Machine Learning 4: Neural Networks

Consider a face recognition problem

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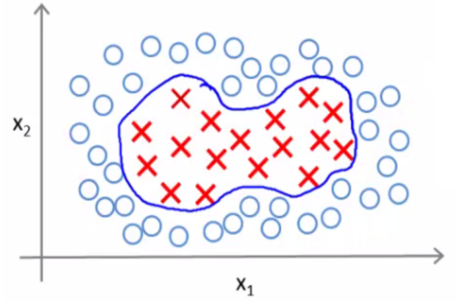
For a 50×50 grayscale image, we have 2500 basic features: $x_{1,\dots,2500} \in \{0, 255\}$

As before, we can map in feature space the 'face' and 'non-face' labels



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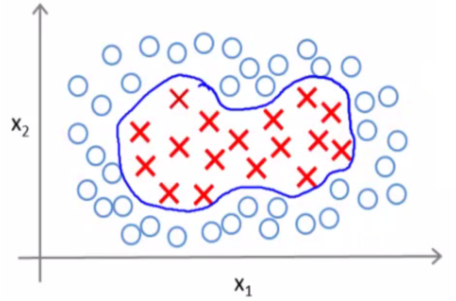

We definitely require polynomial features to map this decision boundary ...

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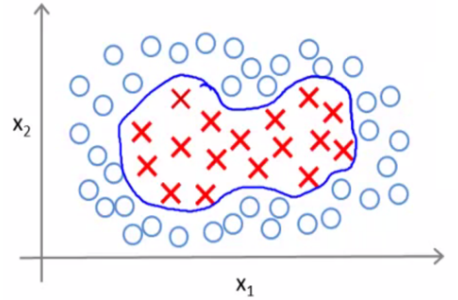

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Up to quadratic order this is already $\sim 3,000,000$ features ....

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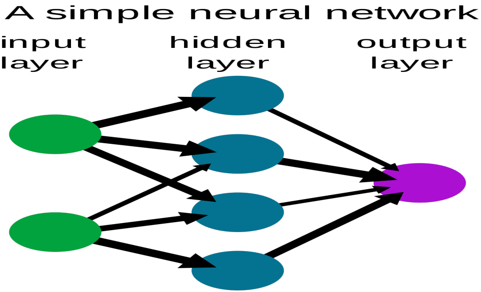
You get the picture (pun intended).

Neural Networks



- A basic neural network can be seen as multi-layered, logistic regression based, map, or a string of **logistic units.**
- Neural networks provide a means to create 'non-linear' features from a set of basic input features. These are called **'activations'**.

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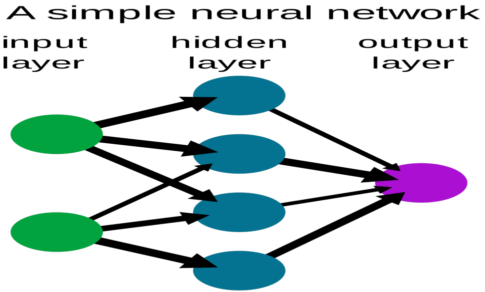

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Let's see what this means more concretely ....

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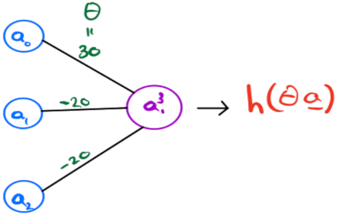

a_1	a_2	h = NAND
0	0	1
0	1	1
1	0	1
1	1	0

$$h(\theta \underline{a}) = \mathrm{sig}(\theta_0 a_0 + \theta_1 a_1 + \theta_2 a_2)$$

$$= \mathrm{sig}(30 - 20 a_1 - 20 a_2)$$

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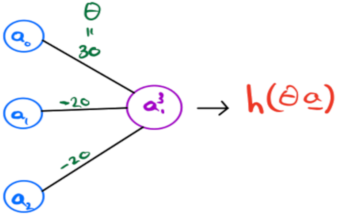

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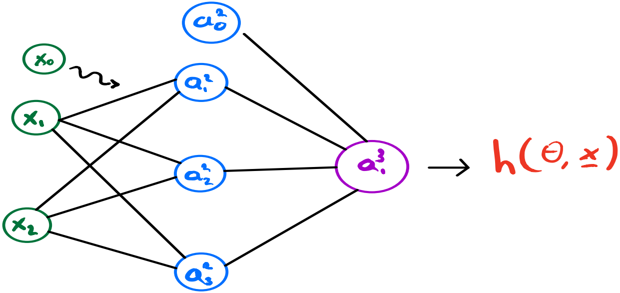
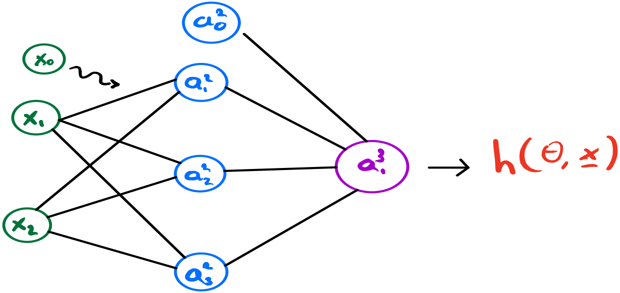
Let's now consider some binary classification as our goal (ex. Dwayne Johnson or not Dwayne Johnson) ....



How is the hypothesis defined?



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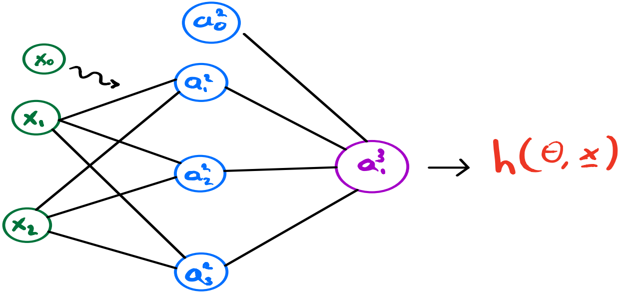
$$h(\Theta, \boldsymbol{x}) = a_1^3 = \mathrm{sig}((\theta^{(2)})^{\mathrm{T}} \boldsymbol{a}),$$
$$(\theta^{(2)})^T \boldsymbol{a} = \theta_{10}^{(2)} a_0^2 + \theta_{11}^{(2)} a_1^2 + \theta_{12}^{(2)} a_2^2 + \theta_{13}^{(2)} a_3^2 \tag{1}$$



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where

$$
\begin{aligned}
\tilde{\boldsymbol{a}} =& [a_1^2, a_2^2, a_3^2] = \mathrm{sig}((\theta^{(1)})^{\mathrm{T}} \boldsymbol{x}), \\
a_i^2 =& \mathrm{sig}(\theta_{\mathrm{i}0}^{(1)} \mathrm{x}_0 + \theta_{\mathrm{i}1}^{(1)} \mathrm{x}_1 + \theta_{\mathrm{i}2}^{(1)} \mathrm{x}_2)
\end{aligned} \tag{2}
$$

Visualising hidden units and building some intuition on their function as new, non-linear features :

https://www.coursera.org/learn/machine-learning/lecture/solUx/examples-and-intuitions-ii

Some points

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So what is our cost function ?

For a neural network with **L** layers, looking to classify into **K** different classes, and a training set of **m** examples, we can extend our old logistic cost function as follows

$$
\begin{aligned}
J(\Theta) = &- \frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{K}\left[y_k^i \log[h_k(x^i)] + (1 - y_k^i)\log[1 - h_k(x^i)]\right] \\
&+ \frac{\lambda}{2m}\sum_{\ell=1}^{L-1}\sum_{i=1}^{s_\ell}\sum_{j=1}^{s_{\ell+1}}(\theta_{ji}^{\ell})^2,
\end{aligned} \tag{3}
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where λ is our regularisation parameter and s_ℓ is the number of units in the ℓ^{th} layer **not** including the bias unit.

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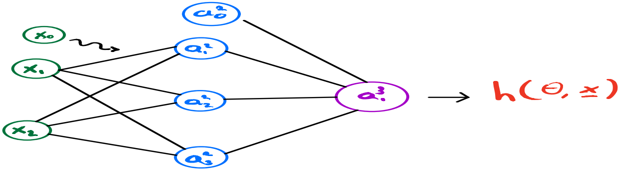
where λ is our regularisation parameter and s_ℓ is the number of units in the ℓ^{th} layer **not** including the bias unit.

We want to minimise the cost by varying the components of *Theta*. To do this, we can use the **back propagation** algorithm. This is a generalisation of gradient descent which requires the partial derivatives $\frac{\partial J(\Theta)}{\partial \theta_{ij}^{\ell}}$.



1. Calculate a_1^3 given some **random** initialisation of Θ (forward propagation).
2. Calculate the error on a_1^3 : $\delta^3 = a_1^3 - y$.
3. Calculate the 'error' on other hidden layers. In this case $\delta_j^2 = \theta_j^{(2)} \delta^3 \mathrm{sig}'(\mathrm{z} = (\theta^{(1)})^{\mathrm{T}} \boldsymbol{x})$. One can show that the derivatives of the sigmoid wrt $z = (\theta^{(1)})^T \boldsymbol{x}$ is just $a_j^2(1 - a_j^2)$.
4. Calculate the derivatives of the cost, D_{ij}^{ℓ}, which can be shown to be :

$$D_{ij}^{\ell} = \frac{1}{m}(\Delta_{ij}^{\ell} + \lambda \theta_{ij}^{\ell}) \qquad \mathrm{for} j \neq 0,$$
$$D_{ij}^{\ell} = \frac{1}{m}\Delta_{ij}^{\ell} \qquad \mathrm{for} j = 0. \tag{4}$$

where $\Delta_{ij}^{\ell} = a_j^{\ell} \delta_i^{\ell+1}$.

Autonomous driving as an example

https://www.coursera.org/learn/machine-learning/lecture/zYS8T/autonomous-driving