Andrew Moore, 10/09/2021

MATH-471, Homework 4

1a.

The Poisson distribution is a potential candidate for the data that's been collected.

1b.

For the Poisson distribution, both the mean and variance are controlled by the same parameter, λ. To estimate these quantities, we will maximize the likelihood function, defined as

$$L(x;\lambda) = f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, ... \text{ and } \lambda > 0.$$

Taking the natural log of L, we have

$$lnL(x;\lambda) = xln(\lambda) - \lambda - ln(x!).$$

Differentiating lnL results in

$$\frac{\partial}{\partial\lambda}\big[lnL(x;\lambda)\big] = \frac{\partial}{\partial\lambda}\big[xln(\lambda) - \lambda - ln(x!)\big] = \frac{x}{\lambda} - 1$$

Setting the equation to 0, we have

$$\begin{aligned} 0 &= \frac{x}{\lambda} - 1 \\ 1 &= \frac{x}{\lambda} \\ \lambda &= x \end{aligned}$$

We can test whether this point is the global maximum by using the 2nd-derivative test. Taking the second derivative of lnL we have

$$\frac{\partial^2}{\partial^2\lambda}\big[ln(L(x;\lambda))\big] = \frac{\partial}{\partial\lambda}\big[\frac{x}{\lambda} - 1\big] = \frac{-x}{\lambda^2}$$

Substituting λ in for x, we receive

$$-\frac{\lambda}{\lambda^2} = -\frac{1}{\lambda}$$

This value is negative, and will be negative for all λ (because, by definition, $\lambda > 0$). This means that we have found the global maximum.

5.1

5.5

5.6

5.8

5.10

5.14

5.15