Orthogonal Contrasts

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A Quick Review of Planned Contrasts

- Contrasts test a *specific* hypothesis related to group means based upon some prior information about the groups.
- Contrasts assign a weight to each of the groups in your predictor variable
    - Weights should add up to zero
    - Assigning a weight of zero means that group will not be included in the contrast
    - Order of the weights corresponds to order of groups in predictor variable
    - Contrasts are one-tailed (we are testing specific hypotheses)

An example of a planned contrast set-up:

	Control	Drug A	Drug B
Contrast 1	-1	1/2	1/2
Contrast 2	0	-1	1

- Contrast 1 tests whether or not the control group differs from the groups that received a drug treatment
- Contrast 2 tests whether or not the two drugs differ in their effect.

Orthogonal Contrasts

A set of contrasts is orthogonal if:

- The number of contrasts = df (number of groups -1)
- You have at least 3 groups to compare
- Two contrasts are *orthogonal* if the pairwise products of the corresponding coefficients for each term sum to zero:

	Control	Drug A	Drug B
Contrast 1	-1	1/2	1/2
Contrast 2	0	-1	1

These contrasts are orthogonal because: (-1 *0) + (1/2 * -1) + (1/2 * 1) = 0

Relevant formulas

We can use orthogonal contrasts to get a lot of information about a dataset, even if we don't have a full ANOVA table!

Sums of Squares for each contrast:

n = n/cell

cj is a set of contrast weights

Ŷj is the mean of the DV in group j

$$SS_{contrast} = \frac{n(\Sigma c\bar{Y}_j)^2}{\Sigma c^2_j}$$

Sum of Squares Treatments:

$$SS_{treatments} = SS_{contrast1} + SS_{contrast2} + \ldots SS_{contrastk}$$

Sum of Squares Total:

$$SS_{total} = SS_{contrast1} + SS_{contrast2} + \ldots SS_{contrastk} + SS_{error}$$

Breaking down the example from Module 2 lecture

	Mean	Contrast 1	Contrast 2
Group 1	35	-1	1
Group 2	40	-1	-1
Group 3	45	2	0

Note: We could use different contrast weights, depending on our specific research question and a priori information.

N for this study was 90 (i.e., 30 subjects/cell in this design)

$SS_{total} = 5000$

Breaking down the example from Module 2 lecture

	Mean	Contrast 1	Contrast 2
Group 1	35	-1	1
Group 2	40	-1	-1
Group 3	45	2	0

Contrast 1 tests whether the mean of Y for treatment group 3 was significantly different than the means of the other two groups.

Contrast 2 tests the hypothesis that the mean of group 1 was significantly different than the mean of group 2.

Breaking down the example from Module 2 lecture

	Mean	Contrast 1	Contrast 2
Group 1	35	-1	1
Group 2	40	-1	-1
Group 3	45	2	0

In this demo we walk through how to test the hypothesis that the mean of group 3 was significantly different from the mean of participants in groups 1 and 2 (i.e., testing the significance of Contrast 1)

We encourage you to go through these steps for Contrast 2 on your own for practice :)

1) Calculate Sums of Squares for the contrast

	Mean	Contrast 1
Group 1	35	-1
Group 2	40	-1
Group 3	45	2

$$SS_{contrast} = \frac{n(\Sigma c\bar{Y}_j)^2}{\Sigma c_j^2}$$

N/cell = 30

$SS_{contrast1}$ = [30 * ((35 * -1) + (40 * -1) + (45 * 2))^2] / (-1)^2 + (-1)^2 + (2)^2

$SS_{contrast1}$ = [30 * (15)^2]/6

$SS_{contrast1}$ = [30 * 225]/6

$SS_{contrast1}$ = 6750/6 = **1125**

2) Calculate eta-squared for the contrast

> *Note: if you are not given SS_{total} you can calculate this from the SD and mean of Y!*

$$\mathbf{Eta^2_{contrast} = SS_{contrast}/SS_{total}}$$

$\mathbf{SS_{total} = SS_{contrast1} + SS_{contrast2} + SS_{error} = 5000}$ (this was given to us)

$\mathbf{Eta^2_{contrast1}} = 1125/5000 = \mathbf{0.225}$

22.5% variance in Y explained by this contrast

2) Calculate eta-squared for contrast

$$\mathbf{Eta^2_{contrast} = SS_{contrast}/SS_{total}}$$

1125 is the $SS_{contrast}$ from the previous slide

5000 is the SS_{total} that was given to us beforehand

$$\mathbf{SS_{total} = SS_{contrast1} + SS_{contrast2} + SS_{erro}}$$

$$\mathbf{Eta^2_{contrast1}} = \boxed{1125}/\boxed{5000} = \mathbf{0.225}$$

22.5% variance in Y explained by this contrast

3) Calculate the F statistic for the contrast

To calculate the F statistic for the contrast:

$$F_{(dfhyp,\ dferr)} = SS_{contrast}/MS_{error}$$

To use this formula, you need to first calculate the MS_{error}, which you can get from the SS_{error}:

$$SS_{error} = SS_{total} - SS_{contrast1} - SS_{contrast2} \longrightarrow MS_{error} = SS_{error}/(n - k)$$

First: $MS_{error} = (5000 - 1125 - 375) / (90 - 3) = 3500/87 = 40.229$

Second: $F_{contrast1(2,87)} = SS_{contrast1}/40.229 = 1125/40.229 = \mathbf{27.96}$

Note: $SS_{contrast2}$ = 375. Calculate this on your own for practice or see the activity answer key.

4) Determine the critical value of F

Use a critical F value table to determine the value your contrast's F statistic needs to exceed in order to be considered significant

- Field, Miles, & Field (2012) Table A.3 pp. 936-939

- df numerator = k -1
- df denominator = a proxy for the df error (n-k)
- Round up to the next highest df denominator if your value isn't listed

A.3 Critical values of the F-distribution

df (Denominator)	p	df (Numerator) 1	2	3	4	5	6	7	8	9	10
1	0.05	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
	0.01	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85
2	0.05	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
	0.01	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
3	0.05	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
	0.01	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	0.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
	0.01	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	0.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
	0.01	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	0.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
	0.01	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	0.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
	0.01	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	0.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
	0.01	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	0.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
	0.01	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	0.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
	0.01	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	0.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
	0.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	0.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
	0.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30

4) Determine the critical value of F

df (Denominator)	p	1	2	3	4	5	6	7	8	9	10
		df (Numerator)									
19	0.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
	0.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	0.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
	0.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
22	0.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
	0.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
24	0.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
	0.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
26	0.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
	0.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
28	0.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
	0.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
30	0.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
	0.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
35	0.05	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11
	0.01	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88
40	0.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
	0.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
45	0.05	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05
	0.01	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74
50	0.05	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
	0.01	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70

Df (Denominator)	p	1	2	3	4	5	6	7	8	9	10
				df (Numerator)							
60	0.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
	0.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
80	0.05	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95
	0.01	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55
100	0.05	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93
	0.01	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50
150	0.05	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89
	0.01	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44
300	0.05	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86
	0.01	6.72	4.68	3.85	3.38	3.08	2.86	2.70	2.57	2.47	2.38
500	0.05	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85
	0.01	6.69	4.65	3.82	3.36	3.05	2.84	2.68	2.55	2.44	2.36
1000	0.05	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84
	0.01	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34

df numerator = 2 (3 groups - 1 = 2)

df denominator = 87 (90 participants - 3 groups), round up to 100

The critical value of F for this ANOVA is **4.82** for an alpha of <0.01

4) Does the F statistic for the contrast exceed the critical value of F?

If yes, we can reject the null hypothesis and interpret the contrast as statistically significant.

$\mathbf{F_{contrast1(2,87)} = 27.96}$

Since 27.96 > 4.82, this contrast is significant, indicating that the mean value of Y for participants in group 3 was significantly different than participants in groups 1 and 2.