MATH 287 HOMEWORK 1

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Exercise 1. Proposition 1.11(vi). If m, n, p, and q are integers, then

$(m(n+p))q = (mn)q + m(pq)$.

Proof. Let $m, n, p, q \in Z$. We will use axioms related to multiplication to show that $(m(n+p))q = (mn)q + m(pq)$:

$$
\begin{aligned}
(m(n+p))q &= (m(n+p))q \\
\text{(Axiom 1.1.3)} \qquad &= (mn+mp)q \\
\text{(Axiom 1.1.3)} \qquad &= mnq + mpq \\
\text{(Axiom 1.1.5)} \qquad &= (mn)q + m(pq).
\end{aligned}
$$

□

Exercise 2. Proposition 1.22(i). For all $m \in \mathbf{Z}$, $-(-m) = m$.

Proof. Let $m \in Z$.

$$-(-m) = -(-m)$$

$$= -1 \cdot (-1 \cdot m)$$

(Axiom 1.1.4) $$= (-1 \cdot -1) \cdot m$$

(Corollary 1.21) $$= 1 \cdot m$$

$$= m$$

$\square$

Exercise 3. Proposition 1.22(ii). $-0 = 0$.

Proof. We can rewrite the equation as $-1 \cdot 0 = 0$. This form now resembles what was/will be proven in Proposition 1.14:

$$-0 = 0$$

$$-1 \cdot 0 = 0$$

(Replacement) $$-1 \cdot (1 + (-1)) = 0$$

(Axiom 1.1.3) $$-1 + (-1)(-1) = 0$$

$$-1 + 1 = 0$$

$$0 = 0$$

$\square$

Exercise 4. Proposition 1.20. In this problem, the textbook gives a proof.

Your homework is to explain the proof in more detail.

Proof. -enter your proof here- $\square$

Exercise 5. Proposition 1.14.

Hint: $0 + 0 = 0$.

Proof.

$$
\begin{aligned}
m \cdot 0 &= m \cdot (1 + (-1)) \\
&= m + m(-1) \\
&= m + (-m) \\
&= m - m \\
&= 0
\end{aligned}
$$

□