Intersection of lines in 2D

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Intersection of Lines in y=ax+b form

If you have the equations in y = ax+b format for both lines (i.e. : none of them is vertical), the lines intersect only if the slopes are different ($a_1 \neq a_2$), and the intersection is:

$$x = (b_2-b_1)/(a_1-a_2)$$

$$y = a_1x + b_1 = a_1(b_2-b_1)/(a_1-a_2) + b_1$$

Intersection of Lines in ax+by +c = 0 form

If you have the equations in the more general case : ax+by+c = 0, the lines intersect only if $a_1b_2 \neq a_2b_1$. If both lines are vertical then by definition they do not intersect, so it can be assumed that at least one of the lines is non-vertical (for example, $a_1 \neq 0$). In this case, the intersection is:

- $y = (c_1a_2 - c_2a_1)/(a_1b_2-a_2b_1)$
- $x = -1/a_1 - b_1y/a_1$

Intersection of Lines defined by four points.

If you have four points : (x_1,y_1) and (x_2,y_2) belonging to the first line and (x_3,y_3) and (x_4,y_4) belonging to the second line, the intersection is given by solving the following equations:

$$\begin{vmatrix} x_i & y_i & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0 \text{ and } \begin{vmatrix} x_i & y_i & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix} = 0$$

where x_i,y_i is the intersection point. Now I assume you came here for the plain results and not for an explanation or derivation of them, so, getting to the point:

The lines intersect if $D \equiv (x_1-x_2)(y_3-y_4) - (y_1-y_2)(x_3-x_4) \neq 0$. The intersection point is

$$x_i = \frac{1}{D}\begin{vmatrix} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} & (x_1-x_2) \\ \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} & (x_3-x_4) \end{vmatrix} = \frac{1}{D}\left[(x_3-x_4)(x_1y_2-y_1x_2)-(x_1-x_2)(x_3y_4-y_3x_4)\right]$$

$$y_i = \frac{1}{D}\begin{vmatrix} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} & (y_1-y_2) \\ \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} & (y_3-y_4) \end{vmatrix} = \frac{1}{D}\left[(y_3-y_4)(x_1y_2-y_1x_2)-(y_1-y_2)(x_3y_4-y_3x_4)\right]$$

where "D" is the above quantity.

The following Java function returns the intersection of two lines defined by four points. Use freely, but please give credit. LGPL license.

intersection-lines.java

```
0001   /**
0002    * Computes the intersection between two lines. The calculated point is approximate,
0003    * since integers are used. If you need a more precise result, use doubles
0004    * everywhere.
0005    * (c) 2007 Alexander Hristov. Use Freely (LGPL license). http://www.ahristov.com
0006    *
0007    * @param x1 Point 1 of Line 1
0008    * @param y1 Point 1 of Line 1
0009    * @param x2 Point 2 of Line 1
0010    * @param y2 Point 2 of Line 1
0011    * @param x3 Point 1 of Line 2
0012    * @param y3 Point 1 of Line 2
0013    * @param x4 Point 2 of Line 2
0014    * @param y4 Point 2 of Line 2
0015    * @return Point where the segments intersect, or null if they don't
0016    */
0017   public Point intersection(
0018     int x1,int y1,int x2,int y2,
0019     int x3, int y3, int x4,int y4
0020   ) {
0021     int d = (x1-x2)*(y3-y4) - (y1-y2)*(x3-x4);
0022     if (d == 0) return null;
0023
0024     int xi = ((x3-x4)*(x1*y2-y1*x2)-(x1-x2)*(x3*y4-y3*x4))/d;
0025     int yi = ((y3-y4)*(x1*y2-y1*x2)-(y1-y2)*(x3*y4-y3*x4))/d;
0026
0027     return new Point(xi,yi);
0028   }
```

Source code

intersection-lines.java (984 bytes)		

Comments

May 30, 2012 at 06:35 Sent by anonymous
Incomplete... this is for line intersections, NOT line segment intersections.
May 14, 2012 at 14:35 Sent by Benjol
Great, but "Intersection of Lines in ax+by +c = 0 form" second equation should be: x = -c1/a1 - b1y/a1 (the c was missing).
May 13, 2012 at 13:02 Sent by anonymous
very nice and thanks
Apr 10, 2011 at 16:50 Sent by cekone