DEEP LEARNING
Backpropagation

ГУУ, 3-й курс 2023, 2-й семестр

ПЛАН

- Алгоритм обратного распространения
- Автоматический градиент

Функция ошибки

Регрессия

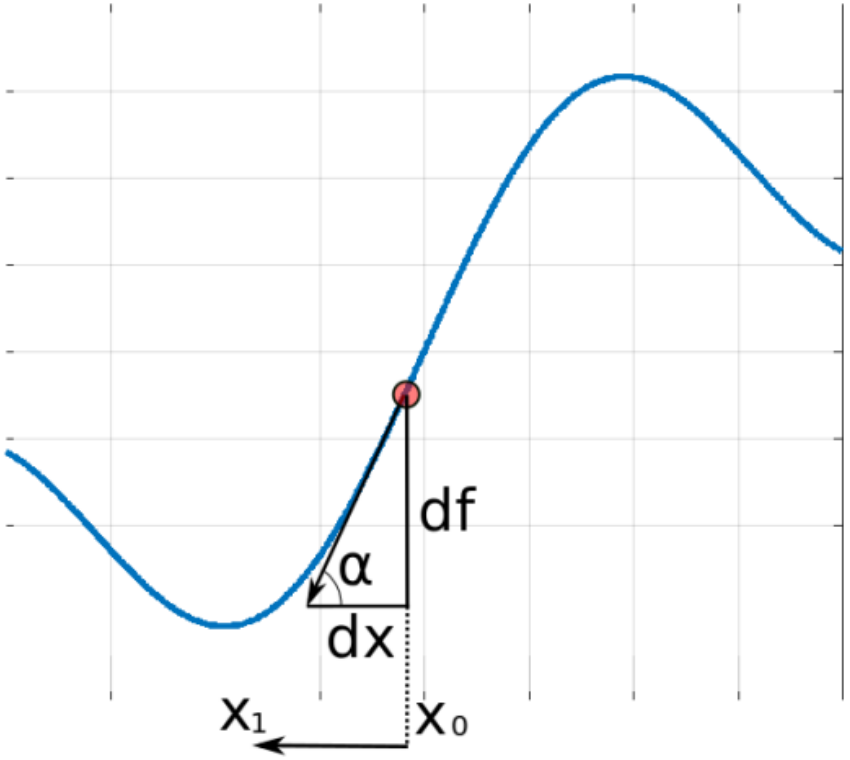
$$E = \sqrt{Y^2 - T^2}$$

Классификация

$$H(p, q) = H(p) + D_{KL}(p.q)$$

$$H(p, q) = -\sum_{x} p(x) \ln(q(x))$$

Оптимизация



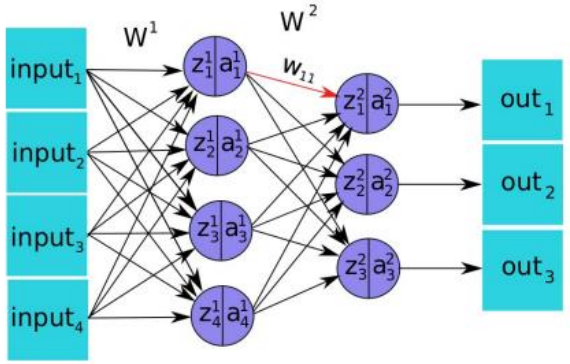
$$x_1 = x_0 - \eta \left. \frac{df}{dx} \right|_{x=x_0}$$

$$\Delta x_0 = -\eta \left. \frac{df}{dx} \right|_{x=x_0}$$

$$x_1 = x_0 + \Delta x_0$$

Алгоритм обратного распространения ошибки: последний слой

$$\Delta w_{11}^2 = -\eta \frac{\partial E}{\partial w_{11}^2}, \qquad \frac{\partial E}{\partial w_{11}^2} = \frac{\partial E}{\partial a_1^2}\frac{\partial a_1^2}{\partial z_1^2}\frac{\partial z_1^2}{\partial w_{11}^2}$$



$$E = \frac{1}{2}\sum_{i=1}^{N}(a_{1i}^2 - t_{1i})^2 \Rightarrow \frac{\partial E}{\partial a_{1i}^2} = \sum_{i=1}^{N} a_1^2 - t_{1i}$$

$$\frac{\partial E}{\partial w_{11}^2} = a_1^2(1-a_1^2)a_1^1\sum_{i=1}^{N} a_1^2 - t_i$$

$$a_1^2 = \frac{1}{1+e^{-z_1^2}} = \sigma(z_1^2) \Rightarrow \frac{\partial a_1^2}{\partial z_1^2} = \sigma(z_1^2)(1-\sigma(z_1^2)) = a_1^2(1-a_1^2)$$

$$\frac{\partial E}{\partial w_{ij}^l} = a_j^l(1-a_j^l)a_i^{l-1}\sum_{k=1}^{N} a_{jk}^l - t_{jk}$$

$$z_1^2 = a_1^1 w_{11}^2 + a_2^1 w_{21}^2 + a_3^1 w_{31}^2 a_4^1 w_{41}^2 + b_1^2 \Rightarrow \frac{\partial z_1^2}{\partial w_{11}^2} = a_1^1$$

$$\frac{\partial E}{\partial a_j^l}\frac{\partial a_j^l}{\partial z_j^l} = \delta_j^l \qquad \frac{\partial E}{\partial w_{ij}^l} = \delta_j^l a_i^{l-1}$$

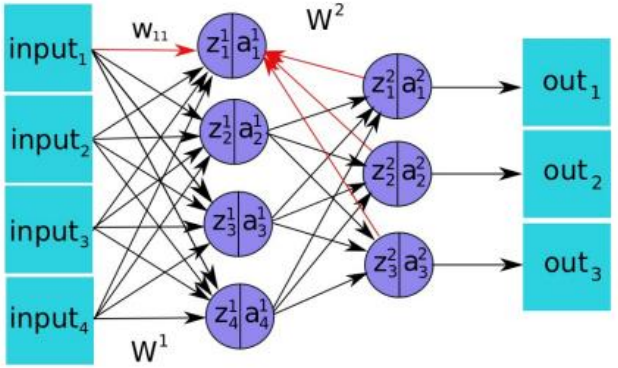
Алгоритм обратного распространения ошибки: внутренний слой

$$\Delta w_{11}^1 = -\eta \frac{\partial E}{\partial w_{11}^1},$$

$$\frac{\partial E}{\partial w_{11}^1} = \frac{\partial E}{\partial a_1^1}\frac{\partial a_1^1}{\partial z_1^1}\frac{\partial z_1^1}{\partial w_{11}^1} = \frac{\partial E}{\partial z_1^1}\frac{\partial z_1^1}{\partial w_{11}^1}$$

$$\frac{\partial E}{\partial z_1^1} = \sum_{k=1}^{3} \frac{\partial E}{\partial z_k^2}\frac{\partial z_k^2}{\partial z_1^1} = \sum_{k=1}^{3} \frac{\partial E}{\partial z_k^2}\frac{\partial z_k^2}{\partial a_1^1}\frac{\partial a_1^1}{\partial z_1^1}$$

$$z_k^2 = a_1^1 w_{1k}^2 + a_2^1 w_{2k}^2 + a_3^1 w_{3k}^2 + a_4^1 w_{4k}^2 + b_k^2 \Rightarrow \frac{\partial z_k^2}{\partial a_1^1} = w_{1k}^2$$

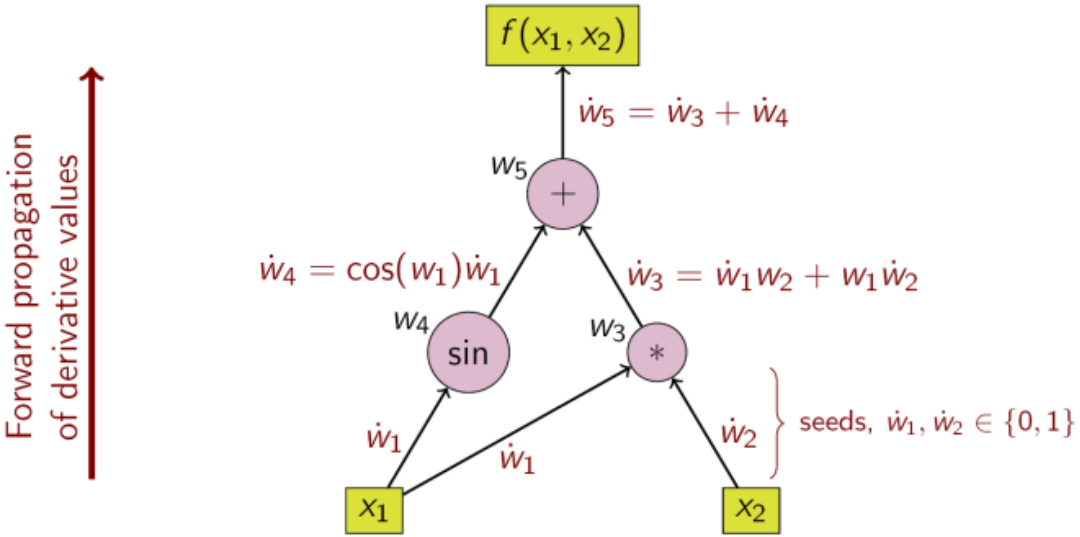


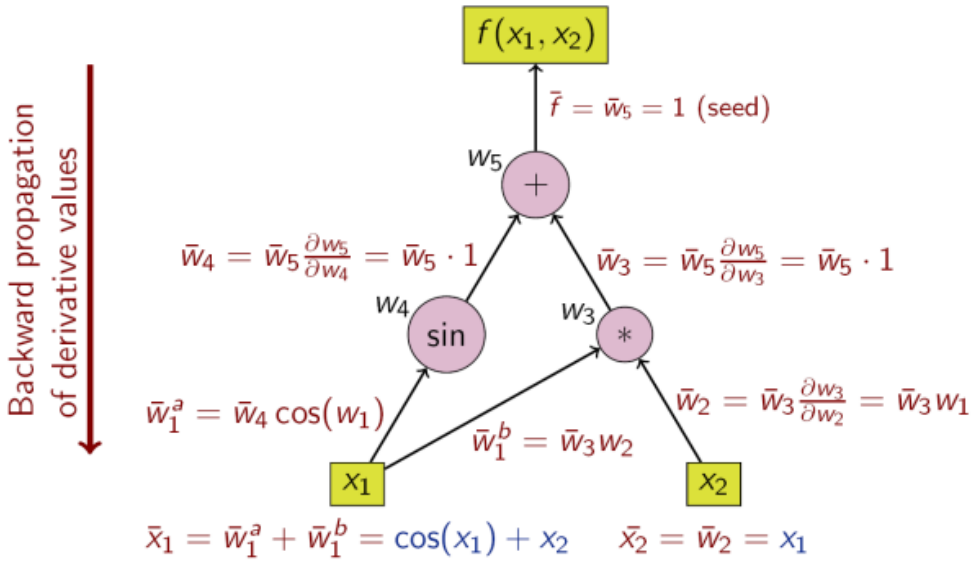
$$\frac{\partial E}{\partial a_j^l}\frac{\partial a_j^l}{\partial z_j^l} = \delta_j^l \Rightarrow \frac{\partial E}{\partial z_k^2} = \delta_k^2$$

$$\frac{\partial E}{\partial w_{11}^1} = input_1 a_1^1 (1 - a_1^1) \sum_{k=1}^{3} \delta_k^2 w_{1k}^2$$

$$\frac{\partial E}{\partial w_{ij}^{(l)}} = a_i^{(l-1)} a_j^{(l)} (1 - a_j^{(l)}) \sum_{k=1}^{n} \delta_k^{(l+1)} w_{jk}^{(l+1)}$$

Автоматический градиент





$$f(x_1, x_2) = x_1 x_2 + \sin(x_1)$$