AE4ASM521 Design Assignment A1: Two-Scale Damage Tolerant Structures

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ABSTRACT

This paper describes the two-scale topology optimization process of an outdoor bench. Different porosities are compared and evaluated on their compliance performance, both without damage and with damage in two different regions to obtain the stiffest design for the bench.

1 INTRODUCTION

The application of topology optimization is becoming increasingly popular in the industry. In the past, most engineering designs were made by designers, with the drawback that the result relies on their creativity, experience and trial-and-error processes. Now, topology optimization enabled a significant reduction in design time. Besides the lower cost, it can also significantly decrease the material usage while still achieving a high performance result [1]. A new introduced technology is two-scale topology optimization. This allows the generation of more porous structures, which are often stiffer in case of damage or material deficiency compared to structures developed with regular topology optimization [2] [3]. Two methods can be used to do so. First, the decoupled approach maps the micro-structure onto the optimized macro-structure. Second, concurrent two-scale optimization directly generates the optimized structure from the principle stress directions.
In this paper, the decoupled two-scale topology optimization approach is used to design a public bench. The goal is to maximize the bench stiffness which is equivalent to minimizing its compliance. To do so, the compliance is evaluated for different structure porosities, both without damage and with damage in two different regions.
This paper is structured as follows. First, section 2 describes how the bench, its load cases and its constraints are modelled, as well as the infill approach for the two-scale topology optimization problem. Next, section 3 evaluates different porosities in terms of compliance and discusses the results. Finally, the conclusion is drawn in section 4.

2 METHOD

2.1 Problem Definition

The model of the bench is shown in Figure 1. Five loads are evenly distributed over the bench width to support different seating positions. These point loads are applied at $x = 0$, $x = 1/4w$, $x = 1/2w$, $x = 3/4w$ and $x = w$. Note that all these loads occur simultaneously which is indicated by F(...,1), because the bench needs to be able to support the seating of multiple people at the same time. The Matlab implementation is shown at the top right.

```
Fsparse1(2*(nelx/4)*(nely + 1) + 2, 1) = -1;
Fsparse2(2*(nelx*3/4)*(nely + 1) + 2, 1) = -1;
Fsparse3(2, 1) = -1;
Fsparse4(2*(nelx)*(nely+1)+2, 1) = -1;
Fsparse5(2*(nelx/2)*(nely + 1) + 2, 1) = -1;
```

In addition, the bench base is fixed to the ground at two points: at $x = 1/10w$ and at $x = 9/10w$.

```
fixeddofs = union([2*(nely+1)*(nelx/10+1) ...
  - 1:2*(nely+1)*(nelx/10+1)], ...
    [2*(nely+1)*(nelx*9/10+1) ...
  - 1:2*(nely+1)*(nelx*9/10+1)]);
```

The design space for the bench is shown in black. In the optimization problem, this space is modelled with 240 x 30 pixels, where all pixels for which $1/4w \leq x \leq 3/4w$ and $5/6h \leq y \leq h$ are removed.

```
xPhys(nely*5/6:nely, nelx/4:nelx*3/4) = 0;
```

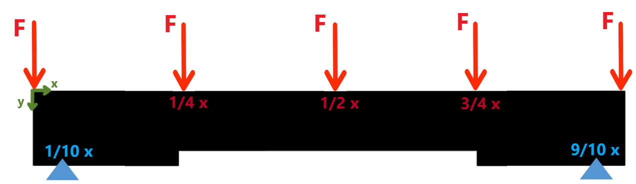

Figure 1: The model of the bench used in the optimization problem.

2.2 Infill Approach

Two-scale topology optimization is used to design the bench. Here, the parts of the bench that do not carry a critical mechanical load are removed, in addition the density of the remaining parts varies. This is achieved by decomposing the structure into unit cells with a circular fill pattern. The fill density determines the circle radius as shown in Figure 2. The radius of each circle is computed by satisfying Equation 1. In this way, the fill percentage of each unit cell is equal to its density fraction.

$$\frac{A_{circ}}{A_{unitcell}} = \frac{\pi R^2}{A_{unitcell}} = 1 - \rho \tag{1}$$

Solving this equation gives the following Matlab implementation. Note that $plotsize$ represents both the width and the height of each unit cell in pixels.

```
R = sqrt((1-rho)*plotsize^2/pi);
```

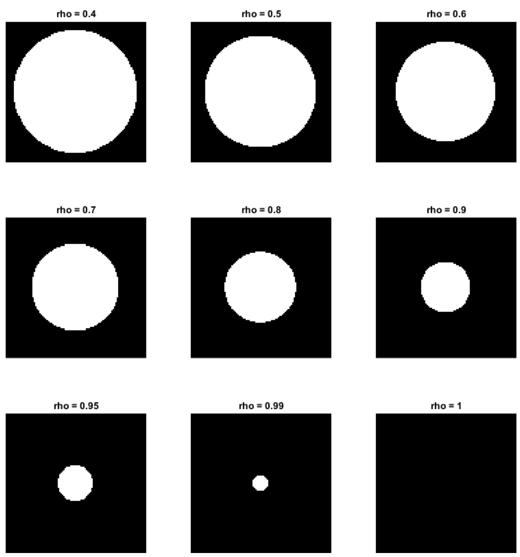

Figure 2: The unit cells with different fill densities.

To visualize the bench composed of these unit cells, an array $topopt2D\_damage$ with size $(nelx * plotsize, nely * plotsize)$ is created. Then, the unit cell arrays are computed with $drawCircle$ and are appended to the full array. The white parts are represented by number 1 and the black parts are represented by a 0.

```
topopt2D_damage = zeros(nelx*plotsize, ...
                        nely*plotsize);
for y = 1:nely
  for x = 1:nelx
    rad = xPhys(y,x);
    circlePixels = drawCircle(rad, plotsize);
    topopt2D_damage(x*plotsize - (plotsize-1): ...
    x*plotsize, y*plotsize - (plotsize-1): ...
    y*plotsize) = circlePixels;
  end
end
```

2.3 Damage region

After executing the optimization process, a damage region is defined in the bench to evaluate its compliance. This is done by removing material from either the top or the bottom of the bench by setting its value to zero.

```
xPhysBackup = xPhys;
 % Top
xPhysBackup(1:nely*1/3, nelx*11/20:nelx*14/20) = 0;
 % Bottom
xPhysBackup(nely*3/6:nely*5/6, nelx*11/20:nelx*14/20) = 0;
xPhys = xPhysBackup;
```

Next, the compliance is evaluated by summing the compliance for every individual load case.

```
ce = reshape(sum((U1(edofMat)*KE).*U1(edofMat),2),nely,nelx)+...
reshape(sum((U2(edofMat)*KE).*U2(edofMat),2),nely,nelx)+...
reshape(sum((U3(edofMat)*KE).*U3(edofMat),2),nely,nelx)+...
reshape(sum((U4(edofMat)*KE).*U4(edofMat),2),nely,nelx)+...
reshape(sum((U5(edofMat)*KE).*U5(edofMat),2),nely,nelx);
c = sum(sum((Emin+xPhys.^penal*(E0-Emin)).*ce));
```

3 Results and Discussion

3.1 Results

To find the stiffest design of the bench, regular topology optimization is compared to two-scale topology optimization for different porosities. A volume fraction of 0.6 is used in all cases. To visualize the difference between the outcomes of these two processes, Figure 3 shows the bench made with regular topology optimization. Here, a penalty score of 5 and a minimum radius of 5 were used.

Figure 3: Regular topology optimization.

The two-scale topology optimization problem is evaluated for different porosities. The minimum radius of the structure was set to do so, where a lower minimum radius results in a more porous structure as shown in Figure 4 till Figure 7. The penalty equals 1 for all structures.

Figure 4: Two-scale topology optimization, min. radius 1.5

Figure 5: Two-scale topology optimization, min. radius 2.5

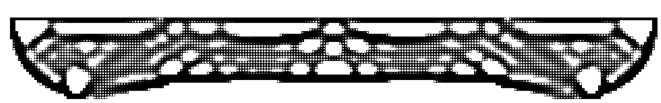

Figure 6: Two-scale topology optimization, min. radius 3.5

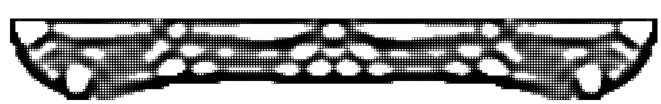

Figure 7: Two-scale topology optimization, min. radius 4.5

Then two damage regions are alternately generated in each bench. The first damage region is shown in Figure 8. The second damage region is shown in Figure 9.

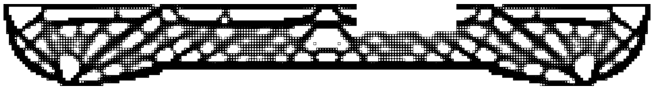

Figure 8: Damage region 1.

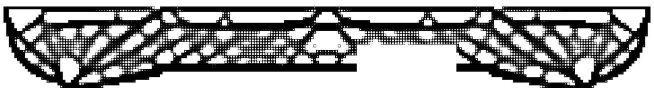

Figure 9: Damage region 2.

The compliance is evaluated for all combinations, and the results are given in Table 1. These results are also plotted in Figure 10.

Two-scale Topology Optimization			
Minimum Radius	**Compliance without damage [m/N]**	**Compliance with damage region 1 [m/N]**	**Compliance with damage region 2 [m/N]**
1.5	279.0	708.5	410.6
2.5	312.0	816.0	420.4
3.5	366.1	912.7	487.4
4.5	388.1	1129.0	756.8
reg. top-opt	221	1699.0	493.9

Table 1: The compliance for a varying minimum radius in different damage regions.

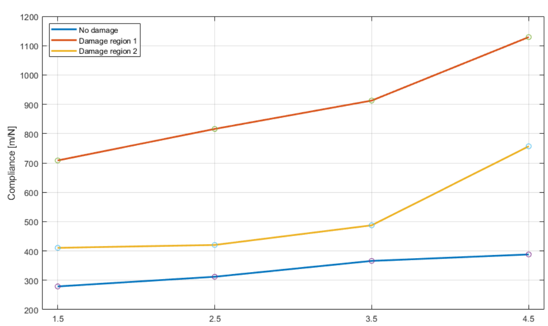

Figure 10: A plot of the compliance over the minimum radius in different damage regions.

3.2 Discussion

First, it is evaluated whether the structures obtained from the topology optimization problems are feasible and make sense. When looking at the material distributions, the micro-structure is more dense in the regions where the load is applied. Also, the micro-structure is aligned with the local stress directions at the application points to prevent local damage. And the structure is symmetric, which is expected in a symmetric load case. Therefore, the shapes obtained from the topology optimization are considered to be feasible outcomes. From the compliance results, it can be noticed that a more porous structure has a smaller compliance. Also, the compliance of the porous structures after damage shows a smaller increase compared to the compliance of less porous structures. This is also confirmed by the results of the regular topology optimization problem, where the compliance increases by 700 % in damage case 1, and by 123 % in damage case 2. For the porous structures obtained from the two-scale topology optimization problem, the largest compliance increases are 190% and 95% for damage case 1 and 2 respectively. Therefore, the most porous bench with a minimum radius of 1.5 is the stiffest structure, and also the most reliable structure in case damage occurs. However, the bench suitability not only depends on the stiffness. Other factors such as manufacturability, stability, and buckling also play a role.

Let's consider manufacturability first. Every 3D printer has a limited extrusion width. If the radius becomes smaller than the minimum extrusion width achievable, the part cannot be manufactured. If the bench is printed with big-area additive manufacturing, then a too small radius might not be achievable due to the manufacturing limitations [4]. On the other hand, if it is printed with a printer that can support a small extrusion width, its manufacturing time will increase significantly, which makes it less attractive for commercial production at a large scale. Dependent on what extrusion widths are available, the two-scale topology optimization bench can be re-generated with the provided code by changing the minimum radius.

Also the overhang needs to be considered. Most 3D printers are able to print an overhang up till approximately 45 without support material. The porous micro-structure, as well as the bench bottom, show an overhang of more than this threshold and therefore require support material. While this support material can be removed at the bottom of the bench, it cannot be removed from the inside if the bench is printed as a closed structure [2]. The current implementation does not account for overhang constraints yet, this is suggested as a topic for further research . Next, stability was considered. [5] showed that the pore shape strongly affects structural stability, where instabilities are triggered by wavelengths that are of the order of the size of the micro-structure. However, only a limited number of pore shapes lead to periodic structures for which microscopic instabilities are critical. It would be recommended to prototype the bench and to test if it suffers from these types of instabilities. If this is the case, the minimum radius shall slightly be increased or decreased to mitigate these issues.

The next factor considered is buckling, which occurs when the external load exceeds the critical buckling load. [6] concluded that the X- unit cell distribution has a higher critical buckling load that the O-type, or circular distribution. Therefore, if the critical buckling load is exceeded, it is recommended to change the unit cell type from an 'O' to an 'X'.

4 Conclusion

This paper used two-scale topology optimization to minimize the compliance of a public bench. The bench was modelled as a structure with five different point loads and two supports, in addition a circular unit cell was used for the two-scale problem. Different porosities were evaluated on their compliance, both in the undamaged and two damaged states. It turned out that the higher the porosity, the higher the compliance and robustness in case of damage, where a lower compliance increment was observed. Therefore, the bench with a minimum radius of 1.5 showed the highest performance. Its feasibility is dependent on manufacturing constraints, in addition its stability and buckling response still need to be evaluated. From these results, it is to be expected that two-scale topology optimization will be applied more in engineering design problems in the future to design porous structures with a high performance.

References

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