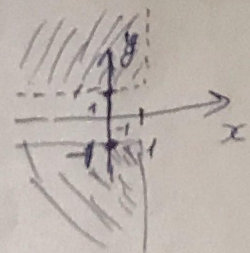
1) $z=\sqrt{1-x^3}+\ln(y^2-1)$

$\begin{cases}1-x^3\geq 0\\ y^2-1\geq 0\end{cases}$ $\begin{cases}x\leq 1\\ y\in(-\infty;-1]\cup[1;+\infty)\end{cases}$



2) $z=\left(1+\frac{\ln x}{\ln y}\right)^3$

$\frac{\partial z}{\partial x}=3\left(1+\frac{\ln x}{\ln y}\right)^2\cdot\frac{1}{\ln y}\cdot\frac{1}{x}=\frac{3}{x\ln y}\left(1+\frac{\ln x}{\ln y}\right)^2$

$\frac{\partial z}{\partial y}=3\left(1+\frac{\ln x}{\ln y}\right)^2\cdot\left(-\frac{\ln x}{(\ln y)^2}\right)\cdot\frac{1}{y}=-\frac{3\ln x}{y(\ln y)^2}\left(1+\frac{\ln x}{\ln y}\right)^2$

3) $z=\sqrt{2xy+\cos\frac{x}{y}}$ $(1;1)$

$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=0{,}364dx-0{,}364dy$

$\frac{\partial z}{\partial x}=\frac{1}{2\sqrt{2xy+\cos\frac{x}{y}}}\cdot\left(2y-\sin\frac{x}{y}\right)\cdot\frac{1}{y}$

$\left.\frac{\partial z}{\partial x}\right|_{(1;1)}=\frac{1}{2\sqrt{2+\underbrace{\cos 1}_{0{,}54}}}\cdot(2-\underbrace{\sin 1}_{0{,}84})\cdot 1=0{,}314\cdot 1{,}16=0{,}364$

$\frac{\partial z}{\partial y}=\frac{1}{2\sqrt{2xy+\cos\frac{x}{y}}}\cdot\left(2x-\sin\frac{x}{y}\right)\cdot\frac{-1}{y^2}\cdot x$

$\left.\frac{\partial z}{\partial y}\right|_{(1;1)}=-0{,}364$

4) $z = x^2 + xy + y^2 - 6x - 9y$

$$\begin{cases} \frac{\partial z}{\partial x} = 0 \\ \frac{\partial z}{\partial y} = 0 \end{cases} \quad \begin{cases} 2x + y - 6 = 0 \\ x + 2y - 9 = 0 \end{cases} \quad \begin{cases} -3y + 12 = 0 \\ 2x + y - 6 = 0 \end{cases} \quad \begin{cases} y = 4 \\ x = 1 \end{cases}$$

$(\cdot)\ (1;4)$

$$\frac{\partial^2 z}{\partial x^2} = 2;\quad \frac{\partial^2 z}{\partial x \partial y} = 1;\quad \frac{\partial^2 z}{\partial y^2} = 2$$

$$\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 2 = 2 > 0 \Rightarrow$$ в $(\cdot)\ (1;4)$ — минимум