1) $U = 3 - 8x + 6y \quad x^2 + y^2 = 36$

$L(\cancel{\lambda}_1, x, y) = 3 - 8x + 6y + \lambda_1(x^2 + y^2 - 36)$

$$\begin{cases} L'(x) = -8 + 2\lambda_1 x = 0 \\ L'(y) = 6 + 2\lambda_1 y = 0 \\ L'_{\lambda_1} = x^2 + y^2 - 36 = 0 \end{cases} \qquad \begin{cases} x = +\dfrac{4}{\lambda_1} \\ y = \dfrac{-3}{\lambda_1} \\ \dfrac{16}{\lambda_1^2} + \dfrac{9}{\lambda^2} = 36 \end{cases}$$

$$\begin{cases} x = +\dfrac{4}{\lambda_1} \\ y = -3/\lambda_1 \\ \lambda^2 = \dfrac{25}{36} \end{cases} \qquad M_1\left(+\frac{24}{5}; -\frac{18}{5}; \frac{5}{6}\right) \quad M_2\left(\frac{-24}{5}; +\frac{18}{5}; -\frac{5}{6}\right)$$

$L''_{xx} = 2\lambda_1 \quad L''_{yy} = 2\lambda_1 \quad L''_{\lambda_1\lambda_1} = 0$

$L''_{xy} = 0 \quad L''_{x\lambda_1} = 2x \quad L''_{\lambda_1 y} = 2y$

$$\begin{vmatrix} 0 & 2x & 2y \\ 2x & 2\lambda_1 & 0 \\ 2y & 0 & 2\lambda_1 \end{vmatrix} = -8\lambda_1(x^2 + y^2) = A$$

это как в презентации

$M_1: A = -8 \cdot 5/6 \cdot \left(\frac{24^2}{5^2} + \frac{18^2}{5^2}\right) < 0$

M_1 - минимум

$M_2: A = -8 \cdot \left(-\frac{5}{6}\right) \cdot (\ ) > 0; \ M_2$ - максимум

3) $u = x^2 + y^2 + z^2 \quad \vec{c}\,(-9, 8, -12) \quad M(8; -12; 9)$

$$\frac{\partial u}{\partial x} = 2x, \quad \frac{\partial u}{\partial x}\Big|_{(M)} = 16$$

$$\frac{\partial u}{\partial y} = 2y, \quad \frac{\partial u}{\partial y}\Big|_{(M)} = -24$$

$$\frac{\partial u}{\partial z} = 2z, \quad \frac{\partial u}{\partial z}\Big|_{(M)} = 18$$

$$|\bar{c}| = \sqrt{81 + 64 + 144} = 17$$

$$\bar{c}_0 = \left(-\frac{9}{17};\ \frac{8}{17};\ -\frac{12}{17}\right)$$

$\cos\alpha \quad \cos\beta \quad \cos\gamma$

$$\frac{\partial u}{\partial c}\Big|_M = -16 \cdot \frac{9}{17} + \frac{8}{17}(-24) + 18 \cdot \left(-\frac{12}{17}\right)$$

4) $u = e^{x^2+y^2+z^2}$; $\vec{d} = (4; -13; -16)$; $L(-16; 4; -13)$

$$\frac{\partial u}{\partial x} = e^{x^2+y^2+z^2} \cdot 2x; \quad \left.\frac{\partial u}{\partial x}\right|_L = e^{441} \cdot (-32)$$

$$\frac{\partial u}{\partial y} = e^{x^2+y^2+z^2} \cdot 2y; \quad \left.\frac{\partial u}{\partial y}\right|_L = e^{441} \cdot 8$$

$$\frac{\partial u}{\partial z} = e^{x^2+y^2+z^2} \cdot 2z; \quad \left.\frac{\partial u}{\partial z}\right|_L = e^{441} \cdot (-26)$$

$$|\vec{d}| = \sqrt{16 + 169 + 256} = 21$$

$$\vec{d}_0 = \left(\underset{\cos\alpha}{\underset{\uparrow}{-\frac{16}{21}}}; \underset{\cos\beta}{\underset{\uparrow}{\frac{4}{21}}}; \underset{\cos\gamma}{\underset{\uparrow}{-\frac{13}{21}}}\right)$$

$$\left.\frac{\partial u}{\partial \vec{d}}\right|_L = \left(\frac{32 \cdot 16}{21} + \frac{8 \cdot 4}{21} + \frac{26 \cdot 13}{21}\right) e^{441} = 35\frac{17}{21} e^{441}$$