Inferences regarding fixed & random effects

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Fixed vs Random effects

- **Fixed Effects:**
  - Interested in the effects of the factors uniquely in our study
  - If we repeated the experiment, the levels would be the same
  - The inferences you make will only be applied to each of the factors included in your ANOVA

- **Random Effects:**
  - Levels of the factors are treated as if they are randomly sampled from a broader population
  - Estimates the variance among treatments (And assumes there are unmeasured groups)
  - The inferences you make will be applied to a wider set of the population

Fixed or **Random**?

- Surveying students at CSU to to understand CSU student opinions

- Collecting wait time data from separate medical centers to estimate how long on average people wait in medical centers

- Studying fertilizers with the intent of making statements about fertilizers in general

Fixed or **Random**?

- **Surveying students at CSU to to understand CSU student opinions**
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Source	df	SS	MS
A	2	200	100
B	3	160	53.33
C	2	400	200
AB	6	60	10
AC	4	80	20
BC	6	70	15
ABC	12	30	2.5
Error	685	420	2

How many cells?

Source	df	SS	MS
A	2	200	100
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How many cells?

- **A: $2_{DF} + 1 = 3$ groups**
- **B: $3_{DF} + 1 = 4$ groups**
- **C: $2_{DF} + 1 = 3$ groups**

3*4*3 = 36 total cells

Source	df	SS	MS
A	2	200	100
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Create EMS tables: A,B,C are all fixed

	σ^2_A	σ^2_B	σ^2_C	σ^2_{AB}	σ^2_{AC}	σ^2_{BC}	σ^2_{ABC}	σ^2_{Error}
EMS_A								
EMS_B								
EMS_C								
EMS_{AB}								
EMS_{AC}								
EMS_{BC}								
EMS_{ABC}								
EMS_{Error}								

Create EMS tables: A,B,C are all fixed

	σ^2_A	σ^2_B	σ^2_C	σ^2_{AB}	σ^2_{AC}	σ^2_{BC}	σ^2_{ABC}	σ^2_{Error}
EMS_A	Yes	No	No	No	No	No	No	Yes
EMS_B	No	Yes	No	No	No	No	No	Yes
EMS_C	No	No	Yes	No	No	No	No	Yes
EMS_{AB}	No	No	No	Yes	No	No	No	Yes
EMS_{AC}	No	No	No	No	Yes	No	No	Yes
EMS_{BC}	No	No	No	No	No	Yes	No	Yes
EMS_{ABC}	No	No	No	No	No	No	Yes	Yes
EMS_{Error}	No	No	No	No	No	No	No	Yes

Identify Equations: A,B,C are all fixed

	σ^2_A	σ^2_B	σ^2_C	σ^2_{AB}	σ^2_{AC}	σ^2_{BC}	σ^2_{ABC}	σ^2_{Error}
EMS_A	$\sigma 2$ A							$\sigma 2$ error
EMS_B		$\sigma 2$ B						$\sigma 2$ error
EMS_C			$\sigma 2$ C					$\sigma 2$ error
EMS_{AB}				$\sigma 2$ AB				$\sigma 2$ error
EMS_{AC}					$\sigma 2$ AC			$\sigma 2$ error
EMS_{BC}						$\sigma 2$ BC		$\sigma 2$ error
EMS_{ABC}							$\sigma 2$ ABC	$\sigma 2$ error
EMS_{Error}								$\sigma 2$ error

Identify variance components for each effect

Write the equations that include the components that go into the EMS for each effect.

$EMS_A = nqr\ \sigma^2A + \sigma^2error$

$EMS_B = npr\ \sigma^2B + \sigma^2error$

Each effect is weighted by the number of observations for each effect

- n = number of subjects/cell
- p = levels of A
- q = levels of B
- r = levels of C

Hint: weight each effect by the variables that are ***NOT*** included in that effect!

Solve for the σ^2_{error} for each effect

1) The best estimate for the σ^2_{error} is the mean square residual for the error
   - Get this value from your ANOVA table

2) The MS for each effect represents the EMS for that effect. You can then solve for the σ^2 of each effect.

-e.g., EMS for A is estimated by 100

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F-Values

You can then use the following formulas to solve for the F and Quasi F ratios for each effect.

E.g.,

$$\frac{MS_A + MSABC}{MS_{AB} + MSAC} = \frac{2\sigma^2_{error} + 2n\sigma^2_{ABC} + nr\sigma^2_{AB} + nq\sigma^2_{AC} + nqr\sigma^2_{A}}{2\sigma^2_{error} + 2n\sigma^2_{ABC} + nr\sigma^2_{AB} + nq\sigma^2_{AC}}$$

Fixed vs. Random Effects

Remember from the lecture, the components that go into each EMS effect are different when the model is fully fixed, fully random, or mixed (i.e., has both fixed and random effects)!