Kuratowski's Theorem

(Toán rời rạc)

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Tóm tắt nội dung

Đây là tóm tắt [1]

[*]K64 ...
[†]K65 ...
[‡]K65 ...

Mục lục

1 Introduction

Theorems can easily be defined

Định lý 1. *Let f be a function whose derivative exists in every point, then f is a continuous function.*

Định lý 2 (Pythagorean theorem)**.** *This is a theorema about right triangles and can be summarised in the next equation*

$$x^2 + y^2 \subsetneq z^2$$

And a consequence of theorem 2 is the statement in the next corollary.

Hệ quả 1. *There's no right rectangle whose sides measure 3cm, 4cm, and 6cm.*

You can reference theorems such as 2 when a label is assigned.

Bổ đề 1. *Given two line segments whose lengths are a and b respectively there is a real number r such that $b = ra$.*

Unnumbered theorem-like environments are also posible.

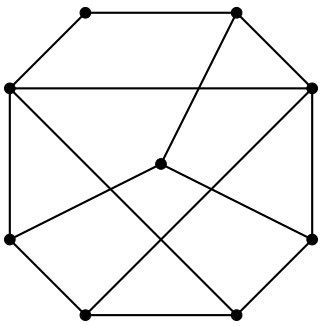
Nhận xét. *This statement is true, I guess.*

2 Defination

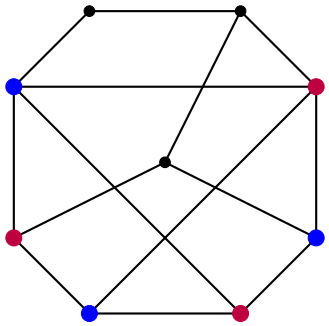
3 Statement of the Theorem

And the next is a somewhat informal definition

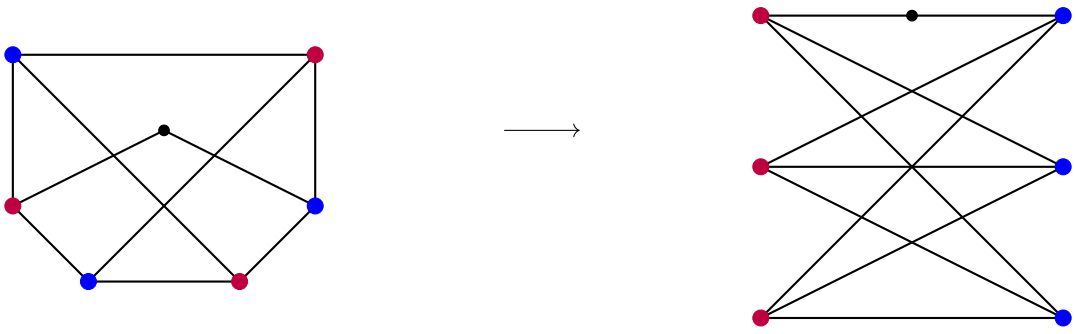
Định lý 3 (Kuratowski)**.** *A graph is nonplanar if and only if it has a subgraph which is a subdivision of K_5 or $K_{3,3}$*



Hình 1: Nonplanar graph G



Hình 2: Nonplanar graph G



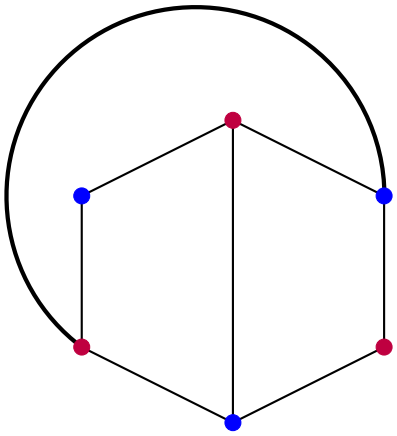
Subgraph of G

Subdivision of $K_{3,3}$

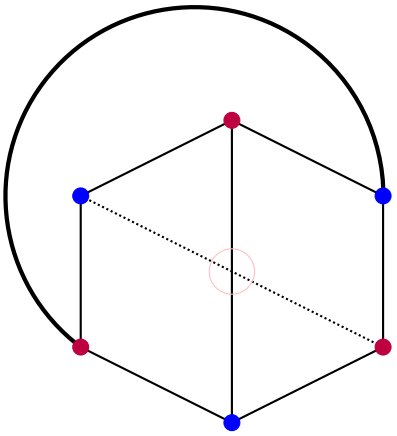
4 Preliminaries

4.1 Planar Graphs and their Properties

Định nghĩa 1 (Planarity)**.** *A graph is planar if some embedding of it onto the plane has no edge intersections.*

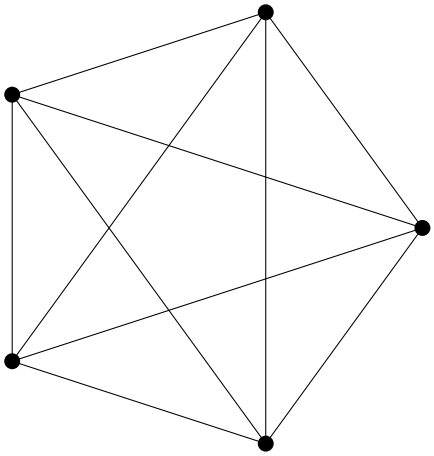


Planar graph

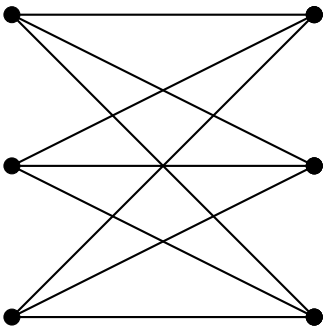


Nonplanar Embedding

4.2 Define K_5 and $K_{3,3}$



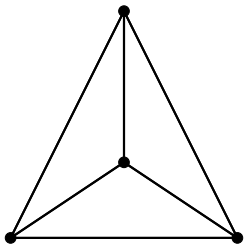
K_5



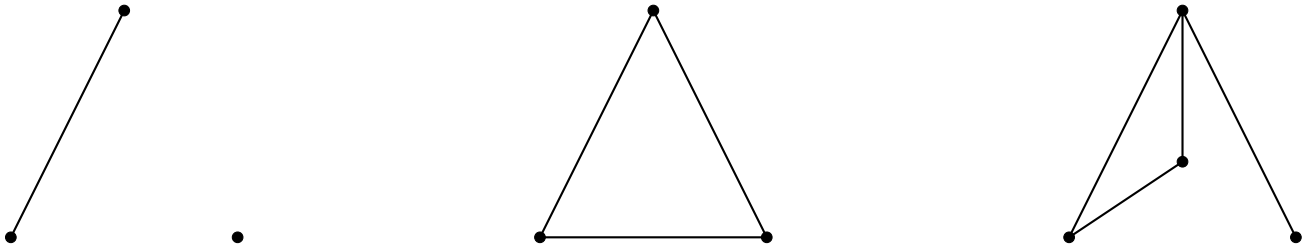
$K_{3,3}$

4.3 Subgraph and Subdivision

Định nghĩa 2. *Subgraphs are subsets of vertices and egdes of some original graphs*



Original graph

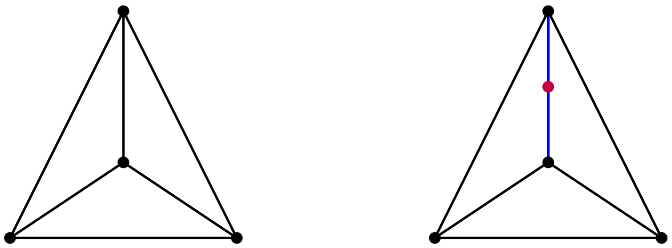


3 Subgraphs

Hệ quả 2. *If graph is planar then all subgraphs are planar*

Chứng minh. Contradiction □

Định nghĩa 3. *Subdivisions are obtained by replacing an edge with 2 edges connected by a new vertex*



Original graph Subdivision graph

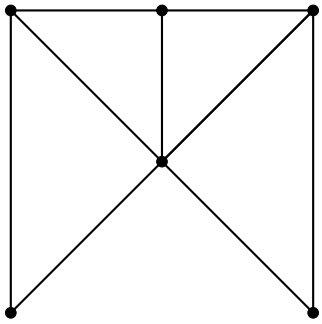
Hệ quả 3. *If some subdivision is planar then graph is planar*

Chứng minh. Ai biết đâu. □

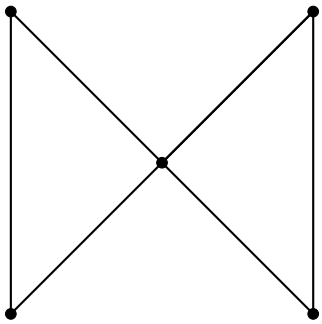
Bổ đề 2. *If graph is nonplanar then all subdivisions are nonplanar*

4.4 2-Connected Graphs and their Properties

Định nghĩa 4. *A graph is 2-connected if it cannot be separated into two components by removing a single vertex*



Example 2-connected graph

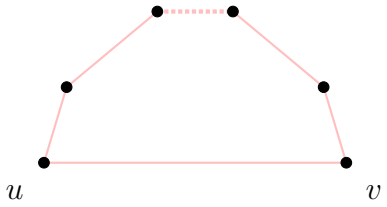


Not 2-connected

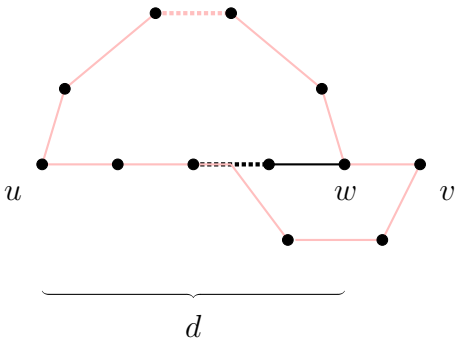
Định lý 4. *In a 2-connected graph, any pair of vertices is contained in a cycle*

Chứng minh. Quy nạp:

Trường hợp cơ bản: u kề v



Quy nạp: u, v có khoảng cách $d+1$



□

5 Graph Theory Background

Định nghĩa 5 (Fibration)**.** *A fibration is a mapping between two topological spaces that has the homotopy lifting property for every space X.*

6 Proof the Theorem

The first direction of Kuratowski's theorem states: If graph G contains a subdivision of K_5 or $K_{3,3}$ then G is nonplanar

Subdivision of Nonplanar is Nonplanar

If a Subgraph is nonplanar then graph is nonplanar

If a subgraph of graph G is a subdivision of nonplanar then G is nonplanar

Bổ đề 3. *$K_{3,3}$ is nonplanar*

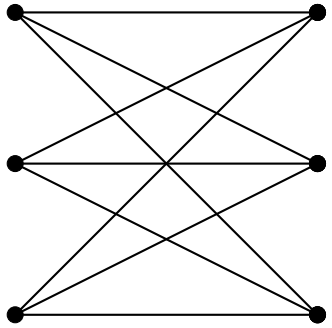
Chứng minh.

$$V - E + F = 2$$

$$6 - E + F = 2$$

$$6 - 9 + F = 2$$

$$F = 5$$



No 3 edge faces

$$4F \leq 2E$$

$$4F \leq 2 \times 9$$

$$F \leq 4.5$$

$$5 \leq 4.5$$

□

Bổ đề 4. *K_5 is nonplanar*

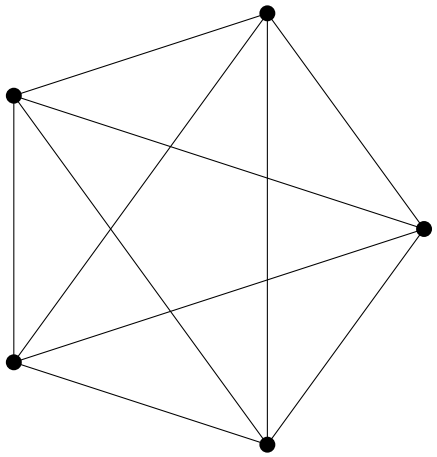
Chứng minh.

$$V - E + F = 2$$

$$5 - E + F = 2$$

$$5 - 10 + F = 2$$

$$F = 7$$



$$3F \leq 2E$$

$$3F \leq 2 \times 10$$

$$F \leq \frac{20}{3}$$

$$7 \leq \tfrac{20}{3}$$

□

Tóm lại. *K_5 và $K_{3,3}$ are nonplanar*

⇒ All of their subdivisions are nonplanar

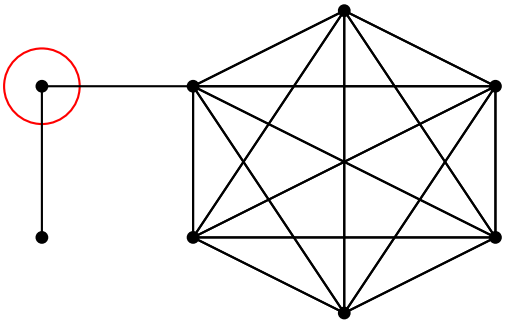
⇒ If graph G contains a subdivision of K_5 or $K_{3,3}$ then G is nonplanar

The second direction of Kuratowski's theorem states: If graph G is nonplanar then G contains a subdivision of K_5 or $K_{3,3}$

Chứng minh. Assume there exist nonplanar graphs which have no subdivisions of K_5 or $K_{3,3}$ as subgraphs.

Let G be the graph of this kind with the $fewest$ edges. Then removing any edge from G gives a *planar* graph

1. G is 2-connected



2. $deg(v) \geq 3$ for all vertex v in G

   Chứng minh phản chứng: assume some vertex $v \in G$ has $deg(v) \leq 2$

3. for some $uv \in G$, $G - uv$ is 2-connected

□

Bổ đề 5. *Given two line segments whose lengths are a and b respectively there is a real number r such that $b = ra$.*

Chứng minh. To prove it by contradiction try and assume that the statement is false, proceed from there and at some point you will arrive to a contradiction. □

Acknowledgement

It is a pleasure to thank my mentor, Reid Harris, for his helpful guidance and advice. I would also like to thank Professor Babai for introducing me to graph theory and Professor May for organizing the REU.