

Университет ИТМО

Факультет программной инженерии и компьютерной техники

Лабораторная работа №2
по «Алгоритмам и структурам данных»
Сортировка

Выполнил:

Студент группы Р3233

Нгуен Нгок Дык

Преподаватели:

Косяков М.С.

Санкт-Петербург

2022

Code: https://github.com/ndwannafly/ITMO_ALGO

Задача № Е. Коровы в стойла

Task reformulation:

- Given array n elements and a number k
- Array is sorted by ascending order.
- Find maximum value M so that we can choose k elements and minimum distance between any of them $\leq M$

Constraint:

- $N \leq 200\,000$
- $1 < K < N$
- $a[i] \leq 10^9$

Keyword: binary search, greedy

My comment: Let's make it complicated :) .

- Choose any distance M , we make edges between elements that have distance $\leq M$.
- We have a graph and our mission is find the "Maximum Independent Set".
- It's a NP-complete problem))) It can't be solved in linear time.
- So here we go to a greedy approach that the task can be solved in linear time.
- Anyway, our task is about finding **the maximum of the minimum**, we should come up with a **binary search** approach.

Solution:

- Binary search for the result of task (the maximum of the minimums), we call it as M
- A greedy observation: Always choose the **first** element is optimal (easy prove).
- Iterate through the array, start with the first element.
- Pick the next element that have distance to the current element $\geq M$.
- If we can pick $\geq k$ element then M is ok, otherwise not.

Complexity:

- Operations: $O(n \log n)$.
- Space: $O(n)$

Задача № F. Число

Task reformulation:

- There are strings of number.
- Combine them in order to obtain the maximum number .

Constraint:

- Number of strings ≤ 100 .
- Each string has ≤ 100 digits

My comment: Nothing special. Just sort it and combine!!!

Keyword: sort, custom sort function.

Solution:

- Sort all the strings.
- Sort function applies this custom $<$ operation for two string X and Y:
 - $X < Y$ if $X + Y > Y + X$
- Combine all them together and we get the optimal number.

Note: I get WA because of return true in case $X + Y == Y + X$. The compare function must return false

Complexity:

- N = number of strings, L = maximum length of strings.
- Operations: $O(N * L * \log(n))$
- Spaces: $O(N * L)$

Задача № G. Кошмар в замке

Task reformulation:

- We have a string with lowercase latin letters.
- Each letter i has its own weight : $w[i]$
- Weight of a string = $\max(\text{maximum distance of any two letters } i * w[i])$
- Arrange the letters by some order so that the weight of the string is maximum.

Constraint:

- $1 \leq |s| \leq 10^5$
- $0 \leq w[i] \leq 2^{31} - 1$

My comment: Эта задача фиговая!!

Keyword: Sort, greedy.

Solution:

- Sort letters by their weight
- Find letter that satisfies these criteria:
 - Its occurrence ≥ 2
 - Its weight is as much as possible.
- Put one at the beginning and one at the last
- Arbitrarily put the others.

Complexity:

- **Operations** : $O(|s| * \log(|s|))$
- **Spaces** : $O(|s|)$

Задача № Н. Магазин

Task reformulation:

- We have an array and a number k
- Divide array into several of segments
- Each segment has weight = sum of size/ k smallest numbers in it.
- Maximize sum of these weights

Constraint:

- $1 \leq n \leq 1\,00\,000$
- $2 \leq k \leq 100$
- $1 \leq a_i \leq 10\,000$

Keyword: Sort, Greedy, Dynamic Programming.

Subtask dp n^2 :

- Sort the array in ascending order.
- Build prefix sum array s .
- DP formula:
 - $Dp[i] = \max(dp[i], dp[j - 1] + s[i] - s[i - free]);$
 - $Free = (i - j + 1) / k$

Complexity:

- **Operation:** $O(n^2)$
- **Spacing:** $O(n^2)$

Subtask greedy $n \log n$:

- Sort the array in ascending order
- Divide them into blocks of size k .
- The minimum element of each block is free, calculate sum of them.
- $Result = sum[n] - sum\ of\ free$

Complexity:

- **Operation:** $O(n \log n)$
- **Spacing:** $O(n)$