Analyzing Paxos with Fault-Tolerant Multiparty Session Types

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1 Introduction

In distributed computing, it is often necessary for coordinating processes to reach consensus, i.e., agree on the value of some data that are needed during computation. These processes agree on the same values to ensure correct computation, which necessitates a correct consensus algorithm. Thus, proving the correctness of consensus algorithms is important. To achieve consensus these algorithms must satisfy the following properties: termination, validity, and agreement [1].

Due to the presence of faulty processes consensus algorithms are designed to be fault-tolerant.

Proving these properties can be complicated. Model checking tools lead to big state-spaces so static analysis is preferable. Multiparty Session Types are particularly interesting since session typing can ensure the absence of communication errors and deadlocks, and protocol conformance [3]. However, to properly model unreliable communication between processes a fault-tolerant extension to Multiparty Session Types is necessary.

Peters, Nestmann, and Wagner developed such an extension.

2 Technical Preliminaries

First, we define the sorts, some additional notation, and use them to define the global type. Afterwards we define some sets and functions to create the processes.

3 Model and Analysis

First, we specify some sorts with which we can then define the global type. Afterwards, we define the processes for the proposer and the acceptor. Finally, we will study an example run of the model.

3.1 Sorts

We assume the following sorts.

```
Maybe a = \{\text{Just } a, \text{Nothing}\}\
```

Value = Set of values.

Promise $a = \{\text{Promise (Maybe (Proposal } a))}, \text{Nack } \mathbb{N}\}$

Proposal $a = \{ \text{Proposal } \mathbb{N} \ a \}$

3.2 Global Type

Since each proposer initiates its own session the global type can be defined for one proposer. A quorum of acceptors ${\cal A}_Q$ is assumed.

The last phase of Paxos contains no inter-process communication, so it is not modeled in the global type.

$$\begin{split} G_{p,A_Q} &= (\mu X) \bigodot_{a \in A_Q} \ p \to_u a : l1a \, \langle \mathbb{N} \rangle \,. \, \bigodot_{a \in A_Q} \ a \to_u p : l1b \, \langle \text{Promise Value} \rangle \,. \\ p \to_w A_Q : Accept. \, \Big(\bigodot_{a \in A_Q} \ p \to_u a : l2a \, \langle \text{Proposal Value} \rangle \Big) \,.end \end{split}$$

- \oplus Restart.X
- \oplus Abort.end

We can distinguish the individual phases of the Paxos algorithm by the labels l1a, l1b, and l2a.

In the first two steps, 1a and 1b, the proposer sends its proposal number to each acceptor in A_Q and listens for their responses. In step 2a the proposer decides whether to send an Accept or Restart message to restart the algorithm. This decision is broadcast to all acceptors in A_Q . Should the proposer crash the algorithm ends for this particular proposer and quorum of acceptors.

3.3 Functions and Sets

```
\label{eq:bool} \text{Bool} = \{\text{true}, \text{false}\} \text{proposalNumber}: \mathbb{N} \times \mathbb{N} \to \mathbb{N} \text{proposalNumber}(a,b) \text{ returns a proposal number when given two natural numbers.} \text{promiseValue}: [\text{Promise } a] \to a
```

promise Value (ps) returns a new value if none of the promises in ps contain a value. Otherwise, the best value is returned. Usually, that means the value with the highest associated proposal number. A promise contains a value v if it is of the form Promise (Just v).

```
\begin{array}{l} \operatorname{anyNack}: [\operatorname{Promise} \ a] \to \operatorname{Bool} \\ \operatorname{anyNack}\left([]\right) = false \\ \operatorname{anyNack}\left((\operatorname{Nack} \ \_: \ \_)\right) = true \\ \operatorname{anyNack}\left((\ \_: xs)\right) = \operatorname{anyNack}\left(xs\right) \end{array}
```

any Nack (ps) returns true if the list contains a promise of the form Nack n. Otherwise, it returns false.

```
\begin{array}{l} \operatorname{promiseCount}: [\operatorname{Promise} \ a] \to \mathbb{N} \\ \operatorname{promiseCount}\left([]\right) = 0 \\ \operatorname{promiseCount}\left((\operatorname{Promise} \ \_: xs)\right) = 1 + \operatorname{promiseCount}\left(xs\right) \\ \operatorname{promiseCount}\left((\_: xs)\right) = \operatorname{promiseCount}\left(xs\right) \end{array}
```

promiseCount (ps) calculates the number of promises in ps of the form Promise m.

```
\begin{array}{l} \operatorname{gt}: a \to \operatorname{Maybe}\ a \to \operatorname{Bool}\\ \operatorname{gt}(\_,\operatorname{Nothing}) = true\\ \operatorname{gt}(a,\operatorname{Just}\ b) = a > b\\ \operatorname{ge}: a \to \operatorname{Maybe}\ a \to \operatorname{Bool}\\ \operatorname{ge}(\_,\operatorname{Nothing}) = true\\ \operatorname{ge}(a,\operatorname{Just}\ b) = a \ge b\\ \operatorname{nFromProposal}:\operatorname{Proposal}\ a \to \mathbb{N}\\ \operatorname{nFromProposal}\left(\operatorname{Proposal}\ n\ \right) = n \end{array}
```

n
FromProposal (p) returns the proposal number n inside proposal p, which has the form
Proposal n pr.

```
genA_O: \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{N}
```

 $\operatorname{genA}_{Q}(i,ac,pc)$ returns a randomly selected set A_{Q} with $A_{Q}\subseteq A=\{1,\ldots,ac\}$ and $|A_{Q}|>\frac{|A|}{2}$.

update (n, m) mutates the value inside n to equal the value of m.

3.4 Processes

3.4.1 System Initialization

$$\begin{split} &\operatorname{Sys}\left(ac,pc\right) = \overline{a}\left[2\right](t) \cdot \operatorname{P}_{\operatorname{init}}^{\operatorname{p}}\left(ac+1,\operatorname{genA}_{\operatorname{Q}}\left(ac+pc,ac,pc\right),ac+pc,ac+pc,\left[\right]\right) \\ &\mid a\left[1\right](t) \cdot \Pi_{ac < i < ac+pc} \cdot \operatorname{P}_{\operatorname{init}}^{\operatorname{p}}\left(ac+1,\operatorname{genA}_{\operatorname{Q}}\left(i,ac,pc\right),i,i,\left[\right]\right) \\ &\mid \Pi_{1 \leq j \leq ac} \cdot \operatorname{P}_{\operatorname{init}}^{\operatorname{a}}\left(j,ac+1,ac,pc,n_{j},pr_{j}\right) \\ &\operatorname{P}_{\operatorname{init}}^{\operatorname{p}}\left(i,A_{Q},n,m,\overrightarrow{V}\right) = \overline{b_{n}}\left[i\right](s) \cdot \operatorname{P}^{\operatorname{p}} \\ &\operatorname{P}_{\operatorname{init}}^{\operatorname{a}}\left(j,i,ac,pc,n,pr\right) = \Pi_{ac < k \leq ac+pc} \cdot b_{k}\left[j\right](s) \cdot \operatorname{P}^{\operatorname{a}} \end{split}$$

Sys (ac, pc), $P_{\text{init}}^{\text{p}}\left(i, A_Q, n, m, \overrightarrow{V}\right)$, and $P_{\text{init}}^{\text{a}}\left(j, i, ac, pc, n, pr\right)$ describe the system initialization. ac and pc are the number of acceptors and proposers respectively.

An outer session is created through shared-point a. This outer session is not strictly necessary but was left in to allow for easier extension of the model. The acceptors are initialized using indices from 1 to ac and the proposers are initialized using indices from ac+1 to ac+pc.

 $\mathrm{P}_{\mathrm{init}}^{\mathrm{p}}\left(i,A_{Q},n,m,\overrightarrow{V}\right)$ is initialized with the proposer's role in its own session i, which is always ac+1, a quorum of acceptors A_{Q} , an index n, and a vector \overrightarrow{V} . Each proposer has the same role i=ac+1 but uses a different shared-point b_{n} according to its index n. \overrightarrow{V} is used in the proposer to collect and evaluate the responses from the acceptors. It is always initialized with an empty list []. Shared-point b_{n} is used to initiate a session. Afterwards, the process behaves like P^{p} .

 $P^{a}_{init}\left(j,i,ac,pc,n,pr\right)$ is initialized with the acceptor's index j, the proposer index i, which is always ac+1, ac, pc, initial knowledge for the highest promised proposal number n, if available, and initial knowledge for the most recently accepted proposal pr, if available. n is of type Maybe $\mathbb N$ and pr is of type Maybe (Proposal Value) thus both can be Nothing. Each of the proposers' session requests are accepted in a separate subprocess. These subprocesses run parallel to each other but still access the same values for n and pr. We observe that each subprocess in an acceptor accesses a different channel s, since it is generated by the proposer and passed through when the proposers' session request is accepted. Afterwards, each subprocess behaves like $\mathbb P^a$.

3.4.2 Proposer

```
\begin{split} & \operatorname{P}^{\mathbf{p}} = (\mu X) \operatorname{update} \left(n, n+1\right). \\ & \left( \bigcirc_{j \in A_Q} \ s \left[i, j\right]!_u l 1a \left\langle \operatorname{proposalNumber} \left(n, m\right) \right\rangle \right). \\ & \left( \bigcirc_{j \in A_Q} \ s \left[j, i\right]?_u l 1b \left\langle \bot \right\rangle \left(v_j\right) \right). \\ & \text{if anyNack} \left( \overrightarrow{V} \right) \ \text{or promiseCount} \left( \overrightarrow{V} \right) < \left\lceil \frac{|A_Q|}{2} \right\rceil \\ & \text{then } s \left[i, A_Q\right]!_w Restart. X \\ & \text{else} \\ & s \left[i, A_Q\right]!_w Accept. \\ & \bigcirc_{j \in A_Q} \ s \left[i, j\right]!_u l 2a \left\langle \operatorname{Proposal proposalNumber} \left(n, m\right) \ \operatorname{promiseValue} \left( \overrightarrow{V} \right) \right\rangle. \\ & end \end{split}
```

At the start of the recursion n is incremented to make sure every run of the recursion uses a different n and thus a different proposal number. The proposal number is sent to every acceptor in A_Q and their replies are gathered in \overrightarrow{V} through v_j . If any of the acceptors responded with Nack x or less than half of the acceptors responded with Promise y the proposer restarts the algorithm. Otherwise, the proposer sends its proposal to the acceptors and terminates.

3.4.3 Acceptor

```
P^{a} = (\mu X) s [i, j]?_{u} l1a \langle \bot \rangle (n').
  if n' = \perp
    then P<sup>a</sup><sub>cont</sub>
    else
      if gt (n', n)
      then update (n, n') .s [j, i]!_u l1b \langle \text{Promise } pr \rangle . P_{\text{cont}}^{\text{a}}
      else s[j,i]!_u l1b \langle \text{Nack } n \rangle . P_{\text{cont}}^{\text{a}}
P_{\text{cont}}^{a} = s[i, j]?_{w}Accept.s[i, j]?_{u}l2a \langle \bot \rangle (pr').
    if pr' = \perp
      then end
      else
        if ge (nFromProposal (pr'), n)
          then update (pr, pr') . update (n, \text{Just nFromProposal}(pr')) . end
          else end
   \oplus Restart.X
   \oplus Abort.end
```

For each proposer an acceptor has a corresponding subprocess, which behaves like P^a . These subprocesses access the same values for n and pr. This means that updating these values with update (n, m) updates them for all subprocesses of an acceptor.

Each subprocess can communicate with one proposer. Thus, if that proposer does not or can not communicate with a particular subprocess of an acceptor then there is no need for that subprocess. It is possible that an acceptor participates in a proposers' session but is not contained in the proposers' quorum of acceptors A_Q , in which case the proposer does not communicate with that acceptor. It is also possible for a proposer to crash or otherwise terminate, in which case the proposer can not communicate with that acceptor.

Each subprocess starts out by potentially receiving a proposal number n' from the corresponding proposer. If the acceptor does receive a proposal number n' it responds with either Promise pr or Nack n, depending on the values of n' and n. If the acceptor does not receive a proposal number then it sends no response to the proposer. In either case the subprocess moves on to receive the proposers' decision in phase 2a.

Since the proposers' decision broadcast is weakly reliable, there are two cases in which the acceptor receives no decision. The proposer might have terminated or this particular acceptor is not in the proposers' quorum of acceptors A_Q . In either case this particular

subprocess of the acceptor is no longer needed, because each subprocess of the acceptor exclusively communicates with one proposer. Thus, the subprocess terminates in the default branch Abort.

In the *Restart* branch this particular subprocess of the acceptor restarts the algorithm to match the corresponding proposer.

In the Accept branch the acceptor potentially receives a proposal pr' from the corresponding proposer. The acceptor updates n and pr if the proposal number in pr' is greater or equal to n. Then the subprocess terminates. If the acceptor does not receive a proposal or the proposal number of pr' is less than n the subprocess terminates without updating n or pr.

3.5 Example

```
ac = 3 pc = 2 V = \{abc\}
  \mathrm{Sys}\left(3,2\right) = a\left[1\right]\left(t\right).\Pi_{3 < i < 5}\ \mathrm{P}^{\mathrm{p}}_{\mathrm{init}}\left(4,\mathrm{genA}_{\mathrm{Q}}\left(i,3,2\right),i,i,\left[\right]\right)
   |\bar{a}[2](t).P_{\text{init}}^{\text{p}}(4,\text{genA}_{\text{Q}}(5,3,2),5,5,[])
   | \Pi_{1 \leq j \leq 3} P_{\text{init}}^{\text{a}}(j, 4, 3, 2, n_j, pr_j) |
  \longmapsto (Init)
 \begin{array}{l} (\nu t) \left( {{{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {4,{A_{Q,1}},4,4,[]} \right)} \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{p}}}\left( {4,{A_{Q,2}},5,5,[]} \right) \\ | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_1}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {2,4,3,2,{n_2},p{r_2}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {3,4,3,2,{n_3},p{r_3}} \right) \\ | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_1}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {2,4,3,2,{n_2},p{r_2}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {3,4,3,2,{n_3},p{r_3}} \right) \\ | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_1}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {2,4,3,2,{n_2},p{r_2}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {3,4,3,2,{n_3},p{r_3}} \right) \\ | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_1}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {2,4,3,2,{n_2},p{r_2}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {3,4,3,2,{n_3},p{r_3}} \right) \\ | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_1}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {2,4,3,2,{n_2},p{r_2}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {3,4,3,2,{n_3},p{r_3}} \right) \\ | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_1}} \right) \; | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_2}} \right) | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_2}} \right) \\ | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_2}} \right) | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_2}} \right) | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_2}} \right) \\ | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,{n_1},p{r_2}} \right) | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,2,{n_1},p{r_2}} \right) | \; {{\rm{P}}_{{\rm{init}}}^{\rm{a}}}\left( {1,4,3,2,2,{n_1},p{r_2}} \right) | \; {{\rm{P}}
  | \Pi_{1 \le k, l \le 2, k \ne l} t_{k \to l} : [] 
  (\nu t) \left(\overline{b_4} \left[4\right] \left(s\right) . P^p \mid \overline{b_5} \left[4\right] \left(r\right) . P^p
   | \Pi_{3 < n \le 5} b_n [1] (s) . P^a
        \Pi_{3 < n < 5} b_n [2] (s) . P^a
       \Pi_{3 < n < 5} b_n [3] (s) . P^a
   | \Pi_{1 < k, l < 2, k \neq l} t_{k \to l} : [])
 (\nu t) (\overline{b_4} [4] (s) . P^p | \overline{b_5} [4] (r) . P^p
   | (b_4 [1] (s) . P^a | b_5 [1] (r) . P^a |
   | (b_4 [2] (s) . P^a | b_5 [2] (r) . P^a)
          (b_4 [3] (s) . P^a | b_5 [3] (r) . P^a)
   | \Pi_{1 \le k, l \le 2, k \ne l} t_{k \to l} : []
```

```
\longmapsto (Init)
\left(\nu t\right)\left(\nu s\right)\left(\nu r\right)\left(\left(\mu X\right)\text{update}\left(n,5\right).\left(\bigcirc_{j\in\left\{ 1,2\right\} }\ s\left[4,j\right]!_{u}l1a\left\langle \text{proposalNumber}\left(n,4\right)\right\rangle \right)...
\mid (\mu X) \text{ update } (n,6) \cdot \left( \bigcirc_{j \in \{2,3\}} r[4,j]!_u l1a \langle \text{proposalNumber } (n,5) \rangle \right) \dots
 |((\mu X) s [4,1]?_u l 1a \langle \perp \rangle (n') . \text{if } \ldots | (\mu X) r [4,1]?_u l 1a \langle \perp \rangle (n') . \text{if } \ldots)
   ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,2]?_u l1a \langle \perp \rangle (n') . if ...)
   ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,3]?_u l1a \langle \perp \rangle (n') . if ...)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
 (\mu X) r [4,2]!_u l 1a \langle \text{proposalNumber } (6,5) \rangle . r [4,3]!_u l 1a \langle \text{proposalNumber } (6,5) \rangle ...
|((\mu X) s [4,1]?_u l 1a \langle \bot \rangle (n') . \text{if } ... | (\mu X) r [4,1]?_u l 1a \langle \bot \rangle (n') . \text{if } ...)
   ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,2]?_u l1a \langle \perp \rangle (n') . if ...)
   ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,3]?_u l1a \langle \perp \rangle (n') . if ...)
 | \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ... [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
   (\mu X) r [4,3]!_u l1a \langle \text{proposalNumber } (6,5) \rangle . r [2,4]?_u l1b \langle \bot \rangle (v_2) \dots
   ((\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,1]?_u l1a \langle \perp \rangle (n') . if ...)
   ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) if 10 = \perp then P_{cont}^a else ...)
   ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,3]?_u l1a \langle \perp \rangle (n') . if ...)
 | \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 1a \langle \text{proposalNumber} (5, 4) \rangle .s [4, 2]!_u l 1a \langle \text{proposalNumber} (5, 4) \rangle ...
   (\mu X) r [4,3]!_u l1a \langle \text{proposalNumber} (6,5) \rangle . r [2,4]?_u l1b \langle \bot \rangle (v_2) \dots
   ((\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,1]?_u l1a \langle \perp \rangle (n') . if ...)
   ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) if gt (10, Nothing) then update <math>(n, n') ... else ...)
   ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,3]?_u l1a \langle \perp \rangle (n') . if ...)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
   (\mu X) r [4,3]!_u l1a \langle \text{proposalNumber} (6,5) \rangle . r [2,4]?_u l1b \langle \bot \rangle (v_2) \dots
   ((\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,1]?_u l1a \langle \perp \rangle (n') . if ...)
   ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [2,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
   ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,3]?_u l1a \langle \perp \rangle (n') . if ...)
 | \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
```

```
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ... [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
 |(\mu X) r[2,4]?_u l1b \langle \bot \rangle (v_2) .r[3,4]?_u l1b \langle \bot \rangle (v_3) ...
 |((\mu X) s [4,1]?_u l 1a \langle \perp \rangle (n') . \text{if } \ldots | (\mu X) r [4,1]?_u l 1a \langle \perp \rangle (n') . \text{if } \ldots)
 |(\mu X) s [4,2]?_u l 1a \langle \perp \rangle (n'). if ... |(\mu X) r [2,4]!_u l 1b \langle \text{Promise Nothing} \rangle \cdot P_{\text{cont}}^a
   ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [3,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USkip)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
 (\mu X) r [2,4]?_u l1b \langle \perp \rangle (v_2) .r [3,4]?_u l1b \langle \perp \rangle (v_3) ...
\mid ((\mu X) s [4,1]?_u l 1a \langle \perp \rangle (n') if ... \mid (\mu X) if \perp = \perp then P_{\text{cont}}^a else ...)
  ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [2,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
  ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [3,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l1a \langle \text{proposalNumber} (5,4) \rangle .s [4,2]!_u l1a \langle \text{proposalNumber} (5,4) \rangle ...
  (\mu X) r [2,4]?_u l1b \langle \perp \rangle (v_2) .r [3,4]?_u l1b \langle \perp \rangle (v_3) ...
 |((\mu X) s [4,1]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,1]?_w Accept ... \oplus Restart.X \oplus Abort.end)|
   ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [2,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
  ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [3,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
|(\mu X) r[3,4]?_u l1b \langle \bot \rangle (v_3) if any Nack (\overrightarrow{V}) or promise Count (\overrightarrow{V}) < 1 then ...
((\mu X) s [4,1]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,1]?_w Accept ... \oplus Restart.X \oplus Abort.end)
|((\mu X) s [4,2]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,2]?_w Accept ... \oplus Restart.X \oplus Abort.end)|
|((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... | (\mu X) r [3,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ... [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
  (\mu X) if false or 2 < 1 then ...else r[4, \{2,3\}]!_w Accept...
   ((\mu X) s [4,1]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,1]?_w Accept ... \oplus Restart.X \oplus Abort.end)
  ((\mu X) s [4,2]?_u l1a \langle \bot \rangle (n').if \dots | (\mu X) r [4,2]?_w Accept \dots \oplus Restart.X \oplus Abort.end)
  ((\mu X) s [4,3]?_u l1a \langle \bot \rangle (n').if \dots | (\mu X) r [4,3]?_w Accept \dots \oplus Restart.X \oplus Abort.end)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
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(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 1a \langle \text{proposalNumber} (5, 4) \rangle .s [4, 2]!_u l 1a \langle \text{proposalNumber} (5, 4) \rangle ...
|(\mu X) r[4, \{2, 3\}]|_w Accept.r[4, 2]|_u l2a \langle Proposal Proposal Number(5, 6) promise Value(<math>\overrightarrow{V}) \rangle \dots
((\mu X) s [4,1]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,1]?_w Accept ... \oplus Restart. X \oplus Abort. end)
   ((\mu X) s [4,2]?_u l1a \langle \bot \rangle (n'). if ... | (\mu X) r [4,2]?_w Accept \cdots \oplus Restart. X \oplus Abort. end)
 ((\mu X) s [4,3]?_u l1a \langle \bot \rangle (n') if ... (\mu X) r [4,3]?_w Accept \cdots \oplus Restart. X \oplus Abort. end)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (WSel + WBran)
(\nu t) \ (\nu s) \ (\nu r) \ (\ (\mu X) \ s \ [4,1]!_u l 1a \ \langle \text{proposalNumber} \ (5,4) \rangle \ .s \ [4,2]!_u l 1a \ \langle \text{proposalNumber} \ (5,4) \rangle \ ...
|(\mu X) r[4,2]!_u l2a \langle \text{Proposal proposalNumber}(5,6) \text{ promiseValue}(\overrightarrow{V}) \rangle.
   r[4,3]!_u l2a \left\langle \text{Proposal proposalNumber}(5,6) \text{ promiseValue}\left(\overrightarrow{V}\right) \right\rangle \dots
((\mu X) s [4,1]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,1]?_w Accept ... \oplus Restart. X \oplus Abort. end)
  ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,2]?_u l2a \langle \perp \rangle (pr') . if ...)
  ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,3]?_u l2a \langle \perp \rangle (pr') . if ...)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
\longmapsto (WSkip)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
|(\mu X) r[4,2]!_u l2a \langle \text{Proposal proposalNumber}(5,6) \text{ promiseValue}(\overrightarrow{V}) \rangle.
    r[4,3]!_u l2a \left\langle \text{Proposal proposalNumber}(5,6) \text{ promiseValue}\left(\overrightarrow{V}\right) \right\rangle \dots
\mid ((\mu X) s [4,1]?_u l 1a \langle \bot \rangle (n') . \text{ if } \ldots \mid (\mu X) end)
  ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,2]?_u l2a \langle \perp \rangle (pr') . if ...)
 ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... | (\mu X) r [4,3]?_u l2a \langle \perp \rangle (pr') . if ...)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
|(\mu X) r[4,3]!_u l2a \langle \text{Proposal proposalNumber}(5,6) \text{ promiseValue}(\overrightarrow{V}) \rangle.end
|(\mu X) s [4,1]?_{n} l1a \langle \bot \rangle (n') . if ...
((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n'). if ... (\mu X) if Proposal 10 abc =\perp then X else ...)
  ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,3]?_u l2a \langle \perp \rangle (pr') . if ...)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l1a \langle \text{proposalNumber} (5,4) \rangle .s [4,2]!_u l1a \langle \text{proposalNumber} (5,4) \rangle ...
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\mid (\mu X) end
|(\mu X) s [4,1]?_u l 1a \langle \perp \rangle (n') if ...
  ((\mu X) s [4,2]?_{u}l1a \langle \bot \rangle (n'). if ... | (\mu X) if Proposal 10 abc = \bot then X else ...)
((\mu X) s [4,3]?_u l 1a \langle \bot \rangle (n') if ... (\mu X) if Proposal 10 abc =\bot then X else ...
\mid \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \mid \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \mid \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ... [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
|(\mu X) s [4,1]?_u l 1a \langle \bot \rangle (n') . if ...
|((\mu X) s [4,2]?_u l 1a \langle \bot \rangle (n') . if ...
     (\mu X) if ge (10, 10) then update (pr, pr') update (n, \text{Just } 10) . X else ...
|((\mu X) s [4,3]?_{\mu} l 1a \langle \perp \rangle (n') . \text{ if } \ldots
     (\mu X) if ge (10, 10) then update (pr, pr') update (n, \text{Just } 10) . X else ...)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ... [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
(\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') if ...
|((\mu X) s [4,2]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) X)|
|((\mu X) s [4,3]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) X)|
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{proposalNumber} (5,4) \rangle ...
(\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') . if ...
|((\mu X) s [4,2]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) X)
|((\mu X) s [4,3]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) X)|
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
\longmapsto (USend + UGet, USend + UGet, USkip)
(\nu t) (\nu s) (\nu r) ((\mu X) s [1,4]?_u l1b \langle \bot \rangle (v_1) .s [2,4]?_u l1b \langle \bot \rangle (v_2) ...
  (\mu X) if 5 = \perp then P_{\text{cont}}^{\text{a}} else ...
\mid ((\mu X) \text{ if } 5 = \perp \text{ then } P_{\text{cont}}^{\text{a}} \text{ else } \dots \mid (\mu X) X)
  ((\mu X)) if \perp = \perp then P_{\text{cont}}^{\text{a}} else ... | (\mu X) X \rangle
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [1, 4]?_u l1b \langle \bot \rangle (v_1) .s [2, 4]?_u l1b \langle \bot \rangle (v_2) ...
(\mu X) if gt (5, Nothing) then update (n, 5) ... else ...
((\mu X)) if gt (5, Just 10) then ... else s[2,4]!_u l1b \langle \text{Nack } 10 \rangle . P_{\text{cont}}^a | (\mu X) X
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| ((\mu X) r [4,3]?_w Accept \cdots \oplus Restart.X \oplus Abort.end | (\mu X) X)
| \Pi_{1 \leq k, l \leq 4, k \neq l} s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [1,4]?_u l 1b \langle \bot \rangle (v_1) . s [2,4]?_u l 1b \langle \bot \rangle (v_2) ...
| (\mu X) s [1, 4]!_u l 1b \langle \text{Promise Nothing} \rangle \cdot P_{\text{cont}}^{\text{a}}
\mid ((\mu X) s [2, 4]!_u l 1b \langle \text{Nack } 10 \rangle . P_{\text{cont}}^{\text{a}} \mid (\mu X) X)
| ((\mu X) r [4,3]?_w Accept \cdots \oplus Restart.X \oplus Abort.end | (\mu X) X)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet, USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X)) if true or 1 < 1 then s [4, \{1, 2\}]!_w Restart X else ...
(\mu X) s [4,1]?_w Accept \cdots \oplus Restart.X \oplus Abort.end
  ((\mu X) s [4,1]?_w Accept \cdots \oplus Restart.X \oplus Abort.end \mid (\mu X) X)
| ((\mu X) r [4,3]?_w Accept \cdots \oplus Restart. X \oplus Abort.end | (\mu X) X) |
| \Pi_{1 \le k, l \le 4, k \ne l} s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, \{1, 2\}]!_w Restart.X
| (\mu X) s [4,1]?_w Accept \cdots \oplus Restart. X \oplus Abort.end
\mid ((\mu X) s [4,1]?_w Accept \cdots \oplus Restart.X \oplus Abort.end \mid (\mu X) X)
  ((\mu X) r [4,3]?_w Accept \cdots \oplus Restart.X \oplus Abort.end \mid (\mu X) X)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (WSel + WBran, WSkip)
(\nu t) (\nu s) (\nu r) ((\mu X) X
\mid (\mu X) X
\mid ((\mu X) X \mid (\mu X) X)
  ((\mu X) end \mid (\mu X) X)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{proposalNumber} (6,4) \rangle .s [4,2]!_u l 1a \langle \text{proposalNumber} (6,4) \rangle ...
|(\mu X) s [4,1]?_u l 1a \langle \bot \rangle (n') if ...
|((\mu X) s [4,2]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) X)|
  (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet, USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [1,4]?_u l1b \langle \perp \rangle (v_1) .s [2,4]?_u l1b \langle \perp \rangle (v_2) .if \dots
(\mu X) if 15 = \perp then P_{\text{cont}}^{\text{a}} else ...
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\mid ((\mu X) \text{ if } 15 = \perp \text{ then } P_{\text{cont}}^{\text{a}} \text{ else } \dots \mid (\mu X) X)
\mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [1,4]?_u l1b \langle \perp \rangle (v_1) .s [2,4]?_u l1b \langle \perp \rangle (v_2) .if \dots
 (\mu X) if gt (15, Just 5) then update (n, 15)...else ...
((\mu X)) if gt (15, Just 10) then update (n, 15) ... else ... (\mu X) X
 \mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [1,4]?_u l1b \langle \perp \rangle (v_1) .s [2,4]?_u l1b \langle \perp \rangle (v_2) .if \dots
 |(\mu X) s [1, 4]!_u l 1b \langle \text{Promise Nothing} \rangle \cdot P_{\text{cont}}^{\text{a}}
|((\mu X) s [2,4]!_u l1b \langle \text{Promise Proposal } 10 \ abc \rangle \cdot P_{\text{cont.}}^a | (\mu X) X)|
\mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet, USend + UGet)
(\nu t)(\nu s)(\nu r)((\mu X)) if false or 2 < 1 then ... else s [4, \{1, 2\}]!_w Accept...
(\mu X) s [4,1]?_w Accept \cdots \oplus Restart.X \oplus Abort.end
 | ((\mu X) s [4,1]?_w Accept \cdots \oplus Restart.X \oplus Abort.end | (\mu X) X) |
 \mid (\mu X) X
| \Pi_{1 \leq k, l \leq 4, k \neq l} s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, \{1, 2\}]!_w Accept...
(\mu X) s [4,1]?_w Accept \cdots \oplus Restart. X \oplus Abort.end
((\mu X) s [4,1]?_w Accept \cdots \oplus Restart. X \oplus Abort.end | (\mu X) X)
\mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (WSel + WBran)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 2a \langle \text{Proposal proposalNumber } (6, 4) \text{ promiseValue } (\overrightarrow{V}) \rangle.
s[4,2]!_u l2a \langle \text{Proposal proposalNumber}(6,4) \text{ promiseValue}(\overrightarrow{V}) \rangle .end
|(\mu X) s [4,1]?_u l2a \langle \bot \rangle (pr'). if ...
|((\mu X) s [4,2]?_u l2a \langle \bot \rangle (pr') . if ... | (\mu X) X)
\mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
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\longmapsto (USend + UGet, USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) end
(\mu X) if Proposal 15 Proposal 15 abc = \perp then ... else ...
|((\mu X)) if Proposal 15 Proposal 15 abc = \bot then ... else ... |(\mu X)X|
\mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) (\mid (\mu X)) if ge (15, Just 15)
     then update (pr, Proposal 15 abc). update (n, 15) . X else . . .
|((\mu X))| if ge (15, Just 15)
     then update (pr, Proposal\ 15\ abc). update (n, 15).X else . . . | (\mu X)X)
\mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) (\mid (\mu X) X
\mid ((\mu X) X \mid (\mu X) X)
 \mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USkip + WSkip, USkip + WSkip, USkip + WSkip, USkip + WSkip)
(\nu t) (\nu s) (\nu r) (\mid (\mu X) end
\mid ((\mu X) end \mid (\mu X) end)
\mid (\mu X) end
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) (\mid \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [\mid \mid \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [\mid \mid \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [\mid \mid ])
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4 Evaluation

RESULTS

5 Discussion

DISCUSSION