MATH 287 HOMEWORK 9

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Exercise 1. Find or create a graphic to include, that you want to share. It can be a math image that you like such as a cool plot or interesting diagram, or a picture of an important mathematician. Or it can be a meme or joke. It doesn't have to be math-related. However all images must be completely appropriate for sharing.

Answer. ◇

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Exercise 2. Proposition 10.8. For all $x, y \in \mathbf{R}$:

(ii) $|xy| = |x| \cdot |y|$.

(iii) $-|x| \leq x \leq |x|$.

Claim 2.1 (10.8 (ii)). $\forall x, y \in \mathbf{R} : |xy| = |x| \cdot |y|$

Proof. Let $x, y \in \mathbf{R}$. We will prove the claim using the following cases:

1. both $x, y > 0$.
2. both $x, y < 0$.
3. one of x or y is < 0.
4. one or both of x and y are equal to 0.

Case 1. Take an element $z \in \mathbf{R}$, which is the product of x and y, i.e., $x \cdot y = z$. Because x and y are positive, we know that z is also positive. Therefore, we can write

$$\begin{aligned} |x| \cdot |y| &= x \cdot y \text{ (definition of absolute value)} \\ &= z \\ &= |z| \\ &= |x \cdot y|. \end{aligned} \tag{1}$$

Case 2. Assume z is the product of two negative real numbers denoted $-x$ and $-y$. We know from Prop. 8.18 that the product of two negative (real) numbers is a positive (real) number. Therefore, z is still positive. Thus, we can write

$$\begin{aligned} |-x| \cdot |-y| &= x \cdot y \text{ (definition of absolute value)} \\ &= z \\ &= |z| \\ &= |-x \cdot -y|. \end{aligned} \tag{2}$$

Case 3. Assume we have two real numbers, $-x$ (negative) and y (positive). From Prop. 8.22 (iii), we know that the product of a negative real number and a positive real number is a negative number. We will call this product $-z$. By the definition of absolute values, we know that $|-z| = z$. So, we can write

$$
\begin{aligned}
|-x|\cdot|y| &= x\cdot y \text{ (definition of absolute value)}\\
&= z\\
&= |-z|\\
&= |-x\cdot y|.
\end{aligned}
\tag{3}
$$

Case 4. In an instance where either or both of x and y are 0, we can apply Proposition 10.8 (i), which lets us conclude $0 = |0| = |0\cdot 0| = |x\cdot 0| = |0\cdot y|$.

Thus we have shown that $|x|\cdot|y| = |x\cdot y|$ for all $x, y \in \mathbf{R}$. This concludes the proof. □

Claim 2.2 (10.8 (iii)). $\forall x, y \in \mathbf{R} : -|x| \leq x \leq |x|$

Proof. □

Exercise 3. Claim: $-y < x < y$ if and only if $|x| < y$.

Proof. $\square$

Exercise 4. Proposition 10.10. Let $x, y, z \in \mathbf{R}$.

(i) $|x - y| = 0$ if and only if $x = y$.

(ii) $|x - y| = |y - x|$.

(iii) $|x - z| \leq |x - y| + |y - z|$.

Proof. $\square$