MATH 287 HOMEWORK 11

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Exercise 1. Suppose $f : A \to B$ is surjective. Then, for any set C and functions $g_1, g_2 : B \to C$, if $g_1 \circ f = g_2 \circ f$, $g_1 = g_2$.

Proof. Let $f : A \to B$ and $g_1, g_2 : B \to C$ be functions. We are examining compositions between f and g_1, g_2. From our hypothesis, we have $g_1 \circ f = g_2 \circ f$. From the definition of a composition, we know $g_1 \circ f : A \to C$ is defined by $(g_1 \circ f)(a) = c$ for all $a \in A$, and $g_2 \circ f : A \to C$ is defined by $(g_2 \circ f)(a) = c$ for all $a \in A$. Additionally, we know that f is surjective. This means that for

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every $b \in B$, there exists some $a \in A$ such that $f(a) = b$. Thus, we can write

$$
\begin{aligned}
&\text{(starting assumption)} && g_1 \circ f = g_2 \circ f \\
& && (g_1 \circ f)(a) = (g_2 \circ f)(a) \\
&\text{(definition of a composition)} && g_1(f(a)) = g_2(f(a)) \\
& && g_1(b) = g_2(b) \\
& && g_1 = g_2.
\end{aligned}
$$

Because for all $b \in B, \exists a \in A$ such that $f(a) = b$ (surjectivity of f), and $g_1 \circ f = g_2 \circ f$, we know that the a being fed to f is the same element of A. This means the result of $f(a)$, is the same b on both sides of the equality. Thus we have shown $g_1 = g_2$. This concludes the proof. $\square$

Exercise 2. a. Find and prove a formula for $2 + 5 + 8 + 11 + \cdots + (3n - 1)$.

b. Prove: for all positive odd integers $n, 5^n - n^2$ is divisible by 4.

Exercise 3. Proposition 11.25.

Exercise 4. Re-do a problem.