MATH 287 HOMEWORK 3

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Exercise 1. Project 3.1. Express each of the following statements using quantifiers.

(iii) Every integer is the product of two integers.

(iv) The equation $x^2 - 2y^2 = 3$ has an integer solution.

Answer.

(iii) -enter your answer here-

(iv) -enter your answer here-

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Exercise 2. Project 3.2. In each of the following cases explain what is meant by the statement and decide whether it is true or false.

(iii) For each $x \in \mathbf{Z}$ there exists $y \in \mathbf{Z}$ such that $xy = x$.

(iv) There exists $y \in \mathbf{Z}$ such that for each $x \in \mathbf{Z}$, $xy = x$.

Answer.

(iii) -enter your answer here-

(iv) -enter your answer here-

$\diamond$

Exercise 3. Project 3.7. Negate the following statements.

(iv) The newspaper article was neither accurate nor entertaining.

(v) If $\gcd(m, n)$ is odd, then m or n is odd.

(vi) (you can enter this one)

(vii) For each $\varepsilon > 0$ there exists $N \in \mathbf{N}$ such that for all $n \geq N$, $|a_n - L| < \varepsilon$.

Answer.

(iv) -enter your answer here-

(v) -enter your answer here-

(vi)

(vii)

$\diamond$

Exercise 4. For all $k \in \mathbf{N}$, $5^k + 3$ is divisible by 4.

(1) Write what this statement says for $k = 1$. Is it true or false? Explain.

(2) Write what this statement says for $k = 2$. Is it true or false? Explain.

(3) Write what this statement says for $k = 3$. Is it true or false? Explain.

Now, prove the statement for all $k \in \mathbf{N}$, using induction.

Proof. (1) For $k = 1$, the statement says that $5^1 + 3$ is divisible by 4. It's true because $5 + 3 = 8$, and $8 = 4 \cdot 2$.

(2)

-enter your proof here- □