FYS3001 – Home Assignment II: Radar, V-2025

This document describes the tasks for the second home assignment in FYS-3001 in spring 2024 and gives an overview of the required data. The data files `image.dat` and `image.txt` are supplied on WISEflow.

Please submit your answers in PDF format on WISEflow. The PDF should include requested figures, maths explanations and code that is explicitly asked for.
Please, optionally, provide your full code, either as a verbatim attachment to the PDF, a single .py or .m file, or a jupyter notebok. The code should be formatted such that we can easily adjust the path to the input image data and then run it to reproduce the results from the PDF document. This is optional and will only be used if we are trying to find out what went wrong for any part.

Opening	Tuesday	22.04.2025 09:00
Closing	Wednesday	07.05.2025 14:00

Objectives of the project work:

- Get experience with complex SAR (Single-look complex (SLC)) data acquired over ocean surface
- Get experienced with speckle theory and statistics of SAR data
- Get experienced with look extraction from SAR SLC data
- Get experienced with information extraction (i.e. spectral estimation) from SAR data, by computation of the SAR image spectra
- The work is divided in three parts, A, B and C. Part A counts 30%, part B counts 60% and part C counts 10%. Each sub-point within A, B and C counts equally.

Data description:

There is one SAR single-look complex (SLC) image (IDL data type 6) (size on ground of around 10kmx5km) stored in the file `image.dat` acquired with the Envisat satellite over ocean. Together with the image file, there is a meta data file (`image.txt`) that describes various imaging and instrument parameters, some of them you will need. The `image.txt` file contains two structures with parameters: the *info* and the *geo* structures. It is a text file and can be read in any standard text editor. In this text file you can find the image size in pixels (`info.xsize`, `info.ysize` corresponds to number of range pixels, and number of azimuth pixels), and you can also compute the range and azimuth pixel sizes (in meters) from the range and azimuth sampling frequency as shown in eq.(1) below.

You can use the following python code to read in the SLC data from the the `image.dat` file.

```
import numpy as np

# define paths to data and header file
data_file = "image.dat"
header_file = "image.txt"

# number of pixels from meta data
Ny = 1759
Nx = 501

# read the image data as vector and reshape to image
dat = np.fromfile(data_file, dtype=np.csingle)
img = dat.reshape(Ny,Nx)
```

Work description:

The work will be to compute some statistical information from the image as well as the image Fourier spectra, and to discuss the results.

1. Read in the SLC data. If you read the data correctly, take the absolute value of the SLC image and plot it, it should look similar the plot in Figure 1 below. Note that matlab and python place [0,0] differently so the image might appear "upside-down" compared to the example in Figure 1.

A. Image Statistics: (counts 30%)

1. Compute and plot the histogram of the real and the imaginary part of the SLC image. What type of statistical distribution do they look like?
2. Compute the image intensity from the complex SLC image, and compute and plot the histogram of the intensity image. What type of statistical distribution does it look like? Compute the normalised variance of the intensity image.
3. Perform a 5x5 smoothing operation on the intensity image (you can use a built in `smooth` of Matlab or find a Python equivalent filter). Redo point 2 on the smoothed intensity image. What type of distribution does it look like? Compute again the normalised variance of the intensity image.
4. Discuss the results of points 1 – 3 in terms of what you have learned about speckle theory.

B. Look Extraction and Fourier Spectral Estimation: (counts 60%)

Extract 3 looks in azimuth from the SLC image and compute the co- and cross-image spectra among the looks. This can be done by first generating 3 intensity images from the SLC image by bandpass filtering the azimuth Fourier domain in 3 equal parts (3 looks), and then performing a co- and cross-image spectra computation among these 3 intensity images.

Following procedure can be used:

1. Perform an azimuth Fourier transform on the SLC image (first normalise the image by dividing it with the square-root of the mean intensity). Shift the Fourier transform such that the zero wavenumber k=(0,0) becomes in the centre of the array. This should give you a complex valued 2D (two-dimensional) array. Plot the absolute value image to view it.
2. Generate the spectral profile in azimuth direction by averaging the absolute value of the 2D array in range direction. Plot the profile and discuss the form. Is it shifted or is it symmetric around the zero frequency?
3. Compute the azimuth shift in pixels of the spectral profile maximum (computed in 2.) from zero frequency, and shift the complex valued 2D spectra (computed in 1.) in azimuth direction by this amount. Plot it with the azimuth frequency [rad/m] on the x-axis. What are the min and max azimuth frequencies of the profile?
4. Split the azimuth Fourier domain (bandpass filtering) of the shifted 2D spectra (from 3.) in three equal parts (looks), and extract each complex look, and go back to spatial domain by an inverse Fourier transformation. You should now have 3 complex images, which can be converted to 3 intensity images by an abs() or conj-square operation. Plot them side-by-side to show that you have three nearly identical images with random speckle.
5. Compute the various (co- and cross-) image spectra between the 3 intensity images. The various spectra can be computed as the various products between the corresponding Fourier transforms. Remove the mean intensity and normalise the intensity images before spectra computation ($I = (I - < I >)/ < I >$). With three intensity images this should give 6 spectra. However, then you should average those with same look separation time (i.e. all three co- spectra can be averaged to one, the one-step cross-spectra between look1 and look2, and between look2 and look3 can be averaged to one, leaving the two-step

cross-spectra between look1 and look3). The finally number should then be 3 complex spectra. Show the equations and key code lines for calculating these co- and cross-spectra.

Note: If you do not manage to do the look extraction properly, you can as an alternative perform the spectral estimation using the original SLC image i.e. jump to point 5 and do only the single co-spectra from the intensity version of the SLC image.

C. Analysis of 2D Spectra: (counts 10%)

1. Compute the wavenumber bins of the 2D spectra (computed in 5.) (see below). Shift the spectra such that wavenumber, k=(0,0) becomes in the centre of the 2D spectral array, and plot the real and imaginary parts (contour or surface plots) of the 3 spectra, and discuss the results. What is the difference between co- and cross-spectra? What is the wavelength where the spectral energy maximises? Is there a unique wave propagation direction of the dominant spectral peak?

The resolution or pixel size in range (x) and azimuth (y) of the input image can be calculated from formulas:

$$\Delta x = \frac{c}{2 f_{\mathrm{sf}} \sin\theta} \quad , \quad \Delta y = \frac{V}{f_{\mathrm{prf}}} \qquad [\mathrm{m}] \tag{1}$$

Here f_{sf} and f_{prf} are the range and azimuth sampling frequencies that can be extracted from the `image.txt` file (info.xsamplefreq, info.ysamplefreq). V is the speed of radar ground velocity that is also given in the `image.txt` file (geo.groundvel). c is the speed of light. θ is the radar incidence angle that is also given in the `image.txt` file (geo.incangle).
The bin size in your 2D spectra should then be:

$$\Delta k_x = \frac{2\pi}{N_x \Delta x} \quad , \quad \Delta k_y = \frac{2\pi}{N_y \Delta y} \qquad [\mathrm{rad/m}] \tag{2}$$

where N_x, N_y are the sizes of your Fourier transform in range and azimuth.
The highest wavenumbers (or shortest wavelength) in your spectra are:

$$k_x^{\max} = \frac{\pi}{\Delta x} \quad , \quad k_y^{\max} = \frac{\pi}{\Delta y} \qquad [\mathrm{rad/m}] \tag{3}$$

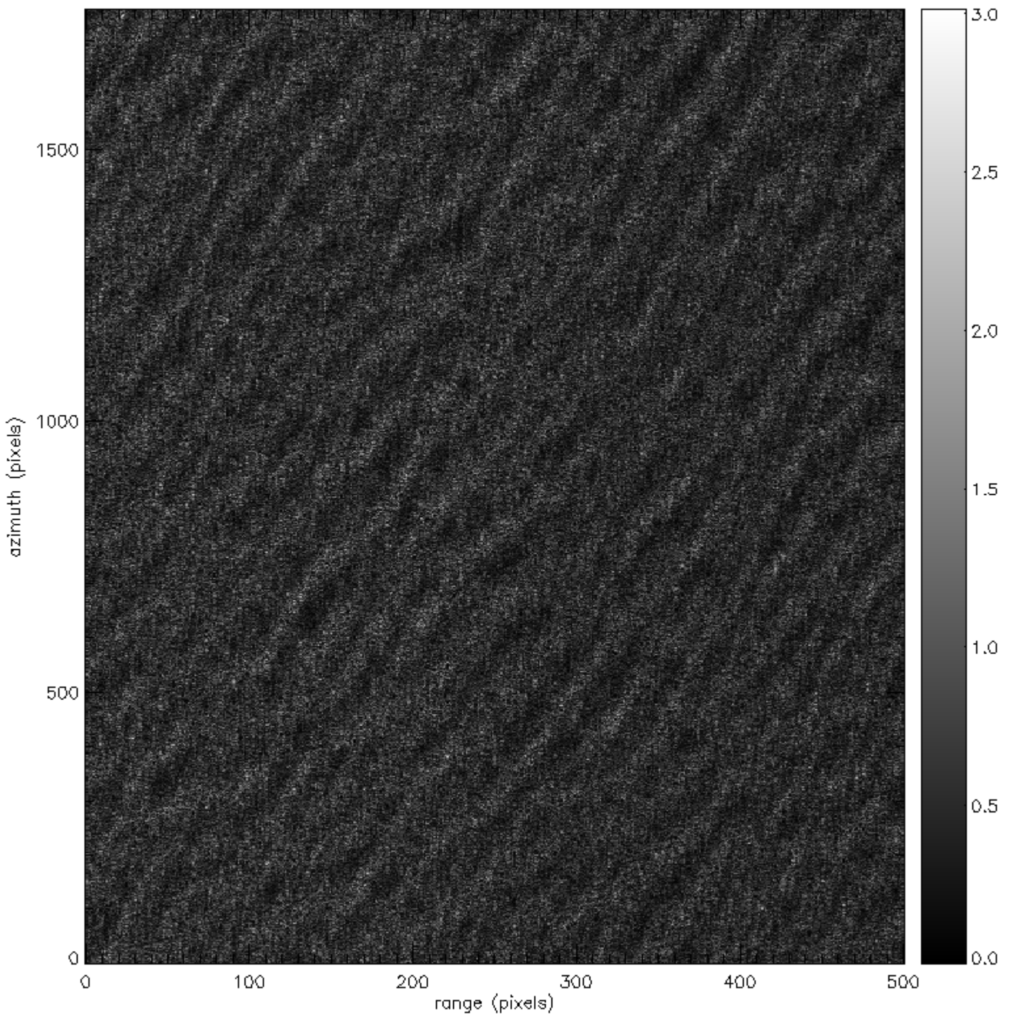

Figure 1: Absolute value of the complex image.