REPORT

Homework III

Numerical Methods

MAT202E - 21257

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1- Lagrange, Linear, Quadratic, Cubic

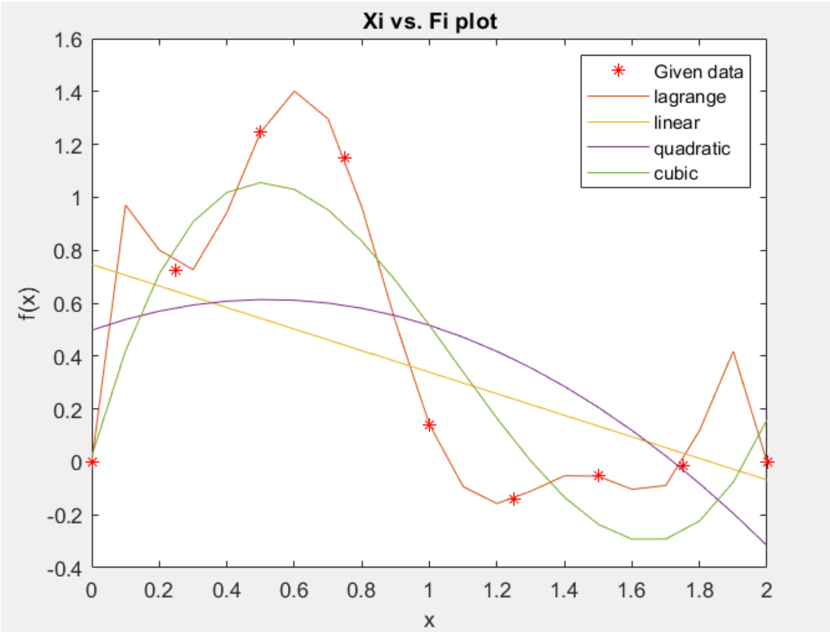
According to the given data, lagrange interpolation gives us the best. We can extract the graph passing through the closest points to the given data graph with lagrange.

MATLAB code as shown below

```
-------------------------------------------------------------
% deltax=0.25
xi=0:0.25:2 ;
% given data
fi=[0 0.72424 1.2456 1.1509 0.14112 -0.14201 -0.054153 -0.012912
-0.0013662];
% deltax=0.1
newxi=0:0.1:2;
% a)lagrange
n=length(xi)-1;
syms xx;
S=0;
for i=1:n+1
   L=1;
   for j=1:n+1
       if j~=i
           L=L*(xx-xi(j))/(xi(i)-xi(j));
       end
   end
   S=S+fi(i)*L;
end
% polynomial
S=expand(S);
lgrn(xx)=S;
for i=1:length(newxi)
   y1(i)=double(lgrn(newxi(i)));
end
```

```
% b)linear
p1=polyfit(xi,fi,1);
y2=polyval(p1,newxi);
% c)quadratic
p2=polyfit(xi,fi,2);
y3=polyval(p2,newxi);
% d)cubic
p3=polyfit(xi,fi,3);
y4=polyval(p3,newxi);
fprintf('\tx, lagrange, linear, quadratic, cubic\n')
%printing
for i=1:length(newxi)
   fprintf('\t%2.2f, %2.4f, %2.4f, %2.4f, 
%2.4f\n',newxi(i),y1(i),y2(i),y3(i),y4(i))
end
%plotting
plot(xi,fi,'r*')
hold on
plot(newxi,y1)
plot(newxi,y2)
plot(newxi,y3)
plot(newxi,y4)
xlabel('x')
ylabel('f(x)')
title('Xi vs. Fi plot')
legend('Given data','lagrange','linear','quadratic','cubic')
----------------------------------------------------------------
```

Plotting



2- Discrete Fourier transform (DFT)

The DFT of any discrete time sequence with sample time

T = 0.5 s is given by :

$$X(k) = \sum_{n=0}^{N-1} x(n) * \exp(-j*2*pi*k*n/N), \text{where } N = 1 + 25\ /T = 51$$

$$= \sum_{n=0}^{N-1} x(n) * \exp(-j*2*pi*k*n/51)$$

$$= \sum_{n=0}^{N-1} x(n) * \cos(2*pi*k*n/51) - j * \sum_{n=0}^{N-1} x(n) * \sin(2*pi*k*n/51)$$

MATLAB code as shown below

```
-----------------------------------------------------------------
N = 51; T = 0.5;

x = [2.542093 0.468238 -1.77268 -5.39948 3.70136 -5.51955 -3.18012
-0.67726 -0.00354 4.438278 4.115519 4.917382 4.215726 0.839853
-0.37754 -3.67276 -4.79094 -4.18733 -5.38507 -1.77695 0.290933
2.40339 5.048626 3.665169 5.291073 2.397254 -0.32691 -0.73802
-5.01271 -4.71194 -4.45979 -4.45855 0.011117 0.308739 3.629702
```

```
4.519921 4.698322 4.513333 1.2386 0.475607 -3.04779 -4.693
-3.97781 -5.71671 -2.01817 -0.12502 1.405532 5.332182 3.726818
5.182152 3.006022];

for i=0 : N-1

   a = 0; b = 0;

   for n = 0 : N-1

      a = a + x(n+1) * cos(2*pi*i*n/51);

      b = b + x(n+1) * sin(2*pi*i*n/51);

   end

   X(i+1) = a - b;

   ampX(i+1) = (a^2 + b^2)^0.5;

   phaseX(i+1) = atand(-b/a);

end

disp ('dominant frequencies (in Hz) are :');

maxamp=max(ampX);

for i=0:N-1

   if ampX(i+1)/maxamp >= 0.5

       disp (i/T)

   end

end

i=0:1/T:N/T - 1;

plot (i,ampX)

xlabel ('frequency (w in Hz)')

ylabel ('Amplitude |X(w)|')

figure

plot (i,phaseX)

xlabel ('frequency (w in Hz)')

ylabel ('Phase (in degrees)')
```

----

dominant frequencies (in Hz) are :
8
94

Those 2 frequencies consist of more than %50 of the maximum amplitude. This can also be observed from the spectral plot that many low frequencies (around 8 Hz) and other very high frequencies (around 94 Hz) have high amplitudes.

3 - Calculate the arc length

A - Trapezoidal Rule - h = 0.2

Since I couldn't take the derivative of the exponential function in matlab, I took it with my own hand and produced f.

MAT202E HW3

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Q3- Given length $s=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$, $y=e^{x^2}$

$$\frac{dy}{dx}=\frac{d}{x}\left(e^{x^2}\right)=e^{x^2}\cdot 2x=\underline{2xe^{x^2}}$$

$$s=\int_a^b \sqrt{1+\left(2xe^{x^2}\right)^2}\,dx=\boxed{\int_a^b \sqrt{1+4x^2e^{2x^2}}\,dx}$$

this is my f

MATLAB code as shown below

h is asked to take 0.2 so n=5, a=1,b=2

```
a=1;
b=2;
n=5;
h=(b-a)/n;
x = linspace (a,b,n+1);
y = exp(x.^2);
dy = 2.*x.*y; % i couldn't take the derivative in matlab so i took
myself
f = sqrt(1+(dy.^2));
answer = h/2 * ((f(1)+f(end))+2*(f(2)+f(3)+f(4)+f(5)));
```

answer = 55.075404502398854.

B - Trapezoidal Rule- h = 0.1

MATLAB code as shown below

h is asked to take 0.1 so n=10, a=1,b=2

```
a=1;
b=2;
n=10;
h=(b-a)/n;
x = linspace (a,b,n+1);
y = exp(x.^2);
dy = 2.*x.*y; % i couldn't take the derivative in matlab so i took
myself
f = sqrt(1+(dy.^2));
answer = h/2 *
((f(1)+f(end))+2*(f(2)+f(3)+f(4)+f(5)+f(6)+f(7)+f(8)+f(9)+f(10)));
-----------------------------------------------------------------
```

answer = 52.708899599702050.

C - Gauss Quadrature Rule 5 point

Since I could not solve the function in matlab, I made it by hand and added it to the code, and I made the t and w points in the same way.

c) Gauss $[a,b] = [1,2]$

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$[a,b] \rightarrow [-1,1]$

$$x = \frac{b+a}{2} + \frac{b-a}{2}t = \frac{1+2}{2} + \frac{2-1}{2}t = \frac{3}{2} + \frac{1}{2}t$$

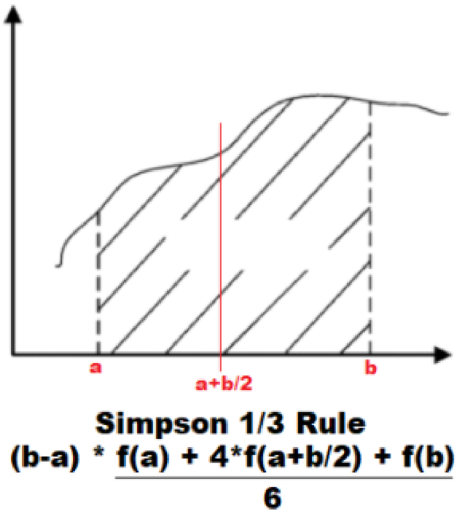
$$dx = \frac{1}{2}dt \qquad \boxed{f = \frac{1}{2}\sqrt{1+(3+t_n)^2 e^{\frac{(3+t_n)^2}{2}}}}$$

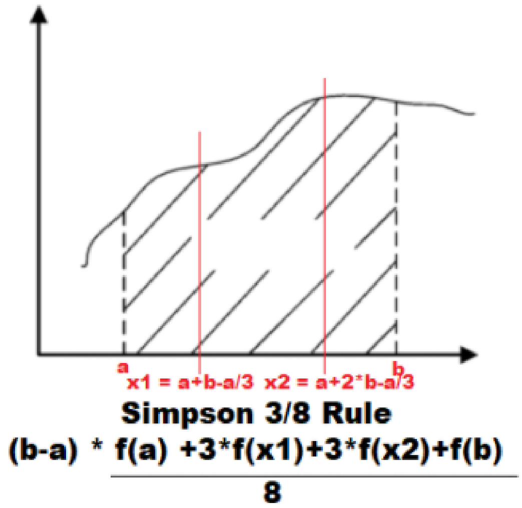
MATLAB code as shown below

```
---------------------------------------------------------
a = -1;
b = 1;
t = [-0.90618 -0.53847 0];    % 3 point
t2 = [0.53847 0.90618]; % 2 point for n = 5
w = [0.23692 0.47863 0.56888];    %
w2 = [0.47863 0.23692]; % weight for n=5
f = 1/2*sqrt(1+((3+t).^2).*exp(((3+t).^2)/2)); %function
f2 = sqrt(1+((3+t2).^2).*exp(((3+t2).^2)/2)); % function for t4
and t5
gaussleg = sum(w.*f);
gaussleg2 = sum(w2.*f2);
answer = 1/2*(gaussleg + gaussleg2);
-----------------------------------------------------------------
```

answer = 46.134565754979000.

4 - Simpson ⅓ and ⅜ Rule





As I showed above, I need 3 parts to apply the 3/8 rule to 2 parts to apply the simpson 1/3 rule. Since the data given to me is 11 intervals, I have to use both rules.

MATLAB code as shown below:

```
%---------------------------------------------------------------
a=0;
b=0.9; % from 0 to 9 for simpson 3/8
c=1.1; % from 9 to 11 for simpson 1/3
f0 = 2.595093;
f3 = 57.72181;
f6 = 95.16424;
f9 = 93.89335;
f10 = 81.54573;
f11 = 75.44009; %values from table
s3 = (b-a)*((f0+(3*f3)+(3*f6)+f9)/8);  % simpson 3/8 rule
s1 = (c-b)*((f9+(4*f10)+f11)/6); % simpson 1/3 rule
answer = s3 + s1;
fprintf('Work: %f', answer);
%---------------------------------------------------------------
```

Work: 78.971204