ARROW SEA 2025: Grid integration session

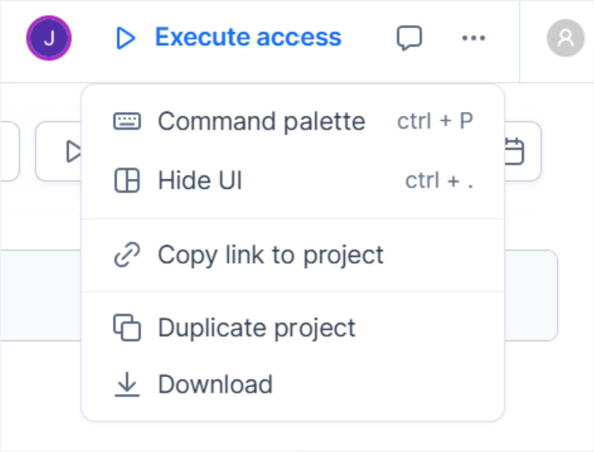
Overview

1. Marginal pricing
2. Optimal power flow formulation
3. Congestion scenarios
4. Unit commitment

Jupyter/Python notebook environment

Duplicate the ARROW SEA project in Deepnote

- Visit https://tinyurl.com/sea2025-deepnote
- Click on three dots `...` in the top-right corner
- Select `Duplicate project`
- Log in / Follow the prompts to create an account
- You can now edit a private copy of the project



Marginal prices

Objectives

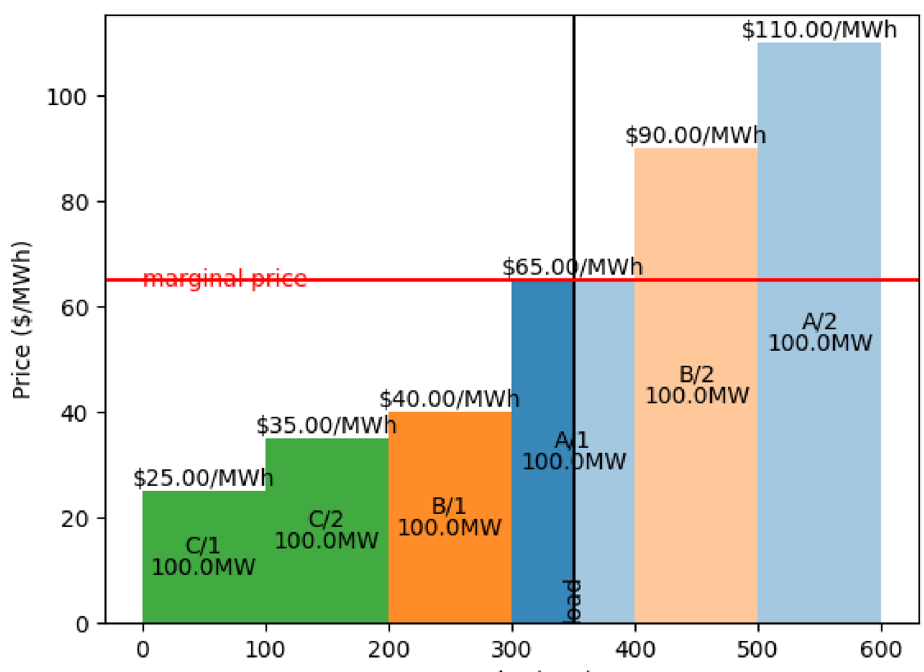
Work through notebook `part1_mp` to:

- Review the graphical method of marginal pricing
- Reproduce this solution via linear programming

(1a) Graphical approach to marginal pricing

1. Stack quantity-price offer pairs, ordered by price
2. Dispatch quantities to left of load
3. Intercept of load and offer price sets marginal price

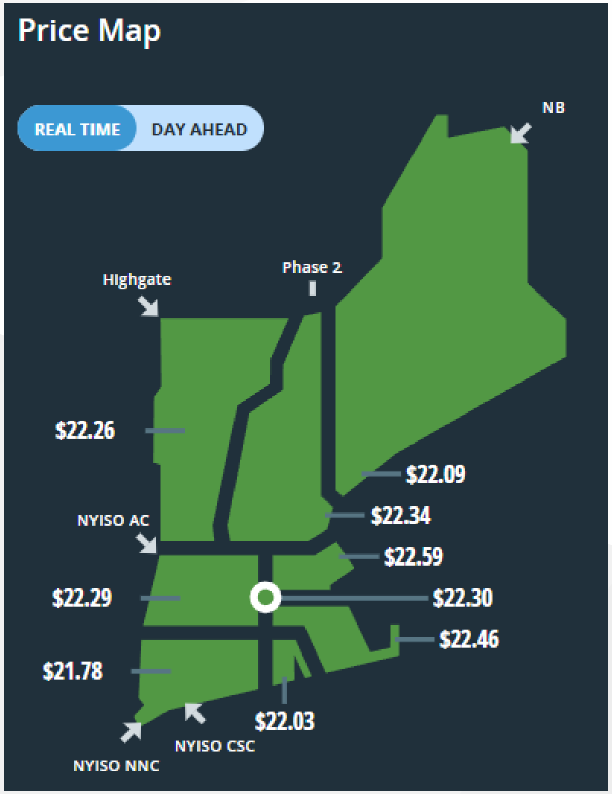
> Does not apply to networks with limited transmission capacity



(1b) Linear programming solution

- Study the CVXPY formulation provided:
    1. Decision variables
    2. Constraints: power balance, bounds
    3. Objective function
- Verify that we reproduce the graphical solution

Context: Locational Marginal Prices



The Optimal Power Flow problem

Objective

Work through Notebook `part2_opf`:

- Extend the LP formulation to account for transmission effects
  - Extra variables: line flows [MW], bus voltage angles [rad]
  - Extra constraint: Power flow on each line
  - Same objective function

OPF problem formulation

Name	Per	Description	Lower	Upper
`p`	offers	dispatched power [MW]	0	offer quantity
`f`	lines	line flow [MW]	-(line capacity)	line capacity
`θ`	buses	bus voltage angle [rad]	$-\pi$	$+\pi$

Power balance constraints:

$$\sum_{o \in \text{Offers @ } b} \mathbf{p}_o + \sum_{\ell \in \text{Lines in } b} \mathbf{f}_\ell - \sum_{\ell \in \text{Lines out } b} \mathbf{f}_\ell = \text{load}_b \quad \text{for each } b \in \text{Buses}$$

Power flow constraints:

$$\mathbf{f}_{ij} = (\theta_i - \theta_j) / \mathtt{reactance}_{ij} \quad \text{for each } (i, j) \in \text{Lines}$$

OPF problem formulation (cont.)

Objective:

$$\text{total cost} = \min_{\mathtt{p},\mathtt{f},\theta} \sum_{o \in \text{Offers}} (\text{offer price})_o \times \mathtt{p}_o$$

Marginal price at bus b:

$$\text{LMP}_b\ [\$/\text{MWh}] = \frac{\partial\, \text{total cost}\ [\$/\text{h}]}{\partial\, \text{load}_b\ [\text{MW}]}$$

AC optimal power flow (OPF) problem

The linearized OPF is an idealization of the AC problem:

$$\min_{S^{\text{gen}} \in \mathbb{C}^{M+N},\, v \in \mathbb{C}^M} \quad \sum_a c_a(S_a^{\text{gen}})$$

$$\begin{aligned}
S_{bc} &= v_a(v_a^* - v_b^*)/z_{bc}^* \quad &&\text{(power flow on line } bc\text{)}\\
S_a^{\text{gen}} &= \sum_b S_{bc} + S_a^{\text{load}} \quad &&\text{power balance at bus } b\\
|S_{bc}| &\le \bar{S}_{bc} \quad &&\text{line capacity}\\
\underline{S}_b \le S_a^{\text{gen}} &\le \bar{S}_b \quad &&\text{injection limits}\\
0 < \underline{v}_b \le |v_b| &\le \bar{v}_b \quad &&\text{voltage limits}\\
v_0 &= 1 + 0i \quad &&\text{voltage at reference bus}
\end{aligned}$$

Symbol	Linearized OPF	AC OPF analog	Assumption	Unit
bus injection	$\mathtt{p}_b$	$\mathrm{real}(S_a^{\mathrm{gen}})$	$\mathrm{imag}(S_a^{\mathrm{gen}}) \approx 0$	MW
line flow	$\mathtt{f}_{bc}$	$\mathrm{real}(S_{bc})$	$\mathrm{imag}(S_{bc}) \approx 0$	MW
bus voltage angle	θ_b	$\mathrm{angle}(v_a)$	$\mathrm{abs}(v_a) \approx 1$	rad
line reactance	x_{bc}	$\mathrm{imag}(z_{bc})$	$\mathrm{real}(z_{bc}) \approx 0$	Ω

Congestion

Objective

Use the small 3-bus networks in Notebook `part3_congestion` to revisit two important observations:

1. Congestion causes LMP separation;
2. LMP separation is a price signal;

The Unit Commitment problem

Context

OPF does not explicitly account for generator dynamics and fixed costs:

- Physical constraints:
  - on ramping
  - on allowable up-time / down-time
- Economic realities:
  - Fixed operating costs
  - Costs of start-up and shut-down

These are captured in **Unit Commitment** formulations run in the day-ahead market:

Review: OPF Formulation

Name	Per	Description	Lower	Upper
`p`	offers	dispatched power [MW]	0	offer quantity
`f`	lines	line flow [MW]	-(line capacity)	line capacity
`θ`	buses	bus voltage angle [rad]	$-\pi$	$+\pi$

Constraints

- Power balance at each bus
- Power flow on each line
- Fixed voltage angle at reference bus

Unit commitment (UC) formulation

Data

- OPF data +
- Load forcast for **each** bus, **each** period

Decision variables

- OPF variables for **each** planning period
- Extra **binary** variables for each generator and **each** period

Description	Name	Costs	Constraints
commitment variable	`x_on`	fixed	(logical)
start-up variable	`x_su`	start-up	minimum up-time
shut-down variable	`x_sd`	shut-down	minimum down-time

Objective function

Cost	Dependence	Unit
energy	output	\$/MWh
operating ("no-load")	on/off	\$/h
start-up, shut-down	transition	\$

Objective: Total cost to be minimized over planning horizon

$$\sum_t \left(\sum_o \texttt{offer\_price}_o \, p_{o,t} + \sum_g \texttt{noload\_cost}_g \, \texttt{x\_on}_{g,t} + \texttt{startup\_cost}_g \, \texttt{x\_su}_{g,t} + \texttt{shutdown\_cost}_g \, \texttt{x\_sd}_{g,t} \right)$$

Unit commitment (UC) formulation (cont.)

Constraints:

- OPF-style constraints at **each period**:
  - Power balance at each bus
  - Power flow on each line
  - Fixed voltage angle at reference bus
- Dynamic constraints: for each generator g at time t

$$|p_{g,t} - p_{g,t-1}| \leq \texttt{max\_ramp}_g$$
$$\texttt{x\_on}_{g,t+\tau} \geq \texttt{x\_su}_{g,t} \quad \text{for each } \tau \in [0, \texttt{min\_uptime}_g]$$
$$\texttt{x\_on}_{g,t+\tau} \leq 1 - \texttt{x\_sd}_{g,t} \quad \text{for each } \tau \in [0, \texttt{min\_downtime}_g]$$

Review questions

Marginal prices

1. Identify products with near-zero marginal price?
2. What is a (locational) marginal energy price (LMP)?
3. Why might LMPs vary across a network?
4. Why do LMPs vary over time?
5. How are LMPs used to settle energy transactions?
6. (How are they settled in New England?)
7. Which operating costs are not captured by marginal pricing?
8. How are these additional costs covered in practice?

Optimal power flow

1. What are the decision variables of the OPF problem?
2. What are the constraints?
3. What is the objective function?
4. What are the key assumptions in the linearized OPF model?
5. Are we always able to solve the linearized OPF?
6. Why not solve the AC-OPF instead?
7. Why is it necessary to prescribe the voltage angle at one bus?

Unit commitment

Contrast the following elements of the UC and OPF problems:

1. Decision variables
2. Constraints
3. Objective function
4. Why do fixed costs difficult to reconcile with marginal prices?

Network economics

1. Do the LMP have economic significance at a bus with zero load? With zero generation capacity?
2. How can load payments exceed generation payments if energy losses are neglected?

Further reading

- Fu & Li (2006) *Different Models and properties on LMP calculations*
  - A description of LMP formulations, including the pricing of congestion and losses
- Krishnamurthy, Li, & Tesfatsion (2016) *An 8-Zone Test System Based on ISO New England Data: Development and Application*
  - Description of the ISO-NE 8-bus model
- Li & Bo (2010) *Small Test Systems for Power System Economic Studies*
  - A description of the PJM 5-bus model
- Gribik, Hogan, & Pope (2007) *Market-Clearing Electricity Prices and Energy Uplift*
  - Explains the difficulty of compensating for fixed costs

Thanks for your time and participation!