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Analyzing Basketball Strategy: Time Management Strategy Profiling

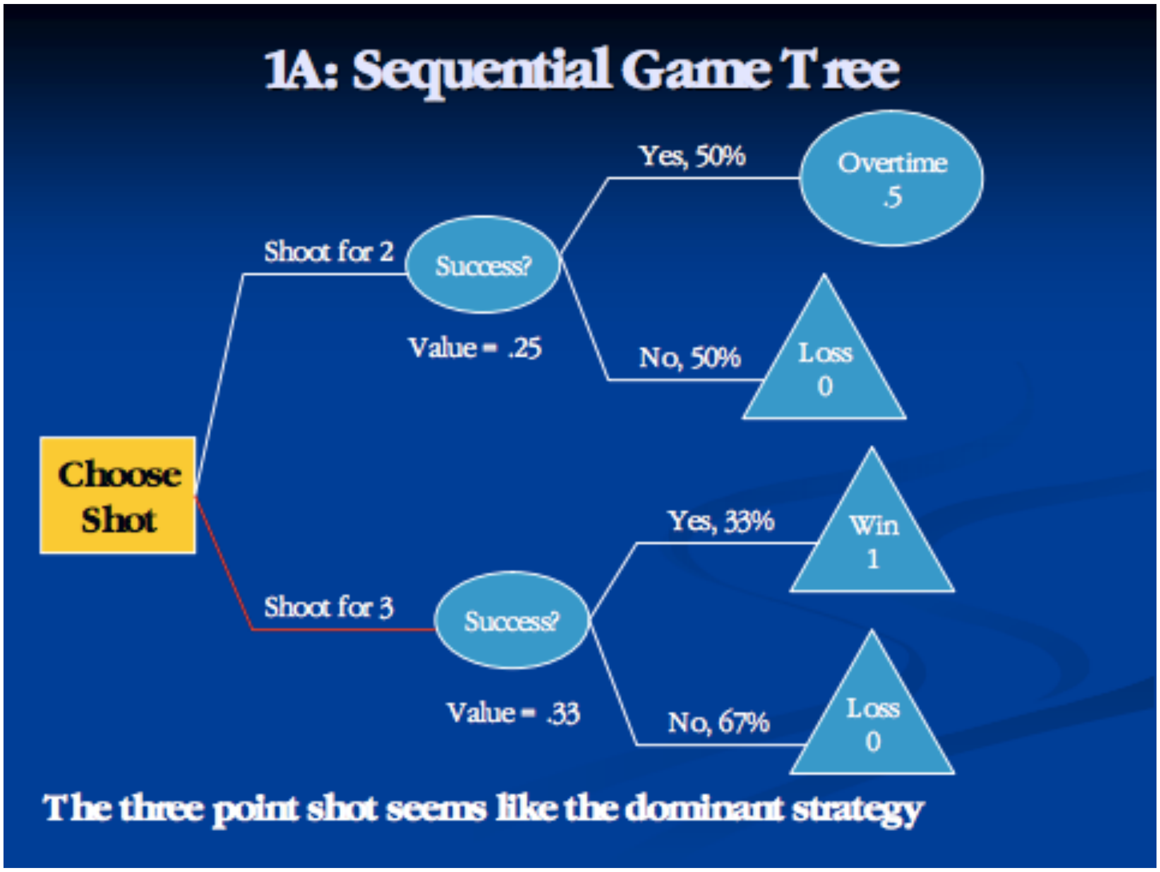
Introduction

One of the most interesting times during a basketball game in terms of strategy is the end of a close game. The reason behind this is that the end of a close basketball game is one of the few times coaches will actively play a role in deciding the strategy of the team. For example, coaches must decide when to foul the other team intentionally to stop the game clock from expiring, when to shoot a two point shot versus a three point shot, and how much time that can be used before shooting. In the paper written by UC-Berkeley students Chow, Miller, Nzima, and Winder on the game theory of basketball, the authors analyze the decision of when to take a two-point shot versus a three-point shot in two specific situations from a game theoretic approach and attempt to provide the best strategies for coaches facing these situations. This paper will use game theory to examine some of the recommendations by the work done by the UC-Berkeley students (Chow et al.) for these end-of-game scenarios and will ultimately expand on and refine their analysis.

Analyzing Situation 1: Down by 2 with under 24 seconds left in game

The first situation analyzed by Chow et al., involves any situation where the team in possession of the ball is losing by two points with less than 24 seconds left in the game. Because basketball possessions are limited by a 24 second game clock, by limiting the analysis to situations with less than 24 seconds in the game, Chow et al. simplified the analysis to rely on the success or failure of the single shot taken by the team in possession of the ball (the shooting team) by assuming that the shooting team would wait until the last second to shoot and thereby ensure the other team (the defending team) did not get a chance to shoot afterwards themselves. In these situations, the coach of the shooting team must determine the appropriate strategy for his team to take. The coach of the shooting team can elect either to take a three-point shot, which if made would win the game for the shooting team, or a two-point shot, which would tie the game and send it into overtime. Based on this decision, Chow et al. first broke down this situation with a simple sequential game analysis. To do their simple sequential game analysis, Chow et al. modeled the situation using the sequential game tree shown in Figure 1.

Figure 1 – Sequential Game Tree for Down by 2 w/ Under 24 Seconds Situation

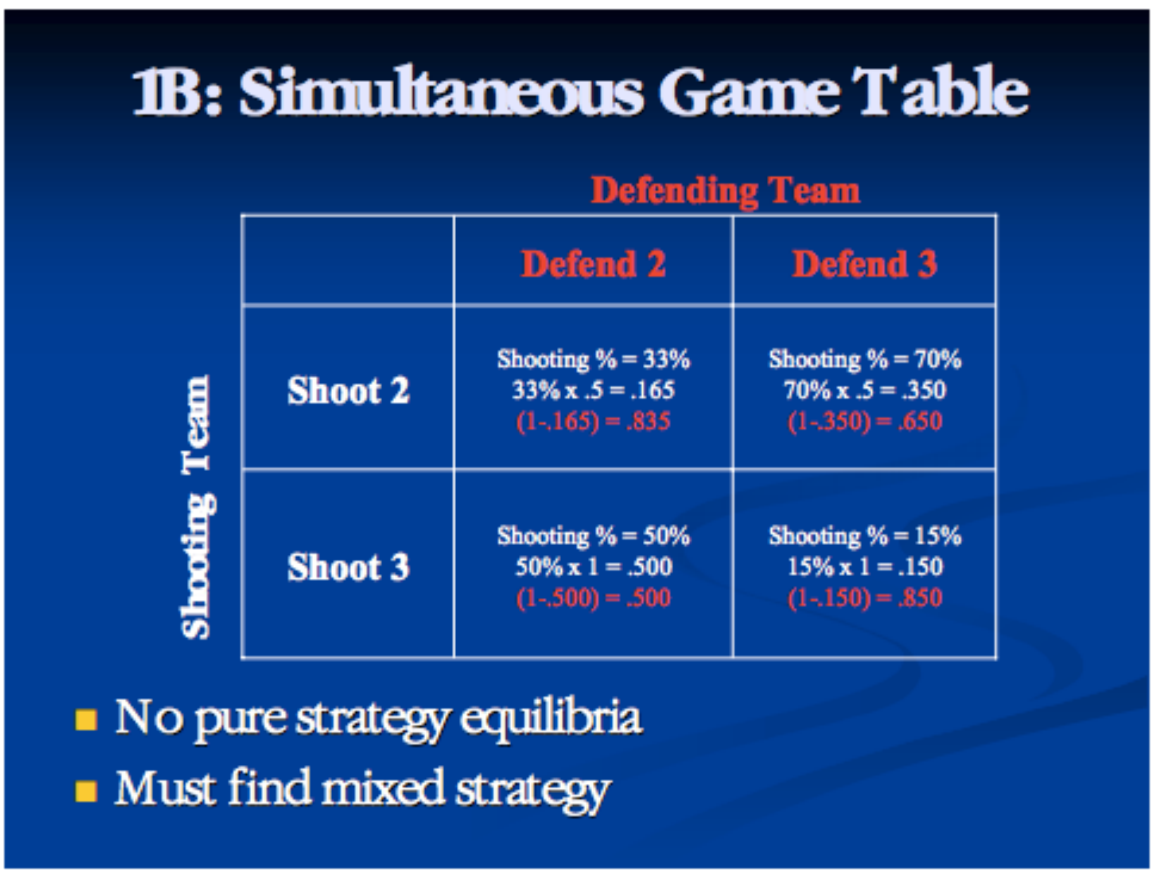


In this tree, the shooting team chooses between attempting a two and three point shot and based on the outcome of the shot receives an appropriate payout based on nature's determination of its success. This analysis comes to the conclusion that attempt a three-point shot would be the dominant strategy for the shooting team. It is easy to deduce this by noticing that shooting the 3 is a weakly dominant strategy. To see this we note that by shooting a 3, the shooting team receives a payoff of 0.33 * 1 = 0.33 if it succeeds whereas the payoff for shooting a successful two-point shot is 0.5 * 0.5 = 0.25. If either shot doesn't succeed, the payoff is 0. However, this analysis does have several flaws with its setup.

Perhaps the biggest flaw with this approach, which Chow et al. recognized, was the lack of an account for the defending team's choice. As Chow et al. state, "the defending team can choose to play aggressively on the perimeter, double covering the shooting team's best three point shooters, or it can play a more balanced defense that also covers the inside players who usually only take close-in two point shots… where the defense concentrates its efforts has a significant effect on the chances of a shot succeeding." Here, they are recognizing the importance of accounting for the defending team's choice in strategy and the lack of this creates a fundamental flaw with this analysis and something they proceed to adjust in their future analyses. However, one flaw they do not notice or account for in their future analyses is the lack of data to motivate the success percentages they assume. In their analysis they claim, "overall shot percentages that are roughly consistent with high-level basketball are assumed, namely 50% for two point shots and 33% for three point shots." However, this is fundamentally a flawed assumption, as the team really should have researched this data to find these probabilities. Also, as will be evident in the next analyses of theirs that is discussed here, these probability assumptions become more egregious when adjusting for the probability of making an open versus contested shot of a particular type.

The next analysis that Chow et al. did of this situation was a simultaneous game analysis. In this setup, the team attempted to model the game as a simultaneous game where the shooting team again had the same action set but the payoff was now dependent on the action of the defending team. They gave the defending team two actions, namely defending the three point shot or defending the two point shot, to parallel the choices of the shooting team. Figure 2 shows the normal form of this model.

Figure 2 – Normal Form Game for Down By 2 w/ Under 24 Seconds Situation



	Defend 2	Defend 3
Shoot 2	Shooting % = 33% 33% x .5 = .165 (1-.165) = .835	Shooting % = 70% 70% x .5 = .350 (1-.350) = .650
Shoot 3	Shooting % = 50% 50% x 1 = .500 (1-.500) = .500	Shooting % = 15% 15% x 1 = .150 (1-.150) = .850

Again, in this analysis the team used assumed success percentages for each of the outcomes, this time settling on a 33% success rate for a defended 2 point shot and a 15% success rate. Based on this, we can easily see that this game thus closely resembles the matching pennies game we looked at in class where one team, the defending team, is rewarded for matched strategies, while the other, the shooting team, is rewarded for mismatched strategies. Interestingly, another real world example of game theory that we discussed in class, namely the soccer penalty kicks example, also demonstrated a similarity to the matching pennies game.

Similarly to the match pennies game, there is no pure strategy Nash Equilibrium for this game and instead we must find a mixed strategy Nash Equilibrium. Appendix 1, taken from the paper by Chow et al. illustrates the mathematics to find this mixed Nash Equilibrium, which was determined to be ((0.346, 0.654), (0.76, 0.24)). Intuitively, this makes sense as if it were obviously always better for the team to take one type of shot as opposed to the other NBA teams would most likely implement such a strategy. As Chow et al. point out in their paper, "the most common outcome for this end-of-game situation will be the defending team tightly guarding the perimeter and the shooting team attempting a higher-percentage two point shot to send the game to overtime." Chow et al. do recognize the flaws in this form of analysis, namely that it simplifies many aspects of the game such as, "the abilities of individual players, the players' ability to dynamically change strategies depending on the defense, the chance of fouls and the opportunities presented by offensive rebounding," it does provide a reasonable approximation for the strategies that can be best taken by coaches.

Analyzing Situation 2: Down by 3 with under 24 seconds left in game

The second situation that is analyzed by this team is the situation in which the shooting team is down by 3 with less than 24 seconds remaining in this game. This situation is now much more complex as it is not necessarily the terminal decision point in the game. That is to say that if the shooting team elects to attempt a two-point shot and succeeds, they will have to foul and have one chance to score again. In this situation, Chow et al. present a similar diagram of a sequential analysis to provide the intuition for the possible scenarios that could play out. This sequential analysis is provided in Appendix 2. However, as Chow et al. realized in their analysis of situation one, to properly model any such situation, it must be modeled as a simultaneous game with two agents. Although it is not portrayed as so by Chow et al., this situation is in fact most accurately modeled by a stochastic game. Namely, based on the choices made by each team in the first game, the game will probabilistically enter a second and potentially third state. Figure 3 provides a cohesive overview of the potential states and transition probability function of this game. Similar assumptions are made as in situation 1.

Figure 3 – Potential States and Transition Probability Function of the Down by 3 Stochastic Game

Transition Probability Function P(x) where x is the shooting percentage
P(x) = (0.25 * 0.25 * x, 0.75 * 0.25 * x * 2, 0.75 * 0.75 * x, 1 - x) = (.0625x, .375x, .5625x, 1-x)

Initial State 0	Defend 2	Defend 3
Shoot 2	Shooting % = 0.33 Payoff = 0 Transition PF = P(0.33)	Shooting % = 0.7 Payoff = 0 Transition PF = P(0.7)
Shoot 3	Shooting % = 0.5 Payoff = (0.25, 0.75)	Shooting % = 0.15 Payoff = (0.075, 0.925)

STATE 1 – Down 1	Defend 2	Defend 3
Shoot 2	Shooting % = 0.33 Payoff = (0.33, 0.66)	Shooting % = 0.7 Payoff = (0.7 0.3)
Shoot 3	Shooting % = 0.5 Payoff = (0.5, 0.5)	Shooting % = 0.15 Payoff = (0.15, 0.85)

STATE 2 – Down 2	Defend 2	Defend 3
Shoot 2	Shooting % = 0.33 Payoff = (0.17, 0.83)	Shooting % = 0.7 Payoff = (0.35, 0.65)
Shoot 3	Shooting % = 0.5 Payoff = (0.5, 0.5)	Shooting % = 0.15 Payoff = (0.15, 0.85)

STATE 3 – Down 3	Defend 2	Defend 3
Shoot 2	Payoff = (0, 1)	Payoff = (0, 1)
Shoot 3	Shooting % = 0.5 Payoff = (0.25, 0.75)	Shooting % = 0.15 Payoff = (0.075, 0.925)

STATE 4 – Shot Missed	Defend 2	Defend 3
Shoot 2	(0, 1)	(0, 1)
Shoot 3	(0, 1)	(0, 1)

To analyze the flow of this game, Chow et al. do the following. First, they assume that each free throw will be made with probability 0.75 (another one of their attempts at a "reasonable approximation" of the average success rate). Although Chow et al. do simplify the free throws to a "one-and-one" scenario where if the first shot is missed, no successive free throw is attempted, I have chosen to redo this scenario without this assumption and instead award the defending team two free throws no matter what. The reason for this is that the "one-and-one" scenario is very uncommon in late game situations as it does not exist in the NBA and can only exist under certain circumstances in college. Thus, the analysis of this game will deviate from the paper by Chow et al.

First we analyze the transition probability function and the successive game states reached through transitions. This probability function is derived from situations where the shooting team decides to shoot a two-point shot. In this case, the game can enter one of the 4 alternate states with the probabilities describe. For the game to enter State 1 the shooting team must successfully make their two-point shot and the defending team must then miss both free throws to result in a one-point deficit. In this scenario, the shooting team can win by making either a two or a three-point shot and thus is awarded payoffs based on the success ratios of the actions played. Similarly, for the game to enter State 2, the shooting team must make their two-point shot and the defending team must miss one free throw and make the other (i.e. miss the first, make the second or vice versa). This game state is akin to situation 1 that was analyzed in the first part of this paper and thus the payoffs reflect the complements of those (since we are now listing payoffs from the perspective of the shooting team). The only way the game can enter State 3 is if the shooting team makes their two-point shot and the defending team makes both of their free throws. In this scenario, the game will enter State 3 and the only reasonable outcome is for both teams to play the three-point shot, as it is the dominant strategy for both teams. Lastly we simplify the game such that if the shooting team misses its shot, which happens with probability 1 – shooting %, they cannot win the game.

Having understood the probability transition function and the payoffs of each the potential game states, we can now holistically analyze the original game. If the shooting team attempts a three-point shot the game will be tied and will thus enter overtime, the results of which is determined by a coin flip. On the other hand if the shooting team attempts a two-point shot the game will probabilistically enter one of the alternate states as described above and the expected payoffs can be determined. Figure 4 shows the expected payoff matrix of the game based on the mathematics shown in Appendix 3.

Figure 4 – Expected Payoffs of the Down by 3 Game

	Defend 2	**Defend 3**
Shoot 2	(0.0625, 0.9375)	(0.13, 0.87)
Shoot 3	(0.25, 0.75)	(0.075, 0.925)

We can now find a mixed Nash Equilibrium for this game using the expected payoffs. Appendix 4 shows the procedure to find this mixed Nash Equilibrium, which we determine to be ((0.72, 0.28), (0.227, 0.773)). In this equilibrium, we see that the defending team has a huge advantage, as one would expect. Because the defending team knows that by preventing the three-point shot they will have an opportunity to shoot high percentage free throw shots, they strongly favor defending the three-point shot. On the flip side, the shooting team will have to adjust to the huge payoffs that the defending team receives by defending the three-point shot and will most likely take a two-point shot since there is a high probability that it will be undefended, thus increasing its chances to win the game. However, the analysis of this situation has shared flaws with the first situation analyzed mostly due to the number of simplifications made and lack of hard data acquired to motivate the percentages.

Conclusion and Future Improvements

The analyses above provide a glimpse of potential applications of game theoretic concepts into basketball decision-making. In particular, these scenarios demonstrate that near the end of a basketball game, best responses can be determined for both the defending and shooting team. However, the model we have used here is slightly limited in its applications to real-life basketball situations because of the number of simplifications made. Beyond improving the percentages used to include hard data on the success rates of different shots of NBA players, this analysis could be improved to include anyone of the number of factors that Chow et al. identify in their paper. In particular, it would be quite doable to account for the abilities of individual players on a team by developing team specific shooting percentages or account for fouls or offensive rebounding by adding in states for the probability of either of these events. In particular, these could be important when analyzing a team like the Golden State Warriors who led the NBA this past season with a 40.3% three-point shooting percentage and widely are considered to have some of the best three-point shooters in the world. On the flip side, a team like the Miami Heat, known for having players who often draw fouls and convert a very high percentage of two-point shots, might be more inclined to shoot more twos. Beyond improving these specific analyses, modifications could be made to attempt to discover optimal strategies for similar but distinct late-game scenarios.

Works Cited

Chow, Ty et al. "Game Theory (MBA 217) Final Paper." *Haas School of Business, UC-Berkeley*.

Appendix 1 – Nash Equilibrium for Situation 1 (modified from Chow et al.)
Let q = probability defender defends 3
The expected payoff to the shooter is:
$q \times .15 + (1-q) * .5$ if shooting a 3
$q \times .35 + (1-q) * .165$ if shooting a 2

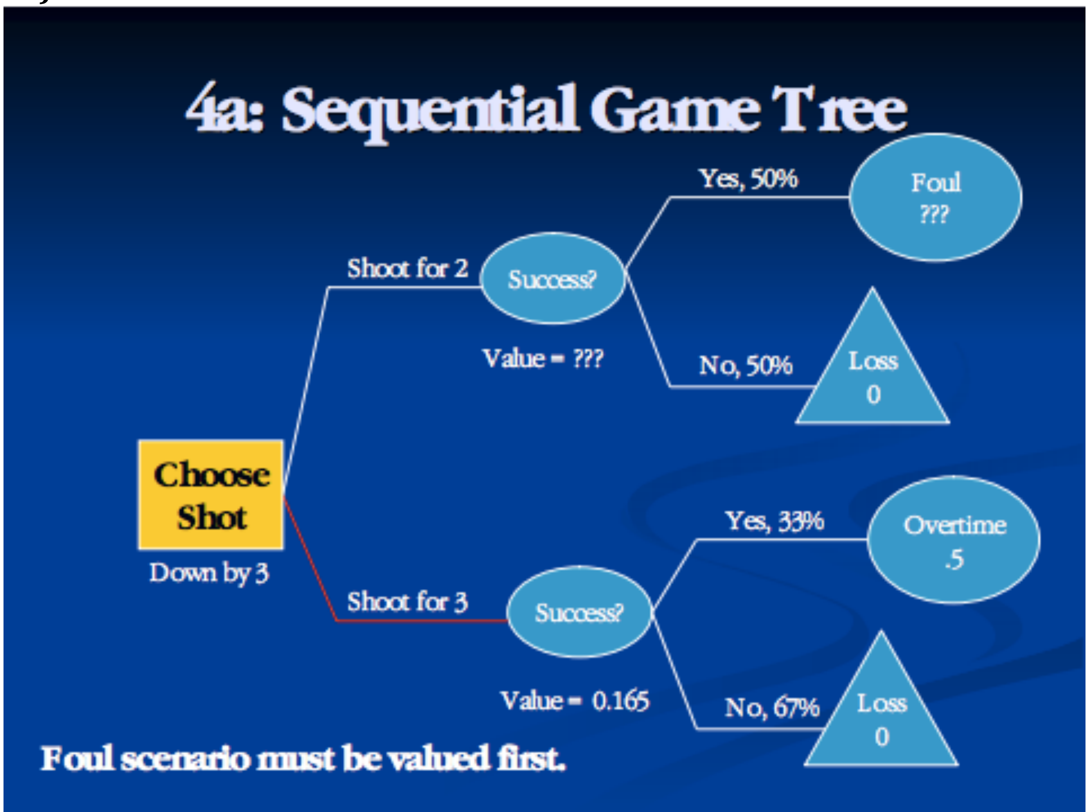
$q * .15 + (1 - q) * .50 = q * .35 + (1 - q) *.165$
$q = .626$

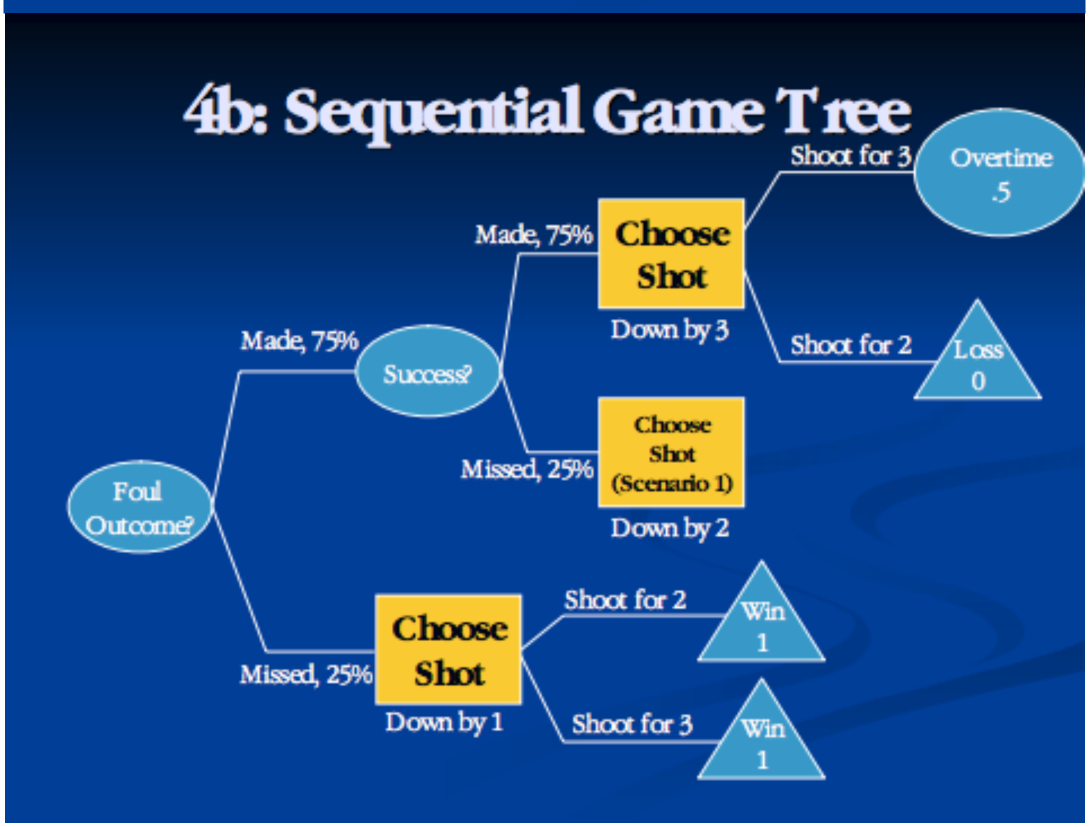
Let p = probability shooter shoots 3
The expected payoff to the defender is:
$p \times .85 + (1 - p) \times .65$ if defending 3
$p \times .50 + (1 - p) \times .835$ if defending 2

$p \times .85 + (1 - p) \times .65 = p \times .50 + (1 - p) \times .835$
$p = .346$

Therefore the mixed Nash Equilibrium is ((0.346, 0.654), (0.626, 0.374))

Appendix 2 – Simplified Sequential Analysis for Situation 2 (taken from Chow et al.)





Appendix 3 – Finding Expected Payoff of Situation 2

State 1 Mixed Nash Equilibrium

Player 2 defends 2 with probability p and 3 with probability 1-p

$$0.33p + 0.7(1-p) = 0.5p + 0.15(1-p)$$
$$p = 0.76$$

Player 1 shoots 2 with probability q and 3 with probability 1-q

$$0.66q + 0.5(1-q) = 0.32 + 0.85(1-q)$$
$$q = 0.49$$

State 2 Mixed Nash Equilibrium

Shown in Appendix 1 to be ((0.346, 0.654), (0.626, 0374))

State 3 Nash Equilibrium

Dominant strategy for both is to play (0,1) so the Nash Equilibrium is ((0,1), (0,1))

State 4 Nash Equilibrium

Only possible payoff is (0, 1) so any play is a Nash Equilibrium.

Expected Payoff for Outcomes Where Shooting Team shoots 2

$$\text{Expected Payoff} = \sum P(x) * EV(q')$$
$$\text{Expected Payoff(Shoot 2, Defend 2)} = 0.0625$$
$$\text{Expected Payoff(Shoot 2, Defend 3)} = 0.13$$

Appendix 4 – Finding Mixed Nash Equilibrium for Situation 2

Situation 2 Mixed Nash Equilibrium

Player 2 defends 2 with probability p and 3 with probability 1-p

$$0.0625p + 0.13(1-p) = 0.25p + 0.075(1-p)$$
$$p = 0.227$$

Player 1 shoots 2 with probability q and 3 with probability 1-q

$$0.9375q + 0.75(1-q) = 0.87 + 0.925(1-q)$$
$$q = 0.72$$

Thus the mixed Nash Equilibrium is ((0.72, 0.28), (0.227, 0.773)).