Analyzing Paxos with Fault-Tolerant Multiparty Session Types

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1 Introduction

In distributed computing, it is often necessary for coordinating processes to reach consensus, i.e., agree on the value of some data that are needed during computation. These processes agree on the same values to ensure correct computation, which necessitates a correct consensus algorithm. Thus, proving the correctness of consensus algorithms is important. To achieve consensus these algorithms must satisfy the following properties: termination, validity, and agreement [1].

Due to the presence of faulty processes consensus algorithms are designed to be fault-tolerant.

Proving these properties can be complicated. Model checking tools lead to big state-spaces so static analysis is preferable. Multiparty Session Types are particularly interesting since session typing can ensure the absence of communication errors and deadlocks, and protocol conformance [3]. However, to properly model unreliable communication between processes a fault-tolerant extension to Multiparty Session Types is necessary.

Peters, Nestmann, and Wagner developed such an extension.

2 Technical Preliminaries

First, we define the sorts, some additional notation, and use them to define the global type. Afterwards we define some sets and functions to create the processes.

3 Model and Analysis

First, we specify some sorts with which we can then define the global type. Afterwards, we define the processes for the proposer and the acceptor. Finally, we will study an example run of the model.

3.0.1 Sorts

We assume the following sorts.

```
Maybe a = \{\text{Just } a, \text{Nothing}\}
Value = Set of values.
Promise a = \{\text{Promise (Maybe (Proposal } a))}, \text{Nack } \mathbb{N}\}
Proposal a = \{\text{Proposal } \mathbb{N} | a\}
```

3.0.2 Global Type

Since each proposer initiates its own session the global type can be defined for one proposer. A quorum of acceptors A_Q is assumed.

The last phase of Paxos contains no inter-process communication, so it is not modeled in the global type.

$$\begin{split} G_{p,A_Q} &= (\mu X) \bigodot_{a \in A_Q} \ p \to_u a : l1a \ \langle \mathbb{N} \rangle \,. \, \bigodot_{a \in A_Q} \ a \to_u p : l1b \ \langle \text{Promise Value} \rangle \,. \\ p \to_w A_Q : Accept. \left(\bigodot_{a \in A_Q} \ p \to_u a : l2a \ \langle \text{Proposal Value} \rangle \right) .end \\ &\oplus Restart. X \\ &\oplus Abort.end \end{split}$$

We can distinguish the individual phases of the Paxos algorithm by the labels l1a, l1b, and l2a.

In the first two steps, 1a and 1b, the proposer sends its proposal number to each acceptor in A_Q and listens for their responses. In step 2a the proposer decides whether to send an Accept or Restart message to restart the algorithm. This decision is broadcast to all acceptors in A_Q . Should the proposer crash the algorithm ends for this particular proposer and quorum of acceptors.

3.0.3 Functions and Sets

```
\begin{aligned} Bool &= \{true, false\} \\ prNumber : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \text{ returns a proposal number when given two natural numbers.} \end{aligned}
```

promValue : [Promise a] $\to a$ if none of the promises in the given list contain a value a new value is returned. A promise contains a value if it is of the form Promise (Just v). v is the value.

```
anyNack : [Promise \ a] \rightarrow Bool
anyNack([]) = false
anyNack((Nack \_: \_)) = true
\operatorname{anyNack}((:xs)) = \operatorname{anyNack}(xs)
promCount : [Promise \ a] \to \mathbb{N}
promCount([]) = 0
promCount((Promise_: xs)) = 1 + promCount(xs)
\operatorname{promCount}((\_:xs)) = \operatorname{promCount}(xs)
gt: a \to Maybe \ a \to Bool
gt(, Nothing) = true
gt(a, Just b) = a > b
ge: a \to \text{Maybe } a \to \text{Bool}
ge(,Nothing) = true
ge(a, Just b) = a \ge b
nFromPr : Proposal a \to \mathbb{N}
nFromPr (Proposal n_{-}) = n
\operatorname{genA}_{Q}(i,ac,pc) returns a set A_{Q} with A_{Q}\subseteq A=\{1,\ldots,ac\} and |A_{Q}|>\frac{|A|}{2}.
```

3.0.4 Processes

System Initialization

$$\begin{split} &\operatorname{Sys}\left(ac,pc\right) = \overline{a}\left[2\right](t) \cdot \operatorname{P}_{\operatorname{init}}^{\operatorname{p}}\left(ac+1,\operatorname{genA}_{\operatorname{Q}}\left(ac+pc,ac,pc\right),ac+pc,ac+pc,\left[\right]\right) \\ &\mid a\left[1\right](t) \cdot \Pi_{ac < i < ac+pc} \cdot \operatorname{P}_{\operatorname{init}}^{\operatorname{p}}\left(ac+1,\operatorname{genA}_{\operatorname{Q}}\left(i,ac,pc\right),i,i,\left[\right]\right) \\ &\mid \Pi_{1 \leq j \leq ac} \cdot \operatorname{P}_{\operatorname{init}}^{\operatorname{a}}\left(j,ac+1,ac,pc,n_{j},pr_{j}\right) \\ &\operatorname{P}_{\operatorname{init}}^{\operatorname{p}}\left(i,A_{Q},n,m,\overrightarrow{V}\right) = \overline{b_{n}}\left[i\right](s) \cdot \operatorname{P}^{\operatorname{p}} \\ &\operatorname{P}_{\operatorname{init}}^{\operatorname{a}}\left(j,i,ac,pc,n,pr\right) = \Pi_{ac < k \leq ac+pc} \cdot b_{k}\left[j\right](s) \cdot \operatorname{P}^{\operatorname{a}} \end{split}$$

 $\operatorname{Sys}\left(ac,pc\right)$, $\operatorname{P}^{\operatorname{p}}_{\operatorname{init}}\left(i,A_{Q},n,m,\overrightarrow{V}\right)$, and $\operatorname{P}^{\operatorname{a}}_{\operatorname{init}}\left(j,i,ac,pc,n,pr\right)$ describe the system initialization. ac and pc are the number of acceptors and proposers respectively.

An outer session is created through shared-point a. This outer session is not strictly necessary but was left in to allow for easier extension of the model. The acceptors are initialized using indices from 1 to ac and the proposers are initialized using indices from ac+1 to ac+pc.

 $\mathrm{P_{init}^p}\left(i,A_Q,n,m,\overrightarrow{V}\right)$ is initialized with the proposer's role in its own session i, which is always ac+1, a quorum of acceptors A_Q , an index n, and a vector \overrightarrow{V} . Each proposer has the same role i=ac+1 but uses a different shared-point b_n according to its index n. \overrightarrow{V} is used in the proposer to collect and evaluate the responses from the acceptors. It is always initialized with an empty list []. Shared-point b_n is used to initiate a session. Afterwards, the process behaves like $\mathrm{P^p}$.

 $P_{\mathrm{init}}^{\mathrm{a}}\left(j,i,ac,pc,n,pr\right)$ is initialized with the acceptor's index j, the proposer index i, which is always ac+1, ac, pc, initial knowledge for the highest promised proposal number n, if available, and initial knowledge for the most recently accepted proposal pr, if available. n is of type Maybe $\mathbb N$ and pr is of type Maybe (Proposal Value) thus both can be Nothing. Each of the proposers' session requests are accepted in a separate subprocess. These subprocesses run parallel to each other but still access the same values for n and pr. Afterwards, each subprocess behaves like P^{a} .

Proposer

```
\begin{split} \mathbf{P}^{\mathbf{p}} &= (\mu X) \operatorname{update}\left(n, n+1\right). \left( \bigcirc_{j \in A_Q} \ s\left[i, j\right]!_u l 1a \left\langle \operatorname{prNumber}\left(n, m\right) \right\rangle \right). \\ &\left( \bigcirc_{j \in A_Q} \ s\left[j, i\right]?_u l 1b \left\langle \bot \right\rangle (v_j) \right). \\ &\text{if anyNack} \left( \overrightarrow{V} \right) \ \text{or promCount} \left( \overrightarrow{V} \right) < \left\lceil \frac{|A_Q|}{2} \right\rceil \\ &\text{then } s\left[i, A_Q\right]!_w Restart. X \\ &\text{else} \\ &s\left[i, A_Q\right]!_w Accept. \bigcirc_{j \in A_Q} \ s\left[i, j\right]!_u l 2a \left\langle \operatorname{Proposal prNumber}\left(n, i\right) \ \operatorname{promValue} \left( \overrightarrow{V} \right) \right\rangle. \\ &end \end{split}
```

Acceptor

```
P^{a} = (\mu X) s [i, j]?_{u} l1a \langle \bot \rangle (n').
   if n' = \perp
     then P<sup>a</sup><sub>cont</sub>
     else
       if gt(n', n)
       then update (n, n') .s [j, i]!_u l1b \langle \text{Promise } pr \rangle . P_{\text{cont}}^{\text{a}}
       else s\left[j,i\right]!_{u}l1b\left\langle \operatorname{Nack}\left[n\right\rangle .\operatorname{P_{cont}^{a}}\right.
\mathbf{P_{cont}^{a}} = s\left[i,j\right]?_{w}Accept.s\left[i,j\right]?_{u}l2a\left\langle \bot\right\rangle \left(pr'\right).
     if pr' = \perp
       then X
       else
         if ge (nFromPr (pr'), n)
           then update (pr, pr') . update (n, \text{Just nFromPr}(pr')) . X
           else X
   \oplus Restart.X
   \oplus Abort.end
```

3.1 Example

```
ac = 3 pc = 2
```

```
\mathrm{Sys}\left(3,2\right) = a\left[1\right](t) . \Pi_{3 < i < 5} \ \mathrm{P_{init}^{p}}\left(4, \mathrm{genA_{Q}}\left(i,3,2\right), i, i, []\right)
| \overline{a} [2] (t) . P_{init}^{p} (4, gen A_{Q} (5, 3, 2), 5, 5, [])
| \Pi_{1 \leq j \leq 3} P_{\text{init}}^{\text{a}} (j, 4, 3, 2, n_j, pr_j) |
\longmapsto (Init)
(\nu t) \left( P_{\text{init}}^{\text{p}} (4, A_{Q,1}, 4, 4, []) \mid P_{\text{init}}^{\text{p}} (4, A_{Q,2}, 5, 5, []) \right)
   P_{\text{init}}^{\text{a}}(1,4,3,2,n_1,pr_1) \mid P_{\text{init}}^{\text{a}}(2,4,3,2,n_2,pr_2) \mid P_{\text{init}}^{\text{a}}(3,4,3,2,n_3,pr_3)
\mid \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \rightarrow l} : \mid )
(\nu t) (\overline{b_4} [4] (s) . P^p | \overline{b_5} [4] (r) . P^p
   \Pi_{3 < n < 5} b_n [1](s) . P^a
  \Pi_{3 < n < 5} b_n [2] (s) . P^a
 | \Pi_{3 < n < 5} b_n [3](s) . P^a
| \Pi_{1 \le k, l \le 2, k \ne l} t_{k \to l} : [])
\left(\nu t\right)\left(\overline{b_{4}}\left[4\right]\left(s\right).\,\mathrm{P^{p}}\ \mid\overline{b_{5}}\left[4\right]\left(r\right).\,\mathrm{P^{p}}
 | (b_4 [1] (s) . P^a | b_5 [1] (r) . P^a)
   (b_4 [2] (s) . P^a | b_5 [2] (r) . P^a)
   (b_4 [3] (s) . P^a | b_5 [3] (r) . P^a)
| \Pi_{1 \le k, l \le 2, k \ne l} t_{k \to l} : [] 
\longmapsto (Init)
\left(\nu t\right)\left(\nu s\right)\left(\nu r\right)\left(\left(\mu X\right)\text{update}\left(n,5\right).\left(\bigcirc_{j\in\left\{ 1,2\right\} }\ s\left[4,j\right]!_{u}l1a\left\langle \text{prNumber}\left(n,4\right)\right\rangle \right)...
\mid (\mu X) \text{ update } (n,6) \cdot \left( \bigcirc_{j \in \{2,3\}} r[4,j]!_u l1a \langle \text{prNumber } (n,5) \rangle \right) \dots
|((\mu X) s [4,1]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,1]?_u l1a \langle \bot \rangle (n') . if ...)
|((\mu X) s [4,2]?_u l 1a \langle \perp \rangle (n') . \text{if } \ldots | (\mu X) r [4,2]?_u l 1a \langle \perp \rangle (n') . \text{if } \ldots)
((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... | (\mu X) r [4,3]?_u l1a \langle \perp \rangle (n') . if ...)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 1a \langle \text{prNumber} (5, 4) \rangle .s [4, 2]!_u l 1a \langle \text{prNumber} (5, 4) \rangle ...
 |(\mu X) r[4,2]!_u l1a \langle \text{prNumber}(6,5) \rangle . r[4,3]!_u l1a \langle \text{prNumber}(6,5) \rangle ...
   ((\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,1]?_u l1a \langle \perp \rangle (n') . if ...)
   ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,2]?_u l1a \langle \perp \rangle (n') . if ...)
   ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,3]?_u l1a \langle \perp \rangle (n') . if ...)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l1a \langle \text{prNumber} (5,4) \rangle .s [4,2]!_u l1a \langle \text{prNumber} (5,4) \rangle ...
```

```
(\mu X) r [4,3]!_{u} l1a \langle \text{prNumber} (6,5) \rangle . r [2,4]?_{u} l1b \langle \bot \rangle (v_{2}) ...
((\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') . if ... | (\mu X) r [4,1]?_u l1a \langle \perp \rangle (n') . if ...)
  ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) if 10 = \perp then P_{cont}^a else ...)
|((\mu X) s [4,3]?_u l 1a \langle \perp \rangle (n') . \text{ if } \ldots | (\mu X) r [4,3]?_u l 1a \langle \perp \rangle (n') . \text{ if } \ldots)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l1a \langle \text{prNumber} (5,4) \rangle .s [4,2]!_u l1a \langle \text{prNumber} (5,4) \rangle ...
  (\mu X) r [4,3]!_u l1a \langle \text{prNumber} (6,5) \rangle . r [2,4]?_u l1b \langle \bot \rangle (v_2) ...
((\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') . if ... | (\mu X) r [4,1]?_u l1a \langle \perp \rangle (n') . if ...)
((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') if ... (\mu X) if gt (10, Nothing) then update (n,n') ... else ...)
|((\mu X) s [4,3]?_u l 1a \langle \perp \rangle (n') . if ... | (\mu X) r [4,3]?_u l 1a \langle \perp \rangle (n') . if ... |
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 1a \langle \text{prNumber} (5, 4) \rangle .s [4, 2]!_u l 1a \langle \text{prNumber} (5, 4) \rangle ...
(\mu X) r [4,3]!_u l1a \langle \text{prNumber} (6,5) \rangle . r [2,4]?_u l1b \langle \bot \rangle (v_2) ...
|((\mu X) s [4,1]?_u l 1a \langle \perp \rangle (n') . if ... | (\mu X) r [4,1]?_u l 1a \langle \perp \rangle (n') . if ...)
  ((\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [2,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
  ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,3]?_u l1a \langle \perp \rangle (n') . if ...)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l1a \langle \text{prNumber} (5,4) \rangle .s [4,2]!_u l1a \langle \text{prNumber} (5,4) \rangle ...
(\mu X) r [2,4]?_u l1b \langle \perp \rangle (v_2) .r [3,4]?_u l1b \langle \perp \rangle (v_3) ...
|((\mu X) s [4,1]?_u l 1a \langle \bot \rangle (n') . \text{if } ... | (\mu X) r [4,1]?_u l 1a \langle \bot \rangle (n') . \text{if } ...)
|(\mu X) s [4,2]?_u l1a \langle \perp \rangle (n') if ... |(\mu X) r [2,4]!_u l1b \langle \text{Promise Nothing} \rangle \cdot P_{\text{cont}}^a
((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... | (\mu X) r [3,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
\longmapsto (USkip)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 1a \langle \text{prNumber} (5, 4) \rangle .s [4, 2]!_u l 1a \langle \text{prNumber} (5, 4) \rangle ...
|(\mu X) r[2,4]?_u l1b \langle \perp \rangle (v_2) .r[3,4]?_u l1b \langle \perp \rangle (v_3) ...
  ((\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) if \perp = \perp then P_{cont}^a else ...)
|((\mu X) s [4,2]?_u l 1a \langle \perp \rangle (n') . if ... | (\mu X) r [2,4]!_u l 1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
|((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... | (\mu X) r [3,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
\mid \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \mid \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \mid \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 1a \langle \text{prNumber} (5, 4) \rangle .s [4, 2]!_u l 1a \langle \text{prNumber} (5, 4) \rangle ...
```

```
|(\mu X) r[2,4]?_u l1b \langle \perp \rangle (v_2) .r[3,4]?_u l1b \langle \perp \rangle (v_3) ...
 \mid ((\mu X) s [4,1]?_u l 1a \langle \bot \rangle (n') if ... \mid (\mu X) r [4,1]?_w Accept \cdots \oplus Restart. X \oplus Abort. end)
  ((\mu X) s [4, 2]?_u l1a \langle \perp \rangle (n'). if \dots \mid (\mu X) r [2, 4]!_u l1b \langle \text{Promise Nothing} \rangle. P_{\text{cont}}^a)
 |((\mu X) s [4,3]?_u l 1a \langle \perp \rangle (n') . if ... | (\mu X) r [3,4]!_u l 1b \langle Promise Nothing \rangle . P_{cont}^a \rangle
 | \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [] ) 
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{prNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{prNumber} (5,4) \rangle ...
|(\mu X) r[3,4]?_u l1b \langle \bot \rangle (v_3) if any Nack (\overrightarrow{V}) or promCount (\overrightarrow{V}) < 1 then ...
((\mu X) s [4,1]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,1]?_w Accept ... \oplus Restart.X \oplus Abort.end)
|((\mu X) s [4,2]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,2]?_w Accept ... \oplus Restart.X \oplus Abort.end)|
  ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [3,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^a)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 1a \langle \text{prNumber} (5, 4) \rangle .s [4, 2]!_u l 1a \langle \text{prNumber} (5, 4) \rangle ...
 (\mu X) if false or 2 < 1 then ...else r[4, \{2,3\}]!_w Accept...
  ((\mu X) s [4,1]?_u l1a \langle \bot \rangle (n'). if ... | (\mu X) r [4,1]?_w Accept \cdots \oplus Restart. X \oplus Abort.end)
 |((\mu X) s [4,2]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,2]?_w Accept ... \oplus Restart. X \oplus Abort. end)|
  ((\mu X) s [4,3]?_u l1a \langle \bot \rangle (n').if \dots | (\mu X) r [4,3]?_w Accept \dots \oplus Restart.X \oplus Abort.end)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 1a \langle \text{prNumber} (5, 4) \rangle .s [4, 2]!_u l 1a \langle \text{prNumber} (5, 4) \rangle ...
|(\mu X) r[4, \{2, 3\}]|_w Accept.r[4, 2]|_u l2a \langle Proposal prNumber(5, 6) promValue(\overrightarrow{V}) \rangle \dots
|((\mu X) s [4,1]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,1]?_w Accept ... \oplus Restart. X \oplus Abort.end)
((\mu X) s [4,2]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,2]?_w Accept ... \oplus Restart. X \oplus Abort. end)
  ((\mu X) s [4,3]?_u l1a \langle \bot \rangle (n'). if ... \mid (\mu X) r [4,3]?_w Accept \cdots \oplus Restart. X \oplus Abort.end)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (WSel + WBran)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 1a \langle \text{prNumber} (5, 4) \rangle .s [4, 2]!_u l 1a \langle \text{prNumber} (5, 4) \rangle ...
|(\mu X) r[4,2]!_u l2a \langle \text{Proposal prNumber}(5,6) \text{ promValue}(\overrightarrow{V}) \rangle.
    r[4,3]!_u l2a \left\langle \text{Proposal prNumber}(5,6) \text{ promValue}\left(\overrightarrow{V}\right) \right\rangle \dots
((\mu X) s [4,1]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,1]?_w Accept ... \oplus Restart.X \oplus Abort.end)
((\mu X) s [4,2]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) r [4,2]?_u l 2a \langle \bot \rangle (pr') . if ...)
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|((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... | (\mu X) r [4,3]?_u l2a \langle \perp \rangle (pr') . if ...)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (WSkip)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l1a \langle \text{prNumber} (5,4) \rangle .s [4,2]!_u l1a \langle \text{prNumber} (5,4) \rangle ...
|(\mu X) r[4,2]!_u l2a \langle \text{Proposal prNumber}(5,6) \text{ promValue}(\overrightarrow{V}) \rangle.
    r[4,3]!_u l2a \langle \text{Proposal prNumber}(5,6) \text{ promValue}(\overrightarrow{V}) \rangle \dots
|((\mu X) s [4,1]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) end)
((\mu X) s [4,2]?_u l 1a \langle \perp \rangle (n') . if ... | (\mu X) r [4,2]?_u l 2a \langle \perp \rangle (pr') . if ...)
  ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... \mid (\mu X) r [4,3]?_u l2a \langle \perp \rangle (pr') . if ...)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 1a \langle \text{prNumber} (5, 4) \rangle .s [4, 2]!_u l 1a \langle \text{prNumber} (5, 4) \rangle ...
|(\mu X) r[4,3]!_u l2a \langle \text{Proposal prNumber}(5,6) \text{ promValue}(\overrightarrow{V}) \rangle.end
(\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') if ...
|((\mu X) s[4,2]?_u l1a \langle \perp \rangle(n'). if \ldots | (\mu X) if Proposal 10 abc = \perp then X else \ldots)
 ((\mu X) s [4,3]?_u l1a \langle \perp \rangle (n') . if ... | (\mu X) r [4,3]?_u l2a \langle \perp \rangle (pr') . if ...)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
\longmapsto (USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{prNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{prNumber} (5,4) \rangle ...
 \mid (\mu X) end
(\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') if ...
 ((\mu X) s [4,2]?_n l1a \langle \bot \rangle (n') if ... (\mu X) if Proposal 10 abc =\bot then X else ...
 \mid ((\mu X) s [4,3]?_u l 1a \langle \bot \rangle (n') if ... \mid (\mu X) if Proposal 10 abc =\bot then X else ...)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l1a \langle \text{prNumber} (5,4) \rangle .s [4,2]!_u l1a \langle \text{prNumber} (5,4) \rangle ...
(\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') if ...
|((\mu X) s [4,2]?_{\mu}l1a \langle \perp \rangle (n').if \dots
     |(\mu X) if ge (10, 10) then update (pr, pr') update (n, \text{Just } 10) X else ...)
|((\mu X) s [4,3]?_u l 1a \langle \bot \rangle (n') . if ...
     (\mu X) if ge (10, 10) then update (pr, pr') update (n, \text{Just } 10) . X else ...)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
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(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{prNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{prNumber} (5,4) \rangle ...
 (\mu X) s [4,1]?_u l 1a \langle \perp \rangle (n') . if ...
  ((\mu X) s [4,2]?_u l1a \langle \bot \rangle (n') . if ... \mid (\mu X) X)
|((\mu X) s [4,3]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) X)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l 1a \langle \text{prNumber} (5,4) \rangle .s [4,2]!_u l 1a \langle \text{prNumber} (5,4) \rangle ...
|(\mu X) s [4,1]?_u l 1a \langle \bot \rangle (n') . if ...
((\mu X) s [4,2]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) X)
  ((\mu X) s [4,3]?_u l1a \langle \bot \rangle (n') . if ... | (\mu X) X)
\mid \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \rightarrow l} : [] \mid \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \rightarrow l} : [] \mid \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \rightarrow l} : [])
\longmapsto (USend + UGet, USend + UGet, USkip)
(\nu t) (\nu s) (\nu r) ((\mu X) s [1,4]?_u l1b \langle \perp \rangle (v_1) .s [2,4]?_u l1b \langle \perp \rangle (v_2) ...
\mid (\mu X) \text{ if } 5 = \perp \text{ then } P_{\text{cont}}^{\text{a}} \text{ else } \dots
\mid ((\mu X) \text{ if } 5 = \perp \text{ then } P_{\text{cont}}^{\text{a}} \text{ else } \dots \mid (\mu X) X)
\mid ((\mu X) \text{ if } \perp = \perp \text{ then } \mathbf{P}_{\text{cont}}^{\mathbf{a}} \text{ else } \dots \mid (\mu X) X)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [1,4]?_u l 1b \langle \bot \rangle (v_1) . s [2,4]?_u l 1b \langle \bot \rangle (v_2) ...
 (\mu X) if gt (5, Nothing) then update (n, 5) \dots else ...
((\mu X)) if gt (5, Just 10) then ... else s[2,4]!_u l1b \langle \text{Nack } 10 \rangle. P_{\text{cont}}^a \mid (\mu X) X
| ((\mu X) r [4,3]?_w Accept \cdots \oplus Restart. X \oplus Abort.end | (\mu X) X) |
| \prod_{1 \le k,l \le 4, k \ne l} s_{k \to l} : [] | \prod_{1 \le k,l \le 4, k \ne l} r_{k \to l} : [] | \prod_{1 \le k,l \le 2, k \ne l} t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [1,4]?_u l 1b \langle \bot \rangle (v_1) . s [2,4]?_u l 1b \langle \bot \rangle (v_2) ...
(\mu X) s [1,4]!_u l1b \langle \text{Promise Nothing} \rangle . P_{\text{cont}}^{\text{a}}
|((\mu X) s [2,4]!_u l 1b \langle \text{Nack } 10 \rangle . P_{\text{cont}}^a | (\mu X) X)|
  ((\mu X) r [4,3]?_w Accept \cdots \oplus Restart.X \oplus Abort.end \mid (\mu X) X)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \ | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \ | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet, USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X)) if true or 1 < 1 then s [4, \{1, 2\}]!_w Restart X else ...
(\mu X) s [4,1]?_w Accept \cdots \oplus Restart. X \oplus Abort.end
| ((\mu X) s [4,1]?_w Accept \cdots \oplus Restart. X \oplus Abort.end | (\mu X) X) |
| ((\mu X) r [4,3]?_w Accept \cdots \oplus Restart.X \oplus Abort.end | (\mu X) X)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
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(\nu t) (\nu s) (\nu r) ((\mu X) s [4, \{1, 2\}]!_w Restart.X
(\mu X) s [4,1]?_w Accept \cdots \oplus Restart.X \oplus Abort.end
| ((\mu X) s [4,1]?_w Accept \cdots \oplus Restart. X \oplus Abort.end | (\mu X) X)
|((\mu X) r [4,3]?_w Accept \cdots \oplus Restart.X \oplus Abort.end | (\mu X) X)|
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (WSel + WBran, WSkip)
(\nu t) (\nu s) (\nu r) ((\mu X) X
\mid (\mu X) X
\mid ((\mu X) X \mid (\mu X) X)
\mid ((\mu X) end \mid (\mu X) X)
| \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \to l} : [] | \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \to l} : [] | \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4,1]!_u l1a \langle \text{prNumber} (6,4) \rangle .s [4,2]!_u l1a \langle \text{prNumber} (6,4) \rangle ...
(\mu X) s [4,1]?_u l1a \langle \perp \rangle (n') . if ...
|((\mu X) s [4,2]?_u l 1a \langle \bot \rangle (n') . if ... | (\mu X) X)
 (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet, USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) s [1, 4]?_u l 1b \langle \perp \rangle (v_1) . s [2, 4]?_u l 1b \langle \perp \rangle (v_2) . if \dots
\mid (\mu X) \text{ if } 15 = \perp \text{ then } P_{\text{cont}}^{\text{a}} \text{ else } \dots
\mid ((\mu X) \text{ if } 15 = \perp \text{ then } P_{\text{cont}}^{\text{a}} \text{ else } \dots \mid (\mu X) X)
  (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [1,4]?_u l1b \langle \perp \rangle (v_1) .s [2,4]?_u l1b \langle \perp \rangle (v_2) .if \dots
(\mu X) if gt (15, Just 5) then update (n, 15)...else ...
((\mu X)) if gt (15, Just 10) then update (n, 15) ... else ... (\mu X)X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [1, 4]?_u l 1b \langle \perp \rangle (v_1) . s [2, 4]?_u l 1b \langle \perp \rangle (v_2) . if \dots
(\mu X) s [1,4]!_u l1b \langle \text{Promise Nothing} \rangle. P_{\text{cont}}^a
|((\mu X) s [2,4]!_u l1b \langle \text{Promise Proposal } 10 \ abc \rangle \cdot P_{\text{cont.}}^a | (\mu X) X)
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\mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet, USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X)) if false or 2 < 1 then ... else s [4, \{1, 2\}]!_w Accept...
(\mu X) s [4,1]?_w Accept \cdots \oplus Restart.X \oplus Abort.end
\mid ((\mu X) s [4,1]?_w Accept \cdots \oplus Restart.X \oplus Abort.end \mid (\mu X) X)
\mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, \{1, 2\}]!_w Accept...
(\mu X) s [4,1]?_w Accept \cdots \oplus Restart. X \oplus Abort.end
 \mid ((\mu X) s [4,1]?_w Accept \cdots \oplus Restart. X \oplus Abort.end \mid (\mu X) X)
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (WSel + WBran)
(\nu t) (\nu s) (\nu r) ((\mu X) s [4, 1]!_u l 2a \langle \text{Proposal prNumber} (6, 4) \text{ promValue} (\overrightarrow{V}) \rangle.
s[4,2]!_u l2a \langle \text{Proposal prNumber}(6,4) \text{ promValue}(\overrightarrow{V}) \rangle .end
|(\mu X) s [4,1]?_u l2a \langle \bot \rangle (pr') . if ...
|((\mu X) s [4,2]?_u l2a \langle \bot \rangle (pr') . if ... | (\mu X) X)
\mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] \ | \ \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
\longmapsto (USend + UGet, USend + UGet)
(\nu t) (\nu s) (\nu r) ((\mu X) end
(\mu X) if Proposal 15 Proposal 15 abc = \perp then ... else ...
|(\mu X) if Proposal 15 Proposal 15 abc = \perp then ... else ... |(\mu X)X|
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
(\nu t) (\nu s) (\nu r) (\mid (\mu X)) if ge (15, Just 15)
    then update (pr, Proposal 15 abc). update (n, 15) . X else . . .
((\mu X)) if ge (15, Just 15)
    then update (pr, Proposal\ 15\ abc). update (n, 15).X else ... | (\mu X)X)
\mid (\mu X) X
| \Pi_{1 \le k, l \le 4, k \ne l} \ s_{k \to l} : [] | \Pi_{1 \le k, l \le 4, k \ne l} \ r_{k \to l} : [] | \Pi_{1 \le k, l \le 2, k \ne l} \ t_{k \to l} : [])
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 = \underbrace{(\nu t) (\nu s) (\nu r) (\mid (\mu X) X} \\ \mid ((\mu X) X \mid (\mu X) X) \\ \mid (\mu X) X \\ \mid \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \rightarrow l} : [] \mid \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \rightarrow l} : [] \mid \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \rightarrow l} : []) \\ \longmapsto (USkip + WSkip, USkip + WSkip, USkip + WSkip, USkip + WSkip) \\ (\nu t) (\nu s) (\nu r) (\mid (\mu X) \ end) \\ \mid ((\mu X) \ end) \mid (\mu X) \ end) \\ \mid (\mu X) \ end \\ \mid \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \rightarrow l} : [] \mid \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \rightarrow l} : [] \mid \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \rightarrow l} : []) \\ = \\ (\nu t) (\nu s) (\nu r) (\mid \Pi_{1 \leq k, l \leq 4, k \neq l} \ s_{k \rightarrow l} : [] \mid \Pi_{1 \leq k, l \leq 4, k \neq l} \ r_{k \rightarrow l} : [] \mid \Pi_{1 \leq k, l \leq 2, k \neq l} \ t_{k \rightarrow l} : [])
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4 Evaluation

RESULTS

5 Discussion

DISCUSSION