| Problem Chosen | 2026 MCM/ICM Summary Sheet | Team Control Number |
| --- | --- | --- |
| A | | 2611613 |

# our's title


**Abstract**

good **good** good *good* *good*

good good good good

- good
- good
  - good
  - good
    - good
    - good
      - good
      - good

1. good
2. good
   (a) good
   (b) good
      i. good
      ii. good
         A. good
         B. good

■
►
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- **Data Pre-processing:** Cleaning the raw data...
- **Model Construction:** Establishing the differential equations...
- **Result Validation:** Comparing with real-world datasets...

**Keywords**: SDCBJHDSBJCDSJBJCDBSJVBSDB;SDCHJSBVSD;SDCDSC;SCSDCDSCS

# Contents

# 1 Introduction

## 1.1 Problem Background

## 1.2 Restatement of the Problem

Considering the background information and restricted conditions identified in the problem statement, we need to solve the following problems:

- Develop a model to describe the frequency of stair usage.
- Create a model to determine whether a certain direction of travel was favored.
- Develop a model to determine whether people traveled single file or side by side.
- Build a model to evaluate whether the observed wear matches historical and environmental factors.
- Create a model to estimate the age of the stairwell and its reliability.
- Design a model to identify signs of repairs or renovations.
- Determine whether the wear is consistent with materials from specific sources.
- Estimate typical usage and identify whether heavy short-term or prolonged lighter usage fits the observed patterns.

## 1.3 Our Work

In this paper, we carried out the following work to study stair usage frequency, service life, current condition, and pedestrian behavior patterns when using the stairs.

➢ **Task 1: Model Preparation**

We determined material and environmental conditions of the target stairs, applied computer vision to obtain and process data, and fitted the wear depth distribution.

➢ **Task 2: Stair Wear Analysis Model**

We modeled pedestrian traffic to characterize usage frequency, inferred preferred walking direction via changes in wear distribution, and computed pedestrian density from traffic flow to judge single-file or side-by-side walking.

➢ **Task 3:**

- Analyze the relation among stair age, wear distribution, and pedestrian traffic.
- Compute theoretical wear from traffic and compare with measured wear; assess error.
- Estimate stair age from wear–traffic relationships and evaluate reliability.

- Identify signs of repairs or renovations via coefficient changes and verification.
- Check material consistency with presumed sources and verify with features.
- Estimate typical usage; distinguish heavy short-term vs. prolonged lighter usage.

## 2 Assumptions and Justifications

- **Assumption 1:** 示例标题

  **Justification:** 示例文本。

- **Assumption 2:** 示例标题

  **Justification:** 示例文本。

- **Assumption 3:** 示例标题

  **Justification:** 示例文本。

- **Assumption 4:** 示例标题

  **Justification:** 示例文本。

# 3 Notations

The key mathematical notations used in this paper are listed in Table 1.

**表 1: Notations used in this paper**

| ***Symbols*** | ***Definition*** | ***Units*** |
|---|---|---|
| $d(x, y)$ | The wear depth at each location $(x, y)$ | $m$ |
| $N$ | Pedestrian flow | $people/s$ |
| $k$ | Wear coefficients (also called Archard's constant) | / |
| $T$ | The time since the steps were completed | $day$ |
| $W$ | The width of the steps | $m$ |
| $L$ | The length of the steps | $m$ |
| $A_i$ | The area of the $i$-th step | $m^2$ |
| $n$ | The total number of steps | / |
| $\rho$ | The pedestrian density | $people/m^2$ |
| $v$ | The velocity vector of pedestrians | $m/s$ |
| $\nabla \cdot (\rho v)$ | The flux divergence | / |
| $b$ | Experience factor | / |

**There are some variables that are not listed here and will be discussed in detail below.*

# 4 Model Preparation

## 4.1 Data Processing

We selected typical ancient architectural stairs as the study object. Computer vision was used to obtain and process images.

- **Step 1: Image processing**
  Original images were converted to grayscale and denoised via Gaussian filtering.

- **Step 2: Feature extraction**
  Edge detection and key feature extraction were performed on the processed images.

- **Step 3: Data processing**
  Structured datasets were formed for subsequent modeling and analysis.

## 4.2 Solution of .....

# 5 Model I:

The core symbols and their definitions used in this study are summarized in Table **??**, providing an overview of the key parameters and their related meanings.

表 2: table

| A | B | C |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |

The core symbols and their definitions used in this study are summarized in Table2, providing an overview of the key parameters and their related meanings.

表 3: table

| Symbol | Description | C |
|---|---|---|
| PSNR | Peak signal-to-noise ratio for image quality evaluation | C |

## 5.1 Proof of Lagrange Mean Value Theorem

### 5.1.1 Step 1: Construct the Auxiliary Function

To apply Rolle's Theorem, we construct an auxiliary function $F(x)$, whose geometric meaning is "the difference between the ordinate of the curve $y = f(x)$ and the ordinate of the chord $AB$" (where $A(a, f(a))$ and $B(b, f(b))$ are the endpoints of the curve on $[a, b]$).

The equation of the straight line passing through $A$ and $B$ is:

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

which can be rewritten as:

$$y = f(a) + \frac{f(b) - f(a)}{b - a}(x - a)$$

Thus, the auxiliary function is defined as:

$$F(x) = f(x) - \left[f(a) + \frac{f(b) - f(a)}{b - a}(x - a)\right]$$

To apply Rolle's Theorem, we construct an auxiliary function $F(x)$, whose geometric meaning is "the difference between

$$F(x)$$

$$y = f(a) + \frac{f(b) - f(a)}{b - a}(x - a) \tag{1}$$

# 6 Model II

## 6.1 Modeling and Solving of Problem 1

### 6.1.1 Problem Analysis

### 6.1.2 Model Preparation

### 6.1.3 Model Construction

### 6.1.4 Model Solution

## 6.2 Modeling and Solving of Problem 1

### 6.2.1 Problem Analysis

### 6.2.2 Model Preparation

### 6.2.3 Model Construction

### 6.2.4 Model Solution

# 7 Sensitivity Analysis

敏感性分析内容

# 8 Model Evaluation and Further Discussion

## 8.1 Model Evaluation

### 8.1.1 Advantages

- 双变量分布刻画磨损形态并匹配参数差异。
- 采用粒子群等优化提升拟合效率与精度。
- 将人群流量抽象为流体并用数值方法求解。
- 引入脉冲函数模拟短时大量人流场景。
- 模型方法成熟、准确性较高、可复用性强。

### 8.1.2 Limitations

- 现场测量手段有限，数据精度受约束。
- 样本场景单一，外推到多类楼梯存在偏差。

## 8.2 Future Work

### 8.2.1 Model extension

### 8.2.2 Model application

# 9 Conclusions

[2]

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# Appendices

# Report on Use of AI

1. Bing AI
   Query1:
   Output:
2. Bing AI
   Query1:
   Output:
3. Bing AI
   Query1:
   Output:
4. Bing AI
   Query1:
   Output: