An Investigation into the Speed-up of a Ray Tracer Application via the use of OpenMP and MPI

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The abstract goes here.

Index Terms—C++11, Ray Tracer, Parallel, OpenMP, MPI, Speed-up.

I. INTRODUCTION

THE aim of this report is to document and analyse the results of an attempt to speed-up a C++11 Ray Tracer application - via the use of concurrency and parallelisation techniques. The methods being tested in this project are OpenMP and MPI.

A. Ray Tracing

Ray Tracing a rendering technique that allows an image to be generated by tracing the path of a ray as it is reflected through a virtual environment, in order to generate an accurate pixel colour on a 2D image plane. Ray Tracing aims to create photo-realistic images but the computation costs of the technique can result in significant run-times for the generation of highly detailed images. An example of how a Ray Tracer operates can be seen in Figure 1.

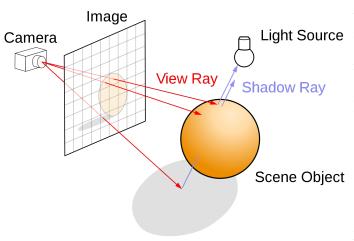


Fig. 1: A diagram of how a Ray Tracer generates an image.

The Ray Tracer being analysed in this project is based on an iterative version of the smallpt Ray Tracer [1]. This Ray Tracer can sample a pixel multiple times to generate a more accurate and detailed image. Two sample images are visible in Figure 2. As can be seen - the accuracy and detail of the final images depends heavily upon the number of ray samples per pixel.

Ray Tracers are ideal candidates for improvement via parallelisation as each individual ray has no dependence upon any other, thereby creating a data-parallel (or embarrassinglyparallel) problem.

B. OpenMP

OpenMP (Open Multi-Processing) is an open source API that allows for the implementation of shared memory multiprocessing with minimal developmental effort. OpenMP makes use of the C++ #pragma directive and the preprocessor to allow developers to flag sections of code (particularly for loops) to be parallelised. A number of different scheduling options can be implemented to alter the way in which OpenMP parallelises an application.

The two schedulers investigated in this project are: Static and inter-weaved Dynamic. The Static scheduler will break a for loop into chunks, each equal to the number of iterations divided by the number of threads. E.g. in the case of a 100 iteration loop split across 4 threads: each thread would run for 25 iterations.

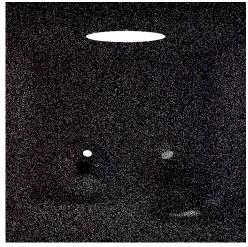
The Dynamic scheduler also breaks a single for loop into chunks, however the chunks are typically much smaller than those produced by the Static scheduler. Threads are then assigned a chunk of work, and upon completion can request a new chunk to work on.

C. MPI

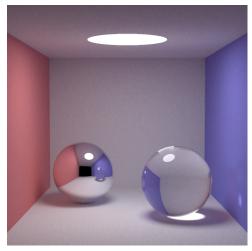
MPI (Message Passing Interface) is a standardised method of distributed parallelism that operates by having multiple processors communicate by sending and receiving signals from one another via communication channels.

MPI allows a developer to build highly scalable systems by simply providing a list of IP addresses when the application is launched. A point that developers must be aware of however is the networking overhead that distributed system innately suffer from. Figure 3 depicts how the amount of time it takes to send data scales with the size of the data being sent. It should be noted that while the time taken to transfer does drop with the size of the data, eventually there is no benefit to adding more

Project available at: github.com/neaop/SET10108Coursework_2



(a) 4 Samples per Pixel



(b) 16384 Samples per Pixel

Fig. 2: Two images produced by the Ray Tracer.



Fig. 3: A line chart depicting the the time required to send data over a network.

PCs to a distributed application. The full data from which the Chart was generated is visible in Appendix II.

II. METHODOLOGY

A. Profiling

Prior to implementing any Speed-up methods, the sequential code must first be analysed. By using the Visual Studio Performance Profiler, it is possible to evaluate the sequential code and locate the functions or methods that use the most CPU time. Once the potentially problematic areas have been identified, a suitable parallelisation method can be implemented to reduce the impact of those areas on the execution time.

B. Data Collection

To ensure fair comparison and accurate results, each implementation was tested using the same parameters. Each solution produced an image 512 pixels high by 512 pixels wide, the

TABLE I: PC Specifications

| CPU | i7-4790k 4 Core HT @ 4.00 ghz |
|-----------|-------------------------------|
| RAM | 16gb Dual Channel DDR3 |
| GPU | Nvidia GeForce GTX 980 |
| OS | Windows 7 64 Bit |
| Bandwidth | 1 Gbit/s |
| Latency | $\sim 129947 \text{ ns}$ |

time taken to do so was recorded. This was repeated onehundred times for each configuration and a mean of the total time taken was produced. All benchmarking was performed on computers with the same technical specifications, which are visible in Table I. It should be noted that all code presented in the report was run without any form of compiler optimisation.

C. Evaluation

As well as the average execution time, speed-up and efficiency are calculated for each technique. Speed-up is defined as:

$$S = \frac{s_t}{p_t}$$

With s_t being sequential time and p_t being parallel time. Once the speed-up of a method has been calculated, the overall efficiency of the parallelisation can be measured as follows:

$$E = \frac{S}{P}$$

S being speed-up from the previous formula and P is the number of physical cores being utilized by the application.

The two equations listed above provide standardised metrics for each method or technology tested - allowing for a fair and simple comparison of the final results.

III. IMPLEMENTATION

As previously stated, the Ray Tracer presented here is a reimplementation of the iterative smallpt system [1].

Several modification had to be made to the original code to allow the implementation of parallelisation. The author of original application had written the Ray Tracer to be under 100 lines of code - which led to said code being highly obfuscated and difficult to read. Much of the development time was spent on expanding, commenting and rewriting the initial code to allow for readability and clearer functionality. Another issue with the original code base was that it was originally designed to be compiled with GCC meaning certain methods calls were unavailable

A. Sequential

Once the program was cleared up, a simple for loop was added to allow multiple iterations to be performed, and a file writer was used to allow the time taken for each iteration to be recorded. This sequential version of the code became the base for both the OpenMP, and MPI implementations.

B. OpenMP

Very little change had to be made to the sequential code in order to implement OpenMP. The most obvious location for parallelisation was the radiance() method, which is called from within five nested for loops. By simply inserting a #pragma parallel for statement above the upper-most loop - the application was made parallel.

C. MPI

IV. RESULTS

A. Sequential

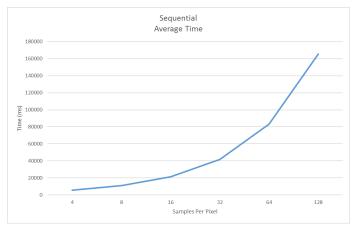


Fig. 4: A line chart indicating the increase in execution time in regard to the number of colour samples per pixel.

B. OpenMP

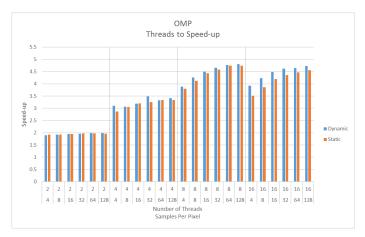


Fig. 5: A bar chart indicating the degree of Speed-up for multiple OpenMP schedule and thread configurations.

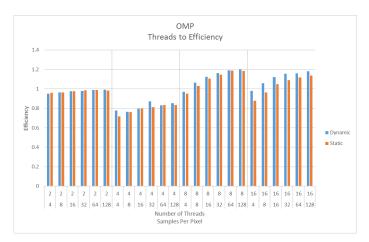


Fig. 6: A bar chart indicating the degree of Efficiency for multiple OpenMP schedule and thread configurations.

C. MPI

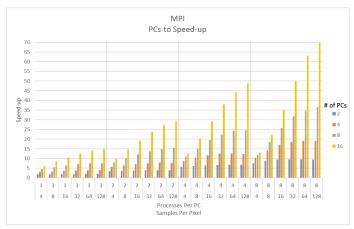


Fig. 7: A bar chart indicating the degree of Speed-up for multiple MPI Node and Host configurations.

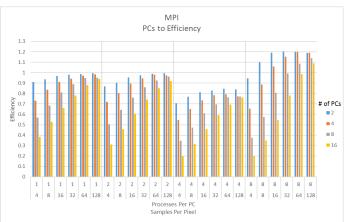


Fig. 8: A bar chart indicating the degree of Efficiency for multiple MPI Node and Host configurations.

D. MPI with OpenMP

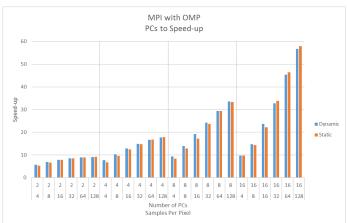


Fig. 9: A bar chart indicating the degree of Speed-up when using MPI and OpenMP with various scheduling types and number of Hosts configurations.

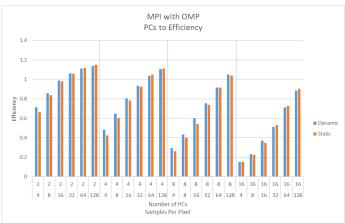


Fig. 10: A bar chart indicating the degree of Efficiency when using MPI and OpenMP with various scheduling types and number of Hosts configurations.

V. Conclusion

REFERENCES

[1] K. Beason. (2014) *smallpt: Global Illumination in 99 lines of C++*. (Accessed on 12/14/2016). [Online]. Available: http://www.kevinbeason.com/smallpt/

APPENDIX A

TABLE II: Data transfer times over a network.

| Nodes | Width | Height | Chunk Size | Pixels Per Node | Vec size (bits) | Total Data (bits) | Bandwidth bits/s | Latency(s) | Time (s) |
|-------|-------|--------|------------|-----------------|-----------------|-------------------|------------------|------------|----------|
| 2 | 512 | 512 | 256 | 131072 | 192 | 25165824 | 1000000000 | 0.00013 | 0.02530 |
| 4 | 512 | 512 | 128 | 65536 | 192 | 12582912 | 1000000000 | 0.00013 | 0.01271 |
| 8 | 512 | 512 | 64 | 32768 | 192 | 6291456 | 1000000000 | 0.00013 | 0.00642 |
| 16 | 512 | 512 | 32 | 16384 | 192 | 3145728 | 1000000000 | 0.00013 | 0.00328 |
| 32 | 512 | 512 | 16 | 8192 | 192 | 1572864 | 1000000000 | 0.00013 | 0.00170 |
| 64 | 512 | 512 | 8 | 4096 | 192 | 786432 | 1000000000 | 0.00013 | 0.00092 |
| 128 | 512 | 512 | 4 | 2048 | 192 | 393216 | 1000000000 | 0.00013 | 0.00052 |

APPENDIX B

TABLE III: Sequential Benchmarks

| Samples | Mean Time |
|---------|-----------|
| 4 | 5735 |
| 8 | 10794 |
| 16 | 21282 |
| 32 | 41829 |
| 64 | 83256 |
| 128 | 165345 |

APPENDIX C

TABLE IV: OMP Speed-up and Efficiency

| Schedule | Samples | Threads | Mean Time | Speed-up | Efficiency |
|----------|---------|---------|-----------|----------|------------|
| Dynamic | 4 | 2 | 3014 | 1.90279 | 0.95139 |
| Dynamic | 8 | 2 | 5600 | 1.92750 | 0.96375 |
| Dynamic | 16 | 2 | 10895 | 1.95337 | 0.97669 |
| Dynamic | 32 | 2 | 21389 | 1.95563 | 0.97782 |
| Dynamic | 64 | 2 | 42033 | 1.98073 | 0.99036 |
| Dynamic | 128 | 2 | 83384 | 1.98293 | 0.99147 |
| Dynamic | 4 | 4 | 1848 | 3.10335 | 0.77584 |
| Dynamic | 8 | 4 | 3525 | 3.06213 | 0.76553 |
| Dynamic | 16 | 4 | 6684 | 3.18402 | 0.79601 |
| Dynamic | 32 | 4 | 11987 | 3.48953 | 0.87238 |
| Dynamic | 64 | 4 | 25062 | 3.32200 | 0.83050 |
| Dynamic | 128 | 4 | 48495 | 3.40953 | 0.85238 |
| Dynamic | 4 | 8 | 1477 | 3.88287 | 0.97072 |
| Dynamic | 8 | 8 | 2538 | 4.25296 | 1.06324 |
| Dynamic | 16 | 8 | 4731 | 4.49841 | 1.12460 |
| Dynamic | 32 | 8 | 8998 | 4.64870 | 1.16217 |
| Dynamic | 64 | 8 | 17454 | 4.77002 | 1.19251 |
| Dynamic | 128 | 8 | 34447 | 4.79998 | 1.20000 |
| Dynamic | 4 | 16 | 1463 | 3.92003 | 0.98001 |
| Dynamic | 8 | 16 | 2551 | 4.23128 | 1.05782 |
| Dynamic | 16 | 16 | 4749 | 4.48136 | 1.12034 |
| Dynamic | 32 | 16 | 9054 | 4.61995 | 1.15499 |
| Dynamic | 64 | 16 | 17938 | 4.64132 | 1.16033 |
| Dynamic | 128 | 16 | 34953 | 4.73050 | 1.18262 |
| Static | 4 | 2 | 2987 | 1.91999 | 0.95999 |
| Static | 8 | 2 | 5604 | 1.92612 | 0.96306 |
| Static | 16 | 2 | 10903 | 1.95194 | 0.97597 |
| Static | 32 | 2 | 21237 | 1.96963 | 0.98481 |
| Static | 64 | 2 | 42101 | 1.97753 | 0.98877 |
| Static | 128 | 2 | 84127 | 1.96542 | 0.98271 |
| Static | 4 | 4 | 1999 | 2.86893 | 0.71723 |
| Static | 8 | 4 | 3538 | 3.05088 | 0.76272 |
| Static | 16 | 4 | 6654 | 3.19838 | 0.79959 |
| Static | 32 | 4 | 12881 | 3.24734 | 0.81184 |
| Static | 64 | 4 | 24933 | 3.33919 | 0.83480 |
| Static | 128 | 4 | 49619 | 3.33229 | 0.83307 |
| Static | 4 | 8 | 1508 | 3.80305 | 0.95076 |
| Static | 8 | 8 | 2621 | 4.11828 | 1.02957 |
| Static | 16 | 8 | 4806 | 4.42821 | 1.10705 |
| Static | 32 | 8 | 9130 | 4.58149 | 1.14537 |
| Static | 64 | 8 | 17541 | 4.74637 | 1.18659 |
| Static | 128 | 8 | 34886 | 4.73958 | 1.18490 |
| Static | 4 | 16 | 1631 | 3.51625 | 0.87906 |
| Static | 8 | 16 | 2799 | 3.85638 | 0.96409 |
| Static | 16 | 16 | 5073 | 4.19515 | 1.04879 |
| Static | 32 | 16 | 9606 | 4.35447 | 1.08862 |
| Static | 64 | 16 | 18609 | 4.47396 | 1.11849 |
| Static | 128 | 16 | 36336 | 4.55045 | 1.13761 |

APPENDIX D

TABLE V: MPI Speed-up and Efficiency (Part 1)

| Samples | Hosts | Nodes | Mean Time | Speed-up | Efficiency |
|---------|-------|-------|-----------|--------------------|------------|
| 4 | 2 | 2 | 3150 | 1.82063 | 0.91032 |
| 8 | 2 | 2 | 5762 | 1.87331 | 0.93665 |
| 16 | 2 | 2 | 11001 | 1.93455 | 0.96728 |
| 32 | 2 | 2 | 21369 | 1.95746 | 0.97873 |
| 64 | 2 | 2 | 42198 | 1.97298 | 0.98649 |
| 128 | 2 | 2 | 83011 | 1.99184 | 0.99592 |
| 4 | 2 | 4 | 1651 | 3.47365 | 0.86841 |
| 8 | 2 | 4 | 2982 | 3.61972 | 0.90493 |
| 16 | 2 | 4 | 5578 | 3.81535 | 0.95384 |
| 32 | 2 | 4 | 10724 | 3.90050 | 0.97513 |
| 64 | 2 | 4 | 21024 | 3.96005 | 0.99001 |
| 128 | 2 | 4 | 41535 | 3.98086 | 0.99521 |
| 4 | 2 | 8 | 1012 | 5.66700 | 0.70837 |
| 8 | 2 | 8 | 1758 | 6.13993 | 0.76749 |
| 16 | 2 | 8 | 3277 | 6.49435 | 0.76749 |
| 32 | 2 | | 6312 | 6.62690 | 0.81179 |
| 64 | 2 | 8 | 12331 | 6.75176 | 0.82836 |
| 128 | 2 | - | 24648 | | |
| 128 | 2 | 8 | 759 | 6.70825 7.55599 | 0.83853 |
| | | 16 | 1225 | | 0.94450 |
| 8 | 2 | 16 | | 8.81143 | 1.10143 |
| 16 | 2 | 16 | 2232 | 9.53495 | 1.19187 |
| 32 | 2 | 16 | 4343 | 9.63136 | 1.20392 |
| 64 | 2 | 16 | 8684 | 9.58729 | 1.19841 |
| 128 | 2 | 16 | 17395 | 9.50532 | 1.18816 |
| 4 | 4 | 4 | 1959 | 2.92751 | 0.73188 |
| 8 | 4 | 4 | 3224 | 3.34801 | 0.83700 |
| 16 | 4 | 4 | 5856 | 3.63422 | 0.90856 |
| 32 | 4 | 4 | 11096 | 3.76974 | 0.94243 |
| 64 | 4 | 4 | 21441 | 3.88303 | 0.97076 |
| 128 | 4 | 4 | 42075 | 3.92977 | 0.98244 |
| 4 | 4 | 8 | 999 | 5.74074 | 0.71759 |
| 8 | 4 | 8 | 1681 | 6.42118 | 0.80265 |
| 16 | 4 | 8 | 2971 | 7.16324 | 0.89541 |
| 32 | 4 | 8 | 5533 | 7.55991 | 0.94499 |
| 64 | 4 | 8 | 10622 | 7.83807 | 0.97976 |
| 128 | 4 | 8 | 21225 | 7.79011 | 0.97376 |
| 4 | 4 | 16 | 656 | 8.74238 | 0.54640 |
| 8 | 4 | 16 | 1038 | 10.39884 | 0.64993 |
| 16 | 4 | 16 | 1812 | 11.74503 | 0.73406 |
| 32 | 4 | 16 | 3344 | 12.50867 | 0.78179 |
| 64 | 4 | 16 | 6569 | 12.67408 | 0.79213 |
| 128 | 4 | 16 | 13404 | 12.33550 | 0.77097 |
| 4 | 4 | 32 | 549 | 10.44627 | 0.65289 |
| 8 | 4 | 32 | 762 | 14.16535 | 0.88533 |
| 16 | 4 | 32 | 1257 | 16.93079 | 1.05817 |
| 32 | 4 | 32 | 2265 | 18.46755 | 1.15422 |
| 64 | 4 | 32 | 4340 | 19.18341 | 1.19896 |
| 128 | 4 | 32 | 8681 | 19.04677 | 1.19042 |

APPENDIX E

TABLE VI: MPI Speed-up and Efficiency (Part 2)

| 4 8 8 1259 4.55520 0.56940 8 8 8 1971 5.47641 0.68455 16 8 8 3286 6.47657 0.80957 32 8 8 5881 7.11257 0.88907 64 8 8 10988 7.57699 0.94712 128 8 8 10988 7.57699 0.94712 128 8 8 16 710 8.07746 0.50484 8 8 16 710 8.07746 0.50484 8 8 16 1049 10.28980 0.64311 16 8 16 1753 12.14033 0.75877 32 8 16 3037 13.77313 0.86082 64 8 16 5633 14.78005 0.92375 128 8 16 10734 15.40386 0.96274 4 8 32 < | Samples | Hosts | Nodes | Mean Time | Speed-up | Efficiency |
|--|---------|-------|-------|-----------|----------|------------|
| 16 8 8 3286 6.47657 0.80957 32 8 8 5881 7.11257 0.88907 64 8 8 10988 7.57699 0.94712 128 8 8 109746 0.95126 4 8 16 710 8.07746 0.50484 8 8 16 1049 10.28980 0.64311 16 8 16 1753 12.14033 0.75877 32 8 16 3037 13.77313 0.86082 64 8 16 5633 14.78005 0.92375 128 8 16 10734 15.40386 0.96274 4 8 32 518 11.07143 0.34598 8 8 32 714 15.11765 0.47243 16 8 32 1882 22.22582 0.69959 32 8 32 1882 22.22582 | 4 | 8 | 8 | 1259 | 4.55520 | 0.56940 |
| 32 8 8 5881 7.11257 0.88907 64 8 8 10988 7.57699 0.94712 128 8 8 21727 7.61012 0.95126 4 8 16 710 8.07746 0.50484 8 8 16 1049 10.28980 0.64311 16 8 16 1073 12.14033 0.75877 32 8 16 3037 13.77313 0.86082 64 8 16 5633 14.78005 0.92375 128 8 16 10734 15.40386 0.96274 4 8 32 518 11.07143 0.34598 8 8 32 714 15.11765 0.47243 16 8 32 1091 19.50687 0.60959 32 8 32 1882 22.22582 0.69456 64 8 32 3417 | 8 | 8 | 8 | 1971 | 5.47641 | 0.68455 |
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| 128 8 8 21727 7.61012 0.95126 4 8 16 710 8.07746 0.50484 8 8 16 1049 10.28980 0.64311 16 8 16 1753 12.14033 0.75877 32 8 16 3037 13.77313 0.86082 64 8 16 5633 14.78005 0.92375 128 8 16 10734 15.40386 0.96274 4 8 32 518 11.07143 0.34598 8 8 32 1091 19.50687 0.60959 32 8 32 1091 19.50687 0.60959 32 8 32 1091 19.50687 0.60959 32 8 32 1882 22.22582 0.69456 64 8 32 3417 24.36523 0.76141 128 8 64 477 </td <td>32</td> <td>8</td> <td>8</td> <td>5881</td> <td>7.11257</td> <td>0.88907</td> | 32 | 8 | 8 | 5881 | 7.11257 | 0.88907 |
| 4 8 16 710 8.07746 0.50484 8 8 16 1049 10.28980 0.64311 16 8 16 1753 12.14033 0.75877 32 8 16 3037 13.77313 0.86082 64 8 16 5633 14.78005 0.92375 128 8 16 10734 15.40386 0.96274 4 8 32 518 11.07143 0.34598 8 8 32 714 15.11765 0.47243 16 8 32 1091 19.50687 0.60959 32 8 32 1882 22.22582 0.69456 64 8 32 3417 24.36523 0.76141 128 8 32 6691 24.71155 0.77224 4 8 64 477 12.02306 0.37572 8 8 64 4825 | 64 | 8 | 8 | 10988 | 7.57699 | 0.94712 |
| 8 8 16 1049 10.28980 0.64311 16 8 16 1753 12.14033 0.75877 32 8 16 3037 13.77313 0.86082 64 8 16 5633 14.78005 0.92375 128 8 16 10734 15.40386 0.96274 4 8 32 518 11.07143 0.34598 8 8 32 518 11.07143 0.34598 8 8 32 1091 19.50687 0.60959 32 8 32 1882 22.22582 0.69456 64 8 32 3417 24.36523 0.76141 128 8 32 6691 24.71155 0.77224 4 8 64 477 12.02306 0.37572 8 8 64 586 18.41980 0.57562 16 8 64 825 | 128 | 8 | 8 | 21727 | 7.61012 | 0.95126 |
| 16 8 16 1753 12.14033 0.75877 32 8 16 3037 13.77313 0.86082 64 8 16 5633 14.78005 0.92375 128 8 16 10734 15.40386 0.96274 4 8 32 518 11.07143 0.34598 8 8 32 714 15.11765 0.47243 16 8 32 1091 19.50687 0.60959 32 8 32 1882 22.22582 0.69456 64 8 32 3417 24.36523 0.76141 128 8 32 6691 24.71155 0.77224 4 8 64 477 12.02306 0.37572 8 8 64 477 12.02306 0.37572 8 8 64 4825 25.79636 0.80614 32 8 64 1320 <td>4</td> <td>8</td> <td>16</td> <td>710</td> <td>8.07746</td> <td>0.50484</td> | 4 | 8 | 16 | 710 | 8.07746 | 0.50484 |
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| 64 8 16 5633 14.78005 0.92375 128 8 16 10734 15.40386 0.96274 4 8 32 518 11.07143 0.34598 8 8 32 518 11.07143 0.34598 8 8 32 1091 19.50687 0.60959 32 8 32 1882 22.22582 0.69456 64 8 32 3417 24.36523 0.76141 128 8 32 6691 24.71155 0.77224 4 8 64 477 12.02306 0.37572 8 8 64 477 12.02306 0.37572 8 8 64 477 12.02306 0.37572 8 8 64 825 25.79636 0.80614 32 8 64 1320 31.68864 0.99027 64 8 64 2397 | 16 | 8 | 16 | 1753 | 12.14033 | 0.75877 |
| 128 8 16 10734 15.40386 0.96274 4 8 32 518 11.07143 0.34598 8 8 32 714 15.11765 0.47243 16 8 32 1091 19.50687 0.60959 32 8 32 1882 22.22582 0.69456 64 8 32 3417 24.36523 0.76141 128 8 32 6691 24.71155 0.77224 4 8 64 477 12.02306 0.37572 8 8 64 586 18.41980 0.57562 16 8 64 825 25.79636 0.80614 32 8 64 3237 34.73342 1.08542 16 8 64 2397 34.73342 1.08542 128 8 64 4353 36.45976 1.13937 4 16 16 934 <td>32</td> <td>8</td> <td>16</td> <td>3037</td> <td>13.77313</td> <td>0.86082</td> | 32 | 8 | 16 | 3037 | 13.77313 | 0.86082 |
| 4 8 32 518 11.07143 0.34598 8 8 32 714 15.11765 0.47243 16 8 32 1091 19.50687 0.60959 32 8 32 1882 22.22582 0.69456 64 8 32 3417 24.36523 0.76141 128 8 32 6691 24.71155 0.77224 4 8 64 477 12.02306 0.37572 4 8 64 477 12.02306 0.37572 8 8 64 586 18.41980 0.57562 16 8 64 825 25.79636 0.80614 32 8 64 1320 31.68864 0.99027 64 8 64 2397 34.73342 1.08542 128 8 64 4535 36.45976 1.13937 4 16 16 934 | 64 | 8 | 16 | 5633 | 14.78005 | 0.92375 |
| 8 8 32 714 15.11765 0.47243 16 8 32 1091 19.50687 0.60959 32 8 32 1882 22.22582 0.69456 64 8 32 3417 24.36523 0.76141 128 8 32 6691 24.71155 0.77224 4 8 64 477 12.02306 0.37572 4 8 64 477 12.02306 0.37572 8 8 64 586 18.41980 0.57562 16 8 64 825 25.79636 0.80614 32 8 64 1320 31.68864 0.99027 64 8 64 2397 34.73342 1.08542 128 8 64 4535 36.45976 1.13937 4 16 16 934 6.14026 0.38377 8 16 16 12.57753 </td <td>128</td> <td>8</td> <td>16</td> <td>10734</td> <td>15.40386</td> <td>0.96274</td> | 128 | 8 | 16 | 10734 | 15.40386 | 0.96274 |
| 16 8 32 1091 19,50687 0.60959 32 8 32 1882 22,22582 0.69456 64 8 32 3417 24,36523 0.76141 128 8 32 6691 24,71155 0.77224 4 8 64 477 12,02306 0.37572 8 8 64 477 12,02306 0.37572 8 8 64 586 18,41980 0.57562 16 8 64 825 25,79636 0.80614 32 8 64 1320 31,68864 0.99027 64 8 64 2397 34,73342 1.08542 128 8 64 4535 36,45976 1.13937 4 16 16 934 6,14026 0.38377 8 16 16 1278 8,44601 0.52788 16 16 3352 12,478 | 4 | 8 | 32 | 518 | 11.07143 | 0.34598 |
| 32 8 32 1882 22.22582 0.69456 64 8 32 3417 24.36523 0.76141 128 8 32 6691 24.71155 0.77224 4 8 64 477 12.02306 0.37572 8 8 64 586 18.41980 0.57562 16 8 64 825 25.79636 0.80614 32 8 64 1320 31.68864 0.99027 64 8 64 2397 34.73342 1.08542 128 8 64 4535 36.45976 1.13937 4 16 16 934 6.14026 0.38377 8 16 16 1278 8.44601 0.52788 16 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 592 | 8 | 8 | 32 | 714 | 15.11765 | 0.47243 |
| 64 8 32 3417 24.36523 0.76141 128 8 32 6691 24.71155 0.77224 4 8 64 477 12.02306 0.37572 8 8 64 586 18.41980 0.57562 16 8 64 586 18.41980 0.57562 16 8 64 325 25.79636 0.80614 32 8 64 1320 31.68864 0.99027 64 8 64 2397 34.73342 1.08542 128 8 64 4535 36.45976 1.13937 4 16 16 934 6.14026 0.38377 8 16 16 1278 8.44601 0.52788 16 16 16 2012 10.57753 0.66110 32 16 16 3352 12.47882 0.77993 64 16 16 5921< | 16 | 8 | 32 | 1091 | 19.50687 | 0.60959 |
| 128 8 32 6691 24.71155 0.77224 4 8 64 477 12.02306 0.37572 8 8 64 586 18.41980 0.57562 16 8 64 825 25.79636 0.80614 32 8 64 1320 31.68864 0.99027 64 8 64 2397 34.73342 1.08542 128 8 64 4535 36.45976 1.13937 4 16 16 934 6.14026 0.38377 8 16 16 1278 8.44601 0.52788 16 16 16 2012 10.57753 0.66110 32 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 10997 15.03546 0.93972 4 16 32 5 | 32 | 8 | 32 | 1882 | 22.22582 | 0.69456 |
| 4 8 64 477 12.02306 0.37572 8 8 64 586 18.41980 0.57562 16 8 64 825 25.79636 0.80614 32 8 64 1320 31.68864 0.99027 64 8 64 2397 34.73342 1.08542 128 8 64 4535 36.45976 1.13937 4 16 16 934 6.14026 0.38377 8 16 16 1278 8.44601 0.52788 16 16 16 2012 10.57753 0.66110 32 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 10997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735< | 64 | 8 | 32 | 3417 | 24.36523 | 0.76141 |
| 8 8 64 586 18.41980 0.57562 16 8 64 825 25.79636 0.80614 32 8 64 1320 31.68864 0.99027 64 8 64 2397 34.73342 1.08542 128 8 64 4535 36.45976 1.13937 4 16 16 934 6.14026 0.38377 8 16 16 1278 8.44601 0.52788 16 16 16 2012 10.57753 0.66110 32 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 16997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 17 | 128 | 8 | 32 | 6691 | 24.71155 | 0.77224 |
| 16 8 64 825 25.79636 0.80614 32 8 64 1320 31.68864 0.99027 64 8 64 2397 34.73342 1.08542 128 8 64 4535 36.45976 1.13937 4 16 16 934 6.14026 0.38377 8 16 16 1278 8.44601 0.52788 16 16 16 2012 10.57753 0.66110 32 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 5921 14.06114 0.87882 128 16 16 10997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 <t< td=""><td>4</td><td>8</td><td>64</td><td>477</td><td>12.02306</td><td>0.37572</td></t<> | 4 | 8 | 64 | 477 | 12.02306 | 0.37572 |
| 32 8 64 1320 31.68864 0.99027 64 8 64 2397 34.73342 1.08542 128 8 64 4535 36.45976 1.13937 4 16 16 934 6.14026 0.38377 8 16 16 1278 8.44601 0.52788 16 16 16 2012 10.57753 0.66110 32 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 10997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 < | 8 | 8 | 64 | 586 | 18.41980 | 0.57562 |
| 64 8 64 2397 34,73342 1.08542 128 8 64 4535 36,45976 1.13937 4 16 16 934 6,14026 0.38377 8 16 16 1278 8,44601 0.52788 16 16 16 2012 10,57753 0.66110 32 16 16 3352 12,47882 0,77993 64 16 16 5921 14,06114 0.87882 128 16 16 10997 15,03546 0,93972 4 16 32 579 9,90501 0.30953 8 16 32 579 9,90501 0.30953 8 16 32 1735 14,68571 0.45893 16 16 32 1103 19,29465 0,60296 32 16 32 1762 23,73950 0,74186 64 16 32 <t< td=""><td>16</td><td>8</td><td>64</td><td>825</td><td>25.79636</td><td>0.80614</td></t<> | 16 | 8 | 64 | 825 | 25.79636 | 0.80614 |
| 128 8 64 4535 36,45976 1.13937 4 16 16 934 6.14026 0.38377 8 16 16 1278 8.44601 0.52788 16 16 16 2012 10.57753 0.66110 32 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 10997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 363 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 | 32 | 8 | 64 | 1320 | 31.68864 | 0.99027 |
| 4 16 16 934 6.14026 0.38377 8 16 16 1278 8.44601 0.52788 16 16 16 2012 10.57753 0.66110 32 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 10997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 3063 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 < | 64 | 8 | 64 | 2397 | 34.73342 | 1.08542 |
| 8 16 16 1278 8.44601 0.52788 16 16 16 2012 10.57753 0.66110 32 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 10997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 3063 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 | 128 | 8 | 64 | 4535 | 36.45976 | 1.13937 |
| 16 16 16 2012 10.57753 0.66110 32 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 10997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 3063 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 | 4 | 16 | 16 | 934 | 6.14026 | 0.38377 |
| 32 16 16 3352 12.47882 0.77993 64 16 16 5921 14.06114 0.87882 128 16 16 10997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 3063 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 | 8 | 16 | 16 | 1278 | 8.44601 | 0.52788 |
| 64 16 16 5921 14.06114 0.87882 128 16 16 10997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 3063 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 | 16 | 16 | 16 | 2012 | 10.57753 | 0.66110 |
| 128 16 16 10997 15.03546 0.93972 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 3063 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 | 32 | 16 | 16 | 3352 | 12.47882 | 0.77993 |
| 4 16 32 579 9.90501 0.30953 8 16 32 735 14.68571 0.45893 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 3063 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 | 64 | 16 | 16 | 5921 | 14.06114 | 0.87882 |
| 8 16 32 735 14.68571 0.45893 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 3063 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 | 128 | 16 | 16 | 10997 | 15.03546 | 0.93972 |
| 16 16 32 1103 19.29465 0.60296 32 16 32 1762 23.73950 0.74186 64 16 32 3063 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 | 4 | 16 | 32 | 579 | 9.90501 | 0.30953 |
| 32 16 32 1762 23,73950 0.74186 64 16 32 3063 27,18119 0.84941 128 16 32 5619 29,42605 0.91956 4 16 64 453 12,66004 0.19781 8 16 64 535 20,17570 0.31525 16 16 64 728 29,23352 0.45677 32 16 64 1104 37,88859 0.59201 64 16 64 1883 44,21455 0.69085 128 16 64 3393 48,73121 0.76143 4 16 128 441 13,00454 0.20320 8 16 128 484 22,30165 0.34846 16 16 128 608 35,00329 0.54693 32 16 128 837 49,97491 0.78086 64 16 128 | 8 | 16 | 32 | 735 | 14.68571 | 0.45893 |
| 64 16 32 3063 27.18119 0.84941 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | 16 | 16 | 32 | 1103 | 19.29465 | 0.60296 |
| 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | 32 | 16 | 32 | 1762 | 23.73950 | 0.74186 |
| 128 16 32 5619 29.42605 0.91956 4 16 64 453 12.66004 0.19781 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | 64 | 16 | 32 | 3063 | 27.18119 | 0.84941 |
| 8 16 64 535 20.17570 0.31525 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | 128 | 16 | 32 | 5619 | | 0.91956 |
| 16 16 64 728 29.23352 0.45677 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | 4 | 16 | 64 | 453 | 12.66004 | 0.19781 |
| 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | 8 | 16 | 64 | 535 | 20.17570 | 0.31525 |
| 32 16 64 1104 37.88859 0.59201 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | 16 | 16 | 64 | | 29.23352 | 0.45677 |
| 64 16 64 1883 44.21455 0.69085 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | 32 | 16 | 64 | 1104 | 37.88859 | 0.59201 |
| 128 16 64 3393 48.73121 0.76143 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | | | 64 | 1883 | | 0.69085 |
| 4 16 128 441 13.00454 0.20320 8 16 128 484 22.30165 0.34846 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | 128 | | 64 | | | |
| 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | | | 128 | | | |
| 16 16 128 608 35.00329 0.54693 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | 8 | 16 | 128 | 484 | 22.30165 | 0.34846 |
| 32 16 128 837 49.97491 0.78086 64 16 128 1320 63.07273 0.98551 | | 16 | 128 | 608 | 35.00329 | 0.54693 |
| 64 16 128 1320 63.07273 0.98551 | - | | | | | |
| | - | | | | | |
| 128 16 128 2376 69.58965 1.08734 | 128 | 16 | 128 | 2376 | 69.58965 | 1.08734 |

APPENDIX F

TABLE VII: MPI with OMP Speed-up and Efficiency

| Samples | Schedule | Hosts | Nodes | Mean Time | Speed-up | Efficiency |
|---------|----------|-------|-------|-----------|----------|------------|
| 4 | Dynamic | 2 | 2 | 1002 | 5.72355 | 0.71544 |
| 8 | Dynamic | 2 | 2 | 1572 | 6.86641 | 0.85830 |
| 16 | Dynamic | 2 | 2 | 2694 | 7.89978 | 0.98747 |
| 32 | Dynamic | 2 | 2 | 4927 | 8.48975 | 1.06122 |
| 64 | Dynamic | 2 | 2 | 9367 | 8.88822 | 1.11103 |
| 128 | Dynamic | 2 | 2 | 18150 | 9.10992 | 1.13874 |
| 4 | Dynamic | 4 | 4 | 744 | 7.70833 | 0.48177 |
| 8 | Dynamic | 4 | 4 | 1044 | 10.33908 | 0.64619 |
| 16 | Dynamic | 4 | 4 | 1652 | 12.88257 | 0.80516 |
| 32 | Dynamic | 4 | 4 | 2797 | 14.95495 | 0.93468 |
| 64 | Dynamic | 4 | 4 | 5014 | 16.60471 | 1.03779 |
| 128 | Dynamic | 4 | 4 | 9351 | 17.68207 | 1.10513 |
| 4 | Dynamic | 8 | 8 | 610 | 9.40164 | 0.29380 |
| 8 | Dynamic | 8 | 8 | 778 | 13.87404 | 0.43356 |
| 16 | Dynamic | 8 | 8 | 1111 | 19.15572 | 0.59862 |
| 32 | Dynamic | 8 | 8 | 1729 | 24.19260 | 0.75602 |
| 64 | Dynamic | 8 | 8 | 2840 | 29.31549 | 0.91611 |
| 128 | Dynamic | 8 | 8 | 4925 | 33.57259 | 1.04914 |
| 4 | Dynamic | 16 | 16 | 591 | 9.70389 | 0.15162 |
| 8 | Dynamic | 16 | 16 | 734 | 14.70572 | 0.22978 |
| 16 | Dynamic | 16 | 16 | 898 | 23.69933 | 0.37030 |
| 32 | Dynamic | 16 | 16 | 1278 | 32.73005 | 0.51141 |
| 64 | Dynamic | 16 | 16 | 1834 | 45.39586 | 0.70931 |
| 128 | Dynamic | 16 | 16 | 2917 | 56.68324 | 0.88568 |
| 4 | Static | 2 | 2 | 1077 | 5.32498 | 0.66562 |
| 8 | Static | 2 | 2 | 1613 | 6.69188 | 0.83648 |
| 16 | Static | 2 | 2 | 2716 | 7.83579 | 0.97947 |
| 32 | Static | 2 | 2 | 4937 | 8.47255 | 1.05907 |
| 64 | Static | 2 | 2 | 9297 | 8.95515 | 1.11939 |
| 128 | Static | 2 | 2 | 17986 | 9.19298 | 1.14912 |
| 4 | Static | 4 | 4 | 845 | 6.78698 | 0.42419 |
| 8 | Static | 4 | 4 | 1116 | 9.67204 | 0.60450 |
| 16 | Static | 4 | 4 | 1701 | 12.51146 | 0.78197 |
| 32 | Static | 4 | 4 | 2825 | 14.80673 | 0.92542 |
| 64 | Static | 4 | 4 | 4963 | 16.77534 | 1.04846 |
| 128 | Static | 4 | 4 | 9288 | 17.80200 | 1.11263 |
| 4 | Static | 8 | 8 | 682 | 8.40909 | 0.26278 |
| 8 | Static | 8 | 8 | 844 | 12.78910 | 0.39966 |
| 16 | Static | 8 | 8 | 1234 | 17.24635 | 0.53895 |
| 32 | Static | 8 | 8 | 1770 | 23.63220 | 0.73851 |
| 64 | Static | 8 | 8 | 2837 | 29.34649 | 0.91708 |
| 128 | Static | 8 | 8 | 4973 | 33.24854 | 1.03902 |
| 4 | Static | 16 | 16 | 587 | 9.77002 | 0.15266 |
| 8 | Static | 16 | 16 | 751 | 14.37284 | 0.22458 |
| 16 | Static | 16 | 16 | 956 | 22.26151 | 0.34784 |
| 32 | Static | 16 | 16 | 1234 | 33.89708 | 0.52964 |
| 64 | Static | 16 | 16 | 1793 | 46.43391 | 0.72553 |
| 128 | Static | 16 | 16 | 2855 | 57.91419 | 0.90491 |