ДЗ №10. Петров Андрей, 132р 3 курс

3) $I(f) = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x \sin x \, dx \quad \varepsilon = 10^{-5} \quad h - ?$

$n = 3$

$$\int_a^b f(x)dx \approx \sum_{i=1}^{N} \frac{h}{2}\left(f\left(a + h\left(i - \frac{1}{2} - \frac{1}{2\sqrt{3}}\right)\right) + f\left(a + h\left(i - \frac{1}{2} + \frac{1}{2\sqrt{3}}\right)\right)\right)$$

$$R = \frac{h^4(b-a)}{4320}\|f^{(4)}(\xi)\| \qquad n = \frac{b-a}{h} \Leftrightarrow h = \frac{b-a}{n} = \frac{\frac{\pi}{2}}{6}$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x \sin x \, dx \approx \frac{\pi}{12}\left[f\left(-\frac{\pi}{4} + \frac{\pi}{6}\left(1 - \frac{1}{2} - \frac{1}{2\sqrt{3}}\right)\right) + f\left(-\frac{\pi}{4} + \frac{\pi}{6}\left(1 + \frac{1}{2\sqrt{3}} + \frac{1}{2}\right)\right)\right] +$$

$$+ \frac{\pi}{12}\left[f\left(-\frac{\pi}{4} + \frac{\pi}{6}\left(2 - \frac{1}{2} - \frac{1}{2\sqrt{3}}\right)\right) + f\left(-\frac{\pi}{4} + \frac{\pi}{6}\left(2 - \frac{1}{2} + \frac{1}{2\sqrt{3}}\right)\right)\right] +$$

$$+ \frac{\pi}{12}\left[f\left(-\frac{\pi}{4} + \frac{\pi}{6}\left(3 - \frac{1}{2} - \frac{1}{2\sqrt{3}}\right)\right) + f\left(-\frac{\pi}{4} + \frac{\pi}{6}\left(3 - \frac{1}{2} + \frac{1}{2\sqrt{3}}\right)\right)\right] \approx$$

$$\approx \frac{\pi}{12}\left(\left[f\left(-\frac{\pi}{6} - \frac{\pi}{12\sqrt{3}}\right) + f\left(-\frac{\pi}{6} + \frac{\pi}{12\sqrt{3}}\right)\right] + \left[f\left(-\frac{\pi}{12\sqrt{3}}\right) + f\left(\frac{\pi}{12\sqrt{3}}\right)\right] +\right.$$

$$\left. + \left[f\left(\frac{\pi}{6} - \frac{\pi}{12\sqrt{3}}\right) + f\left(\frac{\pi}{6} + \frac{\pi}{12\sqrt{3}}\right)\right]\right) = \frac{\pi}{12}\left(\left[0,42151628 + 0,13553318\right] +\right.$$

$$\left. + \left[0,02275941 + 0,02275941\right] + \left[0,13553318 + 0,42151628\right]\right) =$$

$$\approx 0,30358721$$

Точное значение Wolfram $\approx 0,30349$

$f^{(4)}(x) = x\sin x - 4\cos x$ ~~Максимум~~ достигается в т. $x = \frac{\pi}{4}$ и $x = -\frac{\pi}{4}$.

$|f^{(4)}(\frac{\pi}{4})| \approx 2.27306676$

$R \leq 10^{-5}$

$\frac{h^4 \cdot \frac{\pi}{2}}{4320} \cdot 2,27306676 \leq 10^{-5}$

$h \leq \sqrt[4]{4320 \cdot 10^{-5} \cdot \frac{1}{\frac{\pi}{2} \cdot 2,27306676}}$

$h \leq 0,33165605.$

При $n = 4$, $h \simeq \frac{\pi}{8} \approx 0.392699$