MATH-472: Homework 2

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2/9/23

Question 1

The definition of the Gamma function where $n \in \mathbb{Z}^+$ is

$$\Gamma(n) = (n-1)!$$

This can be replicated in R using the `factorial()` function.

```r
posint_gamma <- function(n) {
  if (any(!is.integer(n) | n <= 0)) stop("n must be a positive integer.")

  factorial(n - 1)
}

all(posint_gamma(1:4) == gamma(1:4))
```

```
[1] TRUE
```

Question 2

Question 3

Let X be a discrete random variable with the following cdf:

X	10	30	50	70	90
$F(x) = P(X \leq x)$	0.27	0.41	0.64	0.92	1.00

Now let $x_1 = 10, x_2 = 30, x_3 = 50, x_4 = 70, x_5 = 90$.

```r
x <- c(10, 30, 50, 70, 90)

# the CDF of X
F_X <- function(x) {
  dplyr::case_when(
    x < 10            ~ 0.00,
    x >= 10 & x < 30 ~ 0.27,
    x >= 30 & x < 50 ~ 0.41,
    x >= 50 & x < 70 ~ 0.64,
    x >= 70 & x < 90 ~ 0.92,
    x > 90            ~ 1.00,
  )
}

f <- function(x) {
  u <- runif(n = length(x), 0, 1)

}
```

Question 4

(a) Calculate the inverse function of $u = F(x)$, i.e., $F_X^{-1}(u) = x$.

$$
\begin{aligned}
F(x) = u &\implies 1 - e^{-(0.25x^3)} = u \\
&\implies 1 - u = e^{-(0.25x^3)} \\
&\implies ln(1-u) = -\frac{1}{4}x^3 \\
&\implies -4ln(1-u) = x^3 \\
&\implies (-4ln(1-u))^{\frac{1}{3}} = x \\
&\implies F_X^{-1}(u) = x.
\end{aligned}
$$

(b) Calculate values of x for $u = 0.37, 0.45, 0.82$.

```r
f_inv <- function(u) (-4*log(1 - u))^(1/3)

f_inv(c(0.37, 0.45, 0.82)) |> round(digits = 5)
```

```
[1] 1.22719 1.33726 1.90002
```

Question 5

Question 6

$f(x) = \frac{1}{2}e^{-|x|}$.

Question 7