MATH 287 HOMEWORK 8

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Exercise 1. We need to find the determinants of these matrices:

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} 3 & 5 \\ 5 & 8 \end{bmatrix}, \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$$

Answer. The determinants are

$$\det \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = 1,$$

$$\det \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = -1,$$

$$\det \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} = 1,$$

$$\det \begin{bmatrix} 3 & 5 \\ 5 & 8 \end{bmatrix} = -1,$$

$$\det \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} = 1.$$

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Exercise 2. Proposition 9.7(ii): If $f : A \to B$ is surjective and $g : B \to C$ is surjective, then $g \circ f : A \to C$ is surjective.

Proof. Assume that $f : A \to B$ and $g : B \to C$ are surjective functions. We intend to show that $g \circ f$ is also surjective. That is

$$\exists c \in C, \forall a \in A : (g \circ f)(a) = c.$$

$g \circ f$ is a composition of f and g, defined as

$$g(f(a)) \text{ for all } a \in A.$$

By hypothesis, we know that f maps the entirety of B's elements (f is surjective). This means

$$\forall b \in B, \exists a \in A \text{ such that } f(a) = b.$$

We also know that g maps the entirety of C's elements (g is surjective). That is

$$\forall c \in C, \exists b \in B \text{ such that } g(b) = c.$$

Because $f \circ g$ is a composition, inputs (a's) are evaluated first by f, and the subsequent outputs (b's) are then fed as new inputs for g. Phrased differently, $g \circ f$ maps $A \to C$ by using the range of f as the domain of g. This means, under the composition, all inputs to g are outputs of f, and thus we can write

$$\begin{aligned} f(a) &= b \\ g(b) &= c \\ g(f(a)) &= c \\ (g \circ f)(a) &= c. \end{aligned} \tag{1}$$

Therefore, $g \circ f$ must be surjective. This concludes the proof. $\square$

Exercise 3. Claims: Prove the claims:

For any $n \geq 2$, if $f_1, f_2, \ldots, f_n$ are each injective, then $f_1 \circ f_2 \circ \cdots \circ f_n$ is injective.

For any $n \geq 2$, if $f_1, f_2, \ldots, f_n$ are each surjective, then $f_1 \circ f_2 \circ \cdots \circ f_n$ is surjective.

For any $n \geq 2$, if $f_1, f_2, \ldots, f_n$ are each bijective, then $f_1 \circ f_2 \circ \cdots \circ f_n$ is bijective.

(Hint: Use induction and Proposition 9.7.)

Claim 3.1. ...

Proof. □