Analyzing Paxos with Fault-Tolerant Multiparty Session Types

Bachelor thesis by Nicolas Daniel Torres
Date of submission: October 18, 2021

1. Review: Prof. Dr. Kirstin Peters
Darmstadt

TECHNISCHE UNIVERSITÄT DARMSTADT

Computer Science Department
<institute>
<working group>

Contents

1 Introduction

In distributed computing, it is often necessary for coordinating processes to reach consensus, i.e., agree on the value of some data that are needed during computation. These processes agree on the same values to ensure correct computation, which necessitates a correct consensus algorithm. Thus, proving the correctness of consensus algorithms is important. To achieve consensus these algorithms must satisfy the following properties: termination, validity, and agreement [1].

Due to the presence of faulty processes consensus algorithms are designed to be fault-tolerant.

Proving these properties can be complicated. Model checking tools lead to big state-spaces so static analysis is preferable. Multiparty Session Types are particularly interesting since session typing can ensure the absence of communication errors and deadlocks, and protocol conformance [3]. However, to properly model unreliable communication between processes a fault-tolerant extension to Multiparty Session Types is necessary.

Peters, Nestmann, and Wagner developed such an extension.

2 Technical Preliminaries

First, we define the sorts, some additional notation, and use them to define the global type. Afterwards we define some sets and functions to create the processes.

3 Model and Analysis

First, we specify some sorts with which we can then define the global type. Afterwards, we define the processes for the proposer and the acceptor. Finally, we will study an example run of the model.

3.1 Sorts

We assume the following sorts.

$\text{Maybe } a = \{\text{Just } a, \text{Nothing}\}$

$\text{Value} =$ Set of values.

$\text{Promise } a = \{\text{Promise } (\text{Maybe } (\text{Proposal } a)), \text{Nack } \mathbb{N}\}$

$\text{Proposal } a = \{\text{Proposal } \mathbb{N}\, a\}$

3.2 Global Type

Since each proposer initiates its own session the global type can be defined for one proposer. A quorum of acceptors A_Q is assumed.

The last phase of Paxos contains no inter-process communication, so it is not modeled in the global type.

$$G_{p,A_Q} = (\mu X) \bigodot_{a \in A_Q} p \to_u a : l1a \langle \mathbb{N} \rangle . \bigodot_{a \in A_Q} a \to_u p : l1b \langle \text{Promise Value} \rangle .$$
$$p \to_w A_Q : Accept. \left(\bigodot_{a \in A_Q} p \to_u a : l2a \langle \text{Proposal Value} \rangle \right) .end$$

$\oplus$ $Restart.X$
$\oplus$ $Abort.end$

We can distinguish the individual phases of the Paxos algorithm by the labels $l1a$, $l1b$, and $l2a$.

In the first two steps, $1a$ and $1b$, the proposer sends its proposal number to each acceptor in A_Q and listens for their responses. In step 2a the proposer decides whether to send an $Accept$ or $Restart$ message to restart the algorithm. This decision is broadcast to all acceptors in A_Q. Should the proposer crash the algorithm ends for this particular proposer and quorum of acceptors.

3.3 Functions and Sets

$\text{Bool} = \{\text{true}, \text{false}\}$

$\text{proposalNumber} : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$

$\text{proposalNumber}\,(a, b)$ returns a proposal number when given two natural numbers.

$\text{promiseValue} : [\text{Promise}\ a] \to a$

$\text{promiseValue}\,(ps)$ returns a new value if none of the promises in ps contain a value. Otherwise, the best value is returned. Usually, that means the value with the highest associated proposal number. A promise contains a value v if it is of the form $\text{Promise}\ (\text{Just}\ v)$.

$\text{anyNack} : [\text{Promise}\ a] \to \text{Bool}$
$\text{anyNack}\,([]) = false$
$\text{anyNack}\,((\text{Nack}\ \_ : \_)) = true$
$\text{anyNack}\,((\_ : xs)) = \text{anyNack}\,(xs)$

$\text{anyNack}\,(ps)$ returns true if the list contains a promise of the form $\text{Nack}\ n$. Otherwise, it returns false.

$\text{promiseCount} : [\text{Promise}\ a] \to \mathbb{N}$
$\text{promiseCount}\,([]) = 0$
$\text{promiseCount}\,((\text{Promise}\ \_ : xs)) = 1 + \text{promiseCount}\,(xs)$
$\text{promiseCount}\,((\_ : xs)) = \text{promiseCount}\,(xs)$

$\text{promiseCount}\,(ps)$ calculates the number of promises in ps of the form $\text{Promise}\ m$.

$\mathrm{gt} : a \to \mathrm{Maybe}\ a \to \mathrm{Bool}$
$\mathrm{gt}\,(\_, \mathrm{Nothing}) = true$
$\mathrm{gt}\,(a, \mathrm{Just}\ b) = a > b$

$\mathrm{ge} : a \to \mathrm{Maybe}\ a \to \mathrm{Bool}$
$\mathrm{ge}\,(\_, \mathrm{Nothing}) = true$
$\mathrm{ge}\,(a, \mathrm{Just}\ b) = a \geq b$

$\mathrm{nFromProposal} : \mathrm{Proposal}\ a \to \mathbb{N}$
$\mathrm{nFromProposal}\,(\mathrm{Proposal}\ n\ \_) = n$

$\mathrm{nFromProposal}\,(p)$ returns the proposal number n inside proposal p, which has the form $\mathrm{Proposal}\ n\ pr$.

$\mathrm{genA_Q} : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{N}$

$\mathrm{genA_Q}\,(i, ac, pc)$ returns a randomly selected set A_Q with $A_Q \subseteq A = \{1, \ldots, ac\}$ and $|A_Q| > \frac{|A|}{2}$.

$\mathrm{update}\,(n, m)$ mutates the value inside n to equal the value of m.

3.4 Processes

3.4.1 System Initialization

$\mathrm{Sys}\,(ac, pc) = \overline{a}\,[2]\,(t)\,.\,\mathrm{P^p_{init}}\,(ac+1, \mathrm{genA_Q}\,(ac+pc, ac, pc)\,, ac+pc, ac+pc, [])$
$\mid a\,[1]\,(t)\,.\Pi_{ac<i<ac+pc}\ \mathrm{P^p_{init}}\,(ac+1, \mathrm{genA_Q}\,(i, ac, pc)\,, i, i, [])$
$\mid \Pi_{1\leq j\leq ac}\ \mathrm{P^a_{init}}\,(j, ac+1, ac, pc, n_j, pr_j)$

$\mathrm{P^p_{init}}\left(i, A_Q, n, m, \overrightarrow{V}\right) = \overline{b_n}\,[i]\,(s)\,.\,\mathrm{P^p}$

$\mathrm{P^a_{init}}\,(j, i, ac, pc, n, pr) = \Pi_{ac<k\leq ac+pc}\ b_k\,[j]\,(s)\,.\,\mathrm{P^a}$

$\mathrm{Sys}\,(ac, pc)$, $\mathrm{P^p_{init}}\left(i, A_Q, n, m, \overrightarrow{V}\right)$, and $\mathrm{P^a_{init}}\,(j, i, ac, pc, n, pr)$ describe the system initialization. ac and pc are the number of acceptors and proposers respectively.

An outer session is created through shared-point a. This outer session is not strictly necessary but was left in to allow for easier extension of the model. The acceptors are initialized using indices from 1 to ac and the proposers are initialized using indices from $ac+1$ to $ac+pc$.

$\mathrm{P}^{\mathrm{p}}_{\mathrm{init}}\left(i, A_Q, n, m, \overrightarrow{V}\right)$ is initialized with the proposer's role in its own session i, which is always $ac+1$, a quorum of acceptors A_Q, an index n, and a vector $\overrightarrow{V}$. Each proposer has the same role $i = ac+1$ but uses a different shared-point b_n according to its index n. $\overrightarrow{V}$ is used in the proposer to collect and evaluate the responses from the acceptors. It is always initialized with an empty list $[]$. Shared-point b_n is used to initiate a session. Afterwards, the process behaves like P^{p}.

$\mathrm{P}^{\mathrm{a}}_{\mathrm{init}}\,(j, i, ac, pc, n, pr)$ is initialized with the acceptor's index j, the proposer index i, which is always $ac+1$, ac, pc, initial knowledge for the highest promised proposal number n, if available, and initial knowledge for the most recently accepted proposal pr, if available. n is of type $\mathrm{Maybe}\ \mathbb{N}$ and pr is of type $\mathrm{Maybe}\ (\mathrm{Proposal}\ \mathrm{Value})$ thus both can be $\mathrm{Nothing}$. Each of the proposers' session requests are accepted in a separate subprocess. These subprocesses run parallel to each other but still access the same values for n and pr. We observe that each subprocess in an acceptor accesses a different channel s, since it is generated by the proposer and passed through when the proposers' session request is accepted. Afterwards, each subprocess behaves like P^{a}.

3.4.2 Proposer

$$
\begin{aligned}
&\mathrm{P}^{\mathrm{p}} = (\mu X)\,\mathrm{update}\,(n, n+1)\,.\\
&\quad\left(\bigodot_{j\in A_Q}\ s\,[i,j]!_u l1a\,\langle \mathrm{proposalNumber}\,(n,m)\rangle\right).\\
&\quad\left(\bigodot_{j\in A_Q}\ s\,[j,i]?_u l1b\,\langle\bot\rangle\,(v_j)\right).\\
&\quad \mathrm{if}\ \ \mathrm{anyNack}\left(\overrightarrow{V}\right)\ \ \mathrm{or}\ \ \mathrm{promiseCount}\left(\overrightarrow{V}\right) < \left\lceil \tfrac{|A_Q|}{2}\right\rceil\\
&\quad\ \mathrm{then}\ \ s\,[i,A_Q]!_w Restart.X\\
&\quad\ \mathrm{else}\\
&\quad\ \ s\,[i,A_Q]!_w Accept.\\
&\quad\ \ \bigodot_{j\in A_Q}\ s\,[i,j]!_u l2a\left\langle \mathrm{Proposal}\ \ \mathrm{proposalNumber}\,(n,m)\ \ \mathrm{promiseValue}\left(\overrightarrow{V}\right)\right\rangle.\\
&\quad\ \ end
\end{aligned}
$$

At the start of the recursion n is incremented to make sure every run of the recursion uses a different n and thus a different proposal number. The proposal number is sent to every acceptor in A_Q and their replies are gathered in $\overrightarrow{V}$ through v_j. If any of the acceptors responded with $\mathrm{Nack}\ \ x$ or less than half of the acceptors responded with $\mathrm{Promise}\ \ y$ the proposer restarts the algorithm. Otherwise, the proposer sends its proposal to the acceptors and terminates.

3.4.3 Acceptor

$$
\begin{aligned}
&\mathrm{P}^{\mathrm{a}} = (\mu X)\, s\,[i,j]?_u l1a\,\langle \bot \rangle\,(n')\,. \\
&\quad \text{if } n' = \bot \\
&\quad\quad \text{then } \mathrm{P}^{\mathrm{a}}_{\mathrm{cont}} \\
&\quad\quad \text{else} \\
&\quad\quad\quad \text{if } \mathrm{gt}\,(n', n) \\
&\quad\quad\quad \text{then } \mathrm{update}\,(n, n')\,.s\,[j,i]!_u l1b\,\langle \text{Promise } pr \rangle\,.\,\mathrm{P}^{\mathrm{a}}_{\mathrm{cont}} \\
&\quad\quad\quad \text{else } s\,[j,i]!_u l1b\,\langle \text{Nack } n \rangle\,.\,\mathrm{P}^{\mathrm{a}}_{\mathrm{cont}}
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{P}^{\mathrm{a}}_{\mathrm{cont}} = s\,[i,j]?_w Accept.s\,[i,j]?_u l2a\,\langle \bot \rangle\,(pr')\,. \\
&\quad \text{if } pr' = \bot \\
&\quad\quad \text{then } end \\
&\quad\quad \text{else} \\
&\quad\quad\quad \text{if } \mathrm{ge}\,(\mathrm{nFromProposal}\,(pr'), n) \\
&\quad\quad\quad\quad \text{then } \mathrm{update}\,(pr, pr')\,.\,\mathrm{update}\,(n, \text{Just } \mathrm{nFromProposal}\,(pr'))\,.end \\
&\quad\quad\quad\quad \text{else } end \\
&\quad \oplus Restart.X \\
&\quad \oplus Abort.end
\end{aligned}
$$

For each proposer an acceptor has a corresponding subprocess, which behaves like P^{a}. These subprocesses access the same values for n and pr. This means that updating these values with $\mathrm{update}\,(n, m)$ updates them for all subprocesses of an acceptor.

Each subprocess can communicate with one proposer. Thus, if that proposer does not or can not communicate with a particular subprocess of an acceptor then there is no need for that subprocess. It is possible that an acceptor participates in a proposers' session but is not contained in the proposers' quorum of acceptors A_Q, in which case the proposer does not communicate with that acceptor. It is also possible for a proposer to crash or otherwise terminate, in which case the proposer can not communicate with that acceptor.

Each subprocess starts out by potentially receiving a proposal number n' from the corresponding proposer. If the acceptor does receive a proposal number n' it responds with either $\mathrm{Promise}\ pr$ or $\mathrm{Nack}\ n$, depending on the values of n' and n. If the acceptor does not receive a proposal number then it sends no response to the proposer. In either case the subprocess moves on to receive the proposers' decision in phase $2a$.

Since the proposers' decision broadcast is weakly reliable, there are two cases in which the acceptor receives no decision. The proposer might have terminated or this particular acceptor is not in the proposers' quorum of acceptors A_Q. In either case this particular

subprocess of the acceptor is no longer needed, because each subprocess of the acceptor exclusively communicates with one proposer. Thus, the subprocess terminates in the default branch $Abort$.

In the $Restart$ branch this particular subprocess of the acceptor restarts the algorithm to match the corresponding proposer.

In the $Accept$ branch the acceptor potentially receives a proposal pr' from the corresponding proposer. The acceptor updates n and pr if the proposal number in pr' is greater or equal to n. Then the subprocess terminates. If the acceptor does not receive a proposal or the proposal number of pr' is less than n the subprocess terminates without updating n or pr.

3.5 Failure Patterns

$\mathrm{FP}_{\mathrm{uget}}\left(s, r_1, r_2, l\right)$

$\mathrm{FP}_{\mathrm{uskip}}\left(s, r_1, r_2, l\right)$

$\mathrm{FP}_{\mathrm{ml}}\left(s, r_1, r_2, l\right)$

$\mathrm{FP}_{\mathrm{wskip}}\left(s, r_1, r_2\right)$

$\mathrm{FP}_{\mathrm{crash}}\left(P\right)$

3.6 Example

$ac = 3\ pc = 2\ V = \{\mathrm{abc}\}$

$$
\begin{array}{l}
\mathrm{Sys}\left(3,2\right) = a\left[1\right]\left(t\right).\Pi_{3<i<5}\ \mathrm{P}^{\mathrm{p}}_{\mathrm{init}}\left(4, \mathrm{genA}_{\mathrm{Q}}\left(i,3,2\right), i, i, []\right) \\
\mid \overline{a}\left[2\right]\left(t\right).\mathrm{P}^{\mathrm{p}}_{\mathrm{init}}\left(4, \mathrm{genA}_{\mathrm{Q}}\left(5,3,2\right), 5, 5, []\right) \\
\mid \Pi_{1\leq j\leq 3}\ \mathrm{P}^{\mathrm{a}}_{\mathrm{init}}\left(j, 4, 3, 2, n_j, pr_j\right) \\
\longmapsto (\mathrm{Init}) \\
(\nu t)\left(\mathrm{P}^{\mathrm{p}}_{\mathrm{init}}\left(4, A_{Q,1}, 4, 4, []\right) \mid \mathrm{P}^{\mathrm{p}}_{\mathrm{init}}\left(4, A_{Q,2}, 5, 5, []\right)\right. \\
\mid \mathrm{P}^{\mathrm{a}}_{\mathrm{init}}\left(1, 4, 3, 2, n_1, pr_1\right) \mid \mathrm{P}^{\mathrm{a}}_{\mathrm{init}}\left(2, 4, 3, 2, n_2, pr_2\right) \mid \mathrm{P}^{\mathrm{a}}_{\mathrm{init}}\left(3, 4, 3, 2, n_3, pr_3\right) \\
\left.\mid \Pi_{1\leq k,l\leq 2, k\neq l}\ t_{k\to l} : []\right)
\end{array}
$$

$=$
$(\nu t)\,(\overline{b_4}\,[4]\,(s)\,.\,\mathrm{P}^{\mathrm{p}}\ \mid \overline{b_5}\,[4]\,(r)\,.\,\mathrm{P}^{\mathrm{p}}$
$\mid \Pi_{3<n\leq 5}\ b_n\,[1]\,(s)\,.\,\mathrm{P}^{\mathrm{a}}$
$\mid \Pi_{3<n\leq 5}\ b_n\,[2]\,(s)\,.\,\mathrm{P}^{\mathrm{a}}$
$\mid \Pi_{3<n\leq 5}\ b_n\,[3]\,(s)\,.\,\mathrm{P}^{\mathrm{a}}$
$\mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$
$=$
$(\nu t)\,(\overline{b_4}\,[4]\,(s)\,.\,\mathrm{P}^{\mathrm{p}}\ \mid \overline{b_5}\,[4]\,(r)\,.\,\mathrm{P}^{\mathrm{p}}$
$\mid\ (b_4\,[1]\,(s)\,.\,\mathrm{P}^{\mathrm{a}}\ \mid b_5\,[1]\,(r)\,.\,\mathrm{P}^{\mathrm{a}})$
$\mid\ (b_4\,[2]\,(s)\,.\,\mathrm{P}^{\mathrm{a}}\ \mid b_5\,[2]\,(r)\,.\,\mathrm{P}^{\mathrm{a}})$
$\mid\ (b_4\,[3]\,(s)\,.\,\mathrm{P}^{\mathrm{a}}\ \mid b_5\,[3]\,(r)\,.\,\mathrm{P}^{\mathrm{a}})$
$\mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$
$\longmapsto (\mathit{Init})$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\,\mathrm{update}\,(n,5)\,.\,\Big(\bigodot_{j\in\{1,2\}}\ s\,[4,j]!_u l1a\,\langle \mathrm{proposalNumber}\,(n,4)\rangle\Big)\ldots$
$\mid\ (\mu X)\,\mathrm{update}\,(n,6)\,.\,\Big(\bigodot_{j\in\{2,3\}}\ r\,[4,j]!_u l1a\,\langle \mathrm{proposalNumber}\,(n,5)\rangle\Big)\ldots$
$\mid\ ((\mu X)\,s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots\ \mid\ (\mu X)\,r\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots)$
$\mid\ ((\mu X)\,s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots\ \mid\ (\mu X)\,r\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots)$
$\mid\ ((\mu X)\,s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots\ \mid\ (\mu X)\,r\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots)$
$\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$
$=$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\,s\,[4,1]!_u l1a\,\langle \mathrm{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle \mathrm{proposalNumber}\,(5,4)\rangle\ldots$
$\mid\ (\mu X)\,r\,[4,2]!_u l1a\,\langle \mathrm{proposalNumber}\,(6,5)\rangle\,.r\,[4,3]!_u l1a\,\langle \mathrm{proposalNumber}\,(6,5)\rangle\ldots$
$\mid\ ((\mu X)\,s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots\ \mid\ (\mu X)\,r\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots)$
$\mid\ ((\mu X)\,s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots\ \mid\ (\mu X)\,r\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots)$
$\mid\ ((\mu X)\,s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots\ \mid\ (\mu X)\,r\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots)$
$\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$
$\longmapsto (\mathit{USend} + \mathit{UGet})$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\,s\,[4,1]!_u l1a\,\langle \mathrm{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle \mathrm{proposalNumber}\,(5,4)\rangle\ldots$
$\mid\ (\mu X)\,r\,[4,3]!_u l1a\,\langle \mathrm{proposalNumber}\,(6,5)\rangle\,.r\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\ldots$
$\mid\ ((\mu X)\,s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots\ \mid\ (\mu X)\,r\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots)$
$\mid\ ((\mu X)\,s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots\ \mid\ (\mu X)\,\mathrm{if}\ \ 10 = \bot\ \mathrm{then}\ \ \mathrm{P}^{\mathrm{a}}_{\mathrm{cont}}\,\mathrm{else}\ \ldots)$
$\mid\ ((\mu X)\,s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots\ \mid\ (\mu X)\,r\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\mathrm{if}\ \ldots)$
$\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$
$=$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\,s\,[4,1]!_u l1a\,\langle \mathrm{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle \mathrm{proposalNumber}\,(5,4)\rangle\ldots$

$$
\begin{array}{l}
\mid\ (\mu X)\, r\,[4,3]!_u l1a\,\langle \text{proposalNumber}\,(6,5)\rangle\,.r\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\ldots \\
\mid\ ((\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots) \\
\mid\ ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\,\text{if}\ \text{gt}\,(10,\text{Nothing})\,\text{then}\ \text{update}\,(n,n')\ldots\text{else}\ \ldots) \\
\mid\ ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$=$

$$
\begin{array}{l}
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle \text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle \text{proposalNumber}\,(5,4)\rangle\ldots \\
\mid\ (\mu X)\, r\,[4,3]!_u l1a\,\langle \text{proposalNumber}\,(6,5)\rangle\,.r\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\ldots \\
\mid\ ((\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots) \\
\mid\ ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[2,4]!_u l1b\,\langle \text{Promise}\ \ \text{Nothing}\rangle\,.\,\text{P}^{\text{a}}_{\text{cont}}) \\
\mid\ ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$\longmapsto (USend + UGet)$

$$
\begin{array}{l}
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle \text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle \text{proposalNumber}\,(5,4)\rangle\ldots \\
\mid\ (\mu X)\, r\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\,.r\,[3,4]?_u l1b\,\langle\bot\rangle\,(v_3)\ldots \\
\mid\ ((\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots) \\
\mid\ ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[2,4]!_u l1b\,\langle \text{Promise}\ \ \text{Nothing}\rangle\,.\,\text{P}^{\text{a}}_{\text{cont}}) \\
\mid\ ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[3,4]!_u l1b\,\langle \text{Promise}\ \ \text{Nothing}\rangle\,.\,\text{P}^{\text{a}}_{\text{cont}}) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$\longmapsto (USkip)$

$$
\begin{array}{l}
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle \text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle \text{proposalNumber}\,(5,4)\rangle\ldots \\
\mid\ (\mu X)\, r\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\,.r\,[3,4]?_u l1b\,\langle\bot\rangle\,(v_3)\ldots \\
\mid\ ((\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\,\text{if}\ \ \bot{=}\bot\ \text{then}\ \ \text{P}^{\text{a}}_{\text{cont}}\,\text{else}\ \ldots) \\
\mid\ ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[2,4]!_u l1b\,\langle \text{Promise}\ \ \text{Nothing}\rangle\,.\,\text{P}^{\text{a}}_{\text{cont}}) \\
\mid\ ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[3,4]!_u l1b\,\langle \text{Promise}\ \ \text{Nothing}\rangle\,.\,\text{P}^{\text{a}}_{\text{cont}}) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$=$

$$
\begin{array}{l}
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle \text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle \text{proposalNumber}\,(5,4)\rangle\ldots \\
\mid\ (\mu X)\, r\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\,.r\,[3,4]?_u l1b\,\langle\bot\rangle\,(v_3)\ldots \\
\mid\ ((\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[4,1]?_w \mathit{Accept}\cdots\oplus \mathit{Restart}.X\oplus \mathit{Abort}.\mathit{end}) \\
\mid\ ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[2,4]!_u l1b\,\langle \text{Promise}\ \ \text{Nothing}\rangle\,.\,\text{P}^{\text{a}}_{\text{cont}}) \\
\mid\ ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\text{if}\ \ldots\ \mid\ (\mu X)\, r\,[3,4]!_u l1b\,\langle \text{Promise}\ \ \text{Nothing}\rangle\,.\,\text{P}^{\text{a}}_{\text{cont}}) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$\longmapsto (USend + UGet)$

$$
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle \text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle \text{proposalNumber}\,(5,4)\rangle\ldots
$$

$\mid (\mu X)\, r\,[3,4]?_u l1b\,\langle\bot\rangle\,(v_3)\,.\,\text{if anyNack}\left(\overrightarrow{V}\right)\text{ or promiseCount}\left(\overrightarrow{V}\right) < 1\,\text{then }\ldots$
$\mid ((\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,1]?_w \mathit{Accept}\cdots\oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end})$
$\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,2]?_w \mathit{Accept}\cdots\oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end})$
$\mid ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[3,4]!_u l1b\,\langle\text{Promise Nothing}\rangle\,.\,\mathrm{P}^{\mathrm{a}}_{\mathrm{cont}})$
$\mid \Pi_{1\le k,l\le 4,k\neq l}\, s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\, r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\, t_{k\to l} : [])$

$\longmapsto (\mathit{USend} + \mathit{UGet})$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\ldots$
$\mid (\mu X)\,\text{if false or } 2<1\,\text{then }\ldots\text{else } r\,[4,\{2,3\}]!_w \mathit{Accept}.\ldots$
$\mid ((\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,1]?_w \mathit{Accept}\cdots\oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end})$
$\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,2]?_w \mathit{Accept}\cdots\oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end})$
$\mid ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,3]?_w \mathit{Accept}\cdots\oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end})$
$\mid \Pi_{1\le k,l\le 4,k\neq l}\, s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\, r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\, t_{k\to l} : [])$

$=$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\ldots$
$\mid (\mu X)\, r\,[4,\{2,3\}]!_w \mathit{Accept}.r\,[4,2]!_u l2a\left\langle\text{Proposal proposalNumber}\,(5,6)\ \text{promiseValue}\left(\overrightarrow{V}\right)\right\rangle\ldots$
$\mid ((\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,1]?_w \mathit{Accept}\cdots\oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end})$
$\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,2]?_w \mathit{Accept}\cdots\oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end})$
$\mid ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,3]?_w \mathit{Accept}\cdots\oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end})$
$\mid \Pi_{1\le k,l\le 4,k\neq l}\, s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\, r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\, t_{k\to l} : [])$

$\longmapsto (\mathit{WSel} + \mathit{WBran})$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\ldots$
$\mid (\mu X)\, r\,[4,2]!_u l2a\left\langle\text{Proposal proposalNumber}\,(5,6)\ \text{promiseValue}\left(\overrightarrow{V}\right)\right\rangle.$
$\quad r\,[4,3]!_u l2a\left\langle\text{Proposal proposalNumber}\,(5,6)\ \text{promiseValue}\left(\overrightarrow{V}\right)\right\rangle\ldots$
$\mid ((\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,1]?_w \mathit{Accept}\cdots\oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end})$
$\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,2]?_u l2a\,\langle\bot\rangle\,(pr')\,.\,\text{if }\ldots)$
$\mid ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,3]?_u l2a\,\langle\bot\rangle\,(pr')\,.\,\text{if }\ldots)$
$\mid \Pi_{1\le k,l\le 4,k\neq l}\, s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\, r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\, t_{k\to l} : [])$

$\longmapsto (\mathit{WSkip})$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\ldots$
$\mid (\mu X)\, r\,[4,2]!_u l2a\left\langle\text{Proposal proposalNumber}\,(5,6)\ \text{promiseValue}\left(\overrightarrow{V}\right)\right\rangle.$
$\quad r\,[4,3]!_u l2a\left\langle\text{Proposal proposalNumber}\,(5,6)\ \text{promiseValue}\left(\overrightarrow{V}\right)\right\rangle\ldots$
$\mid ((\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, \mathit{end})$
$\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if }\ldots \mid (\mu X)\, r\,[4,2]?_u l2a\,\langle\bot\rangle\,(pr')\,.\,\text{if }\ldots)$

$\mid ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots \mid (\mu X)\, r\,[4,3]?_u l2a\,\langle\bot\rangle\,(pr')\,.\,\text{if}\ \ldots)$
$\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$

$\longmapsto (USend + UGet)$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\ldots$
$\mid (\mu X)\, r\,[4,3]!_u l2a\,\left\langle\text{Proposal}\ \ \text{proposalNumber}\,(5,6)\ \ \text{promiseValue}\left(\overrightarrow{V}\right)\right\rangle.end$
$\mid (\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots$
$\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots \mid (\mu X)\,\text{if}\ \ \text{Proposal}\ \ 10\ \ \text{abc} =\bot\ \text{then}\ \ X\,\text{else}\ \ldots)$
$\mid ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots \mid (\mu X)\, r\,[4,3]?_u l2a\,\langle\bot\rangle\,(pr')\,.\,\text{if}\ \ldots)$
$\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$

$\longmapsto (USend + UGet)$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\ldots$
$\mid (\mu X)\, end$
$\mid (\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots$
$\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots \mid (\mu X)\,\text{if}\ \ \text{Proposal}\ \ 10\ \ \text{abc} =\bot\ \text{then}\ \ X\,\text{else}\ \ldots)$
$\mid ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots \mid (\mu X)\,\text{if}\ \ \text{Proposal}\ \ 10\ \ \text{abc} =\bot\ \text{then}\ \ X\,\text{else}\ \ldots)$
$\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$

$=$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\ldots$
$\mid (\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots$
$\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots$
$\quad \mid (\mu X)\,\text{if}\ \ \text{ge}\,(10,10)\,\text{then}\ \ \text{update}\,(pr,pr')\,.\,\text{update}\,(n,\text{Just}\ \ 10)\,.X\,\text{else}\ \ldots)$
$\mid ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots$
$\quad \mid (\mu X)\,\text{if}\ \ \text{ge}\,(10,10)\,\text{then}\ \ \text{update}\,(pr,pr')\,.\,\text{update}\,(n,\text{Just}\ \ 10)\,.X\,\text{else}\ \ldots)$
$\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$

$=$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\ldots$
$\mid (\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots$
$\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots \mid (\mu X)\, X)$
$\mid ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots \mid (\mu X)\, X)$
$\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$

$=$
$(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\,.s\,[4,2]!_u l1a\,\langle\text{proposalNumber}\,(5,4)\rangle\ldots$
$\mid (\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots$
$\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots \mid (\mu X)\, X)$

$$
\begin{array}{l}
\mid\ ((\mu X)\, s\,[4,3]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots\ \mid\ (\mu X)\, X) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
\longmapsto (USend + UGet, USend + UGet, USkip) \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,((\mu X)\, s\,[1,4]?_u l1b\,\langle\bot\rangle\,(v_1)\,.s\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\ldots \\
\mid\ (\mu X)\,\text{if}\ \ 5 = \bot\ \text{then}\ \ \mathrm{P}^{\mathrm{a}}_{\mathrm{cont}}\,\text{else}\ \ldots \\
\mid\ ((\mu X)\,\text{if}\ \ 5 = \bot\ \text{then}\ \ \mathrm{P}^{\mathrm{a}}_{\mathrm{cont}}\,\text{else}\ \ldots\ \mid\ (\mu X)\, X) \\
\mid\ ((\mu X)\,\text{if}\ \ \bot = \bot\ \text{then}\ \ \mathrm{P}^{\mathrm{a}}_{\mathrm{cont}}\,\text{else}\ \ldots\ \mid\ (\mu X)\, X) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
= \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,((\mu X)\, s\,[1,4]?_u l1b\,\langle\bot\rangle\,(v_1)\,.s\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\ldots \\
\mid\ (\mu X)\,\text{if}\ \ \mathrm{gt}\,(5,\mathrm{Nothing})\,\text{then}\ \ \mathrm{update}\,(n,5)\ldots\text{else}\ \ldots \\
\mid\ ((\mu X)\,\text{if}\ \ \mathrm{gt}\,(5,\mathrm{Just}\ \ 10)\,\text{then}\ \ldots\text{else}\ \ s\,[2,4]!_u l1b\,\langle\mathrm{Nack}\ \ 10\rangle\,.\,\mathrm{P}^{\mathrm{a}}_{\mathrm{cont}}\ \mid\ (\mu X)\, X) \\
\mid\ ((\mu X)\, r\,[4,3]?_w Accept\cdots\oplus Restart.X \oplus Abort.end \mid\ (\mu X)\, X) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
= \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,((\mu X)\, s\,[1,4]?_u l1b\,\langle\bot\rangle\,(v_1)\,.s\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\ldots \\
\mid\ (\mu X)\, s\,[1,4]!_u l1b\,\langle\mathrm{Promise}\ \ \mathrm{Nothing}\rangle\,.\,\mathrm{P}^{\mathrm{a}}_{\mathrm{cont}} \\
\mid\ ((\mu X)\, s\,[2,4]!_u l1b\,\langle\mathrm{Nack}\ \ 10\rangle\,.\,\mathrm{P}^{\mathrm{a}}_{\mathrm{cont}}\ \mid\ (\mu X)\, X) \\
\mid\ ((\mu X)\, r\,[4,3]?_w Accept\cdots\oplus Restart.X \oplus Abort.end \mid\ (\mu X)\, X) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
\longmapsto (USend + UGet, USend + UGet) \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,((\mu X)\,\text{if}\ \ \mathrm{true}\ \ \mathsf{or}\ \ 1 < 1\,\text{then}\ \ s\,[4,\{1,2\}]!_w Restart.X\,\text{else}\ \ldots \\
\mid\ (\mu X)\, s\,[4,1]?_w Accept\cdots\oplus Restart.X \oplus Abort.end \\
\mid\ ((\mu X)\, s\,[4,1]?_w Accept\cdots\oplus Restart.X \oplus Abort.end \mid\ (\mu X)\, X) \\
\mid\ ((\mu X)\, r\,[4,3]?_w Accept\cdots\oplus Restart.X \oplus Abort.end \mid\ (\mu X)\, X) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
= \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,((\mu X)\, s\,[4,\{1,2\}]!_w Restart.X \\
\mid\ (\mu X)\, s\,[4,1]?_w Accept\cdots\oplus Restart.X \oplus Abort.end \\
\mid\ ((\mu X)\, s\,[4,1]?_w Accept\cdots\oplus Restart.X \oplus Abort.end \mid\ (\mu X)\, X) \\
\mid\ ((\mu X)\, r\,[4,3]?_w Accept\cdots\oplus Restart.X \oplus Abort.end \mid\ (\mu X)\, X) \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
\longmapsto (WSel + WBran, WSkip) \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,((\mu X)\, X \\
\mid\ (\mu X)\, X
\end{array}
$$

$$
\begin{array}{l}
\mid ((\mu X)\, X \mid (\mu X)\, X) \\
\mid ((\mu X)\, end \mid (\mu X)\, X) \\
\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$=$

$$
\begin{array}{l}
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l1a\,\langle \text{proposalNumber}\,(6,4)\rangle\,.s\,[4,2]!_u l1a\,\langle \text{proposalNumber}\,(6,4)\rangle\ldots \\
\mid (\mu X)\, s\,[4,1]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots \\
\mid ((\mu X)\, s\,[4,2]?_u l1a\,\langle\bot\rangle\,(n')\,.\,\text{if}\ \ldots \mid (\mu X)\, X) \\
\mid (\mu X)\, X \\
\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$\longmapsto (USend + UGet, USend + UGet)$

$$
\begin{array}{l}
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[1,4]?_u l1b\,\langle\bot\rangle\,(v_1)\,.s\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\,.\,\text{if}\ \ldots \\
\mid (\mu X)\,\text{if}\ \ 15 = \bot\ \text{then}\ \ \mathrm{P}^{\mathrm{a}}_{\mathrm{cont}}\,\text{else}\ \ldots \\
\mid ((\mu X)\,\text{if}\ \ 15 = \bot\ \text{then}\ \ \mathrm{P}^{\mathrm{a}}_{\mathrm{cont}}\,\text{else}\ \ldots \mid (\mu X)\, X) \\
\mid (\mu X)\, X \\
\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$=$

$$
\begin{array}{l}
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[1,4]?_u l1b\,\langle\bot\rangle\,(v_1)\,.s\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\,.\,\text{if}\ \ldots \\
\mid (\mu X)\,\text{if}\ \ \text{gt}\,(15, \text{Just}\ \ 5)\,\text{then}\ \ \text{update}\,(n, 15)\ldots\text{else}\ \ldots \\
\mid ((\mu X)\,\text{if}\ \ \text{gt}\,(15, \text{Just}\ \ 10)\,\text{then}\ \ \text{update}\,(n, 15)\ldots\text{else}\ \ldots \mid (\mu X)\, X) \\
\mid (\mu X)\, X \\
\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$=$

$$
\begin{array}{l}
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[1,4]?_u l1b\,\langle\bot\rangle\,(v_1)\,.s\,[2,4]?_u l1b\,\langle\bot\rangle\,(v_2)\,.\,\text{if}\ \ldots \\
\mid (\mu X)\, s\,[1,4]!_u l1b\,\langle \text{Promise}\ \ \text{Nothing}\rangle\,.\,\mathrm{P}^{\mathrm{a}}_{\mathrm{cont}} \\
\mid ((\mu X)\, s\,[2,4]!_u l1b\,\langle \text{Promise}\ \ \text{Proposal}\ \ 10\ \ abc\rangle\,.\,\mathrm{P}^{\mathrm{a}}_{\mathrm{cont}} \mid (\mu X)\, X) \\
\mid (\mu X)\, X \\
\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$\longmapsto (USend + UGet, USend + UGet)$

$$
\begin{array}{l}
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\,\text{if}\ \ \text{false}\ \ \text{or}\ \ 2 < 1\,\text{then}\ \ldots\text{else}\ \ s\,[4,\{1,2\}]!_w Accept.\ldots. \\
\mid (\mu X)\, s\,[4,1]?_w Accept\cdots\oplus Restart.X \oplus Abort.end \\
\mid ((\mu X)\, s\,[4,1]?_w Accept\cdots\oplus Restart.X \oplus Abort.end \mid (\mu X)\, X) \\
\mid (\mu X)\, X \\
\mid \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$=$

$$
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,\{1,2\}]!_w Accept.\ldots.
$$

$$
\begin{array}{l}
\mid\ (\mu X)\, s\,[4,1]?_w \mathit{Accept} \cdots \oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end} \\
\mid\ ((\mu X)\, s\,[4,1]?_w \mathit{Accept} \cdots \oplus \mathit{Restart}.X \oplus \mathit{Abort}.\mathit{end} \mid\ (\mu X)\, X) \\
\mid\ (\mu X)\, X \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
\longmapsto (WSel + WBran) \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\, s\,[4,1]!_u l2a\,\left\langle \text{Proposal proposalNumber}\,(6,4)\ \text{promiseValue}\left(\overrightarrow{V}\right)\right\rangle . \\
s\,[4,2]!_u l2a\,\left\langle \text{Proposal proposalNumber}\,(6,4)\ \text{promiseValue}\left(\overrightarrow{V}\right)\right\rangle .\mathit{end} \\
\mid\ (\mu X)\, s\,[4,1]?_u l2a\,\langle\bot\rangle\,(pr')\,.\,\text{if}\ \ldots \\
\mid\ ((\mu X)\, s\,[4,2]?_u l2a\,\langle\bot\rangle\,(pr')\,.\,\text{if}\ \ldots \mid\ (\mu X)\, X) \\
\mid\ (\mu X)\, X \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
\longmapsto (USend + UGet, USend + UGet) \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\,(\mu X)\,\mathit{end} \\
\mid\ (\mu X)\,\text{if Proposal 15 Proposal 15}\ \mathit{abc} = \bot\ \text{then}\ \ldots \text{else}\ \ldots \\
\mid\ ((\mu X)\,\text{if Proposal 15 Proposal 15}\ \mathit{abc} = \bot\ \text{then}\ \ldots \text{else}\ \ldots \mid\ (\mu X)\, X) \\
\mid\ (\mu X)\, X \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
= \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\mid\ (\mu X)\,\text{if ge}\,(15, \text{Just } 15) \\
\quad \text{then update}\,(pr, \text{Proposal } 15\ \mathit{abc})\,.\,\text{update}\,(n, 15)\,.X \text{ else } \ldots \\
\mid\ ((\mu X)\,\text{if ge}\,(15, \text{Just } 15) \\
\quad \text{then update}\,(pr, \text{Proposal } 15\ \mathit{abc})\,.\,\text{update}\,(n, 15)\,.X \text{ else } \ldots \mid\ (\mu X)\, X) \\
\mid\ (\mu X)\, X \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
= \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\mid\ (\mu X)\, X \\
\mid\ ((\mu X)\, X \mid\ (\mu X)\, X) \\
\mid\ (\mu X)\, X \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$
\begin{array}{l}
\longmapsto (USkip + WSkip, USkip + WSkip, USkip + WSkip, USkip + WSkip) \\
(\nu t)\,(\nu s)\,(\nu r)\,\big(\mid\ (\mu X)\,\mathit{end} \\
\mid\ ((\mu X)\,\mathit{end} \mid\ (\mu X)\,\mathit{end}) \\
\mid\ (\mu X)\,\mathit{end} \\
\mid \Pi_{1\le k,l\le 4,k\neq l}\ s_{k\to l} : [] \mid \Pi_{1\le k,l\le 4,k\neq l}\ r_{k\to l} : [] \mid \Pi_{1\le k,l\le 2,k\neq l}\ t_{k\to l} : [])
\end{array}
$$

$$=$$
$$(\nu t)\,(\nu s)\,(\nu r)\,(\ |\ \Pi_{1\leq k,l\leq 4,k\neq l}\ s_{k\to l} : [] \ |\ \Pi_{1\leq k,l\leq 4,k\neq l}\ r_{k\to l} : [] \ |\ \Pi_{1\leq k,l\leq 2,k\neq l}\ t_{k\to l} : [])$$

4 Evaluation

RESULTS

5 Discussion

DISCUSSION