



**ELG 5255 Applied Machine Learning**

**Technical Report**  
**Assignment 2**

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# ## Part 1

predict the class of following example using Naïve Bayes Classification. (Species: 'Setosa' : 0, 'Versicolor' : 1, 'Virginica' : 2)

# Class (0)

⇒ Mean

$$\mu_0_{\text{feature}(1)} = \sum x P(x) = \frac{1}{N} \sum (x) = 5.08$$

$$\mu_0_{\text{feature}(2)} = \sum x P(x) = \frac{1}{N} \sum (x) = 3.7$$

$$\mu_0_{\text{feature}(3)} = \sum x P(x) = \frac{1}{N} \sum (x) = 1.52$$

$$\mu_0_{\text{feature}(4)} = \sum x P(x) = \frac{1}{N} \sum (x) = 0.26$$

# Variance ( $\sigma^2$ )

$$\begin{aligned} \sigma_0^2_{\text{feature}(1)} &= E[(X - \mu_0)^2] = (4.6 - 5.08)^2 + (4.9 - 5.08)^2 + (5.1 - 5.08)^2 \\ &\quad + (5.7 - 5.08)^2 + (4.8 - 5.08)^2 / 5 \\ &= 0.1656 \end{aligned}$$

$$\begin{aligned} \sigma_0^2_{\text{feature}(2)} &= (4.3 - 3.7)^2 + (3 - 3.7)^2 + (3.4 - 3.7)^2 + (4.4 - 3.7)^2 + (3.4 - 3.7)^2 \\ &\quad / 5 \\ &= 0.304 \end{aligned}$$

$$\begin{aligned} \sigma_0^2_{\text{feature}(3)} &= (4.4 - 1.52)^2 + (1.4 - 1.52)^2 + (1.7 - 1.52)^2 + (1.5 - 1.52)^2 + (1.6 - 1.52)^2 \\ &\quad / 5 \\ &= 0.0136 \end{aligned}$$

$$\begin{aligned} \sigma_0^2_{\text{feature}(4)} &= (0.3 - 0.26)^2 + (0.2 - 0.26)^2 + (0.2 - 0.26)^2 + (0.4 - 0.26)^2 \\ &\quad + (0.2 - 0.26)^2 / 5 = 0.0064 \end{aligned}$$

[1]

$$P(\text{Feature 1} | \text{class 0}) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} = \frac{4.445 \times 10^{-5}}{\approx 0}$$

$$P(\text{Feature 2} = 3.1 | \text{class 0}) \approx 0.4$$

$$P(\text{Feature 3} = 5.4 | \text{class 0}) \approx 0$$

$$P(\text{Feature 4} = 2.1 | \text{class 0}) \approx 0$$

# Class 1

⇒ mean

$$\mu_1 = \sum x P(x) = \frac{1}{N} \sum x = 6.24$$

Feature (1)

$$\mu_2 = \sum x P(x) = \frac{1}{N} \sum x = 3.16$$

Feature (2)

$$\mu_3 = \sum x P(x) = \frac{1}{N} \sum x = 4.52$$

Feature (3)

$$\mu_4 = \sum x P(x) = \frac{1}{N} \sum x = 1.56$$

Feature (4)

$$\sigma_{\text{class (1) Feature (1)}}^2 = [E(X - \mu_1)^2] = \frac{(6.3 - 6.24)^2 + (6.4 - 6.24)^2 + (5.9 - 6.24)^2 + (6.7 - 6.24)^2 + (5.9 - 6.24)^2}{5} = 0.0944$$

$$\sigma_{\text{Feature (2)}}^2 = \frac{(3.3 - 3.16)^2 + (3.2 - 3.16)^2 + (3.2 - 3.16)^2 + (3.1 - 3.16)^2 + (3 - 3.16)^2}{5} = 0.0104$$

$$\sigma_{\text{Feature (3)}}^2 = \frac{(4.7 - 4.52)^2 + (4.5 - 4.52)^2 + (4.8 - 4.52)^2 + (4.4 - 4.52)^2 + (4.2 - 4.52)^2}{5} = 0.0456$$

(ii) = Coef:  $\frac{1.6 - 1.8 \cdot (1.5 - 1.8)}{(1.8 - 1.86) + (1.5 - 1.86) + (1.5 - 1.86) + (1.5 - 1.86) + (1.5 - 1.86)} = 0.0184$

$$P(\text{Feature 1} = 6.9 \mid \text{class(1)}) = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}}$$

$$= \frac{1}{\sqrt{2\pi(0.0944)}} e^{-\frac{(6.9-6.24)^2}{2 \times 0.0944}} = 0.129$$

$$P(\text{Feature 2} = 3.1 \mid \text{class(1)}) = \frac{1}{\sqrt{2\pi(0.0104)}} e^{-\frac{(3.1-3.16)^2}{2(0.0104)}}$$

$$= 3.4 \times 0.84 = 2.856$$

$$P(\text{Feature 3} = 5.4 \mid \text{class(1)}) = \frac{1}{\sqrt{2\pi(0.0456)}} e^{-\frac{(5.4-4.52)^2}{2(0.0456)}}$$

$$= 3.8 \times 10^{-4} \approx 0$$

$$P(\text{Feature 4} = 2.1 \mid \text{class(1)}) = \frac{1}{\sqrt{2\pi(0.0184)}} e^{-\frac{(2.1-1.56)^2}{2(0.0184)}}$$

$$= 1.06 \times 10^{-3} \approx 0$$



class(2)

⇒ Mean

$$\mu_2^{\text{feature(1)}} = \sum x P(x) = \frac{1}{N} \sum (x) = (6.08)$$

$$\mu_2^{\text{feature(2)}} = \sum x P(x) = \frac{1}{N} \sum (x) = (2.86)$$

$$\mu_2^{\text{feature(3)}} = \sum x P(x) = \frac{1}{N} \sum (x) = (5.18)$$

$$\mu_2^{\text{feature(4)}} = \sum x P(x) = \frac{1}{N} \sum (x) = (2.02)$$

Variance ( $\sigma^2$ )

$$\sigma_2^2^{\text{feature(1)}} = \left[ E (x - \mu_2)^2 \right] = \frac{(4.9 - 6.08)^2 + (5.8 - 6.08)^2 + (6.4 - 6.08)^2 + (6.4 - 6.08)^2 + (6.4 - 6.08)^2}{5} = (0.4696)$$

$$\sigma_2^2^{\text{feature(2)}} = \frac{(2.5 - 2.86)^2 + (2.7 - 2.86)^2 + (3.2 - 2.86)^2 + (3.2 - 2.86)^2 + (2.7 - 2.86)^2}{5} = (0.0824)$$

$$\sigma_2^2^{\text{feature(3)}} = \frac{(4.5 - 5.18)^2 + (5.1 - 5.18)^2 + (5.7 - 5.18)^2 + (5.3 - 5.18)^2 + (5.3 - 5.18)^2}{5} = (0.15872)$$

$$\sigma_2^2^{\text{feature(4)}} = \frac{(1.7 - 2.02)^2 + (1.9 - 2.02)^2 + (2.3 - 2.02)^2 + (2.3 - 2.02)^2 + (1.9 - 2.02)^2}{5} = (0.05704)$$

$$P(\text{Feature 1} = 6.9 | \text{class}(2)) = \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}$$

$$= \frac{1}{\sqrt{2\pi}(0.4696)} \times e^{-\frac{(6.9 - 6.08)^2}{2(0.4696)^2}} = 0.415$$

$$P(\text{Feature 2} = 3.1 | \text{class}(2)) = \frac{1}{\sqrt{2\pi}(0.0824)} \times e^{-\frac{(3.1 - 2.86)^2}{2(0.0824)^2}}$$

$$= 0.9798$$

$$P(\text{Feature 3} = 5.4 | \text{class}(2)) = \frac{1}{\sqrt{2\pi}(0.15872)} \times e^{-\frac{(5.4 - 5.18)^2}{2(0.15872)^2}}$$

$$= 0.8598$$

$$P(\text{Feature 4} = 2.1 | \text{class}(2)) = \frac{1}{\sqrt{2\pi}(0.05904)} \times e^{-\frac{(2.1 - 2.02)^2}{2(0.05904)^2}}$$

$$= 1.579$$

$$\begin{aligned}
 \text{evidence}_0 &= P(\text{class}_0) * P(\text{feature}_1 | \text{class}_0) * \\
 &P(\text{feature}_2 | \text{class}_0) * P(\text{feature}_3 | \text{class}_0) * P(\text{feature}_4 | \text{class}_0) \\
 &= \frac{1}{3} * 4.4 * 10^{-5} * 0.4 * 0 * 0 = \boxed{\text{Zero}}
 \end{aligned}$$

$$\begin{aligned}
 \text{evidence}_1 &= P(\text{class}_1) * P(\text{feature}_1 | \text{class}_1) \\
 &* P(\text{feature}_2 | \text{class}_1) * P(\text{feature}_3 | \text{class}_1) * \\
 &P(\text{feature}_4 | \text{class}_1) \\
 &= \frac{1}{3} * 0.129 * 3.3 * 0 * 0 = \boxed{\text{Zero}}
 \end{aligned}$$

$$\begin{aligned}
 \text{evidence}_2 &= P(\text{class}_2) * P(\text{feature}_1 | \text{class}_2) \\
 &* P(\text{feature}_2 | \text{class}_2) * P(\text{feature}_3 | \text{class}_2) \\
 &* P(\text{feature}_4 | \text{class}_2) \\
 &= \frac{1}{3} * 0.415 * 0.9 * 0.8 * 1.5 = \boxed{0.147}
 \end{aligned}$$

→ According to Gaussian function and evidence the test sample belongs to class\_2



After these calculations, we found that the probability of the test sample belonging to class 2 is greater than it belongs to class 1 and 0 .



# Another solution

①

/ /

Sepal length

0	1	2
4.6	6.3	4.9
4.9	6.4	5.8
5.4	5.9	6.9
5.7	6.7	6.4
4.8	5.9	6.4

mean

$$\frac{\sum x}{n} = \left[ \frac{25.4}{5} \right], \left[ \frac{31.2}{5} \right], \left[ \frac{30.4}{5} \right] \Rightarrow \text{class 0, 1, 2}$$

5.08      6.24      6.08

$$\frac{\sum (x - m)^2}{n} = \frac{(5.08 - 4.6)^2 + (5.08 - 4.9)^2 + (5.08 - 5.4)^2 + (5.08 - 5.7)^2 + (5.08 - 4.8)^2}{5}$$

$$= \frac{(0.48)^2 + (0.18)^2 + (0.32)^2 + (0.62)^2 + (0.28)^2}{5}$$

$$= \frac{0.828}{5} = \underline{\underline{0.1656}}$$

spread length

(2)

$$\begin{aligned} \sigma_1^2 &= \frac{(6.3-6.24)^2 + (6.4-6.24)^2 + (5.9-6.24)^2 + (6.7-6.24)^2 + (5.9-6.24)^2}{5} \\ &= \frac{(0.06)^2 + (0.16)^2 + (0.34)^2 + (0.46)^2 + (0.34)^2}{5} \\ &= \frac{0.472}{5} = \underline{\underline{0.0944}} \end{aligned}$$

$$\begin{aligned} \sigma_2^2 &= \frac{(6.08-4.9)^2 + (6.08-5.8)^2 + (6.08-6.9)^2 + (6.08-6.4)^2 + (6.08-6.4)^2}{5} \\ &= \frac{(1.18)^2 + (0.28)^2 + (0.82)^2 + (0.32)^2 + (0.32)^2}{5} \\ &= \frac{2.348}{5} = \underline{\underline{0.4696}} \end{aligned}$$

$$P = SL_0(6.9) = \frac{1}{\sqrt{2\pi} \sigma_0} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$\begin{aligned} &\frac{1}{\sqrt{2\pi} \sqrt{0.1656}} \exp\left(-\frac{(6.9-5.08)^2}{2 \times 0.1656}\right) = \\ &\frac{1}{1.02} \exp\left(-\frac{(1.82)^2}{0.3312}\right) = \frac{1}{1.02} e^{-\frac{3.3124}{0.3312}} = \\ &\frac{1}{1.02} e^{-10.0012} = \underline{\underline{0.0000444}} \end{aligned}$$

$$\begin{aligned} SL_1(6.9) &= \frac{1}{\sqrt{2\pi} \times 0.0944} \exp\left(-\frac{(6.9-6.24)^2}{2 \times 0.0944}\right) = \\ &\frac{1}{0.77} e^{-\frac{0.4356}{0.1888}} = \underline{\underline{0.1292}} \end{aligned}$$

$$\begin{aligned}
 S_2(6.9) &= \frac{1}{\sqrt{2\pi} \sqrt{0.4696}} e^{-\frac{(6.9-6.08)^2}{2 \cdot 0.4696}} \\
 &= \frac{1}{1.7177} e^{-\frac{(0.82)^2}{0.9392}} = \\
 &= \frac{1}{1.7177} e^{-0.7159} = \underline{\underline{0.2845}}
 \end{aligned}$$



3

Sepal width

0	1	2
3.4	3.3	2.5
3	3.2	2.7
3.4	3.2	3.2
3.4	3.1	3.2
4.4	3.0	2.7

$$\mu_0 = \frac{17.6}{5} \quad \mu_1 = \frac{15.8}{5} \quad \mu_2 = \frac{14.3}{5}$$

$$\mu_0 = 3.52, \mu_1 = 3.16, \mu_2 = 2.86$$

$$s_0^2 = \frac{(3.52-3.4)^2 + (3.52-3)^2 + (3.52-3.4)^2 + (3.52-3.4)^2 + (3.52-4.4)^2}{5}$$

$$= \frac{(0.12)^2 + (0.52)^2 + (0.12)^2 + (0.12)^2 + (0.88)^2}{5}$$

$$= \frac{1.088}{5} = 0.2176$$

$$s_1^2 = \frac{(3.16-3.3)^2 + (3.16-3.2)^2 + (3.16-3.2)^2 + (3.16-3.1)^2 + (3.16-3)^2}{5}$$

$$= \frac{(0.14)^2 + (0.04)^2 + (0.04)^2 + (0.06)^2 + (0.16)^2}{5}$$

$$= \frac{0.052}{5} = 0.0104$$



[4]

$$a_2^2 = \frac{(2.86-2.5)^2 + (2.86-2.7)^2 + (2.86-3.2)^2 + (2.86-3.2)^2 + (2.86-2.7)^2}{5}$$

$$= \frac{(0.36)^2 + (0.16)^2 + (0.34)^2 + (0.34)^2 + (0.16)^2}{5}$$

$$= \frac{0.412}{5} = 0.0824$$

$$Sw_0(3.1) = \frac{1}{\sqrt{2\pi} \sqrt{0.2176}} e^{-\frac{(3.1-3.52)^2}{2 \times 0.2176}}$$

$$= \frac{1}{1.1692} e^{-\frac{(-0.42)^2}{0.4352}}$$

$$= \frac{1}{1.1692} \times 0.66675 = 0.57$$

$$Sw_1(3.1) = \frac{1}{\sqrt{2\pi} \sqrt{0.0104}} e^{-\frac{(3.1-3.16)^2}{2 \times 0.0104}}$$

$$= \frac{1}{0.2556} e^{-\frac{0.0036}{0.0208}} = 0.173$$

$$= \frac{0.841}{3.29}$$

$$Sw_2(3.1) = \frac{1}{\sqrt{2\pi} \sqrt{0.0824}} e^{-\frac{(3.1-2.86)^2}{2 \times 0.0824}}$$

$$= 0.979$$

5

Petal length

0	1	2
1.4	4.7	4.5
1.4	4.5	<del>4.5</del> 5.1
1.7	4.8	5.7
1.5	4.4	5.3
1.6	4.2	5.3

$$\mu_0 = \frac{7.6}{5}, \mu_1 = \frac{22.6}{5}, \mu_2 = \frac{25.9}{5}$$

$$\mu_0 = 1.52, \mu_1 = 4.52, \mu_2 = 5.18$$

$$\begin{aligned} s_0^2 &= \frac{(1.52-1.4)^2 + (1.52-1.4)^2 + (1.52-1.7)^2 + (1.52-1.5)^2 + (1.52-1.6)^2}{5} \\ &= \frac{(0.12)^2 + (0.12)^2 + (0.18)^2 + (0.02)^2 + (0.08)^2}{5} \\ &= \frac{0.068}{5} = 0.0136 \end{aligned}$$

$$\begin{aligned} s_1^2 &= \frac{(4.52-4.7)^2 + (4.52-4.5)^2 + (4.52-4.8)^2 + (4.52-4.4)^2 + (4.52-4.2)^2}{5} \\ &= \frac{(0.18)^2 + (0.02)^2 + (0.28)^2 + (0.12)^2 + (0.32)^2}{5} \\ &= \frac{0.228}{5} = 0.0456 \end{aligned}$$

(6)

$$\begin{aligned}
 a_2^2 &= \frac{(5.18-4.5)^2 + (5.18-5.1)^2 + (5.18-5.7)^2 + (5.18-5.3)^2 + (5.18-5.3)^2}{5} \\
 &= \frac{(0.68)^2 + (0.08)^2 + (0.52)^2 + (0.12)^2 + (0.12)^2}{5} \\
 &= \frac{0.768}{5} = 0.1536
 \end{aligned}$$

$$PL_0(5.4) = \frac{1}{\sqrt{2\pi} \sqrt{0.0136}} e^{-\frac{(5.4-1.52)^2}{2 \times 0.0136}} = 1.4 \times 10^{-24} \approx 0$$

$$PL_1(5.4) = \frac{1}{\sqrt{2\pi} \sqrt{0.0456}} e^{-\frac{(5.4-4.52)^2}{2 \times 0.0456}} = 0.000383 \approx 0$$

$$\begin{aligned}
 PL_2(5.4) &= \frac{1}{\sqrt{2\pi} \sqrt{0.1536}} e^{-\frac{(5.4-5.18)^2}{2 \times 0.1536}} \\
 &= 0.869
 \end{aligned}$$

7

Petal width

0	1	2
0.3	1.6	1.7
0.2	1.5	1.9
0.2	1.8	2.3
0.4	1.4	2.3
0.2	1.5	1.9

$$\mu = \left[ \frac{1.3}{5} \right] \left[ \frac{7.8}{5} \right] \left[ \frac{10.1}{5} \right]$$

$$\mu_0 = 0.26, \mu_1 = 1.56, \mu_2 = 2.02$$

$$\begin{aligned} \sigma_0^2 &= \frac{(0.26-0.3)^2 + (0.26-0.2)^2 + (0.26-0.2)^2 + (0.26-0.4)^2 + (0.26-0.2)^2}{5} \\ &= \frac{(0.04)^2 + (0.06)^2 + (0.06)^2 + (0.14)^2 + (0.06)^2}{5} \\ &= \frac{0.032}{5} = 0.0064 \end{aligned}$$

$$\begin{aligned} \sigma_1^2 &= \frac{(1.56-1.6)^2 + (1.56-1.5)^2 + (1.56-1.8)^2 + (1.56-1.4)^2 + (1.56-1.5)^2}{5} \\ &= \frac{(0.04)^2 + (0.06)^2 + (0.24)^2 + (0.06)^2 + (0.16)^2}{5} \\ &= \frac{0.092}{5} = 0.0184 \end{aligned}$$



(8)

$$\begin{aligned}
 a_2^2 &= \frac{(2.02-1.7)^2 + (2.02-1.9)^2 + (2.02-2.3)^2 + (2.02-1.9)^2 + (0.28)^2}{5} \\
 &= \frac{(0.32)^2 + (0.12)^2 + (0.28)^2 + (0.12)^2 + (0.28)^2}{5} \\
 &= \frac{0.288}{5} = \underline{\underline{0.0576}}
 \end{aligned}$$

$$\begin{aligned}
 Pw_0(2.1) &= \frac{1}{\sqrt{2\pi} \sqrt{0.0064}} e^{-\frac{(2.1-0.28)^2}{2 \times 0.0064}} \\
 &= 67.13 \times 10^{-11.6} = 0
 \end{aligned}$$

$$\begin{aligned}
 Pw_1(2.1) &= \frac{1}{\sqrt{2\pi} \sqrt{0.0184}} e^{-\frac{(2.1-1.56)^2}{2 \times 0.0184}} \\
 &= 0.06106
 \end{aligned}$$

$$\begin{aligned}
 Pw_2(2.1) &= \frac{1}{\sqrt{2\pi} \sqrt{0.0576}} e^{-\frac{(2.1-2.02)^2}{2 \times 0.0576}} \\
 &= 1.57
 \end{aligned}$$

A

$$P(\text{class}=0) = P(\text{class}=1) \cdot P(\text{class}=2) = 5/15$$

$$\textcircled{1} P(\text{class}=0 \mid \begin{matrix} SL=6.9 \\ SW=3.1 \\ PL=5.4 \\ PW=2.1 \end{matrix}) =$$

$$P(\text{class}=0) \cdot SL_0(6.9) \cdot SW_0(3.1) \cdot PL_0(5.4) \cdot PW_0(2.1) =$$

$$\frac{5}{15} \cdot 0.0000444 \cdot 0.57 \cdot 0 \approx 0$$

$$\textcircled{2} P(\text{class}=1 \mid \begin{matrix} SL=6.9 \\ SW=3.1 \\ PL=5.4 \\ PW=2.1 \end{matrix}) = P(\text{class}=1) \cdot SL_1(6.9) \cdot SW_1(3.1) \cdot PL_1(5.4) \cdot PW_1(2.1) =$$

$$\frac{5}{15} \cdot 0.1292 \cdot 3.29 \cdot 0.000383 \cdot 0.00106 \cdot 0.000005752 \approx 0$$

$$\textcircled{3} P(\text{class}=2 \mid \begin{matrix} SL=6.9 \\ SW=3.1 \\ PL=5.4 \\ PW=2.1 \end{matrix}) = P(\text{class}=2) \cdot SL_2(6.9) \cdot SW_2(3.1) \cdot PL_2(5.4) \cdot PW_2(2.1)$$

$$\frac{5}{15} \cdot 0.2845 \cdot 0.979 \cdot 0.869 \cdot 1.57 = 0.1266$$

Belong To class(2)

After these calculations of mean and variance for each feature within the three classes , we found that the probability of the test sample belonging to class 2 which is "Virginica" is greater than it belongs to class 1 and 0

# ## Part 2

## 1. Load the Iris dataset

```
import numpy as np
import pandas as pd
import json
from pandas import Series, DataFrame
import seaborn as sns
import matplotlib.pyplot as plt
from sklearn.datasets import load_iris

%matplotlib inline
```

```
#Load our dataset
iris = load_iris()
print(iris)
```

```
{'data': array([[5.1, 3.5, 1.4, 0.2],
               [4.9, 3. , 1.4, 0.2],
               [4.7, 3.2, 1.3, 0.2],
               [4.6, 3.1, 1.5, 0.2],
               [5. , 3.6, 1.4, 0.2],
               [5.4, 3.9, 1.7, 0.4],
               [4.6, 3.4, 1.4, 0.3],
               [5. , 3.4, 1.5, 0.2],
               [4.4, 2.9, 1.4, 0.2],
               [4.9, 3.1, 1.5, 0.1],
               [5.4, 3.7, 1.5, 0.2],
               [4.8, 3.4, 1.6, 0.2],
               [4.8, 3. , 1.4, 0.1],
               [4.3, 3. , 1.1, 0.1],
               [5.8, 4. , 1.2, 0.2],
               [5.7, 4.4, 1.5, 0.4],
               [5.4, 3.9, 1.3, 0.4],
               [5.1, 3.5, 1.4, 0.3],
               .
               .
               .])
```

## 2. Drop the petal length and petal width features to form a 2D Iris dataset

```
#(1)Drop the first&second features
X = iris.data[:, :2] # we only take the first two features.
Y = iris.target
print(X)
print(y)
```

```
[[5.1 3.5]
 [4.9 3. ]
 [4.7 3.2]
 [4.6 3.1]
 [5.  3.6]
 [5.4 3.9]
 [4.6 3.4]
 [5.  3.4]
 [4.4 2.9]
 [4.9 3.1]
 [5.4 3.7]
 [4.8 3.4]
 [4.8 3. ]
 [4.3 3. ]
 [5.8 4. ]
 [5.7 4.4]
 [5.4 3.9]
 [5.1 3.5]
 [5.7 3.8]
 [5.1 3.8]
 [5.4 3.4]
 [5.1 3.7]
```

3.

### 3.1 Plot the likelihoods of first feature (Sepal length) for each class.

```
from scipy import stats
from scipy.stats import norm
import scipy

data_column = X[:, :1]
#print(data_column)

class_0=data_column[0:50]
class_1=data_column[51:100]
class_2=data_column[101:150]

mean=data_column.mean()
mean_0=class_0.mean()
mean_1=class_1.mean()
mean_2=class_2.mean()

variance=data_column.var()
variance_0=class_0.var()
variance_1=class_1.var()
variance_2=class_2.var()

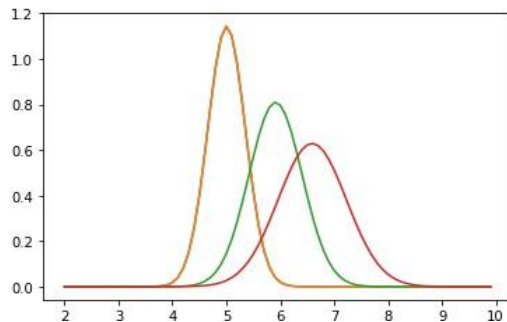
x_values = np.arange(2, 10, 0.1)
y_values = scipy.stats.norm(mean_0, np.sqrt(variance_0))

plt.plot(x_values, y_values.pdf(x_values))

lh1=scipy.stats.norm(mean_0, np.sqrt(variance_0))
lh2=scipy.stats.norm(mean_1, np.sqrt(variance_1))
lh3=scipy.stats.norm(mean_2, np.sqrt(variance_2))

plt.plot(x_values, lh1.pdf(x_values))
plt.plot(x_values, lh2.pdf(x_values))
plt.plot(x_values, lh3.pdf(x_values))
```

[<matplotlib.lines.Line2D at 0x2526052b070>]





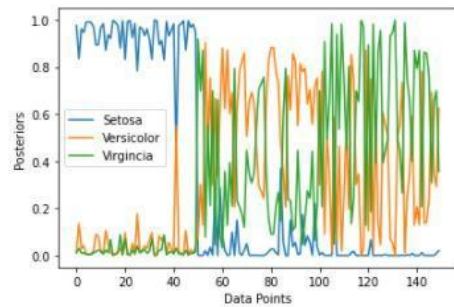
### 3.2 Naïve Bayes Classifier to 2D Iris dataset to predict classes. Plot posterior probabilities

```
from sklearn.naive_bayes import GaussianNB
from sklearn.metrics import accuracy_score

gnb = GaussianNB()
y_pred = gnb.fit(X, Y).predict_proba(X)

plt.plot(range(0,150),y_pred[:, 0], label="Setosa")
plt.plot(range(0,150),y_pred[:, 1], label="Versicolor")
plt.plot(range(0,150),y_pred[:, 2], label="Virginica")
plt.ylabel('Posteriors')
plt.xlabel('Data Points')

plt.legend()
plt.show()
```



### 3.3 calculate the accuracy =0.78

```
from sklearn.naive_bayes import GaussianNB
from sklearn.metrics import accuracy_score

gnb = GaussianNB()
y_pred = gnb.fit(X, Y).predict(X)
print(accuracy_score(Y,y_pred))

0.78
```

4. and values must already be obtained to solve previous question. Now change the actual values of  $\mu$  and  $\sigma$  to the given values below for each class. Plot the likelihoods of the first feature (Sepal length) with updated  $\mu$  and  $\sigma$  values for each class and apply Naïve Bayes Classifier.

#### 4.1

- 4.1.1  $\mu = 5.5$  and keep the actual values of variance for the first feature. Plot the likelihoods.

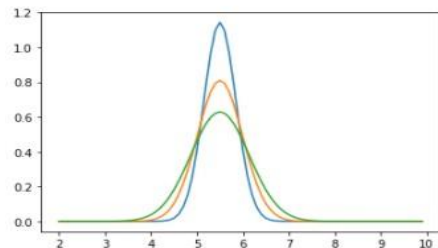
```
from scipy import stats
from scipy.stats import norm
import numpy as np

mean=5.5

lh1=scipy.stats.norm(mean, np.sqrt(variance_0))
lh2=scipy.stats.norm(mean, np.sqrt(variance_1))
lh3=scipy.stats.norm(mean, np.sqrt(variance_2))

plt.plot(x_values, lh1.pdf(x_values))
plt.plot(x_values, lh2.pdf(x_values))
plt.plot(x_values, lh3.pdf(x_values))
```

[<matplotlib.lines.Line2D at 0x2526077ed90>]



- 4.1.2 Naïve Bayes Classifier. Plot posterior probabilities

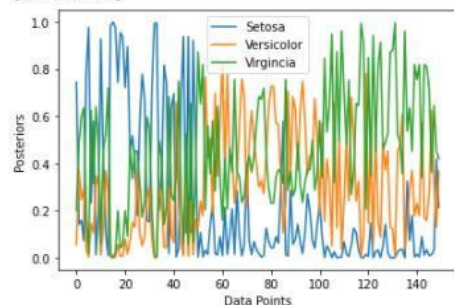
```
gnb.theta[:,0]=[5.5, 5.5, 5.5]
print(gnb.theta[:,0])

y_pred = gnb.predict_proba(X)

plt.plot(range(0,150),y_pred[:, 0], label="Setosa")
plt.plot(range(0,150),y_pred[:, 1], label="Versicolor")
plt.plot(range(0,150),y_pred[:, 2], label="Virginica")
plt.ylabel('Posteriors')
plt.xlabel('Data Points')

plt.legend()
plt.show()
```

[5.5 5.5 5.5]



- 4.1.3 calculate the accuracy =0.62

```
y_pred = gnb.predict(X)
print(accuracy_score(Y,y_pred))

0.6266666666666667
```

## 4.2

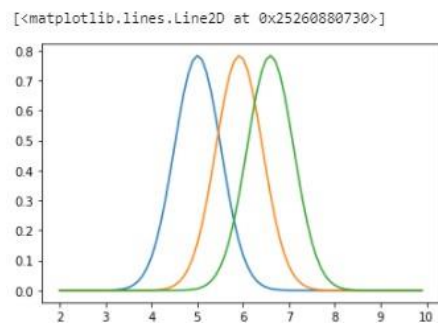
4.2.1 variance = 0.26 and keep the actual values of mean for the first feature. Plot the likelihoods.

```
from scipy import stats
from scipy.stats import norm
import scipy

variance=0.26

lh1=scipy.stats.norm(mean_0, np.sqrt(variance))
lh2=scipy.stats.norm(mean_1, np.sqrt(variance))
lh3=scipy.stats.norm(mean_2, np.sqrt(variance))

plt.plot(x_values, lh1.pdf(x_values))
plt.plot(x_values, lh2.pdf(x_values))
plt.plot(x_values, lh3.pdf(x_values))
```



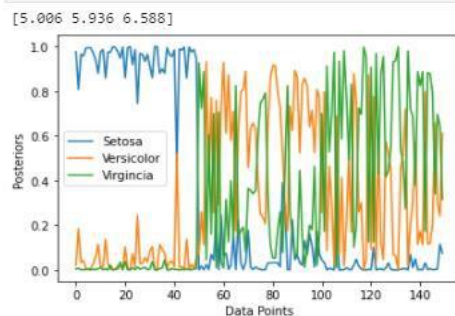
4.2.2 Naïve Bayes Classifier. Plot posterior probabilities

```
gnb.fit(X,Y)
gnb.sigma[:,0]=[0.26, 0.26, 0.26]
print(gnb.theta[:,0])

y_pred = gnb.predict_proba(X)

plt.plot(range(0,150),y_pred[:, 0], label="Setosa")
plt.plot(range(0,150),y_pred[:, 1], label="Versicolor")
plt.plot(range(0,150),y_pred[:, 2], label="Virginica")
plt.ylabel('Posteriors')
plt.xlabel('Data Points')

plt.legend()
plt.show()
```

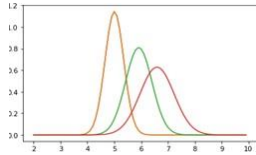
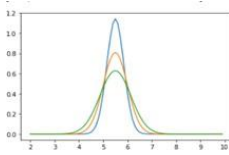
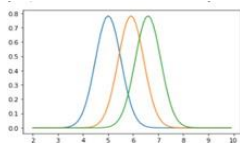
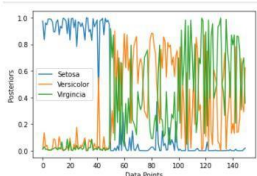
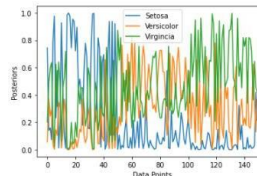
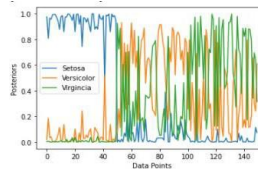


4.2.3 calculate the accuracy =0.8

```
y_pred = gnb.predict(X)
print(accuracy_score(Y,y_pred))

0.6266666666666667
```

### 4.3 Compare the accuracy values and make a comment based on it

Points of Comparison	Point 3	Point 4.1	Point 4.2
Mean	5.006, 5.914, 6.593	5.5 for all	5.006, 5.914, 6.593
variance	0.12, 0.24, 0.40	0.12, 0.24, 0.40	0.26 for all
Likelihood			
Posterior			
Accuracy	0.78	0.62	0.8
Comment	<p>as mean represents the expected class of the variable, which is the centroid of the pdf and same the point at which the pdf is maximum, so as the mean for each class be separated from the other classes the accuracy increased and become easily for our model to predict the right class.</p>	<p>when we let mean for the 3 classes to be the same =5.5 the accuracy decreased cause now it is more difficult to predict the output to which class</p> <p>-Cause the mean of gaussian means the average of many samples (observations) and all of the 3 model has the same average of observations</p>	<p>when we let var=0.26 for the 3 classes to be the same, and the mean is the same as point 3 and as the variance <math>\sigma^2</math> is a measure of the dispersion of the random variable around the mean. The accuracy increased and be the higher one.</p> <p>-Cause here the average of many samples (observations) for every curve are far from each other and the interactions between them becomes lower</p>