Introduction to Multilevel Modeling

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PSY 653 Module 12 Lab

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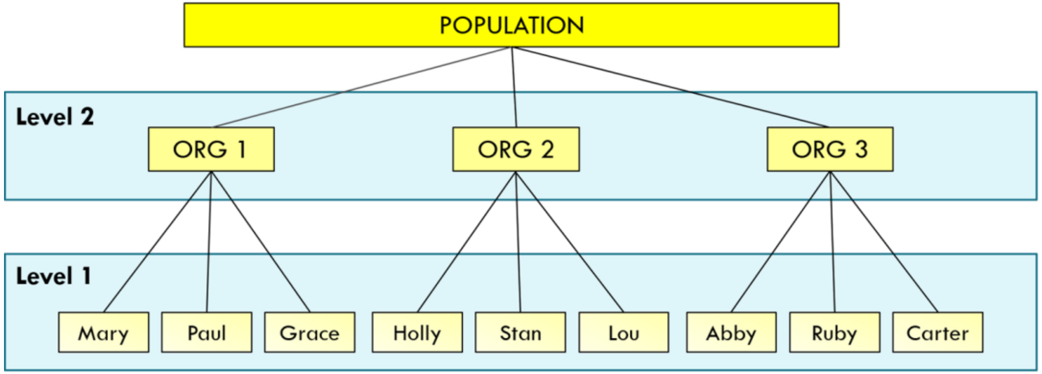
A quick note:

Many of the materials in this demo were created by Dr. Kim Henry.
We gratefully acknowledge use of her materials!

Multilevel modeling

A multilevel model (also commonly called a random effects model, a mixed effects model, and a hierarchical linear model) is used to model hierarchical data. In data that arise from a hierarchical design, the upper level units (e.g., organizations) are selected from the population. Then, cases (e.g., employees) are selected from within these upper level units. In this way, employees (Level 1 of the hierarchy) are nested in organizations (Level 2 of the hierarchy).



Load Libraries

```r
# Load Libraries
```{r}
library(tidyverse)
library(lme4)
library(psych)
library(MuMIn)

```
```

We'll use the lme4 package to conduct multilevel models

We'll use the MiMIn package to calculate model pseudo R^2 values

Read in data

```{r}
teams <- read_csv("mlm_teams.csv")
```

```
Parsed with column specification:
cols(
  kid_id = col_double(),
  team_id = col_double(),
  txcond = col_double(),
  risk = col_double(),
  score = col_double(),
  comafrd = col_double()
)
```

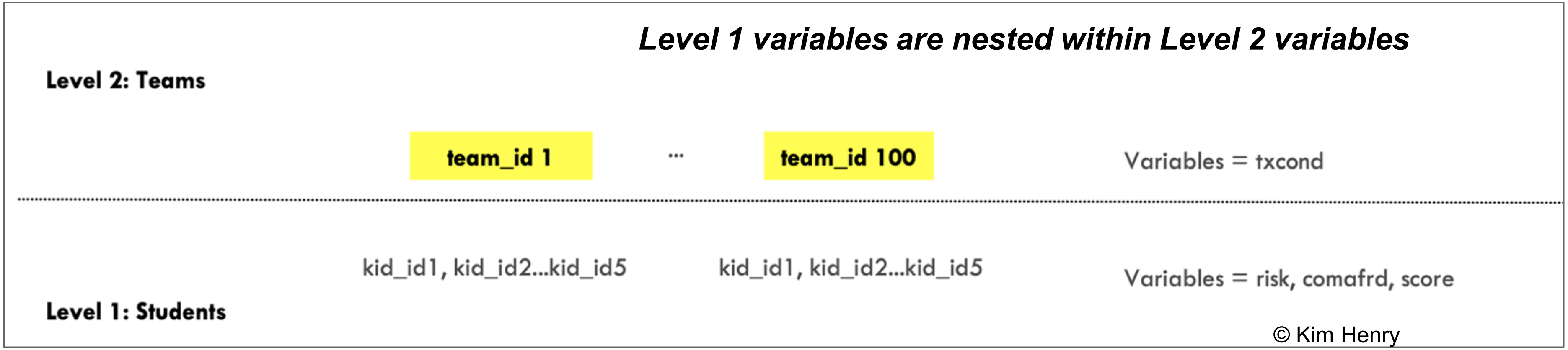
This dataset includes 500 female high school students who participated in a summer science program.

Students' "risk index" for likelihood of success in the program was calculated from previous academic data.

Students were randomly assigned to teams of 5, and each team was randomly assigned to different version of the summer program:

- Control condition: individually-focused work
- Treatment condition: teams-based work

Structure of the data



Level 1 variables = individual level = each student's id number (kid_id), risk index (risk), belief in communal affordances of STEM career (comafrd), and final exam score (score)

Level 2 variables = upper/group level = each student's team number (team_id), the treatment condition the team was assigned to (txcond)

Before running analyses, we need to factor our categorical variables

````
## Factor team_id
```{r}
teams <- mutate(teams, team_id.f = factor(team_id))
```
````

Get Mean of Means

```
# Get Mean of Means
```{r}

team_means  <- group_by(teams, team_id)
team_means  <- summarize(team_means, mean_score = mean(score))
meanofmeans <- summarize(team_means, meanofmeans = mean(mean_score))
meanofmeans

```
```

meanofmeans <dbl>
73.216

1 row

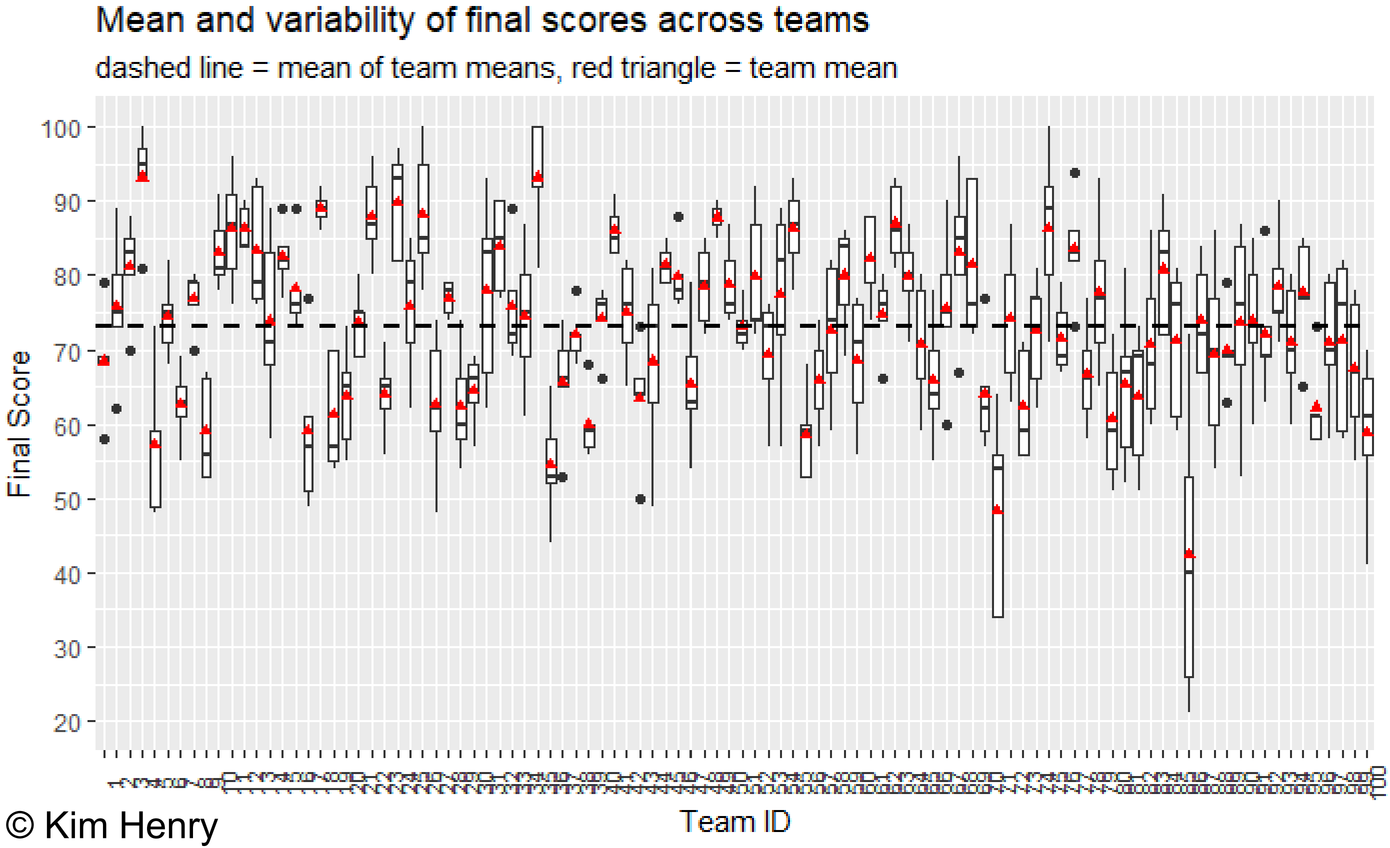
We saved this value as the object “meanofmeans”, which we’ll use in upcoming figures, and we’ll see again in some model output

Visualize how final scores varied across the teams

```
ggplot(data = teams, aes(x = team_id.f, y = score)) +
  geom_boxplot() +
  stat_summary(aes(y = score, group = team_id.f), fun = mean, color = "red",
geom = "point", pch = 17, size = 1.5) +
  geom_hline(yintercept = meanofmeans$meanofmeans, linetype="dashed", color
= "black", size = 1) +
  scale_y_continuous(limits = c(20,100), breaks = seq(20, 100, 10)) +
  labs(title = "Mean and variability of final scores across teams",
       subtitle = "dashed line = mean of team means, red triangle = team mean",
       x = "Team ID", y = "Final Score") +
  theme(axis.text.x = element_text(colour="grey20", size=8, angle=90, hjust=.5))
```

(Note: this plot is optional for the try-it-yourself activity, but recommended for extra ggplot practice!)

Visualize how final scores varied across the teams



Notice all of the variability between Teams.

The lmer() function

Similar to a lm, the dependent variable is listed, then a tilde. Since, this is an unconditional model, there are no predictors, but we include a 1 to denote the intercept. This is called the fixed effects part of the model and will provide us with the mean means across the groups.

lmer is the function used to specify a of multilevel model (it stands for linear mixed effects regression).

```
mod1 = lmer(score ~ 1 + (1|team_id), REML = FALSE, data = teams)
summary(mod1)
```

After the fixed effects, we provide the random effects. Here we list

the effects that we want to denote as random. In this case it is just the intercept (1), which will capture the between group variability. The bar (|) and then team_id denotes the Level 2 grouping variable.

- Kim Henry

Defining **fixed** and **random** effects in the lmer package

These definitions are a little different in multilevel analyses than in ANOVAs.

Fixed effect = does not vary over subjects of groups – average value of slope or intercept (i.e., *what is the estimate of the effect across all of the groups?*)

Random effect = might vary across subjects or groups – intercepts and slopes might be calculated for each group or each subject to see if they vary meaningfully (i.e., *how much does the estimate for the effect vary across the groups?*)

Useful effect sizes in multilevel modeling

Intraclass correlation (ICC): the proportion of variance in a Level 1 variable (i.e., individual-level variable) that is accounted for by a Level 2 variable (i.e, between-group differences)

- Another way to interpret ICC: the average correlation of a Level 1 variable between two individuals in the *same Level 2 group*.
- ICC > 0.2 generally indicates a meaningful Level 2 effect, ICC > 0.05 is worth further investigation
- The ICC is calculated by dividing the random effect variance by the total variance (i.e. the sum of the random effect variance and the residual variance)

Useful effect sizes in multilevel modeling

Pseudo R^2 values: will not be comparable to OLS R^2 values on same data

- Marginal R^2 = amount of variance explained in Y by fixed effects only
- Conditional R^2 = amount of variance explained in Y by fixed and random effects

Model 1: Random intercept only

```{r}
# Random intercept
randint <- lmer(score ~ 1 + (1|team_id.f), data = teams, REML = FALSE)
summary(randint)
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: score ~ 1 + (1 | team_id.f)
   Data: teams

     AIC      BIC   logLik deviance df.resid
  3749.0   3761.6  -1871.5   3743.0      497

Scaled residuals:
     Min       1Q   Median       3Q      Max
-3.08009 -0.59814 -0.01048  0.70383  2.95197

Random effects:
 Groups    Name        Variance Std.Dev.
 team_id.f (Intercept) 80.62    8.979
 Residual              71.48    8.455
Number of obs: 500, groups:  team_id.f, 100

Fixed effects:
            Estimate Std. Error t value
(Intercept)  73.2160     0.9743   75.15
```

Mean of Means

Model 1: Random intercept only

```{r}
# Random intercept
randint <- lmer(score ~ 1 + (1|team_id.f), data = teams, REML = FALSE)
summary(randint)
```

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Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: score ~ 1 + (1 | team_id.f)
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Fixed effects:
            Estimate Std. Error t value
(Intercept)  73.2160     0.9743   75.15
```

- **Random intercept: On average, the mean of each team varies from the grand mean by 8.979 standard deviations**

- ***Fixed Intercept: In the absence of any fixed effects*, this intercept represents the "mean of means" of our outcome variable.**

Model 1: Random intercept only

Calculate ICC & Pseudo R^2

$$ICC = \frac{\text{var}(u_{oj})}{\text{var}(u_{0j}) + \text{var}(r_{ij})}$$

ICC = 80.62 / (80.62 + 71.48)

ICC = ***.53***

53% of the variance in final test scores can be attributed to differences between teams.

```{r}
r.squaredGLMM(randint)
```

```
     R2m       R2c
[1,]   0 0.5300362
```

R2m (Marginal R^2): NA, no fixed effects in this model

R2c (Conditional R^2): 53% of the variance in scores can be explained by the model’s fixed and random effects

Model 2: Random Intercept, Fixed Slope

```
rifs<-lmer(score ~ 1 + risk + (1|team_id.f),data=teams, REML = FALSE)
summary(rifs)
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: score ~ 1 + risk + (1 | team_id.f)
   Data: teams

     AIC      BIC   logLik deviance df.resid
  3667.1   3683.9  -1829.5   3659.1      496

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.2366 -0.5766 -0.0013  0.6421  2.7827

Random effects:
 Groups    Name        Variance Std.Dev.
 team_id.f (Intercept) 83.33    9.128
 Residual              57.96    7.613
Number of obs: 500, groups:  team_id.f, 100

Fixed effects:
            Estimate Std. Error t value
(Intercept)  77.8680     1.0867  71.653
risk         -2.3260     0.2407  -9.662

Correlation of Fixed Effects:
     (Intr)
risk -0.443
```

Here, we added risk as a fixed level 1 predictor

- **Random Intercept:** On average, team intercepts vary by 9.128 standard deviations
- **Fixed Intercept:** The average intercept, while incorporating risk, is 77.868
- **Fixed Slope:** On average, team scores decreased at a rate of 2.326 units for a 1-unit increase in risk

Plot: Random Intercept, Fixed Slope

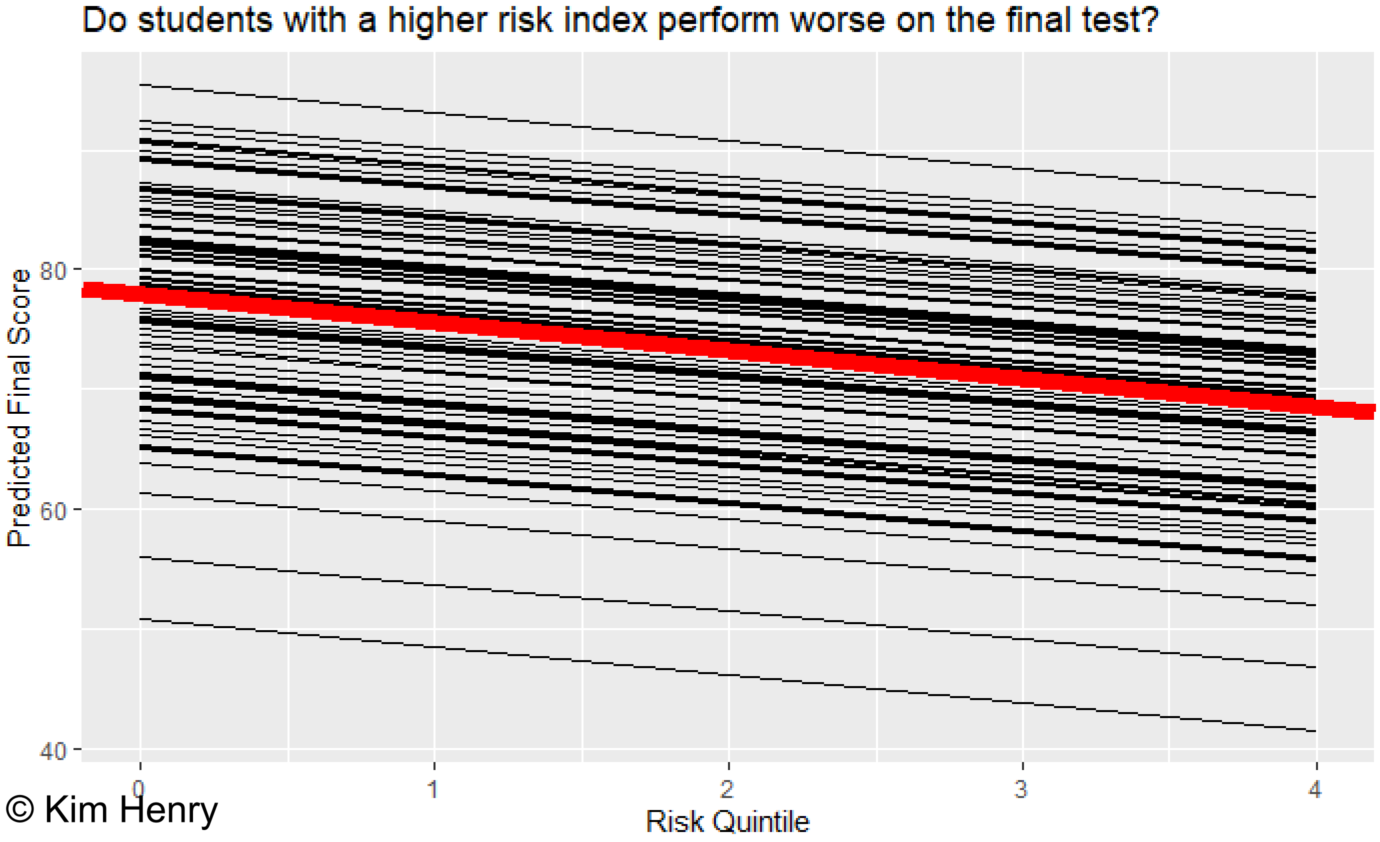
```
library(modelr)

# Model without random slope
mod2_b.plot <- add_predictions(data = teams, model = rifs)

ggplot(data = mod2_b.plot, aes(x = risk, y = pred, group = team_id.f)) +
  geom_line() +
  geom_abline(intercept = 77.868, slope = -2.326, color="red", size=3) +
  labs(title = "Do students with a higher risk index perform worse on the final test?",
x = "Risk Quintile", y =
        "Predicted Final Score")
```

(Again, these plots are optional for the try-it-yourself activity)

Plot: Random Intercept, Fixed Slope



Notice the slopes are all exactly the same.

Red line = Fixed Effects

Model 2: Random Intercept, Fixed Slope

Calculate ICC & Pseudo R^2

$$ICC = \frac{\mathrm{var}(u_{oj})}{\mathrm{var}(u_{0j}) + \mathrm{var}(r_{ij})}$$

ICC = 83.33 / (83.33 + 57.96)

ICC = ***.59***

59% of the variance in final test scores can be attributed to differences between teams.

```{r}
r.squaredGLMM(rifs)
```

```
            R2m       R2c
[1,] 0.07127094 0.6190137
```

R2m (Marginal R^2): 7% of the variance in scores can be explained by the model's fixed effects

R2c (Conditional R^2): 62% of the variance in scores can be explained by the model's fixed and random effects

Model 3: Random Slope, Fixed Intercept

```{r}
# random slopes fixed intercepts
rsfi<- lmer(score ~ 1 + risk + (0 + risk|team_id.f),data=teams, REML = FALSE)
summary(rsfi)
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: score ~ 1 + risk + (0 + risk | team_id.f)
   Data: teams

     AIC      BIC   logLik deviance df.resid
  3747.5   3764.3  -1869.7   3739.5      496

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.3701 -0.5836  0.0391  0.5992  2.5838

Random effects:
 Groups    Name Variance Std.Dev.
 team_id.f risk 11.32    3.364
 Residual       73.37    8.566
Number of obs: 500, groups:  team_id.f, 100

Fixed effects:
            Estimate Std. Error t value
(Intercept)  77.8680     0.6635 117.359
risk         -2.3260     0.4319  -5.385

Correlation of Fixed Effects:
     (Intr)
risk -0.512
```

Model 3: Random Slope, Fixed Intercept

```
```{r}
# random slopes fixed intercepts
rsfi<- lmer(score ~ 1 + risk + (0 + risk|team_id.f),data=teams, REML = FALSE)
summary(rsfi)
```
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: score ~ 1 + risk + (0 + risk | team_id.f)
   Data: teams

     AIC      BIC   logLik deviance df.resid
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Fixed effects:
            Estimate Std. Error t value
(Intercept)  77.8680     0.6635 117.359
risk         -2.3260     0.4319  -5.385

Correlation of Fixed Effects:
     (Intr)
risk -0.512
```

Here, we plug in a 0 to our random effects to indicate the intercept is fixed.

- **Random Slope:** On average, team slopes vary by 3.364 standard deviations
- **Fixed Intercept:** The average intercept, while incorporating risk, is 77.868
- **Fixed Slope:** On average, team scores decreased at a rate of 2.326 units for a 1-unit increase in risk

Plot: Fixed Intercept, Random slope

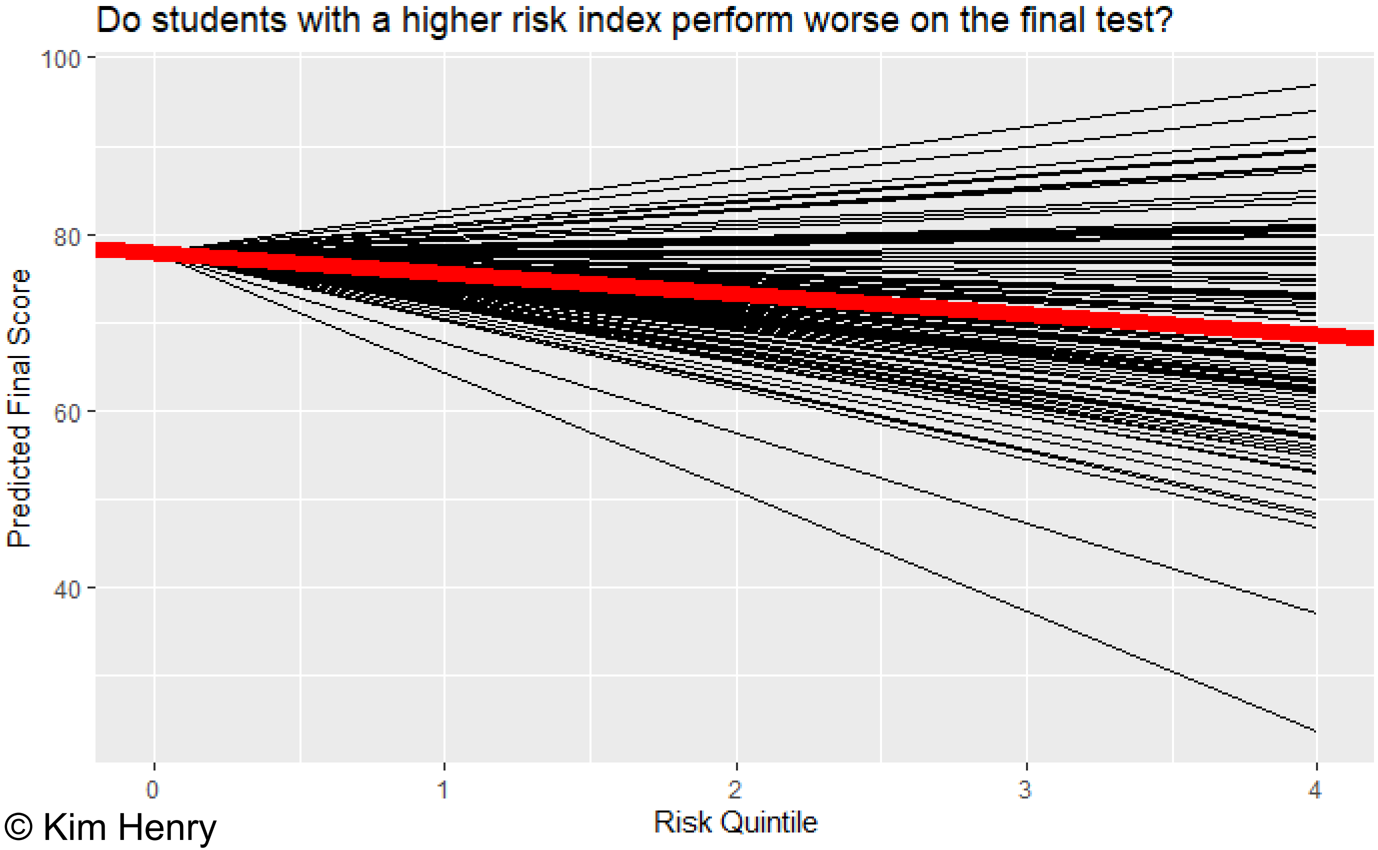
```
library(modelr)

# Model without random intercept
mod2_b.plot <- add_predictions(data = teams, model = rsfi)

ggplot(data = mod2_b.plot, aes(x = risk, y = pred, group = team_id.f)) +
  geom_line() +
  geom_abline(intercept = 77.868, slope = -2.326, color="red", size=3) +
  labs(title = "Do students with a higher risk index perform worse on the final test?",
x = "Risk Quintile", y =
        "Predicted Final Score")
```

(Again, these plots are optional for the try-it-yourself activity)

Plot: Fixed Intercept, Random slope



Notice all slopes come from the same intercept

Red line = Fixed Effects

Model 3: Random Slope, Fixed Intercept

Calculate ICC & Pseudo R^2

$$ICC = \frac{\mathrm{var}(u_{oj})}{\mathrm{var}(u_{0j}) + \mathrm{var}(r_{ij})}$$

ICC = 11.32 / (11.32 + 73.37)

ICC = ***.13***

13% of the variance in final test scores can be attributed to differences between teams.

```
```{r}
r.squaredGLMM(rsfi)
```
```

```
           R2m       R2c
[1,] 0.07127094 0.5176855
```

R2m (Marginal R^2): 7% of the variance in scores can be explained by the model's fixed effects

R2c (Conditional R^2): 52% of the variance in scores can be explained by the model's fixed and random effects

Model 4: All Random

```{r}
# all random
allrand<- lmer(score ~ risk +  (1 + risk|team_id.f),data=teams, REML = FALSE)
summary(allrand)
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: score ~ risk + (1 + risk | team_id.f)
   Data: teams

     AIC      BIC   logLik deviance df.resid
  3618.7   3644.0  -1803.4   3606.7      494

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.52404 -0.55663  0.02976  0.58267  2.14656

Random effects:
 Groups    Name        Variance Std.Dev. Corr
 team_id.f (Intercept) 79.603   8.922
           risk         7.101   2.665    -0.22
 Residual              40.206   6.341
Number of obs: 500, groups:  team_id.f, 100

Fixed effects:
            Estimate Std. Error t value
(Intercept)  77.8680     1.0185  76.456
risk         -2.3260     0.3335  -6.975

Correlation of Fixed Effects:
     (Intr)
risk -0.392
```

Model 4: All Random

```
```{r}
# all random
allrand<- lmer(score ~ risk +  (1 + risk|team_id.f),data=teams, REML = FALSE)
summary(allrand)
```
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: score ~ risk + (1 + risk | team_id.f)
   Data: teams

     AIC      BIC   logLik deviance df.resid
  3618.7   3644.0  -1803.4   3606.7      494

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.52404 -0.55663  0.02976  0.58267  2.14656

Random effects:
 Groups    Name        Variance Std.Dev. Corr
 team_id.f (Intercept) 79.603   8.922
           risk         7.101   2.665    -0.22
 Residual              40.206   6.341
Number of obs: 500, groups:  team_id.f, 100

Fixed effects:
            Estimate Std. Error t value
(Intercept)  77.8680     1.0185  76.456
risk         -2.3260     0.3335  -6.975

Correlation of Fixed Effects:
     (Intr)
risk -0.392
```

- **Random Intercept:** On average, team intercepts vary by 8.922 standard deviations
- **Random Slope:** On average, team slopes vary by 2.665 standard deviations

- **Fixed Intercept:** The average intercept, while incorporating risk, is 77.868
- **Fixed Slope:** On average, team scores decreased at a rate of 2.326 units for a 1-unit increase in risk

Plot: All random

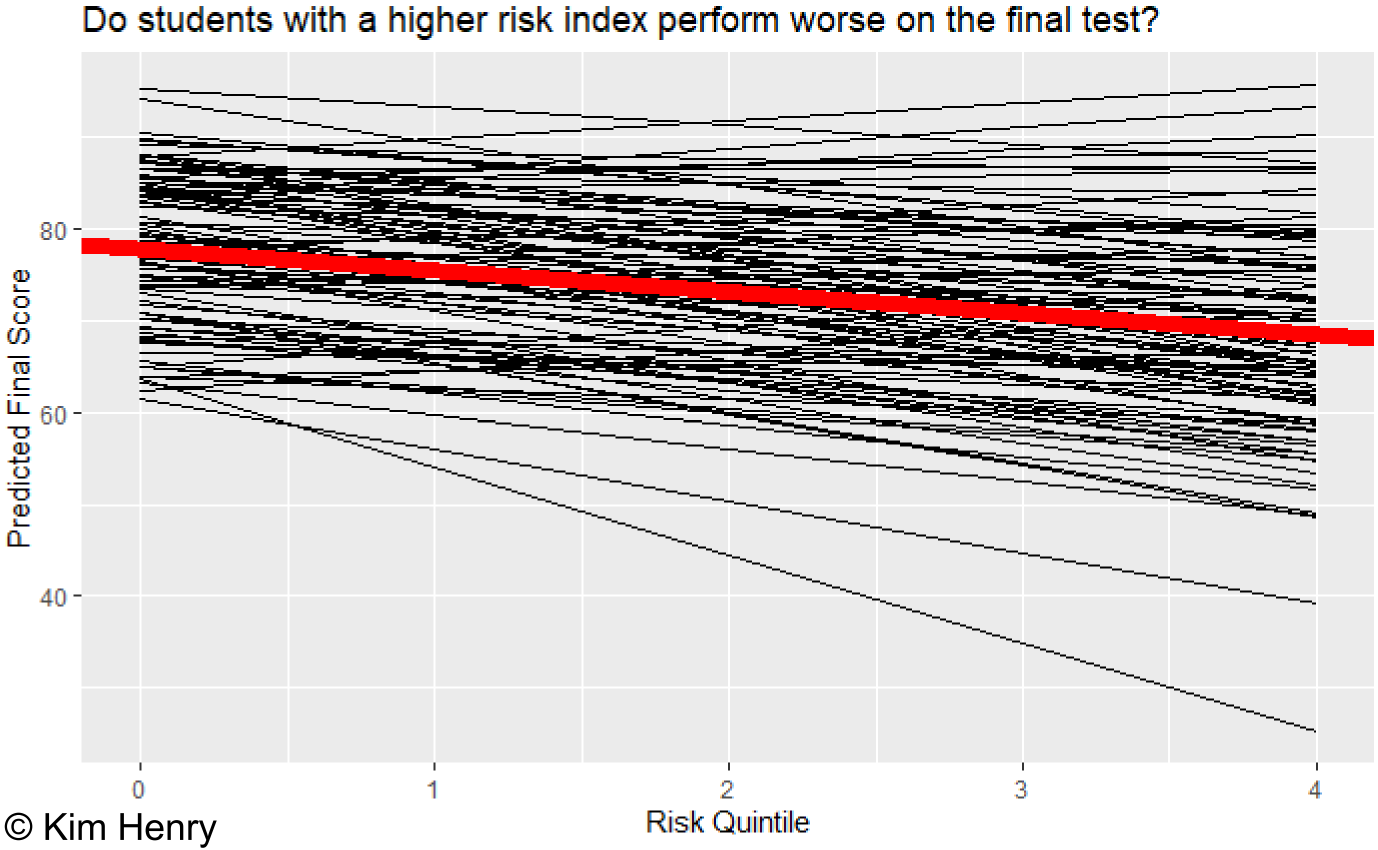
```
library(modelr)

# Model with random slope & Intercept
mod2_a.plot <- add_predictions(data = teams, model = allrand)

ggplot(data = mod2_a.plot, aes(x = risk, y = pred, group = team_id.f)) +
  geom_line() +
  geom_abline(intercept = 77.868, slope = -2.326, color="red", size=3) +
  labs(title = "Do students with a higher risk index perform worse on the final test?",
x = "Risk Quintile", y = "Predicted Final Score")
```

(Again, these plots are optional for the try-it-yourself activity)

Plot: All random



Notice that all slopes and intercepts are different

Red line = Fixed Effects

Model 4: All Random

Calculate ICC & Pseudo R^2

$$ICC = \frac{\mathrm{var}(u_{oj})}{\mathrm{var}(u_{0j}) + \mathrm{var}(r_{ij})}$$

ICC = (79.60 + 7.21) / (79.60 + 7.21 + 40.21)

ICC = ***.68***

68% of the variance in final test scores can be attributed to differences between teams.

```{r}
r.squaredGLMM(allrand)
```

```
           R2m       R2c
[1,] 0.0712698 0.7357103
```

R2m (Marginal R^2): 7% of the variance in scores can be explained by the model's fixed effects

R2c (Conditional R^2): 73% of the variance in scores can be explained by the model's fixed and random effects

Log likelihood test: Random Intercept-Only vs. All Random

Pairwise comparison
```{r}

anova(randint,allrand)

```

```
Data: teams
Models:
rifs: score ~ 1 + risk + (1 | team_id.f)
allrand: score ~ risk + (1 + risk | team_id.f)
        Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
rifs     4 3667.1 3683.9 -1829.5   3659.1
allrand  6 3618.7 3644.0 -1803.3   3606.7 52.371      2  4.243e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The inclusion of the random effects significantly improves model fit

Log likelihood test: Random Intercept Fixed Slope vs. All Random

```
## Pairwise comparison
```{r}
anova(rifs,allrand)
```
```

```
Data: teams
Models:
rifs: score ~ 1 + risk + (1 | team_id.f)
allrand: score ~ risk + (1 + risk | team_id.f)
        Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)    
rifs     4 3667.1 3683.9 -1829.5   3659.1                             
allrand  6 3618.7 3644.0 -1803.3   3606.7 52.371      2  4.243e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The inclusion of the random effects significantly improves model fit

Log likelihood test: Fixed Intercept Random Slope vs. All Random

```{r}
anova(rsfi,allrand)
```

```
Data: teams
Models:
rsfi: score ~ 1 + risk + (0 + risk | team_id.f)
allrand: score ~ risk + (1 + risk | team_id.f)
        Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)    
rsfi     4 3747.5 3764.3 -1869.7   3739.5                             
allrand  6 3618.7 3644.0 -1803.3   3606.7 132.79      2  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The inclusion of the random effects significantly improves model fit