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MATH-471, Homework 4

The Poisson distribution is a potential candidate for the data that's been collected. We will maximize the likelihood function, which has one parameter of interest, λ.

$$L(x;\lambda) = f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, ... \text{ and } \lambda > 0.$$

Taking the natural log of L, we have

$$lnL(x;\lambda) = xln(\lambda) - \lambda - ln(x!).$$

Differentiating lnL results in

$$\frac{\partial}{\partial\lambda}\big[lnL(x;\lambda)\big] = \frac{\partial}{\partial\lambda}\big[xln(\lambda) - \lambda - ln(x!)\big] = \frac{x}{\lambda} - 1$$

Setting the equation to 0, we have

$$0 = \frac{x}{\lambda} - 1$$
$$1 = \frac{x}{\lambda}$$
$$\lambda = x$$

We can test whether this point is the global maximum by using the 2nd-derivative test. Taking the second derivative of lnL we have

$$\frac{\partial^2}{\partial^2\lambda}\big[ln(L(x;\lambda))\big] = \frac{\partial}{\partial\lambda}\big[\frac{x}{\lambda} - 1\big] = \frac{-x}{\lambda^2}$$

Substituting λ in for x, we receive

$$-\frac{\lambda}{\lambda^2} = -\frac{1}{\lambda}$$

This value is negative, and will be negative for all λ (because, by definition, $\lambda \geq 0$). This means that we have found the global maximum.