MATH 287 HOMEWORK 1

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Exercise 1. Proposition 1.11(vi). If m, n, p, and q are integers, then

$(m(n+p))q = (mn)q + m(pq)$.

Proof. Let $m, n, p, q \in \mathbf{Z}$. We will use axioms related to multiplication to show that $(m(n+p))q = (mn)q + m(pq)$:

$$
\begin{aligned}
(m(n+p))q &= (m(n+p))q \\
\text{(Axiom 1.1.3)} \qquad &= (mn+mp)q \\
\text{(Axiom 1.1.3)} \qquad &= mnq + mpq \\
\text{(Axiom 1.1.5)} \qquad &= (mn)q + m(pq).
\end{aligned}
$$

$\square$

Exercise 2. Proposition 1.22(i). For all $m \in \mathbf{Z}$, $-(-m) = m$.

Proof. Let $m \in Z$. We can start by re-expressing $-(-m)$ as $-1 \cdot (-1 \cdot m)$.

Rearranging the terms using our axioms, and simplifying, we get:

$$-(-m) = -1 \cdot (-1 \cdot m)$$

$$\text{(Axiom 1.1.4)} \qquad = (-1 \cdot -1) \cdot m$$

$$\text{(Corollary 1.21)} \qquad = 1 \cdot m$$

$$\text{(Axiom 1.3)} \qquad = m.$$

It must be for any integer m, $-(-m) = m$. $\square$

Exercise 3. Proposition 1.22(ii). $-0 = 0$.

Proof. We can rewrite the equation as $0 = -1 \cdot 0$. Also observe, as defined in Axiom 1.4, that $1 + (-1) = 0$. With this in hand, we can see that

$$
\begin{aligned}
0 &= -0 \\
&= -1 \cdot 0 \\
\text{(Replacement)} \qquad &= -1 \cdot (1 + (-1)) \\
\text{(Axiom 1.1.3)} \qquad &= -1 + (-1)(-1) \\
\text{(Simplifying, using Corollary 1.21)} \qquad &= -1 + 1 \\
&= 0.
\end{aligned}
$$

$\square$

Exercise 4. Proposition 1.20. For all $m, n \in \mathbf{Z}, (-m)(-n) = mn$.

Proof. Let $m, n \in \mathbf{Z}$. By Axiom 1.4, $m + (-m) = 0$ and $n + (-n) = 0$. Multiplying both sides of the first equation (on the right) by n and the second (on the left) by $-m$ gives, after applying Proposition 1.14 on the right-hand sides,

$(m + (-m))n = 0$ and $(-m)(n + (-n)) = 0$.

With Axiom 1.1(iii) and Proposition 1.6 we deduce

$mn + (-m)n = 0$ and $(-m)n + (-m)(-n) = 0$.

It remains to use Axiom 1.1(i) on the left and then Proposition 1.10 to conclude

$mn = (-m)(-n)$. $\square$

Exercise 5. Proposition 1.14. For all $m \in \mathbf{Z}, m \cdot 0 = 0 = 0 \cdot m$.

Hint: $0 + 0 = 0$.

Proof. Observe that, by Axiom 1.4: $1 + (-1) = 0$. Replacing this fact into the equation, and distributing we get

$$m \cdot 0 = m \cdot (1 + (-1))$$

(Axiom 1.1.3) $\qquad = m + m(-1)$

(Axiom 1.1.3) $\qquad = m + (-m)$

(Axiom 1.4) $\qquad = 0.$

It must be that for any integer $m, m \cdot 0 = 0$. $\square$