  -1.961   0.0633 .
## qsec         0.82104    0.73084   1.123   0.2739
## vs           0.31776    2.10451   0.151   0.8814
## amManual     2.52023    2.05665   1.225   0.2340
## gear         0.65541    1.49326   0.439   0.6652
## carb        -0.19942    0.82875  -0.241   0.8122
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.65 on 21 degrees of freedom
## Multiple R-squared:  0.869,  Adjusted R-squared:  0.8066
## F-statistic: 13.93 on 10 and 21 DF,  p-value: 3.793e-07
```

```
summary(mStepF) # Forward stepwise model
```

```
##
## Call:
## lm(formula = mpg ~ am + hp + wt + qsec, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.4975 -1.5902 -0.1122  1.1795  4.5404
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 17.44019    9.31887   1.871  0.07215 .
## amManual     2.92550    1.39715   2.094  0.04579 *
## hp          -0.01765    0.01415  -1.247  0.22309
## wt          -3.23810    0.88990  -3.639  0.00114 **
## qsec         0.81060    0.43887   1.847  0.07573 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.435 on 27 degrees of freedom
## Multiple R-squared:  0.8579, Adjusted R-squared:  0.8368
## F-statistic: 40.74 on 4 and 27 DF,  p-value: 4.589e-11
```

```
summary(mStepB) # Backward stepwise model
```

```
##
## Call:
## lm(formula = mpg ~ wt + qsec + am, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.4811 -1.5555 -0.7257  1.4110  4.6610
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   9.6178     6.9596   1.382 0.177915
## wt           -3.9165     0.7112  -5.507 6.95e-06 ***
## qsec          1.2259     0.2887   4.247 0.000216 ***
## amManual      2.9358     1.4109   2.081 0.046716 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.459 on 28 degrees of freedom
## Multiple R-squared:  0.8497, Adjusted R-squared:  0.8336
## F-statistic: 52.75 on 3 and 28 DF,  p-value: 1.21e-11
```

```
summary(mInt) # Final model
```

```
##
## Call:
## lm(formula = mpg ~ am + wt + qsec + am:wt, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.5076 -1.3801 -0.5588  1.0630  4.3684
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)    9.723      5.899   1.648 0.110893
## amManual      14.079      3.435   4.099 0.000341 ***
## wt            -2.937      0.666  -4.409 0.000149 ***
## qsec           1.017      0.252   4.035 0.000403 ***
## amManual:wt   -4.141      1.197  -3.460 0.001809 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.084 on 27 degrees of freedom
## Multiple R-squared:  0.8959, Adjusted R-squared:  0.8804
## F-statistic: 58.06 on 4 and 27 DF,  p-value: 7.168e-13
```

```
anova(mInt)
```

```
## Analysis of Variance Table
##
## Response: mpg
##           Df Sum Sq Mean Sq F value    Pr(>F)
## am         1 405.15  405.15  93.276 2.980e-10 ***
## wt         1 442.58  442.58 101.892 1.161e-10 ***
## qsec       1 109.03  109.03  25.102 2.963e-05 ***
## am:wt      1  52.01   52.01  11.974  0.001809 **
## Residuals 27 117.28    4.34
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```