ECE 556— Microwave Engineering I
UNIVERSITY OF VIRGINIA

Lecture 1— Circuits and Electromagnetics

Low-frequency circuit theory is based on the concept of a "lumped element" — a circuit component in which the electromagnetic fields (and, consequently, the voltage and current) do not vary appreciably with position.

Advantages of Lumped-Element Circuit Theory

- Cause-effect relationships readily determined; Element functions and behavior easily understood.
- Fields associated with circuit elements are "quasi-static" $\longrightarrow$ although $\vec{E}$ and $\vec{H}$ vary in time, the fields have the spatial form of static (i.e., non time-varying fields). This permits the circuit to be described in terms of voltage and current (scalars) rather than vector fields.
- A collection of powerful and simple techniques exist for understanding and analyzing lumped-element circuits (Kirchhoff's Laws, Norton and Thévenin's Theorems).

Unfortunately, at sufficiently high frequencies, we cannot treat circuit elements (including the interconnects such as wires and cables) in this way and must allow for the possibility that the fields will vary over the spatial extent of the components. In this regime, lumped elements no longer exhibit ideal behavior and must be treated as having "parasitics" (extraneous circuit elements that account for resistive losses and the storage of electric and magnetic energy). In addition, you might recall that electrical energy travels at the speed of light. Consequently, when the time-period over which the electrical signals are changing is comparable to (or less than) the time it takes them to propagate through the circuit, we will notice a delayed response due to the finite propagation time. This is a manifestation of the electromagnetic waves that are always generated by time-varying fields.

Kirchhoff's Laws and Maxwell's Equations

All (classical) electromagnetic phenomena can be understood as a consequence of Maxwell's Equations – the laws that govern electromagnetic fields. This includes elementary circuit theory and Kirchhoff's Laws (which are just Maxwell's equations expressed in the simplified form appropriate to lumped-element circuits). You should have already seen Maxwell's equations in your previous courses and you might want to take a quick review if you are a bit "rusty" on this material.

To summarize, Maxwell's equations are a set of four vector equations that give the curl and divergence of the electric and magnetic fields. In integral form, they can be written as (in MKS units),

$$\oint_L \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{a} \qquad \text{(Faraday's Law)}$$

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0}\int_V \rho \, dv \qquad \text{(Gauss's Law)}$$

$$\oint_L \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{a} + \epsilon_0 \frac{d}{dt}\int_S \vec{E} \cdot d\vec{a} \qquad \text{(Ampere's Law)}$$

$$\oint_S \vec{B} \cdot d\vec{a} = 0 \qquad \text{(no name)}$$

Combining Ampere's Law (applied to a closed surface) and Gauss's Law leads to the "continuity equation,"

$$\oint_S \vec{J} \cdot d\vec{a} = -\frac{d}{dt}\int_V \rho \, dv$$

Using Maxwell's equations, we can see where Kirchhoff's Laws come from:

Kirchhoff's Voltage Law (KVL)

This states that the algebraic sum of the voltages around a closed loop is zero and is the basis of "mesh analysis" used to analyze circuits. Referring the figure 1, we can state KVL mathematically as,

$$\sum_k V_k = 0 \qquad \text{(KVL)}$$

KVL is a result of Faraday's Law. Recalling that the voltage drop (a.k.a. the "potential difference") between two points is given by,

$$V_a - V_b \equiv -\int_a^b \vec{E} \cdot d\vec{l},$$

we can re-write Faraday's Law for the closed path of the circuit in figure 1 as,

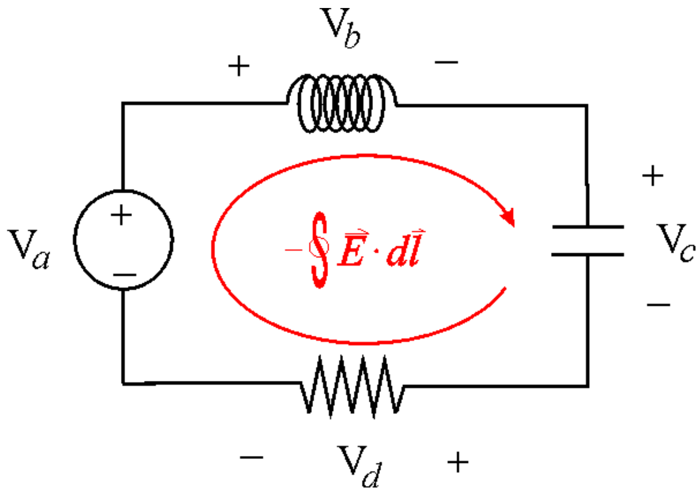

Fig. 1. Circuit illustrating "mesh analysis."

$$-\oint_L \vec{E} \cdot d\vec{l} \quad = \quad \underbrace{\sum_k V_k}_{\text{voltage drop}} \quad = \quad \frac{d}{dt} \int \vec{B} \cdot d\vec{a} \quad = \quad \underbrace{\frac{d\Phi_B}{dt}}_{\text{emf generated}} \tag{1}$$

where Φ_B is the magnetic flux through the circuit loop. For static (non time-varying) fields, equation (1) reduces to the usual form of KVL. Notice that, strictly speaking, KVL is not valid for time-varying electrical signals! However, if the time variations are *sufficiently slow* then KVL is a good approximation. We start to run into trouble when the electromotive force (or "emf") generated by the time-varying currents circulating around the circuit loop is comparable to the voltage drops encountered across the various circuit elements (V_a, V_b, etc...).

As a result, KVL is only valid when the right side of expression (1) is negligible. This is true if the spatial extent of the circuit loop is small in terms of a wavelength and the magnetic flux through the circuit changes slowly. In the microwave region of the spectrum, we begin to violate these assumptions and KVL (as it is normally applied) is no longer valid.

Kirchhoff's Current Law (KCL)

Kirchhoff's current law states that the sum of the currents flowing into a node is zero. Algebraically, we have (referring to fig. 2),

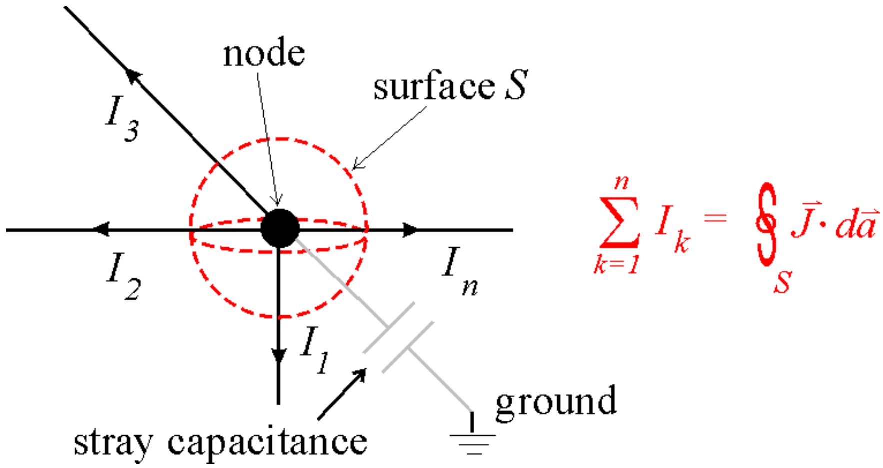

Fig. 2. Circuit illustrating "node analysis."

$$\sum_k I_k = 0 \tag{KCL}$$

KCL is the foundation of "node analysis" for elementary circuits and is based on the continuity equation. Applying the continuity equation to a closed surface S surrounding a circuit node, we have

$$\oint_S \vec{J} \cdot d\vec{a} \quad = \quad \underbrace{\sum_k I_k}_{\text{current into node}} \quad = \quad -\frac{d}{dt}\int_V \rho \, dv \quad = -\frac{d}{dt} \underbrace{Q}_{\text{charge inside S}} \tag{2}$$

where Q is the total charge inside surface S. Clearly, (2) reduces to KCL for static circuits. As before, we see that KCL is not strictly true for time-varying voltages and currents. However, it is approximately true provided the right side of equation (2) can be neglected. Again, is the case provided the time-varying quantities change sufficiently slow. The right side of equation (2) represents *displacement current* that is associated with the "stray" capacitance between the node and ground. Generally, this is not important for low-frequency circuits, but it can have a significant effect as the operating frequency approaches the microwave region.

Summary

- The fields that give rise to a circuit element's electrical behavior are distributed spatially.
- Electrical signals take a finite amount of time (based on the speed of light) to propagate through a circuit. For time-harmonic (AC) signals, this gives rise to a phase lag.
- When a circuit's spatial extent is comparable to a wavelength (or, equivalently, the propagation delay through the circuit is comparable to the time period over which the signal changes), Kirchhoff's laws must be modified or reformulated.
- Rather than treating microwave circuits as electromagnetic structures (described by Maxwell's equations), we can augment our circuit models and circuit theory to account for parasitic reactances and wave propagation. This is known as the "distributed circuit" or "transmission-line approach."