# Matching with Costly Interviews: The Benefits of Asynchronous Offers

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**Abstract**

In many matching markets, matches are formed after costly interviews. We develop a model of worker-firm matching that incorporates this feature, and examine the trade-offs between a centralized matching system and a decentralized one, where matches can occur at any time. Centralized matching with a common offer date leads to coordination issues in the interview stage. Each firm must incorporate the externality imposed by the interview decisions of the firms ranked above it when deciding on its interview list. As a result, low-ranked firms often fail to interview some candidates that ex-ante have high match quality. In a decentralized setting with exploding offers, the set of candidates who receive interviews differs, but the welfare generated is weakly greater than in the centralized setting. Total welfare is highest with a system that ensures firms interview and match in sequence, clearing the market for the next firm. Such asynchronicity reduces interview congestion. This system can be implemented by encouraging top firms to interview and match early and allowing candidates to renege on offers.

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# 1 Introduction

The National Resident Matching Program (NRMP) was established in 1952 to match medical graduates with residency programs across the United States. Each year, the NRMP facilitates the matching of tens of thousands of aspiring doctors with suitable training positions via the Gale-Shapley deferred acceptance algorithm. The success of the NRMP match has led to an expanding interest in the centralization of other matching markets, ranging from the academic job market to college admissions to the market for college athletes.

Much of the theoretical analysis of centralized, two-sided matching processes has focused on settings where agents have complete information about their preferences.[1] However, a fundamental feature of labor markets is that agents are initially uncertain about their preferences and so invest in costly information acquisition — generally in the form of interviews — *prior to* submitting their rank-order lists. This process imposes significant costs on agents. For instance, in the NRMP match, medical school graduates often have to pay interview expenses out of their pocket, and the residency program doctors conducting the interviews cannot do surgeries on interview days. Moreover, due to competition for candidates, hospital interviewing decisions become calculated decisions, and much effort is devoted to thinking about which candidates are "gettable" and worth interviewing (Wapnir et al. (2021)).

We focus on how the design of matching markets affects how firms acquire information through interviews. These choices are not just about *how much* information to acquire, but *how* to acquire it: firms must not only choose how many interview invitations to issue but also which candidates will receive them. Moreover, these decisions are strategic: while a given firm cares only about the quality of the workers it is matched to, the final assignment is determined by the information that it acquires *and* the information acquired by its competitors. Consequently, the welfare produced by a matching mechanism is inextricably linked with its impact on firms' interviewing incentives.

To analyze this link, we develop a novel model of two-sided matching between firms and workers, where firms conduct costly interviews before offers are made. Interview decisions are decentralized: there is no restriction on how interviews are assigned as they are a strategic choice by firms. We show that centralized matching mechanisms where workers are matched to firms via deferred acceptance have a drawback: firms must decide which candidates to interview without knowing whether those candidates' interviews with more preferred firms will convert to offers. As a result, firms fail to interview some candidates that ex-ante have high match quality, leading to high-quality workers "falling through the cracks".

To evaluate the magnitude of this inefficiency, we contrast such a centralized system with two types of decentralized systems:

1. A decentralized system where firms can interview at any time and make binding offers.

[1] The assumption of complete information of preferences is substantive. Fernandez et al. (2022) highlight the fragility of classical theorems to this assumption.

2. A decentralized system where firms can interview at any time and make public, non-binding offers.

The first is representative of the status quo in many markets, including the market for investment banking analysts, corporate law associates, and university professors. The second is a hybrid regime of sorts. Firms are free to interview at any time and make public offers, but the offers are non-binding. Effectively, workers can hold onto offers and need not make a decision until the very end of the hiring cycle. Such a system does exist, for example, in the college athletic scholarship market: high school athletes are permitted to publicly receive scholarship offers at any point after the start of their junior year. However, they cannot officially accept them until the December of their senior year.

In the decentralized regime where firms can interview at any time and make binding offers (e.g. exploding offers), we show that all firms are *ex ante* no worse off in equilibrium than in the centralized setting. However firms can be worse off ex post. In the decentralized regime where offers are public and non-binding, worker welfare is maximized in the equilibrium where firms interview and match sequentially according to their ranking. Total firm surplus is also higher in such a setting than in the equilibrium of the centralized setting. Sequential matching removes the uncertainty about which candidates will receive offers from higher-ranked competitors *before* lower-ranked firms have to make their interview decisions. The asynchronicity of offers removes firms' incentives to skip over higher-quality applicants and leads to more efficient interview decisions and final matches.

## 1.1 Related Literature

There is an extensive literature on two-sided matching in labor markets, particularly on the NRMP (Roth (1984); Roth (2008); Roth and Sotomayor (1992)). In much of the theoretical analysis of such models, there is complete information about preferences and no uncertainty regarding match quality (See Kojima et al. (2020) for a comprehensive survey of this literature).

Illustrative examples of the effect of incomplete information on traditional notions of stability and other classical matching properties are provided by Roth (1989) and Fernandez et al. (2022). Liu et al. (2014) and Liu (2020) also focus on a two-sided matching market with incomplete information, developing a notion of stability to accommodate such an environment. In their setting, agents draw inferences about the value of matching with others by observing their cooperative deviations. Because of workers' common preferences, this channel does not play a role in our setting. Instead, we focus on firms' incentives to gather information through interviews *before* matching.

There is an emerging body of literature on matching with search. Chade and Smith (2006) and Ali and Shorrer (2025) analyze simultaneous search problems where a student applies to colleges but is unsure whether a particular college will admit them. Our model can be viewed as generalizing both of these in three ways: firms in our model can hire multiple workers, firms must compete with other firms for the workers, and the probability of a worker being hired is *endogenous*.

Within the search and matching literature, our model is closest to Chade et al. (2014), who (like us) consider a model where firms compete for multiple workers of unknown value, observe a costly signal about the value of some (but not all) workers, and hire the workers with the highest signals. The key differences between our papers are the presence of private information on one side of the market, the commonality of workers' values to different firms, the nature of firms' signals, and the side of the market that decides which firms will be informed about which workers. Specifically, in their model, each worker's value is *private information*, their values to different firms are *identical*, and *workers* decide which firms to reveal *noisy signals* of that value to. In our model, there is a *public noisy signal* (application rank) of each worker's value, their values to different firms are *conditionally independent* of one another, and *firms* decide which workers to acquire *perfect information* about.

Immorlica et al. (2020) also examines a college admissions setting, but students can first acquire information about a college before applying. They develop a notion of regret-free stability, which incorporates student information acquisition decisions. In their model, agents acquiring information are part of a continuum, and so each agent's decision to gather information does not influence another agent's incentives to acquire information. This is not the case in our model: a given firm's interview decision imposes a direct externality on other firms.

A nascent literature on two-sided matching with interviews has recently received growing interest. Lee and Schwarz (2017) investigates the welfare consequences of interviewing in centralized mechanisms but focuses on how interview decisions affect total matches. They find that in balanced markets, maximizing total matches is achieved when firm interview sets have perfect overlap. In our model, the market is unbalanced, and no firm must worry about not filling its capacity. Instead, each firm only cares about the match value generated less the cost of interviews. Consequently, perfect overlap in our setting is suboptimal. Echenique et al. (2022) and Manjunath and Morrill (2023) also examine a two-sided matching setting where firms want to maximize match value less interview costs. However, they assume interview assignments are determined via a many-to-many deferred acceptance algorithm, which takes as inputs the *ex-ante* rank order list. The interviews do not provide any added information. Rather, whom a firm selects to interview restricts which agents it can list in the rank order list it submits to the clearinghouse. We do not use an exogenous interview assignment protocol. Interview assignments are determined in equilibrium. It is not the case that the outcome of a many-to-many deferred-acceptance algorithm on interview preferences aligns with the outcome in a game where firms select whom to interview.

Along these lines, there are two other works, Kadam (2015) and Erlanson and Gottardi (2023), which look at centralized matching environments where firms are free to select whom to interview. These two papers investigate different questions. The former assumes a specific functional form on the interview technology and analyzes the effect of interview capacity constraints. He shows that relaxing interview capacities can reduce welfare due to over-interviewing. The latter looks at a two-firm environment where interviews are informative for workers and firms, and the interviewing technology for firms is equivalent to our "interviewing for bad news" in Example 1. Our paper differs because we use a general interview technology and focus on equilibrium outcomes *across* different matching protocols. In addition, in our centralized setting, equilibria are inefficient even when worker preferences are common and fixed due to the externality higher-ranked firms place on lower-ranked ones.

Lastly, Ferdowsian et al. (2022) presents a model similar to our decentralized setting except there are no exploding offers, workers can hold on to offers for as long as they like, and if a worker accepts an offer, they cannot renege. Notably, while private information and uncertainty exist in their model, there is no information acquisition stage.

## 2 Model

**Firms and Workers** There is a finite set of $F$ firms, indexed by $\{1, \dots, F\}$, and a unit measure of workers. Each worker can work at one firm; each firm can hire at most $\Delta \in (0, 1)$ workers. We assume that there are more workers than slots: $F\Delta < 1$.

Workers have common preferences over firms: when matched with firm $f$, she receives a payoff of $z_f$, where $z_1 > z_2 > \cdots > z_F > 0$. Each worker prefers working for firm $F$ (and thus any other firm) to remaining unmatched: If a worker does not match with any firm, she receives a payoff of $u \leq 0$.

**Application Ranks and Match Values** Workers are identified by their application score $a \in [0, 1]$, which is common knowledge and is distributed continuously. Without loss of generality, we assume this distribution is uniform, and so the application score is simply the application rank.[2]

Hiring a given worker yields different payoffs for different firms, and these *match values* $s \geq 0$ are not known *ex-ante* by either party. Instead, conditional on the worker's application rank, they are i.i.d. with distribution $G(\cdot|a)$.[3] Workers with higher application ranks are more likely to yield higher match values, in the sense of first-order stochastic dominance: for all $a' > a$ and all $s \geq 0$, $G(s|a') \leq G(s|a)$.

**Interviews and Job Offers** Firms must interview workers before making offers to them. These interviews have constant marginal cost: interviewing a measure $\mu$ of workers costs $c\mu$. When a firm interviews a worker, it learns the value of matching with her. We assume that firms cannot discriminate among workers with the same application rank (and, as we will describe later, existing offers), and so their interview decisions are described by *interview sets* $I_f \subseteq [0, 1]$. We also assume that the lowest-ranked worker is at least potentially worth interviewing: $\int_0^\infty s dG(s|0) > c$.

[2] Since the application score distribution is continuous, we can always index applications by their quantile, or percentile rank, which must be uniformly distributed on the unit interval.

[3] It is without loss to consider distributions with support on a subset of $[0, \infty)$ since a firm will never hire a worker with a negative match value. Given a distribution $G$ with negative values in its support, one can simply consider a distribution $\hat{G}$ with support on a subset of $[0, \infty)$ such that $\hat{G}(s) = G(s)$ for $s \geq 0$ and $\hat{G}(s) = 0$ for all $s < 0$.

A firm's *hiring rule* describes the job offers that it makes to the workers it interviews. For tractability, we assume that firms cannot discriminate between workers with identical match values: if a firm interviews two workers, and observes that they have the same match value, its hiring rule must independently offer each a job with the same probability. Accordingly, we formally define a hiring rule as follows:

**Definition 1.** A *hiring rule* is a measurable function $x : [0,1] \to [0,1]$. A *greedy hiring rule* is a hiring rule $x$ such that for some $\bar{s} \in [0,1]$, $x(s) = 1$ for all $s > \bar{s}$ and $x(s) = 0$ for all $s < \bar{s}$.

A firm's hiring rule maps each match value $s$ that an available worker might have to the probability $x(s)$ with which the firm will make him a job offer. Greedy hiring rules are those that make offers to workers with higher match values first; since higher match values are better, firms are always better off using a greedy rule than a hiring rule that is not greedy. We use $\mathcal{X}$ to denote the set of all greedy hiring rules. This set has a natural total order: We say that a greedy hiring rule $x$ is *more permissive* than another $\hat{x}$, and write $x >_{\mathcal{X}} \hat{x}$, if there is a match value $s \in [0,1]$ such that $x(s) > \hat{x}(s)$.

**Outcomes and Firm Payoffs** Given a tuple of interview sets $\{I_f\}_{f=1}^F$, the *outcome* of any of the matching regimes we consider (described in Section 2.1) is tuple of positive Borel measures $\mathcal{M} = \{\mu_f\}_{f=1}^F$ on $[0,1] \times \mathbb{R}_+$, where $\mu_f(S)$ describes the mass of workers hired by firm $f$ that have combinations of match values and application ranks $(a, s) \in S$.

Each firm's payoff is separable in the values of the workers that it hires: given an interview set $I_f$ and an outcome $\mathcal{M}$, its payoff is

$$\int s d\mu_f - c \int_{I_f} da.$$

## 2.1 Matching Regimes

We consider three types of matching regimes: *centralized*, *decentralized with binding offers*, and *decentralized with nonbinding offers*. The solution concept in each is subgame perfect equilibrium.

**Centralized Matching** The centralized matching game takes place in two stages. In the first stage (the *interview stage*), firms simultaneously and publicly choose which workers $I_f \subseteq [0,1]$ to interview. Then, in the second stage (the *matching stage*) both sides submit preferences to a centralized clearinghouse, which runs a worker-proposing Gale-Shapley algorithm. As is well known, this algorithm is strategy-proof for the workers; hence, we assume they submit their true preferences, and consider the resulting game among the firms. This mechanism is equivalent to the one used in the NRMP. Furthermore, because of our independence assumption, it is also equivalent to a regime in which firms interview simultaneously, offers are private, and offers can be held until a common, fixed deadline.[4]

[4]Our assumption that a firm can only hire workers it has interviewed means that it can only list interviewed workers in its submitted preferences. This is what occurs in practice. We can motivate it by incorporating a small probability that a candidate

**Decentralized Matching** In a decentralized matching regime, there are $T \geq F$ time periods, indexed by $t \in \{1, \ldots, T\}$. Each period $t$ takes place in three stages. In the first (the *entry stage*), each firm $f$ that has not already interviewed simultaneously and publicly decides whether to interview in period $t$. Then, in the second stage (the interview stage), each firm that decided to interview in period $t$ simultaneously and publicly chooses a set of candidates $I_f^t \subseteq [0, 1]$ to interview. Finally, in the third stage (the matching stage), the firms that interviewed in period $t$ simultaneously choose hiring rules $x_f^t$, and publicly make employment offers to each worker they interviewed whose match value is $s$ with probability $x_f^t(s)$.[5]

We consider two varieties of decentralized matching regimes:

- **With Nonbinding Offers** Offers made by the firms are nonbinding. Equivalently, all offers expire only after the last period $T$, and so at the end of period $T$, each worker matches with the highest-ranked firm that has made her an offer.[6] Since offers are public, when firms choose a time $t$ and an interview set $I_f^t$, they only interview those candidates with application ranks $a \in I_f^t$ *who have not received an offer from a higher-ranked firm* in a previous period $t' < t$.

- **With Binding Offers** Offers made by firms are binding and expire at the end of the period. For tractability, we consider the extreme case where workers are sufficiently risk averse and so they are never willing to decline all the offers they receive in a period in hopes of receiving a better offer later. Formally:

  **Assumption 2.1.** $u < -\frac{1-G(\underline{s}|1)}{G(\underline{s}|1)} \cdot z_F$, *where* $\underline{s} = \inf\left\{\int_0^{1-\Delta\cdot(F-1)} 1 - G(\underline{s}|a)da \leq \Delta\right\}$.

  Hence, at the end of *each* period $t$, each worker that received an offer matches with the highest-ranked firm that has made her an offer *in that period*, and exits the game. Thus, when firms choose an interview set $I_f^t$, they only interview those candidates with application ranks $a \in I_f^t$ *who have not matched with another firm* in a previous period $t' < t$.

# 3 Analysis

## 3.1 Centralized Matching

We begin our analysis with the centralized regime: As it turns out, most of the results and intuition needed to analyze the two decentralized regimes can be imported from the centralized case. Since the centralized regime takes place in two stages, our analysis proceeds by backward induction.

is not a good fit, in which case a firm would never want to hire them.

[5] No extra information is available about candidates at later periods. The reason for this assumption is to identify the inefficiencies that arise solely due to costly interviews rather than unraveling.

[6] We avoid allowing workers to accept offers before period $T$ because doing so would not change our analysis; indeed, it would make waiting until period $T$ to accept offers a weakly dominant strategy for the workers.

### 3.1.1 Centralized Regime: Matching Stage

We first consider the centralized regime's matching (i.e., second) stage, in which firms simultaneously submit preference orderings to a standard worker-offering Gale-Shapley deferred acceptance algorithm.[7] Proposition 1 shows that since the workers' preferences are identical, the matching stage has the same unique equilibrium outcome as a serial dictatorship in which firms move in rank order, make offers to available workers according to hiring rules $x_f$, and each of those offers is immediately accepted. That outcome is characterized by each firm choosing the most permissive greedy hiring rule that hires no more than $\Delta$ workers, given the hiring rules chosen by higher-ranked firms.

**Proposition 1.** *Fixing an interview profile $\{I_f\}_{f=1}^F$, the matching stage of the centralized regime has the same unique equilibrium outcome $\{\mu_f^*\}_{f=1}^F$ as a* serial dictatorship *where in each period $f \in \{1, \dots, F\}$, firm $f$ chooses a hiring rule $x_f$ and uses it to hire from the pool of workers in $I_f$ that have not already been hired by another firm. Moreover, this outcome is described by*

$$\mu_f^*(A \times S) = \int_{I_f \cap A} \underbrace{\int_S x_f^*(s, \{I_h\}_{h=1}^f) dG(s|a)}_{\substack{\text{fraction of available rank } a \text{ workers} \\ \text{with } s \in S \text{ matched to firm } f}} \underbrace{\left( \prod_{h<f, I_h \ni a} \int_0^\infty (1 - x_h^*(s, \{I_k\}_{k=1}^h)) dG(s|a) \right)}_{\substack{\text{fraction of rank } a \text{ workers} \\ \text{not matched to firms } h<f}} da, \tag{1}$$

*where the hiring rules $\{x_f^*(\cdot, \{I_h\}_{h=1}^f)\}_{f=1}^F$ are defined recursively by*

$$x_f^*(\cdot, \{I_h\}_{h=1}^f) \equiv \max_{\succ_{\mathcal{X}}} \left\{ x \in \mathcal{X} | \int_{I_f} \int_0^\infty x(s) dG(s|a) \left( \prod_{h<f, I_h \ni a} \int_0^\infty (1 - x_h^*(s, \{I_k\}_{k=1}^h)) dG(s|a) \right) da \leq \Delta \right\}.$$

Intuitively, in the first round of the deferred acceptance algorithm, all workers make offers to firm 1, so it acts as a dictator, and greedily holds on to offers from the best $\Delta$ workers that it interviewed. The remaining workers each make offers to firm 2 in round 2, so it acts as a *serial* dictator, and greedily holds on to offers from the best $\Delta$ workers that it interviewed who do not have an outstanding offer to firm 1. This process continues until round $F$, in which the lowest-ranked firm greedily dictates the workers that it hires from among those remaining.

### 3.1.2 Centralized Regime: Interview Stage

Next, we examine the centralized regime's interview stage. Observe that each $x_f^*$ depends only on firm $f$'s interview set and the interview sets of higher-ranked firms. Thus, following the interview profile

[7] Recall that since the algorithm is strategy-proof for workers, we assume that they make offers according to their true preferences.

$\{I_f\}_{f=1}^F$, firm $f$'s equilibrium payoff in the centralized regime is pinned down *recursively*, as follows:

$$\pi_f(\{I_h\}_{h=1}^f) \equiv \int_{I_f} \underbrace{\int_0^\infty s x_f^*(s, \{I_h\}_{h=1}^f) dG(s|a)}_{\textit{match value from hiring rule}} \cdot \underbrace{\left( \prod_{h<f, I_h \ni a} \int_0^\infty (1 - x_h^*(s, \{I_k\}_{k=1}^f)) dG(s|a) \right)}_{\substack{\textit{fraction of rank a workers} \\ \textit{not matched to firms h<f}}} - c\, da$$

The recursive structure of the firms' payoffs allows equilibrium interview sets to also be pinned down recursively, beginning with the highest-ranked firm, firm 1. Since the workers unanimously prefer it to every other firm, its payoff is independent of the other firms' interview sets and hiring rules. Hence, in any equilibrium, its interview set $I_1^*$ solves

$$\max_{I_1} \int_{I_1} \int_0^\infty s x_1^*(s, I_1) dG(s|a) - c\, da, \tag{2}$$

$$\text{where } x_1^*(\cdot, I_1) = \max_{\succ_{\mathcal{X}}} \left\{ x \in \mathcal{X} | \int_{I_1} \int_0^\infty x(s) dG(s|a) da \leq \Delta \right\}. \tag{3}$$

Recall that the match value distributions $G(\cdot|a)$ of workers with different application ranks are ordered by first-order stochastic dominance. It follows that, *holding its hiring rule fixed*, firm 1 receives a greater return from interviewing higher-ranked applicants than from interviewing lower-ranked ones. This suggests that it should interview greedily, i.e., choose $I_1^* = [a_1, 1]$ for some $a_1$. But, as the dependence of $x_1^*$ on $I_1$ illustrates, if firm 1 replaces lower-ranked applicants with higher-ranked ones, then unless it is interviewing fewer applicants than it has positions available (and thus hires all of them), its hiring rule cannot remain fixed. Instead, it must become *less permissive* in order to hire the same mass $\Delta$ of workers.

It turns out that it is still optimal for the highest-ranked firm to interview greedily. To see why, first observe that any interview set $I$ induces a distribution of match values — where match values of uninterviewed applicants are set to zero — given by $G(s|I) \equiv \int_I G(s|a)da + \int_{[0,1]\setminus I} da$. Then we can rewrite

$$\pi_1(I_1) = \int_0^\infty s x_1^*(s, I_1) dG(s|I_1); \qquad x_1^*(\cdot, I_1) = \max_{\succ_{\mathcal{X}}} \left\{ x \in \mathcal{X} | \int_0^\infty x(s) dG(s|I_1) \leq \Delta \right\}.$$

Since firm 1's hiring rule is defined so that it hires the top $\Delta$ workers from $G(\cdot|I)$, we can write its payoff in terms of the *quantile function* $G^{-1}(v|I) \equiv \inf\{s | G(s|I) \geq v\}$:

$$\pi_1(I_1) = \int_{1-\Delta}^1 G^{-1}(v|I) dv.$$

Since first-order stochastic dominance is equivalent to an upward shift in the quantile function, it follows that *the top ranked firm is never worse off when it replaces interviews of lower-ranked candidates with* *interviews of higher-ranked ones.*

**Proposition 2** (Equilibrium in the Centralized Regime: Greedy Interviews at the Top)**.** *Given the firms' equilibrium strategies in the centralized regime's matching stage, the highest-ranked firm has a dominant strategy in the interview stage in which it interviews greedily: There exists* $I_1^* = [a_1, 1]$ *for some* $a_1 \in [0, 1]$ *such that* $\pi_1(I_1^*) \geq \pi_1(I_1)$ *for all* $I_1 \subseteq [0, 1]$.

Next, consider the lower-ranked firms, focusing initially on the second-ranked firm (firm 2). Unlike firm 1, firm 2 does not face a single-agent decision problem. Instead, it must take into account the fact that some of the workers it interviews may receive offers from firm 1 — in which case firm 2 will be unable to hire them. In particular, its interview set $I_2^*$ solves

$$\max_{I_2} \int_{I_2} \int_0^\infty s x_2^*(s, I_1^*, I_2) dG(s|a) \psi_2(a) - c\, da,$$

$$\text{where } x_2^*(\cdot, I_1^*, I_2) = \max_{\succ_{\mathcal{X}}} \left\{ x \in \mathcal{X} | \int_{I_2} \int_0^\infty x(s) dG(s|a) \psi_2(a) da \leq \Delta \right\},$$

$$\text{and } \psi_2(a) = \begin{cases} 1, & a \notin I_1^*; \\ \int_0^\infty (1 - x_1^*(s, I_1^*)) dG(s|a), & a \in I_1^*. \end{cases}$$

Because firm 2 does not know which workers with rank $a \in I_1^*$ will receive offers from firm 1, it cannot discriminate between them when it makes its own interview decisions. Instead, it must interview all rank-$a$ workers or none of them, and adjust the total match value it gets from those workers (but *not* the cost of interviewing them) by the probability $\psi_2(a)$ that they are available.

This adjustment to the benefit of an interview — but not its cost — distorts the interview decisions of the second-ranked firm (and of the lower-ranked firms more generally). In particular, unlike the top-ranked firm, firm 2 does not necessarily interview greedily: even though the applicants interviewed by firm 1 have more favorable match value distributions than the applicants that are not, the former are less likely to be available. However, when the match value distribution satisfies two regularity conditions, we can say that interviewing greedily is optimal *on firm 1's interview set* $I_1^*$ and, separately, *below firm 1's interview set*. More generally, we show that for each firm $f > 1$, interviewing greedily is optimal *on each set of candidates that are interviewed by the same set of higher-ranked firms*.

To understand why, observe that interviewing a set $I'$ of higher-ranked applicants instead of an equal-mass set $I$ of lower-ranked ones changes the firm's hiring rule $x_f^*$ in two ways. First, there is a *quality effect*: the distribution of match values is shifted to the right, making $x_f^*$ more stringent. (This is the effect we discussed in the context of Firm 1's problem.) Second, there is an *availability effect*: if the applicants in $I'$ and $I$ are interviewed by the same set of higher-ranked firms, the higher-ranked applicants in $I'$ are less likely to be available, since they are more likely to have met those other firms' hiring thresholds; consequently, interviewing $I'$ instead of $I$ makes $x_f^*$ more permissive. The regularity property guaranteeing that interviewing $I'$ is better than interviewing $I$ depends on which of these effects dominates.

Our first regularity condition — which is intermediate between first-order stochastic dominance and the monotone likelihood ratio property — plays this role when the quality effect dominates.

**Regularity Condition #1.** $G(\cdot|a)$ is increasing in $a$ in the hazard rate order: For all $a' > a$ and $s' > s$, $(1-G(s'|a'))(1-G(s|a)) \geq (1-G(s|a'))(1-G(s'|a))$.

Regularity Condition #1 guarantees that, holding the firm's hiring rule constant, an applicant's expected match value *conditional on being hired* is increasing in their rank $a$. Interviewing higher-ranked applicants under a hiring rule that is at least as stringent — as the firm does when the quality effect dominates — can only increase this conditional expectation further. Since the firm's hiring rule $x_f^*$ is constructed to yield an equal mass of hired workers from any interview sets with the same mass, this relationship extends to the *total* value of the workers hired from higher- and lower-ranked sets of applicants.

Our second regularity condition ensures that interviewing higher-ranked applicants is more valuable when the availability effect dominates.

**Regularity Condition #2.** $G$ has *increasing k-adjusted yields* for some $k$: For any profile of greedy hiring rules $\{x, \{x_f\}_{f=1}^k\} \subset \mathcal{X}$, any set $A \subseteq [0,1]$ such that $\int x_f(s)dG(s|a) > 0$ for each $a \in A$, and $f \in \{1, \ldots, k\}$,

$$\int x(s)sdG(s|a) \prod_{f=1}^{k} \left( \int (1-x_f(s))dG(s|a) \right) \text{ is non-decreasing in } a \text{ on } \operatorname{int} A.^{8}$$

Recall that first-order stochastic dominance ensures that, fixing a greedy hiring rule $x$, $\int x(s)sdG(s|a)$ is increasing in $a$. When the match value distribution $G$ has increasing $k$-adjusted yields, the value created by interviewing an applicant is still increasing in her rank even when it is adjusted for the probability that she will be hired by one of $k$ higher-ranked firms that have also interviewed her.[9] Setting a more permissive hiring rule for higher-ranked applicants — as the firm does when the availability effect dominates — can only increase this value further.

We can interpret $k$ as the *largest* number of firms whose decisions to interview a pair of applicants cannot reverse their relative attractiveness to lower-ranked firms; that is, if the property holds for $k$, it holds for each $k' < k$. Formally:

**Lemma 1.** *If $G$ has increasing k-adjusted yields, then it has increasing $k'$-adjusted yields for each $k' < k$.*

*All* distributions satisfying first-order stochastic dominance have increasing 0-adjusted yields. For some canonical distributions, the property holds for higher $k$. We provide two such examples, which are each also increasing in $a$ in the hazard rate order (Regularity Condition #1).[10]

[9]Or more precisely, it is increasing on the interior of the set of applicants interviewed by those $k$ firms.

[10]For a proof, see Lemma 3 in the appendix.

**Example 1** (Exponential Distribution)**.** Suppose that given an applicant's rank, match values are exponentially distributed and the distribution has longer tails for higher-ranked applicants. Then we can write the match value distribution as $G_\lambda(s|a) = 1 - e^{s\lambda(a)}$, where $\lambda : [0, 1] \to \mathbb{R}_+$ is a decreasing function. Lemma 3 shows that $G_\lambda$ has increasing k-adjusted yields for some $k \geq 1$.

**Example 2** (Interviewing for "Bad News")**.** Suppose interviewing an applicant either reveals that they are unacceptable ($s = 0$) with probability $\beta$ or their pre-interview rank was accurate ($s = a$). The match value distribution is given by $G_\beta(s|a) = \begin{cases} \beta, & s < a \\ 1, & s \geq a. \end{cases}$ Such a technology appears in Chade and Smith (2006) and Erlanson and Gottardi (2023). Lemma 3 shows that $G_\beta$ has increasing adjusted yields.

Proposition 3 shows that when the match value distribution is increasing in $a$ in the hazard rate order and exhibits increasing $k$-adjusted yields, the highest-ranked $k + 1$ firms will *overlap* greedily with the interview decisions of higher-ranked firms.

**Proposition 3** (Equilibrium in the Centralized Regime: Interview Stage)**.** *Suppose G is increasing in a in the hazard rate order and has increasing k-adjusted yields. There is a Nash equilibrium* $\left\{I_f^*\right\}_{f=1}^F$ *in which*

*i. The highest-ranked firm interviews greedily:* $I_1^* = [a_1, 1]$ *for some* $a_1 \in [0, 1]$.

*ii. Each firm* $f \in \{2, \ldots, k\}$

  *(a) Overlaps greedily with each set of firms above them: For* $S \subseteq \{1, \ldots, f-1\}$,
  $I_f^* \cap \bigcap_{j \in S} I_j^* = [a_f^S, 1] \cap \bigcap_{j \in S} I_j^*$ *for some* $a_f^S \in [0, 1]$.

  *(b) Interviews greedily* below *the firm above them: There exists a decreasing sequence* $\{a_f\}_{f=1}^F$ *such that* $I_f^* \cap [0, a_{f-1}) = [a_f, a_{f-1})$.

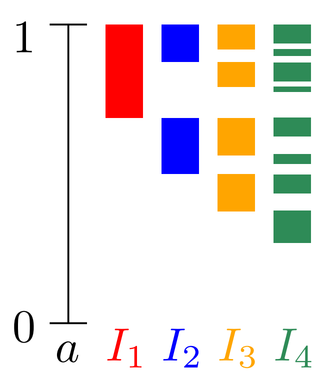


**Figure 1: Equilibrium interview sets in the centralized regime.** Proposition 3 shows that in the centralized regime, the highest-ranked firm interviews greedily, while lower-ranked firms interview greedily *on each set of applicants that is interviewed by the same higher-ranked firms.*

Figure 1 illustrates the conclusions of Proposition 3. The gaps in the lower-ranked firms' interview sets arise because of a coordination issue: they do not know which of the workers interviewed by higher-ranked firms will end up receiving job offers from those firms. Hence, interviewing such a worker carries a risk: she may be unavailable in the matching stage because she has already received an offer from a higher-ranked firm. This prompts the lower-ranked firms to skip over some of these workers, and interview those that are less highly ranked instead. Our two regularity conditions ensure that this skipping is greedy, even if the firm's overall interview set is not.

By creating uncertainty about workers' availability, each firm's interview decision imposes an externality on the firms ranked below it. As we show in sections 3.2 and 3.3, allowing the firms to interview and make offers over time either mitigates this externality or eliminates it entirely. Consequently, total surplus is lower in the centralized regime's equilibrium than in *any* equilibrium of *either* decentralized regime. However, even when interview decisions must be made simultaneously, the centralized regime's equilibrium does not necessarily maximize total surplus. Example 3 illustrates.

**Example 3** (Inefficient Equilibrium Interviews in the Centralized Regime)**.** Consider a setting with two firms, each with a capacity $\Delta = \frac{1}{2}$ and interview cost $c = \frac{12}{5}$. Suppose that the pool of workers consists of two applicant types, I and II, each with equal measure $\frac{1}{2}$: Type I applicants provide a match value of either 1, 9, or 10 with equal probability ($\frac{1}{3}$ probability each), while type II applicants provide a match value of 0, 1, or 9 with equal probability. In the language of the model, applicants of rank $a \in [\frac{1}{2}, 1]$ are of type I and those of rank $a \in [0, \frac{1}{2})$ are of type II.

In the centralized regime, equilibrium interview sets are $I_1^* = [0, 1]$ and $I_2^* = \emptyset$. In other words, firm 1 interviews everyone and firm 2 interviews no one. Total surplus is then just firm 1's payoff:

$$\underbrace{\frac{1}{6} \cdot 10 + \frac{1}{3} \cdot 9}_{\text{total match value for firm 1}} \quad - \quad \underbrace{1 \cdot \frac{12}{5}}_{\text{firm 1's cost of interviewing}} \quad = \frac{34}{15}.$$

This is inefficient, since firm 1's interview decisions crowd firm 2 out of the market completely. Consider an interview profile where Firm 1 interviews only type I applicants (i.e., $I_1 = [\frac{1}{2}, 1]$) and Firm 2 interviews only type II applicants (i.e., $I_2 = [0, \frac{1}{2})$). Each firm then hires every applicant it interviewed. Total firm surplus is then

$$\underbrace{\frac{1}{2}}_{\substack{\text{mass}\\ \text{of } I_1}} \Bigg( \underbrace{\frac{20}{3}}_{\substack{\text{match value}\\ \text{of type I}}} - \underbrace{\frac{12}{5}}_{c} \Bigg) + \underbrace{\frac{1}{2}}_{\substack{\text{mass}\\ \text{of } I_2}} \Bigg( \underbrace{\frac{10}{3}}_{\substack{\text{match value}\\ \text{of type II}}} - \underbrace{\frac{12}{5}}_{c} \Bigg) = \frac{13}{5} > \frac{34}{15}.$$

In addition, worker surplus also increases: Unlike in the equilibrium of the centralized regime, no workers go unmatched.

## 3.2 Decentralized Matching with Binding Offers

We next turn to the two decentralized regimes, in which offers take place over $T$ time periods, rather than all at once as part of a centralized matching mechanism. We begin with the case where offers are *binding*, in the sense that workers cannot hold an offer past the end of the current period without accepting it, and cannot renege on an accepted offer if they receive a better one later on. This is the status quo environment in many markets, including the academic job market in economics.

As stated earlier (Assumption 2.1), we assume workers are sufficiently risk averse — in the sense that the payoff they receive when they are unmatched is sufficiently low — that each will always opt to match with the highest-ranked firm that has made them an offer in the current period, rather than risk being unmatched at the end of the game.[11] This means that in any given period, once firms have committed to their entry decisions, the decentralized regime functions like a miniature version of the centralized regime: The firms that entered must simultaneously make decisions on who to interview and (afterward) what rule to use for making offers, while workers behave as if the alternative to accepting an offer is remaining unmatched. In particular, since no workers have accepted offers in the first period, the firms that interview in that period get equilibrium outcomes that are *identical* to the outcomes of the centralized market — just with their *overall* rankings replaced by their rankings *among firms that interview in period 1*. Since the latter ranking must be higher than the former, and higher-ranked firms get higher payoffs in the centralized regime (Lemma 6 in the Appendix), the expected payoff that each firm receives in the binding-offer regime must be at least as high as their payoff in the centralized regime.

**Proposition 4** (Decentralized Matching with Binding Offers: Equilibrium and Welfare)**.** *A subgame perfect equilibrium exists. In any equilibrium, each firm's ex ante expected payoff is weakly greater than its payoff in the centralized regime.*

In equilibrium, firm $f$ randomizes over the times $t \in \{1, \ldots, f\}$ at which to interview. The probability of entering at time $t$ is based on the observed history up until that time. While the ex-ante expected payoff for each firm is weakly higher than in the centralized setting, there may be an outcome realization such that a firm is left worse off. Furthermore, while the top-ranked firm interviews the same workers that it does in the centralized setting, lower-ranked firms generally interview different ones.

**Example 4** (Equilibrium with Binding Offers)**.** Suppose there are three firms. Let $I_1^*$, $I_2^*$, and $I_3^*$ be each firm's equilibrium interviewing strategy in the centralized setting. In the equilibrium of the decentralized setting with binding offers, Firm 1 interviews $I_1^* = [a_1, 1]$ in period 1. Firm 2 randomizes between interviewing $I_2^*$ in period 1 and interviewing workers $[a_2, 1]$ *of the available workers in period 2*, where $a_2$ is determined based on that pool. Firm 3 randomizes between interviewing $\hat{I}_3$ (not necessarily equal to $I_3^*$)

[11] This is necessary to make exploding offers viable in equilibrium: there is no point in front-running (i.e. interviewing before another firm) to make an exploding offer if said offer will never be accepted.

in period 1 and interviewing workers $[a_3, 1]$ *of the available workers in period 3*, where $a_3$ is determined based on that pool.

In equilibrium, the ex-ante expected payoff of each firm is weakly greater than in the centralized setting. However, one can see that there is a realization in which Firm 2 is worse off ex-post than in the centralized setting: when Firm 1 and Firm 3 interview and match in the first period, and Firm 2 interviews and matches in the second period.

The externality imposed by higher-ranked firms on lower-ranked firms when multiple firms interview in the same period also potentially reduces worker welfare: As Example 3 illustrates, firms are not guaranteed to fill their slots in the equilibrium of the centralized setting. Since the decentralized regime with binding offers operates like a series of centralized markets, the same is true in its equilibrium.

## 3.3 Decentralized Matching with Nonbinding Offers

In the centralized matching regime, the firms' interview decisions are distorted by the need to adjust the benefit of an interview, but not its cost, by the probability that the interviewee will receive an offer from a higher-ranked firm. Decentralized matching regimes can potentially eliminate this distortion. In particular, if the firms interview one at a time in rank order, they do not need to worry about interviewing candidates that could still receive a dominating offer.

When offers are *nonbinding*, Proposition 5 shows that this is achievable. More precisely, there is a *sequential-hiring equilibrium* such that on the equilibrium path, each firm $f$ interviews in period $f$. However, because firms are indifferent about when to interview as long as they do so after higher-ranked firms, there are also other equilibria, including one that mimics the outcome of the centralized regime. We focus on the sequential-hiring equilibrium because it is the unique equilibrium that survives the introduction of a small cost of delay.[12]

**Proposition 5** (Decentralized Matching with Nonbinding Offers: Equilibrium Timing)**.** *In the decentralized regime with nonbinding offers,*

*i.* *There is a* sequential-hiring equilibrium *in which each firm's strategy is to interview immediately after all higher-ranked firms have interviewed, or in the final period (whichever comes first). Hence, on the equilibrium path, each firm $f$ interviews in period $f$.*

*ii.* *There is a* centralized equilibrium *such that each firm waits to interview at period $T$, and chooses the same interview set $I_f^*$ as it does in the equilibrium of the centralized regime described in Proposition 3.*

[12]That is, if firms discount the future at rate $\delta$, then for $\delta$ close enough to 1, the sequential-hiring equilibrium is the unique subgame perfect equilibrium: When firms are arbitrarily patient, they still prefer to wait until after higher-ranked firms have interviewed, but discounting makes them strictly prefer to interview immediately when they would otherwise be indifferent.

For intuition, recall that in the centralized regime, the firms' payoffs had a recursive structure: the strategies of lower-ranked firms did not affect the payoffs of higher-ranked firms, because an offer from a lower-ranked firm would never affect a worker's decision to accept an offer from a higher-ranked one. When offers are nonbinding, the same is true in the decentralized regime: Since workers do not accept offers until the end of the last period, the payoff of a higher-ranked firm does not depend on either the set of workers interviewed by a lower-ranked firm *or the timing of the lower-ranked firm's interviews.*[13] Thus, once all higher-ranked firms have conducted their interviews, a firm becomes indifferent between interviewing in the current period and delaying until a later period. In contrast, if a firm can delay its interviews until after those of firms ranked above it, it is always in its interest to do so, in order to avoid the cost of interviewing workers that end up receiving dominating offers from those firms. Thus, if firms interview immediately whenever they are indifferent about timing, the sequential-hiring equilibrium results (i); if they delay whenever they are indifferent, the centralized equilibrium results (ii).

Like in the centralized regime, equilibrium interview sets can be pinned down recursively in the sequential-hiring equilibrium. But unlike in the centralized regime, the interview sets of higher-ranked firms do not distort a firm's interview decisions: Because firms interview in rank order, each firm can observe the offers made by higher-ranked firms when it makes its interview decisions, and avoid selecting applicants that receive dominating offers. Hence, in contrast to the centralized regime, the possibility of such offers does not deter them from interviewing applicants that are also interviewed by higher-ranked firms.

**Proposition 6** (Decentralized Matching with Nonbinding Offers: Sequential-Hiring Equilibrium)**.** *In the sequential-hiring equilibrium, on the equilibrium path:*

*i. Each firm interviews greedily: For each firm $f$, $I_f = [a_f, 1]$ for some $a_f \in [0,1]$.*

*ii. There is maximal employment: in the equilibrium outcome $\{\mu_f^{SH}\}_{f=1}^F$, $\mu_f^{SH}([0,1]\times\mathbb{R}_+) = \Delta$.*

Proposition 6 shows that the equilibrium interview sets are *greedy all the way down*: no workers "fall through the cracks." As a result, when each firm interviews, it will always choose to interview enough applicants to fill all of their available positions. Hence, the total payoff of the workers is weakly greater

[13]Formally, firm $f$'s payoffs following a terminal history in which each firm $k$ interviews the set of workers $I_k$ in period $t_k$ and chooses hiring rule $x_k$ can be written

$$\pi_f^{NB}(\{I_k, x_k, t_k\}_{k=1}^f) \equiv \int_{I_f}\left(\int_0^\infty s x_f(s)dG(s|a)\psi_f(a, \{I_k, x_k, t_k\}_{k=1}^{f-1}, t_f) - c\right)\phi_f(a, \{I_k, x_k, t_k\}_{k=1}^{f-1}, t_f)da, \tag{4}$$

where $\psi_f(a, \{I_k, x_k, t_k\}_{k=1}^{f-1}, t_f) = \prod_{\substack{h<f, I_h\ni a,\\ t_h\geq t_f}}\int_0^\infty(1-x_h(s))dG(s|a)$, and $\phi_f(a, \{I_k, x_k, t_k\}_{k=1}^{f-1}, t_f) = \prod_{\substack{h<f, I_h\ni a,\\ t_h< t_f}}\int_0^\infty(1-x_h(s))dG(s|a)$.

Here, $\phi_f(a, \{I_k, x_k, t_k\}_{k=1}^{f-1}, t_f)$ is the probability that a rank-$a$ worker will receive an offer from a higher-ranked firm *before* firm $f$ conducts its interviews, whereas $\psi_f(a, \{I_k, x_k, t_k\}_{k=1}^{f-1}, t_f)$ is the probability that she will receive such an offer *afterward*.

than in the centralized regime. At the same time, the asynchronicity of the interviews and offers benefits the firms. Since each firm waits for the higher-ranked firms to "clear the market" before interviewing, each firm avoids the cost of interviewing workers who receive a dominating offer. This reduces the interview congestion and coordination frictions present in the centralized regime. With two firms, this leads to a Pareto improvement: both firms are better off.

**Proposition 7** (Welfare in the Sequential-Hiring Equilibrium)**.** *In the sequential-hiring equilibrium of the decentralized regime with nonbinding offers,*

*i. The top-ranked firm receives the same payoff as in the equilibrium of the centralized regime.*

*ii. The second-ranked firm receives a weakly higher payoff than in the equilibrium of the centralized regime.*

For (i), observe that the top-ranked firm faces the same single-agent decision problem in the first period of the sequential-hiring equilibrium that it did in the centralized regime. For (ii), observe that in the second period of the sequential-hiring equilibrium, it cannot be profitable for the second-ranked firm to deviate by interviewing the same set of workers that it did in the equilibrium of the centralized regime. But since the top-ranked firm interviews and hires the same way in both regimes, doing so would give the second-ranked firm the same total match value as in the centralized regime, but lower interview costs.

## 3.4 Efficiency

Suppose that a social planner can choose the time period in which a firm interviews and matches, and which candidates it can interview (but not which candidates the firm matches with after its interviews). The following observation is immediate:

**Observation 3.1.** *In either decentralized regime, total surplus is maximized when firms interview sequentially.*

The planner prefers that interviews and matches occur sequentially so that interviewing is more efficient. More formally, suppose $k$ firms are interviewing and matching in a given period. One can strictly improve welfare by assigning the same interview sets but with one firm moving after the other matches. Each firm will match with the same match-value distribution of workers but will interview less of them.

Given this, and the results from Proposition 6, one may assume that the sequential hiring equilibrium is efficient. However, that is not necessarily the case.

**Proposition 8.** *In any of the three regimes, equilibria may fail to maximize total surplus.*

The fact that the equilibrium in the centralized regime does not maximize total surplus follows from Observation 3.1 and Proposition 5 (ii) (in fact, Example 3 shows it is not even necessarily constrained efficient). Any equilibrium of the decentralized regime with binding offers is inefficient if, with positive probability, at least two firms interview in the same period. If no two firms are interviewing in the same period with positive probability, it means the equilibrium coincides with the sequential hiring equilibrium of the decentralized regime with non-binding offers. Thus, it suffices to provide an example of a setting where the latter is inefficient.

**Example 5** (Inefficiency in the Sequential-Hiring Equilibrium)**.** Suppose there are two firms, each in need of a measure $\frac{2}{5}$ of workers. There are two types of applicants (Type I and Type II). There are a measure of $\frac{1}{5}$ Type I applicants and a measure of $\frac{4}{5}$ Type II applicants. In other words, applicant $a \in [\frac{4}{5}, 1]$ is Type I and applicant $a \in [0, \frac{4}{5})$ is Type II. The match value distribution for each type is as follows:

- Type I applicants ha a match value of 10 with probability $\beta_I \approx 1$, and 0 otherwise.
- Type II applicants provide a match value of 9 with probability $\beta_{II} = 0.5$, and 0 otherwise.

If interview costs are sufficiently low ($c \to 0$), then in the sequential matching equilibrium of the decentralized setting with non-binding offers:

- Firm 1 interviews everyone in period 1, and matches with a $\frac{1}{5}$ measure of applicants with match value 10 and a $\frac{1}{5}$ measure of applicants with match value 9.
- Firm 2 interviews all remaining applicants in period 2, and matches with a $\frac{3}{10}$ measure of applicants with match value 9 and a $\frac{1}{10}$ measure of applicants with match value 0.

Total match value across both firms is $10 \cdot \frac{1}{5} + 9 \cdot \frac{1}{2} = 6.5$. The total number of interviews conducted is $1 + (1 - \frac{2}{5}) = \frac{8}{5}$.

Now, consider the following interview assignment: Firm 1 interviews all the Type II applicants in the first period, and Firm 2 interviews all the remaining applicants in the second period. Consequently, firm 1 matches with a $\frac{2}{5}$ measure of candidates with match value 9, while firm 2 matches with a $\frac{1}{5}$ measure of applicants with match value 10 and a $\frac{1}{5}$ measure with match value 9. Total match value across both firms is thus $10 \cdot \frac{1}{5} + 9 \cdot \frac{3}{5} = 7.4$, while the total number of interviews conducted is $\frac{4}{5} + (1 - \frac{2}{5}) = \frac{7}{5}$.

Example 5 highlights the key reason the planner's incentives do not align with the firms'. The "10's" are likely to be "10's" everywhere: there's little variance. The Type II applicants have the most variance, so the planner prefers as many draws from this pool as possible. In other words, it is more efficient for a type II applicant to have more interviews than a Type I applicant! Firms do not internalize this particular mean-variance trade-off when they make their interview decisions.

# 4 Discussion

## 4.1 Assumptions

**Common Ranking of Firms** Workers have a common ranking of firms unaffected by the interviews. This is partially for tractability but also to isolate the inefficiencies created by strategic interviewing *even when* firms know where they stand. To some degree, this feature resembles many of the motivating labor markets, such as medical residency and academia. One possible way to generalize our model while maintaining some tractability is to include a tiered ranking of firms. Formally, consider tiers $T_1, \ldots, T_k$, where $T_i \subset \{1, \ldots, n\}$ for all $i$ and $T_i \cap T_j = \emptyset$ for all $i \neq j$. While each worker strictly prefers firms in $T_i$ to firms in $T_j$ for $i < j$, worker preferences over firms *within* a given tier are dependent on the interviews. As the results in Erlanson and Gottardi (2023) suggest, any centralization of interviews within a given tier would lead to potential multiplicity of equilibria, as well as inefficiency. In such a world, our results indicate that total surplus in a setting where each tier of firms interviews in sequence would be higher than in a centralized setting. However, firms within the same tier would have an incentive to front-run each other.

**Independent Match Values** In our model, the match value generated by a particular worker for a firm is independent across firms. With correlated match values, an adverse selection issue arises. To make this concrete, consider a common-values environment where a worker's match value to a given firm is the same across all firms. If a candidate is available to a low-ranked firm, either that candidate was not interviewed by better firms or was unsuccessful in their interviews, meaning they have low match quality. Thus, in the centralized regime, lower-ranked firms become more averse to interviewing candidates whom top firms are interviewing.

This adverse selection issue has important implications in the decentralized regime as well. When match values were independent, the sequential interviewing procedure from Section 3.3 was an equilibrium in the decentralized regime with non-binding offers.

**Observation 4.1.** *The sequential-hiring strategy profile is not an equilibrium of the decentralized regime with non-binding offers when match values are common across firms.*

Why? The top-ranked firm has no incentive to interview and screen first. Instead, it would prefer to wait until the end to see whom the other firms extended offers to. Such an economic force mirrors that in Ely and Siegel (2013). We conjecture that this free-riding issue from public offers leads to equilibria where all firms interview and match with the same set of workers as they would have in the centralized regime.

**Risk Aversion** In the decentralized regime with binding offers, we assumed workers were sufficiently risk averse. Such an environment generates the most concern amongst labor market participants and organizers, as it makes exploding offers an effective tactic. In our model, risk aversion is captured by the relative size of the workers' payoffs from being matched to different firms and the payoff associated with being unmatched. These payoffs determine workers' incentives to accept or reject early offers. To understand the crucial role of risk aversion, consider the opposite extreme, where any worker will reject an offer from a firm if they know they will be interviewed by a better firm later. Then, it can be shown that in any equilibrium of the decentralized regime with binding offers, no firm will front-run a higher-ranked one. Hence, such a regime is equivalent to the decentralized setting with non-binding offers.

## 4.2 Interpretation of Results

To understand the welfare consequences of any matching mechanism and the trade-offs between matching mechanisms, one must consider their effect on interviewing decisions. A common criticism of a decentralized environment with binding offers is that exploding offers prohibit workers from interviewing at more preferable firms later in the hiring cycle. Colloquially, the usual story is "Person $A$ received an exploding offer from Firm $X$, and could not interview at Firm $Y$. We must centralize the market so that Person A can interview at $X$ and $Y$, and compare offers." But it is not necessarily the case that in the centralized system, Person $A$ would have received an interview from any of those firms! Interviewing decisions are an equilibrium object sensitive to the matching mechanism.

In particular, the nature of a centralized matching process to make the timing of decisions simultaneous generates coordination issues at the interview stage. Each firm must incorporate the externality imposed by the interview decisions of its competitors when deciding on its interview list. This is not to say that centralized mechanisms should be done away with. For instance, recall our assumption of a continuum of workers in our analysis. Such an assumption is substantive because it implies no uncertainty about yield: firms can choose a hiring rule that ensures they match with exactly $\Delta$ applicants. In a discrete setting, firms must be concerned about whether more than $\Delta$ candidates accept or less than $\Delta$ accept. An important property of centralized matching mechanisms is they guarantee that a firm will never be matched with more than its capacity.

Nevertheless, one benefit of a decentralized system is that the interview coordination issues can be mitigated due to matches occurring over time. Firm and worker exit can lead to more effective interviewing for the remaining firms. As an illustrative example, early-action programs in US undergraduate college admissions function to reduce application congestion in the regular admissions cycle. Now, in a completely decentralized environment where firms can make binding offers (e.g. exploding offers), the incentive of firms to front-run can reduce some of these benefits. Hence, the optimal system is a hybrid of these two systems. On the one hand, we want to allow firms to interview and make public offers at different times, but we also want to prohibit front-running. If this is achievable, we then need firms to interview in rank order by nudging top firms to interview first.

How would such a system be implemented in practice? Theoretically, one could institute a rule that offers are non-binding until a given date (e.g. ban exploding offers). This exists in the college athletic scholarship market. Unlike this market, though, most labor markets lack a governing body that can enforce such a rule. Thus, an alternative is to encourage a culture of reneging: explain to candidates that before a common date, they are free to change their mind about an accepted offer if a more preferred offer comes along.

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# A Proofs

Before proving Proposition 1, we first define the deferred acceptance procedure that occurs in the matching stage of the centralized regime:

- First, each firm submits a complete and transitive strict preference order $\succ_f$ over match scores.
- Then, in each round, the workers make offers to the highest-ranked firm that has not rejected them, and each firm $f$ holds the $\succ_f$-highest-ranked offers from the workers in $I_f$, up to a mass $\Delta$, and rejects the rest, breaking ties uniformly at random.
- The game ends after any round in which no new offers are made.

**Proof of Proposition 1 (Equilibrium in the Centralized Regime: Matching Stage)** Our proof relies on two claims.

**Claim 1**: *Suppose that $\mathcal{M} = \{\mu_f\}_{f=1}^F$ is the outcome of a Nash equilibrium $\{\succ_f^*\}_{f=1}^F$ of the matching stage of the centralized game following the interview profile $\{I_f\}_{f=1}^F$. Then each firm $f$ receives offers in period $f$ from each worker that has not had an offer held by a firm $h < f$, and that (1) holds for each $A \subseteq [0,1]$ and $S \subseteq \mathbb{R}_+$.*

For $f = 1$, Claim 1 follows immediately from the fact that firm 1 immediately receives offers from each worker in $I_1$. Suppose Claim 1 holds for $f < k$; then in period $k$, no worker has made an offer to firm $k$, and so it receives offers from each worker that has not had an offer held by a firm $h < k$. Then if (1) does not hold, firm $k$'s payoff would be higher if it had submitted the preference order $\succ$ instead of $\succ_f^*$, and so $\{\succ_f^*\}_{f=1}^F$ is not an equilibrium, a contradiction. The claim follows by induction.

**Claim 2**: *Suppose that $\{\tilde{x}_f : [0,1] \times ([0,1]^{[0,1]})^{f-1} \to [0,1]\}_{f=1}^F$ is a subgame perfect equilibrium of the serial dictatorship game. Then on the equilibrium path, strategies must match the hiring rules $x_f^*(\cdot, \{I_h\}_{h=1}^f)$; that is, $\tilde{x}_f(s, \{\tilde{x}_h\}_{h=1}^{f-1}) = x_f^*(s, \{I_h\}_{h=1}^f)$ for each $f$.*

Observe that firm $f$'s payoffs do not depend on the hiring rules chosen by firms $h > f$. Then it is immediate that the claim holds for $f = 1$, and if it holds for $f < k$, it follows that it holds for $f = k$. The claim follows by induction.

The proposition then follows immediately from Claims 1 and 2. □

**Lemma 2.** *Suppose that $G$ has increasing* 1*-adjusted yields. Then whenever* $1 > a' > a > 0$,

*i. If $G(\cdot|a)$ does not place positive probability on* $\max\operatorname{supp} G(\cdot|a)$, *then* $\min\operatorname{supp} G(\cdot|a') < \max\operatorname{supp} G(\cdot|a)$.

*ii. If $G(\cdot|a)$ places positive probability on* $\max\operatorname{supp} G(\cdot|a)$, *then* $G(\max\operatorname{supp} G(\cdot|a)|a'') > 0$.

*Proof.* (i): Suppose that $G(\cdot|a)$ does not place positive probability on $\max\operatorname{supp} G(\cdot|a)$, and that $\min\operatorname{supp} G(\cdot|a') \geq \max\operatorname{supp} G(\cdot|a)$. Choose $\varepsilon > 0$ such that $0 < \int_{\max\operatorname{supp} G(\cdot|a)-\varepsilon}^{\infty} dG(s|a) < 1$, and let

$$x_1(s) = \begin{cases} 1, & s \geq \max\operatorname{supp} G(\cdot|a) - \varepsilon, \\ 0, & s < \max\operatorname{supp} G(\cdot|a) - \varepsilon. \end{cases}$$

We have $\int x_1(s)dG(s|a) = \int_{\max\operatorname{supp} G(\cdot|a)-\varepsilon}^{\infty} dG(s|a) \in (0,1)$, and $\int x_1(s)dG(s|a') = 1$. Let $x(s) = 1$ for all $s$. Then $\int x(s)sdG(s|a) = \int sdG(s|a)$. Then since $G$ has increasing 1-adjusted yields,

$$\begin{aligned} \int x(s)sdG(s|a') \int (1 - x_1(s))dG(s|a') = 0 \geq & \int x(s)sdG(s|a) \int (1 - x_1(s))dG(s|a) \\ = & \int sdG(s|a) \int_{\max\operatorname{supp} G(\cdot|a)-\varepsilon}^{\infty} dG(s|a) \\ > & \, c \int_{\max\operatorname{supp} G(\cdot|a)-\varepsilon}^{\infty} dG(s|a) > 0, \end{aligned}$$

a contradiction. It follows that $\min\operatorname{supp} G(\cdot|a') < \max\operatorname{supp} G(\cdot|a)$.

(ii): Suppose that $G(\cdot|a)$ places positive probability on $\max\operatorname{supp} G(\cdot|a)$, and that $G(\max\operatorname{supp} G(\cdot|a)|a') = 0$. Choose $x_1(s) = \begin{cases} 1, & s > \max\operatorname{supp} G(\cdot|a), \\ \frac{1}{2}, & s = \max\operatorname{supp} G(\cdot|a), \\ 0, & s < \max\operatorname{supp} G(\cdot|a). \end{cases}$ We have $\int x_1(s)dG(s|a) = \frac{1}{2}\int_{\{\max\operatorname{supp} G(\cdot|a)\}} dG(s|a) \in (0,1)$, and $\int x_1(s)dG(s|a') = 1$. Let $x(s) = 1$ for all $s$. Then $\int x(s)sdG(s|a) = \int sdG(s|a)$. Then since $G$ has increasing 1-adjusted yields,

$$\begin{aligned}\int x(s)sdG(s|a')\int(1-x_1(s))dG(s|a') = 0 \geq \int x(s)sdG(s|a)\int(1-x_1(s))dG(s|a) \\ = \int sdG(s|a)\left(1-\frac{1}{2}\int_{\{\max\operatorname{supp} G(\cdot|a)\}} dG(s|a)\right) \\ > c\left(1-\frac{1}{2}\int_{\{\max\operatorname{supp} G(\cdot|a)\}} dG(s|a)\right) > 0,\end{aligned}$$

a contradiction. It follows that $G(\max\operatorname{supp} G(\cdot|a)|a') > 0$.

□

**Proof of Lemma 1:** Without loss, let $k > 0$. Suppose that we have $a'', a' \in \operatorname{int}\bigcap_{f=1}^{k'}\left\{a \mid \int x_f(s)dG(s|a) > 0\right\}$ with $a'' > a'$. Choose $\underline{a} \in (0, a')$ and $\overline{a} \in (a'', 1)$. We consider two cases.

**Case 1:** *$G(\cdot|\underline{a})$ does not place positive probability on* $\max\operatorname{supp} G(\cdot|\underline{a})$. By Lemma 2 (i), $\min\operatorname{supp} G(\cdot|\overline{a}) < \max\operatorname{supp} G(\cdot|\underline{a})$. Then let $z = \frac{1}{2}(\min\operatorname{supp} G(\cdot|\overline{a}) + \max\operatorname{supp} G(\cdot|\underline{a}))$; by FOSD, for each $a \in [\underline{a}, \overline{a}]$,

$$\min\operatorname{supp} G(\cdot|a) \leq \min\operatorname{supp} G(\cdot|\overline{a}) < z < \max\operatorname{supp} G(\cdot|\underline{a}) \leq \max\operatorname{supp} G(\cdot|a),$$

so we have $0 < \int_z^\infty dG(s|\underline{a}) \leq \int_z^\infty dG(s|a) \leq \int_z^\infty dG(s|\overline{a}) < 1$ for all $a \in [\underline{a}, \overline{a}]$.

Now for each $f > k'$, choose $x_f(s) = \begin{cases} 1, & s \geq z, \\ 0, & s < z. \end{cases}$ Then for each $f > k'$ and each $a \in [\underline{a}, \overline{a}]$, $\int x_f(s)dG(s|a) = \int_z^\infty dG(s|a) \in (0,1)$. Then $a'', a' \in \operatorname{int}\bigcap_{f=1}^{k}\left\{a \mid \int x_f(s)dG(s|a) > 0\right\}$.

Then since $G$ has increasing $k$-adjusted yields, we have

$$\int x(s)sdG(s|a'')\prod_{f=1}^{k}\left(\int(1-x_f(s))dG(s|a'')\right) \geq \int x(s)sdG(s|a')\prod_{f=1}^{k}\left(\int(1-x_f(s))dG(s|a')\right). \tag{5}$$

or equivalently,

$$\begin{aligned}\int x(s)sdG(s|a'')\left(1-\int_z^\infty dG(s|a'')\right)^{k-k'}\prod_{f=1}^{k'}\left(\int(1-x_f(s))dG(s|a'')\right) \\ \geq \int x(s)sdG(s|a')\left(1-\int_z^\infty dG(s|a')\right)^{k-k'}\prod_{f=1}^{k'}\left(\int(1-x_f(s))dG(s|a')\right)\end{aligned}$$

By FOSD, $\left(1-\int_z^\infty dG(s|a'')\right)^{k-k'} \leq \left(1-\int_z^\infty dG(s|a')\right)^{k-k'}$. Dividing through then gives us

$$\int x(s)sdG(s|a'')\prod_{f=1}^{k'}\left(\int(1-x_f(s))dG(s|a'')\right) \geq \int x(s)sdG(s|a')\prod_{f=1}^{k'}\left(\int(1-x_f(s))dG(s|a')\right),$$

and so $G$ has increasing $k'$-adjusted yields.

**Case 2:** *$G(\cdot|\underline{a})$ places positive probability on* $\max\operatorname{supp} G(\cdot|\underline{a})$. Let $z = \max\operatorname{supp} G(\cdot|\underline{a})$. By Lemma 2 (ii), $G(z|a) > 0$ for each $a \in [\underline{a},\bar{a}]$. Now for each $f > k'$, choose $x_f(s) = \begin{cases} 1, & s > z, \\ \frac{1}{2}, & s = z, \\ 0, & s < z. \end{cases}$ It follows that for each $a \in [\underline{a},\bar{a}]$, $\int x_f(s)dG(s|a) < 1$. Since $G(\cdot|\underline{a})$ places positive probability on $z$, we have $\int x_f(s)dG(s|\underline{a}) > 0$; it follows from FOSD that for each $a \in [\underline{a},\bar{a}]$, $\int x_f(s)dG(s|a) > 0$. Then $a'', a' \in \operatorname{int}\bigcap_{f=1}^k \left\{a \mid \int x_f(s)dG(s|a) > 0\right\}$.

Then since $G$ has increasing $k$-adjusted yields, (5) holds. By FOSD, for each $f > k'$, since $x_f$ is nondecreasing, $0 < \int(1-x_f(s))dG(s|a'') \leq \int(1-x_f(s))dG(s|a')$. Dividing the left and right sides of (5) by $\prod_{f=k'+1}^k \int(1-x_f(s))dG(s|a'')$ and $\prod_{f=k'+1}^k \int(1-x_f(s))dG(s|a')$, respectively, gives us

$$\int x(s)sdG(s|a'')\prod_{f=1}^{k'}\left(\int(1-x_f(s))dG(s|a'')\right) \geq \int x(s)sdG(s|a')\prod_{f=1}^{k'}\left(\int(1-x_f(s))dG(s|a')\right).$$

Hence, $G$ has increasing $k'$-adjusted yields. □

**Lemma 3.** *For any $\beta \in (0,1)$ $G_\beta$ has increasing adjusted yields. For any decreasing $\lambda : [0,1] \to \mathbb{R}_+$, $G_\lambda$ has increasing $k$-adjusted yields for some $k \geq 1$.*

*Proof.* Given a hiring rule $x_f \in \mathcal{X}$, let $\bar{s}_f = \inf\left\{s|x_f(s) > 0\right\}$.

For an exponential distribution $H(t|\lambda) = 1 - e^{-\lambda t}$:

$$\left(\int(1-x_1(s))dH(s|\lambda)\right)\cdot\int x(s)sdH(s|\lambda) = (1-e^{-\bar{s}_1\lambda})\cdot e^{-\lambda\bar{s}}\cdot(\bar{s}+\frac{1}{\lambda})$$

Differentiating with respect to $\lambda$ yields:

$$\frac{e^{-\lambda(\bar{s}_1+\bar{s})}}{\lambda^2}\cdot\left[\lambda(\lambda x+1)(\bar{s}_1+\bar{s}) - e^{\lambda\bar{s}_1}(\lambda^2\bar{s}^2+\lambda\bar{s}+1)+1\right]$$

This quantity is less than or equal to 0 if and only if:

$$\lambda(\lambda\bar{s}+1)(\bar{s}_1+\bar{s}) - e^{\lambda\bar{s}_1}(\lambda^2\bar{s}^2+\lambda\bar{s}+1)+1 \leq 0$$

$$\iff (\lambda^2\bar{s}^2+\lambda\bar{s}+1)(1-e^{-\lambda\bar{s}_1}) + \lambda\bar{s}_1(\lambda\bar{s}+1) \leq 0 \iff 1-e^{-\lambda\bar{s}_1} > 1+\lambda\bar{s}_1$$

Thus, $H(\bar{s}_1|\lambda) \cdot (1 - H(\bar{s}|\lambda)) \cdot \mathbb{E}_H[t|t \geq \bar{s}]$ is decreasing in $\lambda$ for all $\bar{s}$ and $\bar{s}_1$. Since $G_\lambda(s|a) = H(s|\lambda(a))$ for some decreasing function $\lambda(\cdot) : [0,1] \to (0,\infty)$, it follows that $G_\lambda$ has increasing 1-adjusted yields.

Now observe that for any greedy hiring rule $y$, $\int y(s)dG_\beta(s|a) = (1-\beta)y(a) + \beta y(0) > 0 \Leftrightarrow y(a) > 0$. If this holds on a set $A$ for each $y \in \{x_f\}_{f=1}^k \subset \mathcal{X}$, then since $\{x_f\}_{f=1}^k$ are greedy, $x_f(a) = 1$ for each $a \in \operatorname{int} A$ and $f \in \{1,\ldots,k\}$. It follows that for each such $a$,

$$\int x(s)dG_\beta(s|a) \cdot \prod_{f=1}^{k}\left(\int (1-x_f(s))dG_\beta(s|a)\right) = (1-\beta)\cdot x(a)\cdot a \cdot \prod_{f=1}^{k}[\beta \cdot (1-x_f(0))]$$

Since $x$ is greedy, this expression must be nondecreasing in $a$; it follows that $G_\beta$ has increasing adjusted yields. $\square$

**Lemma 4** (Greedy Interviewing is Optimal)**.** *Let $\underline{a}, \overline{a} \in [0,1]$ and suppose that $\phi : [0,1] \to [0,1]$ and $\psi : [0,1] \to [0,1]$ are such that either (i) $\psi(a) = 1$ on $[\underline{a}, \overline{a}]$, or (ii) $\phi(a) = 1$, $G(\cdot|a)$ is increasing in a in the hazard rate order $\succeq_{HR}$, and for each greedy hiring rule x, $\int x(s)s\psi(a)dG(s|a)$ is nondecreasing in a on $(\underline{a}, \overline{a})$. Then for any interview set $I_f \subseteq [0,1]$ and greedy hiring rule $x_f$, there exists $a_f \in [\underline{a}, \overline{a}]$ and a greedy hiring rule $x'_f$ such that*

$$\begin{aligned}
&\int_{(I_f\setminus[\underline{a},\overline{a}])\cup[a_f,\overline{a}]}\left(\int_0^\infty x'_f(s)s\psi(a)dG(s|a) - c\right)\phi(a)da \geq \int_{I_f}\left(\int_0^\infty x_f(s)s\psi(a)dG(s|a) - c\right)\phi(a)da,\\
&\text{and } \int_{(I_f\setminus[\underline{a},\overline{a}])\cup[a_f,\overline{a}]}\left(\int x'_f(s)dG(s|a)\right)\psi(a)\phi(a)da \leq \int_{I_f}\left(\int x_f(s)dG(s|a)\right)\psi(a)\phi(a)da. \qquad (6)
\end{aligned}$$

*Proof.* Choose $a_f$ so that $\int_{a_f}^{\overline{a}} \phi(a)da = \int_{I_f\cap[\underline{a},\overline{a}]}\phi(a)da$. Let $\overline{S}$ be a random variable with distribution $\overline{G}$, and $\underline{S}$ be a random variable with distribution $\underline{G}$, where

$$\overline{G}(s) \equiv \frac{\int_{[a_f,\overline{a}]\setminus I_f} G(s|a)\psi(a)\phi(a)da}{\int_{[a_f,\overline{a}]\setminus I_f}\psi(a)\phi(a)da}, \qquad \underline{G}(s) \equiv \frac{\int_{[\underline{a},a_f]\cap I_f} G(s|a)\psi(a)\phi(a)da}{\int_{[\underline{a},a_f]\cap I_f}\psi(a)\phi(a)da}.$$

**Step 1:** $\overline{S} \succeq_{FOSD} \underline{S}$. Since $G(s|a') \leq G(s|a)$ for all $a' > a$, for all $s \geq 0$, we have

$$\begin{aligned}
&G(s|a')\psi(a')\phi(a')\psi(a)\phi(a) \leq G(s|a)\psi(a')\phi(a')\psi(a)\phi(a) \text{ for all } a' > a\\
\Rightarrow &\int_{[a_f,\overline{a}]\setminus I_f} G(s|a)\psi(a)\phi(a)da \int_{[\underline{a},a_f]\cap I_f}\psi(a)\phi(a)da \leq \int_{[\underline{a},a_f]\cap I_f} G(s|a)\psi(a)\phi(a)da \int_{[a_f,\overline{a}]\setminus I_f}\psi(a)\phi(a)da\\
&\overline{G}(s) \leq \underline{G}(s),
\end{aligned}$$

as desired.

**Step 2:** *In case (ii),* $\overline{S} \succeq_{HR} \underline{S}$. Given $s' > s$, since $(1 - G(s'|a'))(1 - G(s|a)) \geq (1 - G(s|a'))(1 - G(s'|a))$ for all $a' > a$, we have

$$
\begin{aligned}
&(1-G(s'|a'))\psi(a')\phi(a')(1-G(s|a))\psi(a)\phi(a) \geq (1-G(s'|a))\psi(a)\phi(a)(1-G(s|a'))\psi(a')\phi(a') \text{ for all } a' > a \\
&\int_{[a_f,\overline{a}]\setminus I_f}(1-G(s'|a))\psi(a)\phi(a)da \int_{[\underline{a},a_f]\cap I_f}(1-G(s|a))\psi(a)\phi(a)da \\
&\qquad\geq \int_{[\underline{a},a_f]\cap I_f}(1-G(s'|a))\psi(a)\phi(a)da \int_{[a_f,\overline{a}]\setminus I_f}(1-G(s|a))\psi(a)\phi(a)da \\
&\left(1-\frac{\int_{[a_f,\overline{a}]\setminus I_f} G(s'|a)\psi(a)\phi(a)da}{\int_{[a_f,\overline{a}]\setminus I_f}\psi(a)\phi(a)da}\right)\left(1-\frac{\int_{[\underline{a},a_f]\cap I_f} G(s|a)\psi(a)\phi(a)da}{\int_{[\underline{a},a_f]\cap I_f}\psi(a)\phi(a)da}\right) \\
&\qquad\geq \left(1-\frac{\int_{[\underline{a},a_f]\cap I_f} G(s'|a)\psi(a)\phi(a)da}{\int_{[\underline{a},a_f]\cap I_f}\psi(a)\phi(a)da}\right)\left(1-\frac{\int_{[a_f,\overline{a}]\setminus I_f} G(s|a)\psi(a)\phi(a)da}{\int_{[a_f,\overline{a}]\setminus I_f}\psi(a)\phi(a)da}\right)
\end{aligned}
$$

$$\implies (1-\overline{G}(s'))(1-\underline{G}(s)) \geq (1-\underline{G}(s'))(1-\overline{G}(s)) \text{ as desired}$$

**Step 3:** *There exists a greedy hiring rule $\overline{x_f}$ such that*

$$\int_{[a_f,\overline{a}]\setminus I_f}\int_0^\infty \overline{x_f}(s)s\psi(a)\phi(a)dG(s|a)da \geq \int_{[\underline{a},a_f]\cap I_f}\int_0^\infty x_f(s)s\psi(a)\phi(a)dG(s|a)da, \tag{7}$$

$$\text{and} \int_{[a_f,\overline{a}]\setminus I_f}\left(\int \overline{x_f}(s)dG(s|a)\right)\psi(a)\phi(a)da \leq \int_{[\underline{a},a_f]\cap I_f}\left(\int x_f(s)dG(s|a)\right)\psi(a)\phi(a)da. \tag{8}$$

Let $M = \int_{[\underline{a},a_f]\cap I_f}\left(\int x_f(s)dG(s|a)\right)\psi(a)\phi(a)da$, and

$$
\begin{aligned}
\overline{\Psi} &= \int_{[a_f,\overline{a}]\setminus I_f}\psi(a)\phi(a)da, & \underline{\Psi} &= \int_{[\underline{a},a_f]\cap I_f}\psi(a)\phi(a)da, \\
\overline{s_f} &\equiv \overline{G}^{-1}\left(\max\{1-M/\overline{\Psi},0\}\right), & \underline{s_f} &\equiv \underline{G}^{-1}\left(1-M/\underline{\Psi}\right), \\
\overline{p_f} &\equiv \min\left\{\frac{M/\overline{\psi}-(1-\overline{G}(\overline{s_f}))}{\int_{\{\overline{s_f}\}}d\overline{G}(s)},1\right\}, & \underline{p_f} &\equiv \frac{M/\underline{\Psi}-(1-\underline{G}(\underline{s_f}))}{\int_{\{\underline{s_f}\}}d\underline{G}(s)},
\end{aligned}
\qquad
\overline{x_f}(s) = \begin{cases} 1, & s > \overline{s_f}; \\ \overline{p_f}, & s = \overline{s_f}; \\ 0, & s < \overline{s_f}, \end{cases}
$$

Observe that since $x_f$ is a greedy hiring rule, we must have $x_f(s) = \begin{cases} 1, & s > \underline{s_f}; \\ \underline{p_f}, & s = \underline{s_f}; \\ 0, & s < \underline{s_f}, \end{cases}$

Moreover, by construction,

$$\int_{[a_f,\bar{a}]\setminus I_f}\left(\int \overline{x_f}(s)dG(s|a)\right)\psi(a)\phi(a)da = \min\{M,\overline{\Psi}\} \tag{9}$$

$$\int_{[a_f,\bar{a}]\setminus I_f}\left(\int \overline{x_f}(s)sdG(s|a)\right)\psi(a)\phi(a)da = \overline{\Psi}\int \overline{x_f}(s)sd\overline{G}(s) = \overline{\Psi}\int_{\max\{1-M/\overline{\Psi},0\}}^{1}\overline{G}^{-1}(q)dq; \tag{10}$$

$$\int_{[\underline{a},a_f]\cap I_f}\left(\int x_f(s)sdG(s|a)\right)\psi(a)\phi(a)da = \underline{\Psi}\int x_f(s)sd\underline{G}(s) = \underline{\Psi}\int_{1-M/\underline{\Psi}}^{1}\underline{G}^{-1}(q)dq. \tag{11}$$

Hence, (8) holds.

For (7), first consider case (i). By construction of $a_f$,

$$\overline{\Psi} = \int_{[a_f,\bar{a}]\setminus I_f}\phi(a)da = \int_{[\underline{a},a_f]\cap I_f}\phi(a)da = \underline{\Psi}.$$

Then $M/\overline{\Psi} = M/\underline{\Psi} \le 1$. By Step 1, $\overline{S} \ge_{FOSD} \underline{S}$, and so $\overline{G}^{-1}(q) \ge \underline{G}^{-1}(q)$ for all $q \in [0,1]$. Then

$$\int_{1-M/\overline{\Psi}}^{1}\overline{G}^{-1}(q)dq \ge \int_{1-M/\overline{\Psi}}^{1}\underline{G}^{-1}(q)dq = \int_{1-M/\underline{\Psi}}^{1}\underline{G}^{-1}(q)dq,$$
$$\Rightarrow \overline{\Psi}\int_{1-M/\overline{\Psi}}^{1}\overline{G}^{-1}(q)dq \ge \underline{\Psi}\int_{1-M/\underline{\Psi}}^{1}\underline{G}^{-1}(q)dq.$$

The claim then follows from (10) and (11).

Now consider case (ii). First suppose that either $\overline{s_f} > \underline{s_f}$, or $\overline{s_f} = \underline{s_f}$ and $\overline{p_f} < \underline{p_f}$. Then $\overline{x_f}(s) \le x_f(s)$ for each $s$. Moreover, we must have $M \le \overline{\Psi}$: Suppose not. Then $\underline{s_f} \le \overline{s_f} = \overline{G}^{-1}(0) = 0$. So we must have $\overline{s_f} = \underline{s_f}$ and $\overline{p_f} = 1 < \underline{p_f}$, a contradiction.

Define the weighted random variables $\underline{S}_{x_f} \sim \underline{G}_{x_f}$, $\overline{S}_{x_f} \sim \overline{G}_{x_f}$, and $\overline{S}_{\overline{x_f}} \sim \overline{G}_{\overline{x_f}}$, where

$$\underline{G}_{x_f}(s) = \frac{\int_0^s x_f(t)d\underline{G}(t)}{\int_0^\infty x_f(t)d\underline{G}(t)},\quad \overline{G}_{x_f}(s) = \frac{\int_0^s x_f(t)d\overline{G}(t)}{\int_0^\infty x_f(t)d\overline{G}(t)},\quad \overline{G}_{\overline{x_f}}(s) = \frac{\int_0^s \overline{x_f}(t)d\overline{G}(t)}{\int_0^\infty \overline{x_f}(t)d\overline{G}(t)} = \frac{\int_0^s w(t)x_f(t)d\overline{G}(t)}{\int_0^\infty w(t)x_f(t)d\overline{G}(t)},$$
$$\text{where } w(t) = \begin{cases} \overline{x_f}(t)/x_f(t), & t \ge \overline{s_f}; \\ 0, & t < \overline{s_f}. \end{cases}$$

From Step 2 and Bartoszewicz and Skolimowska (2006) Theorem 9, $\overline{S}_{x_f} \ge_{HR} \underline{S}_{x_f}$, and hence $\overline{S}_{x_f} \ge_{FOSD} \underline{S}_{x_f}$. Since $\overline{x_f}(t) \le x_f(t)$ for each $t$, $w$ is nondecreasing; then by Theorem 1 in Błażej (2008)[14] $\overline{S}_{\overline{x_f}} \ge_{FOSD} \overline{S}_{x_f}$. It follows

[14] Since $w$ is nondecreasing, we have (in the notation of Błażej (2008))

$$\overline{G}^*_{x_f}(u) - u = \frac{1}{\int w(s)d\overline{G}_{x_f}(s)}\left((1-u)\int_0^u w(\overline{G}_{x_f}^{-1}(z))dz - u\int_u^1 w(\overline{G}_{x_f}^{-1}(z))dz\right) \le \frac{(1-u)uw(\overline{G}_{x_f}^{-1}(u)) - (1-u)uw(\overline{G}_{x_f}^{-1}(u))}{\int w(s)d\overline{G}_{x_f}(s)} = 0.$$

that

$$\frac{\int_{[a_f,\bar{a}]\setminus I_f}\left(\int s\overline{x_f}(s)dG(s|a)\right)\psi(a)\phi(a)da}{\int_{[a_f,\bar{a}]\setminus I_f}\left(\int \overline{x_f}(s)dG(s|a)\right)\psi(a)\phi(a)da} = E[\overline{S}_{\overline{x_f}}] \geq E[\underline{S}_{x_f}] = \frac{\int_{[\underline{a},a_f]\cap I_f}\left(\int sx_f(s)dG(s|a)\right)\psi(a)\phi(a)da}{\int_{[\underline{a},a_f]\cap I_f}\left(\int x_f(s)dG(s|a)\right)\psi(a)\phi(a)da}.$$

Since $M \leq \overline{\Psi}$, the claim then follows from (9).

Alternatively, consider the case where either $\overline{s_f} < \underline{s_f}$, or $\overline{s_f} = \underline{s_f}$ and $\overline{p_f} \geq \underline{p_f}$. Then $\overline{x_f}(s) \geq x_f(s)$ for each $s$, and we have

$$\int_{[a_f,\bar{a}]\setminus I_f}\int \overline{x_f}(s)s\psi(a)\phi(a)dG(s|a)da \geq \int_{[a_f,\bar{a}]\setminus I_f}\int x_f(s)s\psi(a)\phi(a)dG(s|a)da \geq \int_{[\underline{a},a_f]\cap I_f} sx_f(s)\psi(a)\phi(a)dG(s|a)da,$$

where the second inequality holds since, by assumption, $\phi(a) = 1$ and $\int x_f(s)s\psi(a)dG(s|a)$ is nondecreasing in $a$ on $(\underline{a}, \bar{a})$. The claim then follows from (9).

**Step 4:** $\int_{(I_f\setminus[\underline{a},\bar{a}])\cup[a_f,\bar{a}]} c\phi(a)da = \int_{I_f} c\phi(a)da$. Follows from construction of $a_f$.

**Step 5.** We now prove the statement of Lemma 4. By Step 3, there is a greedy hiring rule $\overline{x_f}$ such that

$$\begin{aligned} &\begin{array}{l}\int_{I_f\setminus[\underline{a},a_f]}\int x_f(s)s\psi(a)\phi(a)dG(s|a)da\\ +\int_{[a_f,\bar{a}]\setminus I_f}\int \overline{x_f}(s)s\psi(a)\phi(a)dG(s|a)da\end{array} \geq \int_{I_f}\int_{s_f}^{\infty} s\psi(a)\phi(a)dG(s|a)da, \qquad (12)\\ \text{and } &\begin{array}{l}\int_{I_f\setminus[\underline{a},a_f]}\int x_f(s)dG(s|a)\psi(a)\phi(a)da\\ +\int_{[a_f,\bar{a}]\setminus I_f}\int \overline{x_f}(s)dG(s|a)\psi(a)\phi(a)da\end{array} \leq \int x_f(s)dG(s|a)\psi(a)\phi(a)da.\end{aligned}$$

Then the left-hand side of (12) must be no greater than the value of

$$\max_{x,y\in\mathcal{X}} \int_{I_f\setminus[\underline{a},a_f]}\int x(s)s\psi(a)\phi(a)dG(s|a)da + \int_{[a_f,\bar{a}]\setminus I_f}\int y(s)s\psi(a)\phi(a)dG(s|a)da \qquad (13)$$

$$\text{s.t.} \begin{array}{l}\int_{I_f\setminus[\underline{a},a_f]}\int x(s)dG(s|a)\psi(a)\phi(a)da\\ +\int_{[a_f,\bar{a}]\setminus I_f}\int y(s)dG(s|a)\psi(a)\phi(a)da\end{array} \leq \int_{I_f}\int x_f(s)dG(s|a)\psi(a)\phi(a)da. \qquad (14)$$

Clearly, it is without loss to consider $x = y$ in (13): Then let $x^*$ be such that $(x^*, x^*)$ solves (13), and choose $x'_f = x^*$; the statement of Lemma 4 then follows from Step 4. □

The intuition for Lemma 4 is simplest when $\phi(a) = \psi(a) = 1$ for all $a$ — the relevant case for firm 1. Given an interview set $I$, we can construct a greedy interview set $[1 - \mu(I), 1]$ with the same mass, and keep the hiring rule the same for those workers interviewed in both sets. Then, for the workers that are interviewed in the greedy set but not in $I$, we choose a new greedy hiring rule, so that the firm hires the same mass of them as it would have hired from $I \setminus [1 - \mu(I), 1]$ (the workers interviewed in $I$ but not in the greedy set). First-order stochastic dominance then ensures that conditional on being hired, the average match value of workers only interviewed in the greedy set is higher than the average match value of workers only interviewed in the original set $I$.

**Lemma 5** (Existence of Optimal Interview Sets). *For each measurable $\psi : [0,1] \to [0,1]$ and $\phi : [0,1] \to [0,1]$, there exists a solution*

$$(I^*, x^*) \in \arg\max_{\substack{I \subseteq [0,1] \\ x \in \mathcal{X}}} \int_I \left( \int x(s) s dG(s|a) \psi(a) - c \right) \phi(a) da \text{ s.t. } \int_I \left( \int x(s) dG(s|a) \right) \phi(a) \psi(a) da \leq \Delta,$$
$$\text{where } x^* = \max_{\succ_{\mathcal{X}}} \left\{ x \in \mathcal{X} \mid \int_I \left( \int x(s) dG(s|a) \right) \phi(a) \psi(a) da \leq \Delta \right\}.$$

*Proof.* Let $[0,1]^{[0,1]}$ be the set of all functions $y : [0,1] \to [0,1]$; $\mathcal{Y} \subseteq [0,1]^{[0,1]}$, the set of all characteristic functions of Borel sets $S \subseteq [0,1]$; and $\mathcal{X} \subseteq [0,1]^{[0,1]}$, the set of greedy hiring rules, each given the topology of pointwise convergence. Since $\mathcal{X}$ and $\mathcal{Y}$ are closed under pointwise limits, and by Tychonoff's theorem, $[0,1]^{[0,1]}$ is compact, $\mathcal{X}$ and $\mathcal{Y}$ are compact.

By Lebesgue's dominated convergence theorem, the functions $\tilde{\pi}, \delta : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$ given by

$$\tilde{\pi}(x, y) \equiv \int_0^1 y(a) \left( \int_0^\infty s x(s) dG(s|a) \psi(a) - c \right) \phi(a) da,$$
$$\delta_f(x, y) \equiv \int_0^1 y(a) \left( \int_0^\infty x(s) dG(s|a) \psi(a) - c \right) \phi(a) da$$

are continuous. Then by Weierstrass' theorem, the problem

$$\max_{x \in \mathcal{X}, y \in \mathcal{Y}} \tilde{\pi}(x, y) \text{ s.t. } \delta(x, y) \leq \Delta$$

has a solution. Since replacing $x$ with $x' \succ_{\mathcal{X}} x$ can only increase the value of the objective, and $\succ_{\mathcal{X}}$ is a total order, it is without loss to consider solutions $(\tilde{x}, y^*)$ where

$$\tilde{x} = \max_{\succ_{\mathcal{X}}} \left\{ x \in \mathcal{X} \mid \int_0^1 y^*(a) \left( \int x(s) dG(s|a) \right) \phi(a) \psi(a) da \leq \Delta \right\}.$$

The claim follows immediately for $I_f^* = y^{*-1}(1)$.

$\square$

**Corollary 1.** *For any $\phi : [0,1] \to [0,1]$, there exists $a^* \in [0,1]$ such that*

$$([a^*, 1], x^*) \in \arg\max_{\substack{I \subseteq [0,1] \\ x \in \mathcal{X}}} \int_I \left( \int x(s) s dG(s|a) - c \right) \phi(a) da \text{ s.t. } \int_I \left( \int x(s) dG(s|a) \right) \phi(a) da \leq \Delta, \tag{15}$$
$$\text{where } x^* = \max_{\succ_{\mathcal{X}}} \left\{ x \in \mathcal{X} \mid \int_I \left( \int x(s) dG(s|a) \right) \phi(a) da \leq \Delta \right\}.$$

*Proof.* By Lemma 5, (15) has a solution. It is immediate from case (i) of Lemma 4 that it is without loss to consider solutions of the form $([a^*, 1], x)$ for some $a^* \in [0,1]$. The claim follows by noting that replacing $x$ with $x' \succ_{\mathcal{X}} x$ can only increase the value of the objective in (15). $\square$

**Proof of Proposition 2 (Centralized Regime: Greedy Interviews at the Top)** Follows immediately from (2) and Corollary 1. $\square$

**Proof of Proposition 3 (Equilibrium in the Centralized Regime: Interview Stage)** (i): Follows immediately from Proposition 2.

(iia) and (iib): We proceed by induction, beginning with $f = 2$. Let

$$\psi_2(a) = \begin{cases} \int (1 - x_1^*(s))dG(s|a), & a \geq a_1^*; \\ 1, & a < a_1^*. \end{cases}$$

By Lemma 5, the problem

$$\max_{\substack{I \subseteq [0,1] \\ x \in \mathcal{X}}} \int_I \int x(s) s dG(s|a)\psi_2(a) - c da \text{ s.t. } \int_I \int x(s) dG(s|a)\psi_2(a) da \leq \Delta \tag{16}$$

has a solution.

Since $G$ has increasing $k$-adjusted yields, by Lemma 1, it has increasing 1-adjusted yields, and so for any greedy hiring rule $x$, $\int x(s)s\psi_2(a)dG(s|a)$ is nondecreasing on $[a_1^*, 1]$. Then by case (ii) of Lemma 4 (letting $\underline{a} = a_1^*$ and $\overline{a} = 1$), for any $I_2 \subseteq [0, 1]$ and $x \in \mathcal{X}$, there exists $a_2^{\{1\}} \in [a_1^*, 1]$ and $x' \in \mathcal{X}$ such that

$$\begin{aligned} \int_{(I_2 \setminus I_1^*) \cup [a_2^{\{1\}}, \overline{a}]} \left( \int x'(s) s \psi_2(a) dG(s|a) \right) - c da &\geq \int_{I_2} \left( \int x(s) s \psi_2(a) dG(s|a) \right) - c da, \\ \text{and } \int_{(I_2 \setminus I_1^*) \cup [a_2^{\{1\}}, \overline{a}]} \int x'(s) dG(s|a) \psi_2(a) da &\leq \int_{I_2} \int x(s) dG(s|a) \psi_2(a) da. \end{aligned}$$

Applying case (i) of Lemma 4, letting $\underline{a} = 0$ and $\overline{a} = a_1^*$, shows that there exists $a_2^* \in [0, a_1^*]$ and $x'' \in \mathcal{X}$ such that

$$\begin{aligned} \int_{[a_2, a_1^*) \cup [a_2^{\{1\}}, \overline{a}]} \left( \int x''(s) s \psi_2(a) dG(s|a) \right) - c da &\geq \int_{I_2} \left( \int x(s) s \psi_2(a) dG(s|a) \right) - c da, \\ \text{and } \int_{[a_2, a_1^*) \cup [a_2^{\{1\}}, \overline{a}]} \int x''(s) dG(s|a) \psi_2(a) da &\leq \int_{I_2} \int x(s) dG(s|a) \psi_2(a) da. \end{aligned} \tag{17}$$

It follows that for some greedy hiring rule $x_2$ and some $a_2^{\{1\}*} \in [a_1^*, 1]$ and $a_2^* \in [0, a_1^*]$, $(I_2^*, x_2)$ solves (16), where $I_2^* = [a_2^*, a_1^*) \cup [a_2^{\{1\}*}, 1]$. Since replacing $x''$ with $\tilde{x} >_{\mathcal{X}} x''$ cannot decrease the value of the objective in (16), it is without loss to let $x_2 = \max_{>_{\mathcal{X}}} \left\{ x \in \mathcal{X} \mid \int_{[a_2^*, a_1^*] \cup [a_2^{\{1\}*}, \overline{a}]} \left( \int x(s) \psi_2(a) dG(s|a) \right) da \leq \Delta \right\}$. Consequently, $I_2^* \in \arg\max_{I_2} \pi_2(I_1^*, I_2)$. Hence, (iia) and (iib) hold for $f = 2$.

Now consider $2 < f \leq k$, and suppose that (iia) and (iib) hold for each $2 \leq f' < f$. Then for each $S \subseteq \{1, \ldots, f-1\}$, $\bigcap_{j \in S} I_j^*$ is an interval. Let $\psi_f(a) = \prod_{h < f, a \in I_h^*} \left( \int (1 - x_h(s)) dG(s|a) \right)$. By Lemma 5, the problem

$$\max_{\substack{I \subseteq [0,1] \\ x \in \mathcal{X}}} \int_I \int x(s) s dG(s|a)\psi_f(a) - c da \text{ s.t. } \int_I \int x(s) dG(s|a)\psi_f(a) da \leq \Delta \tag{18}$$

has a solution.

Since $G$ has increasing $k$-adjusted yields, by Lemma 1, it has increasing $k'$-adjusted yields for each $k' < k$.

Then for any greedy hiring rule $x$ and any $S \subseteq \{1, \ldots, f-1\}$, $\int x(s)s\psi_f(a)dG(s|a)$ is nondecreasing in $a$ on $\bigcap_{j\in S} I_j^*$. Then applying case (ii) of Lemma 4 once for each $S \subseteq \{1, \ldots, f-1\}$ (letting $\underline{a} = \min \bigcap_{j\in S} I_j^*$ and $\overline{a} = \max \bigcap_{j\in S} I_j^*$), for any $I_f \subseteq [0,1]$ and $x \in \mathcal{X}$, there exist $\{a_f^S\}_{S\subseteq\{1,\ldots,f-1\}}$ and $x' \in \mathcal{X}$ such that for $I_f' = \bigcup_{S\subseteq\{1,\ldots,f-1\}} \left([a_f^S, 1] \cap \bigcap_{j\in S} I_j^*\right)$,

$$\int_{I_f'} \left(\int x'(s)s\psi_f(a)dG(s|a)\right) - cda \geq \int_{I_f} \left(\int x(s)s\psi_f(a)dG(s|a)\right) - cda, \tag{19}$$

$$\text{and } \int_{I_f'} \left(\int x'(s)\psi_f(a)dG(s|a)\right) da \leq \int_{I_f} \left(\int x(s)\psi_f(a)dG(s|a)\right) da.$$

It follows that for some greedy hiring rule $x_f$ and some $\{a_f^{S*}\}_{S\subseteq\{1,\ldots,f-1\}}$, $(I_f^*, x_f)$ solves (18), where $I_f^* = \bigcup_{S\subseteq\{1,\ldots,f-1\}} \left([a_f^{S*}, 1] \cap \bigcap_{j\in S}\right.$ Since replacing $x'$ with $\tilde{x} >_{\mathcal{X}} x'$ cannot decrease the value of the objective in (18), it is without loss to let $x_f = \max_{>_{\mathcal{X}}} \left\{x \in \mathcal{X} \mid \int_{I_f^*} \left(\int x(s)\psi_f(a)dG(s|a)\right) da \leq \Delta\right\}$. Consequently, $I_f^* \in \arg\max_{I_f} \pi_f(I_f, \{I_h^*\}_{h<f})$. The claim follows by induction. □

**Lemma 6** (Equilibrium Payoffs in Centralized Matching)**.** *For each $f$ and each $f' > f$, $\pi_f(\{I_k^*\}_{k=1}^f) \geq \pi_{f'}(\{I_k^*\}_{k=1}^{f'})$.*

*Proof.* We show that $\pi_f(\{I_k^*\}_{k=1}^f) \geq \pi_{f+1}(\{I_k^*\}_{k=1}^{f+1})$ for each $f < F$. Since $I_f^*$ is a best response to $\{I_k^*\}_{k=1}^{f-1}$ in the interview stage,

$$\begin{aligned}
\pi_f(\{I_k^*\}_{k=1}^f) \geq & \pi_f(\{I_k^*\}_{k=1}^{f-1}, I_{f+1}^*) \\
= & \max_{x\in\mathcal{X}} \left\{ \begin{array}{l} \int_{I_{f+1}^*} \int_0^\infty sx(s)dG(s|a) \left(\prod_{h<f, I_h^*\ni a} \int_0^\infty (1 - x_h^*(t, \{I_k^*\}_{k=1}^h)dG(t|a))\right) da \\ \text{s.t. } \int_{I_{f+1}^*} \int_0^\infty x(s)dG(s|a) \left(\prod_{h<f, I_h^*\ni a} \int_0^\infty (1 - x_h^*(t, \{I_k^*\}_{k=1}^h))dG(t|a)\right) da \leq \Delta \end{array} \right\} \\
= & \max_{\mu_f\in\mathcal{M}_f} \left\{ \begin{array}{l} \int_{I_{f+1}^*\times\mathbb{R}} sd\mu_f - c\int_{I_{f+1}^*} da \text{ s.t. } \int_{I_f^*\times\mathbb{R}} d\mu_f \leq \Delta, \\ \mu_f(S\times[0,s]) \leq \int_{I_{f+1}^*} G(s|a) \left(\prod_{h<f, I_h^*\ni a} \int_0^\infty (1 - x_h^*(t, \{I_k^*\}_{k=1}^h))dG(t|a)\right) da \ \forall S \subseteq [0,1], s \geq 0 \end{array} \right\}; \\
& \hspace{30em} (20) \\
\geq & \int_{I_{f+1}^*} \int_0^\infty sx_{f+1}^*(s, \{I_k^*\}_{k=1}^{f+1})dG(s|a) \left(\prod_{h\leq f, I_h^*\ni a} \int_0^\infty (1 - x_h^*(t, \{I_k^*\}_{k=1}^h))dG(t|a)\right) da \qquad (21) \\
= & \pi_{f+1}(\{I_k^*\}_{k=1}^{f+1}),
\end{aligned}$$

where (20) follows from the fact that the only payoff-relevant part of the outcome $\mu_f$ is the marginal distribution of match values $s \in \mathbb{R}$, while (21) follows by choosing $\mu_f$ defined by

$$\mu_f(S \times [0, s]) = \int_S \int_0^s x^*_{f+1}(t, \{I^*_k\}_{k=1}^{f+1}) dG(t|a) \left( \prod_{h \leq f, I^*_h \ni a} \int_0^\infty (1 - x^*_h(r, \{I^*_k\}_{k=1}^h)) dG(r|a) \right) da$$

$$\leq \int_{I^*_{f+1}} G(s|a) \left( \prod_{h < f, I^*_h \ni a} \int_0^\infty (1 - x^*_h(t, \{I^*_k\}_{k=1}^h)) dG(t|a) \right) da,$$

for each $S \subseteq [0, 1]$ and $s \geq 0$. $\square$

**Proof of Proposition 4 (Decentralized Matching with Binding Offers: Equilibrium and Welfare)** Define the following objects:

1. For any $S \subset \{1, \ldots, n\}$ define $r(S, i)$ to be the rank of firm $i$ in S.
2. For any $\psi : [0, 1] \to [0, 1]$ such that $\int_0^1 \psi(a)da \leq 1$, define the candidate pool associated with $\psi$ to be the pair $(W_\psi, \int_0^1 \psi(a)da)$, where $W_\psi(a) = \frac{a}{\int_0^a \psi(s)ds}$. In other words, there is a measure $\int_0^1 \psi(a)da$ of applicants on $[0, 1]$, and the conditional distribution is given by $W_\psi$. Due to the one-to-one correspondence, we will refer to $\psi$ as the applicant pool. Thus, $\psi(a) = 1$ is the original applicant pool. In an abuse of notation, we will also use $\psi$ to refer to the actual set of applicants.
3. For any $i \in \{1, \ldots, n\}$ and candidate pool $\psi$, let $\sigma(i, \psi)$ be the optimal interview strategy for an $i$-th ranked firm in a centralized setting with candidate pool $\psi$.
4. Given a subset of firms $S$ and applicant pool $\psi$, for each $i \in S$, let $C_i(S, \psi)$ denote the equilibrium payoff in a centralized matching setting where firms $S$ face candidate pool $\psi$. $C_i(S, \psi)$ is pinned down due to firm $j \in S$ using equilibrium strategy $\sigma(r(S, j), \psi)$.
5. Consider a centralized matching setting with firms $S \subset \{1, \ldots, F\}$ and an applicant pool $\psi$, Let $\mu(S, \psi)$ be the set of applicants firms in $S$ match with in equilibrium. Define $\Xi(S, \psi) = \psi \setminus \mu(S, \psi)$.

We proceed by induction on the number of time periods.

**Base Case:** $T = 2$

Firms only have two choices: interview in time period 1 or 2. Let $A \in \{0, 1\}^n$ denote the choice profile for each firm, where $A_i = 0$ means firm $i$ is choosing to interview in period 1. Notice then, that for any $A$, firm $i$'s payoff is uniquely pinned down:

1. If $A_i = 0$, then firm $i$'s payoff is $\pi_i(A) = C_i(\{j|A_j = 0\}, \psi)$
2. If $A_i = 1$, then firm $i$'s payoff is $\pi_i(A) = C_i\Big(\{j|A_j = 1\}, \Xi(\{j|A_j = 0\}, \psi)\Big)$

A mixed strategy equilibrium exists since there are a finite number of players and actions!

**Inductive Step: Assume that a subgame perfect equilibrium exists for all $T \leq k$**

For any subset of firms $S$ and applicant pool $\psi$, let $E_i(T, S, \psi)$ denote the equilibrium payoff to firm $i$ in a game with $T$ time periods.

**Case $T = k + 1$:**

Now, consider the following game: firms can choose to interview in time period 1 or defer to a later period. Let $A \in \{0, 1\}^F$ denote the choice profile for each firm, where $A_i = 0$ means firm $i$ is choosing to interview in period 1. Notice, then, that for any $A$, firm $i$'s payoff is uniquely pinned down by the inductive hypothesis:

1. If $A_i = 0$, then firm $i$'s payoff is $\pi_i(A) = C_i\left( \left\{j|A_j = 0\right\}, \psi\right)$
2. If $A_i = 1$, then firm $i$'s payoff is $\pi_i(A) = E_i\left(k, \left\{j|A_j = 1\right\}, \Xi(\left\{j|A_j = 0\right\}, \psi)\right)$

A mixed strategy equilibrium exists in this game as there are two actions and $F$ players. Equilibrium payoff must be the subgame perfect equilibrium payoffs.

Finally, in any subgame perfect equilibrium $\sigma^B$, each firm $f$ must receive a payoff $\pi_f^B(\sigma^B)$ weakly higher than their centralized market payoff $\pi_f(\{I_k^*\}_{k=1}^f)$: Since $\sigma^B$ is a subgame perfect equilibrium, there cannot be a profitable one-shot deviation by firm $f$ in the interview stage of period 1. Consider some terminal history following such a deviation, and suppose that $S \ni f$ is the set of firms that interview in period 1 in that terminal history. Since $\sigma^B$ is subgame perfect, in that terminal history, each firm $j \in S$ must play $\sigma(r(S, j), 1)$ in the interview and hiring stages of period 1. Then firm $f$'s payoff in that terminal history is $\pi_{r(S,f)}(\{I_k^*\}_{k=1}^{r(S,f)})$. Observe that for any $S$, $r(S, f) \leq f$; it follows from Lemma 6 that $\pi_f^B(\sigma^B) \geq \min_{S \ni f} \pi_{r(S,f)}(\{I_k^*\}_{k=1}^{r(S,f)}) = \pi_f(\{I_k^*\}_{k=1}^f)$.

□

**Proof of Proposition 5 (Decentralized Matching with Nonbinding Offers: Equilibrium Timing)** Let $\mathcal{H}_{Ent}^t$, $\mathcal{H}_{Int}^t$, and $\mathcal{H}_{Hire}^t$ be the set of partial histories ending just before the period-$t$ entry, interview, and hiring stages, respectively; let $\mathcal{H}^t = \mathcal{H}_{Ent}^t \cup \mathcal{H}_{Int}^t \cup \mathcal{H}_{Hire}^t$. For each partial history $h \in \mathcal{H}^t$, let $\mathcal{F}(h) \subseteq \{1, \ldots, F\} \setminus \{f\}$ be the set of firms that have interviewed in periods $t' < t$, and for each partial history $h \in \mathcal{H}_{Int}^t \cup \mathcal{H}_{Hire}^t$, let $\mathcal{F}^t(h) \subseteq \{1, \ldots, F\} \setminus (\mathcal{F})$ be the set of firms that have chosen to interview in the entry stage of period $t$.

(i): Let each firm $f$'s *sequential-hiring strategy* $\sigma_f^{SH}$ be as follows.

- In the entry stage of period $t$, after a history $h \in \mathcal{H}_{Ent}^t$ with $f \notin \mathcal{F}(h)$,
  - do not interview if $t < T$ and there is some $f' < f$ with $f' \notin \mathcal{F}(h)$;
  - interview if $t < T$ and for each $f' < f$, we have $f' \in \mathcal{F}(h)$, or if $t = T$.

Hence, for each firm $k$ and partial history $h \in \mathcal{H}^t$, we can define $t_k(h)$ as the period in which firm $k$ interviews when all firms play the strategy profile $\sigma^{SH}$ in the subgame following $h$.

- In the interview stage of period $t$, after a history $h \in \mathcal{H}^t_{Int}$ with $f \in \mathcal{F}^t(h)$, choose $I^t_f = I^t_f(h)$, and
- In the hiring stage of period $t$, after a history $h \in \mathcal{H}^t_{Hire}$ with $f \in \mathcal{F}^t(h)$, choose $x^t_f = \hat{x}^t_f(h)$,

where $\{\{I^t_k(h), x^t_k(h)\}_{k\in\mathcal{F}^t(h), h\in\mathcal{H}^t_{Int}}\}_{t=1}^T$ and $\{\hat{x}^t_k(h)\}_{k\in\mathcal{F}^t(h), h\in\mathcal{H}^t_{Hire}}$ are defined recursively on $t$ as follows:

- $t = T$: for each $h \in \mathcal{H}^T_{Int}$ in which firms $\mathcal{F}(h)$ have interviewed in previous periods $\{t_k\}_{k\in\mathcal{F}(h)}$ and chosen the interview sets $\{I_k\}_{k\in\mathcal{F}(h)}$ and hiring rules $\{x_k\}_{k\in\mathcal{F}}$, define $I^T_f(h), x^T_f(h)$ recursively on $f \in \mathcal{F}^T(h)$ as follows:

$$(I^T_f(h), x^T_f(h)) \in \arg\max_{\substack{I\subseteq[0,1]\\ x\in\mathcal{X}}} \pi^{NB}_f\left(\{I_k, x_k, t_k\}_{\substack{k\in\mathcal{F}(h)\\ k<f}}, \{I^T_k(h), x^T_k(h), T\}_{\substack{k\in\mathcal{F}^T(h)\\ k<f}}, \{\emptyset, 1, T\}_{\substack{k\notin\mathcal{F}(h)\cup\mathcal{F}^T(h)\\ k<f}}, I, x, T\right)$$

$$\text{s.t.} \int_I \left(\int x(s)dG(s|a)\right)\begin{pmatrix}\phi_f\left(a, \{I_k, x_k, t_k\}_{\substack{k\in\mathcal{F}(h)\\ k<f}}, \{I^T_k(h), x^T_k(h), T\}_{\substack{k\in\mathcal{F}^T(h)\\ k<f}}, \{\emptyset, 1, T\}_{\substack{k\notin\mathcal{F}(h)\cup\mathcal{F}^T(h)\\ k<f}}, T\right)\\ \times\,\psi_f\left(a, \{I_k, x_k, t_k\}_{\substack{k\in\mathcal{F}(h)\\ k<f}}, \{I^T_k(h), x^T_k(h), T\}_{\substack{k\in\mathcal{F}^T(h)\\ k<f}}, \{\emptyset, 1, T\}_{\substack{k\notin\mathcal{F}(h)\cup\mathcal{F}^T(h)\\ k<f}}, T\right)\end{pmatrix} da \le \Delta, \tag{22}$$

where $\pi^{NB}_f$ is as in (4); by Lemma 5, each $(I^T_f(h), x^T_f(h))$ is well defined. Then for each $h \in \mathcal{H}^T_{Hire}$ in which firms $\mathcal{F}(h) \subseteq \{1, \dots, F\} \setminus \{f\}$ have interviewed in previous periods $\{t_k\}_{k\in\mathcal{F}(h)}$ and chosen the interview sets $\{I_k\}_{k\in\mathcal{F}(h)}$ and hiring rules $\{x_k\}_{k\in\mathcal{F}(h)}$, and firms $\mathcal{F}^t(h)$ have chosen to interview in the entry stage of period $t$ and chosen the interview sets $\{I_k\}_{k\in\mathcal{F}^t(h)}$, define $\hat{x}^T_f(h)$ recursively on $f \in \mathcal{F}^t(h)$ as follows:

$$\hat{x}^T_f(\cdot, h) \equiv \max_{\succ\mathcal{X}}\left\{x \in \mathcal{X} \middle| \int_{I_f}\int_0^\infty x(s)dG(s|a)\left(\left(\prod_{\substack{k\in\mathcal{F}(h)\\ k<f, I_k\ni a}}\int_0^\infty (1 - x_k(s))dG(s|a)\right)\left(\prod_{\substack{k\in\mathcal{F}^T(h)\\ k<f, I_k\ni a}}\int_0^\infty (1 - \hat{x}^T_k(s))dG(s|a)\right)\right)da \le \Delta\right\},$$

which is well defined, since by Lebesgue's dominated convergence theorem, the integral in the definition is continuous in $x$.

- $t < T$, given $\sigma^{SH}(h')$ for all histories $h' \in \mathcal{H}^{t'}$, $t' > t$: For each $h \in \mathcal{H}^t_{Int}$ in which firms in $\mathcal{F}(h)$ have interviewed in previous periods $\{t_k\}_{k\in\mathcal{F}(h)}$ and chosen the interview sets $\{I_k\}_{k\in\mathcal{F}(h)}$ and hiring rules $\{x_f\}_{f\in\mathcal{F}(h)}$, define $\hat{h}^{t'}_{Int}(\sigma^{SH}|h, \{I_k, x_k\}_{k\in\mathcal{F}^t(h)})$ as the partial history in $\mathcal{H}^{t'}_{Int}$ reached under $\sigma^{SH}$ following history $h$ and the play of $\{I_k\}_{k\in\mathcal{F}^t(h)}$ and $\{x_k\}_{k\in\mathcal{F}^t(h)}$ in the period-$t$ interview and hiring stages.[15] Then define $I^t_f(h), x^t_f(h)$ recursively on $f \in \mathcal{F}^t$ as follows:

[15] That is, the partial history reached under $\sigma^{SH}(h, \{I_k\}_{k\in\mathcal{F}^t(h)}, \{x_k\}_{k\in\mathcal{F}^t(h)})$.

$$(I_f^t(h), x_f^t(h)) \in \arg\max_{\substack{I\subseteq[0,1]\\ x\in\mathcal{X}}} \pi_f^{NB}\left(\begin{array}{l}\{I_k^{t_k(h)}(\hat{h}_{Int}^{t_k(h)}(\sigma^{SH}|h,\{I_k,x_k\}_{k\in\mathcal{F}^t(h)})), x_k^{t_k(h)}(\hat{h}_{Int}^{t_k(h)}(\sigma^{SH}|h,\{I_k,x_k\}_{k\in\mathcal{F}^t(h)})), t_k(h)\}_{\substack{k\notin\mathcal{F}(h)\cup\mathcal{F}^t(h),\\ k<f}},\\ \{I_k,x_k,t_k\}_{\substack{k\in\mathcal{F}(h),\\ k<f}}, \{I_k^t(h),x_k^t(h),t\}_{\substack{k\in\mathcal{F}^t(h),\\ k<f}}, t\end{array}\right)$$

$$\text{s.t.} \int_I \left(\int x(s)dG(s|a)\right)\left(\begin{array}{l}\phi_f\left(a, \begin{array}{l}\{I_k^{t_k(h)}(\hat{h}_{Int}^{t_k(h)}(\sigma^{SH}|h,\{I_k,x_k\}_{k\in\mathcal{F}^t(h)})), x_k^{t_k(h)}(\hat{h}_{Int}^{t_k(h)}(\sigma^{SH}|h,\{I_k,x_k\}_{k\in\mathcal{F}^t(h)})), t_k(h)\}_{\substack{k\notin\mathcal{F}(h)\cup\mathcal{F}^t(h)\\ k<f}}\\ \{I_k,x_k,t_k\}_{\substack{k\in\mathcal{F}(h),\\ k<f}}, \{I_k^t(h),x_k^t(h),t\}_{\substack{k\in\mathcal{F}^t(h),\\ k<f}}, t\end{array}\right)\\ \times\,\psi_f\left(a, \begin{array}{l}\{I_k^{t_k(h)}(\hat{h}_{Int}^{t_k(h)}(\sigma^{SH}|h,\{I_k,x_k\}_{k\in\mathcal{F}^t(h)})), x_k^{t_k(h)}(\hat{h}_{Int}^{t_k(h)}(\sigma^{SH}|h,\{I_k,x_k\}_{k\in\mathcal{F}^t(h)})), t_k(h)\}_{\substack{k\notin\mathcal{F}(h)\cup\mathcal{F}^t(h)\\ k<f}}\\ \{I_k,x_k,t_k\}_{\substack{k\in\mathcal{F}(h),\\ k<f}}, \{I_k^t(h),x_k^t(h),t\}_{\substack{k\in\mathcal{F}^t(h),\\ k<f}}, t\end{array}\right)\end{array}\right)da \leq \Delta, \tag{23}$$

where $\pi_f^{NB}$ is as in (4); by Lemma 5, each $(I_f^t(h), x_f^t(h))$ is well defined. Then for each $h \in \mathcal{H}_{Hire}^t$ in which firms in $\mathcal{F}(h)$ have interviewed in previous periods $\{t_k\}_{k\in\mathcal{F}(h)}$ and chosen the interview sets $\{I_k\}_{k\in\mathcal{F}(h)}$ and hiring rules $\{x_f\}_{f\in\mathcal{F}(h)}$, and firms $\mathcal{F}^t(h)$ have chosen to interview in period $t$ and chosen the interview sets $\{I_k\}_{k\in\mathcal{F}^t(h)}$, define $\hat{h}_{Int}^{t'}(\sigma^{SH}|h,\{x_k\}_{k\in\mathcal{F}^t(h)})$ as the partial history in $\mathcal{H}_{Int}^{t'}$ reached under $\sigma^{SH}$ following history $h$ and the play of $\{x_k\}_{k\in\mathcal{F}^t(h)}$ in the period-$t$ hiring stage,[16] and define $\hat{x}_f^t(h)$ recursively on $f \in \mathcal{F}^t(h)$ as follows:

$$\hat{x}_f^t(\cdot,h) \equiv \max_{\succ_{\mathcal{X}}}\left\{x\in\mathcal{X} | \int_{I_f}\int_0^\infty x(s)dG(s|a)\left(\begin{array}{l}\left(\prod_{\substack{k\in\mathcal{F}(h)\\ k<f, I_k\ni a}}\int_0^\infty(1-x_k(s))dG(s|a)\right)\times\left(\prod_{\substack{k\in\mathcal{F}^t(h)\\ k<f, I_k\ni a}}\int_0^\infty(1-\hat{x}_k^t(s))dG(s|a)\right)\\ \times\left(\prod_{\substack{k\notin\mathcal{F}(h)\cup\mathcal{F}^t(h)\\ k<f, I_k\ni a}}\int_0^\infty(1-x_k^{t_k(h)}(s,\hat{h}_{Hire}^{t_k(h)}(\sigma^{SH}|h,\{\hat{x}_j^t(h)\}_{\substack{j\in\mathcal{F}^t(h)\\ j<k}})))dG(s|a)\right)\end{array}\right)da\leq\Delta\right\},$$

which is well defined, since by Lebesgue's dominated convergence theorem, the integral in the definition is continuous in $x$.

Note that by Lemma 5, $x_f^t(h) = \hat{x}_f^t(h, \{I_k^t(h)\}_{k\in\mathcal{F}^t(h)})$ for each $t$, each $h\in\mathcal{H}_{Int}^t$, and each $f\in\mathcal{F}^t(h)$.

It is immediate that given $\sigma_{-f}^{SH}$, there is no profitable one-shot deviation from $\sigma_f^{SH}$ in the hiring stage after any $h\in\mathcal{H}_{Hire}^t$ for each $t$: $\succ_{\mathcal{X}}$ is a total order on $\mathcal{X}$, and any $x' \prec_{\mathcal{X}} \hat{x}_f^t(h)$ would yield a weakly lower payoff for firm $f$, while any $x' \succ_{\mathcal{X}} \hat{x}_f^t(h)$ is not feasible since it would lead to $f$ hiring more than $\Delta$ workers. Similarly, given $\sigma_{-f}^{SH}$, there is no profitable one-shot deviation from $\sigma_f^{SH}$ in the interview stage after any $h\in\mathcal{H}_{Int}^t$ for each $t$.

To show that given $\sigma_{-f}^{SH}$, there is no profitable one-shot deviation from $\sigma_f^{SH}$ in the entry stage, first observe that $I_f^t(h) = I_f^{t'}(h')$ and $x_f^t(h) = x_f^{t'}(h')$ for all $t, t'$ and all $h\in\mathcal{H}_{Int}^t$ and $h'\in\mathcal{H}_{Int}^{t'}$ with $k\in\mathcal{F}(h)$ and $k\in\mathcal{F}(h')$ for all $k<f$ but $f\notin\mathcal{F}(h)$ and $f\notin\mathcal{F}(h')$. It follows that for any $t<T$ and after any $h\in\mathcal{H}_{Ent}^t$ with $k\in\mathcal{F}(h)$ for all $k<f$ but $f\notin\mathcal{F}(h)$, firm $f$ receives the same payoff whether it interviews in period $t$ (as specified by $\sigma_f^{SH}$), or makes a one-shot deviation to not interview in period $t$, in which case $\sigma_f^{SH}$ leads it to interview the same set of workers in period $t+1$. And after any $h\in\mathcal{H}_{Ent}^T$ with $f\notin\mathcal{F}(h)$, firm $f$ receives a higher payoff by interviewing in period $T$ (as specified by $\sigma_f^{SH}$) than by making a one-shot deviation to not interview at all.

[16] That is, the partial history reached under $\sigma^{SH}(h,\{x_k\}_{k\in\mathcal{F}^t(h)})$.

Now consider a history $h \in \mathcal{H}^t_{Int}$, $t < T$, such that $f \notin \mathcal{F}(h)$ but $f' \notin \mathcal{F}(h)$ for some $f' < f$, and consider a one-shot deviation by $f$ to interview in period $t$. Let $\bar{h}$ be the terminal history that results from that one-shot deviation, and $\hat{h}^T(\sigma^{SH}|h)$ be the terminal history when all firms play according to $\sigma^{SH}$ following $h$. For each firm $k$, let $\bar{I}_k$ be the set of workers interviewed by firm $k$ in the terminal history $\bar{h}$, let $\bar{x}_k$ be its hiring rule in $\bar{h}$, and let $\bar{t}_k$ be the period in which it interviews in $\bar{h}$; similarly, let $\hat{I}_k$, $\hat{x}_k$, and $\hat{t}_k$ be its interview set, hiring rule, and interview period in the terminal history $\hat{h}^T(\sigma^{SH}|h)$. Note that by definition of $\sigma^{SH}$, if the partial histories $h'$ and $h''$ are identical except for the actions of firm $f$, then for any $k < f$, $\sigma^{SH}_k(h') = \sigma^{SH}_k(h'')$. Then for each $k < f$, $(\bar{I}_k, \bar{x}_k, \bar{t}_k) = (\hat{I}_k, \hat{x}_k, \hat{t}_k)$. Hence, $\psi_f(a, \{\bar{I}_k, \bar{x}_k, \bar{t}_k\}^{f-1}_{k=1}, \bar{t}_f)\phi_f(a, \{\bar{I}_k, \bar{x}_k, \bar{t}_k\}^{f-1}_{k=1}, \bar{t}_f) = \psi_f(a, \{\hat{I}_k, \hat{x}_k, \hat{t}_k\}^{f-1}_{k=1}, \hat{t}_f)\phi_f(a, \{\hat{I}_k, \hat{x}_k, \hat{t}_k\}^{f-1}_{k=1}, \hat{t}_f)$ for all $a$.

It follows from (4) and the definition of $\sigma^{SH}$ that firm $f$'s payoffs from $\bar{h}$ and $\hat{h}^T(\sigma^{SH}|h)$ can only differ if $\hat{t}_{f'} = t$ for some $f' < f$, in which case $\phi_f(a, \{\bar{I}_k, \bar{x}_k, \bar{t}_k\}^{f-1}_{k=1}, \bar{t}_f) \geq \phi_f(a, \{\hat{I}_k, \hat{x}_k, \hat{t}_k\}^{f-1}_{k=1}, \hat{t}_f)$ for all $a$. Then for any $(I_f, x_f)$, $\pi^{NB}_f(\{\bar{I}_k, \bar{x}_k, \bar{t}_k\}^{f-1}_{k=1}, I_f, x_f, t) \leq \pi^{NB}_f(\{\hat{I}_k, \hat{x}_k, \hat{t}_k\}^{f-1}_{k=1}, I_f, x_f, \hat{t}_f)$; it follows that

$$\max_{I,x}\left\{\pi^{NB}_f(\{\bar{I}_k, \bar{x}_k, \bar{t}_k\}^{f-1}_{k=1}, I, x, t) \text{ s.t. } \int_I\left(\int x(s)dG(s|a)\right)\phi_f(\{\bar{I}_k, \bar{x}_k, \bar{t}_k\}^{f-1}_{k=1}, t)\psi_f(\{\bar{I}_k, \bar{x}_k, \bar{t}_k\}^{f-1}_{k=1}, t) \leq \Delta\right\}$$
$$\leq \max_{I,x}\left\{\pi^{NB}_f(\{\hat{I}_k, \hat{x}_k, \hat{t}_k\}^{f-1}_{k=1}, I, x, \hat{t}_f) \text{ s.t. } \int_I\left(\int x(s)dG(s|a)\right)\phi_f(\{\hat{I}_k, \hat{x}_k, \hat{t}_k\}^{f-1}_{k=1}, \hat{t}_f)\psi_f(\{\hat{I}_k, \hat{x}_k, \hat{t}_k\}^{f-1}_{k=1}, \hat{t}_f) \leq \Delta\right\}.$$

Then by definition of $\sigma^{SH}$, firm $f$'s payoff from $\bar{h}$ can be no greater than its payoff from $\hat{h}^T(\sigma^{SH}|h)$.

Then for each $f \in \{1, \ldots, F\}$, there are no profitable one-shot deviations from $\sigma^{SH}_f$ given $\sigma^{SH}_{-f}$. It follows that $\sigma^{SH}$ is a subgame perfect equilibrium.

(ii): Let each firm $f$'s *centralized strategy* $\sigma^C_f$ be as follows.

- In the entry stage of period $t$, after a history $h \in \mathcal{H}^t_{Ent}$ with $f \notin \mathcal{F}(h)$,
  - do not interview if $t < T$;
  - interview if $t = T$.

  Hence, for each firm $k$ and partial history $h \in \mathcal{H}^t$, when all firms play the strategy profile $\sigma^C$ in the subgame following $h$, each interviews in period $T$.

- In the interview stage of period $t$, after a history $h \in \mathcal{H}^t_{Int}$ with $f \in \mathcal{F}^t(h)$, choose $I^t_f = I^t_f(h)$, and
- In the hiring stage of period $t$, after a history $h \in \mathcal{H}^t_{Hire}$ with $f \in \mathcal{F}^t(h)$, choose $x^t_f = \hat{x}^t_f(h)$,

where $\{\{I^t_k(h), x^t_k(h)\}_{k \in \mathcal{F}^t(h), h \in \mathcal{H}^t_{Int}}\}^T_{t=1}$ and $\{\hat{x}^t_k(h)\}_{k \in \mathcal{F}^t(h), h \in \mathcal{H}^t_{Hire}}$ are defined in the same way as in (i), but with $t_k(h)$ replaced by $T$.

By an identical argument to (i), for each firm $f$, given $\sigma^C_{-f}$, there are no profitable one-shot deviations from $\sigma^C_f$ in the interview or hiring stages after any partial history. A one-shot deviation in the interview stage of period $T$ results in firm $f$ not interviewing at all, which cannot increase its payoff. Consider a one-shot deviation for firm $f$ in period $t < T$ after some $h \in \mathcal{H}^t_{Ent}$. Let $\bar{h}$ be the terminal history that results from that one-shot deviation, and $\hat{h}^T(\sigma^C|h)$ be the terminal history when all firms play according to $\sigma^C$ following $h$. For each firm $k$, let $\bar{I}_k$ be the set of workers interviewed by firm $k$ in the terminal history $\bar{h}$, let $\bar{x}_k$ be its hiring rule in $\bar{h}$, and let $\bar{t}_k$ be the period in which it interviews in $\bar{h}$; similarly, let $\hat{I}_k$, $\hat{x}_k$, and $\hat{t}_k$ be its interview set, hiring rule, and interview period in the terminal history

$\hat{h}^T(\sigma^C|h)$. Note that by definition of $\sigma^C$, if the partial histories $h'$ and $h''$ are identical except for the actions of firm $f$, then for any $k < f$, $\sigma_k^C(h') = \sigma_k^C(h'')$. Then for each $k < f$, $(\bar{I}_k, \bar{x}_k, \bar{t}_k) = (\hat{I}_k, \hat{x}_k, \hat{t}_k)$. Moreover, by definition of $\sigma_C$, for each $k < f$, either $\hat{t}_k < t$ or $\hat{t}_k = T$. It follows that $\psi_f(a, \{\bar{I}_k, \bar{x}_k, \bar{t}_k\}_{k=1}^{f-1}, t) = \psi_f(a, \{\hat{I}_k, \hat{x}_k, \hat{t}_k\}_{k=1}^{f-1}, T)$ and $\phi_f(a, \{\bar{I}_k, \bar{x}_k, \bar{t}_k\}_{k=1}^{f-1}, t) = \phi_f(a, \{\hat{I}_k, \hat{x}_k, \hat{t}_k\}_{k=1}^{f-1}, T)$ for all $a$. Then for any $(I_f, x_f)$, $\pi_f^{NB}(\{\bar{I}_k, \bar{x}_k, \bar{t}_k\}_{k=1}^{f-1}, I_f, x_f, t) = \pi_f^{NB}(\{\hat{I}_k, \hat{x}_k, \hat{t}_k\}_{k=1}^{f-1}, I_f, x_f, T)$; it follows that

$$\max_{I,x}\left\{\pi_f^{NB}(\{\bar{I}_k, \bar{x}_k, \bar{t}_k\}_{k=1}^{f-1}, I, x, t) \text{ s.t. } \int_I\left(\int x(s)dG(s|a)\right)\phi_f(\{\bar{I}_k, \bar{x}_k, \bar{t}_k\}_{k=1}^{f-1}, t)\psi_f(\{\bar{I}_k, \bar{x}_k, \bar{t}_k\}_{k=1}^{f-1}, t) \le \Delta\right\}$$
$$= \max_{I,x}\left\{\pi_f^{NB}(\{\hat{I}_k, \hat{x}_k, \hat{t}_k\}_{k=1}^{f-1}, I, x, T) \text{ s.t. } \int_I\left(\int x(s)dG(s|a)\right)\phi_f(\{\hat{I}_k, \hat{x}_k, \hat{t}_k\}_{k=1}^{f-1}, T)\psi_f(\{\hat{I}_k, \hat{x}_k, \hat{t}_k\}_{k=1}^{f-1}, T) \le \Delta\right\}.$$

Then by definition of $\sigma^C$, firm $f$'s payoff from $\bar{h}$ can be no greater than its payoff from $\hat{h}^T(\sigma^C|h)$.

Then for each $f \in \{1, \ldots, F\}$, there are no profitable one-shot deviations from $\sigma_f^C$ given $\sigma_{-f}^C$. It follows that $\sigma^C$ is a subgame perfect equilibrium. □

**Proof of Proposition 6 (Decentralized Matching with Nonbinding Offers: Sequential-Hiring Equilibrium)** Let $h^{SH}$ be the terminal history that results from the strategy profile $\sigma^{SH}$, and for each firm $k$, let $I_k^{SH}$ be the set of workers interviewed by firm $k$ in $h^{SH}$, let $x_k^{SH}$ be its hiring rule in $h^{SH}$, and let $t_k^{SH} = k$ be the period in which it interviews in $h^{SH}$.

(i): Follows immediately by applying Corollary 1 recursively for each firm $f$, letting $\phi(a) = \phi_f(a, \{I_k^{SH}, x_k^{SH}, k\}_{k=1}^{f-1}, f)$.

(ii): Follows immediately from the definition of $\sigma^{SH}$, given the assumption that $\int_0^\infty s dG(s|0) > c$, and hence (since $G(\cdot|a) \succeq_{FOSD} G(\cdot|0)$) $\int_0^\infty s dG(s|a) > c$ for all $a$. □

**Proof of Proposition 7 (Welfare in the Sequential-Hiring Equilibrium)** (i): Follows immediately from (2) and the definition of $\sigma^{SH}$.

(ii): Let $I_2^*$ be firm 2's equilibrium interview set in the centralized regime, and $I_2^{SH}, x_2^{SH}$ be firm 2's interview set and hiring rule in the outcome of the sequential-hiring equilibrium; for each $I_1, I_2$, let $x_1^*(I_1)$ and $x_2^*(I_1, I_2)$ be as defined in Proposition 1. By (i), the definition of $\sigma^{SH}$, and since $I_2^{SH}$ is optimal in the period-2 interview stage on the equilibrium path in the sequential-hiring equilibrium, $\pi_2^{NB}((I_1^*, x_1^*, 1), (I_2^{SH}, x_2^{SH}, 2)) \ge \pi_2^{NB}((I_1^*, x_1^*, 1), (I_2^*, x_2^*(I_1^*, I_2^*), 2)$. By definition of $\pi_2^{NB}$ and $\pi_2$,

$$\pi_2^{NB}((I_1^*, x_1^*, 1), (I_2^*, x_2^*(I_1^*, I_2^*), 2) = \pi_2(I_1^*, I_2^*) + c\int_{I_2^*\cap I_1^*}\int_0^\infty (1 - x_1^*(s, I_1^*))dG(s|a)da \ge \pi_2(I_1^*, I_2^*).$$

The claim follows. □