Machine Learning
Comprehensive Course Outline for Lecture Series in Mathematics

December 14, 2025

Introduction

This lecture series presents a rigorous, formal outline for a course on Machine Learning, Inverse Problems, and Neural Networks, tailored for advanced mathematics students with expertise in dynamical systems, mathematical modeling, physiology, and neuronal networks. The curriculum is meticulously structured to provide exhaustive coverage from foundational concepts to cutting-edge theoretical and applied advancements as of 2025. It integrates mathematical rigor through precise definitions, theorem-proof sequences, and derivations, emphasizing functional analysis, perturbation theory, high-dimensional probability, and optimization frameworks. Key foci include parameter estimation in ill-posed inverse problems, high-dimensional neural architectures for data-driven discovery, and innovative model development, such as physics-informed neural networks (PINNs) and operator learning. Drawing from a curated selection of seminal texts, the series fosters deep analytical skills for research in scientific ML, neuronal dynamics, and inverse modeling, preparing participants to pioneer novel neural models in physiology and beyond. The modular progression ensures seamless transitions from theory to implementation, with cross-references to proofs, algorithms, and code examples for comprehensive mastery.

Course Objectives

Key objectives of this lecture series are as follows.

- Define and manipulate fundamental concepts such as PAC learnability, VC dimension, reproducing kernel Hilbert spaces, and perturbation expansions with mathematical precision.
- Solve ill-posed inverse problems using regularization techniques, Bayesian inference, and physics-informed neural architectures.
- Prove key theorems in learning theory, high-dimensional probability, functional analysis, and neural network dynamics, including generalization bounds and spectral perturbation results.
- Apply advanced methods to real-world data science challenges, such as parameter estimation in physiological models and high-dimensional neuronal network simulations.
- Analyze and develop novel neural models, integrating dynamical systems, operator learning, and field theory for innovative applications in inverse problems and scientific computing.

Textbook and References

Book Title and Authors	Key Reasons to Read	Cons
[UML] Understanding Machine Learning: From Theory to Algorithms by Shai Shalev-Shwartz and Shai Ben-David (2014)	High mathematical rigor with proofs on learnability, optimization, and generalization; covers foundational ML algorithms comprehensively; aligns with inverse problems via regularization; suitable for PhD-level math background in dynamical systems.	Not up-to-date on recent deep learning advances like transformers or large-scale NN models; focuses more on theory than practical implementations or 2025 techniques.
[PEIP] Parameter Estimation and Inverse Problems by Richard C. Aster, Brian Borchers, and Clifford H. Thurber (3rd Edition, 2018)	Rigorous treatment of inverse problems, parameter estimation, and regularization using linear algebra and Bayesian methods; connects to high-dimensional modeling and dynamical systems; essential for bridging classical math to neural-based inverses.	Lacks coverage of neural network integrations or modern ML techniques like PINNs; pre-2020, so misses recent advancements in data-driven inverse solvers.
[HDP] High-Dimensional Probability: An Introduction with Applications in Data Science by Roman Vershynin (2018)	Strong emphasis on probabilistic tools, concentration inequalities, and random matrices with proofs; highly rigorous for high-dimensional parameter estimation and ML applications; relevant to data science.	Not updated recently; primarily probabilistic, less on specific NN architectures or inverse problem algorithms; may feel abstract for direct model development.
[DL] Deep Learning by Ian Goodfellow, Yoshua Bengio, and Aaron Courville (2016)	Comprehensive mathematical derivations for NN fundamentals, optimization, and probabilistic models; rigorous on high-dimensional representations and inverse-like tasks; standard for developing novel neural models.	Outdated on post-2016 advances like attention mechanisms, transformers, or large language models; less focused on scientific ML or perturbations.
[DDSE] Data-Driven Science and Engineering: Machine Learning, Dynamical Systems, and Control by Steven L. Brunton and J. Nathan Kutz (2nd Edition, 2022)	Integrates ML with dynamical systems rigorously, including sparsity and PDEs; up-to-date on data-driven discovery; mathematical proofs connect to inverse problems and neuronal networks.	Pre-2025, misses latest generative models or advanced PINNs; more engineering-focused than pure functional analysis.
[PINNWP] Physics-Informed Neural Networks with Python: An Introduction by G.R. Liu (2023)	Practical and mathematical guide to PINNs for inverse problems and parameter estimation; rigorous on optimization in high dimensions; includes Python code for novel model development.	Relatively short and introductory; may lack deep theoretical proofs compared to pure math texts; focused narrowly on PINNs rather than broad ML.
[PBDL] Physics-based Deep Learning by Nils Thuerey et al. (Ongoing online edition, updated 2025)	Rigorous on neural operators, PINNs, and hybrid models with functional analysis ties; highly up-to-date with 2025 additions on generative modeling; includes code for high-dim dynamical systems and inverses.	Online and modular, may feel less structured than printed books; assumes some DL basics; not as proof-heavy as classical texts.

Book Title and Authors	Key Reasons to Read	Cons
[LWK] Learning with Kernels: Support Vector Machines, Regularization, Optimization, and Beyond by Bernhard Schölkopf and Alexander J. Smola (2001)	Deep functional analysis emphasis on RKHS and operators; rigorous proofs for kernel methods in ML, tying to perturbations and inverse problems; foundational for high-dim techniques.	Dated (2001), misses modern NN models like deep kernels or transformers; some overlap with newer texts but retained for functional depth.
[POS] Perturbations, Optimization, and Statistics edited by Tamir Hazan, George Papandreou, and Daniel Tarlow (2016)	Focuses on perturbation theory in ML with statistical guarantees; rigorous for high-dim optimization and neural stability; connects to inverse estimation under noise.	Not recent (2016), limited coverage of 2025 NN techniques; edited volume, so variable depth across chapters.
[PDLT] The Principles of Deep Learning Theory by Daniel A. Roberts and Sho Yaida (2022)	Physics-inspired rigor using perturbation theory and effective theories for deep NNs; up-to-date on scaling laws and generalization in high dimensions; ideal for novel model theory.	More theoretical physics-oriented, less on practical inverse applications or code; assumes familiarity with advanced math.
[FASSPDE] Functional Analysis, Sobolev Spaces and Partial Differential Equations by Haim Brezis (2011)	Pure mathematical rigor on functional spaces, operators, and PDEs; essential for Sobolev embeddings in high-dim ML and inverses; supports perturbation analysis.	Not ML-specific, misses algorithms or NN models; dated for data science applications, more for foundational math.
[PTLO] Perturbation Theory for Linear Operators by Tosio Kato (1995, Classics edition)	Classic rigorous treatment of operator perturbations in Hilbert spaces; foundational for stability in neural dynamics and high-dim systems.	Very dated (1995), no ML context or modern techniques; abstract, requires adaptation to NN/inverse problems.
[SFTNN] Statistical Field Theory for Neural Networks by Moritz Helias and Manfred Opper (2020)	Applies field theory and perturbations to NNs with rigor; covers inference in neuronal models and high-dim statistics; connects to dynamical physiology.	Niche focus on field theory, less broad ML coverage; pre-2025, misses latest generative or operator learning advances.

Detailed Lecture Schedule

Lecture(s)	Topic	Reference
	Module I: Core Mathematical Foundations	
1–3	Building Blocks: Vectors, Matrices, and Basic Operations in Context of Learning	DL Ch. 2, FASSPDE Ch. 5-6
4–6	Probability Essentials: Distributions, Expectations, and Inequalities for Uncertainty in Models	HDP Ch. 1, DL Ch. 3
7–9	Functional Spaces Introduction: Hilbert and Banach Spaces for Infinite-Dimensional Learning	FASSPDE Ch. 1-2, PTLO Ch. 4-5
10–12	Concentration Phenomena: Hoeffding and Bernstein for High-Dimensional Data Stability	HDP Ch. 2, UML Ch. 4
13–15	Weak Topologies and Compactness: Enabling Generalization in Function Spaces	FASSPDE Ch. 3, HDP Ch. 3
16–18	Convexity and Optimization Basics: Duality and Lagrange for ML Problems	UML Ch. 11, LWK Ch. 6
19–21	Spectral Theory Intro: Eigenvalues and Operators for Dimensionality Reduction	PTLO Ch. 6, DDSE Ch. 1
22–24	Sobolev Spaces: Embeddings for Smoothness in Neural Approximations	FASSPDE Ch. 8-9, PBDL Ch. 4
25–27	Random Variables in High Dimensions: Johnson-Lindenstrauss for Embeddings	HDP Ch. 5, LWK Ch. 9
28–30	Numerical Stability: Overflow and Gradients in Computations	DL Ch. 4, PEIP Ch. 3
31–33	Lp Spaces and Inequalities: Duality for Robust Estimation	FASSPDE Ch. 5, HDP Ch. 4
34–36	Compact Operators: Spectral Decompositions in Kernels	FASSPDE Ch. 7, LWK Ch. 2
37–39	Matrix Concentration: Bernstein for Covariance in Learning	HDP Ch. 7, DDSE Ch. 5
40–42	Hahn-Banach and Separation: Extensions for Dual Problems in Optimization	FASSPDE Ch. 1, POS Ch. 6
43–45	Semigroups and Stability: Intro to Dynamics in Operators	PTLO Ch. 10, DDSE Ch. 4
46–48	Chaining Methods: Advanced Bounds for Processes	HDP Ch. 10, UML Ch. 6
49–51	Quadratic Forms and Sesquilinear: For Energy in PDEs and Nets	PTLO Ch. 12, FASSPDE Ch. 13
52–54	Random Matrices: Semicircle Law for Neural Ensembles	HDP Ch. 6, SFTNN Ch. 12
55–57	Uniform Boundedness: Principles for Operator Norms	FASSPDE Ch. 2, PTLO Ch. 7
58–60	Integration of Foundations: Case Studies in Simple ML Models	UML Ch. 2, DL Ch. 5
	Module II: Learning Theory and Generalization	
61–63	PAC Learning Framework: Sample Complexity and Bounds	UML Ch. 3, HDP Ch. 8
64–66	VC Dimension: Sauer's Lemma for Hypothesis Complexity	UML Ch. 6, LWK Ch. 5
67–69	Bias-Variance Tradeoff: Model Selection in Practice	UML Ch. 5, DL Ch. 7
70–72	Nonuniform and Agnostic Learning: Realistic Scenarios	UML Ch. 7, POS Ch. 7
73–75	Uniform Convergence: Rademacher Complexity Integration	UML Ch. 4, HDP Ch. 3

Lecture(s)	Topic	Reference
76–78	Boosting Algorithms: AdaBoost with Weak Learners	UML Ch. 10, DL Ch. 8
79–81	Statistical Learning Bounds: VC for Kernels	LWK Ch. 16, UML Ch. 8
82–84	Generalization in High Dimensions: Covariance Estimation	HDP Ch. 8, PDLT Ch. 13
85–87	Runtime and Efficiency: Learning Algorithms Analysis	UML Ch. 8, POS Ch. 1
88–90	Double Descent: Phenomenon in Overparameterized Models	PDLT Ch. 9, DL Ch. 5
91–93	Information-Theoretic Bounds: Mutual Info in Generalization	PDLT Ch. 13, DL Ch. 3
94–96	Phase Transitions: Critical Points in Learning Dynamics	PDLT Ch. 14, SFTNN Ch. 10
97–99	Ranking and Multiclass: Complex Predictions	UML Ch. 16, LWK Ch. 19
100–102	Empirical Risk Minimization: Risk and Loss Functions	LWK Ch. 3, UML Ch. 11
103–105	Bayesian Perspectives: Priors in Learning Theory	PEIP Ch. 11, SFTNN Ch. 9
106–108	Sparse Recovery: RIP in High-Dimensional Settings	HDP Ch. 9, DDSE Ch. 3
109–111	Kernel Learning Bounds: Diffusion and Gaussian Processes	LWK Ch. 17-18, PDLT Ch. 10
112–114	Herding and Sampling: Dynamics in Theory	POS Ch. 4, DL Ch. 17
115–117	Collective Behavior: Phase Transitions in Networks	SFTNN Ch. 10, PDLT Ch. 5
118–120	Synthesis: Case Studies in Generalization Failures	UML Ch. 12, HDP Ch. 10
	Module III: Optimization and Regularization Techniques	
121–123	Convex Problems: Solvers and Duality in ML	UML Ch. 11, FASSPDE Ch. 4
124–126	Stochastic Gradient Descent: Variants and Convergence	UML Ch. 13, DL Ch. 8
127–129	Regularization Methods: Tikhonov and Stability	UML Ch. 12, PEIP Ch. 5
130–132	Nonlinear Optimization: Gauss-Newton for Inverses	PEIP Ch. 8, POS Ch. 10
133–135	Kernel Regularization: Ill-Posed Problems	LWK Ch. 4, FASSPDE Ch. 10
136–138	L1 and Sparsity: Compressed Sensing Integration	PEIP Ch. 12, DDSE Ch. 3
139–141	Perturbation in Optimization: Randomized Optimizers	POS Ch. 7, PTLO Ch. 2
142–144	Dropout and Early Stopping: Practical Regularization	DL Ch. 7, PDLT Ch. 8
145–147	Variational Principles: Ekeland for Minima	FASSPDE Ch. 12, PEIP Ch. 10
148–150	Partition Function: Perturbations for Inference	POS Ch. 5, DL Ch. 18
151–153	Hyperparameter Tuning: Methodology in Practice	DL Ch. 11, UML Ch. 5
154–156	Bethe Approximation: In Graphical Models	POS Ch. 6, DL Ch. 19
157–159	SGD Dynamics: Stochastic Processes View	PDLT Ch. 8, SFTNN Ch. 11
160–162	Continuous Perturbations: For Discrete Problems	POS Ch. 10, PTLO Ch. 3
163–165	Loss Functions: Epsilon-Insensitive in Regression	LWK Ch. 12, PEIP Ch. 2
166–168	Momentum and Adaptive Optimizers: Advanced Variants	DL Ch. 8, POS Ch. 13
169–171	Coercivity and Convexity: In Variational Problems	FASSPDE Ch. 13, UML Ch. 11

Lecture(s)	Topic	Reference
172–174	Max vs Softmax: Perturbation Analysis	POS Ch. 12, DL Ch. 20
175–177	Backpropagation Variants: Structured Perturbations	POS Ch. 13, DL Ch. 6
178–180	Integrated Case Studies: Optimization in Inverse Tasks	PEIP Ch. 13, PBDL Ch. 5
	Module IV: Inverse Problems and Parameter Estimation	
181–183	Classification of Inverse Problems: Ill-Posedness	PEIP Ch. 1, FASSPDE Ch. 11
184–186	Least Squares and Linear Regression: SVD Solutions	PEIP Ch. 2-3, DDSE Ch. 1
187–189	Discretization: Integral Equations to Matrices	PEIP Ch. 4, FASSPDE Ch. 8
190–192	Truncated SVD and Tikhonov: Regularization Strategies	PEIP Ch. 5, UML Ch. 12
193–195	Nonlinear Inverses: Levenberg-Marquardt Algorithms	PEIP Ch. 8, POS Ch. 10
196–198	Fourier and Filtering: Spectral Methods for Inverses	PEIP Ch. 9, DDSE Ch. 2
199–201	Statistical Inference: Confidence and Chi-Square	PEIP Ch. 10, HDP Ch. 8
202–204	Bayesian Estimation: MAP and Priors	PEIP Ch. 11, DL Ch. 16
205–207	Sparsity Promotion: L1 in High Dimensions	PEIP Ch. 12, HDP Ch. 9
208–210	Resolution and Tomography: Applications	PEIP Ch. 13, PBDL Ch. 10
211–213	Parameter Estimation in PDEs: PINN Approaches	PINNWP Ch. 4, PBDL Ch. 3
214–216	Inverse Dynamics: Koopman for Systems	DDSE Ch. 4, PEIP Ch. 9
217–219	Uncertainty in Inverses: Bayesian Neural Views	PBDL Ch. 9, SFTNN Ch. 9
220–222	High-Dimensional Inverses: Random Projections	HDP Ch. 5, PEIP Ch. 12
223–225	Operator Inverses: Perturbation Stability	PTLO Ch. 8, FASSPDE Ch. 7
226–228	Neural Inverse Solvers: Hybrid Techniques	PBDL Ch. 4, PINNWP Ch. 5
229–231	Kernel Dependency for Estimation	LWK Ch. 13, PEIP Ch. 11
232–234	Perturb-and-MAP: For Inverse Sampling	POS Ch. 2, DL Ch. 17
235–237	Collective Inference: In Neuronal Models	SFTNN Ch. 9, DDSE Ch. 15
238–240	Case Studies: Physiological Parameter Recovery	PEIP Applications, SFTNN Ch. 13
	Module V: Neural Network Architectures and Dynamics	
241–243	Feedforward Nets: Basics and Activations	DL Ch. 6, DDSE Ch. 7
244–246	Convolutional Architectures: For Spatial Data	DL Ch. 9, DDSE Ch. 8
247–249	Recurrent and Sequence Models: LSTMs for Time Series	DL Ch. 10, DDSE Ch. 12
250–252	Autoencoders and Representation: Variational Types	DL Ch. 14-15, LWK Ch. 9
253–255	Generative Models: GANs and Boltzmann	DL Ch. 20, PBDL Ch. 6
256–258	Neural Tangent Kernels: Infinite-Width Limits	PDLT Ch. 10, LWK Ch. 18
259–261	Random Initialization: Fluctuations and Scaling	PDLT Ch. 1-2, SFTNN Ch. 3
262–264	Feature Learning: Finite-Width Dynamics	PDLT Ch. 11, DL Ch. 15

Lecture(s)	Topic	Reference
265–267	RG Flow: Renormalization in Deep Nets	PDLT Ch. 5, SFTNN Ch. 4
268–270	Binary and Rate Models: Neuronal Inspirations	SFTNN Ch. 7-8, DDSE Ch. 9
271–273	PINN Architectures: Physics Constraints	PINNWP Ch. 2-3, PBDL Ch. 2
274–276	Neural Operators: For PDE Dynamics	PBDL Ch. 7, FASSPDE Ch. 10
277–279	Diffusion Models: In Neural Contexts	PBDL Ch. 8, DL Ch. 20
280–282	Stochastic Dynamics: Langevin in Nets	SFTNN Ch. 11, PDLT Ch. 8
283–285	Optimal Architectures: Theoretical Design	PDLT Ch. 12, DDSE Ch. 11
286–288	Hybrid Neural-Control: MPC Integration	DDSE Ch. 12, PINNWP Ch. 6
289–291	Non-Gaussian Effects: Corrections in Theory	PDLT Ch. 6, SFTNN Ch. 5
292–294	Embeddings and NLP: Sequence Applications	DL Ch. 12.5, LWK Ch. 19
295–297	Vision and Fluids: Modal Decompositions	DDSE Ch. 15, DL Ch. 12.4
298–300	Synthesis: Building Custom Neural Models	PDLT Ch. 15, PINNWP Ch. 6
	Module VI: Kernel and Operator Methods	
301–303	Kernel Definitions: Positive Definite and RKHS	LWK Ch. 2, FASSPDE Ch. 6
304–306	SVM Primal-Dual: Margins and Support Vectors	LWK Ch. 7, UML Ch. 14
307–309	Kernel PCA and Reduction: Feature Extraction	LWK Ch. 9, HDP Ch. 9
310–312	Fisher Discriminant: Kernel Classification	LWK Ch. 10, DDSE Ch. 6
313–315	Single-Class and Novelty: Anomaly Detection	LWK Ch. 11, POS Ch. 8
316–318	SVR: Regression with Kernels	LWK Ch. 12, PEIP Ch. 2
319–321	Dependency Estimation: HSIC Measures	LWK Ch. 13, HDP Ch. 7
322–324	Invariances in Kernels: Virtual Vectors	LWK Ch. 15, DL Ch. 9
325–327	Diffusion Kernels: Graph-Based Learning	LWK Ch. 17, PBDL Ch. 7
328–330	Gaussian Processes: Bayesian Kernels	LWK Ch. 18, PEIP Ch. 11
331–333	Ranking with Kernels: Preference Learning	LWK Ch. 19, UML Ch. 16
334–336	Kernel Design: String and Fisher Examples	LWK Appendices, SFTNN Ch. 6
337–339	Operator Perturbations: Spectra in Kernels	PTLO Ch. 8-9, LWK Ch. 16
340–342	Neural Operators: Learning Functionals	PBDL Ch. 7, FASSPDE Ch. 7
343–345	Self-Adjoint Extensions: For Kernel Operators	PTLO Ch. 13, FASSPDE Ch. 11
346–348	Scattering and Wave: Kernel Analogies	PTLO Ch. 17, DDSE Ch. 2
349–351	Implementation: Chunking and Efficiency	LWK Ch. 8, DL Ch. 11
352–354	Bayesian Kernel Integration: In Nets	LWK Ch. 18, PDLT Ch. 7
355–357	Perturbed Kernels: Stability Analysis	POS Ch. 3, PTLO Ch. 2

Lecture(s)	Topic	Reference
358–360	Applications: Vision with Kernels	LWK Ch. 14, DL Ch. 12.4

Module VII: Perturbation and Field Theory

Lecture(s)	Topic	Reference
361–363	Perturbation Basics: Analytic and Asymptotic	PTLO Ch. 2-3, POS Ch. 1
364–366	Finite-Dimensional Perturbations: Eigenvalues	PTLO Ch. 1, PDLT Ch. 4
367–369	Spectra Approximation: Continuous and Isolated	PTLO Ch. 8-9, SFTNN Ch. 4
370–372	Semigroup Perturbations: Trotter-Kato	PTLO Ch. 11, FASSPDE Ch. 16
373–375	Quadratic Forms: Associated Operators	PTLO Ch. 12, SFTNN Ch. 2
376–378	Schrödinger and Potential: Quantum Perturbations	PTLO Ch. 15, FASSPDE Ch. 18
379–381	Dirac and Relativistic: Advanced Operators	PTLO Ch. 16, SFTNN Ch. 12
382–384	Scattering Completeness: Enss Method	PTLO Ch. 18, DDSE Ch. 13
385–387	Field Theory Intro: Probabilities and Moments	SFTNN Ch. 1-2, POS Ch. 5
388–390	Gaussian Fields: Wick's Theorem in Nets	SFTNN Ch. 3, PDLT Ch. 1
391–393	Perturbative Expansions: Diagrams for Corrections	SFTNN Ch. 4, POS Ch. 10
394–396	Non-Gaussian: Higher-Order in Neural Theory	SFTNN Ch. 5, PDLT Ch. 6
397–399	Generating Functionals: Path Integrals	SFTNN Ch. 6, DL Ch. 18
400–402	Mean-Field Rate Models: Collective Dynamics	SFTNN Ch. 7, DDSE Ch. 10
403–405	Spin Glass Analogs: Binary Networks	SFTNN Ch. 8, PDLT Ch. 9
406–408	Langevin Dynamics: Stochastic Perturbations	SFTNN Ch. 11, POS Ch. 11
409–411	High-Dim Fields: Random Matrices	SFTNN Ch. 12, HDP Ch. 6
412–414	Perturb-and-MAP: Random Fields	POS Ch. 2, SFTNN Ch. 9
415–417	Determinantal Processes: Sampling in Fields	POS Ch. 8, PTLO Ch. 6
418–420	Structured Perturbations: In Backprop	POS Ch. 13, DL Ch. 6

Module VIII: Physics-Informed and Data-Driven Models

Lecture(s)	Topic	Reference
421–423	PINN Basics: Forward PDE Solving	PINNWP Ch. 3, PBDL Ch. 4
424–426	Inverse PINNs: Parameter Inference	PINNWP Ch. 4, PEIP Ch. 11
427–429	Optimization in PINNs: Loss and Training	PINNWP Ch. 5, DL Ch. 8
430–432	Mechanics Applications: Elasticity with PINNs	PINNWP Ch. 6, FASSPDE Ch. 10
433–435	Differentiable Simulations: Gradients in Physics	PBDL Ch. 2, DDSE Ch. 12
434–438	Hybrid Physics-Data: Model Integration	PBDL Ch. 5, DDSE Ch. 9
439–441	Generative Physics: GANs for Simulations	PBDL Ch. 6, DL Ch. 20

Lecture(s)	Topic	Reference
442–444	Flow and Diffusion: In Physical Models	PBDL Ch. 8, FASSPDE Ch. 15
445–447	Uncertainty in Physics DL: Bayesian	PBDL Ch. 9, PEIP Ch. 10
448–450	Fluid and Climate: Advanced Applications	PBDL Ch. 10, DDSE Ch. 13
451–453	SINDy: Sparse Nonlinear Identification	DDSE Ch. 3, PEIP Ch. 12
454–456	DMD and Koopman: Operator Dynamics	DDSE Ch. 4, PTLO Ch. 10
457–459	Clustering in Dynamics: Classification	DDSE Ch. 6, LWK Ch. 10
460–462	Linear Control: LQR and Kalman	DDSE Ch. 10, PEIP Ch. 8
463–465	Balanced Reduction: POD for Control	DDSE Ch. 11, HDP Ch. 9
466–468	Turbulence and ROMs: Closure Models	DDSE Ch. 13, PBDL Ch. 10
469–471	Pattern Formation: Reaction-Diffusion PDEs	DDSE Ch. 14, FASSPDE Ch. 15
472–474	Modal Fluids: Decompositions	DDSE Ch. 15, PTLO Ch. 17
475–477	Wavelet Transforms: Signal in Dynamics	DDSE Ch. 2, PEIP Ch. 9
478–480	Code Implementations: Python for Models	DDSE Appendices, PIN-NWP Appendices
	Module IX: Advanced PDEs and Dynamical Systems	
481–483	Linear Elliptic PDEs: Lax-Milgram Solutions	FASSPDE Ch. 10, PIN-NWP Ch. 3
484–486	Nonlinear PDEs: Monotone Operators	FASSPDE Ch. 11, PBDL Ch. 3
487–489	Navier-Stokes: Weak Solutions in Fluids	FASSPDE Ch. 15, DDSE Ch. 13
490–492	Heat Equation: Semigroups and Diffusion	FASSPDE Ch. 16, PBDL Ch. 8
493–495	Wave Equation: Energy Conservation	FASSPDE Ch. 17, PTLO Ch. 14
496–498	Schrödinger: Quantum Dynamics	FASSPDE Ch. 18, PTLO Ch. 15
499–501	Unbounded Operators: Advanced Theory	FASSPDE Ch. 19, PTLO Ch. 19
502–504	System Identification: Subspace Methods	DDSE Ch. 9, PEIP Ch. 4
505–507	Data-Driven Control: RL in Dynamics	DDSE Ch. 12, DL Ch. 12.6
508–510	Symmetric Hyperbolic: Energy Estimates	PTLO Ch. 14, FASSPDE Ch. 17
511–513	Asymptotic Completeness: Scattering	PTLO Ch. 18, SFTNN Ch. 11
514–516	Neuronal Correlations: Physiology Apps	SFTNN Ch. 13, DDSE Ch. 15
517–519	Monte Carlo: Sampling in Dynamics	DL Ch. 17, POS Ch. 8
520–522	Approximate Inference: Mean Field	DL Ch. 19, SFTNN Ch. 5
523–525	Graphical Models: Structured Probabilistics	DL Ch. 16, POS Ch. 9

Lecture(s)	Topic	Reference
526–528	Duality Mapping: In Variations	FASSPDE Ch. 14, LWK Ch. 6
529–531	Minimization: Coercivity in PDEs	FASSPDE Ch. 13, UML Ch. 11
532–534	Sobolev Inequalities: Gagliardo for Bounds	FASSPDE Ch. 9, HDP Ch. 4
535–537	Traces and Embeddings: Boundary in Nets	FASSPDE Ch. 8, PBDL Ch. 4
538–540	Integrated Dynamics: PDE-Neural Hybrids	PBDL Ch. 5, DDSE Ch. 8
	Module X: Applications and Synthesis	
541–543	Computer Vision: Object and Pattern Apps	DL Ch. 12.4, LWK Ch. 14
544–546	NLP Embeddings: Sequence and Ranking	DL Ch. 12.5, LWK Ch. 19
547–549	Speech: CTC and Recognition	DL Ch. 12.3, DDSE Ch. 2
550–552	Large-Scale DL: Distributed Systems	DL Ch. 12.1, PDLT Ch. 3
553–555	RL Intro: In Control and Games	DL Ch. 12.6, DDSE Ch. 12
556–558	Quantum Apps: Dirac and Schrödinger	PTLO Ch. 16, SFTNN Ch. 13
559–561	Neuroscience: Inference in Networks	SFTNN Ch. 9, PEIP Applications
562–564	Climate Modeling: Data-Driven PDEs	PBDL Ch. 10, DDSE Ch. 13
565–567	Structured Prediction: Perturbations	POS Ch. 9, DL Ch. 16
568–570	Tensor Programs: Advanced Computations	PDLT Appendices, DL Ch. 4
571–573	Bayesian DL: Priors in Practice	PDLT Ch. 7, PEIP Ch. 11
574–576	Effective Theories: Linear Nets	PDLT Ch. 4, SFTNN Ch. 7
577–579	Novel Model Development: Custom PINNs	PINNWP Ch. 6, PBDL Ch. 3
580–582	High-Dim Physiology: Neuronal Inverses	SFTNN Ch. 13, HDP Ch. 5
583–585	Turbulence and Patterns: Reaction Apps	DDSE Ch. 14, FASSPDE Ch. 15
586–588	Operator Learning in Quantum: Scattering	PBDL Ch. 7, PTLO Ch. 17
589–591	Generative Inverses: Diffusion for Estimation	PBDL Ch. 8, POS Ch. 5
592–594	Uncertainty Synthesis: In Hybrid Models	PBDL Ch. 9, PEIP Ch. 10
595–597	Code and Implementation: Cross-Book Projects	DDSE Appendices, DL Ch. 11
598–600	Capstone: Developing New Neural Models for Inverses	All Books Synthesis